Microscopic Description of Brane Gauginos

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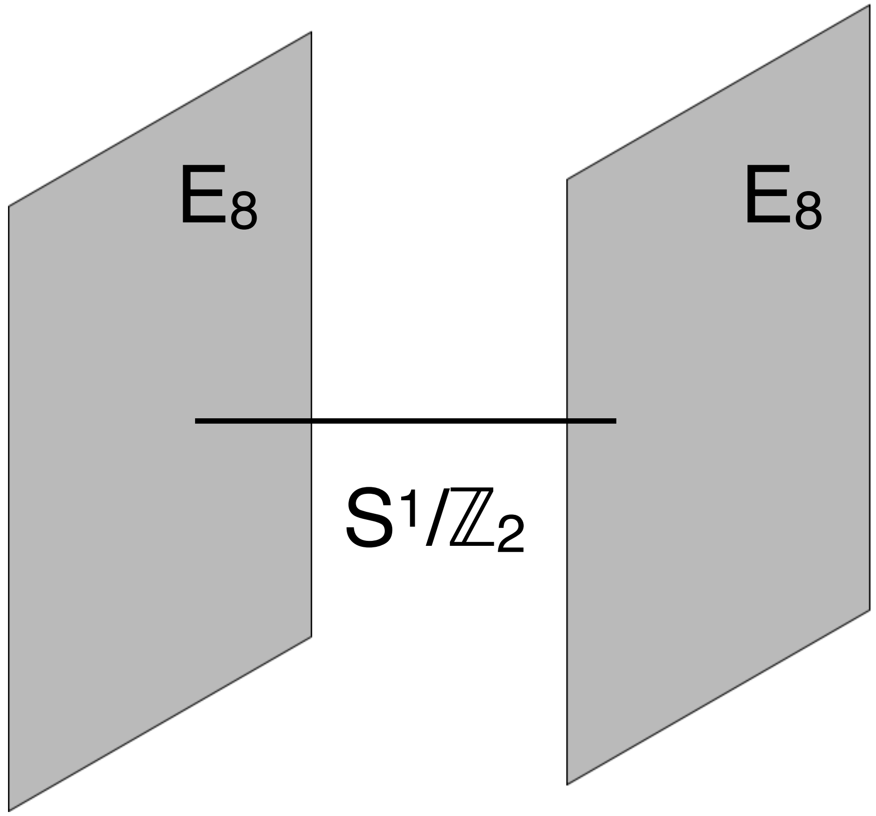
Based on joint work with Yuta Hamada, Arthur Hebecker, and Pablo Soler:

- "On brane gaugino condensates in 10d," JHEP 1904, 008 (2019) [arXiv:1812.06097 [hep-th]].
- "Understanding KKLT from a 10d perspective," JHEP 1906, 019 (2019) [arXiv:1902.01410 [hep-th]].
- "Completing the D7-brane local gaugino action," [arXiv:2105.11467 [hep-th]].

Motivation

- Non-perturbative effects play a decisive role in string phenomenology. Since the 80s, they have been used to stabilize moduli.
- This role continues in recent times, e.g. gaugino condensation on branes is a key element in many proposed dS constructions e.g., KKLT/LVS.
- Non-perturbative effects localized on branes introduce subtleties:



$$S_G \sim -\int_{M^{11}} d^{11}x\,\sqrt{-g}\,\left(G_{IJKL}G^{IJKL} - \delta(x^{11})G_{ABC11}j^{ABC}\right)$$

$$G_{ABC11} \sim \delta(x^{11})j_{ABC} \qquad \Rightarrow \qquad S_G \sim \delta(0)$$

Supersymmetry suggests how to regularize the action

$$S \sim -\int_{M^{11}} d^{11}x\,\sqrt{-g}\left(G_{ABC11} - \frac{1}{2}\delta(x^{11})j_{ABC}\right)^2$$ [Horava, Witten, '96]

Only if this UV divergence is properly regularized can we extract physically meaningful results.

