Understanding Q-balls and Q-shells Analytically

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12 July 2021

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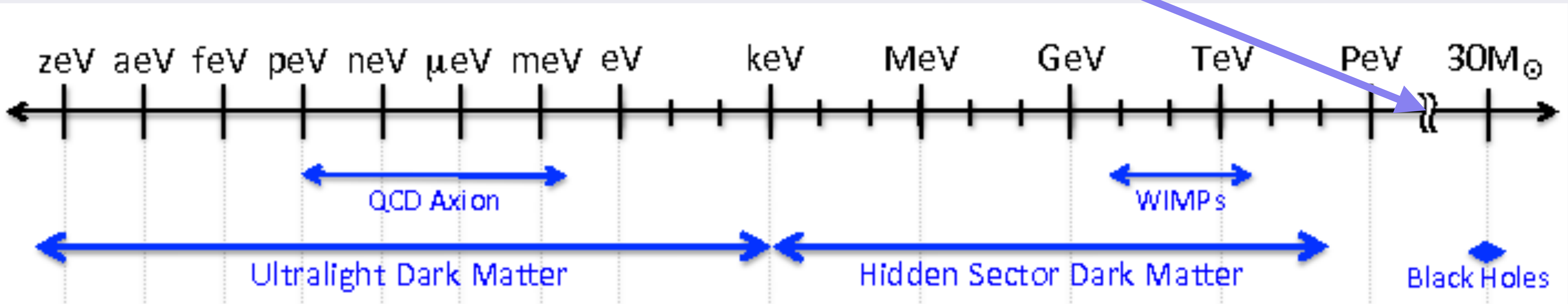
arXiv:2009.08462
arXiv:2103.06905
arXiv:2105.02893

