# $(g-2)_\mu$ from lattice QCD and experiments: 4.2 sigma, indeed?

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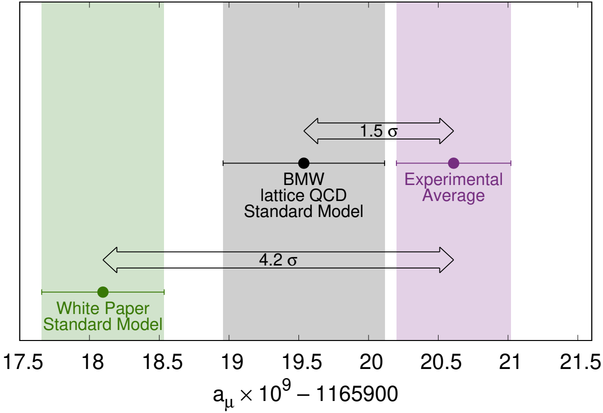
Budapest–Marseille–Wuppertal collaboration(BMW)

DPF 2021

July 13, 2021, Florida State University

# Tensions in $(g-2)_\mu$

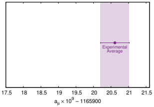


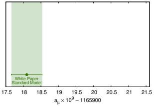

[Muon g-2 Theory Initiative, Phys.Rept. 887 (2020) 1-166]

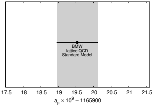
[Budapest–Marseille–Wuppertal-coll., Nature (2021)]

# Outline

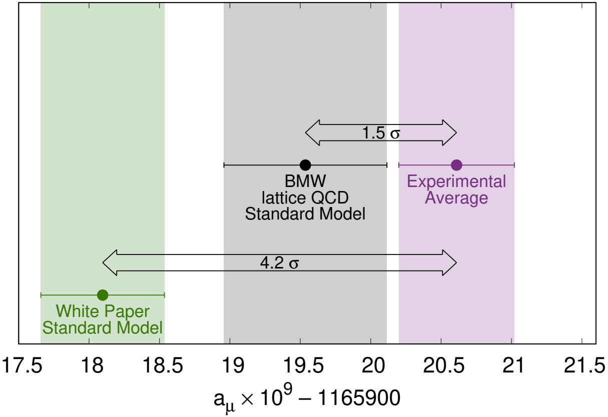
1. $(g-2)_\mu$

2. 


3. 

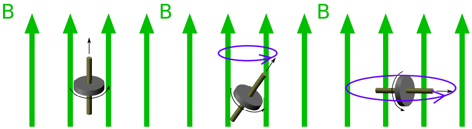

4. 


5. Summary



# Anomalous magnetic moment

- Charged top, with homogeneous charge and mass distribution, $Q$,$M$




- Circular current $\longrightarrow$ magnetic moment $\boldsymbol{\mu}$

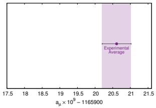
$$\boldsymbol{\mu} = \frac{Q}{2M}\mathbf{L}$$

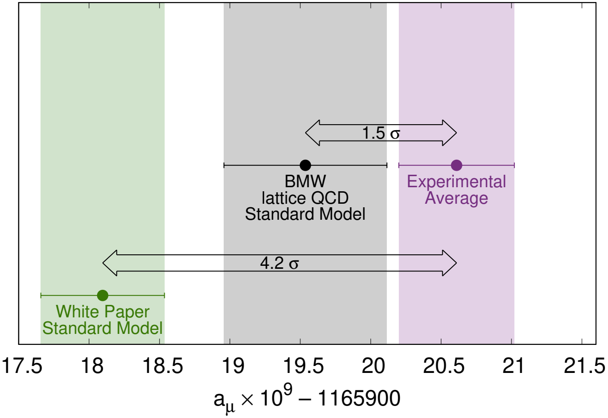
- $\boldsymbol{\mu}$ is proportional to angular momentum $\mathbf{L}$
- Muon has spin $\frac{1}{2}$ $\longrightarrow$ described by the Dirac equation

$$\boldsymbol{\mu}_\mu = 2\,\frac{e}{2m_\mu}\mathbf{S}_\mu = g\,\frac{e}{2m_\mu}\mathbf{S}_\mu$$

