# Exploring Measuring the Momentum Compaction Factor from Turn-by-Turn Data

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**Acknowledgements:**

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ABP Meeting

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# Introduction Mom. Comp. Factor

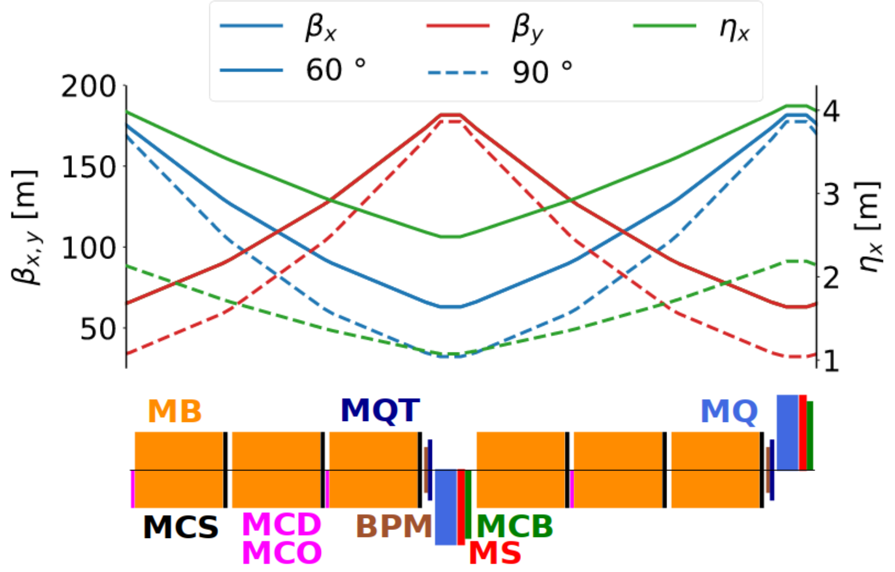
- Describes path length change for off-momentum particles
- Link between transverse and longitudinal optics
- Transverse optics: Typically momentum compaction factor used ($\alpha_C$)
- Longitudinal optics: Typically phase slip factor used ($\eta_C$)

*Relativistic gamma factor*

$$\eta_C = \gamma_{\mathrm{rel}}^{-2} - \alpha_C$$

$$\alpha_C = \frac{\Delta C / C}{\delta_p} = \frac{I_1}{C} = \frac{1}{C} \oint \frac{\eta_x}{\rho} \, \mathrm{d}s$$

***Generated by dispersion in bending structures***
*Dispersion depends on quadrupole strengths*
*For example: 60° vs 90° phase advance in LHC FODO cell*
*60°: About 2 times higher dispersion leads to about 2 times larger momentum compaction (~7 x 10$^{-4}$)*




*Phys. Rev. Accel. Beams, 23, pp. 101602, 2020*

# Nonlinear Mom. Comp. Factor

- Momentum compaction and phase slip factors depend on momentum offset

$$\alpha_C = \sum_{i \geq 1} \alpha_C^{(i)} \delta^{i-1} \quad \text{and} \quad \eta_C = \sum_{i \geq 1} \eta_C^{(i)} \delta^{i-1}$$

***Second order momentum compaction***

*Depends on $\eta'_x = d\eta_x / ds$ and second-order dispersion*

*Proc. 10th Int. Part. Accel. Conf. (IPAC'19), MOPMP027, 2019*

$$\alpha_C^{(2)} = \frac{1}{C} \oint \left( \frac{\eta_x'}{2} + \frac{\eta_x^{(2)}}{\rho} \right) \mathrm{d}s$$

*Particle Accelerators, 18, pp. 183-201, 1986*

- $\alpha_C$ ~3.2 x $10^{-4}$ to 3.5 x $10^{-4}$ in LHC
- $\alpha_C^{(2)}$ ~-3 x $10^{-4}$ in LHC