Non-Topological Solitons

Clumps of matter that are stabilized by self-interactions

Interesting field theory objects

Provide a concrete model for macroscopic dark matter



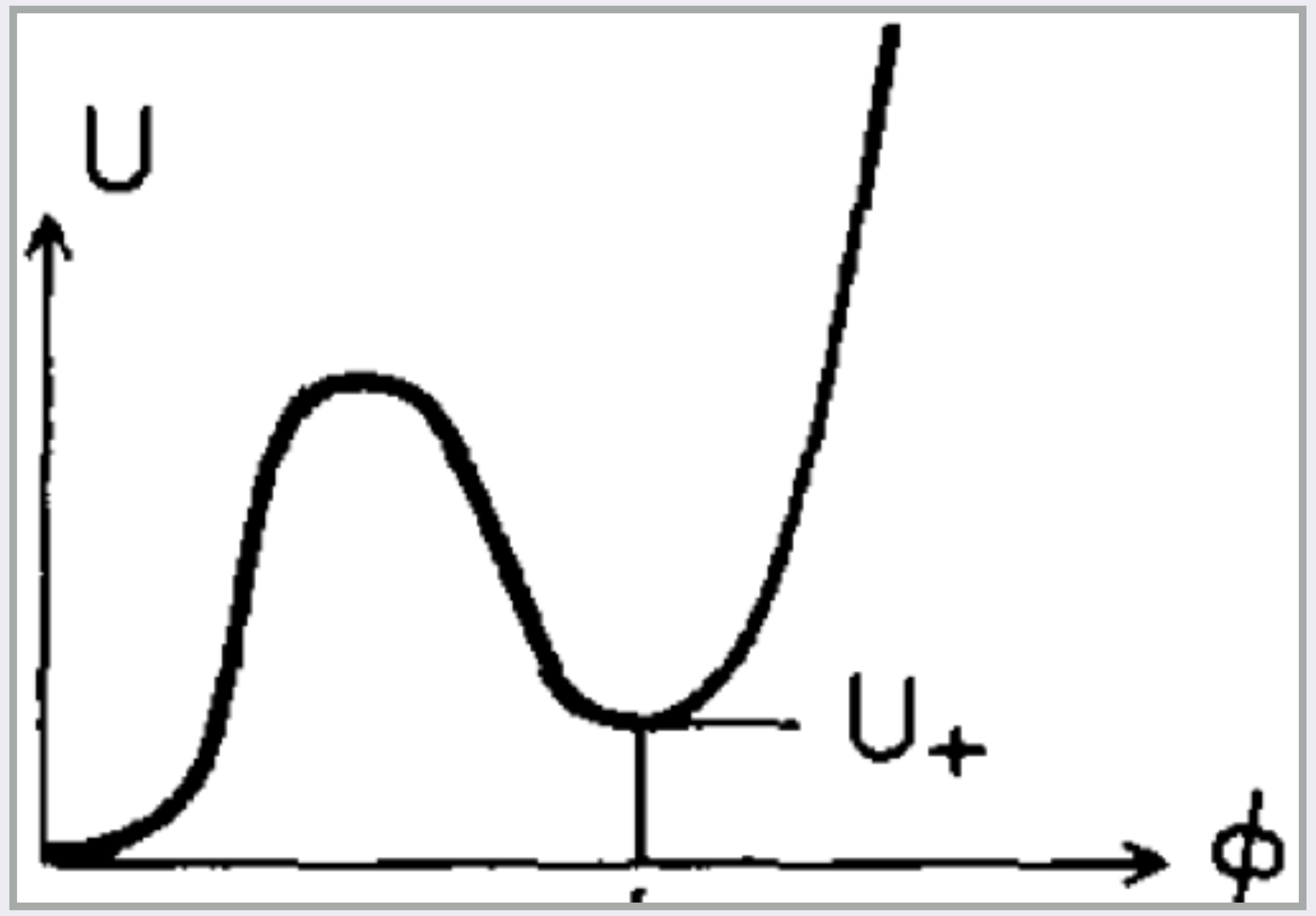
Nonlinear equations

Numerical analysis

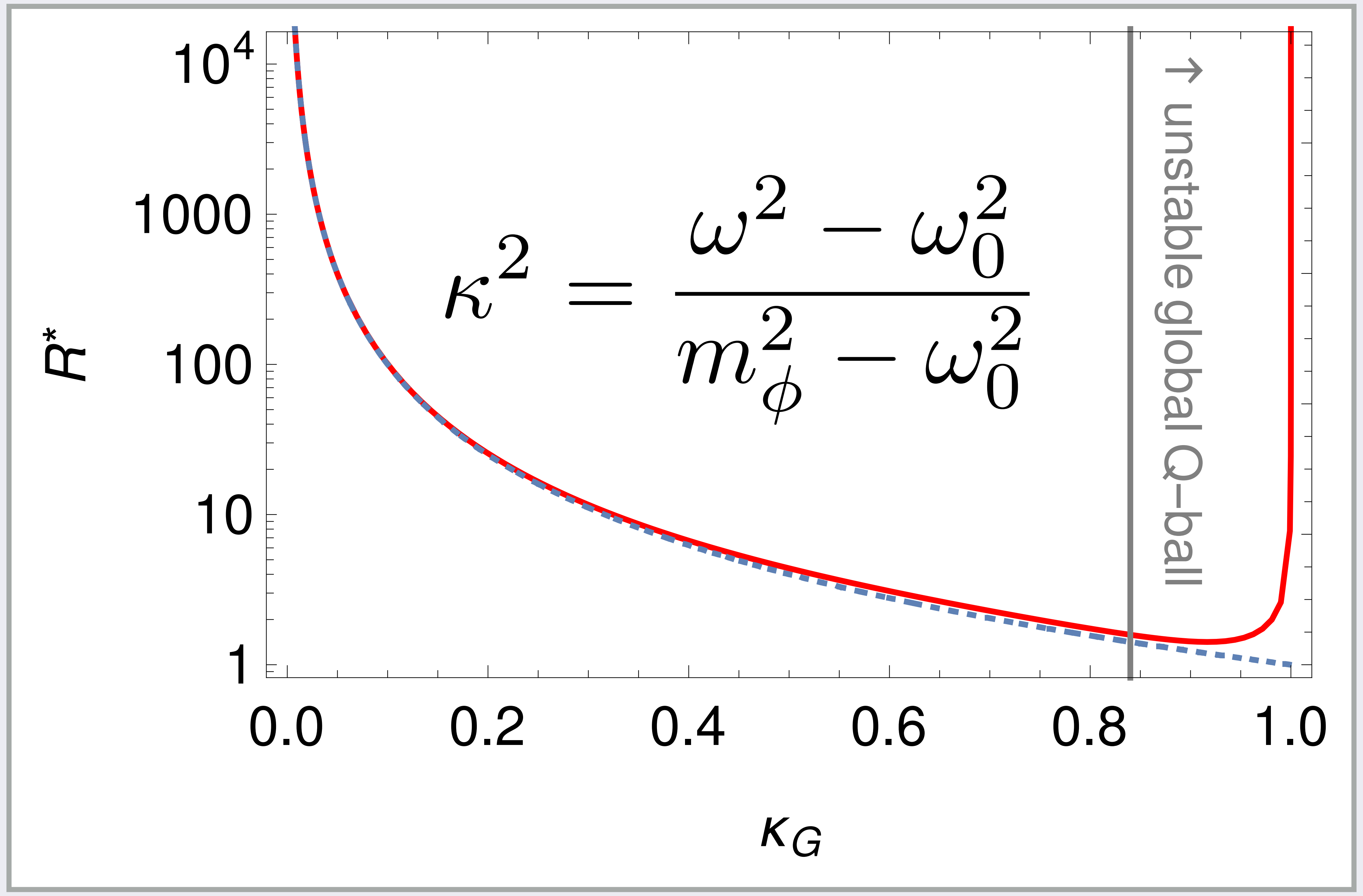
Q-balls

Simple Lagrangian $\mathcal{L} = |\partial_\mu \phi|^2 - U(|\phi|)$

EFT inspired potential $U = m_\phi^2 |\phi|^2 - \beta |\phi|^4 + \frac{\xi}{m_\phi^2} |\phi|^6$