- Quantum mechanics predicts: $g = 2$
- In reality: $g_\mu \approx 2.00233\ldots$
- Deviation: Anomalous Magnetic Moment $a = \dfrac{g-2}{2}$

# Outline

2. 
Experimental
Average
17.5
18
18.5
19
19.5
20
20.5
21
21.5
$a_\mu \times 10^9 - 1165900$


1.5 σ
BMW
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$a_\mu \times 10^9 - 1165900$

# Experimental result

- Newly announced result at Fermilab

$$a_\mu(\text{FNAL}) = 11\,659\,204.0(5.4) \cdot 10^{-10} \quad (0.46\,\text{ppm})$$

- Equivalent to: bathroom scale sensitive to weight of a single eyelash.

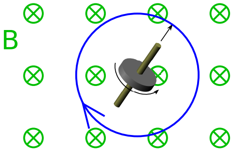
- Fully agrees with the BNL E821 measurement

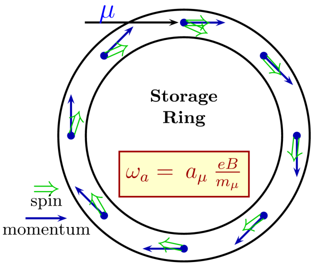
$$\begin{aligned} a_\mu(\text{BNL}) &= 11\,659\,209.1(6.3) \cdot 10^{-10} \quad (0.54\,\text{ppm}) \\ a_\mu(\text{combined}) &= 11\,659\,206.1(4.1) \cdot 10^{-10} \quad (0.35\,\text{ppm}) \end{aligned}$$

- Target uncertainty: (1.6)

# Measurement principle

- Cyclotron motion frequency: $\omega_c = \frac{eB}{m_\mu \gamma}$ with $\gamma = \frac{1}{\sqrt{1-v^2}}$
- Spin precession frequency: $\omega_s = \frac{eB}{m_\mu \gamma} + a_\mu \frac{eB}{m_\mu}$



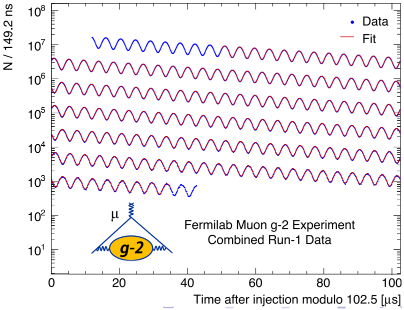

- Measure: $\omega_a = \omega_s - \omega_c = a_\mu \frac{eB}{m_\mu}$
- Gives $a_\mu = \frac{g_\mu - 2}{2}$ directly
- During each circle spin axis changes $12'$

# Measurement principle

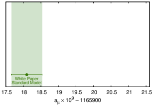
- Pions produced at fixed target $\quad p + p \rightarrow p + n + \pi^+$
- Pion decays through weak interaction $\longrightarrow$ highly polarized muons

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

- Muons decay after several circles: $\quad \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$
- Strong correlation between $\mu^+$ spin and $e^+$ momentum
- Detect emitted $e^+$



# Outline

3. 
White Paper
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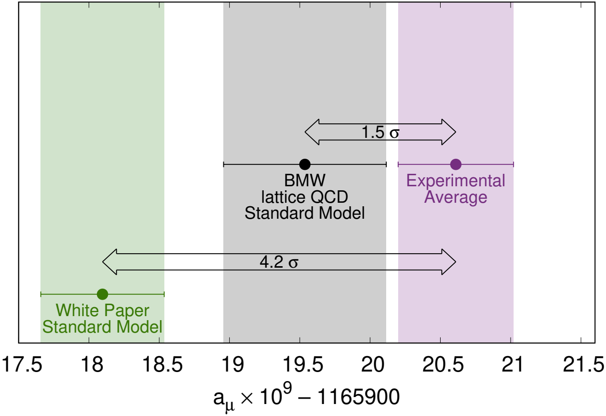
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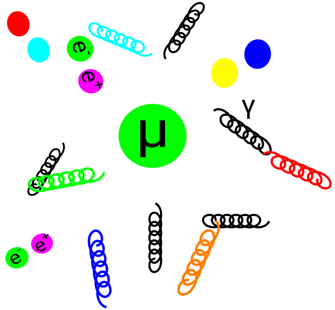
# Theory: Standard Model




Sum over all known physics:

1. quantum electrodynamics (QED): photons, leptons
2. electroweak (EW): W, Z bosons, neutrinos, Higgs
3. strong (QCD): quarks and gluons

- [2006.04822] White Paper of Muon g-2 Theory Initiative

| | $a_\mu \times 10^{-10}$ |
|---|---|
| QED | 11658471.9(0.0) |
| electroweak | 15.4(0.1) |
| strong | 693.7(4.3) |
| total | 11659181.0(4.3) |

# Theory: QED

- $\alpha = \frac{e^2}{4\pi} \ll 1 \rightarrow$ rapidly converging series, was a key to success of QED

$$\left(\frac{g-2}{2}\right) = \left(\frac{\alpha}{\pi}\right) a^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 a^{(2)} + \left(\frac{\alpha}{\pi}\right)^3 a^{(3)} + \ldots$$

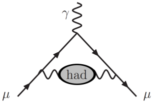
- all contributions with photons and leptons ($e$, $\mu$, $\tau$)

| n-loop | $a_\mu^{\rm QED} \times 10^{-10}$ |
|---|---|
| 1 | 11614097.330(0.008) |
| 2 | 41321.762(0.010) |
| 3 | 3014.190(0.000) |
| 4 | 38.081(0.030) |
| 5 | 0.448(0.140) |
| total | 11658471.811(0.160) |

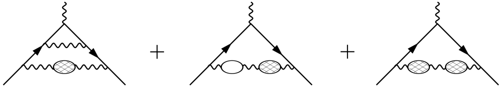
- inputs are $m_\mu/m_e$, $m_\mu/m_\tau$ and $\alpha$

# Hadronic contributions

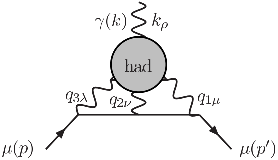
- LO hadron vacuum polarization (LO-HVP, $(\frac{\alpha}{\pi})^2$)




- NLO hadron vacuum polarization (NLO-HVP, $(\frac{\alpha}{\pi})^3$)



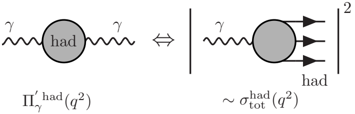

- Hadronic light-by-light (HLbL, $(\frac{\alpha}{\pi})^3$)




- pheno $a_\mu^{\text{HLbL}} = 9.2(1.9)$
  [Colangelo, Hoferichter, Kubis, Stoffer et al '15–'20 ]
- lattice $a_\mu^{\text{HLbL}}$=7.9(3.1)(1.8) or 10.7(1.5) [RBC/UKQCD '19 and Mainz '21]

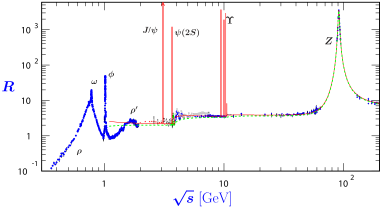
# HVP from R-ratio

- Optical theorem




Use $e^+e^- \to$ had data of CMD, SND, BES, KLOE, BABAR, ...
systematics limited

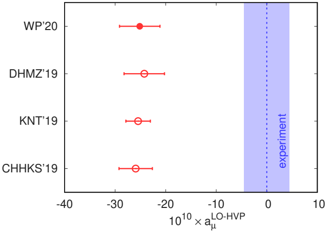
$$a_\mu^{\text{LO-HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int \frac{ds}{s^2} K_\mu(s) R(s)$$




| | | | |
|---|---|---|---|
| LO | [Jegerlehner '18] | 688.1(4.1) | 0.60% |
| LO | [Davier et al '19] | 693.9(4.0) | 0.58% |
| LO | [Keshavarzi et al '19] | 692.78(2.42) | 0.35% |
| LO | [Hoferichter et al '19] | 692.3(3.3) | 0.48% |
| NLO | [Kurz et al '14] | -9.87(0.09) | |
| NNLO | [Kurz et al '14] | 1.24(0.01) | |

# Discrepancy

- $a_\mu^{\text{exp}} - a_\mu^{\text{theory}} = 25.1(6.0)$ around $4.2\sigma$ significance




error budget:

$(4.1)_{\text{exp}}(0.1)_{\text{QED}}(0.1)_{\text{weak}}(4.0)_{\text{HVP}}(1.8)_{\text{HLbL}}$