***Defined by tune and chromaticity***

$$\alpha_C^{(2)} = \frac{1 - 2Q_x' Q_x - Q_x^2}{Q_x^4}$$

*CERN-ACC-NOTE-2018-0062, 2018*

- Second-order momentum compaction negligible

- Higher order momentum compaction more important in smaller machines

# Errors Impacting Mom. Comp.

- Model:
- Linear momentum compaction depends on quadrupole strengths and dispersion
- → Measurement settings reproduced in model (orbit, tune)
- Aim to measure momentum compaction factor for seven different off-momentum optics measurements

# Measurement Settings

- 7 different off-momentum optics measurements of Run 2 used
- Already summarized in proceedings of IPAC’19

| Date | Fill | Type | E [TeV] | $\beta^*$ [m] | Tunes | | | | Xing [μrad] | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | $Q_x^n$ | $Q_y^n$ | $Q_x^d$ | $Q_y^d$ | IP1 | IP2 | IP5 | IP8 |
| 26th Mar. ’16 | 4729 | physics | 6.5 | 11.0 | 0.28 | 0.31 | 0.268 | 0.325 | 0 | 0 | 0 | 0 |
| 10th May ’16 | 4908 | VdM | 6.5 | 19.2 | 0.31 | 0.32 | 0.298 | 0.335 | -140 | 200 | 140 | -170 |
| 16th Jun. ’16 | 5023 | high $\beta^*$ | 6.5 | 2500 | 0.28 | 0.31 | 0.271 | 0.325 | 0 | 200 | 0 | -200 |
| 28th Jul. ’16 | 5124 | ATS | 6.5 | 0.40 | 0.28 | 0.31 | 0.265 | 0.322 | 0 | 0 | 0 | 0 |
| 3rd Oct. ’16 | 5356 | ATS | 6.5 | 0.21 | 0.28 | 0.31 | 0.265 | 0.322 | 0 | 200 | 0 | -250 |
| 20th Oct. ’16 | 5434 | ions | 6.5 | 0.60 | 0.31 | 0.32 | 0.295 | 0.332 | 0 | 138 | 0 | -180 |
| 3rd Nov. ’18 | 7396 | ions | 6.5 | 0.50 | 0.31 | 0.32 | 0.298 | 0.335 | 160 | 137[a] | 160 | -170 |

[a] The measurements of Beam 1 are performed after a crossing angle switch to −137 μrad.

*Proc. 10th Int. Part. Accel. Conf. (IPAC’19), MOPMP027, 2019*

# Errors Impacting Mom. Comp.

- Model:
- Linear momentum compaction depends on quadrupole strengths and dispersion
- → Measurement settings reproduced in model (orbit, tune)
- Aim to measure momentum compaction factor for seven different off-momentum optics measurements

- Quadrupole errors:
- Quadrupolar errors at regions with large dispersion lead to a shift

$$\Delta\alpha_C = -K_1 \frac{\eta_x^2 - \eta_y^2}{C_0}$$

- Known errors: 60 WISE tables
- Include quadrupole errors

*Proc. 10th Europ. Part. Accel. Conf. (EPAC'06), TUPP091*

*Proc. 11th Europ. Part. Accel. Conf. (EPAC'08)*

# Quadrupole Errors

- 60 different model configurations from 60 WISE tables
- Quadrupole and skew quadrupole errors applied to main dipoles (K1 and K1S)
- Corrections applied with trim quadrupoles and skew quadrupoles

| Date | Type | $\beta^*$ [m] | Model | |
|---|---|---|---|---|
| | | | $\alpha_C^{\mathrm{mdl}}$ | $\alpha_C^{\mathrm{mdl}+\Delta K_1}$ |
| 26/03/16 | physics | 11 | 3.22 | $3.22 \pm 0.001$ |
| 05/10/16 | VdM | 19.2 | 3.20 | $3.21 \pm 0.008$ |
| 06/16/16 | high $\beta^*$ | 2500 | 3.18 | $3.20 \pm 0.001$ |
| 28/07/16 | ATS | 0.4 | 3.49 | $3.48 \pm 0.003$ |
| 03/10/16 | ATS | 0.21 | 3.49 | $3.50 \pm 0.004$ |
| 16/10/16 | ions | 0.6 | 3.21 | $3.23 \pm 0.002$ |
| 03/11/18 | ions | 0.5 | 3.49 | $3.49 \pm 0.003$ |

