Line Bundle Hidden Sectors for the B-L MSSM and Gaugino Condensation

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The B-L MSSM Heterotic Standard Model:

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The CY compactification 3-fold:

The Calabi–Yau manifold X is chosen to be a torus-fibered threefold with fundamental group $\pi_1(X) = Z_3 \times Z_3$. More specifically, the Calabi–Yau threefold X is the fiber product of two rationally elliptic dP_9 surfaces, that is, a self-mirror Schoen threefold quotiented with respect to a freely acting $Z_3 \times Z_3$ isometry. Its Hodge data is $h^{1,1} = h^{1,2} = 3$, so there are three Kähler and three complex structure moduli.

The Kahler moduli are denoted by a^1, a^2, a^3 and the intersection numbers are

$$(d_{ijk}) = \begin{pmatrix} (0,\frac{1}{3},0) & (\frac{1}{3},\frac{1}{3},1) & (0,1,0) \\ (\frac{1}{3},\frac{1}{3},1) & (\frac{1}{3},0,0) & (1,0,0) \\ (0,1,0) & (1,0,0) & (0,0,0) \end{pmatrix}$$

The Observable Sector Bundle:

On the observable orbifold plane, the vector bundle $V^{(1)}$ on X is chosen to be a specific holomorphic bundle with structure group $SU(4) \subset E_8$. **It can be shown to be slope-stable and, hence, satisfy the HYM equations in the region of Kahler moduli space given by**

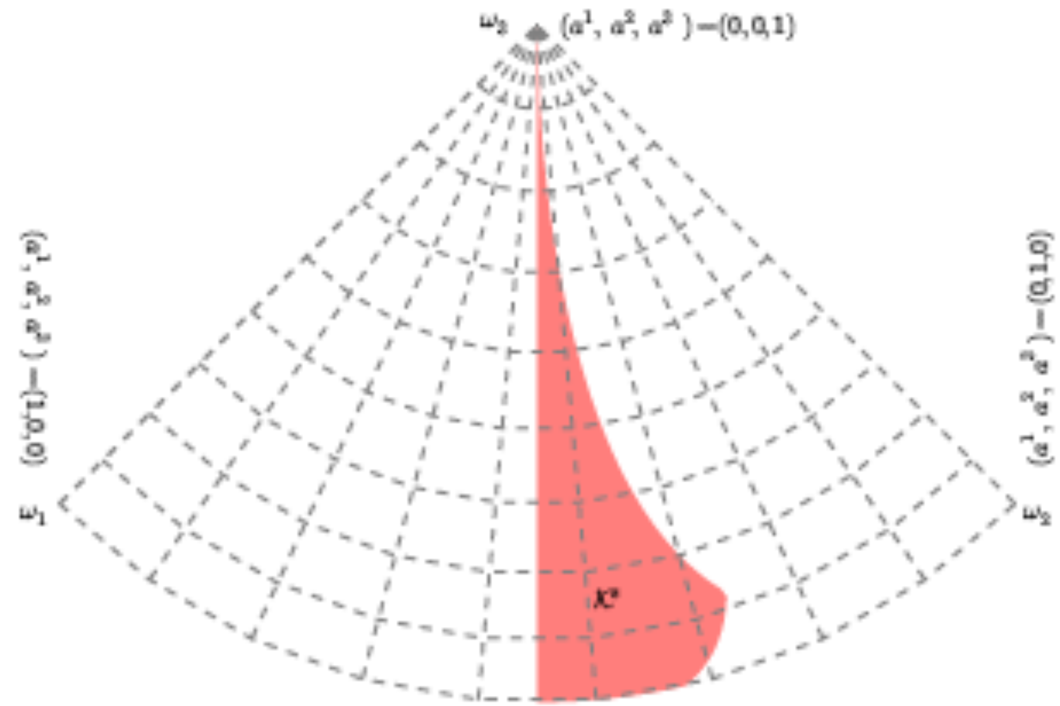

Figure 1: The observable sector stability region in the Kähler cone.

This SU(4) bundle breaks

$$E_8 \to Spin(10)$$

However, to proceed further, one must break this $Spin(10)$ "grand unified" group down to the gauge group of the MSSM. This is accomplished by turning on two *flat* Wilson lines, each associated with a different $\mathbb{Z}_3$ factor of the $\mathbb{Z}_3 \times \mathbb{Z}_3$ holonomy of X. Doing this preserves the $N = 1$ supersymmetry of the effective theory, but breaks the observable gauge group down to

$$Spin(10) \to SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

Low Energy Spectrum:

The particle spectrum of the B-L MSSM is EXACTLY that of the MSSM- three families of quarks and leptons, including three right-handed neutrino chiral supermultiplets – one per family – and exactly one pair of Higgs-Higgs conjugate chiral superfields. There are no vector-like pairs of particles and no exotics of any kind.

The Hidden Sector Bundle:

Generically, the hidden sector bundle can have the form of a Whitney sum

$$V^{(2)} = \mathcal{V}_N \oplus \mathcal{L}\ , \qquad \mathcal{L} = \bigoplus_{r=1}^{R} L_r$$

where $\mathcal{V}_N$ is a slope-stable, non-abelian bundle and each L_r, $r = 1, \ldots, R$, is a holomorphic line bundle with structure group $U(1)$. However, in this talk, we will simply this bundle to be defined by a *single* holomorphic line bundle L, in such a way that its $U(1)$ structure group embeds into E_8. A line bundle L is associated with a divisor of X and is conventionally expressed as

$$L = \mathcal{O}_X(l^1, l^2, l^3) ,$$

where the l^i are integers satisfying the condition

$$(l^1 + l^2) \bmod 3 = 0 .$$

This additional constraint is imposed in order for these bundles to arise from $\mathbb{Z}_3 \times \mathbb{Z}_3$ equivariant line bundles on the covering space of X. The structure group of L is $U(1)$. However, there are many distinct ways in which this $U(1)$ subgroup can be embedded into the hidden-sector E_8 group. The choice of embedding determines two important properties of the effective low-energy theory. First, a specific embedding will define a commutant subgroup of E_8, which appears as the symmetry group for the four-dimensional effective theory. Second, the explicit choice of embedding will determine a real numerical constant

$$a = \frac{1}{4} \operatorname{tr}_{E_8} Q^2 ,$$

where Q is the generator of the $U(1)$ factor embedded in the $\underline{\mathbf{248}}$ adjoint representation of the hidden sector E_8, and the trace tr includes a factor of $1/30$. This coefficient will enter several of the consistency conditions, such as the anomaly cancellation equation, required for an acceptable vacuum solution.

