Predictions for the Leptonic Dirac CP-Violating Phase

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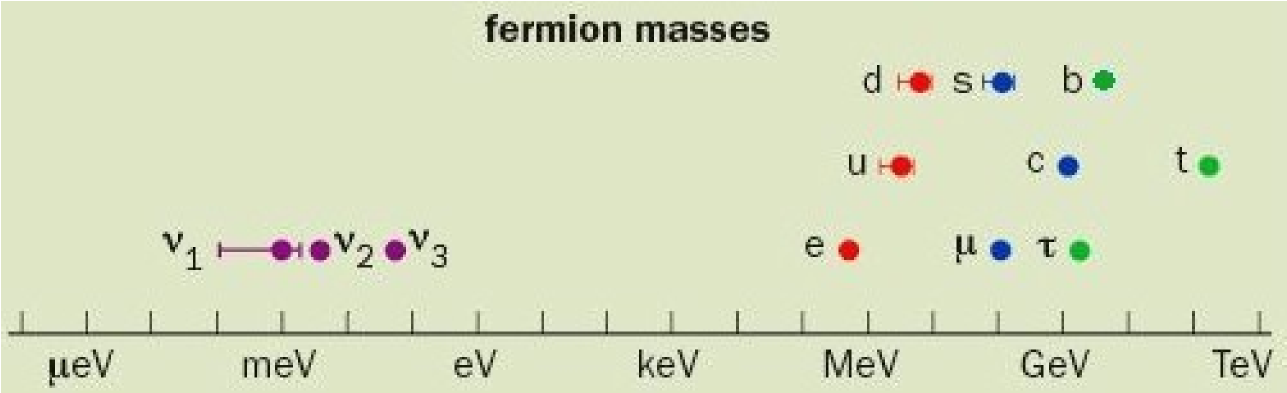
DPF21

Based on: L.A. Delgadillo, L.L. Everett, R. Ramos and AS, Phys.Rev. D97 (2018) no.9, 095001 [arXiv:1801.06377] and L.L Everett, R. Ramos, A.B. Rock, and AS [arXiv:1912.10139]

What We Measure



Quark Mixing

$$U_{CKM} = R_1(\theta_{23}^{CKM})R_2(\theta_{13}^{CKM}, \delta_{CKM})R_3(\theta_{12}^{CKM})$$

$$\theta_{13}^{CKM} = 0.2^\circ \pm 0.1^\circ$$

$$\theta_{23}^{CKM} = 2.4^\circ \pm 0.1^\circ$$

$$\theta_{12}^{CKM} = 13.0^\circ \pm 0.1^\circ$$

$$\delta_{CKM} = 60^\circ \pm 14^\circ$$

Quarks look like deviations from unity.

Lepton Mixing

$$U_{MNSP} = R_1(\theta_{23})R_2(\theta_{13}, \delta_{CP})R_3(\theta_{12})P$$

NuFIT 4.1 (2019): 1811.05487

$$\theta_{13}^{MNSP} = (8.61^\circ)^{+0.13}_{-0.13}$$

$$\theta_{23}^{MNSP} = (48.3^\circ)^{+1.1}_{-1.9}$$

$$\theta_{12}^{MNSP} = (33.82^\circ)^{+0.78}_{-0.76}$$

$$\delta_{CP} = (222^\circ)^{+38}_{-28}$$

What about the leptons?