- Other field-errors not applied
- No misalignment errors

*Error bar is the standard deviation of 60 seeds after applying errors and using exisiting correction routines*

Take away:
✔ Momentum compaction shift below 1 %

# Mom. Comp. Factor Measurements

- Fit of relative energy (momentum) offset over frequency
- Problem: no device in LHC to measure energy → Use TbT measurements

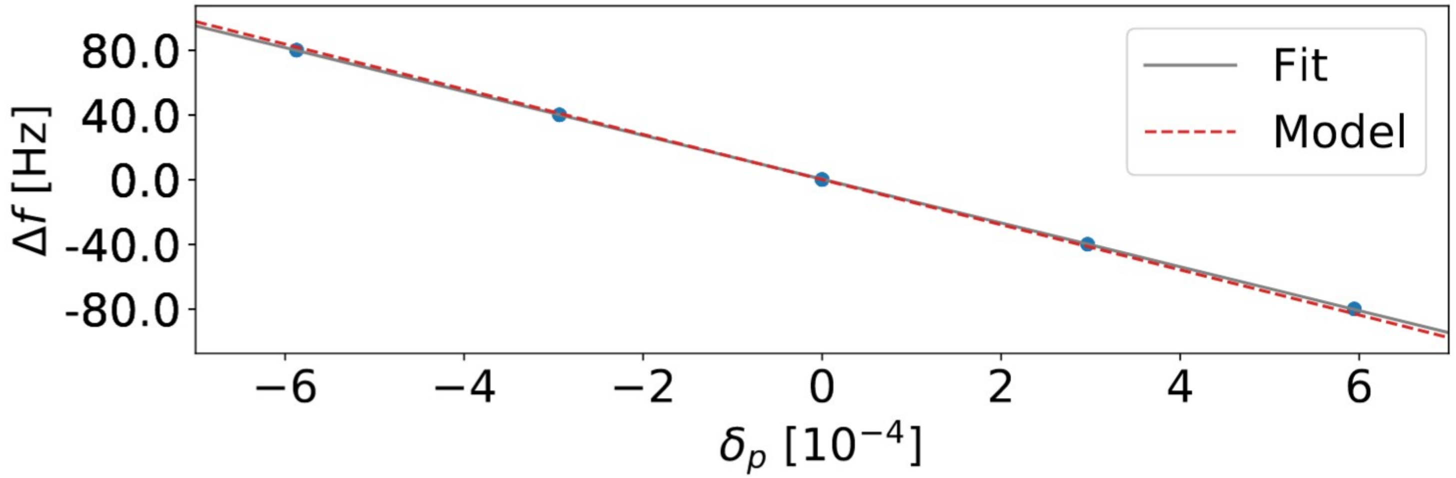
$$\delta_p = \frac{\langle \eta_x^{\mathrm{mdl}} CO_x \rangle}{\langle (\eta_x^{\mathrm{mdl}})^2 \rangle}$$

***Measured closed orbit and model dispersion at arc BPMs***

- Fit using

$$\delta_p = -\left( \frac{1}{\gamma_{\mathrm{rel}}^{-2} + \alpha_C} \right) \frac{\Delta f}{f}$$