- HUGE: is about $2\times$ electroweak contribution

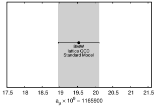
For new physics:

- FNAL(plan) + same theory errors $6\sigma$
- FNAL(plan) + HLbL 10% + HVP 0.2% $11\sigma$

For no new physics:

- 4% larger HVP, $a_\mu^{\text{LO-HVP}} = 720.0(6.8)$
- 360% larger HLbL, $a_\mu^{\text{HLbL}} = 37.9(7.1)$

# Outline

4. 
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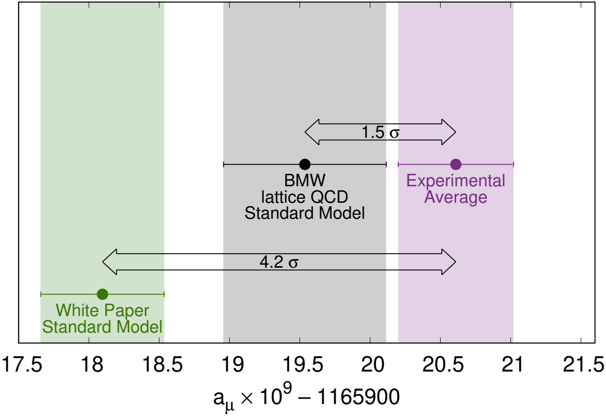
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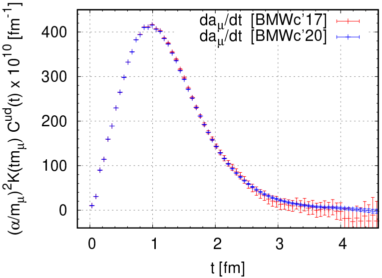
# $a_\mu^{\text{LO-HVP}}$ from lattice QCD Nature 593 (2021) 7857, 51

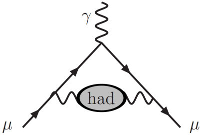
- Compute electromagnetic current-current correlator

$$C(t) = \langle J_\mu(t) J_\nu(0) \rangle$$

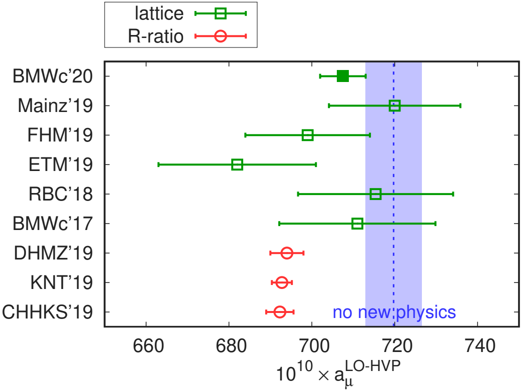
$$a_\mu^{\text{LO-HVP}} = \alpha^2 \int_0^\infty dt\, K(t)\, C(t)$$




$K(t)$ describes the leptonic part of diagram



# Final result



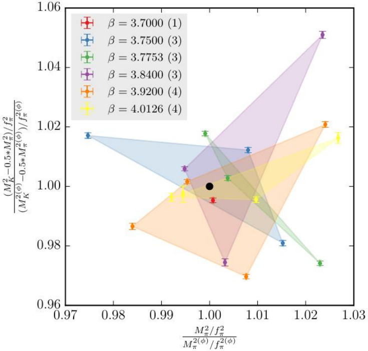

- $a_\mu^{\text{LO-HVP}} = 707.5(2.3)(5.0)[5.5]$ with 0.8% accuracy
- consistent with new FNAL experiment
- $2.0\sigma$ larger than [DHMZ'19], $2.5\sigma$ than [KNT'19]

# Simulation setup

- 6 lattice spacings: 0.13 fm – 0.064 fm $\longrightarrow$ controlled continuum limit
- Box size: $L \sim 6$ fm

