

Explicit Construction of KKLT Vacua

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Summary

We give explicit constructions of $\mathcal{N} = 1$ supersymmetric AdS_4 vacua, along lines proposed by KKLT, in orientifolds of Calabi-Yau hypersurfaces in toric varieties.

We find quantized fluxes for which $W_0 \ll 1$ and $g_s \ll 1$. W_{np} arises from rigid prime toric divisors with constant Pfaffians. All tadpoles are cancelled, and all moduli are stabilized. We explicitly include worldsheet instantons in the Kähler potential.

We relied on new tools for computing Gopakumar-Vafa invariants, constructing orientifolds, uplifting to F-theory, and enumerating Euclidean D3/M5-branes, all at $h^{1,1} \gg 1$.

We have not yet considered arranging a suitable warped throat for uplifting to de Sitter.

Collaboration

Mehmet Demirtas, Manki Kim, Jakob Moritz, Andres Rios-Tascon

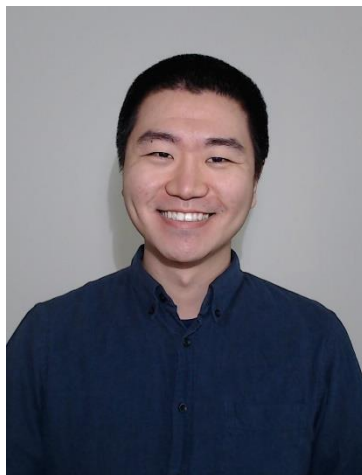
Demirtas, Kim, L.M., Moritz 19

Demirtas, Kim, L.M., Moritz 20

Demirtas, L.M., Rios-Tascon, 20

Demirtas, Kim, L.M., Moritz, Rios-Tascon, 2107.NNNNN

+works in progress with Demirtas, Gendler, Heidenreich, Kim, Moritz, Rios-Tascon, Rudelius, Stillman



Broader Goal

Computation of the String Landscape

Understand *through enumeration* what is possible in quantum gravity

Construct ensembles of vacua whose phenomenology can be studied

Plan

- I. Exponentially small flux superpotential Demirtas, Kim, L.M., Moritz 19
- II. Nonperturbative superpotential from rigid divisors
- III. Supersymmetric vacua
- IV. Control of α' corrections

Superpotential

First task: establish

$$W = W_{\text{flux}}(z, \tau) + \sum_D \mathcal{A}_D(z, \tau) \exp\left(-\frac{2\pi}{c_D} T_D\right) + \dots$$

with

- $W_0 := \langle |W_{\text{flux}}| \rangle \ll 1$ Use mechanism of Demirtas, Kim, L.M., Moritz 19
- At least $h^{1,1}$ nonzero Pfaffians $\mathcal{A}_D(z, \tau)$

Find rigid prime toric divisors D_I

We will require a stronger condition:

- At least $h^{1,1}$ nonzero Pfaffians \mathcal{A}_D that are pure numbers

Find toric uplift to F-theory, compute $h^{2,1}(\widehat{D}_I)$ by stratification Kim, to appear

$h^{2,1}(\widehat{D}_I) = 0 \Rightarrow \mathcal{J}$ trivial $\Rightarrow \mathcal{A}_{D_I}$ constant Witten 96

Small Flux Superpotentials

Statistical arguments suggest $|W_0| \ll 1$ should occur,
but exponentially rarely,
and preferentially when moduli space dimension is large.

Our tools permit exploring such moduli spaces.
But brute-force search still very challenging.

Previous record: $\mathcal{O}(0.01)$

Denef, Douglas, Florea 04

Denef, Douglas, Florea, Grassi, Kachru 05

Better to invent and apply a **mechanism**.

Giryavets, Kachru, Tripathy, Trivedi 03

Denef, Douglas, Florea 04

Demirtas, Kim, L.M., Moritz 19



Racetrack Superpotential

$$W = W_{\text{inst}}(z)$$

$$W_{\text{inst}}(z) = \mathcal{N}_1 e^{-p_1 z} + \mathcal{N}_2 e^{-p_2 z} + \dots$$

$$\rightarrow 0 \text{ for } z \rightarrow \infty \quad p_1, p_2 > 0$$

$$W_{\text{inst}}(z_{\text{min}}) = \frac{\mathcal{N}_2 (p_1 - p_2)}{p_1} \left(-\frac{\mathcal{N}_2 p_2}{\mathcal{N}_1 p_1} \right)^{\frac{p_2}{p_1 - p_2}}$$

$$\ll 1 \text{ if } |p_1 - p_2| \ll p_2, \quad \mathcal{N}_2 \ll \mathcal{N}_1$$

Incarnation as Flux Superpotential

Need to show that, in a bona fide solution of string theory:

(1) $W_{\text{flux}}(z) = W_{\text{inst}}(z) = \mathcal{N}_1 e^{-p_1 z} + \mathcal{N}_2 e^{-p_2 z} + \dots$

(2) with $|p_1 - p_2| \ll p_2$ and $\mathcal{N}_2 \ll \mathcal{N}_1$

Then we'll have succeeded:

$$\langle W_{\text{flux}} \rangle = W_{\text{inst}}(z_{\text{min}}) = \frac{\mathcal{N}_2(p_1 - p_2)}{p_1} \left(-\frac{\mathcal{N}_2 p_2}{\mathcal{N}_1 p_1} \right)^{\frac{p_2}{p_1 - p_2}} \ll 1$$

(1) we established a sufficient condition on topological data

(2) we found explicit examples

Demirtas, Kim, L.M., Moritz 19

By analytic continuation to conifold, mechanism yields vacua with **small W and a warped conifold**: $e^{2A} \approx |W_0| \ll 1$

Flux Superpotential in a Calabi-Yau

$$(h^{2,1}, h^{1,1}) = (5, 81)$$

We find quantized fluxes,

$$\mathbf{M} = \begin{pmatrix} 3 & -5 & 2 & -2 & -5 \end{pmatrix}^T, \quad \mathbf{K} = \begin{pmatrix} -5 & 5 & -4 & -1 & 5 \end{pmatrix}^T,$$

s.t. $\mathbf{z} = \mathbf{p} \tau$, $\mathbf{p} = \left(\frac{13}{8} \quad \frac{59}{24} \quad \frac{5}{4} \quad \frac{5}{4} \quad \frac{5}{12} \right)$ is a perturbatively flat vacuum.

The leading instantons have GV invariants

$$\mathcal{N}_{\tilde{\mathbf{q}}} = \begin{pmatrix} 2 & 56 & 2 & 2 \end{pmatrix}.$$

$$W_{\text{flux}}(\tau) = \frac{2}{(2\pi)^{5/2}} \left(2 e^{2\pi i \tau \cdot \frac{3}{8}} + 324 e^{2\pi i \tau \cdot \frac{5}{12}} \right)$$

Moduli are stabilized at $g_s \approx 0.05$ and

$$W_0 \approx \sqrt{\frac{2}{\pi}} \frac{1}{(2\pi i)^2} \times 36 \times 180^{-10} \approx 2.04 \times 10^{-23}.$$

Manufacturing small W_0

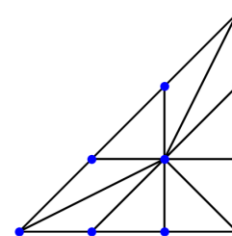
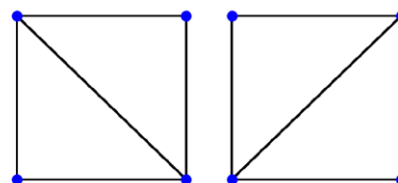
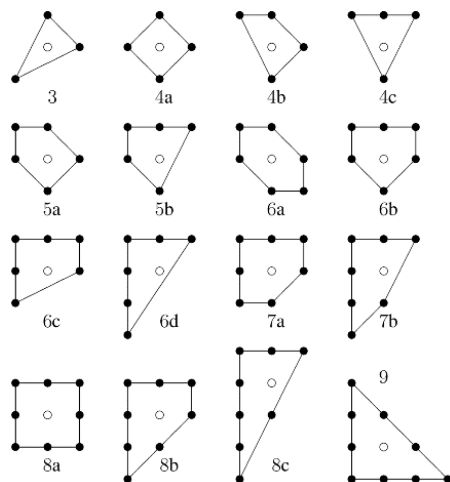
1. Begin with an O3/O7 orientifold of a CY₃, X
2. Compute prepotential via mirror symmetry
3. Find quantized fluxes F_3, H_3 that align with GV invariants of \tilde{X} to give IIA worldsheet instanton racetrack along flat direction (within the D3-brane tadpole!)

Step 3 is a search in $\mathbb{Z}^{2h^{2,1}(X)}$.

Feasible for $h^{2,1}(X) \lesssim 5$ (laptop), $\lesssim 7$ (cluster).

Setting: Hypersurfaces

We will work with mirror pairs of CY_3 hypersurfaces X, \tilde{X}
in toric varieties V, \tilde{V}
obtained from triangulations of 4d polytopes Δ°, Δ .