*E = 6.5 TeV and therefore the relativistic gamma is negligible*

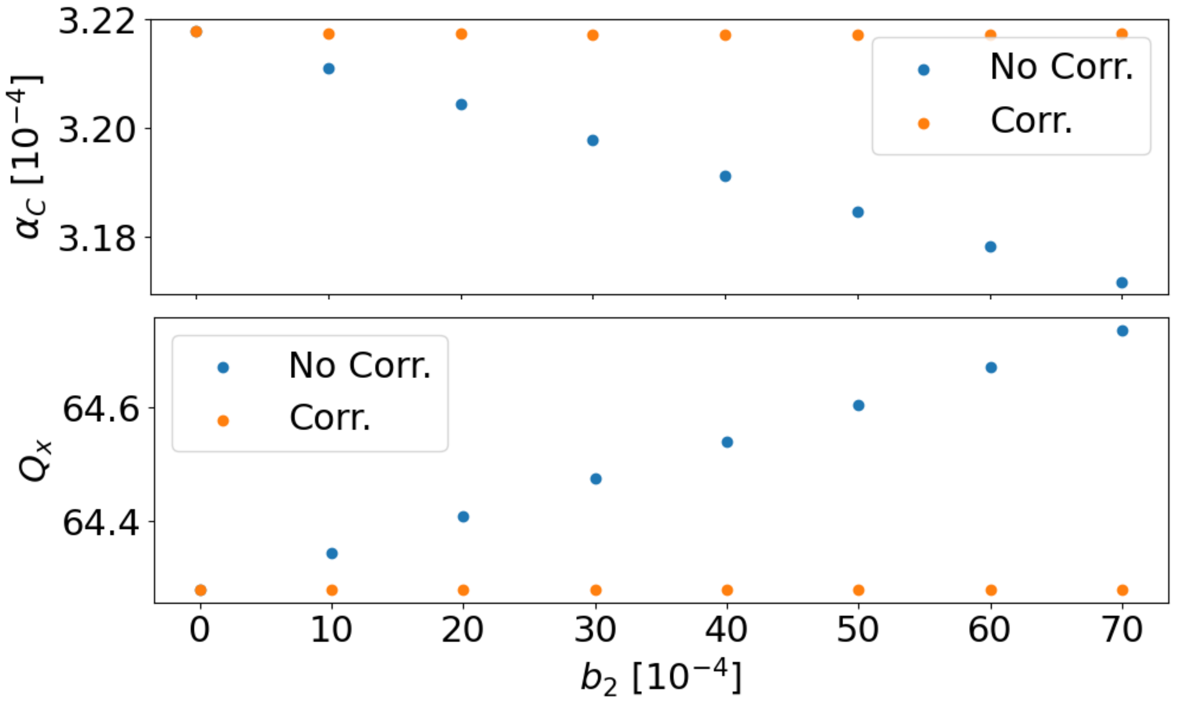



*Relative error between measurement and model about **-3 %***

# Systematic Quadrupole Errors in Arcs

- Horizontal dispersion in arc always positive
- Vertical dispersion in arc oscillating around 0
- Negative $\Delta\alpha_C$ measured → would result from positive $K_1$

$$\Delta\alpha_C = -K_1 \frac{\eta_x^2 - \eta_y^2}{C_0}$$




- Including quadrupole errors ($b_2$) in arc quadrupoles
  - Decreases $\alpha_C$
  - Increases horizontal tune
- Tune matching with trim quadrupoles shifts $\alpha_C$ back to model

***Quadrupole errors in arc quadrupoles cannot explain measured error of - 3%***

# Beam Position Monitor Errors

- Measured closed orbit used for momentum offset calculation

$$\delta_p = \frac{\langle \eta_x^{\mathrm{mdl}} CO_x \rangle}{\langle (\eta_x^{\mathrm{mdl}})^2 \rangle}$$