The Fifth Dimension $S^1/\mathbb{Z}_2$:

The physical length of this orbifold interval is then given by $\pi\rho\hat{R}$. It is convenient to define a new coordinate z by $z = \frac{x^{11}}{\pi\rho}$, which runs over the interval $z \in [0,1]$.

Five-Branes:

In addition to the holomorphic vector bundles on the observable and hidden orbifold planes, the bulk space between these planes can contain five-branes wrapped on two-cycles $\mathcal{C}_2^{(n)}$, $n = 1, \ldots, N$ in X. Cohomologically, each such five-brane is described by the $(2,2)$-form Poincaré dual to $\mathcal{C}_2^{(n)}$, which we denote by $W^{(n)}$. Note that to preserve $N = 1$ supersymmetry in the four-dimensional theory, these curves must be holomorphic and, hence, each $W^{(n)}$ is an effective class. **In this paper, we consider** only a *single* five-brane. We denote its location in the bulk space by z_1, where $z_1 \in [0,1]$. When convenient, we will re-express this five-brane location in terms of the parameter $\lambda = z_1 - \frac{1}{2}$, where $\lambda \in [-\frac{1}{2}, \frac{1}{2}]$.

Having expressed the constituents of the observable, hidden and orbifold sectors of the B-L MSSM vacuum, we now must solve all vacuum, dimensional reduction and physical constraints on the theory.

The Vacuum Constraints:

There are three fundamental constraints that any consistent vacuum state of the $B-L$ MSSM must satisfy. These are the following.

1) The SU(4) Slope Stability Constraint

The Kahler moduli must satisfy

$$\left(a^1 < a^2 \leq \sqrt{\tfrac{5}{2}}a^1 \quad \text{and} \quad a^3 < \frac{-(a^1)^2 - 3a^1a^2 + (a^2)^2}{6a^1 - 6a^2}\right) \quad \text{or}$$

$$\left(\sqrt{\tfrac{5}{2}}a^1 < a^2 < 2a^1 \quad \text{and} \quad \frac{2(a^2)^2 - 5(a^1)^2}{30a^1 - 12a^2} < a^3 < \frac{-(a^1)^2 - 3a^1a^2 + (a^2)^2}{6a^1 - 6a^2}\right) .$$

2) The Anomaly Cancellation Constraint

$$W_i - \left(\tfrac{4}{3}, \tfrac{7}{3}, -4\right)\Big|_i + a\, d_{ijk} l^j l^k \geq 0 \qquad i = 1, 2, 3$$

3) Positive Unified Gauge Coupling Constraint

$g_1^2 > 0 \ \Rightarrow$

$$d_{ijk}a^ia^ja^k - 3\epsilon'_S \frac{\hat{R}}{V^{1/3}}\Big(-\big(\tfrac{8}{3}a^1 + \tfrac{5}{3}a^2 + 4a^3\big) + 2(a^1 + a^2) - \big(\tfrac{1}{2} - \lambda\big)^2 a^i W_i\Big) > 0 \, ,$$

$g_2^2 > 0 \ \Rightarrow$

$$d_{ijk}a^ia^ja^k - 3\epsilon'_S \frac{\hat{R}}{V^{1/3}}\Big(a\, d_{ijk}a^il^jl^k + 2(a^1 + a^2) - \big(\tfrac{1}{2} + \lambda\big)^2 a^i W_i\Big) > 0$$

Solution to the Vacuum Constraints:

The solution to the SU(4) slope stability constraint 1) was given above and can be presented as

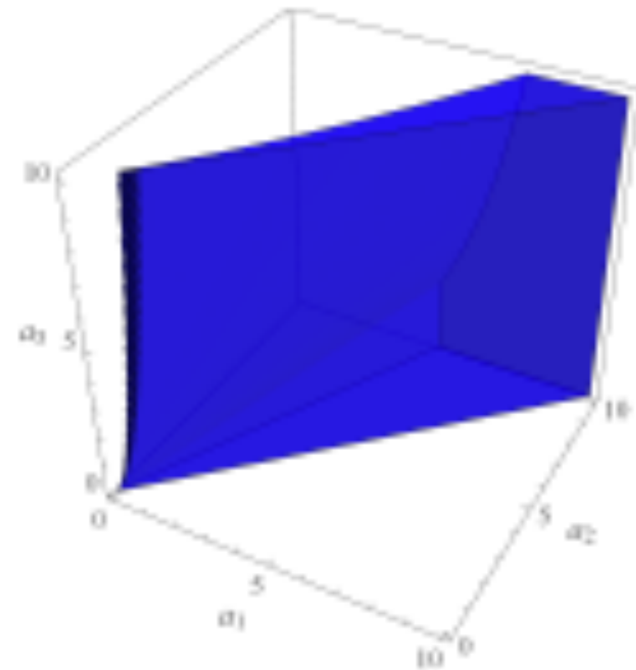

Figure 2: The region of slope-stability for the $SU(4)$ observable-sector bundle, restricted to $0 \leq a^i \leq 10$ for $i = 1, 2, 3$.

However, the anomaly conditions 2) and 3) require one to choose the line bundle $L = \mathcal{O}_X(l^1, l^2, l^3)$ **and to compute its embedding coefficien** $a = \frac{1}{4}\mathrm{tr}_{E_8} Q^2$ **.**

In this talk, as a simple example, we will choose

$$L = \mathcal{O}_X(2, 1, 3) .$$

There are many possible embeddings of L into E_8 **.**

Here, for specificity, we choose the following. Recall

$$SU(2) \times E_7 \subset E_8$$

is a maximal subgroup. With respect to $SU(2) \times E_7$, the $\underline{\mathbf{248}}$ representation of E_8 decomposes as

$$\underline{\mathbf{248}} \rightarrow (\underline{\mathbf{1}}, \underline{\mathbf{133}}) \oplus (\underline{\mathbf{2}}, \underline{\mathbf{56}}) \oplus (\underline{\mathbf{3}}, \underline{\mathbf{1}}) \ .$$

Now choose the generator of the $U(1)$ structure group of L in the fundamental representation of $SU(2)$ to be $(1, -1)$. It follows that under $SU(2) \rightarrow U(1)$

$$\underline{\mathbf{2}} \rightarrow 1 \oplus -1 \ ,$$

and, hence, under $U(1) \times E_7$

$$\underline{\mathbf{248}} \rightarrow (0, \underline{\mathbf{133}}) \oplus ((1, \underline{\mathbf{56}}) \oplus (-1, \underline{\mathbf{56}})) \oplus ((2, \underline{\mathbf{1}}) \oplus (0, \underline{\mathbf{1}}) \oplus (-2, \underline{\mathbf{1}})).$$

It follows from the above expression that the embedding coefficient is

$$a = 1.$$

Clearly, the low energy gauge group is given by

$$H = E_7 \times U(1) \ ,$$

where the second factor is an "anomalous" $U(1)$.