473,800,776 4d reflexive polytopes

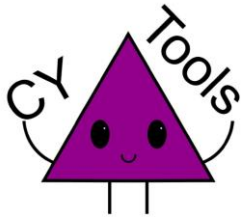
Kreuzer and Skarke 2000

$< 10^{428} \text{ CY}_3$

Demirtas, L.M., Rios-Tascon 2020

Computation in Kreuzer-Skarke

Given a 4d polytope from Kreuzer-Skarke list:



NEW:

Fine regular star triangulation \Rightarrow toric variety V

Intersection numbers κ_{ijk} of CY_3 $X \subset V$

Kähler cone of V , $\mathcal{K}(V) \subset \mathcal{K}(X)$

Gopakumar-Vafa invariants of curves in X

Orientifolds of X

F-theory uplifts Y of orientifolds of X

Divisors supporting $EM5 \subset Y$ or $ED3 \subset X$

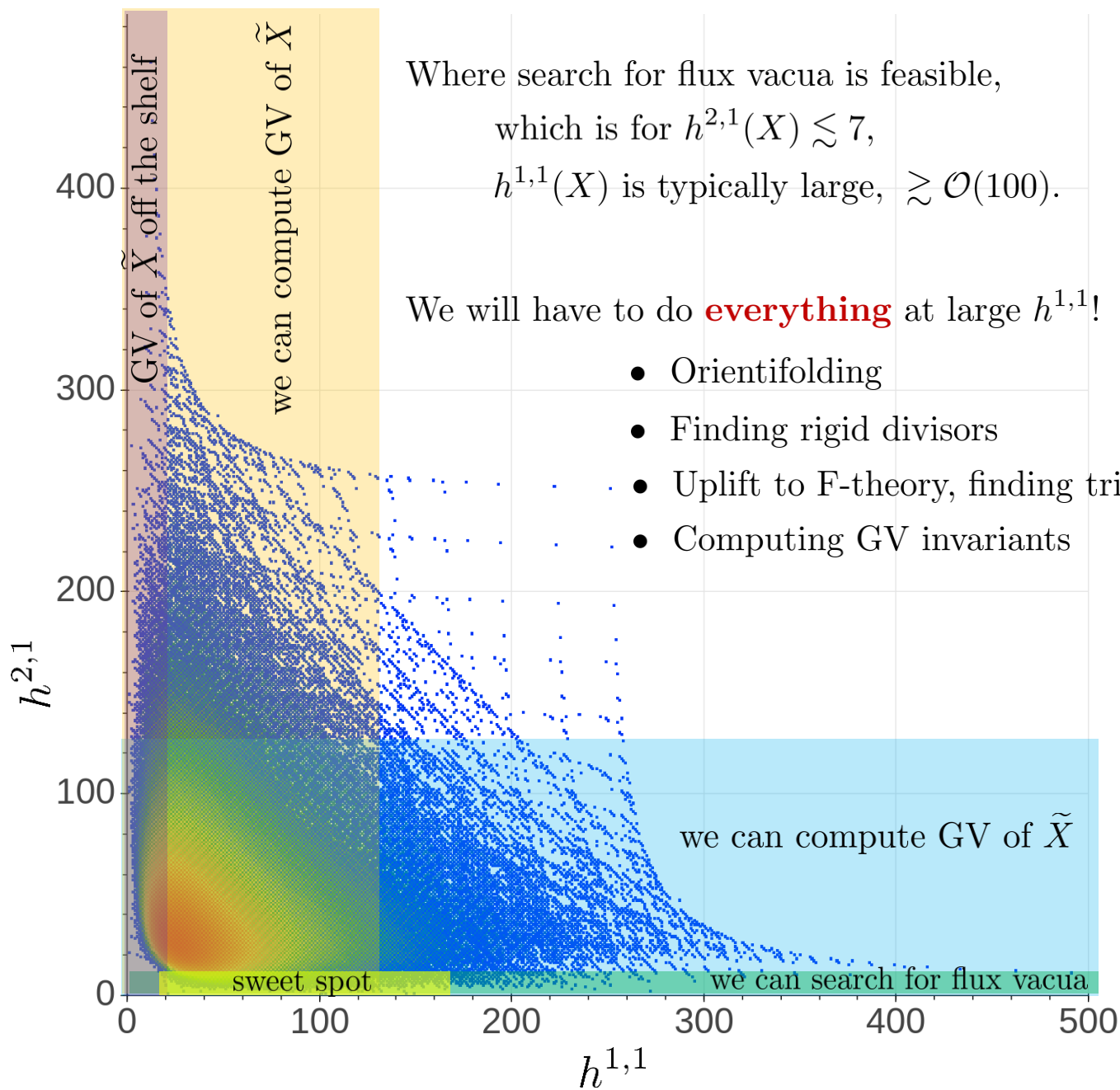
Now easily triangulate for any $h^{1,1}$; GV up to $h^{1,1} \sim 100$.