*Measured closed orbit not necessary real orbit*

$$CO_x^{\mathrm{meas}} = C \times CO_x^{\mathrm{real}}$$

*BPM calibration C can modify real orbit to measured one*

- What would calibration errors do?
- If average $C_i$ over all used BPMs is 1.01
- → $\delta_p^{\mathrm{meas}}$ would be 1.01 larger
- → Slope of $\delta_p$ over $\Delta f/f$ 0.99 smaller
- → Momentum compaction factor smaller

$$\delta_p^{\mathrm{meas}} = \frac{\langle \eta_{x,i}^{\mathrm{mdl}} \, C_i \, CO_{x,i} \rangle}{\langle (\eta_{x,i}^{\mathrm{mdl}})^2 \rangle}$$

*Phys. Rev. Accel. Beams, 23, pp. 042801, 2020*

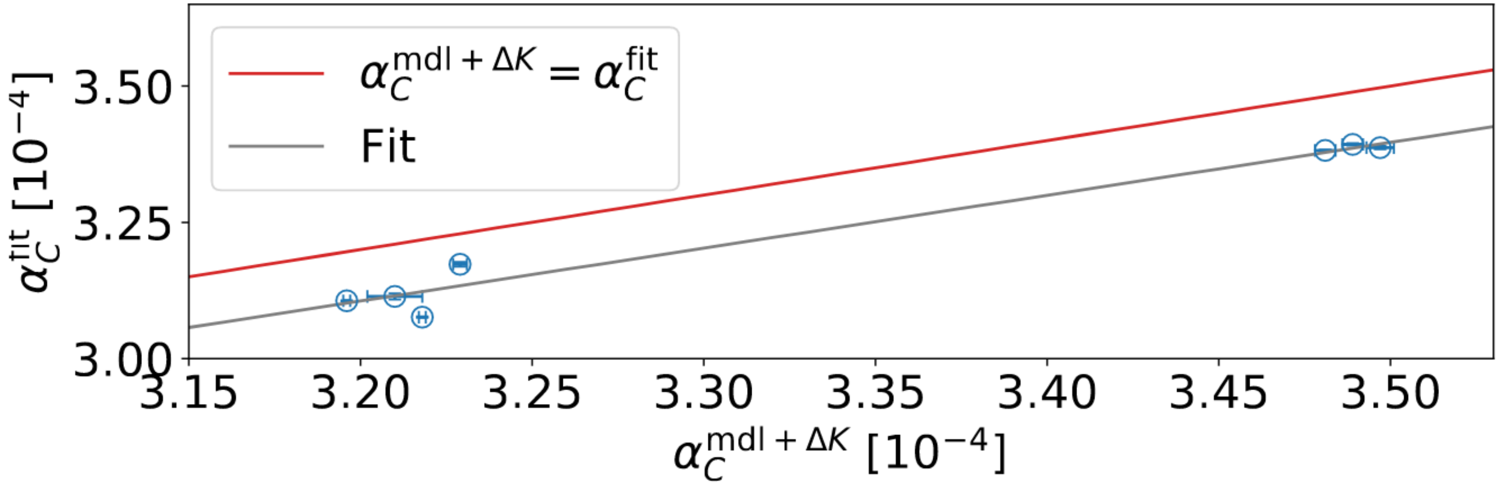
# Arc Calibration from Mom. Comp.

| Date | Type | $\beta^*$ [m] | Model $\alpha_C^{mdl}$ | Model $\alpha_C^{mdl+\Delta K_1}$ | Measurements $\alpha_C^{fit}$ | Measurements $\sigma^{fit}$ [%] |
|---|---|---|---|---|---|---|
| 26/03/16 | physics | 11 | 3.22 | 3.22 ± 0.001 | 3.08 ± 0.001 | -4.4 |
| 05/10/16 | VdM | 19.2 | 3.20 | 3.21 ± 0.008 | 3.11 ± 0.010 | -3.0 |
| 06/16/16 | high $\beta^*$ | 2500 | 3.18 | 3.20 ± 0.001 | 3.11 ± 0.001 | -2.8 |
| 28/07/16 | ATS | 0.4 | 3.49 | 3.48 ± 0.003 | 3.38 ± 0.001 | -2.8 |
| 03/10/16 | ATS | 0.21 | 3.49 | 3.50 ± 0.004 | 3.39 ± 0.003 | -3.1 |
| 16/10/16 | ions | 0.6 | 3.21 | 3.23 ± 0.002 | 3.17 ± 0.005 | -1.8 |
| 03/11/18 | ions | 0.5 | 3.49 | 3.49 ± 0.003 | 3.39 ± 0.002 | -2.8 |