To appropriately embed the line bundle as above, it is necessary to extend it to the "induced" rank 2 bundle

$$V = L \oplus L^{-1}$$

For this explicit example, the vacuum constraints 2) and 3) above become

2) The Anomaly Cancellation Constraint

$$W_i = (9, 17, 0)|_i \geq 0 \quad \text{for each } i = 1, 2, 3$$

which is satisfied.

3) Positive Unified Gauge Coupling Constraint

$$g_1^2 > 0 \;\Rightarrow\; (a^1)^2 a^2 + a^1 (a^2)^2 + 6a^1 a^2 a^3 + 2a^1 - a^2 + 12a^3 + \\ + 3\left(\frac{1}{2} - \lambda\right)^2 (9a^1 + 17a^2) > 0$$

$$g_2^2 > 0 \;\Rightarrow\; (a^1)^2 a^2 + a^1 (a^2)^2 + 6a^1 a^2 a^3 - 29a^1 - 50a^2 - 12a^3 + \\ + 3\left(\frac{1}{2} + \lambda\right)^2 (9a^1 + 17a^2) > 0$$

Fixing, for example, the location of the 5-brane to be at

$$\lambda = 0.49$$

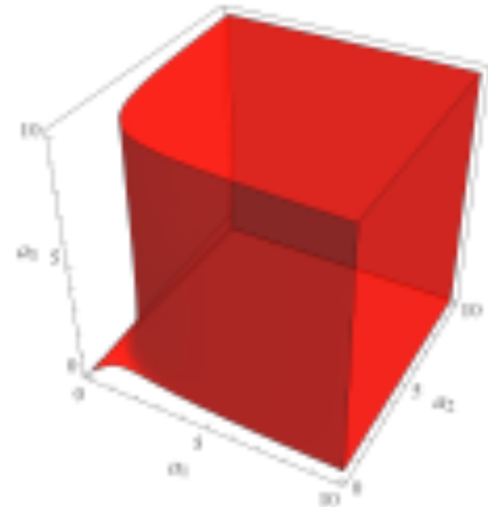

Figure 3: Simultaneous solution to both $(g^{(1)})^2 > 0$ and $(g^{(3)})^2 > 0$ gauge coupling constraints with $\lambda = 0.49$, restricted to the region $0 \le a^i \le 10$ for $i = 1, 2, 3$.

We find that all three Vacuum Constraints are solved within the region of Kahler moduli space given by

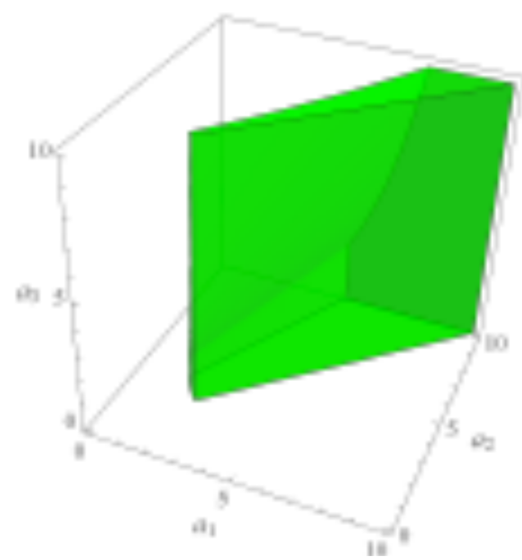

Figure 4: The region of Kähler moduli space where the $SU(4)$ slope-stability conditions, the anomaly cancellation constraint, and the positive squared gauge coupling constraints with $\lambda = 0.49$ are simultaneously satisfied in unity gauge, restricted to $0 \le a^i \le 10$ for $i = 1, 2, 3$. This amounts to the intersection of Figures 2 and 3.

The Dimensional Reduction Constraint:

The fifth-dimensional length must be larger than the CY length. This implies

$$\frac{\pi\rho\hat{R}V^{-1/3}}{(vV)^{1/6}} > 1$$

where $V = \frac{1}{6}d_{ijk}a^i a^j a^k$.

The Physical Constraint:

In the “simultaneous” Wilson line scenario, the “unified” SO(10) gauge coupling parameter g_u in the observable sector should satisfy the constraint

$$\langle\alpha_u\rangle = \frac{1}{26.46} \quad , \quad \langle M_U\rangle = 3.15 \times 10^{16}\ \mathrm{GeV}$$

Note that using

$$\langle\alpha_u\rangle = \frac{1}{26.46} = \frac{\hat{\alpha}_{GUT}}{\mathrm{Re}\,f_1} \quad \textbf{and} \quad \mathrm{Re}\,f_1 = V + \tfrac{1}{3}a^1 - \tfrac{1}{6}a^2 + 2a^3 + \tfrac{1}{2}(\tfrac{1}{2} - \lambda)^2(9a^1 + 17a^2)$$

and

$$\langle\alpha_2\rangle = \frac{\alpha_{\hat{GUT}}}{Ref_2} \quad \textbf{and} \quad \mathrm{Re}\,f_2 = V - \tfrac{29}{6}a^1 - \tfrac{25}{3}a^2 - 2a^3 + \tfrac{1}{2}(\tfrac{1}{2} + \lambda)^2(9a^1 + 17a^2)$$

allows one to compute g_2 at $\langle M_U\rangle = 3.15 \times 10^{16}\ \mathrm{GeV}$ in the hidden sector.