Motivation

- Our work is a continuation of this quest. Codimension ≥ 2 branes introduce new subtleties, but a properly regularized, local action is finally obtained.
- FAQ: Gaugino condensation is an IR phenomenon, what is the point of this microscopic (10d) treatment?
- What this 10d treatment teaches us is how the brane gauginos interact with the bulk fields regardless of whether the gauginos condense or not.
- 4d EFT arguments alone do not tell us anything about small couplings, e.g.,

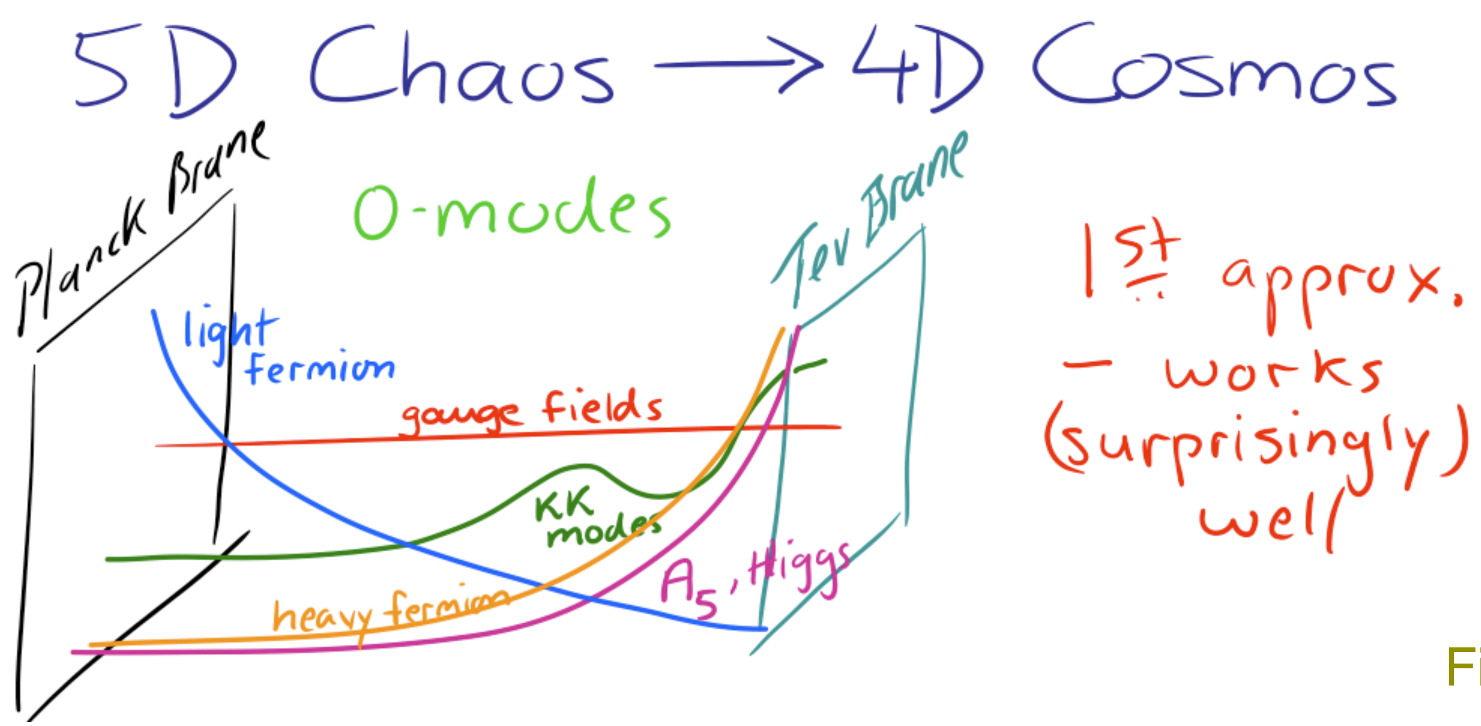
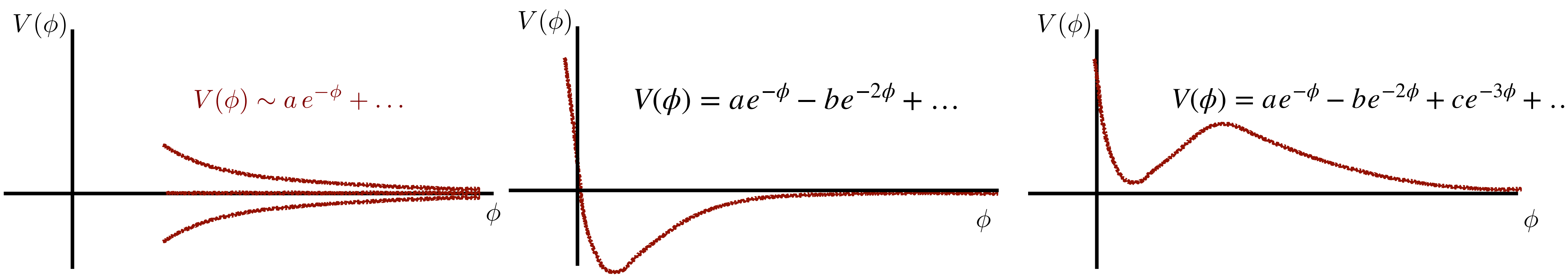


