Muon-electron scattering at NNLO for the MUonE experiment

DPF2021

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In collaboration with

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μONe

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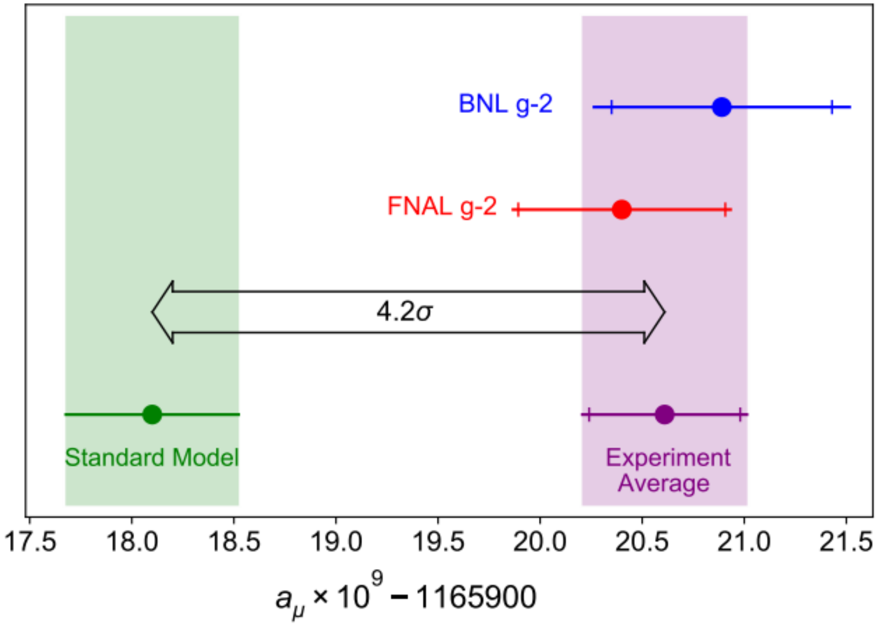
Outline

- Motivation
- MUonE
- NNLO Photonic corrections
- NNLO Leptonic corrections
- Outlook

Motivation



$$a_\mu^{EXP} = (116592089 \pm 63) \times 10^{-11} [0.54ppm]$$ BNL E821

$$a_\mu^{EXP} = (116592040 \pm 54) \times 10^{-11} [0.46ppm]$$ FNAL E989 Run 1

$$a_\mu^{EXP} = (116592061 \pm 41) \times 10^{-11} [0.35ppm]$$ World Average

- First 3 runs of FNAL is completed and the 4th is underway. Finally aiming at 16×10^{-11}.
- Muon g-2 proposal at J-PARC: Phase-1 with ∼ BNL precision

Theory:

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{HVP, LO}} + a_\mu^{\text{HVP, NLO}} + a_\mu^{\text{HVP, NNLO}} + a_\mu^{\text{HLbL}} + a_\mu^{\text{HLbL, NLO}}$$
$$= 116\,591\,810(43) \times 10^{-11} \,.$$

[WP20]

WP20 = White paper of the Muon g-2 Theory Initiative: arXiv:2006.04822

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{HVP, LO}} + a_\mu^{\text{HVP, NLO}} + a_\mu^{\text{HVP, NNLO}} + a_\mu^{\text{HLbL}} + a_\mu^{\text{HLbL, NLO}}$$
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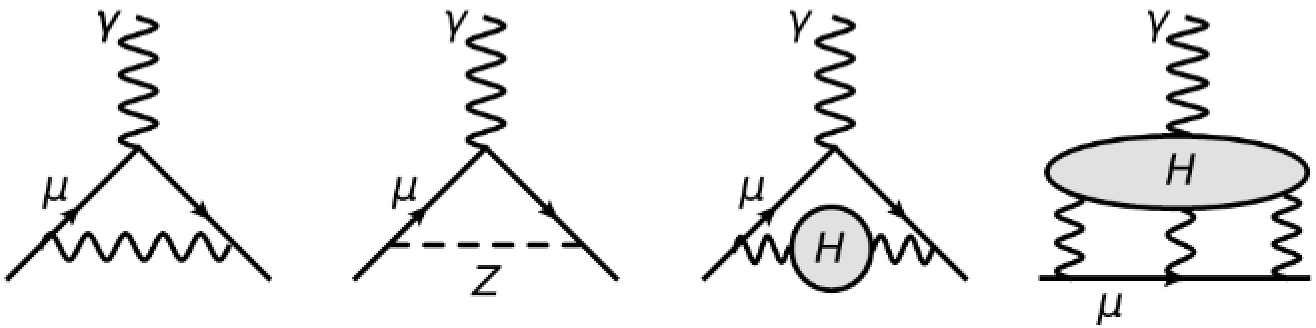
[WP20]



Types of corrections that goes inside SM prediction

WP20 = White paper of the Muon g-2 Theory Initiative: arXiv:2006.04822

$$a_\mu^{\text{QED}}(\alpha(\text{Cs})) = 116\,584\,718.931(104) \times 10^{-11}\,.$$

$$a_\mu^{\text{EW}} = 153.6(1.0) \times 10^{-11}$$

$$a_\mu^{\text{HVP, LO}} = 6931(40) \times 10^{-11}$$

$$a_\mu^{\text{HVP, NLO}} = -98.3(7) \times 10^{-11}$$

$$a_\mu^{\text{HVP, NNLO}} = 12.4(1) \times 10^{-11}$$

$$a_\mu^{\text{HLbL}}(\text{phenomenology + lattice QCD}) = 90(17) \times 10^{-11}$$

$$a_\mu^{\text{HLbL, NLO}} = 2(1) \times 10^{-11}$$

[WP20]