Solving all Vacuum, Dimensional Reduction and Physical constraints simultaneously has the solution space

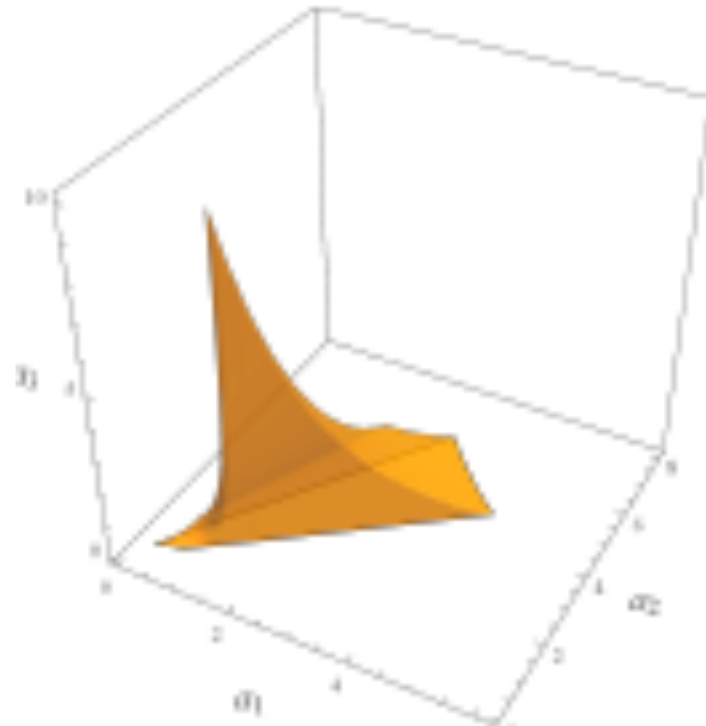

Figure 5: The region of Kähler moduli space where the $SU(4)$ slope-stability conditions, the anomaly cancellation constraint and the positive squared gauge coupling constraint from Figure 4 are satisfied, in addition to the dimensional reduction and the phenomenological constraints. The results are valid for a hidden sector line bundle $L = \mathcal{O}_X(2,1,3)$ with $a = 1$ and for a single five-brane located at $\lambda = 0.49$. The results are for simultaneous Wilson lines with $\langle \alpha_u \rangle = \frac{1}{20.86}$.

Having solved all fundamental constraints required in the B-L MSSM, we must now discuss the slope stability and N=1 supersymmetry of the hidden sector of the vacuum.

The Genius-One Adjusted Slope:

For the specific rank 2 bundle

$$V = L \oplus L^{-1} \ , \ L = \mathcal{O}_X(2,1,3) \, .$$

on the B-L MSSM CY threefold, the $\mathcal{O}(\kappa_{11}^{2/3})$ corrected Fayet-Iliopoulos term is

$$FI = \frac{\epsilon_S \epsilon_R^2}{2\kappa_4^2} \frac{1}{\hat{R} V^{2/3}} \left(\tfrac{1}{3}(a^1)^2 + \tfrac{2}{3}(a^2)^2 + 8a^1a^2 + 4a^2a^3 + 2a^1a^3 - 13.35\right)$$

It follows that the genus-one corrected slope is given by

$$\mu(L) = \tfrac{1}{3}(a^1)^2 + \tfrac{2}{3}(a^2)^2 + 8a^1a^2 + 4a^2a^3 + 2a^1a^3 - 13.35$$

Hence, $V = L \oplus L^{-1}$ will be polystable if and only if the Kahler moduli satisfy

$$\tfrac{1}{3}(a^1)^2 + \tfrac{2}{3}(a^2)^2 + 8a^1a^2 + 4a^2a^3 + 2a^1a^3 - 13.35 = 0$$

That is

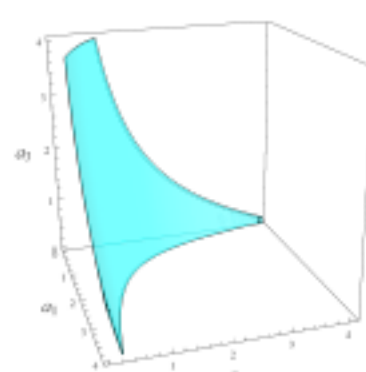

Figure 6: The surface in Kähler moduli space where the genus-one corrected slope of the hidden sector line bundle $L = \mathcal{O}_X(2,1,3)$ vanishes.

Intersecting the region of Figure 5 satisfying all required constraints and the region of Figure 6 required for slope-stability gives

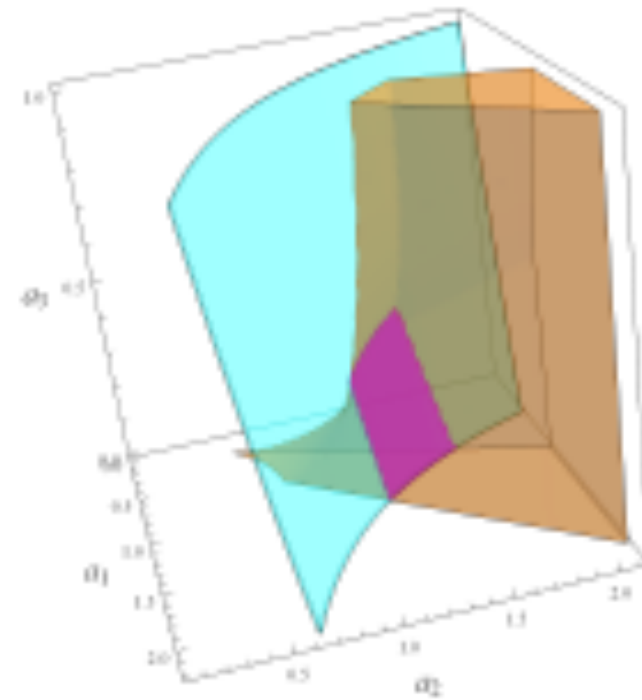

Figure 7: The magenta region shows the intersection between the brown region of Figure 5 and the two-dimensional cyan surface in Figure 6.

The “magenta” region solves all required vacuum, dimensional and physical constraints with the hidden sector rank 2 bundle satisfying the Hermitian Yang-Mills equations and being N=1 supersymmetric!

Having found a hidden sector vector bundle satisfying all mathematical and physical requirements we can now compute its low energy spectrum.

Low Energy Spectrum:

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Using the Euler Characteristic we find that

$U(1) \times E_7$	Cohomology	Index χ
$(0, \underline{133})$	$H^*(X, \mathcal{O}_X)$	0
$(0, \underline{1})$	$H^*(X, \mathcal{O}_X)$	0
$(-1, \underline{56})$	$H^*(X, L)$	8
$(1, \underline{56})$	$H^*(X, L^{-1})$	-8
$(-2, \underline{1})$	$H^*(X, L^2)$	58
$(2, \underline{1})$	$H^*(X, L^{-2})$	-58

and, hence, that the $U(1) \times E_7$ hidden sector massless spectrum for $L = \mathcal{O}_X(2,1,3)$ is

$$1 \times (0, \underline{133}) + 1 \times (0, \underline{1}) + 8 \times (1, \underline{56}) + 58 \times (2, \underline{1})$$

corresponding to one vector supermultiplet transforming in the adjoint representation of E_7, one $U(1)$ adjoint representation vector supermultiplet, eight chiral supermultiplets transforming as $(1, \underline{56})$ and 58 chiral supermultiplets transforming as $(2, \underline{1})$.