- Systematic error of about 3 %
- Other field and misalignment errors and corrections are assumed to only minor impact moment compaction

***Error between expectation and measurement: (-2.95±0.003)%***

<u>Take away:</u>
✔ ~3 % error attributed tentatively attributed to arc BPM calibration



# Conclusion and Outlook

- Conclusion:
- Error between measured and model momentum compaction factor:

  **(-2.95±0.003) %**
- Only arc BPMs are used → **Error could arise from arc BPM calibration**

- Outlook and further studies:
- Systematic analysis of Run 2 off-momentum measurements
- Aim to reproduce measured momentum compaction factor in simulations
- Study effect of other field and misalignment errors
- Effects on IR BPM calibration to be evaluated
- Optics with significantly different momentum compaction could help clarifying

FUTURE CIRCULAR COLLIDER

TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

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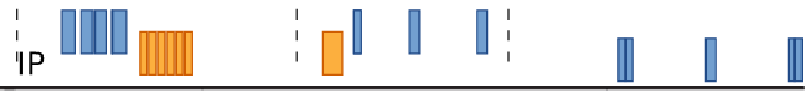
# BPM Calibration Errors

***Results taken from***

*Phys. Rev. Accel. Beams, 23, pp. 042801, 2020*

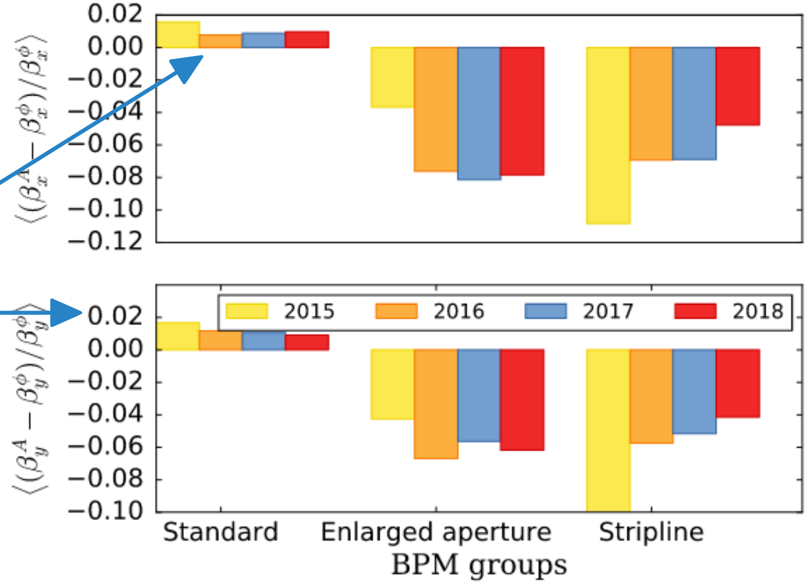
*A. Garcia-Tabares, PhD Thesis, CERN-THESIS-2019-211*

- Recent study investigated BPM calibration errors in LHC (and PSB)
- Different BPMs in LHC, in total 506 BPMs, 428 (84 %) standard BPMs




| Name | Stripline | Stripline | Enlarged aperture | Standard |
|---|---|---|---|---|
| Geometry | Stripline | Stripline | Button | Button |
| LHC name | BPMS | BPMSX | BPMW | BPM |
| Diameter | 61 mm | 81 mm | 61 mm | 49 mm |

- Motivation: systematic offset in interaction region between $\beta^A$ and $\beta^{ph}$