Popular Patterns (before 2012)

$$\tilde{U}_\nu = R_{23}\left(\theta_{23}^\nu\right) R_{12}\left(\theta_{12}^\nu\right) = \begin{pmatrix} \cos\theta_{12}^\nu & \sin\theta_{12}^\nu & 0 \\ -\frac{\sin\theta_{12}^\nu}{\sqrt{2}} & \frac{\cos\theta_{12}^\nu}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sin\theta_{12}^\nu}{\sqrt{2}} & \frac{\cos\theta_{12}^\nu}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(Marzocca, et al. (2013);
Petcov (2014);
Girardi, Petcov, Titov (2015))

TriBiMaximal (TBM) Mixing: $\theta_{12}^\nu \approx 35.26^\circ$ (P. Harrison, D. Perkins, W. Scott (2002); Z. Xing (2002); X. He, A. Zee (2003))

BiMaximal (BM) Mixing: $\theta_{12}^\nu = 45^\circ$ (F. Vissani (1997); V. Barger, S. Pakvasa, T. Weiler, K. Whisnant, (1998); A. Baltz, A. Goldhaber, M. Goldhaber (1998))

Golden Ratio 1 (GR1) Mixing: $\theta_{12}^\nu \approx 31.72^\circ$ (A. Datta, F. Ling, P. Ramond (2003); L. Everett, AS (2008))

Golden Ratio 2 (GR2) Mixing: $\theta_{12}^\nu = 36^\circ$ (W. Rodejohann (2009))

HexaGonal (HG) Mixing: $\theta_{12}^\nu = 30^\circ$ (C. Albright, A. Dueck and W. Rodejohann (2010); J. E. Kimand M. Seo (2011))

However, they all have a vanishing reactor mixing angle θ_{13}. How can we fix this?

Charged Lepton Corrections

$$U_{\rm MNSP} = U_e^\dagger U_\nu$$

As on previous slide, assume the neutrino mixing matrix is given as $U_\nu = R_{23}(\theta_{23}^\nu) R_{12}(\theta_{12}^\nu)$.

$$R_{23}^\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23}^\nu & s_{23}^\nu \\ 0 & -s_{23}^\nu & c_{23}^\nu \end{pmatrix} \qquad R_{12}^\nu = \begin{pmatrix} c_{12}^\nu & s_{12}^\nu & 0 \\ -s_{12}^\nu & c_{12}^\nu & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Postulate a theoretical form for U_e, *i.e.,* one rotation, two rotations, etc. For simplicity assume U_e consists of only a 1-2 rotation:

$$U_{12}^e = \begin{pmatrix} c_{12}^e & s_{12}^e e^{-i\delta_{12}^e} & 0 \\ -s_{12}^e e^{i\delta_{12}^e} & c_{12}^e & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad s_{ij}^e = \sin\theta_{ij}^e \text{ and } c_{ij}^e = \cos\theta_{ij}^e$$

How does this charged lepton rotation change the initial mixing predictions?

$$U_e = U^e_{12}(\theta^e_{12}, \delta^e_{12})$$

The Effect of a 1-2 Rotation

$$U_{\mathsf{MNSP}} \equiv U = U_e^\dagger U_\nu = U^{e\dagger}_{12} R^\nu_{23} R^\nu_{12}$$

$$U_{e1} = c^e_{12} c^\nu_{12} + c^\nu_{23} e^{-i\delta^e_{12}} s^e_{12} s^\nu_{12}, \quad U_{e2} = c^e_{12} s^\nu_{12} - c^\nu_{12} c^\nu_{23} e^{-i\delta^e_{12}} s^e_{12};$$

$$U_{e3} = -e^{-i\delta^e_{12}} s^e_{12} s^\nu_{23}, \quad U_{\mu 1} = -c^e_{12} c^\nu_{23} s^\nu_{12} + c^\nu_{12} e^{i\delta^e_{12}} s^e_{12}$$

$$U_{\mu 2} = c^e_{12} c^\nu_{12} c^\nu_{23} + e^{i\delta^e_{12}} s^e_{12} s^\nu_{12}, \quad U_{\mu 3} = c^e_{12} s^\nu_{23},$$

$$U_{\tau 1} = s^\nu_{12} s^\nu_{23}, \quad U_{\tau 2} = -c^\nu_{12} s^\nu_{23},$$

$$U_{\tau 3} = c^\nu_{23}.$$

Compare this to the PDG parameterization of the MNSP Matrix:

$$U^{\mathsf{PDG}} = \begin{pmatrix} c_{12} c_{13} & c_{13} s_{12} & e^{-i\delta} s_{13} \\ -c_{23} s_{12} - c_{12} e^{i\delta} s_{13} s_{23} & c_{12} c_{23} - e^{i\delta} s_{12} s_{13} s_{23} & c_{13} s_{23} \\ s_{12} s_{23} - c_{12} c_{23} e^{i\delta} s_{13} & -c_{12} s_{23} - c_{23} e^{i\delta} s_{12} s_{13} & c_{13} c_{23} \end{pmatrix} P_{\mathsf{Maj}}$$

By equating both parameterizations, it should become clear that it is possible to express the PDG parameters in terms of the model parameters….

Enter Sum Rules

Specifically, one can find a relationship between the Dirac CP-violating phase δ, the experimentally measured PDG matrix, and the model parameters, i.e.,

$$\cos\delta = \frac{(1/t_{23} + s_{13}^2 t_{23})(s_{12}^\nu)^2 - (s_{12}^2/t_{23} + c_{12}^2 s_{13}^2 t_{23})}{s'_{12} s_{13}}$$

(D. Marzocca, S.T. Petcov, et al. (2011); D. Marzocca, S.T. Petcov, et al. (2013); P. Ballet, S.F. King, et al. (2014) S.T. Petcov (2015); I. Girardi, S.T. Petcov, et al. (2015);)

Can be derived easily by taking the ratio (P. Ballet, S.F. King, et al. (2014)):

$$\frac{|U_{\tau 1}|}{|U_{\tau 2}|} = t_{12}^\nu \qquad \frac{|U_{\tau 1}^{PDG}|}{|U_{\tau 2}^{PDG}|} = \frac{|s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta}|}{|c_{12}s_{23} + c_{23}s_{12}s_{13}e^{i\delta}|}$$

This sum rule can in principle be applied to reveal how this 1-2 rotation affects the predictions for the leptonic Dirac CP-violating phase. In principle there exists 9!/2!7!-1=35 other additional ratios that must hold to guarantee the unitarity of the matrices.

A Simple set of 4 Sum Rules

The 36 sum rules, upon equating parameters, reduce to only 4 sum rules. The original:

$$\cos\delta = \frac{(1/t_{23} + s_{13}^2 t_{23})(s_{12}^\nu)^2 - (s_{12}^2/t_{23} + c_{12}^2 s_{13}^2 t_{23})}{s_{12}' s_{13}}$$

And 3 others which give the angles of the MNSP matrix in terms of model parameters:

$$s_{13}^2 = (s_{12}^e)^2 (s_{23}^\nu)^2, \qquad s_{23}^2 = \frac{(s_{23}^\nu)^2 - (s_{12}^e)^2 (s_{23}^\nu)^2}{1 - (s_{12}^e)^2 (s_{23}^\nu)^2},$$

$$s_{12}^2 = \frac{(c_{12}^\nu)^2 (c_{23}^\nu)^2 (s_{12}^e)^2 + (c_{12}^e)^2 (s_{12}^\nu)^2 - 2 c_{12}^e c_{12}^\nu c_{23}^\nu s_{12}^e s_{12}^\nu \cos\delta_{12}^e}{1 - (s_{12}^e)^2 (s_{23}^\nu)^2}$$

The value of $\cos\delta$ is subject to the constraints in these 3 additional sum rules. How does this value look for the different mixing patterns:

BM Mixing and a 1-2 Rotation

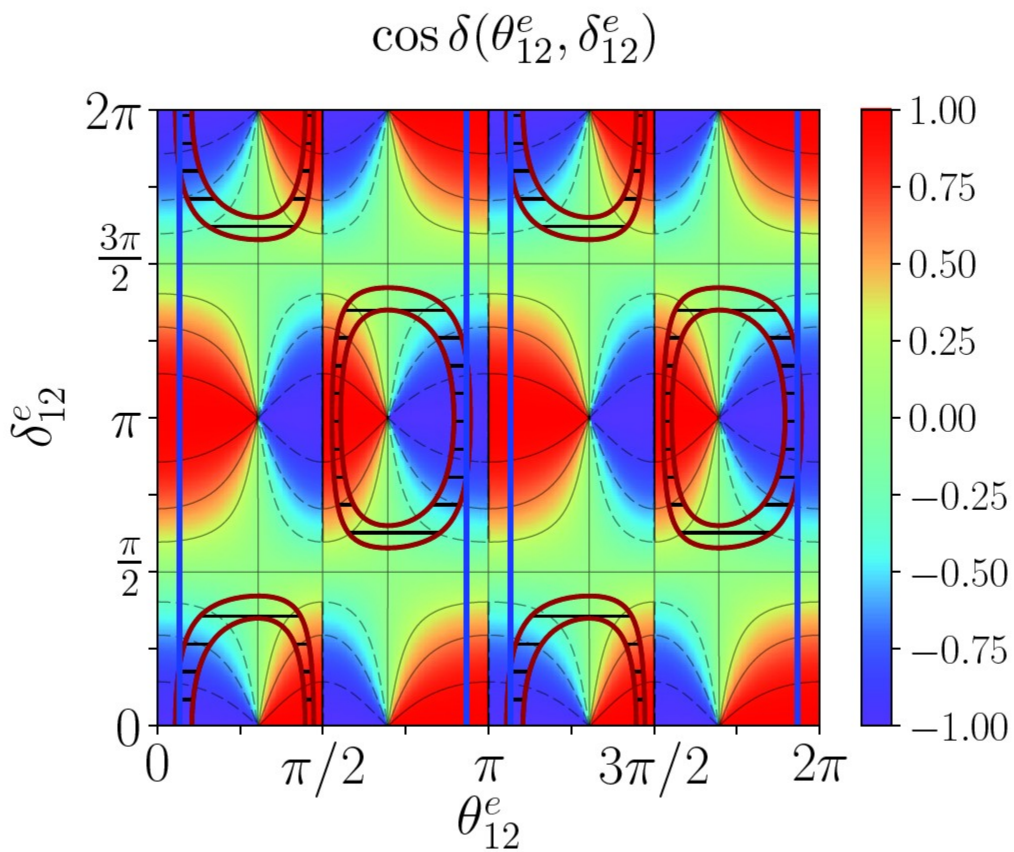
It is possible to showcase the allowed regions of cos(δ) by using a contour plot:

$$\sin^2(\theta_{13}) \equiv s_{13}^2 = (s_{12}^e)^2/2$$

$$s_{23}^2 = \frac{(c_{12}^e)^2/2}{(1-(s_{12}^e)^2/2)} = \frac{1-2s_{13}^2}{2(1-s_{13}^2)}.$$

$$0.4878 \leq s_{23}^2 \leq 0.4904$$

(First octant!)



Blue bands and regions between red contours represent regions allowed by $\sin^2(\theta_{13})$
And $\sin^2(\theta_{12})$ at 3σ, respectively. Notice cos(δ)=~-1 is preferred value.
How about for another popular mixing scenario?