CYTools: A Software Package for Analyzing Calabi-Yau Hypersurfaces

Demirtas, L.M., Rios-Tascon

Demirtas, Kim, L.M., Moritz, Rios-Tascon, work in progress

complexity of analyzing \tilde{X} grows exponentially \longrightarrow



complexity of analyzing X grows exponentially \longrightarrow

Superpotential so far

After flux choice: $W = W_0 + \dots$

Goal: $W = W_0 + \sum_D \mathcal{A}_D \exp\left(-\frac{2\pi}{c_D} T_D\right) + \dots$

- Find at least $h^{1,1}$ rigid prime toric divisors D_I , $I \in \{1, \dots, h^{1,1} + 4\}$
- whose uplifts \widehat{D}_I to F-theory have trivial intermediate Jacobian \mathcal{J}
- in an orientifold where all seven-branes lie in $\mathfrak{so}(8)$ stacks

D_I rigid $\Leftrightarrow h^\bullet(\widehat{D}_I) = (1, 0, 0, 0) \Leftrightarrow h_+^\bullet(D_I) = (1, 0, 0)$ and $h_-^\bullet(D_I) = (0, 0, 0)$

Witten 96a: Nonperturbative superpotentials in string theory

$h^{2,1}(\widehat{D}_I) = 0 \Rightarrow \mathcal{J}$ trivial $\Rightarrow \mathcal{A}_{D_I}$ constant

Witten 96b: Five-brane effective action in M theory

So we need to compute the Hodge numbers of divisors **at large $h^{1,1}$** .

Find a **combinatorial formula** via stratification. Kim, to appear

Thus equipped, we carried out a search.

Search for vacua

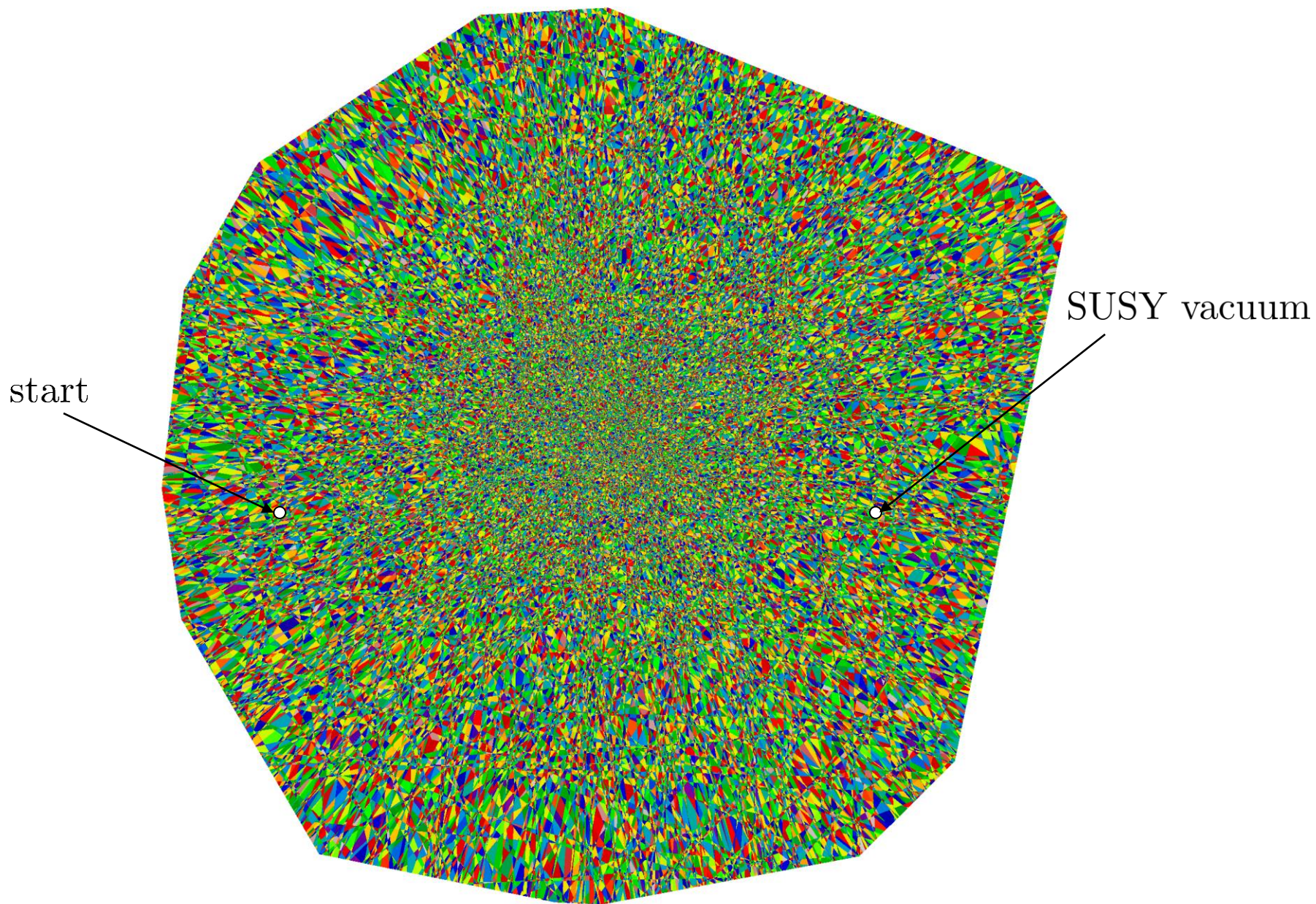
$$W = W_0 + \sum_i \mathcal{A}_{D_i} \exp\left(-\frac{2\pi}{c_i} T_i\right) + \dots \quad i = 1, \dots, h^{1,1}$$