N=1 SUSY Breaking via Gaugino Condensation:

In the previous section, we reviewed the constraints imposed on a heterotic M-theory vacuum whose hidden sector is defined by a single line bundle L with its $U(1)$ structure group embedded into the $SU(2)$ subgroup of $SU(2) \times E_7 \subset E_8$ via the induced vector bundle $L \oplus L^{-1}$. We demanded that $d = 4$, $N = 1$ supersymmetry be exactly preserved. In this section, however, we will analyze how spontaneous supersymmetry breaking in four dimensions can occur due to gaugino condensation of E_7 in the hidden sector.

Gaugino Condensation:

As is well known, if the E_7 gauge group of the hidden sector becomes strongly coupled below the compactification scale, then the associated gauginos condense and produce an effective superpotential for the relevant geometric moduli of the theory – in our case, the dilaton S and the complexified Kähler moduli T^i, $i = 1, 2, 3$. The form of this gaugino condensate superpotential is given by

$$W = \langle M_U \rangle^3 \exp\left(-\frac{6\pi}{b_L \hat{\alpha}_{\mathrm{GUT}}} f_2\right)$$

where for any line bundle L embedded as above

$$f_2 = S + \frac{\epsilon_S'}{2}\left(-(2,2,0)_i - d_{ijk} l^j l^k\right) T^i$$

Note that

$$\mathrm{Re}\, T^i = t^i = \frac{\hat{R}}{V^{1/3}} a^i, \;\; i = 1,2,3 \;\; \text{and} \;\; \mathrm{Re}\, S = V + \frac{\epsilon'_S}{2}(\tfrac{1}{2}+\lambda)^2 W_i t^i$$

where ϵ'_S is the strong coupling parameter $\propto \kappa_{11}^{2/3}$. As above $\langle M_U \rangle = 3.15 \times 10^{16}$ GeV.

The beta function coefficient b_L depends explicitly on the low energy spectrum of the hidden sector bundle L.

The Condensation Scale:

For an arbitrary momentum p below the unification scale, the renormalization group equation for the hidden sector gauge parameter $\alpha^{(2)}$ is given by

$$\alpha^{(2)}(p)^{-1} = \langle \alpha_u^{(2)} \rangle^{-1} - \frac{b_L}{2\pi} \ln\left(\frac{\langle M_U \rangle}{p} \right)$$

When $b_L > 0$ condensation can, in principle, occur.

In this case, roughly speaking, the hidden sector E_7 gauge theory becomes strongly coupled and, hence, its gauginos condense, at a momentum $p \approx \Lambda$ where $\alpha^{(2)}(\Lambda)^{-1}$ can be well approximated by 0. It then follows from the above that

$$\langle \alpha_u^{(2)} \rangle^{-1} = \frac{b_L}{2\pi} \ln\left(\frac{\langle M_U \rangle}{\Lambda} \right)$$

The condensation scale Λ can then be expressed as

$$\Lambda = \langle M_U \rangle e^{\frac{-2\pi}{b_L} \langle \alpha_u^{(2)} \rangle^{-1}} = \langle M_U \rangle e^{\frac{-2\pi}{b_L} \frac{\mathrm{Re}\, f_2}{\mathrm{Re}\, f_1 \langle \alpha_u \rangle}}$$

Note that the condensate superpotential above can now be expressed as

$$W = \Lambda^3 e^{-i \frac{6\pi}{b_L \hat{\alpha}_{GUT}} \mathrm{Im}\, f_2}$$

The supersymmetry breaking in the S and T^i moduli is then gravitationally mediated to the observable matter sector. The scale of SUSY breaking in the low-energy observable matter sector is then of order

$$m_{\mathrm{susy}} \sim \kappa_4^2 \Lambda^3 = 8\pi \frac{\Lambda^3}{M_P^2}$$

Our Specific $L = \mathcal{O}_X(2,1,3)$ Example:

In this case $\langle \alpha_u \rangle = \frac{1}{26.46}$ **and**

$$\mathrm{Re}\, f_1 = V + \frac{1}{3}a^1 - \frac{1}{6}a^2 + 2a^3 + \frac{1}{2}(\tfrac{1}{2} - \lambda)^2(9a^1 + 17a^2)$$

$$\mathrm{Re}\, f_2 = V - \frac{29}{6}a^1 - \frac{25}{3}a^2 - 2a^3 + \frac{1}{2}(\tfrac{1}{2} + \lambda)^2(9a^1 + 17a^2)$$

where $\lambda = 0.49$ **and**

$$V = \frac{1}{6}((a^1)^2 a^2 + a^1 (a^2)^2 + 6a^1 a^2 a^3)$$

For the low energy spectrum given above, we find that

$$b_L = 6 \ (> 0)$$

Plotting $m_{\text{susy}} \sim \kappa_4^2 \Lambda^3 = 8\pi \frac{\Lambda^3}{M_P^2}$ over the the viable "magenta" region of Kahler moduli space gives