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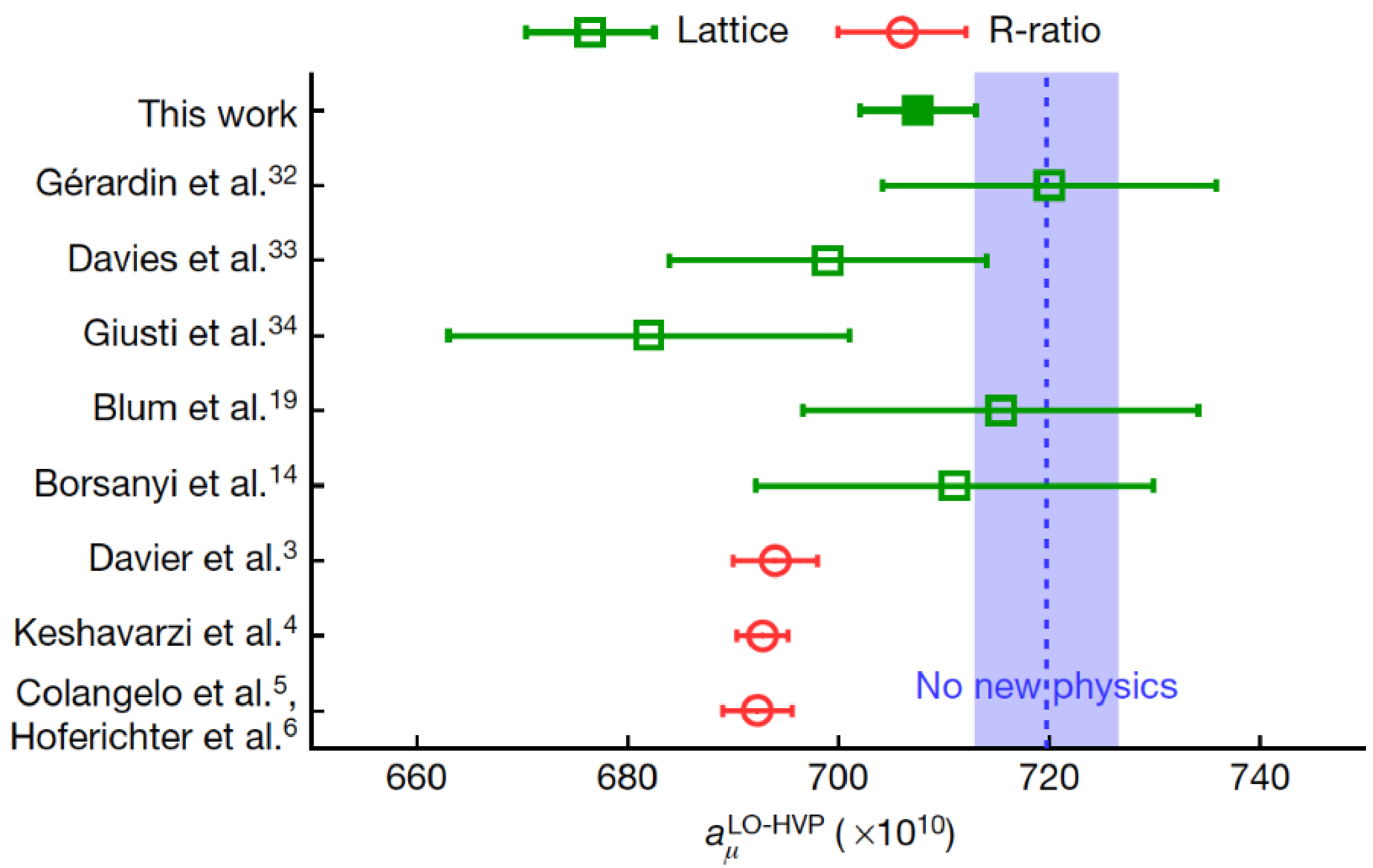
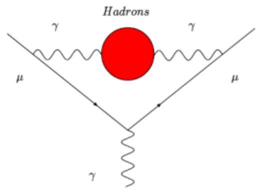
Types of corrections that goes inside SM prediction



[Borsanyi et. al. (BMWc), Nature 2021]

⤳ In the following, focus on a_μ^{HLO}, which contributes (with a_μ^{HLbL}) to the SM uncertainty



- Using dispersion relations and the Optical Theorem

$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^0_{e^+e^-\to\text{had}}(s) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \frac{K(s) R^{\text{had}}(s)}{s^2} =$$

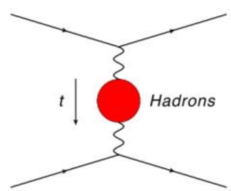
$$= \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left[\int_{4m_\pi^2}^{E^2_{\text{cut}}} ds \frac{K(s) R^{\text{had}}_{\text{data}}(s)}{s^2} + \int_{E^2_{\text{cut}}}^{\infty} ds \frac{K(s) R^{\text{had}}_{\text{pQCD}}(s)}{s^2}\right]$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{m_\mu^2}} \sim \frac{1}{s} \qquad R^{\text{had}}(s) = \frac{\sigma^0_{e^+e^-\to\text{had}}(s)}{\frac{4}{3}\frac{\pi\alpha^2}{s}}$$

- Alternatively (exchanging s and x integrations in a_μ^{HLO})

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\text{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$



e.g. **Lautrup, Peterman, De Rafael, Phys. Rept. 3 (1972) 193**

⤳ Essentially the same formula used in lattice QCD calculation of a_μ^{HLO}

★ $\Delta\alpha_{\text{had}}(t)$ (and a_μ^{HLO}) can be directly measured in a (single) experiment involving a space-like scattering process

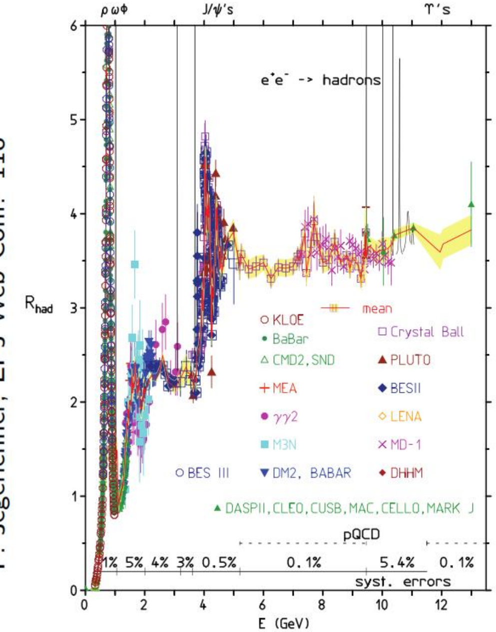
CC, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

Arbuzov *et al.* EPJC 34 (2004) 267

Abbiendi *et al.* (OPAL) EPJC 45 (2006) 1

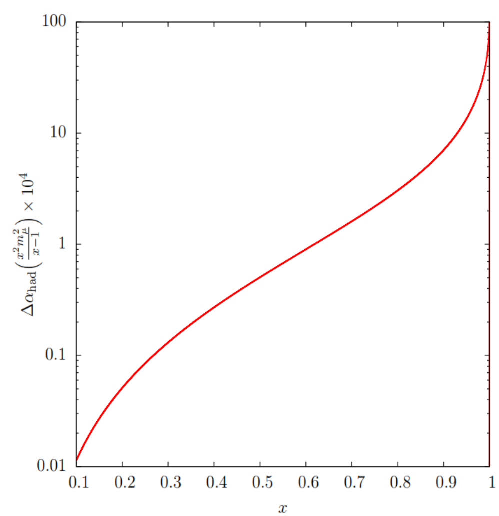
★ **Still a data-driven evaluation of a_μ^{HLO}, but with space-like data**