***Standard BPMs mostly in arc***

*Measure small difference between beta calculations*

# Calibration Factor

***Results taken from***

*Phys. Rev. Accel. Beams, 23, pp. 042801, 2020*

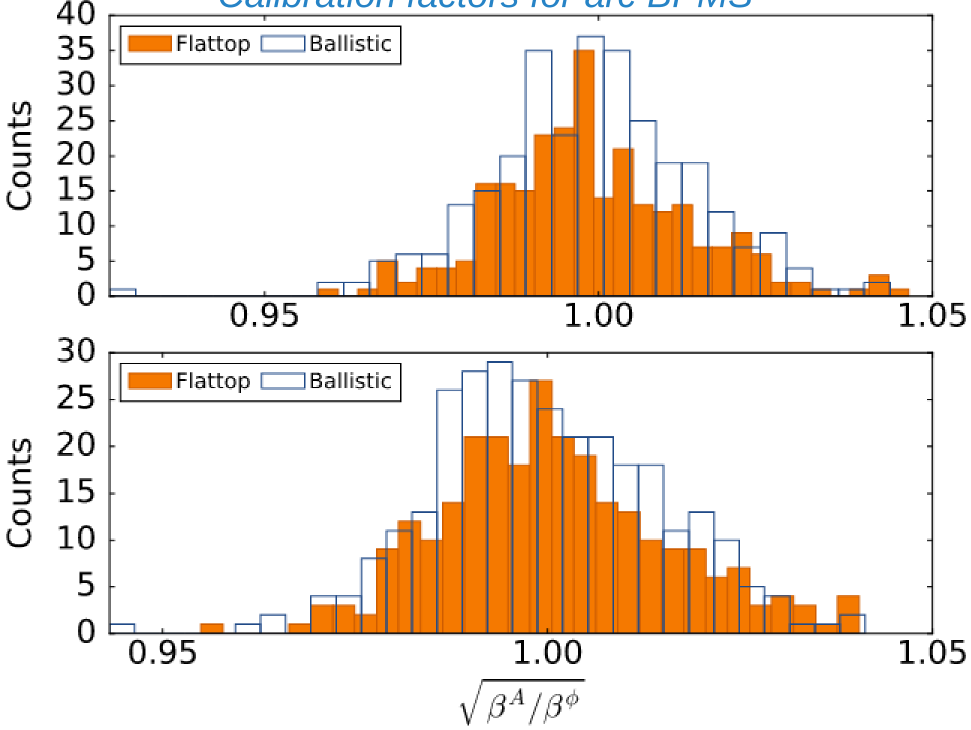
*A. Garcia-Tabares, PhD Thesis, CERN-THESIS-2019-211*

- Calibration factor from β-functions

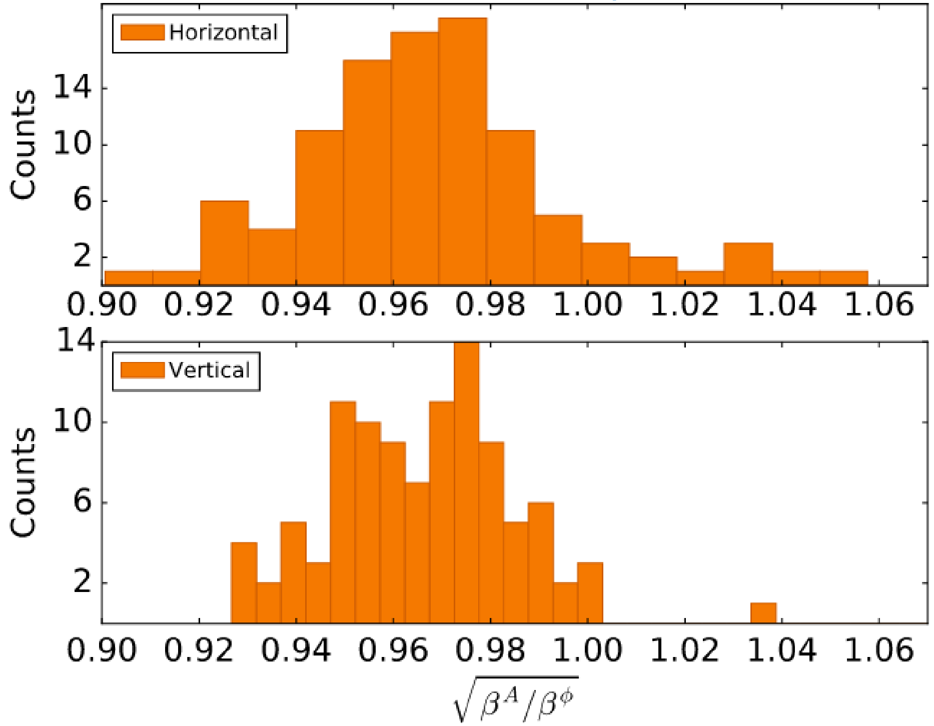
$$C_{\beta,i} = \sqrt{\frac{\beta_i^{\mathrm{A}}}{\beta_i^{\phi}}} = \frac{C_i}{\sqrt{\overline{(C_{i,\mathrm{ARCS}})^2}}}$$