TBM Mixing and a 1-2 Rotation

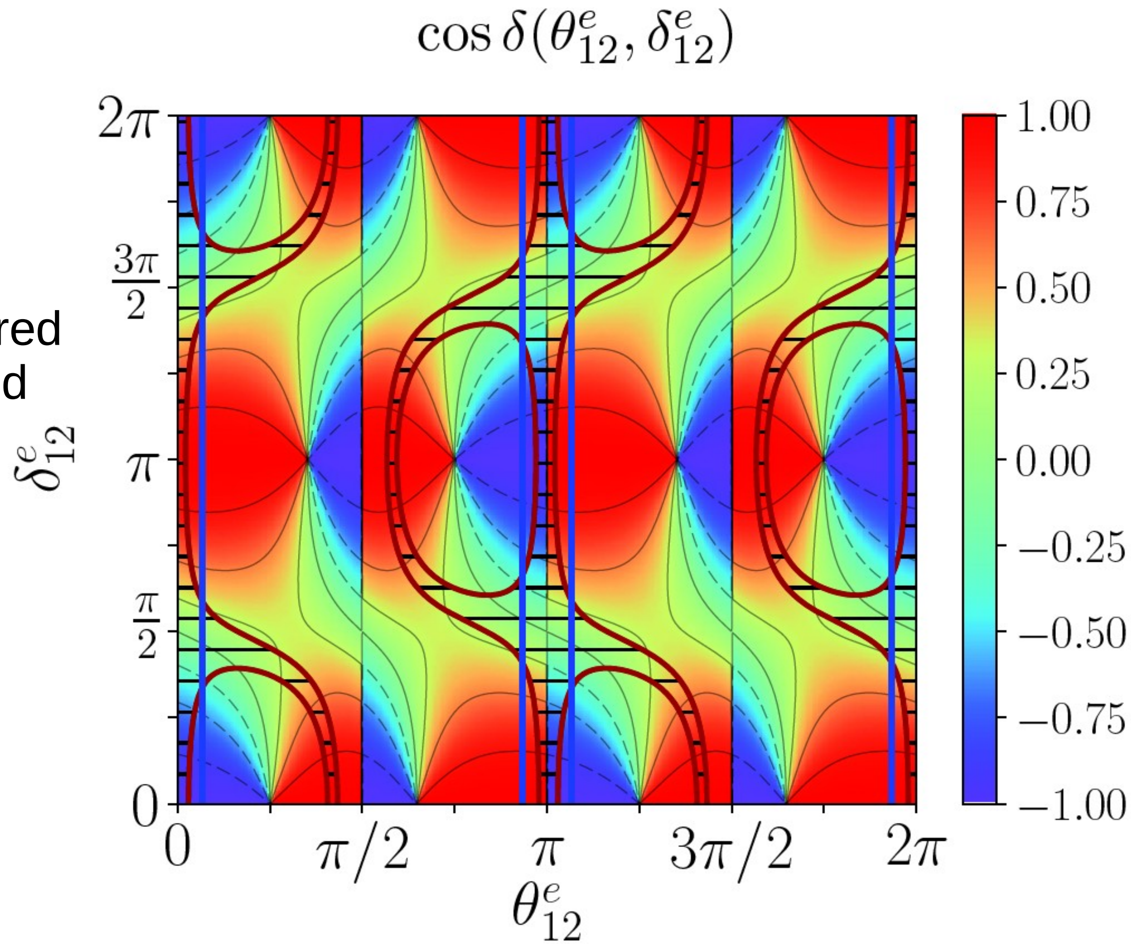
Blue bands and regions between red contours represent regions allowed by $\sin^2(\theta_{13})$ and $\sin^2(\theta_{12})$ at 3σ, respectively.

$$\sin^2(\theta_{13}) \equiv s_{13}^2 = (s_{12}^e)^2/2$$

$$s_{23}^2 = \frac{(c_{12}^e)^2/2}{(1-(s_{12}^e)^2/2)} = \frac{1-2s_{13}^2}{2(1-s_{13}^2)}.$$