Profile defined by $\phi(x) = \frac{\phi_0}{\sqrt{2}} f(r) e^{i\omega t}$



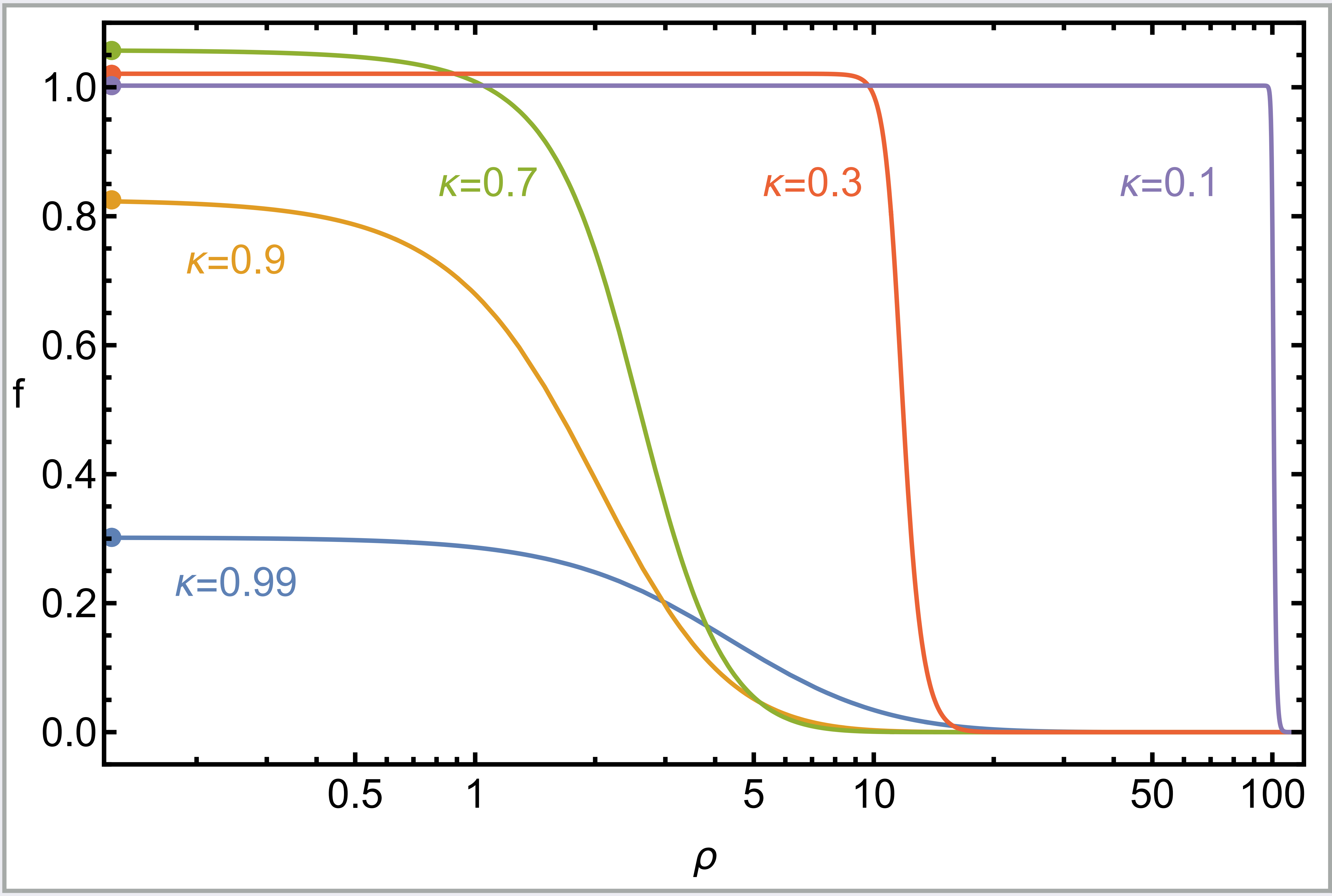
ϕ_0 min of $\frac{U(|\phi|)}{\phi^2}$

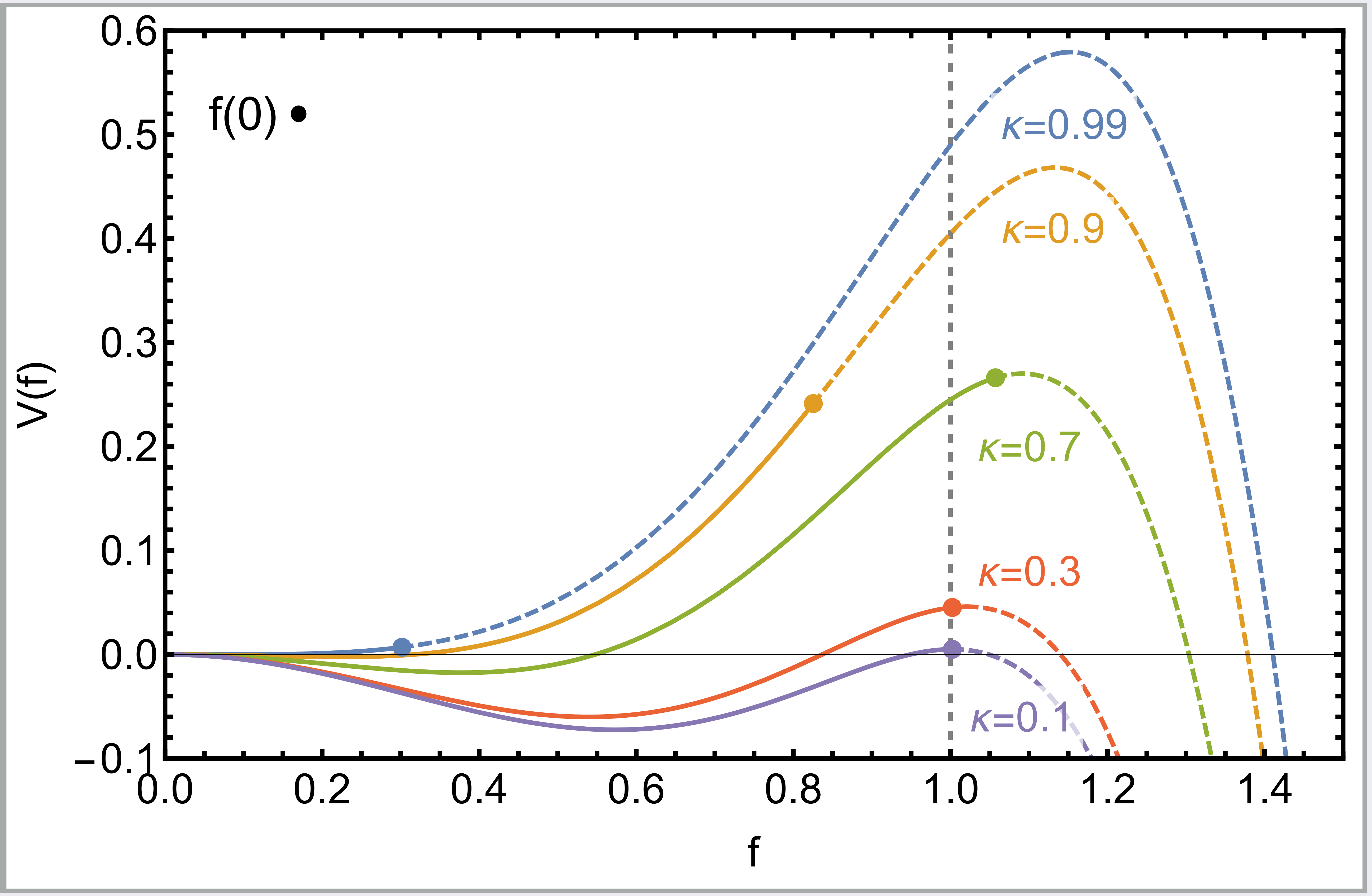
$\omega_0 < m_\phi$ min frequency

Mechanical Analogy

Particle rolling in potential with time dependent friction

$$f'' + \frac{2}{\rho} f' + \frac{dV}{df} = 0 \qquad V(f) = \frac{f^2}{2}\left[\kappa^2 - \left(1 - f^2\right)^2\right]$$





Thin-Wall Limit

When $\kappa \to 0$ the scale of the wall is much less than the soliton radius

For large radius, neglect friction term

$$V(f) = \frac{f^2}{2}\left[\kappa^2 - \left(1 - f^2\right)^2\right]$$

$$f'' + \frac{dV}{df} = \frac{1}{f'}\frac{d}{d\rho}\left(\frac{1}{2}f'^2 + V(f)\right) = 0$$

$$\mathcal{E} = \frac{1}{2}f'^2 + V(f) = 0$$

First order equation for transition region after "infinite" radius

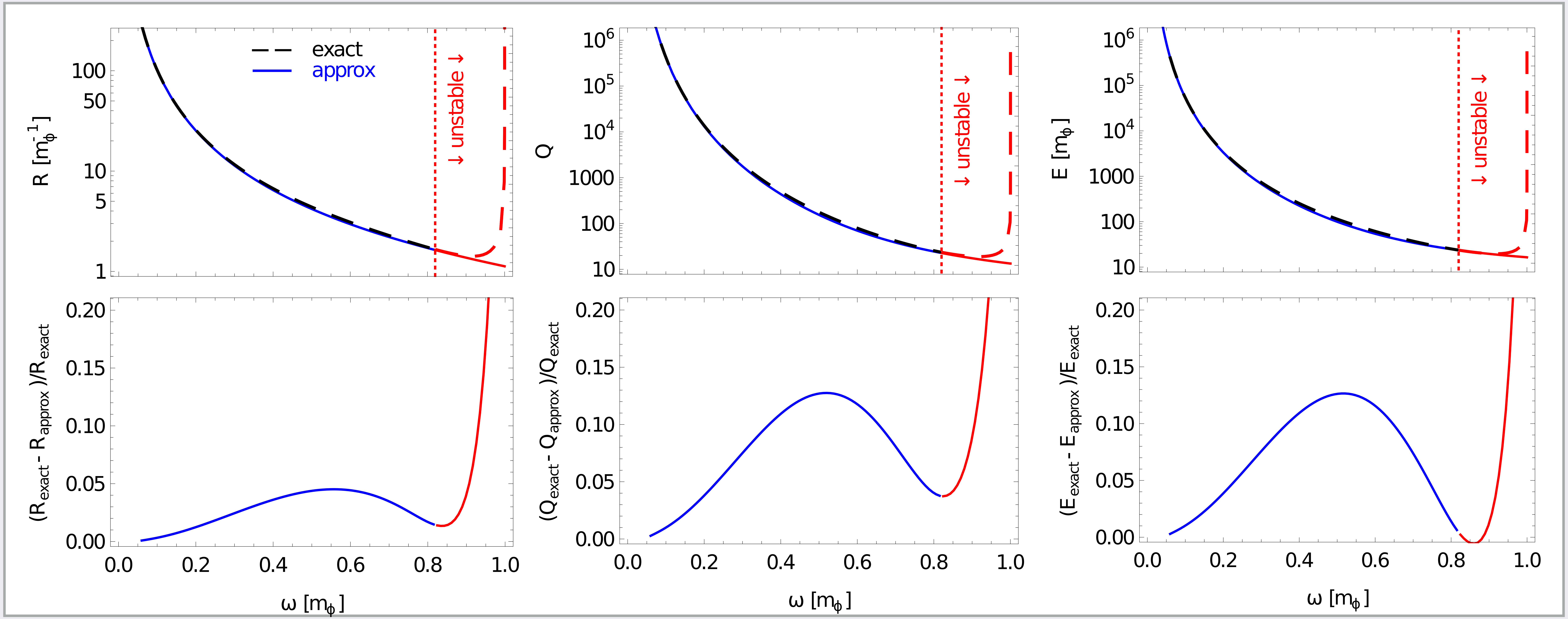
$$f''(R^*) = 0$$

$$\frac{df}{d\rho} = \pm f(1 - f^2) \quad \Rightarrow \quad f = \left[1 + 2e^{2(\rho - R^*)}\right]^{-1/2}$$

Justified for transition region and small kappa, but excellent approximation for most stable Q-balls

Global Q-ball Results

Profile leads to accurate prediction of Radius, Charge, and Energy



Gauged Q-balls

Gauge the symmetry $$\mathcal{L} = |D_\mu \phi|^2 - U(|\phi|) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

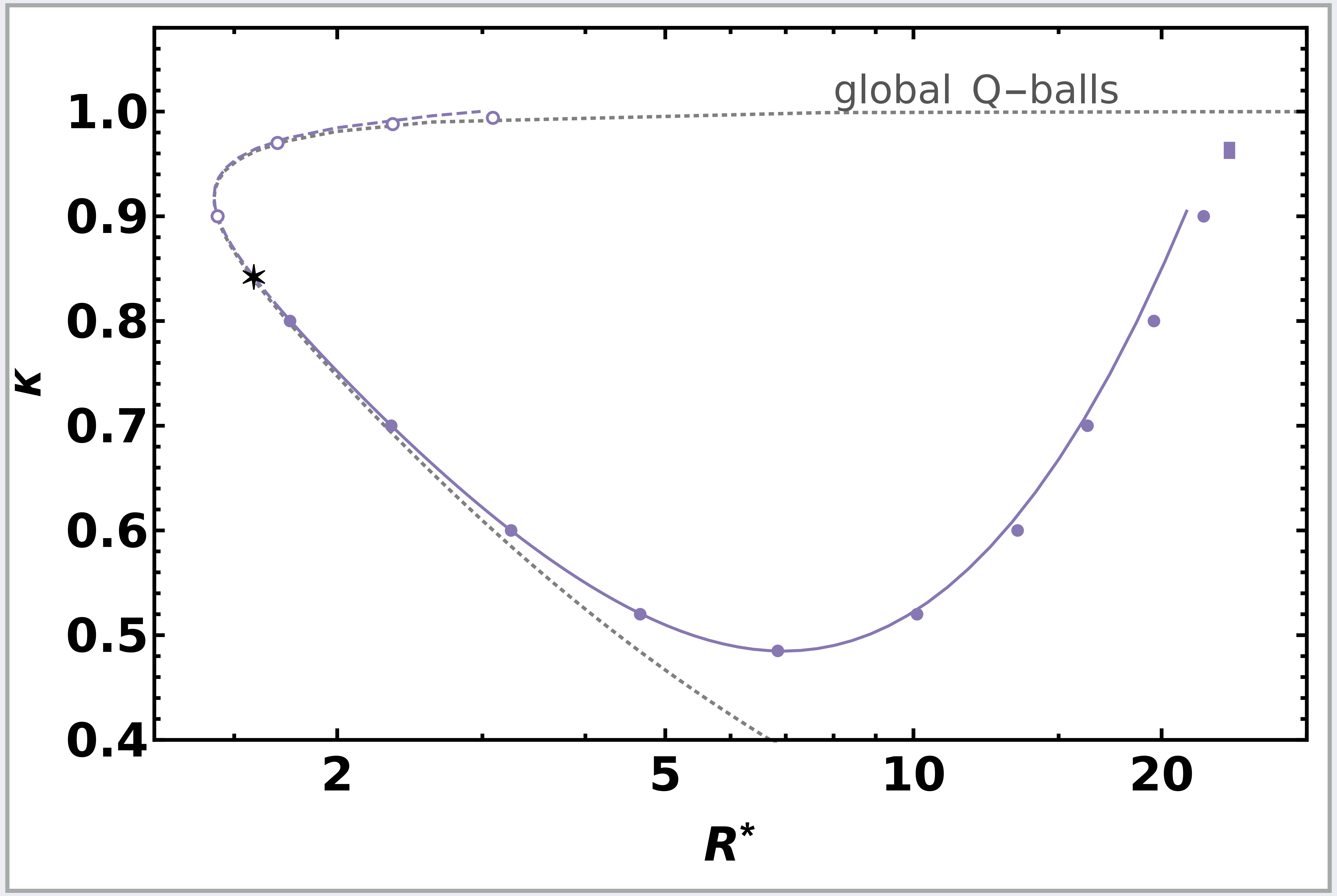
2 Scalar fields $$\phi(x) = \frac{\phi_0}{\sqrt{2}} f(r) e^{i\omega t}, \quad A_0(x) = A(r), \quad A_i(x) = 0$$