Time-like



F. Jegerlehner, EPJ Web Conf. 118

→ **Space-like**



Smooth function

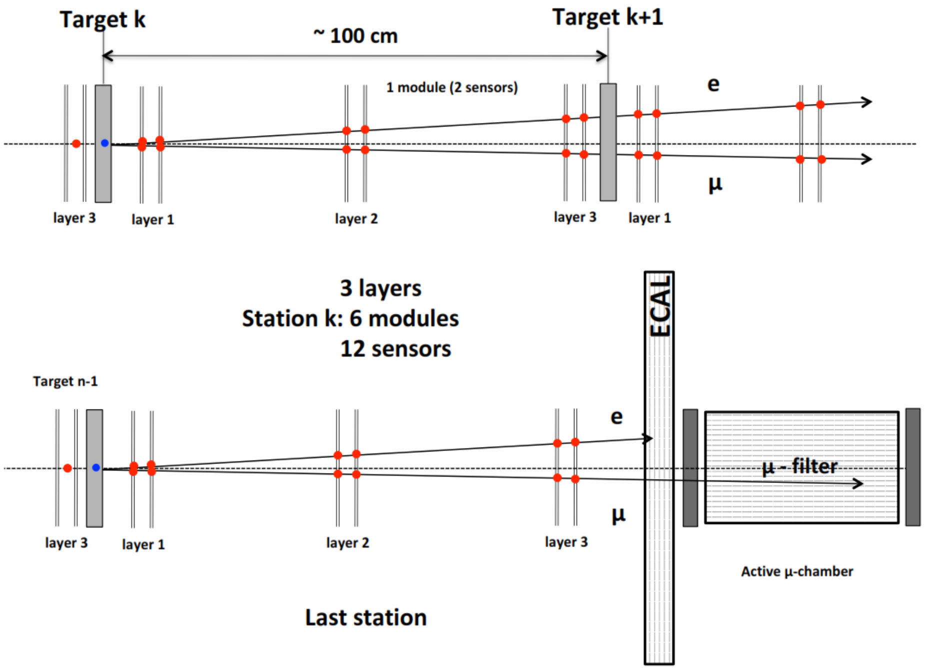
CC, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

From Carlo's talk, INFN Roma Tre, 2019

MUonE

Abbendi, Carloni Calame, Marconi, Matteuzzi, Montagna,
Nicrosini, Passera, Piccinini, Tenchini, Trentadue, Venanzoni
EPJC 2017 – arXiv:1609.08987

$\Delta\alpha_{had}(t)$ can be measured from elastic $\mu e \to \mu e$ scattering.



Abbendi, Carloni Calame, Marconi, Matteuzzi, Montagna, Nicrosini, Passera, Piccinini, Tenchini, Trentadue, Venanzoni
EPJC 2017 – arXiv:1609.08987

-> 150 GeV muon beam on a fixed electron target.
->Each module consists of a low-Z target (Berillium) and two silicon tracking stations located at a distance of one meter.

-> Systematic effects must be known at ≤ 10 ppm
-> Test run approved for 2022.
> Hopefully full run from 2023-25.

- Fully differential fixed-order MC@NLO (Pavia & PSI 2018-19)
- NNLO QED: MI for 2 loop box are computed.
  Amplitude for 4 fermion 2 loop process is computed. (Padova 2017-2021)
- Two MC is built including partial subsets of the NNLO QED corrections due to electron and muon radiation. (Pavia & PSI 2020)
- NNLO hadronic effect is computed. (Padova & KIT 2019)
- Extraction of the leading electron mass effects from the massless muon-electron scattering amplitudes (PSI 2019-present)
- New physics extracting $\Delta\alpha_{had}(t)$ at MUonE (Padova & Heidelberg)
- and so on…

The ratio of the SM cross section in the signal and the normalization region must be known at ≤ 10 ppm

A report of the MUonE theory initiative: arXiv:2004.13663

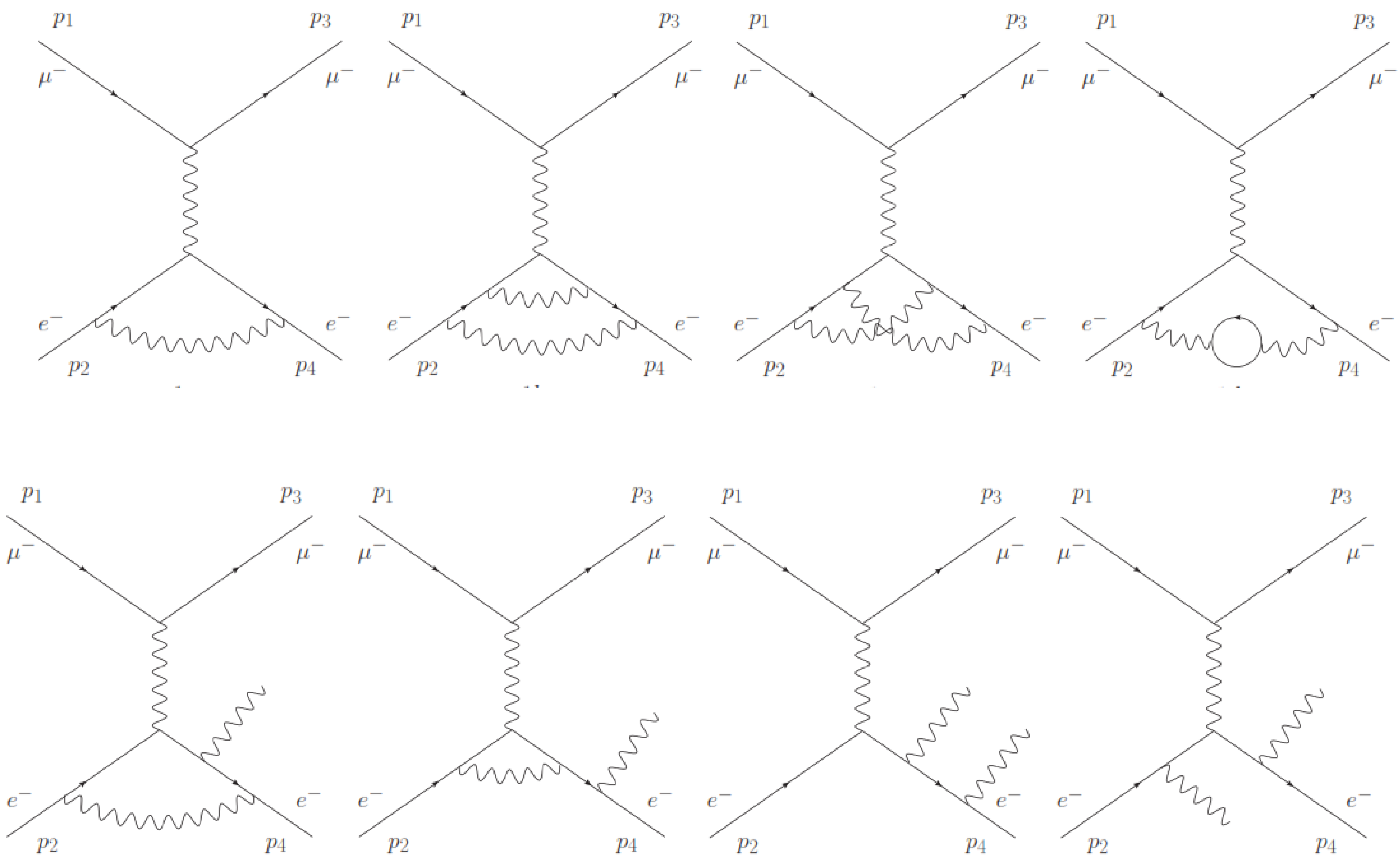
- It has been found that the muon-electron scattering in the context of MUonE experiment is dominated by QED effects.
- The tree level Z exchange has to be accounted since the leading order effect is non-negligible.
- The NLO weak corrections are negligible.
- The NLO and NNLO QED corrections are needed to be accounted.