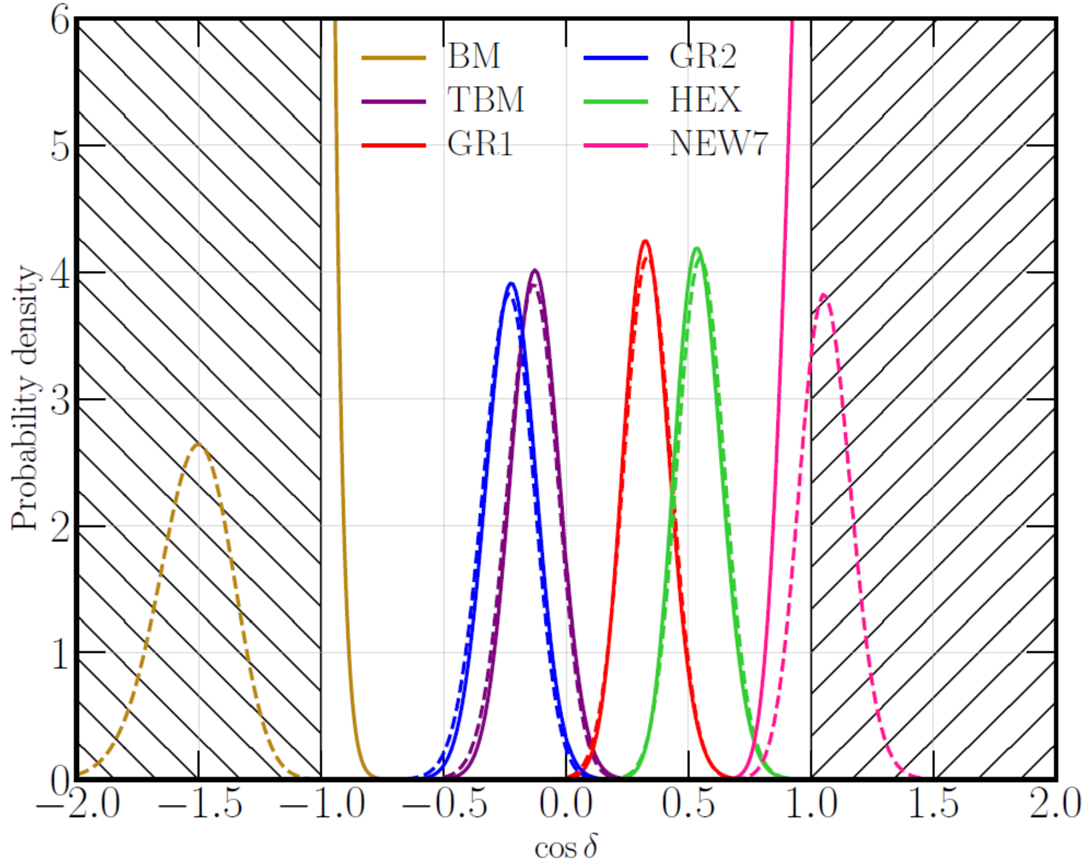
$$0.4878 \le s_{23}^2 \le 0.4904$$

(First octant!)



Because the solar mixing angle does not start as maximal (like in BM mixing) there is a larger region of parameter space in which the reactor and solar mixing angle constraints can be satisfied. This also happens for GR1, GR2, and Hexagonal Mixing. Thus, let this case serve as a representative for them. **Why do I keep mentioning unitarity?**

The Elephant in the Room



See **1912.10139** for details describing how to generate plot.

Solid lines are when original sum rule is applied with 3 additional sum rules correlating experimental input.
Dashed lines are obtained from treating the experimental distributions as uncorrelated inputs, i.e., not properly using unitarity constraints.

Conclusion

- The question of why particles have the masses and mixings that they do still remains unsolved, i.e., the flavor problem. However, both current and future experiments are beginning to shine light on possible solutions.
- By assuming a well-known starting point for U_ν it is possible to analyze the phenomenological predictions of this starting point by applying the unitary matrix U_e (see **1801.06377** for the double-rotation cases and more details of the single rotation cases).
- This additional charged lepton rotation gives rise to a nonzero reactor mixing angle and sum rules which allow for correlations between parameters. Perhaps the most important of these correlations are the correlations between the atmospheric and reactor angles.
- Furthermore, even if atmospheric constraints are satisfied in such a simple model, the results for the solar mixing and CP-violating parameter δ will further separate these scenarios.
- Studies such as **1801.06377** and **1912.10139** highlight that with the anticipated improvements and measurements of lepton mixing, we may be on the verge of making great progress in understanding the flavor problem.