Qualitatively different solution space

two “branches”

maximum radius

minimum kappa



Mechanical Analogy

Particle rolling in 2D potential

$$V(f,A) = \frac{f^2}{2}\left[\kappa^2 + \alpha A(\alpha A - 2\Omega) - \left(1-f^2\right)^2\right]$$

Rolls uphill in the A direction

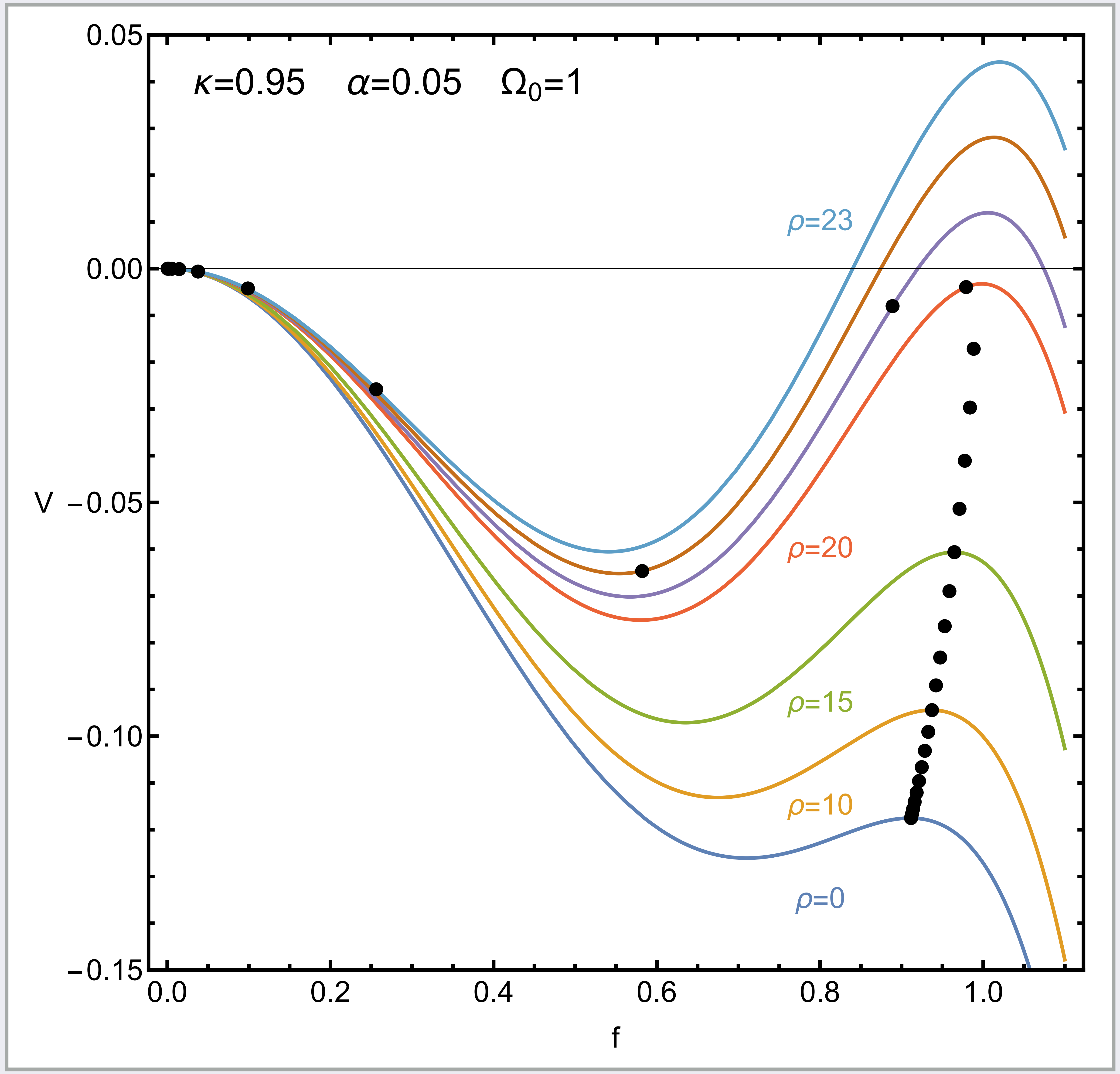
$$L \propto \int d\rho \rho^2 \left[-\frac{1}{2}f'^2 + \frac{1}{2}A'^2 + \frac{1}{2}f^2\left(\Omega - \alpha A\right)^2 - U(f)\right]$$

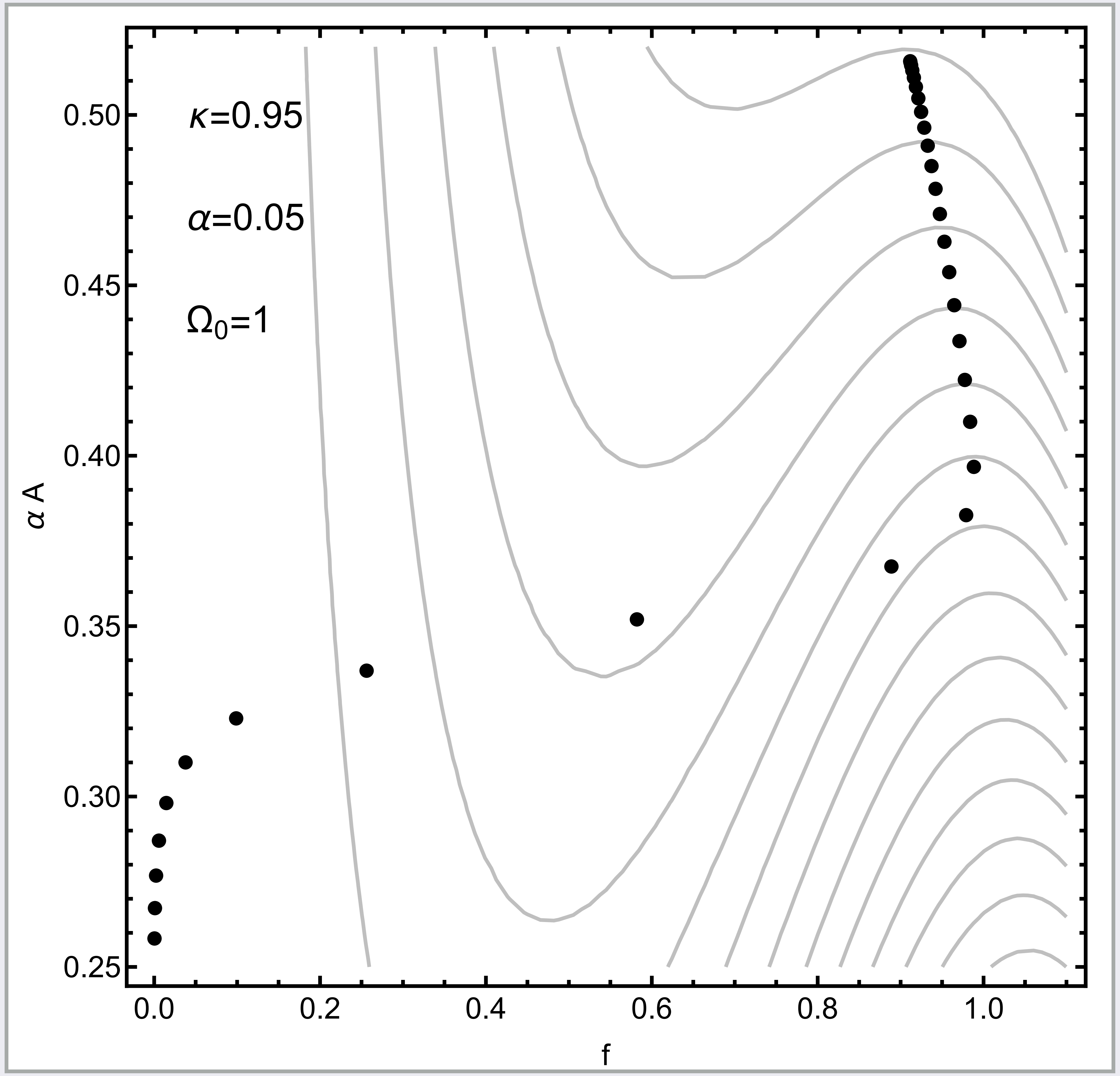
Work due to friction is also opposite for gauge field

