A Minimal Structure for the String Landscape

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Based on:

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2105.02232 with Brice Bastian, Damian van de Heisteeg

2107.nnnn with Erik Plauschinn, Damian van de Heisteeg

21xx.nnnn with Christian Schnell, Ben Bakker, Jacob Tsmierman

Outline

Part 1: Four lessons about the complex structure moduli space and moduli stabilization

→ Scalar field spaces and scalar potentials in Type IIB flux compactifications are remarkably constrained.

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→ Constraints are 'just enough' to ensure non-trivial finiteness property.

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Part 3: Structure ensuring finiteness

→ o-minimal structures and tame topology to describe the landscape

Flux compactifications: some lessons we learned

Type IIB / F-theory flux compactifications

- Type IIB flux compactifications review: [Graña] [Kachru,Douglas] …

background flux: $F_3, \; H_3 \in H^3(Y_3, \mathbb{Z})$ $\quad \int_{Y_3} F_3 \wedge H_3 < K$

vacuum condition: $*G_3 = iG_3$ $\quad G_3 = F_3 - \tau H_3$

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→ well studied set of N=0,1 vacua with (partially) fixes complex structure moduli, backreaction under control, higher-derivative corrections consistently included

[Becker,Becker]…[TG,Pugh,Weissenbacher]…[Cicoli,Quevedo,Savelli,Schachner,Valandro]

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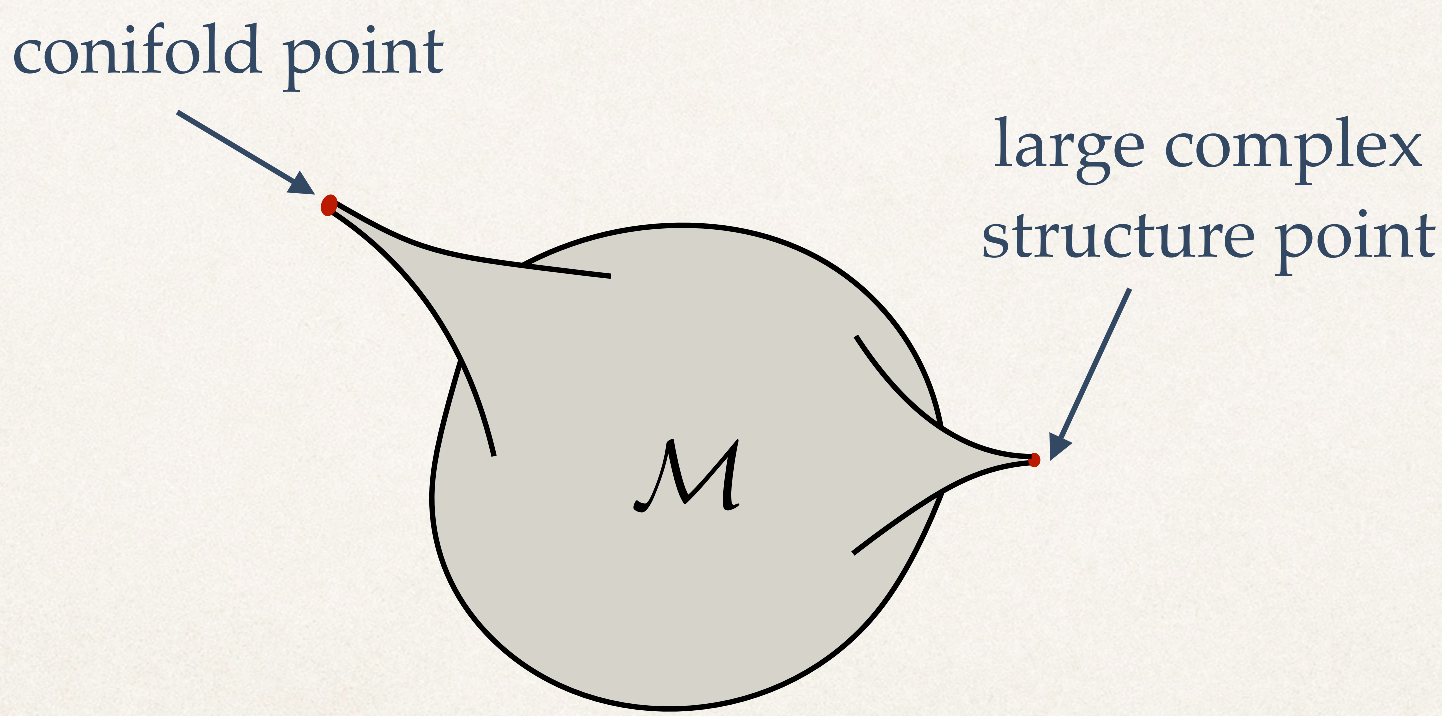