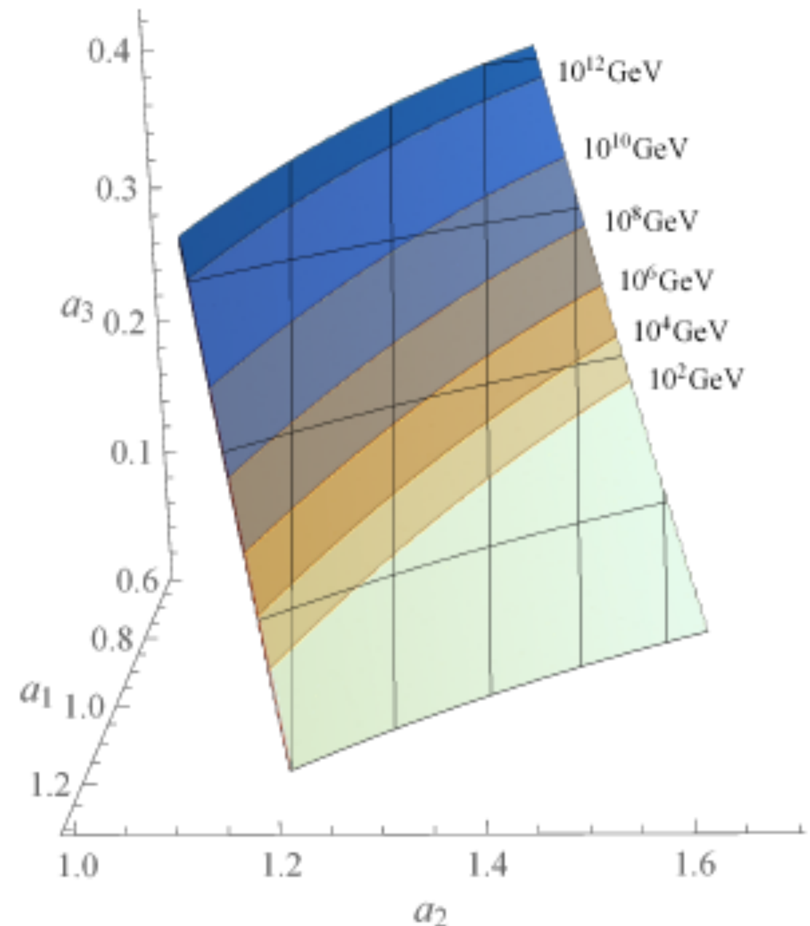

Figure 8: Variation of the mass scale $m_{\text{susy}} \sim 8\pi\Lambda^3/M_p^2$ of the soft breaking terms across the "viable" region of Kähler moduli space space that satisfies all constraints for the line bundle $L = \mathcal{O}_X(2,1,3)$. The numbers indicate the m_{susy} value corresponding to each contour. m_{susy} scales below the EW scale $\sim 10^2$ GeV become unphysically small and, therefore, are not displayed.

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Soft SUSY Breaking in the Low Energy Effective Theory:

Defining the index a which runs over (S, T^1, T^2, T^3, Z), the F-terms are given by

$$\bar{F}^{\bar{b}} = \kappa_4^2 e^{\hat{K}/2} \hat{K}^{\bar{b}a} (\partial_a W + W \partial_a \hat{K}) \ ,$$

where **the Kahler potential** $\hat{K}$ is given by

with

$$\hat{K} = \tilde{K}_S + K_T \ ,$$

$$\tilde{K}_S = -\ln\left(S + \bar{S} - \frac{\epsilon'_S}{2} \frac{(Z+\bar{Z})^2}{W_i (T+\bar{T})^i}\right) \ , \quad K_T = -\ln\left(\tfrac{1}{48} d_{ijk} (T+\bar{T})^i (T+\bar{T})^j (T+\bar{T})^k\right)$$

and $W_i - (9, 17, 0)|_i \geq 0$ for each $i = 1, 2, 3$.

The superpotential of the observable sector evaluated at $\langle M_U \rangle$ is given by

$$W = \mu H_u H_d + Y_u Q H_u u^c - Y_d Q H_d d^c - Y_e Q H_d e^c + Y_\nu Q H_u \nu^c$$

which we rewrite in the form

$$W = \frac{1}{2} \hat{\mu}_{IJ} C^I C^J + \frac{1}{3} \hat{Y}^{IJK} C^I C^J C^K \ ,$$

where C^I are the chiral superfields associated with the top and bottom quarks, the tau lepton and the up- and down-Higgs particles.

$\hat{\mu}_{IJ}$ is symmetric and all components vanish with the exception of $\hat{\mu}_{H_u H_d} = \mu$. Similarly, the coefficients $\hat{Y}_{IJK}$ are completely symmetric and are given by

$$\hat{Y}_{IJK} = 2\sqrt{2\pi\hat{\alpha}_{GUT}}\, y_{IJK} ,$$

where y_{IJK} are the physical Yukawa parameters for the top, bottom and tau particles scaled up to $\langle M_U \rangle$.

The soft supersymmetry breaking terms associated with the observable sector of the $B-L$ MSSM were found to be of the form

$$\begin{aligned}
-\mathcal{L}_{\text{soft}} = & \left(\frac{1}{2}M_3\tilde{g}^2 + \frac{1}{2}M_2\tilde{W}^2 + \frac{1}{2}M_R\tilde{W}_R^2 + \frac{1}{2}M_{BL}\tilde{B}'^2\right. \\
& \left. + a_u\tilde{Q}H_u\tilde{u}^c - a_d\tilde{Q}H_d\tilde{d}^c - a_e\tilde{L}H_d\tilde{e}^c + a_\nu\tilde{L}H_u\tilde{\nu}^c + bH_uH_d + \text{h.c.}\right) \\
& + m_{\tilde{Q}}^2|\tilde{Q}|^2 + m_{\tilde{u}^c}^2|\tilde{u}^c|^2 + m_{\tilde{d}^c}^2|\tilde{d}^c|^2 + m_{\tilde{L}}^2|\tilde{L}|^2 + m_{\tilde{\nu}^c}^2|\tilde{\nu}^c|^2 + m_{\tilde{e}^c}^2|\tilde{e}^c|^2 \\
& + m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 .
\end{aligned}$$

which we rewrite in the form

$$-\mathcal{L}_{\text{soft}} = \left(\frac{1}{2}M_i(\lambda^i)^2 + \frac{1}{3}a_{IJK}\tilde{C}^I\tilde{C}^J\tilde{C}^K + \frac{1}{2}B_{IJ}\tilde{C}^I\tilde{C}^J + \text{h.c.}\right) + m_{I\bar{J}}^2\tilde{C}^I\bar{\tilde{C}}^{\bar{J}} ,$$

where λ^i are the gauginos for $i = 3, 2, 3R, BL$, and $\tilde{C}^I$ are the scalar components of the chiral superfields associated with the top and bottom quarks, the tau lepton and up- and down-Higgs particles.