$$\mathcal{K} = -2 \log\left(\mathcal{V}(T_i, \overline{T_i})\right)$$

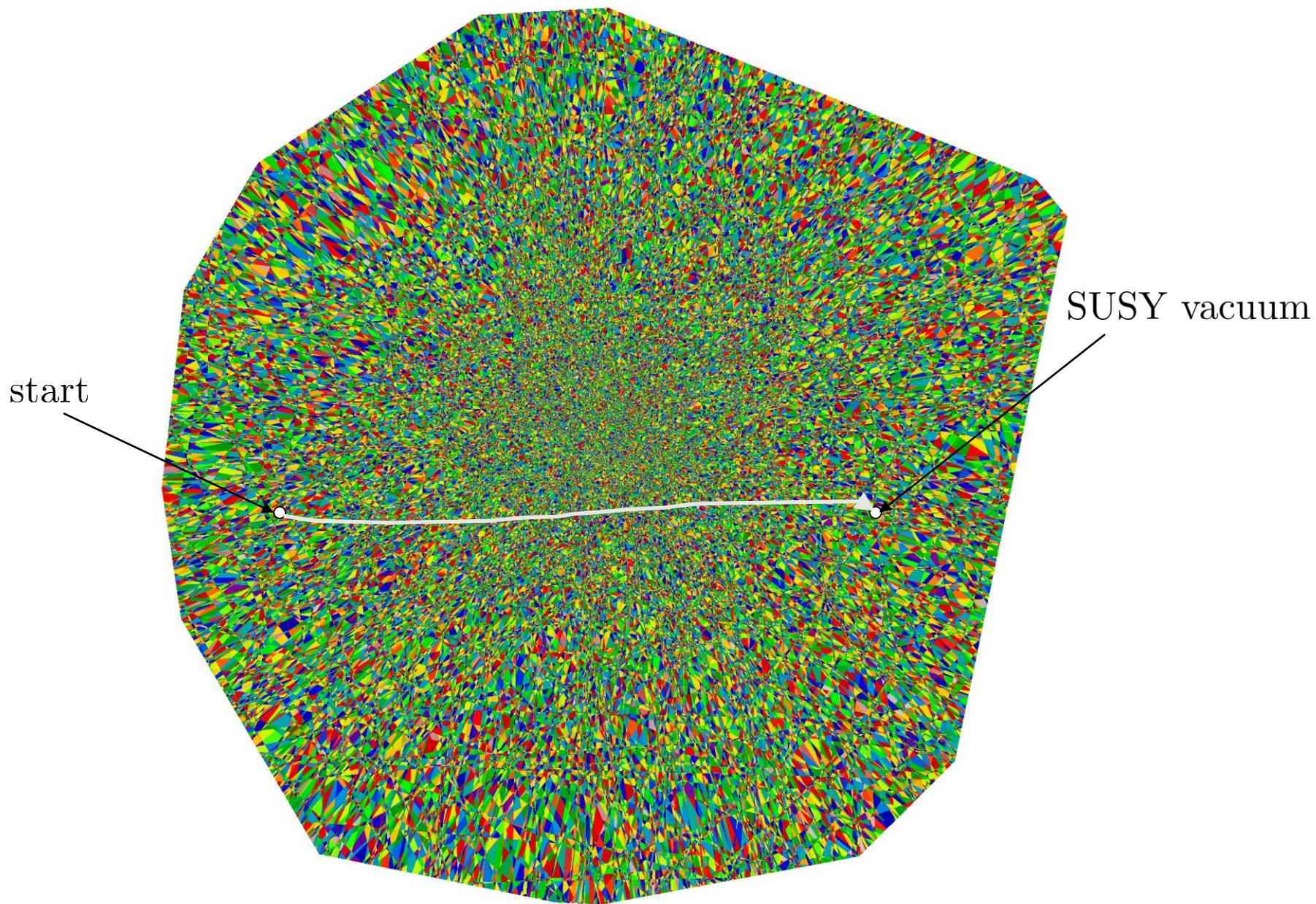
$$\text{Want: } D_{T_i} W = 0 \Rightarrow \text{Re}(T_i) \approx \frac{c_i}{2\pi} \log(W_0^{-1}) + \dots$$

But where in the Kähler cone is this point?