***Calibration factors normalized by average arc calibration***

*Calibration factors for arc BPMS*




*Calibration factors for stripline BPMS*




***$C_{i,ARCS}$ centered by reference or defintion around 1.0***

***$\langle C_i \rangle$ about 3 % attributed to IR BPMs***

# Approximation of Mom. Comp.

- Use avarage measured dispersion

$$\alpha_C \approx \alpha_C^{\text{ap.}} = \frac{2\pi\langle\eta_x\rangle}{C}$$

- Model: About 2% larger than expected
- Measurements: agrees almost perfectly with model approximation ($\sigma^{\text{ap.}}$)

Take away:
✔ Approximation not suitable for estimation of calibration

| Date | Type | $\beta^*$ [m] | Model | | | Measurements | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | $\alpha_C^{\text{mdl}}$ | $\alpha_C^{\text{mdl}+\Delta K_1}$ | $\alpha_C^{\text{mdl}+\Delta K,\text{ap.}}$ | $\alpha_C^{\text{fit}}$ | $\sigma^{\text{fit}}$ [%] | $\alpha_C^{\text{ap.}}$ | $\sigma^{\text{ap.}}$ [%] |
| 26/03/16 | physics | 11 | 3.22 | 3.22 ± 0.001 | 3.28 ± 0.008 | 3.08 ± 0.001 | -4.4 | 3.28 | 0.01 |
| 05/10/16 | VdM | 19.2 | 3.20 | 3.21 ± 0.008 | 3.26 ± 0.002 | 3.11 ± 0.010 | -3.0 | 3.28 | 0.6 |
| 06/16/16 | high $\beta^*$ | 2500 | 3.18 | 3.20 ± 0.001 | 3.26 ± 0.001 | 3.11 ± 0.001 | -2.8 | 3.26 | 0.1 |
| 28/07/16 | ATS | 0.4 | 3.49 | 3.48 ± 0.003 | 3.51 ± 0.002 | 3.38 ± 0.001 | -2.8 | 3.51 | -0.3 |
| 03/10/16 | ATS | 0.21 | 3.49 | 3.50 ± 0.004 | 3.56 ± 0.003 | 3.39 ± 0.003 | -3.1 | 3.57 | 0.1 |
| 16/10/16 | ions | 0.6 | 3.21 | 3.23 ± 0.002 | 3.29 ± 0.002 | 3.17 ± 0.005 | -1.8 | 3.32 | +1.4 |
| 03/11/18 | ions | 0.5 | 3.49 | 3.49 ± 0.003 | 3.54 ± 0.004 | 3.39 ± 0.002 | -2.8 | 3.53 | -0.4 |

*Error resulting from unexpected quadrupolar error sources somewhere in the lattice*