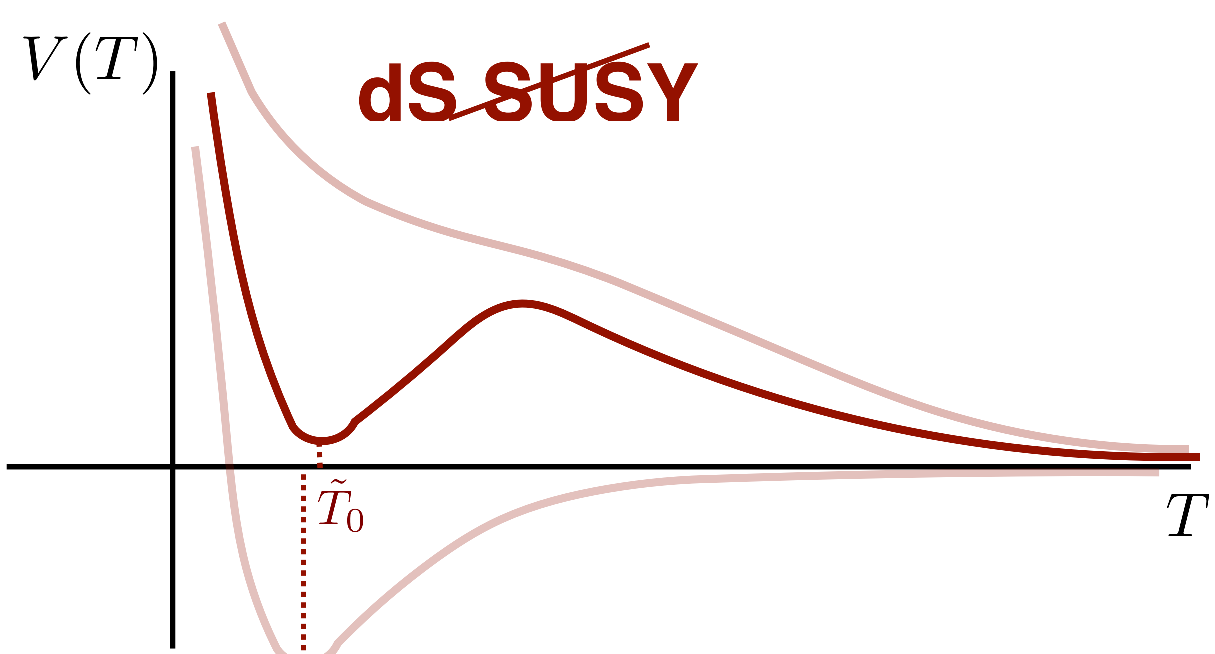
Figure from Sundrum

Motivation

- An accurate description of brane-bulk interactions is particularly crucial in string theory because of the Dine-Seiberg problem:



- Weak coupling vacua necessitate small numbers in the EFT, e.g., in KKLT:



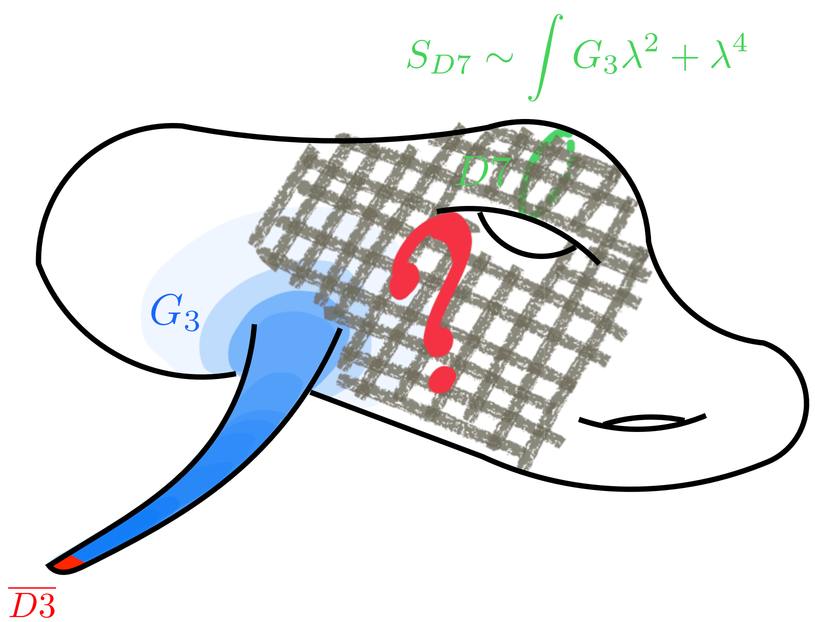
$$V(T) \sim \left(-\frac{2}{T^2} W_0\, e^{-T} + \frac{1}{T} e^{-2T} \right) + \frac{\Omega^4 \mu_3}{T^2}$$

$\Omega =$ warp factor: $\Omega^4 \sim W_0 \sim e^{-\tilde{T}_0} \ll 1$

A microscopic treatment is necessary for substantiating these constructions as well as quantifying corrections.

Motivation

- It has been argued that an $\mathcal{O}(1)$ fraction of the compactification is singular [Gao-Hebecker-Junghans, '20]. Resolution of such singularity would necessarily modify the compactification geometry [Carta-Moritz, '21].



[See Hebecker's talk on the "singular bulk" problem]

- A regularized, local brane action is needed to capture the interactions among the brane and bulk fields which are smeared in the 4d EFT.
- Our results are general for flux compactifcation with branes, but for the sake of presentation, KKLT will be used as a proxy.

Brane Gauginos in 10d

10d no-go theorems

- Consider Einstein equation in 10d and its trace over 4d indices:

$$\mathcal{R}_{MN} = T_{MN} - \frac{1}{8} g_{MN} T^L_{\ L} \implies \mathcal{R}^\mu_{\ \mu} = \frac{1}{2}\left(T^\mu_{\ \mu} - T^m_{\ m}\right) \equiv -\Delta$$

- For a warped ansatz: $ds^2_{10} = \Omega^2(y)\left(\eta_{\mu\nu}\, dx^\mu\, dx^\nu + g_{mn}\, dy^m\, dy^n\right)$

$$\mathcal{V}_6\, \mathcal{R}(\eta) = \int d^6y \sqrt{g}\ \Omega^8(y)\, \mathcal{R}(\eta) = -\int d^6y \sqrt{g}\ \Omega^{10}(y)\, \Delta$$

Useful for no-go theorems: most sources have Δ>0!

[Maldacena, Nuñez '00]

$\Delta \sim (T^m_{\ m} - T^\mu_{\ \mu})$

10d no-go theorems

- **KKLT:** uplift from gaugino condensate and anti D3-branes

$$\mathscr{V}_6\,\mathscr{R}(\eta) \approx -\int d^6y \sqrt{g}\ \Omega^{10}(y)\left(\Delta^{\langle\lambda\lambda\rangle} + \Delta^{\overline{D3}}\right)$$

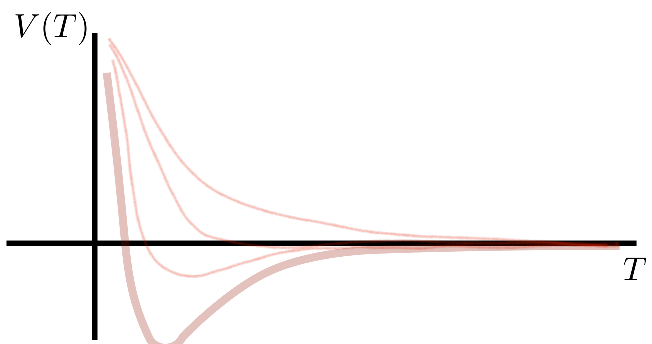
$\Delta^{\overline{D3}}$: easily computed from the $\overline{\text{D3}}$-brane world-volume action

$\Delta^{\langle\lambda\lambda\rangle}$: inferred from the D7-brane action with $\lambda\lambda \to \langle\lambda\lambda\rangle \sim e^{-T}$

Claim: both $\Delta^{\langle\lambda\lambda\rangle}$ and $\Delta^{\overline{D3}}$ are strictly positive!? "Flattening"??