2d cross-section of Kähler cone in $h^{1,1} = 491$ threefold



2d cross-section of Kähler cone in $h^{1,1} = 491$ threefold



Control checks

$$W = W_0 + \sum_i \mathcal{A}_{D_i} \exp\left(-\frac{2\pi}{c_i} T_i\right) + \dots \quad i = 1, \dots, h^{1,1}$$

Control of superpotential

- Ensured that \mathcal{A}_{D_i} are **nonzero numbers** by standard zero-mode counting.
Their numerical values have small effects: $\Delta T_i / T_i \sim \log(\Delta \mathcal{A}_D) / \log(W_0)$
We checked that our vacua persist for $\mathcal{A}_{D_i} \in [10^{-4}, 10^4]$
- Further instantons (e.g., ED3 on other divisors) negligible
- $W_{\text{ED}(-1)} = \sum_{k=1}^{\infty} B_k(z) e^{2\pi i k \tau}$ negligible due to $g_s \ll 1$

Control of Kähler potential

- Einstein-frame volumes are *large* $\text{Re}(T_i) \approx \frac{c_i}{2\pi} \log(W_0^{-1})$
- String-frame volumes are order-unity : $g_s \propto \frac{2\pi}{\log(W_0^{-1})} \ll 1$
- Weak string coupling \Rightarrow leading corrections are at **string tree level**, to all orders in α'

$$\mathcal{K} = -2 \log \left(g_s^{-2} \mathcal{V}(T_i, \overline{T}_i) \right)$$

Gopakumar-Vafa invariants

$$\begin{aligned} \mathcal{V}(T_i, \overline{T}_i) = & \frac{1}{6} \kappa_{ijk} t^i t^j t^k - \frac{\zeta(3) \chi(X)}{4(2\pi)^3} \\ & + \frac{1}{2(2\pi)^3} \sum_{\mathbf{q} \in \mathcal{M}(X)} \mathcal{N}_{\mathbf{q}} \left(\text{Li}_3 \left((-1)^{\gamma \cdot \mathbf{q}} e^{-2\pi \mathbf{q} \cdot \mathbf{t}} \right) + 2\pi \mathbf{q} \cdot \mathbf{t} \text{Li}_2 \left((-1)^{\gamma \cdot \mathbf{q}} e^{-2\pi \mathbf{q} \cdot \mathbf{t}} \right) \right), \\ \text{Re}(T_i) = & \frac{1}{2} \kappa_{ijk} t^j t^k - \frac{\chi(D_i)}{24} + \frac{1}{(2\pi)^2} \sum_{\mathbf{q} \in \mathcal{M}(X)} q_i \mathcal{N}_{\mathbf{q}} \text{Li}_2 \left((-1)^{\gamma \cdot \mathbf{q}} e^{-2\pi \mathbf{q} \cdot \mathbf{t}} \right). \end{aligned}$$

Convergence of worldsheet instanton sum

$$\xi_n := \mathcal{N}_{n\mathbf{q}} e^{-2\pi n \mathbf{q} \cdot \mathbf{t}}$$

\mathbf{q} : curve class in X

\mathbf{t} : Kähler parameters in KKLТ vacuum

Convergence if $\xi_n \rightarrow 0$ for $n \rightarrow \infty$.

So we compute the GV invariants $\mathcal{N}_{n\mathbf{q}}$ for small curves, and check.

I'll present one example.

Of the 175,772 *smallest* curves, all but 25 have $\mathcal{N}_{\mathbf{q}} = 0$.