[Alacevic, Calame, Chiesa, Montagna, Nicrosini, Piccinini, ’18] (arXiv:2004.13663)

NNLO Photonic Corrections

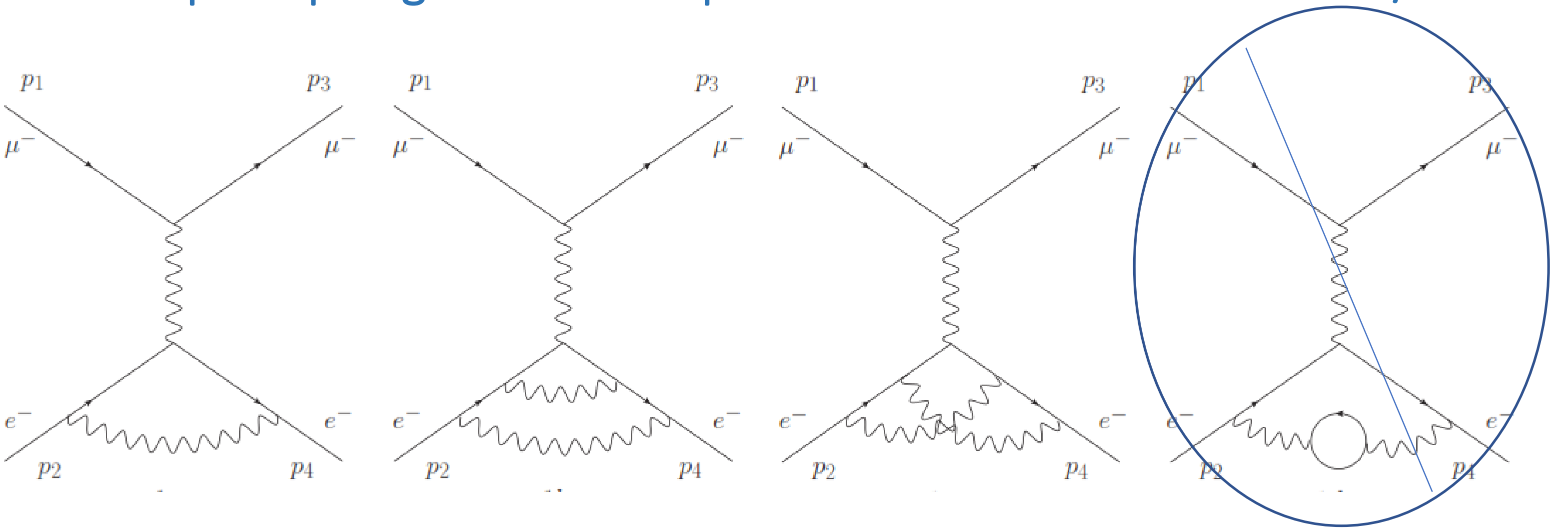
Published in --
JHEP 11 (2020) 028
arXiv:2007.01586

Sample topologies for NNLO QED corrections on electron/muon line



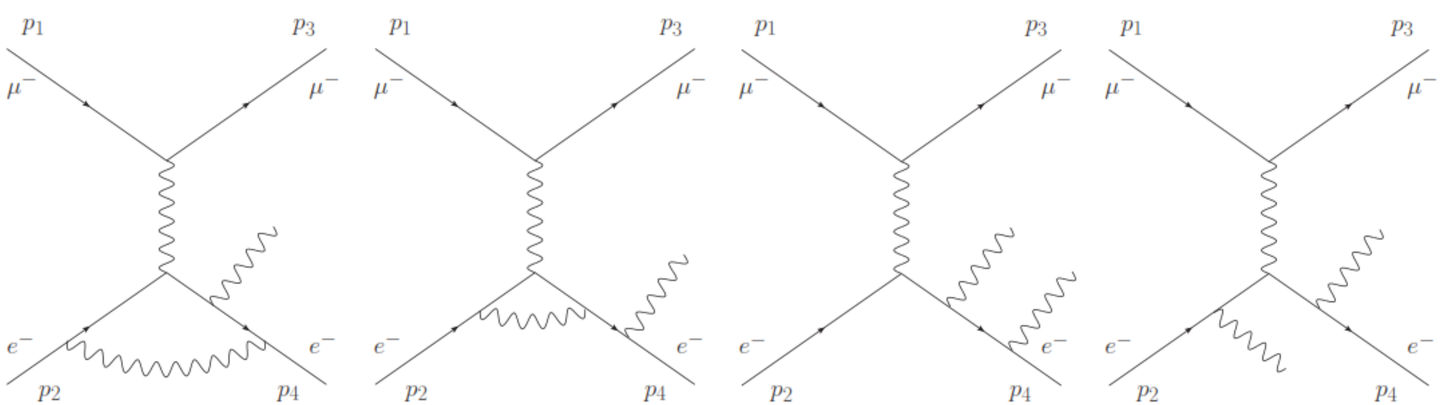
[Carlo Calame et. Al. ‘20]

Sample topologies for NNLO photonic corrections on electron/muon line



Nf=1 subset is removed

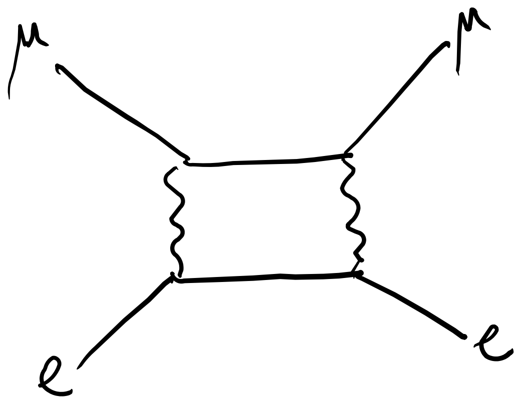
Exact calculation



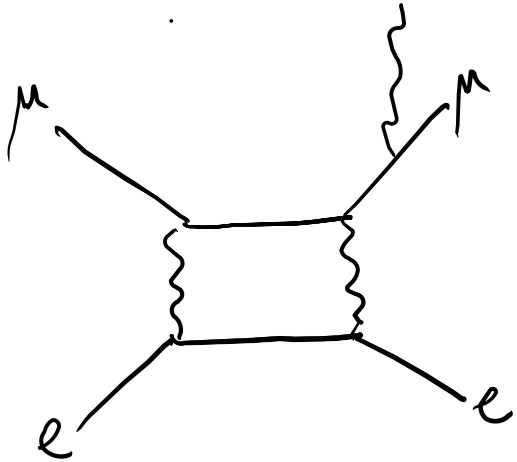
Two loop Formfactors are taken from Mastrolia et. al. arXiv:hep-ph/0302162