Soft Breaking Terms:

The gravitino mass

$$m_{3/2} = \kappa_4^2 \mathrm{e}^{\hat{K}/2} |W|$$

The gaugino masses

The gaugino masses are universal $M_3 = M_2 = M_{3R} = M_{BL} = M_{1/2}$ where

$$M_{1/2} = \frac{\frac{1}{2}F^S + \epsilon'_S \frac{29}{12} F^{T^1} + \epsilon'_S \frac{25}{6} F^{T^2} + \epsilon'_S F^{T^3} - \epsilon'_S \frac{1}{2} F^Z}{2\left(V + \frac{1}{3}a^1 - \frac{1}{6}a^2 + 2a^3 + \frac{1}{2}\left(\frac{1}{2} - \lambda\right)(9a^1 + 17a^2)\right)}$$

The Yukawa couplings

$$a_{IJK} = \mathcal{A}(S, T^i, Z)\; y_{IJK} \;,$$

where y_{IJK} are the Yukawa couplings at the unification scale and $\mathcal{A}$ is the specific function of the S, T^i and Z moduli given by

$$\mathcal{A}(S, T^i, Z) = 2\sqrt{2\pi\hat{\alpha}_{GUT}}\, \mathrm{e}^{\hat{K}/2} F^a \partial_a \left(\tilde{K}_S - \frac{3}{2}\epsilon'_S \frac{(T+\bar{T})^i}{(S+\bar{S})} [X]_i \right) .$$

where

$$[X]_i = \left(\frac{2}{3}, -\frac{1}{3}, 4\right)_i + \left(1 - \frac{Z+\bar{Z}}{W_l (T+\bar{T})^l}\right)^2 W_i$$

The scalar masses

$$m^2_{I\bar{J}} = m^2_s(a^1, a^2, a^3)\mathcal{G}_{I\bar{J}} \ ,$$

where m_s^2 is a moduli-dependent function, independent of the $I, \bar{J}$ indices, given by

$$m_s^2(a^1, a^2, a^3) = \frac{1}{\hat{R}}\Big(m^2_{3/2} - \tfrac{1}{3}F^a\bar{F}^{\bar{b}}(\partial_a\partial_{\bar{b}}K_T)\Big) + \frac{m^2_{3/2}}{2\hat{R}V}\Big[\Big(\tfrac{2}{3} + 9\big(\tfrac{1}{2}-\lambda\big)^2\Big)a^1 + \Big(-\tfrac{1}{3} + 17\big(\tfrac{1}{2}-\lambda\big)^2\Big)a^2 + 4a^3\Big] - F^a\bar{F}^{\bar{b}}M^{(\epsilon'_S)}_{a\bar{b}} \ .$$

Presently, there is no way to compute $\mathcal{G}_{I\bar{J}}$ explicitly. However, the absence of flavor changing neutral currents in low-energy experiments restricts this matrix to be diagonal **and of the form**

$$\begin{gathered}
\mathcal{G}_{\tilde{Q}_1} = \mathcal{G}_{\tilde{Q}_2} = \mathcal{G}_{\tilde{Q}_3} \ , \\
\mathcal{G}_{\tilde{u}^c_1} = \mathcal{G}_{\tilde{u}^c_2} = \mathcal{G}_{\tilde{u}^c_3} \ , \qquad \mathcal{G}_{\tilde{d}^c_1} = \mathcal{G}_{\tilde{d}^c_2} = \mathcal{G}_{\tilde{d}^c_3} \ , \\
\mathcal{G}_{\tilde{L}_1} = \mathcal{G}_{\tilde{L}_2} = \mathcal{G}_{\tilde{L}_3} \ , \\
\mathcal{G}_{\tilde{\nu}^c_1} = \mathcal{G}_{\tilde{\nu}^c_2} = \mathcal{G}_{\tilde{\nu}^c_3} \ , \qquad \mathcal{G}_{\tilde{e}^c_1} = \mathcal{G}_{\tilde{e}^c_2} = \mathcal{G}_{\tilde{e}^c_3} \ , \\
\mathcal{G}_{\tilde{H}_u} \neq \mathcal{G}_{\tilde{H}_d} \ .
\end{gathered}$$

We choose each of these values independently and statistically in the range

$$\mathcal{G}_{\tilde{Q}_1, \tilde{u}^c_1, \tilde{d}^c_1, \tilde{L}_1, \tilde{\nu}^c_1, \tilde{e}^c_1, \tilde{H}_u, \tilde{H}_d} \in \left[\frac{1}{10}, 10\right] \ .$$

There is one final parameter that is thrown at random; that is

$$\tan\beta \in [1.2, 65] \ .$$

The W, Z Mass Constraint:

At low energy, to get the correct W and Z masses, the parameters must satisfy

$$\mu^2 = \frac{m_{H_u}^2 \tan^2\beta - m_{H_d}^2}{1-\tan^2\beta} - \frac{1}{2}M_Z^2 \quad , \quad \frac{2b}{\sin 2\beta} = 2\mu^2 + m_{H_u}^2 + m_{H_d}^2$$

Scaling μ and b **to the unification scale, they must satisfy the ratio**

$$\frac{b}{\mu} = \mathcal{B}(S, T^i, Z)$$

where

$$\mathcal{B}(S, T^i, Z) = \mathrm{e}^{\hat{K}/2}\left[F^a\partial_a\left(\tilde{K}_S - \frac{3}{2}\epsilon_S' \frac{(T+\bar{T})^i}{(S+\bar{S})}[X]_i\right) - m_{3/2}\right] .$$

Results:

To begin, we choose a point

$$(a^1, a^2, a^3) = (0.910, 1.401, 0.163)$$

inside the "viable" region of Kähler moduli space. **We find**

$$M_{1/2} = -13{,}290 \text{ GeV} , \quad \mathcal{A} = 2{,}170 \text{ GeV} , \quad m_s = 4{,}079 \text{ GeV} .$$