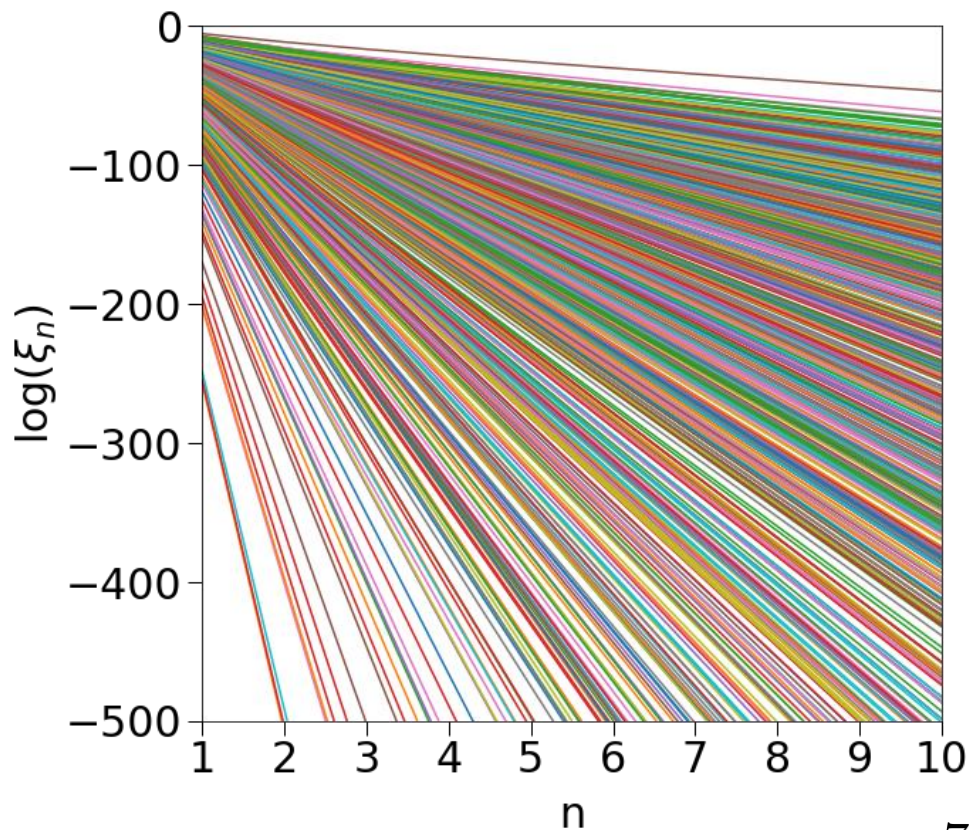
Of the curves with $\mathcal{N}_{\mathbf{q}} \neq 0$, almost all have $\mathcal{N}_{k\mathbf{q}} = 0 \ \forall k > k_{\max} \in \mathbb{N}$.

These are safely collapsible, but we do include them.

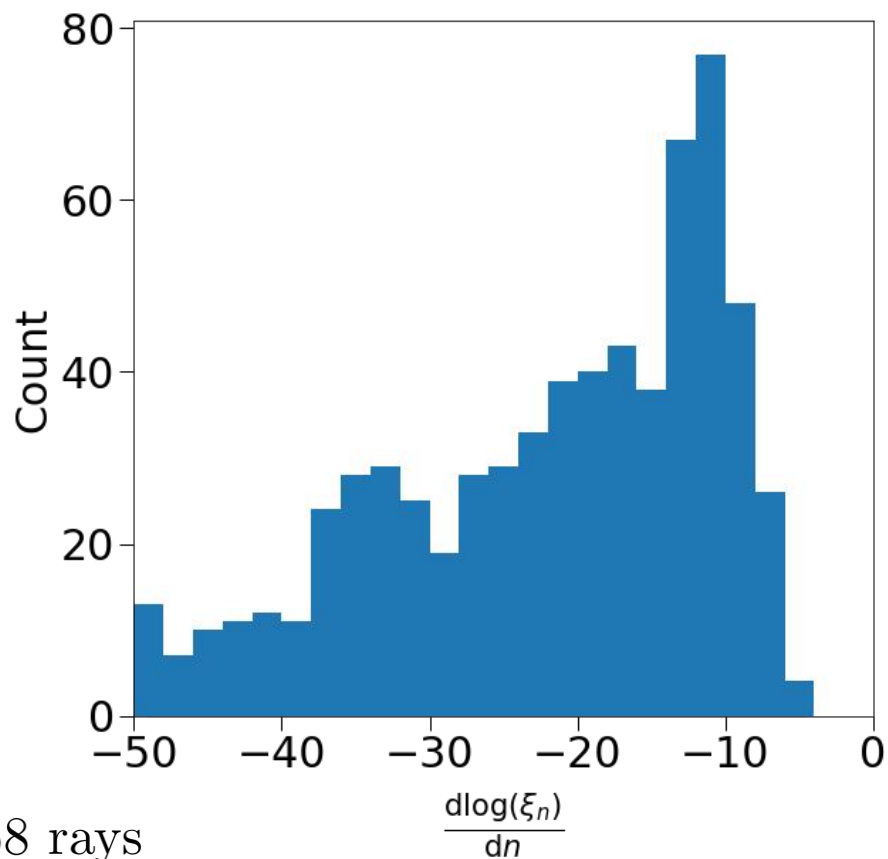
But we keep searching outward until we find 758 rays of curves with increasing (and plausibly infinite) series of nonzero $\mathcal{N}_{k\mathbf{q}}$.