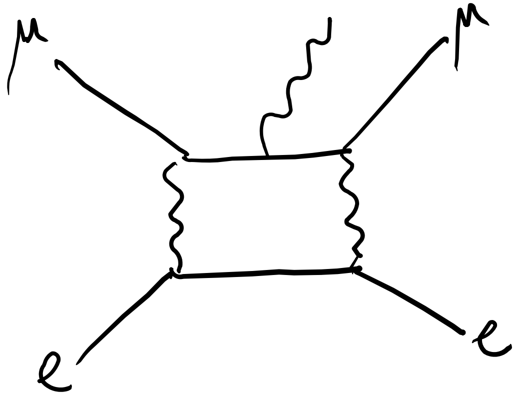
[Carlo Calame et. Al. '20]

Sample topologies for one loop boxes

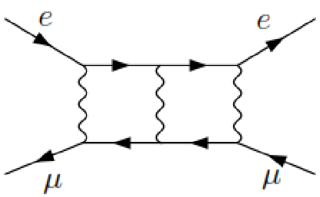


All relevant one loop boxes and pentagons are calculated exactly

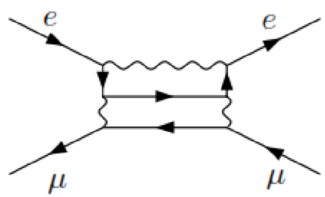




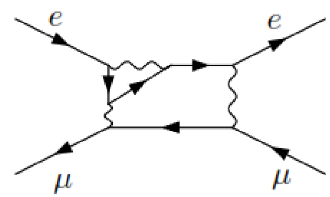
Sample topologies for NNLO photonic corrections to box like structure



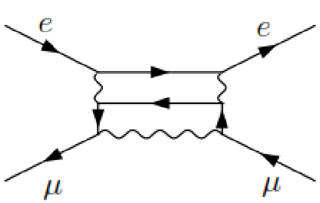
T_1



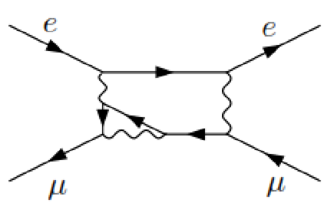
T_2



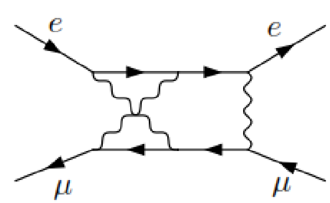
T_3



T_4



T_5



T_6

→ Not known exactly with full mass effect. Partial results are available.

[Heller, 2021. arXiv:2105.08046]

There is significant progress in calculation of MIs and 4-fermion amplitudes keeping the muon mass and neglecting electron mass.

[Mastrolia et. al.,2021 arXiv:2106.13179]

Massification

Engel, Gnendiger, Signer, Ulrich, arXiv:1811.06461
Becher, Melnikov, arXiv:0704.3582
Mitov, Moch, arXiv:hep-ph/0612149
A. Penin, arXiv:hep-ph/0501120

→ Yennie-Frautschi-Suura (YFS) approximation used including full mass dependence

NNLO photonic corrections

$$\mathcal{M}^{\alpha^0} = \mathcal{T}$$

$$\widetilde{\mathcal{M}}^{\alpha^2} = \mathcal{M}_e^{\alpha^2} + \mathcal{M}_\mu^{\alpha^2} + \mathcal{M}_{e\mu,\,1\mathrm{L}\times 1\mathrm{L}}^{\alpha^2} + \frac{1}{2} Y_{e\mu}^2 \mathcal{T} + Y_{e\mu}\left(Y_e + Y_\mu\right)\mathcal{T} + (Y_e + Y_\mu)\mathcal{M}_{e\mu}^{\alpha^1,\mathrm{R}} + Y_{e\mu} M^{\alpha^1,\mathrm{R}}.$$

Only non-IR remnant of the two loop boxes are approximated

$$Y = \sum_{i,j=1,4}^{j\geq i} Y_{ij} = Y_e + Y_\mu + Y_{e\mu}$$

$$Y_{ij} = \begin{cases} \frac{1}{8}\frac{\alpha}{\pi} Q_i^2 \left[B_0\left(0, m_i^2, m_i^2\right) - 4 m_i^2 C_0\left(m_i^2, 0, m_i^2, \lambda^2, m_i^2, m_i^2\right)\right] & \text{for } i = j \\ \frac{\alpha}{\pi} Q_i Q_j \vartheta_i \vartheta_j \Big[p_i \cdot p_j\ C_0\left(m_i^2, (\vartheta_i p_i + \vartheta_i p_j)^2, m_j^2, \lambda^2, m_i^2, m_j^2\right) + \\ \qquad + \frac{1}{4} B_0\left((\vartheta_i p_i + \vartheta_j p_j)^2, m_i^2, m_j^2\right)\Big] & \text{for } i \neq j \end{cases}$$

$$Y_e = Y_{24} + Y_{22} + Y_{44}$$

$$Y_\mu = Y_{13} + Y_{11} + Y_{33}$$

$$Y_{e\mu} = Y_{12} + Y_{14} + Y_{23} + Y_{34}$$

[Carlo Calame et. Al. ‘20]

NNLO photonic corrections

- Phenomenological results are obtained by using fully differential MC code, MESMER.
- Structure of the code is completely general. YFS can be replaced by exact calculation.
- We adopt the typical running condition of the MUonE experiment. Energy of the incoming muon beam is taken to be 150 GeV.
- The electron is assumed to be in rest inside a bulk target and thus $\sqrt{s} \simeq 0.405541$ GeV

[Carlo Calame et. al. '19]

1. $\vartheta_e, \vartheta_\mu < 100$ mrad and $E_e > 1$ GeV (i.e. $t_{ee} \lesssim -1.02 \cdot 10^{-3}$ GeV^2). The angular cuts model the typical acceptance conditions of the experiment and the electron energy threshold is imposed to guarantee the presence of two charged tracks in the detector (Setup 1);

2. the same criteria as above, with the additional acoplanarity cut $|\pi - |\phi_e - \phi_\mu|| \leq$ 3.5 mrad. We remind the reader that this event selection is considered in order to mimic an experimental cut which allows to stay close to the elasticity curve given by the tree-level relation between the electron and muon scattering angles (Setup 2)

where $t_{ee} = (p_2 - p_4)^2$, $(\vartheta_e, \phi_e, E_e)$ and $(\vartheta_\mu, \phi_\mu, E_\mu)$ are the scattering and azimuthal angles and the energy, in the laboratory frame, of the outgoing electron and muon, respectively.