$$W_f \propto 2\int \frac{d\rho}{\rho}\left(f'^2 - A'^2\right)$$

Mechanical Analogy

Two views of the potential





Mapping

Gauge field does not change much during f transition

$$f'' + \frac{2}{\rho} f' + f\left[\Omega - \alpha A(R^*)\right]^2 = \frac{dU}{df}$$

$$\Omega_G = \Omega - \alpha A(R^*)$$

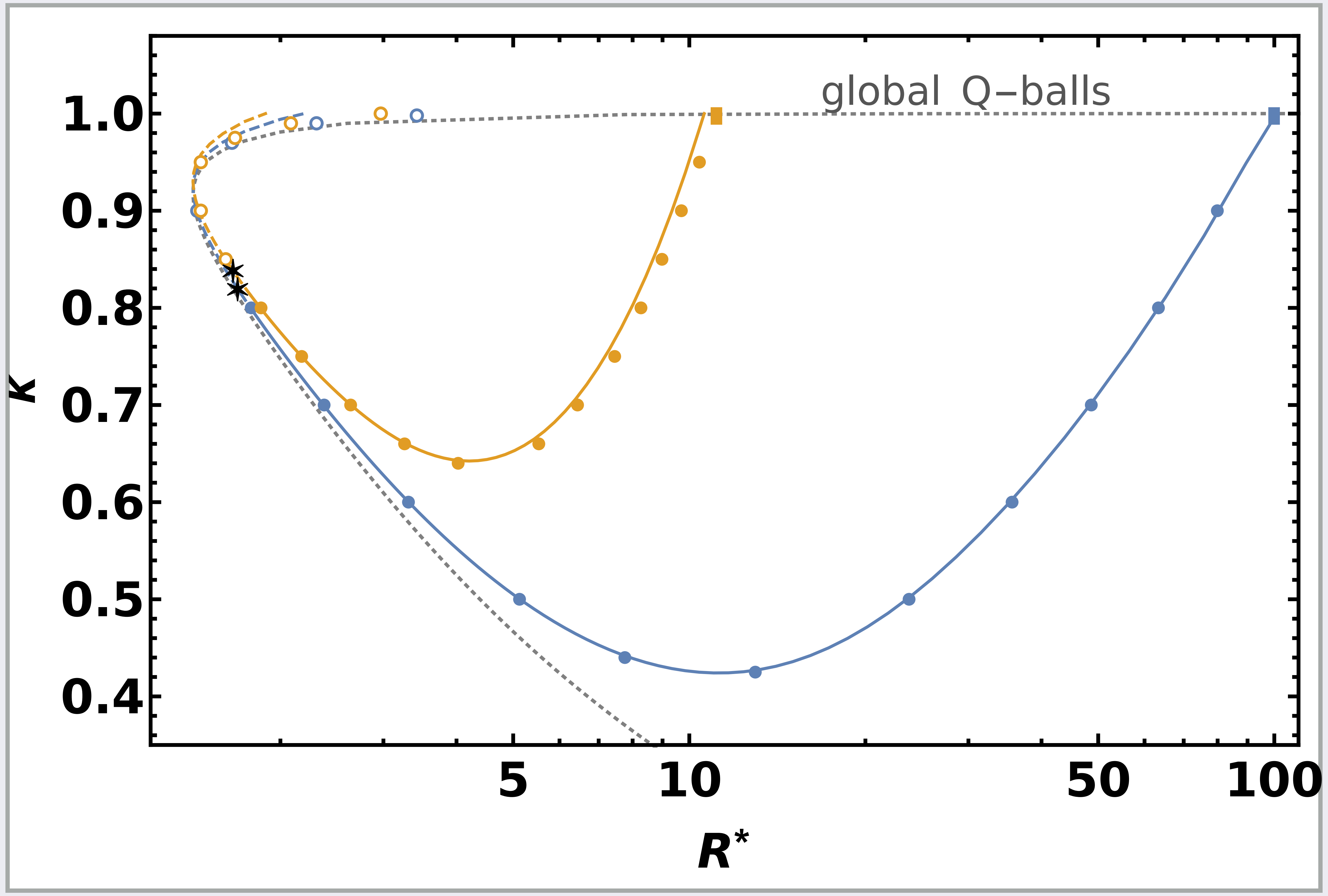
Use thin-wall ansatz to determine gauge field

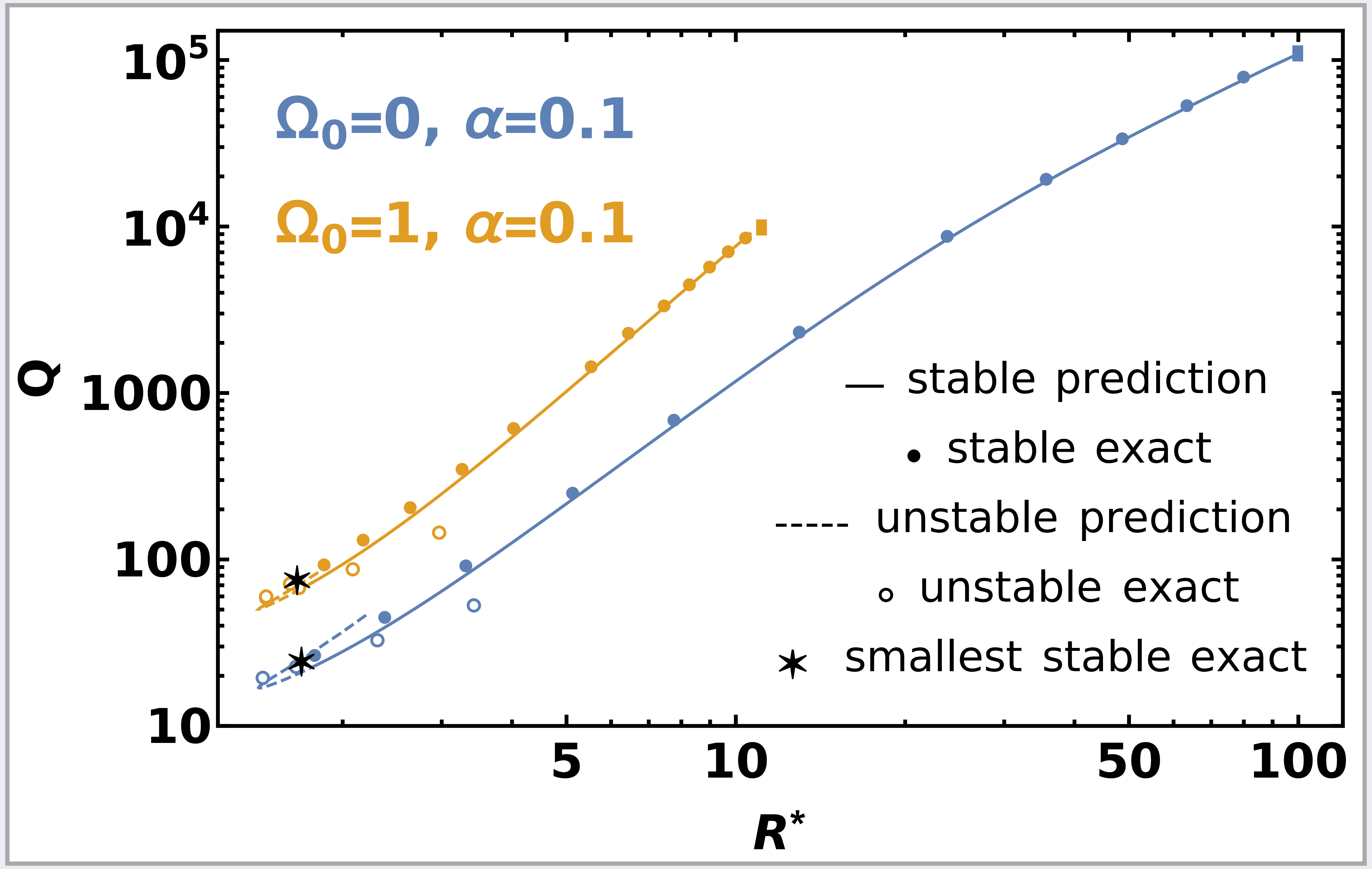
$$\Omega(R^*) = \Omega_G(R^*)\alpha R^* \coth(\alpha R^*)$$