[Moritz, Retolaza, Westphal '17, '18]; [Moritz, Van Riet, '18]; [Gautason, Van Hemelryck, Van Riet '18]

Problem: calculations of $\Delta^{\langle\lambda\lambda\rangle}$ were UV divergent and regularization dependent!!



[Hamada, Hebecker, GS, Soler, '18]

Brane Gaugino Action

- A goal of our work [Hamada, Hebecker, GS, Soler, '18, '19, '21] is to properly regularize the brane action such that physically meaningful finite quantities e.g. $\Delta^{\langle\lambda\lambda\rangle} \sim (T^m_{\ m} - T^\mu_{\ \mu})$ from the D7-action can be computed and compared with 4d SUGRA expectation:

$$\mathscr{L}_{4d} = e^K \left(|D_T W_0 + e^{-K/2} \lambda\lambda|^2 - 3|W_0|^2 \right)$$

[See e.g. Wess & Bagger]

The action was known up to order $\lambda\lambda$ in gauginos

Camara, Ibanez, Uranga '04
Dymarsky, Martucci, '10
Grana, Kovensky, Retolaza '20
...

$$S_{\lambda\lambda} \sim -\int |G_3|^2 - \overline{\lambda\lambda}\, \delta_{D7}\, (G_3 \cdot \Omega_3) + \text{c.c.} + \ldots$$

Problem: this action diverges if used to order $|\lambda\lambda|^2$

(Schematically) $\quad G_3 \sim \lambda\lambda\, \delta_{D7} + \ldots \quad \Longrightarrow \quad S_{\lambda\lambda} \sim \int |\lambda\lambda|^2 \delta_{D7}^2 \to \infty$

For clarity: $g_s = \Omega(y) = 1$

Brane Gaugino Action

- Unlike the codimension 1 case (c.f. Horava-Witten), the "perfect square structure" leaves us with a non-local tail [Hamada, Hebecker, GS, Soler, '18]:

$$d * G = 2\, d * j = 2\, e^{-\phi/2} \sum_i \lambda_i^2\, d\left(* \overline{\Omega}\, \delta_i^{(0)} \right) , \qquad\qquad d\, G = 0\,.$$

$$|G - 2j|^2 \sim |2P_c(j)|^2 \longrightarrow 1/z^2$$

approaching the brane locus

- Similar non-locality appears in [Kachru, Kim, McAllister, Zimet, '19] where the brane-localized quartic gaugino term depends on the brane transverse volume.

- These actions are at best approximations where effects that should be localized to D7-branes have been smeared over the internal space.

- For reasons discussed above, we need a microscopic description to accurately capture the interactions among localized sources and bulk fields.

$(*G_\pm = \pm i G_\pm)$

Brane Gaugino Action

- Regularizing the D7 action while respecting locality turns out to be very challenging and has been completed only recently

$$\mathscr{L}_{\lambda\lambda} = -\frac{1}{2}|G_3|^2 + \sum_{i\in D7} \overline{\lambda}\overline{\lambda}_i \delta_i \,(G_3\cdot\Omega_3) \,+\, \text{c.c.} \,-\, \frac{i}{2} G_3 \wedge \overline{G}_3$$

$$= -\left| G_+ - \sum_{i\in D7} \lambda\lambda_i \,\delta_i \,\overline{\Omega}_3 \right|^2 - |G_-^{(0)}|^2 + |G_+^{(0)}|^2 + \sum_{ij} \delta_i\delta_j \,\lambda\lambda_i \overline{\lambda\lambda}_j |\Omega|^2$$