[Carlo Calame et. Al. '20]

NNLO photonic corrections

$$\Delta^i_{\mathrm{NLO}} = 100 \times \frac{d\sigma^i_{\mathrm{NLO}} - d\sigma_{\mathrm{LO}}}{d\sigma_{\mathrm{LO}}}$$

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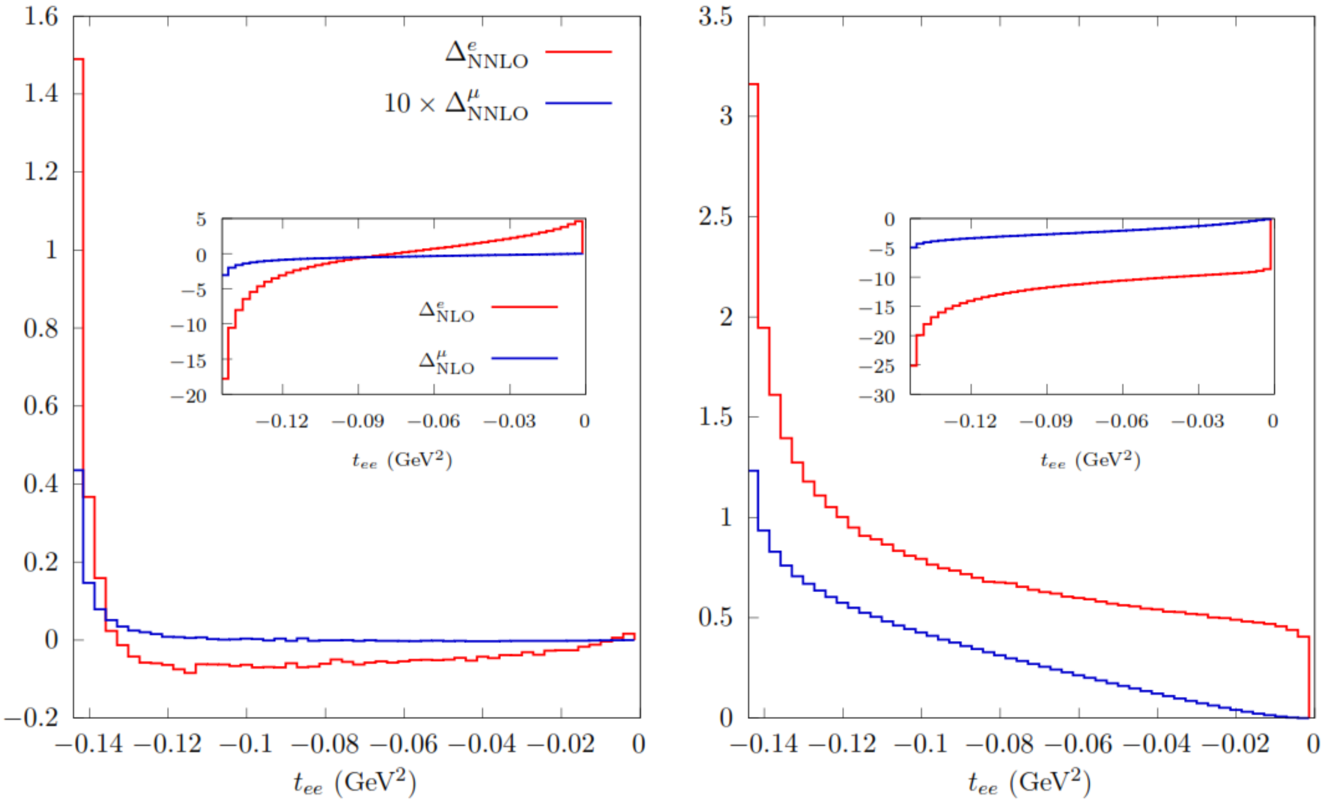
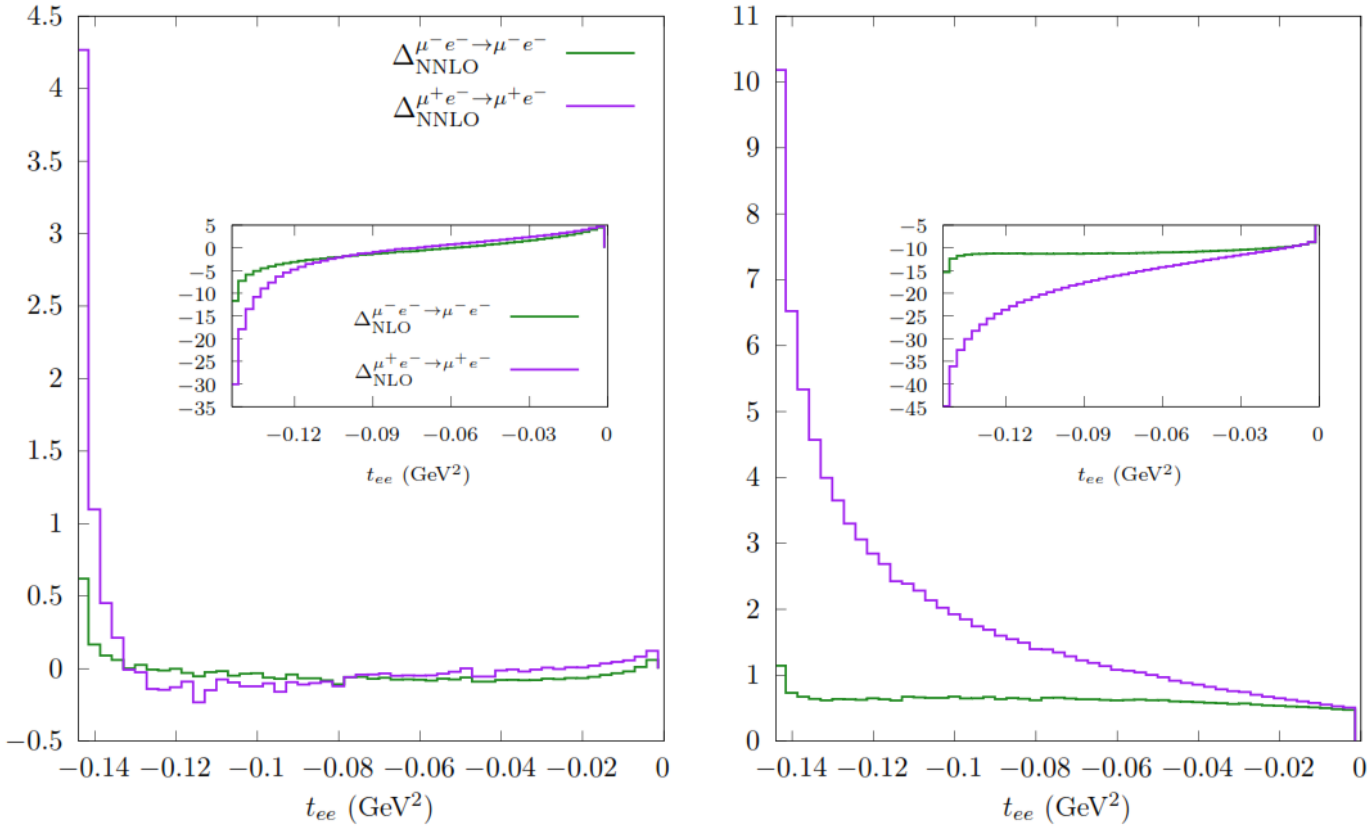


[Carlo Calame et. Al. '20]

NNLO photonic corrections



[Carlo Calame et. Al. '20]

NNLO Leptonic Corrections

(Work in progress)