Can use “known” global function to determine the gauged version

Gauged Results

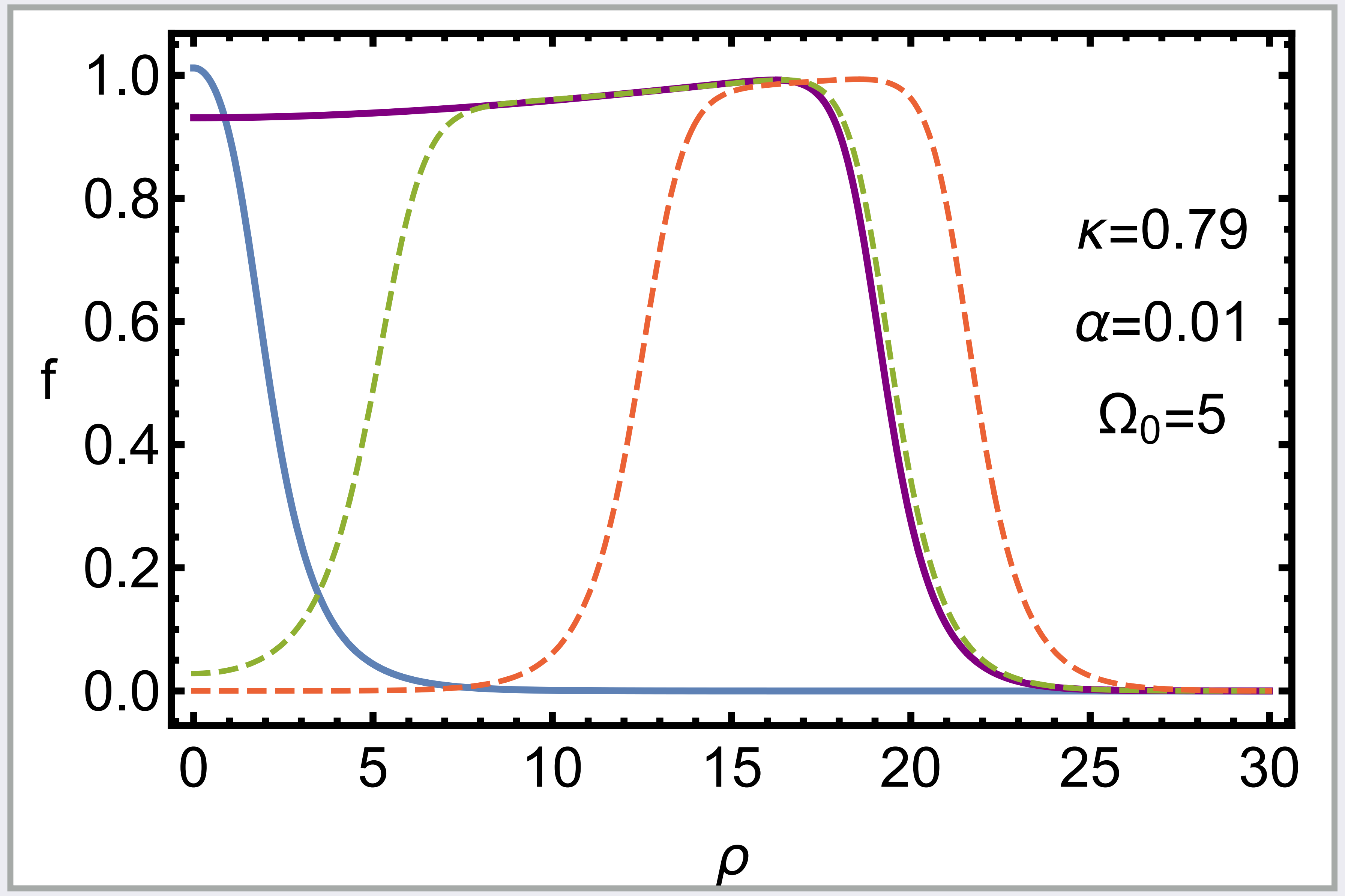
Excellent agreement between analytic prediction and numerical results!