Convergence of worldsheet instanton sum

$$\xi_n := \mathcal{N}_{n\mathbf{q}} e^{-2\pi n \mathbf{q} \cdot \mathbf{t}}$$



758 rays



largest correction: $\mathcal{O}(10^{-7})$

An example with $(h^{2,1}, h^{1,1}) = (5, 81)$

The vertices of Δ are the columns of

$$\begin{pmatrix} 1 & -2 & -2 & -2 & -2 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & -1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 & -1 \end{pmatrix}.$$

There are $h^{1,1} + 3 = 18_{\mathfrak{so}(8)} + 66_{\text{ED3}}$ rigid prime divisors with $h^{2,1}(\widehat{D}_I) = 0$.
The fluxes

$$\mathbf{M} = \begin{pmatrix} 3 & -5 & 2 & -2 & -5 \end{pmatrix}^T, \quad \mathbf{K} = \begin{pmatrix} -5 & 5 & -4 & -1 & 5 \end{pmatrix}^T,$$

carry D3-brane charge $\frac{71}{2}$, and the D3-brane tadpole is 44, so there are 8 D3-branes and one ‘half’ D3-brane. The leading instantons have GV invariants

$$\mathcal{N}_{\mathbf{q}} = \begin{pmatrix} 2 & 56 & 2 & 2 \end{pmatrix}.$$

and the flux superpotential is

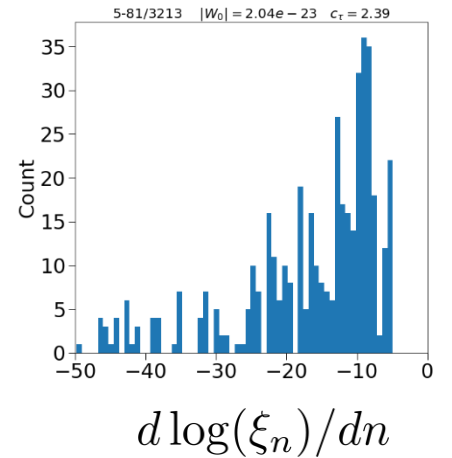
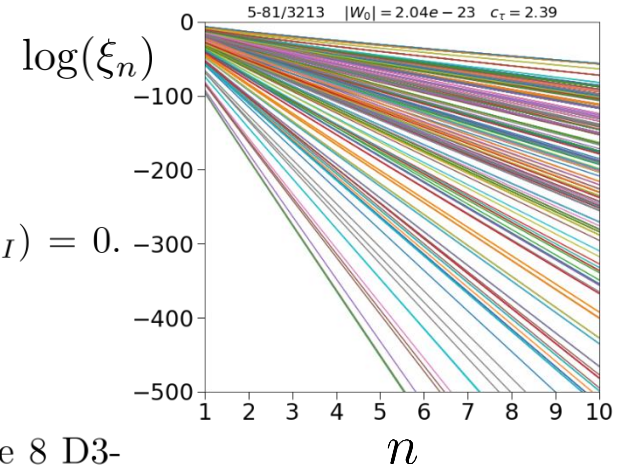
$$W_{\text{flux}}(\tau) = \sqrt{\frac{2}{\pi}} \frac{1}{(2\pi i)^2} \left(2 e^{2\pi i \tau \cdot \frac{3}{8}} + 324 e^{2\pi i \tau \cdot \frac{5}{12}} \right) + \mathcal{O} \left(e^{2\pi i \tau \cdot \frac{5}{6}} \right),$$

which stabilizes the moduli at $g_s \approx 0.05$ and

$$W_0 \approx \sqrt{\frac{2}{\pi}} \frac{1}{(2\pi i)^2} \times 36 \times 180^{-10} \approx 2.04 \times 10^{-23}.$$

We find a supersymmetric AdS_4 vacuum with volume $\mathcal{V} \approx 507.4$ and with vacuum energy

$$V_0 = -3e^{\mathcal{K}} |W|^2 \approx -8.6 \times 10^{-63} M_{\text{pl}}^4.$$



Conclusions

We have given explicit constructions of supersymmetric AdS_4 vacua in CY_3 orientifolds with $h^{2,1} \leq 7$, $h^{1,1} \geq 51$.

Stabilization is at $g_s \ll 1$, LCS, large Einstein-frame volume.

Heavily tested. We judge them to be robust.

These are incarnations of the KKLT scenario.

Supersymmetric KKLT vacua are in the landscape.

Mechanism for small W_0 led to giant hierarchies.

Search is automated; large-scale studies possible.

Uplift is a question for the future.