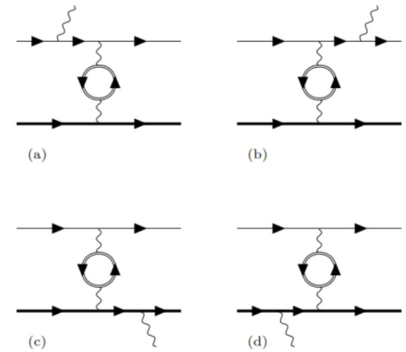
NNLO Leptonic Corrections

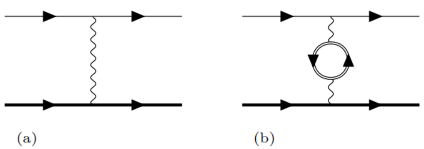
$$d\sigma^{\alpha^2}_{N_f} = d\sigma^{\alpha^2}_{\rm virt} + d\sigma^{\alpha^2}_{\gamma} + d\sigma^{\alpha^2}_{\rm real}$$

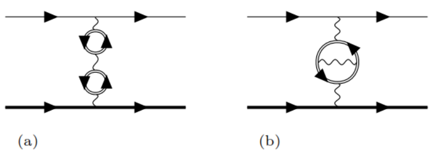
2 loop virtual

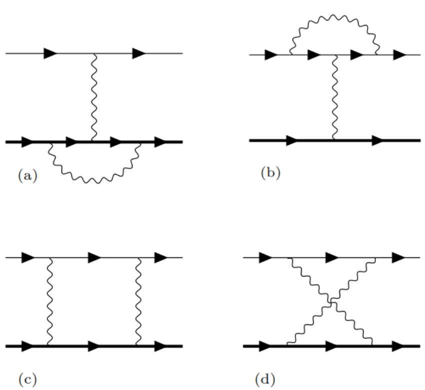
1 virtual 1 real

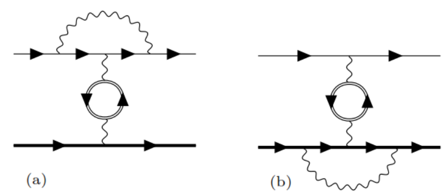
double real

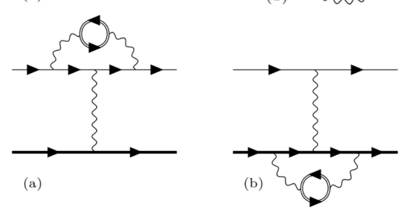


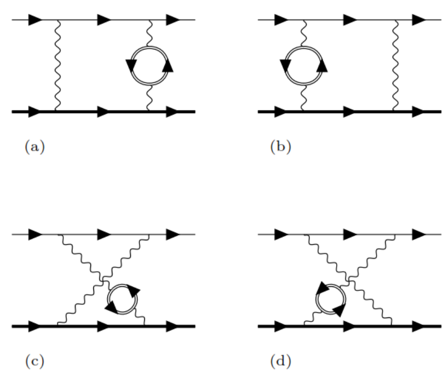


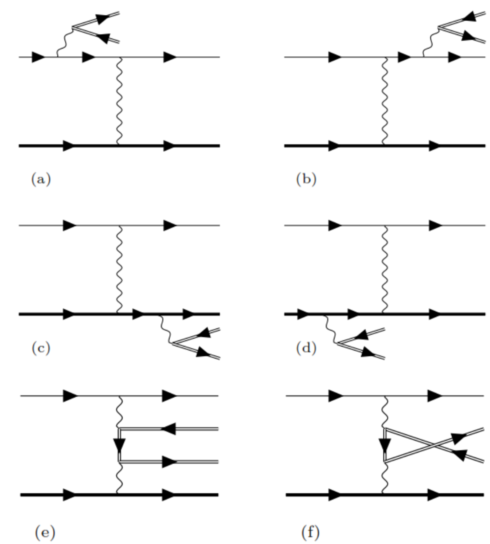












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NNLO virtual Leptonic Corrections

Dispersion Relation Technique

$$\frac{-ig_{\mu\nu}}{q^2+i\epsilon} \to \frac{-ig_{\mu\delta}}{q^2+i\epsilon} i \left(q^2 g^{\delta\lambda} - q^\delta q^\lambda\right) \Pi(q^2) \frac{-ig_{\lambda\nu}}{q^2+i\epsilon}$$

Replace photon propagator to capture bubble insertion

$$\Pi(q^2) = -\frac{q^2}{\pi} \int_{4m_l^2}^{\infty} \frac{dz}{z} \frac{\mathrm{Im}\Pi(z)}{q^2 - z + i\epsilon}$$

Modified Dispersion relation

$$\mathrm{Im}\Pi(z) = -\frac{\alpha}{3} R(z) \,,$$

$$R(z) = \left(1 + \frac{4m_l^2}{2z}\right) \sqrt{1 - \frac{4m_l^2}{z}}$$

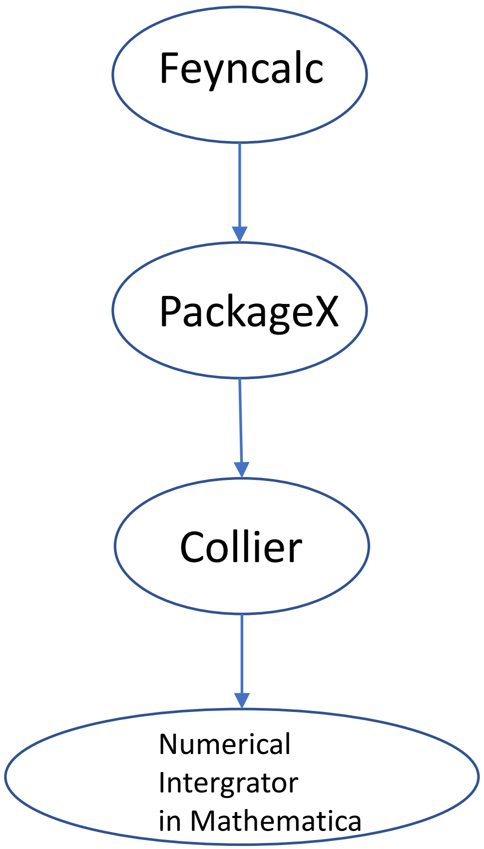
Two loop bubble inserted diagrams can be calculated using one loop technologies

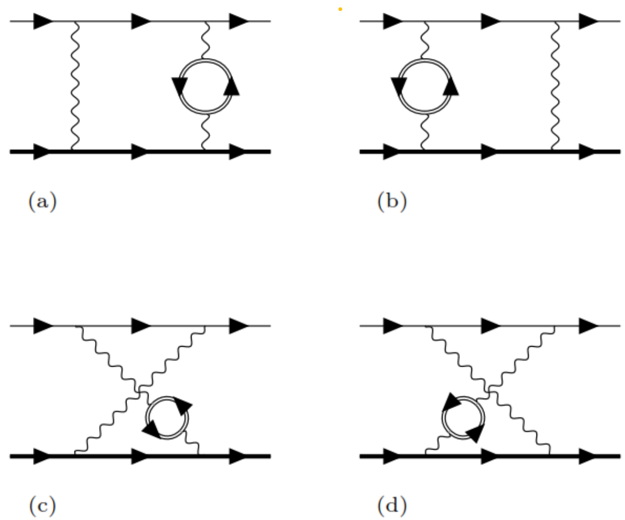
$$\frac{-ig_{\mu\nu}}{q^2+i\epsilon} \to -ig_{\mu\nu} \left(\frac{\alpha}{3\pi}\right) \int_{4m_l^2}^{\infty} \frac{dz}{z} \frac{1}{q^2 - z + i\epsilon} \left(1 + \frac{4m_l^2}{2z}\right) \sqrt{1 - \frac{4m_l^2}{z}}$$

[Actis, Czakon, Gluza, Reimann. 2008]
[Actis, Gluza, Reimann. 2008]
[Kuhn, Uccirati. 2009]

NNLO Leptonic Corrections

Bubble inserted double virtual box diagrams are checked using two independent calculation based on dispersion relation technique.