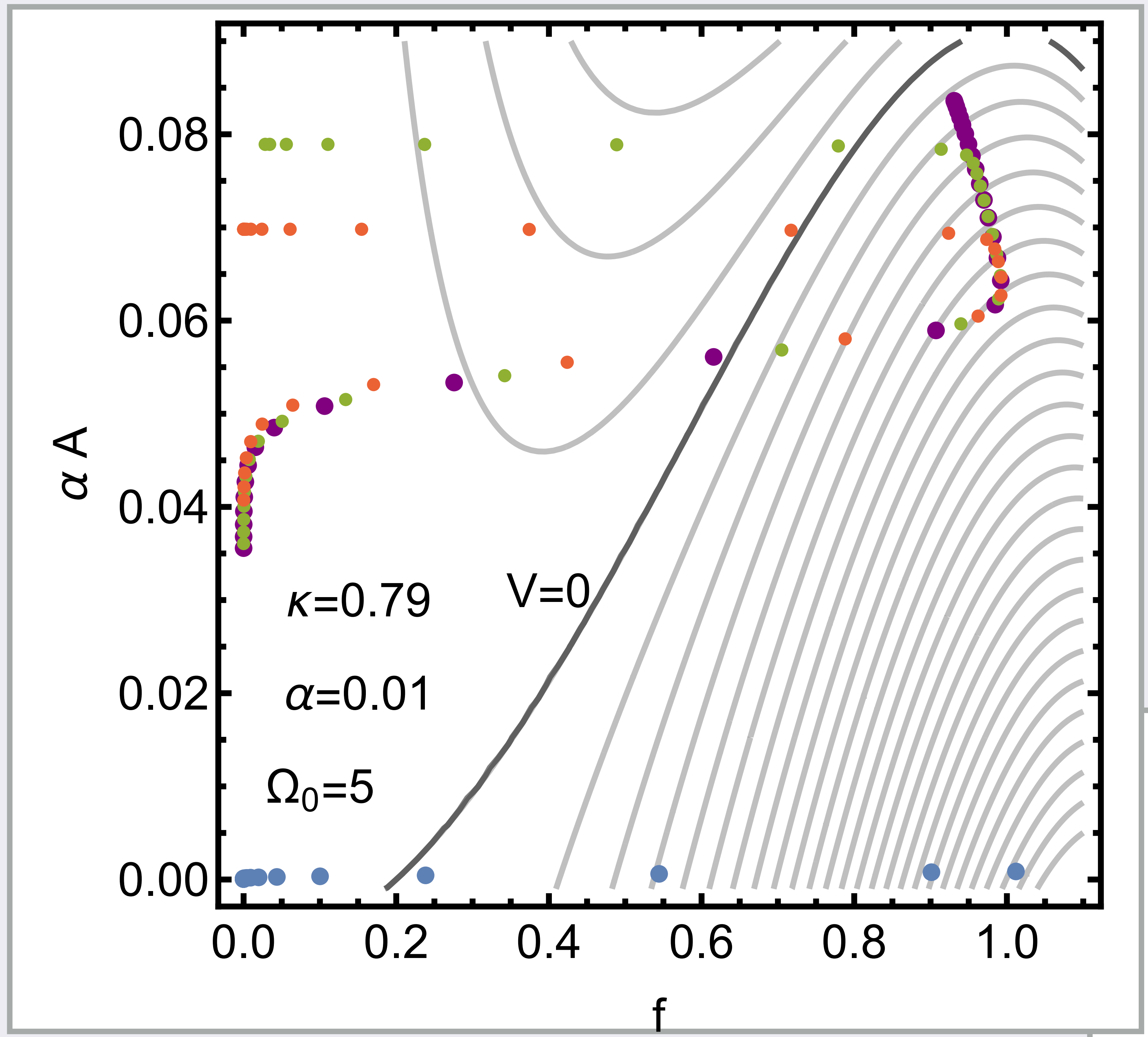


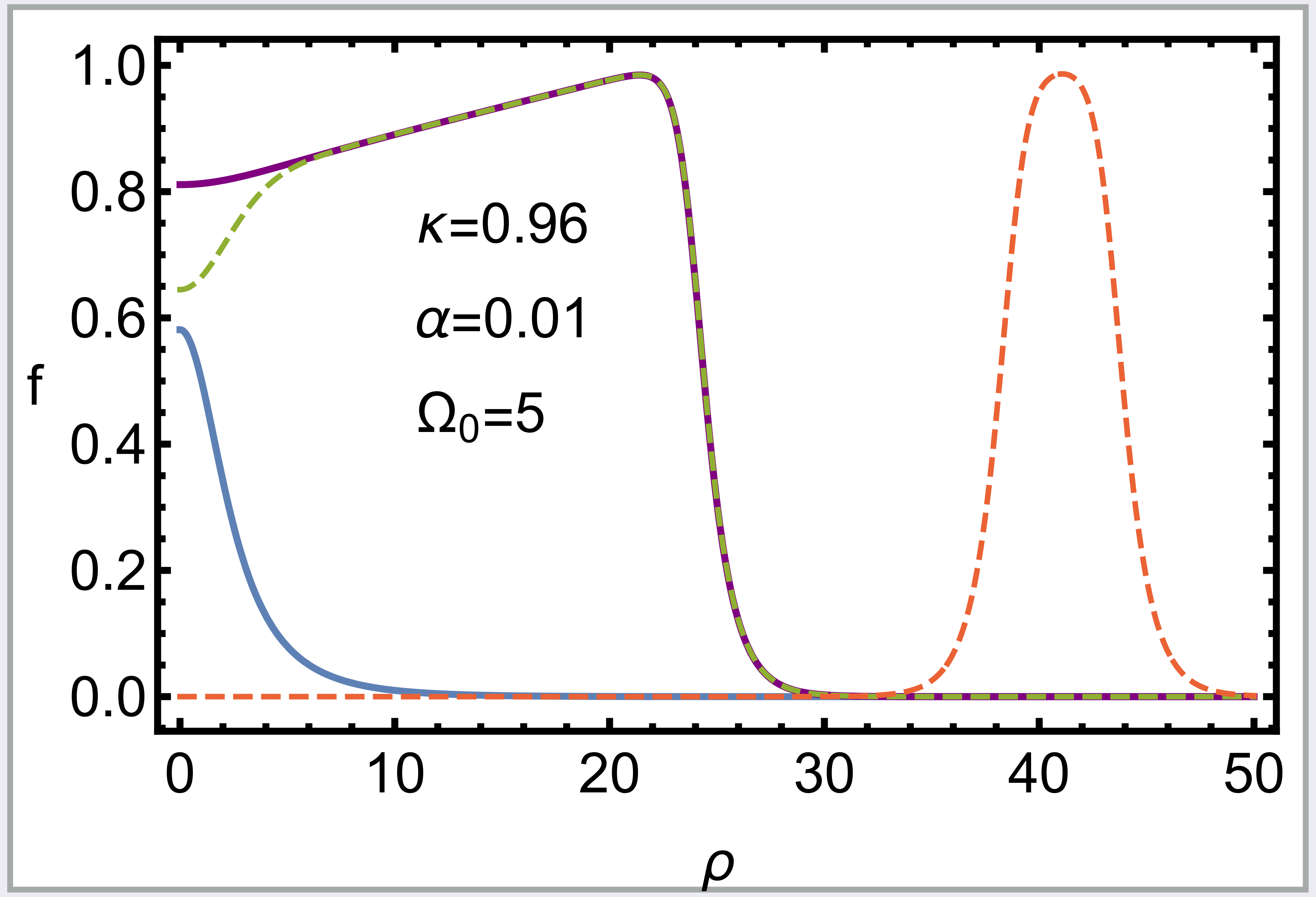


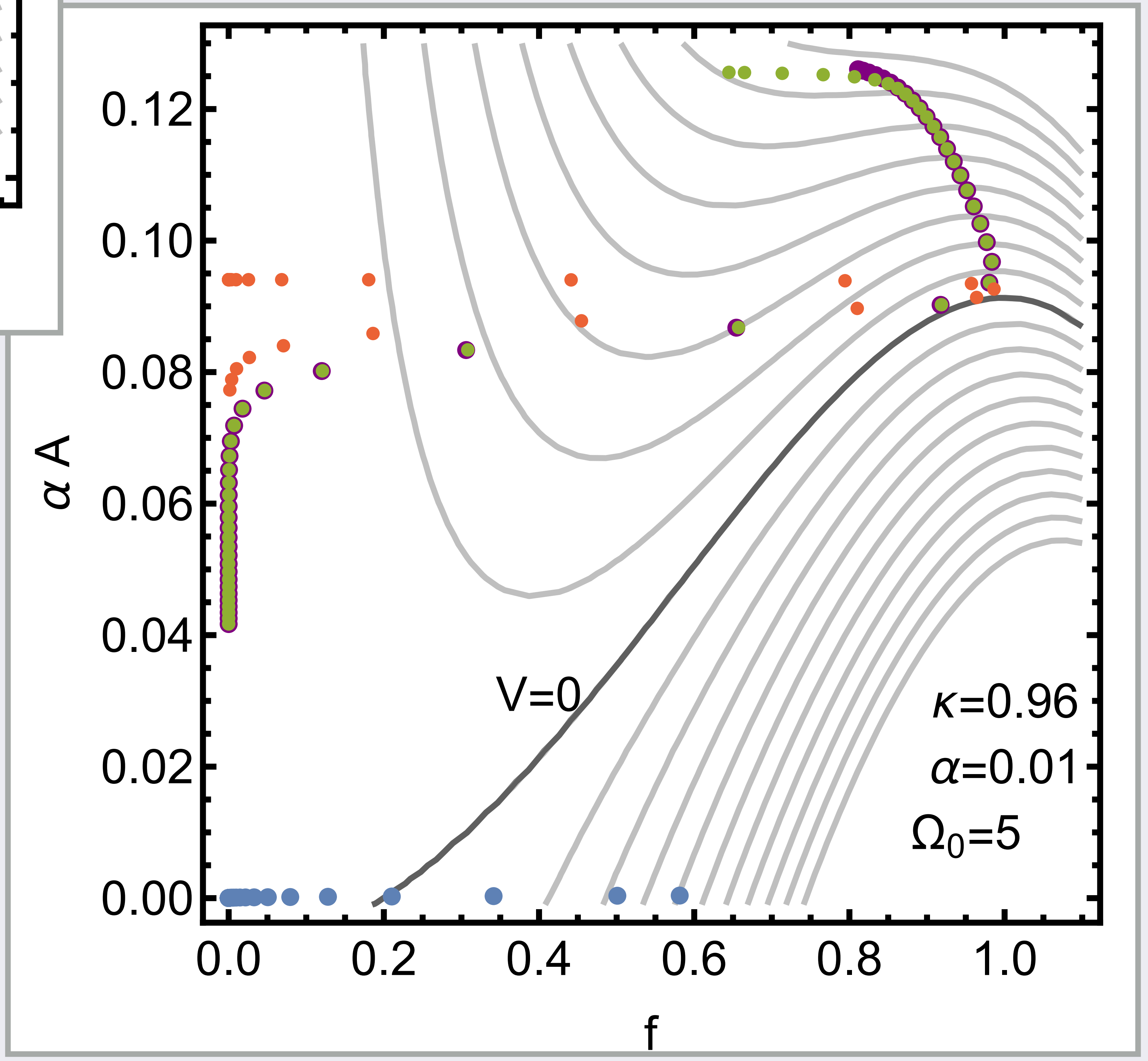
Gauged Q-shells

No global limit







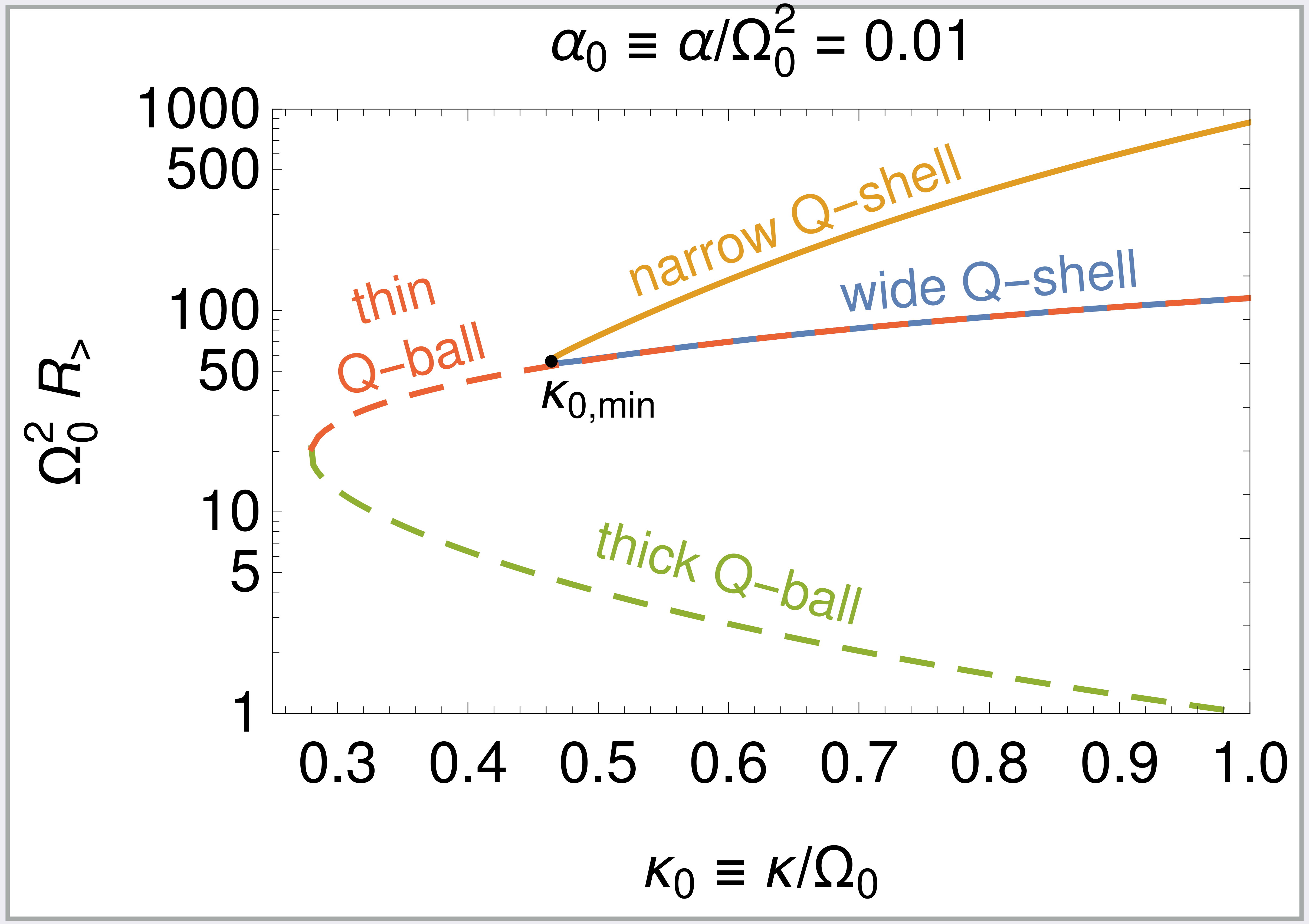


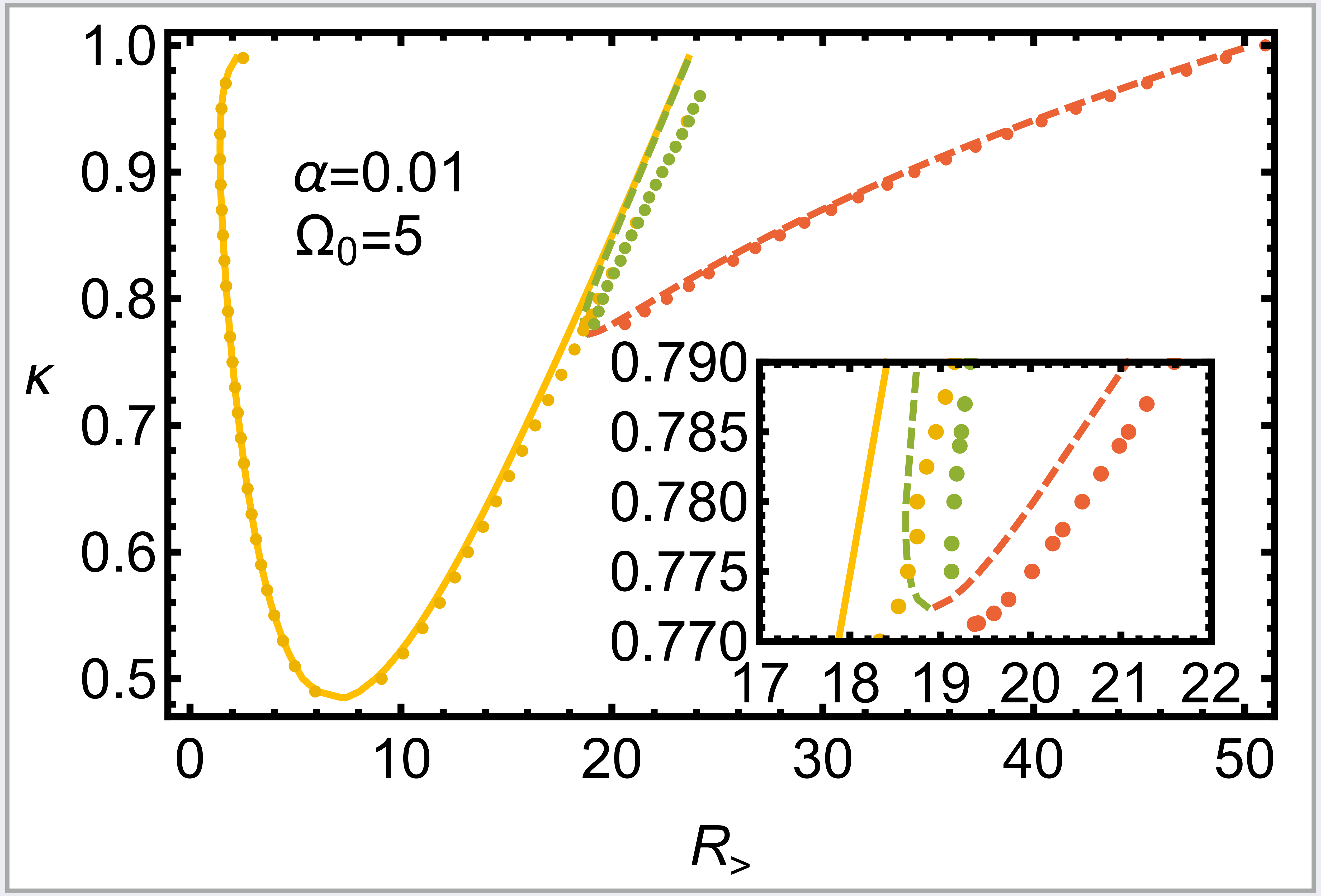
Q-shell Ansatz

Similar to thin-wall

$$f(\rho) = \begin{cases} 0 & \rho < R_< \\ 1 & R_< \leq \rho \leq R_> \\ 0 & R_> < \rho \end{cases}$$

Leads to predictive results and understanding





Conclusion

Non-topological are interesting field theory objects and a macroscopic dark matter candidate

For a motivated potential, much of the numerical results can be understood analytically

Analytic predictions are shown to agree well with the numerics

New Q-shell solutions discovered and characterized

Paves the way for phenomenological study of these objects without the need to continually solve the coupled nonlinear equations