  - Only the last term diverges (for i=j), but it also contains crucial finite contributions from (self) intersections of D7-stacks

$$\int \delta_i\,\delta_j\,|\Omega|^2 \rightarrow \int \delta_i\delta_j\, J\wedge J\wedge J = 3!\int \delta_i^{(2)}\wedge\delta_j^{(2)}\wedge J = 3!\mathscr{K}_{ij}$$

  - CY intersection numbers: $\mathscr{K}_{ij} = \int \omega_i\wedge\omega_j\wedge J\,,\qquad [\omega_i]\in H_+^{(1,1)}$

Hamada, Hebecker, GS, Soler ’18, ’19, ’21

A finite local brane gaugino action

- With this insight, we can write a local finite action:

$$\boxed{S = -\int_{10d} \left| G_+ - \sum_i \lambda\lambda_i \delta_i \overline{\Omega}_3 \right|^2 - \int_{10d} \left(|G_-^{(0)}|^2 - |G_+^{(0)}|^2 \right) + 3! \sum_{i,j} \lambda\lambda_i \overline{\lambda\lambda}_j \, \mathcal{K}_{ij}}$$

Hamada, Hebecker, GS, Soler, '18, '19, '21
c.f. Kallosh '19; Carta, Moritz, Westphal '19;
Gautason, Van Hemelryck, Van Riet, Venken '19;
Kachru, Kim, McAllister, Zimet '19; Retolaza, Rogers, Tatar, Tonioni '21

$\Delta \sim (T^m_{\ m} - T^\mu_{\ \mu})$

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With this regularized action, we have done further to:

1. Write S in a 10d covariant way: $\{\lambda, \Omega_3, J\} \to \Psi_8$
2. Check that S reduces to $S_{4d,\, sugra}$ upon CY compactification
3. Compute $\Delta^{\langle \lambda\lambda \rangle}$ and check that: $\mathscr{R}_\eta \sim \int \Delta = \mathscr{R}_{KKLT}$

Hamada, Hebecker, GS, Soler, ’18, ’19, ’21
c.f. Kallosh ’19; Carta, Moritz, Westphal ’19;
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10d Covariant Action

Hamada, Hebecker, Shiu, PS, ’18, ’21

Covariant action

$$S_{10d} = -\frac{1}{2}\int_{10d} e^{\phi}\left| G_+ - e^{-\phi/2}\sum_i \lambda\lambda_i\,\delta_{D7_i}\overline{\Omega}_3\right|^2 - \frac{1}{2}\int_{10d} e^{\phi}\,|\,G_-^{(0)}|^2 + \int_{10d} e^{\phi}\,|\,G_+^{(0)}|^2 + 3\sum_{ij}(\lambda\lambda)_i\,(\overline{\lambda\lambda})_j\,\mathscr{K}_{ij}$$

- This is not 10d covariant: λ_i are 4d gauginos; J , Ω_3 and $\mathscr{K}_{ij}$ are defined with respect to the internal CY. We should use

$$\Psi_i = \lambda_i \otimes \chi_i$$

χ_i is a covariantly const. spinor on the holomorphic D7$_i$-cycle

$$\Omega_{mnz} = \chi_i^T \Gamma_{mn}\chi_i$$
$$J_{mn} = \chi_i^\dagger \Gamma_{mn}\chi_i$$
$$\mathscr{K}_{ii} \sim \int_{D7_i}\left(\chi_i^\dagger[\nabla_m, \nabla_n]\chi_i\right)\left(\chi_i^\dagger \Gamma^{mn}\chi_i\right)$$

Hamada, Hebecker, GS, Soler, ‘21

Covariant action

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- To see how $\mathscr{K}_{ij}$ can be expressed in terms of the internal spinor:

$$\mathcal{K}_{\Sigma\Sigma} \equiv \int_X [\Sigma]\wedge[\Sigma]\wedge J = \int_\Sigma c_1(\mathcal{O}(\Sigma))\wedge J = \int_\Sigma F(N)\wedge J\,.$$

- The covariantly constant internal spinor χ satisfies:

$$\overline{\chi}^c[D_m, D_n]\chi = 0 \qquad \Rightarrow \qquad \overline{\chi}^c R_{mn}\chi + \overline{\chi}^c F_{mn}\chi = 0$$