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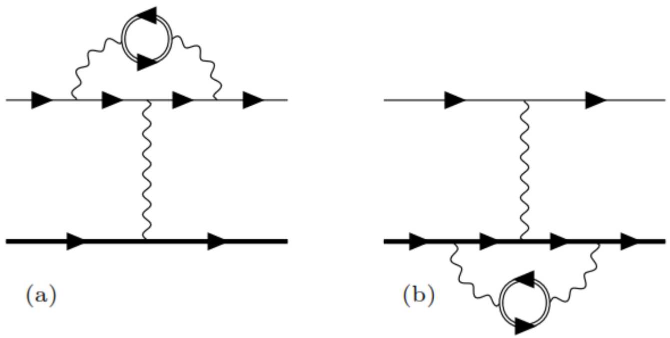
Fulvio Picinini. (Work in progress)

NNLO Leptonic Corrections

Bubble inserted vertex diagrams are checked against analytic result and dispersion relation technique.



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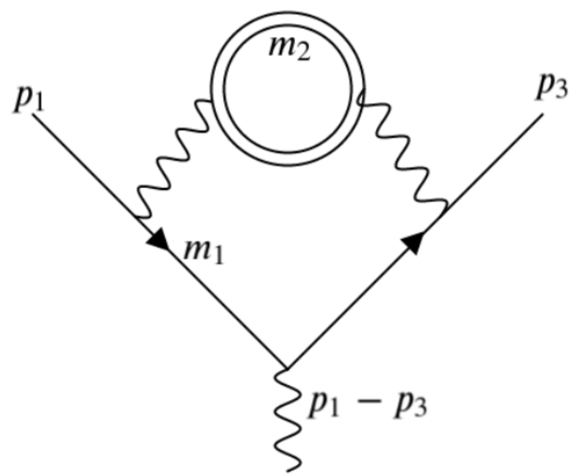
Outlook

- MUonE is on track. It aims to provide an independent determination of a_μ^{HLO}.
- From theory perspective high precision is needed for the simulation of muon-electron scattering.
- NNLO Photonic corrections are presented
- NNLO Leptonic corrections are in progress
- New multi scale Master integrals are calculated

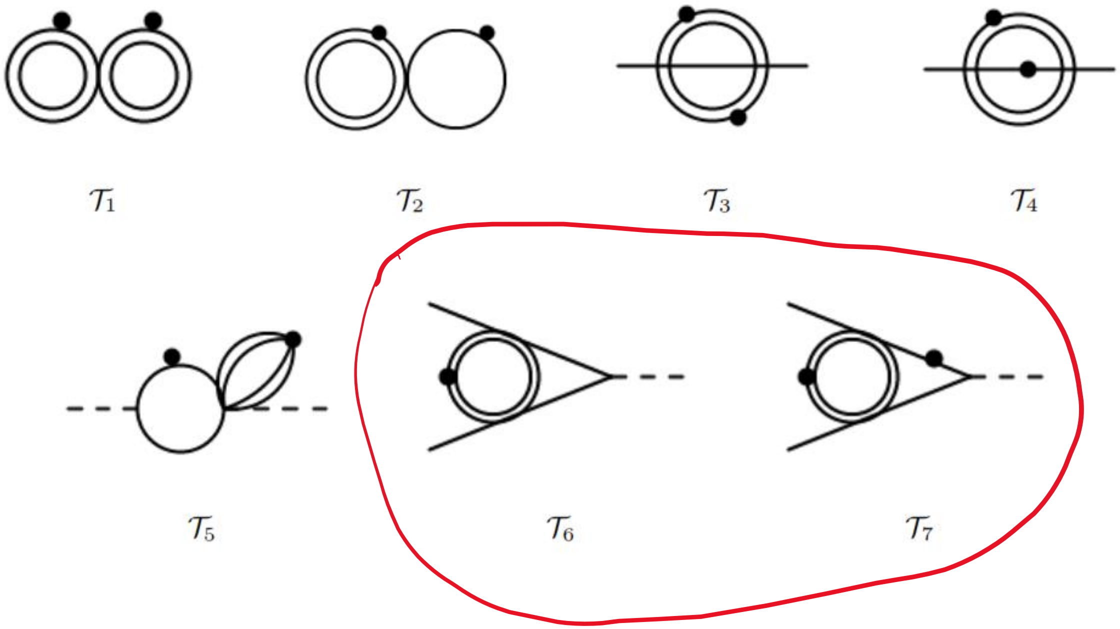
- Thank you
  for the attention

- Extra Slides

Vertex with two different mass:



MIs:



Boundary Constants are fixed using PSLQ

Numerically Checked against SecDec 3
(Borowka et. al arXiv:1502.06595)

NNLO photonic corrections

$\alpha = 1/137.03599907430637 \qquad m_e = 0.510998928$ MeV $\qquad m_\mu = 105.6583715$ MeV

σ (μb)	Setup 1		Setup 2	
	$\mu^- e^- \to \mu^- e^-$	$\mu^+ e^- \to \mu^+ e^-$	$\mu^- e^- \to \mu^- e^-$	$\mu^+ e^- \to \mu^+ e^-$
σ_{LO}	245.038910(1)			
σ^e_{NLO}	255.5500(7)		223.4387(6)	
$\sigma^\mu_{\mathrm{NLO}}$	244.9707(1)		244.4136(1)	
σ^f_{NLO}	255.1176(5)	255.8437(5)	222.8545(3)	222.7714(3)
σ^e_{NNLO}	255.5725(5)		224.4796(4)	
$\sigma^\mu_{\mathrm{NNLO}}$	244.9706(1)		244.4154(1)	
σ^f_{NNLO}	*255.205(1)*	*256.092(1)*	*224.041(1)*	*224.088(1)*

Cross sections (in μb) and relative corrections for the processes $\mu^- e^- \to \mu^- e^-$ and $\mu^+ e^- \to \mu^+ e^-$, in the two different setups described in the text. The symbols $\sigma^{e/\mu/f}_{\mathrm{(N)(N)LO}}$ stand for the cross sections with corrections along the electron line only, along the muon line only and the full approximate contributions, respectively, with the perturbative accuracy given by the subscripts. The digits in parenthesis correspond to 1σ MC error. Italicized numbers in the last row indicate that in this cross-section the full two-loop amplitude is approximated

[Carlo Calame et. Al. ‘20]