Backup Slides

NEW7 Mixing Pattern

Predicts a 0 reactor angle, a maximal atmospheric angle and a solar angle of $\pi/7$

Possible to generate by preserving the following Klein Symmetry Group:

(L. Everett, AS (2015))

$$G_2 = \begin{pmatrix} -\sin\left(\frac{3\pi}{14}\right) & \frac{\cos\left(\frac{3\pi}{14}\right)}{\sqrt{2}} & \frac{\cos\left(\frac{3\pi}{14}\right)}{\sqrt{2}} \\ \frac{\cos\left(\frac{3\pi}{14}\right)}{\sqrt{2}} & -\sin^2\left(\frac{\pi}{7}\right) & \cos^2\left(\frac{\pi}{7}\right) \\ \frac{\cos\left(\frac{3\pi}{14}\right)}{\sqrt{2}} & \cos^2\left(\frac{\pi}{7}\right) & -\sin^2\left(\frac{\pi}{7}\right) \end{pmatrix} \qquad G_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$G_1 = G_2 G_3$$

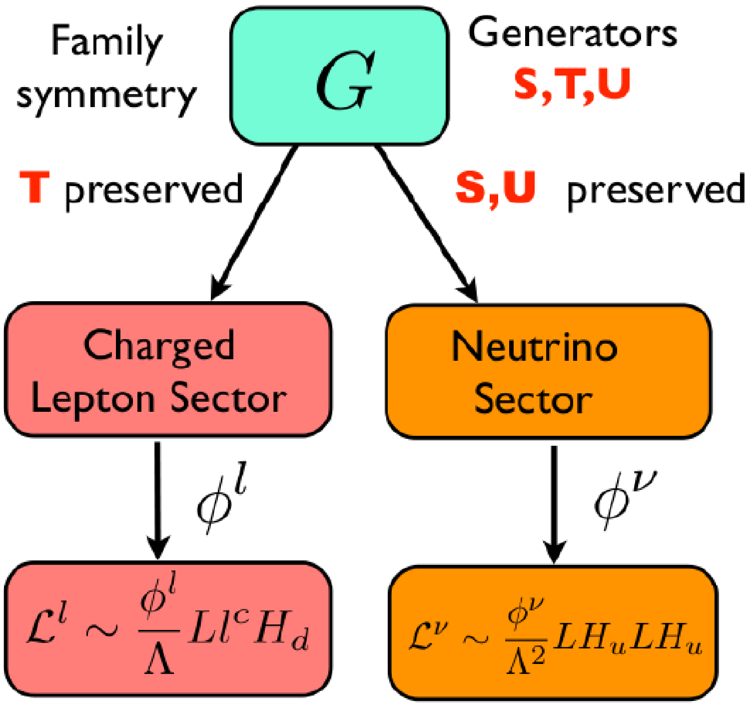
It could also have origins in the symmetry group of the tetradecagon, D_{14}, because a regular tetradecagon has an exterior angle of $\pi/7$.

Motivated by Symmetry

Introduce set of flavon fields (e.g. ϕ^ν and ϕ^ℓ) whose vevs break G_f to G_ν in the neutrino sector and G_e in the charged lepton sector.

$$T\langle\phi^\ell\rangle \approx \langle\phi^\ell\rangle \qquad S\langle\phi^\nu\rangle = U\langle\phi^\nu\rangle = \langle\phi^\nu\rangle$$

Non-renormalizable couplings of flavons to mass terms can be used to explain the smallness of Yukawa Couplings.



S.F. King, C. Luhn (2013)

What are some popular examples of flavor symmetries?