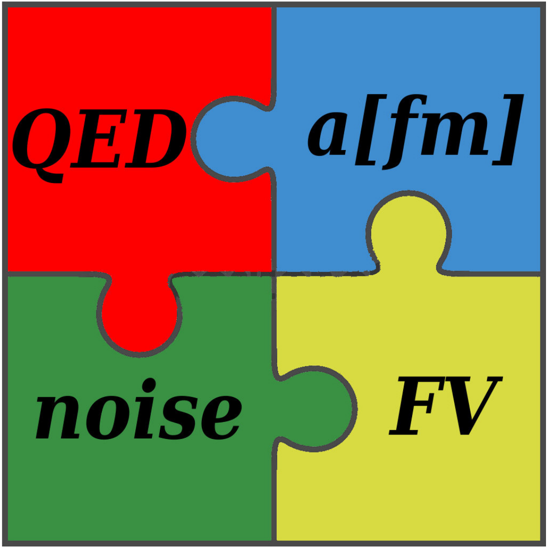
  $L \sim 11$ fm at one lattice spacing $\longrightarrow$ FV effects

  1 fm $= 10^{-15}$ m $\sim$ size of proton
- Quark masses bracketing their physical values




| $\beta$ | $a$[fm] | $L \times T$ | #conf |
|---|---|---|---|
| 3.7000 | 0.1315 | $48 \times 64$ | 904 |
| 3.7500 | 0.1191 | $56 \times 96$ | 2072 |
| 3.7753 | 0.1116 | $56 \times 84$ | 1907 |
| 3.8400 | 0.0952 | $64 \times 96$ | 3139 |
| 3.9200 | 0.0787 | $80 \times 128$ | 4296 |
| 4.0126 | 0.0640 | $96 \times 144$ | 6980 |

# New challenges



# Scale determination

Lattice spacing $a$ enters into $a_\mu$ determination:

- physical value of $m_\mu$
- physical values of $m_\pi, m_K$

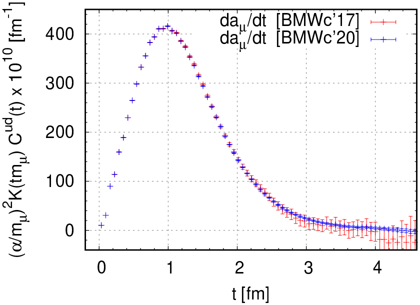
$\longrightarrow$ $\Delta_{\text{scale}} a_\mu \sim 2 \cdot \Delta(\text{scale})$

1. For final results: $M_\Omega$ scale setting $\longrightarrow$ $a = (aM_\Omega)^{\text{lat}} / M_\Omega^{\text{exp}}$
   - Experimentally well known: 1672.45(29) MeV [PDG 2018]
   - Moderate $m_q$ dependence
   - Can be precisely determined on the lattice
2. For separation of isospin breaking effects: $w_0$ scale setting
   - Moderate $m_q$ dependence
   - Can be precisely determined on the lattice
   - No experimental value  
     $\longrightarrow$ Determine value of $w_0$ from $M_\Omega \cdot w_0$

$$w_0 = 0.17236(29)(63)[70] \text{ fm}$$

# Noise reduction

- noise/signal in $C(t) = \langle J(t)J(0)\rangle$ grows for large distances



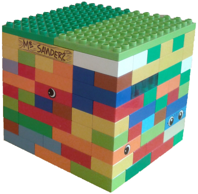

- Low Mode Averaging: use exact (all2all) quark propagator in IR and stochastic in UV
- decrease noise by replacing $C(t)$ by upper/lower bounds above $t_c$

$$0 \le C(t) \le C(t_c)e^{-E_{2\pi}(t-t_c)}$$

$\longrightarrow$ few permil level accuracy on each ensemble

# Finite-size effects

- Typical lattice runs use $L < 6\,\mathrm{fm}$, earlier model estimates gave $O(2)\%$ FV effect.

$L_{\mathrm{ref}} = 6.272\,\mathrm{fm}$   $L_{\mathrm{big}} = 10.752\,\mathrm{fm}$