- Putting things together: $\mathcal{K}_{\Sigma\Sigma} = -\frac{i}{4}\int_\Sigma d^4y\,\sqrt{g}\,\left(\overline{\chi}^c\,[\nabla_m,\nabla_n]\,\chi\right)\left(\overline{\chi}^c\,\gamma_{kl}\,\chi\right)\epsilon^{mnkl}$

Hamada, Hebecker, GS, Soler, ‘21

Covariant action

$$S_{10d} = -\frac{1}{2}\int_{10d} e^{\phi}\left|G_+ - e^{-\phi/2}\sum_i \lambda\lambda_i\,\delta_{D7_i}\overline{\Omega}_3\right|^2 - \frac{1}{2}\int_{10d} e^{\phi}\,|\,G_-^{(0)}|^2 + \int_{10d} e^{\phi}\,|\,G_+^{(0)}|^2 + 3\sum_{ij}(\lambda\lambda)_i\,(\overline{\lambda\lambda})_j\,\mathscr{K}_{ij}$$

- After some very messy (Fierz) manipulations:

$$\begin{aligned} S_{10d} &= \mathscr{L}_{\lambda^2} - \mathscr{L}_{div} + \mathscr{L}_{\lambda^4} \\ &= -\frac{1}{4}e^{\phi}\int G\wedge *\overline{G} - \frac{i}{4}e^{\phi}\int G\wedge\overline{G} - \frac{1}{2}e^{\phi/2}\sum_i\left(\int_{D7_i}\overline{G}_{MNz_i}\overline{\Psi}_i\,\Gamma^{MN}\,\Psi_i + \text{c.c.}\right) \\ &\quad + \frac{1}{2}\sum_i\int_{D7_i}\delta_i^{(0)}\,\left(\overline{\Psi}_i^c\,\Gamma_{MN}\,\Psi_i^c\right)\left(\overline{\Psi}_i\,\Gamma^{MN}\Psi_i\right) \\ &\quad + \frac{3\,i}{16}\sum_i\int_{D7_i} d^4y\sqrt{g}\,\left(\overline{\Psi}_i^c\,[\,\nabla_M,\nabla_N\,]\,\Gamma^{KL}\Gamma^{MN}\Psi_i^c\right)\left(\overline{\Psi}_i\,\Gamma_{KL}\Psi_i\right)\,, \end{aligned}$$

Hamada, Hebecker, GS, Soler, ‘21

$S_{10d} \to S_{4d,sugra}$ upon CY compactification

Hamada, Hebecker, GS, Soler, ’18, ’21

$$S_{10d} \longrightarrow S_{4d,sugra}$$

$$S_{10d} = -\frac{1}{2}\int_{10d} e^{\phi}\left|G_+ - e^{-\phi/2}\sum_i \lambda\lambda_i\,\delta_{D7_i}\overline{\Omega}_3\right|^2 - \frac{1}{2}\int_{10d} e^{\phi}\,|\,G_-^{(0)}|^2 + \int_{10d} e^{\phi}\,|\,G_+^{(0)}|^2 + 3\sum_{ij}(\lambda\lambda)_i\,(\overline{\lambda\lambda})_j\,\mathscr{K}_{ij}$$

- Solving for G, this action can be casted in 4d N=1 SUGRA with:

$$K = -\,2\log\left[\mathscr{K}(T_\alpha,\bar{T}_\beta)\right] - \log\,\left[-i(\tau-\bar{\tau})\right]\,, \qquad W = \int G\wedge\Omega\,, \qquad f_i(T) = T_i \sim \mathscr{K}_i$$

where $\mathscr{K}\,,\,\mathscr{K}_\alpha\,,\mathscr{K}_{\alpha\beta}\,,\mathscr{K}_{\alpha\beta\gamma}$ are 6-, 4-, 2-, 0-volumes (intersection numbers) of the CY

$$\boxed{S_{10d} \rightarrow -\,e^{-K}\left(\left|\frac{e^{-K/2}}{4}\left(\partial_{T_\alpha}f_i\right)\lambda\lambda_i + D_{T_\alpha}W\right|^2 + |\,D_\tau W\,|^2 - 3\,|\,W\,|^2\right) = S_{4d,sugra}}$$

✓

Hamada, Hebecker, Shiu, PS, '21

Compute $\Delta^{\langle\lambda\lambda\rangle}$ (and $\Delta^{\overline{D3}}$) from 10d