Then, throwing $\mathcal{G}_{I\bar{J}}$ and $\tan\beta$ **statistically gives**

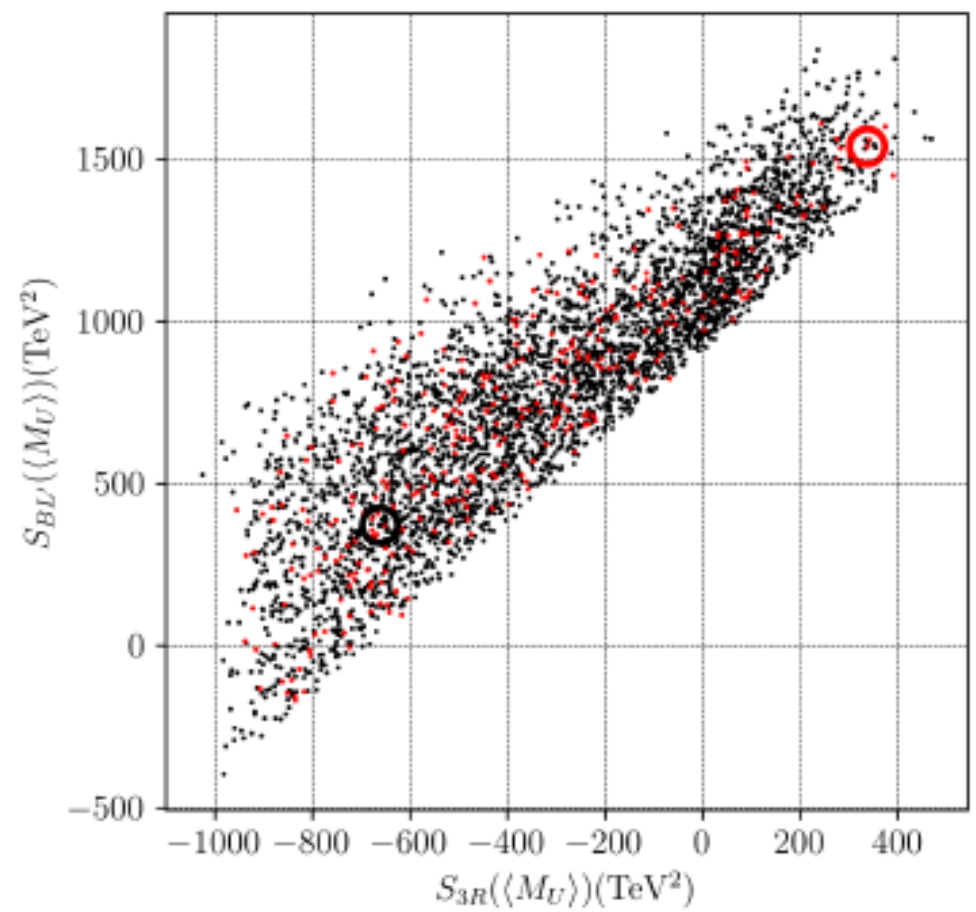

Figure 9: Plot of the "black" points obtained after running the RGE simulation at the point $(a^1, a^2, a^3) = (0.910, 1.401, 0.163)$. We statistically throw 10 million sets of random initial data. The 3,330 "black" points we obtain are further divided into two sets by color. With red we display the *acceptable* "black" points that, in addition, have the correct $\frac{b}{\mu}$ value at the unification scale. The remaining 2,974 *unacceptable* "black" points that do not satisfy this constraint are shown in black.

Full Scan:

Repeating this analysis for ALL points in the viable "magenta" region of Kahler moduli space we find

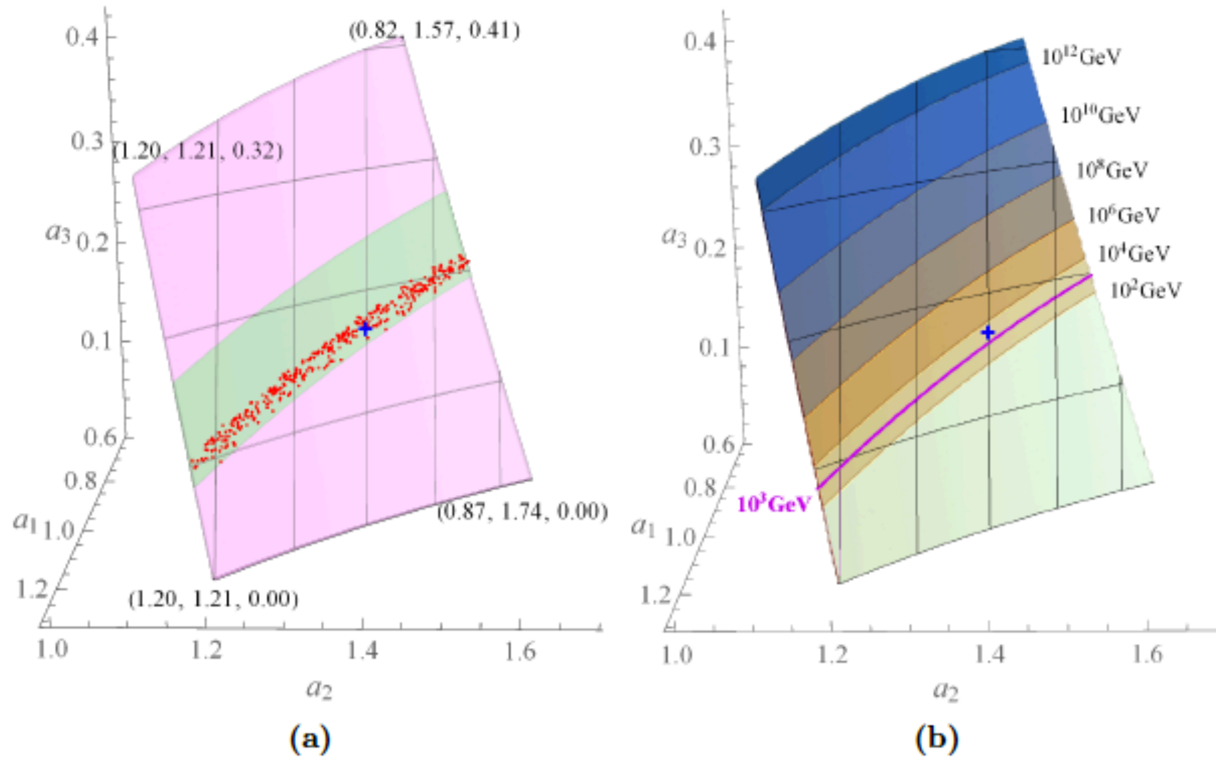

Figure 10: (a) In magenta we show the "viable" region of Kähler moduli space that satisfies all constraints for the line bundle $L = \mathcal{O}_X(2,1,3)$. The RGE simulation produces "black" points when (a^1, a^2, a^3) are sampled within the green subregion. With red we show points in this green subspace where we find at least one *acceptable* "black" point–that is, a "red" point. The region of possible *acceptable* "black" points forms a strip. The blue cross indicates the point $(a^1, a^2, a^3) = (0.910, 1.401, 0.163)$ we used for the previous examples where we found the "black" and "red" points shown in Figure 9. (b) Plot of the variation of the soft SUSY breaking parameter m_{susy} across the "viable" region of Kähler moduli space. The blue cross indicates the point $(a^1, a^2, a^3) = (0.910, 1.401, 0.163)$. The purple line is the $m_{\text{susy}} = 1$ TeV contour.