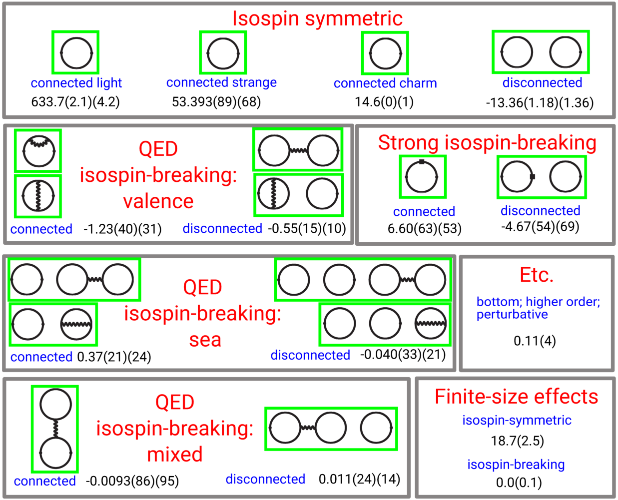
1. $a_\mu(\mathrm{big}) - a_\mu(\mathrm{ref})$
   - perform numerical simulations in $L_{\mathrm{big}} = 10.752\,\mathrm{fm}$
   - perform analytical computations to check models

| lattice | NLO XPT | NNLO XPT | MLLGS | HP | RHO |
|---|---|---|---|---|---|
| $18.1(2.0)_{\mathrm{stat}}(1.4)_{\mathrm{cont}}$ | 11.6 | 15.7 | 17.8 | 16.7 | 15.2 |

2. $a_\mu(\infty) - a_\mu(\mathrm{big})$
   - use models for remnant finite-size effect of “big” $\sim 0.1\%$

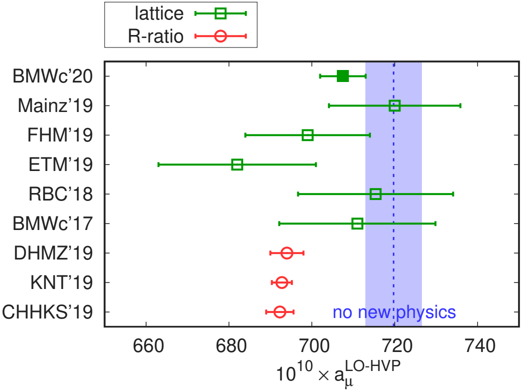
# Isospin breaking effects

- Include leading order IB effects: $\mathcal{O}(e^2)$, $\mathcal{O}(\delta m)$




$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$

# Final result

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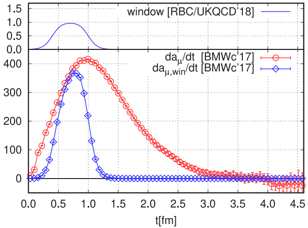
# Window observable

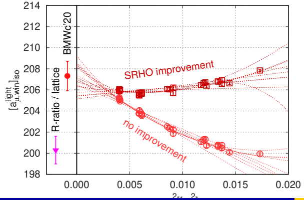
- Restrict correlator to window between $t_1 = 0.4\,\mathrm{fm}$ and $t_2 = 1.0\,\mathrm{fm}$

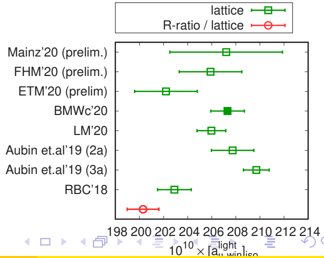



(144 × 96$^3$, $a \sim 0.064$ fm, $M_\pi \sim 135$ MeV)

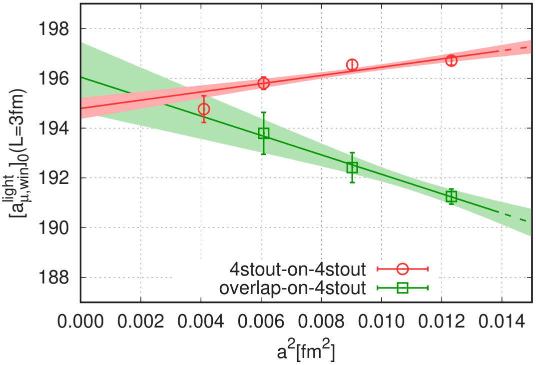
[RBC/UKQCD'18]

- Less challenging than full $a_\mu$
  - signal/noise
  - finite size effects
  - lattice artefacts (short & long)



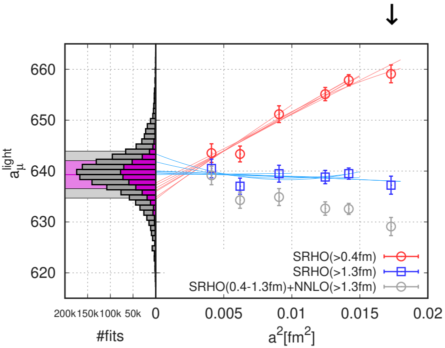



# Crosscheck – overlap




- compute $a_{\mu,\text{win}}$ with overlap valence
- local current instead of conserved $\longrightarrow$ had to compute $Z_V$
- cont. limit in $L = 3\,\text{fm}$ box consistent with staggered valence