Hamada, Hebecker, GS, Soler, '19

$\Delta \sim (T^m_{\ m} - T^\mu_{\ \mu})$

$$\mathscr{V}_6 \mathscr{R}(\eta) = -\int d^6y \sqrt{g}\ \Omega^{10}(y)\,\Delta$$

- Given our finite action, one can revisit the 10d analysis of KKLT

$$\Delta = \frac{1}{2}\left(-T^\mu_\mu + T^m_m\right) = \left(\mathscr{L} - \underbrace{g^{mn}\frac{\partial \mathscr{L}}{\partial g^{mn}}}\right)$$

= scaling w/ volume T

10d KKLT **step 0:** $\Delta = \Delta^{GKP} = 0$

$$\mathscr{R}_\eta = 0$$

(No-scale) Minkowski vacua

$\Delta \sim (T^m_{\ m} - T^\mu_{\ \mu})$

$$\mathscr{V}_6 \mathscr{R}(\eta) = -\int d^6y \sqrt{g}\ \Omega^{10}(y)\, \Delta$$

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10d KKLT **step 1:** $\Delta = \Delta^{\langle\lambda\lambda\rangle}$ computed from $S_{\langle\lambda\lambda\rangle\to e^{-T}}$

$$\mathscr{R}_\eta \sim -\left(e^{-2T} - \frac{1}{T}W_0\, e^{-T}\right) + \frac{1}{2T}\, e^{-2T}$$

Same as KKLT AdS result on-shell

$$\mathscr{R}_\eta \sim \frac{1}{2}\, T\, \frac{\partial V_{\langle\lambda\lambda\rangle}}{\partial T} + V_{\langle\lambda\lambda\rangle} \quad \xrightarrow[T\to T_0]{} \quad V_{\langle\lambda\lambda\rangle}\Big|_{T_0}$$

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Subleading in 1/T terms come from renormalization of gauge couplings [Kaplunovsky, Louis, '91]

Same as KKLT AdS result on-shell

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$$\Delta = \frac{1}{2}\left(-T^\mu_\mu + T^m_m\right) = \left(\mathscr{L} - g^{mn}\frac{\partial\mathscr{L}}{\partial g^{mn}}\right)$$

= scaling w/ volume T

10d KKLT **step 2:** $\Delta = \Delta^{\langle\lambda\lambda\rangle} + \Delta^{\overline{D3}}$

Contribution from $\Delta^{\overline{D3}}$ is negligible: $\Omega^8(y_{\overline{D3}})\,\Delta^{\overline{D3}} \approx 0$

Same off-shell formula as in step 1, different on-shell result

$$\mathscr{R}_\eta \sim \frac{1}{2}T\frac{\partial V_{\langle\lambda\lambda\rangle}}{\partial T} + V_{\langle\lambda\lambda\rangle} \underset{[V=V_{\lambda\lambda}+V_{\overline{D3}}]}{=} -\frac{1}{2}T\frac{\partial V_{\overline{D3}}}{\partial T} + V_{\langle\lambda\lambda\rangle} \xrightarrow[V_{\overline{D3}}\sim T^{-2}]{} V_{\overline{D3}} + V_{\langle\lambda\lambda\rangle}\Big|_{\tilde{T}_0}$$

Summary

Summary

- We have discussed several motivations that necessitate a microscopic description of brane-localized non-perturbative effects.
- We have completed the D7-brane action for gauginos and their coupling to 3-form flux up to the quartic order.
- The properly regularized action is free of divergences & local: a necessary first step for *quantitative* studies of flux compactification with branes.
- This microscopic action 1) reduces upon Calabi-Yau compactification to:

$$\mathscr{L}_{4d} = e^K \left(|D_T W_0 + e^{-K/2} \lambda\lambda|^2 - 3|W_0|^2 \right)$$

and 2) the 10d computation of $\mathscr{R}_4$ matches with 4d results of KKLT.

- It would be interesting to confirm our results with a 10d Noether procedure [Horava, Witten, '96] or by completing the κ-symmetric D7-brane action [Grana, Kovensky, Retolaza, '20];[Retolaza, Rogers, Tatar, Tonioni, '21] to quartic order.