# Continuum limit



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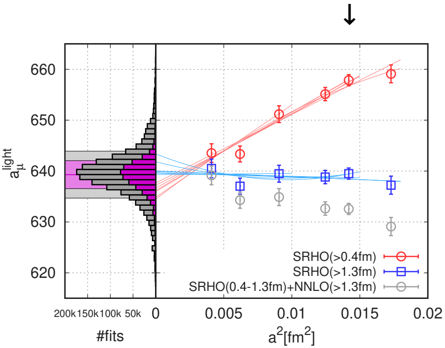


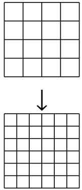

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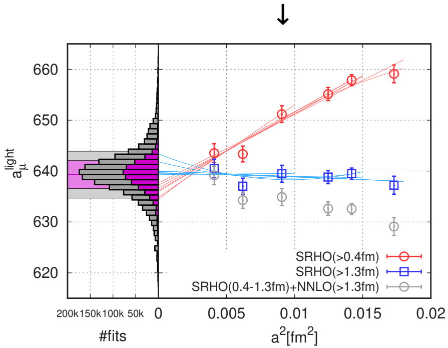
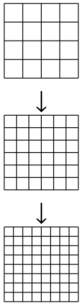
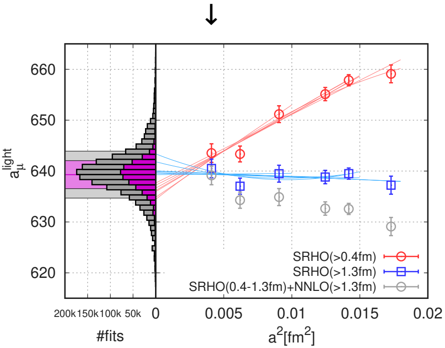
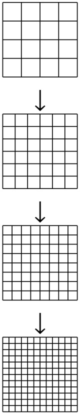


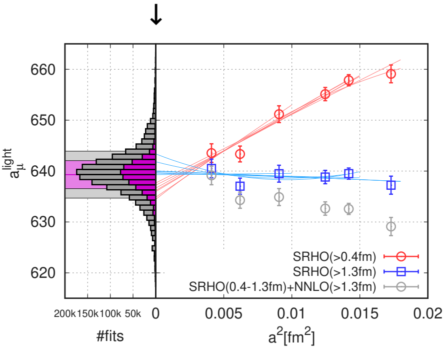



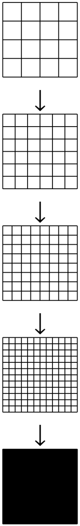
# Continuum limit



# Continuum limit

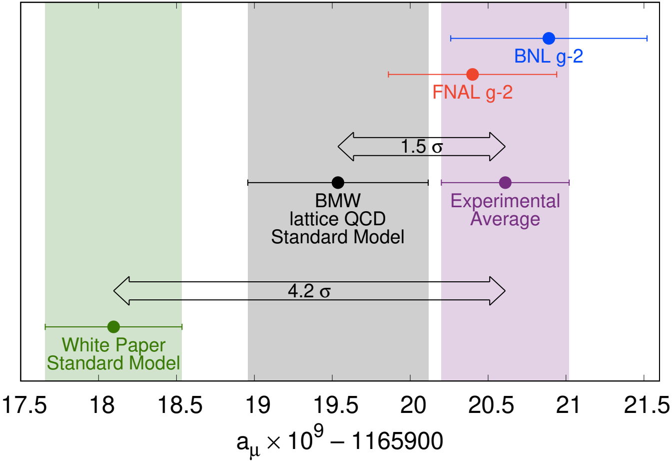


# Outline

5. Summary

# Tensions



BNL g-2
FNAL g-2
1.5 σ
BMW
lattice QCD
Standard Model
Experimental
Average
4.2 σ
White Paper
Standard Model
17.5
18
18.5
19
19.5
20
20.5
21
21.5
$a_\mu \times 10^9 - 1165900$