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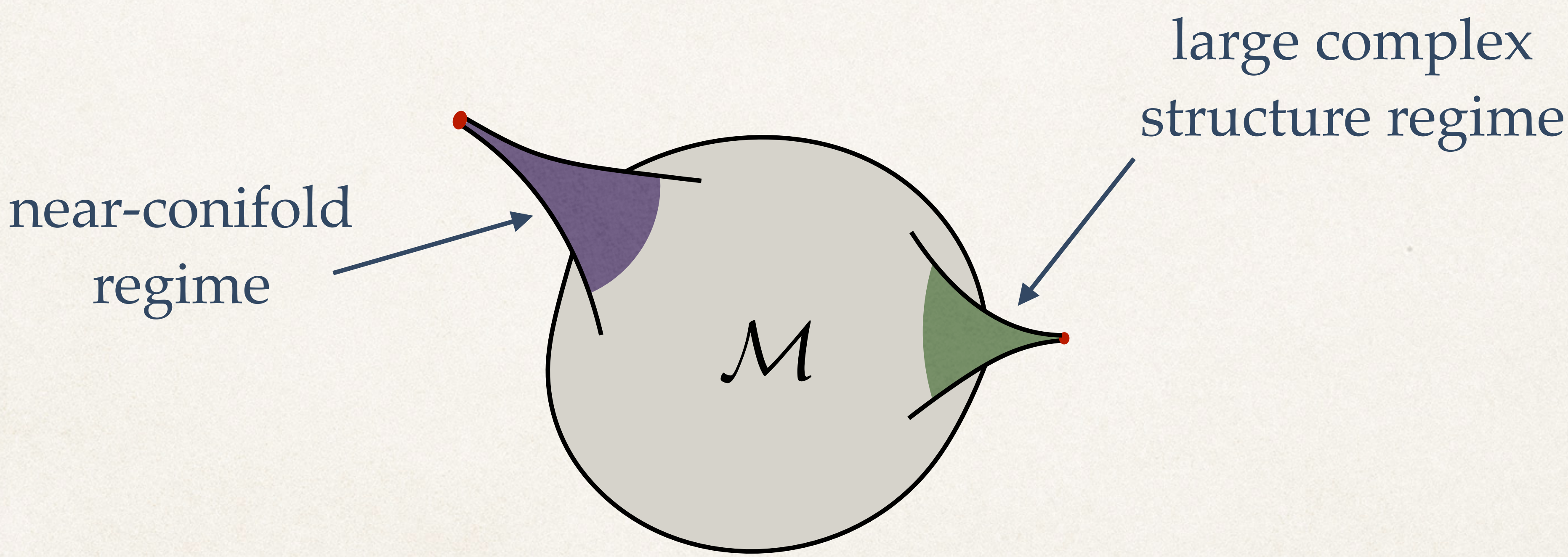

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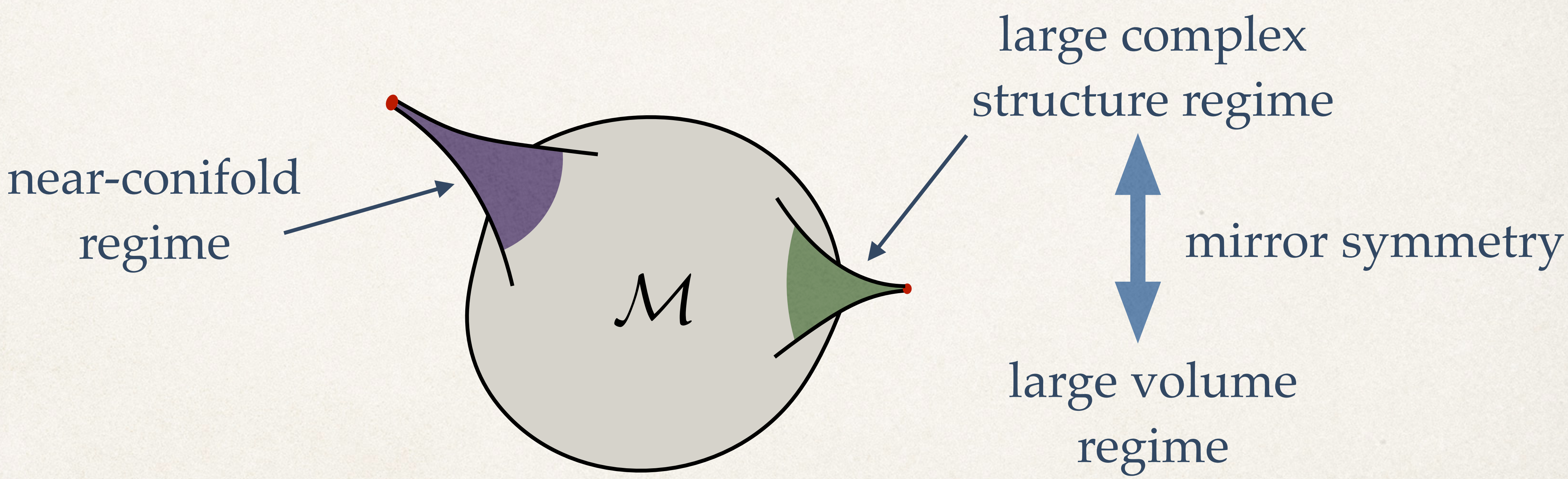
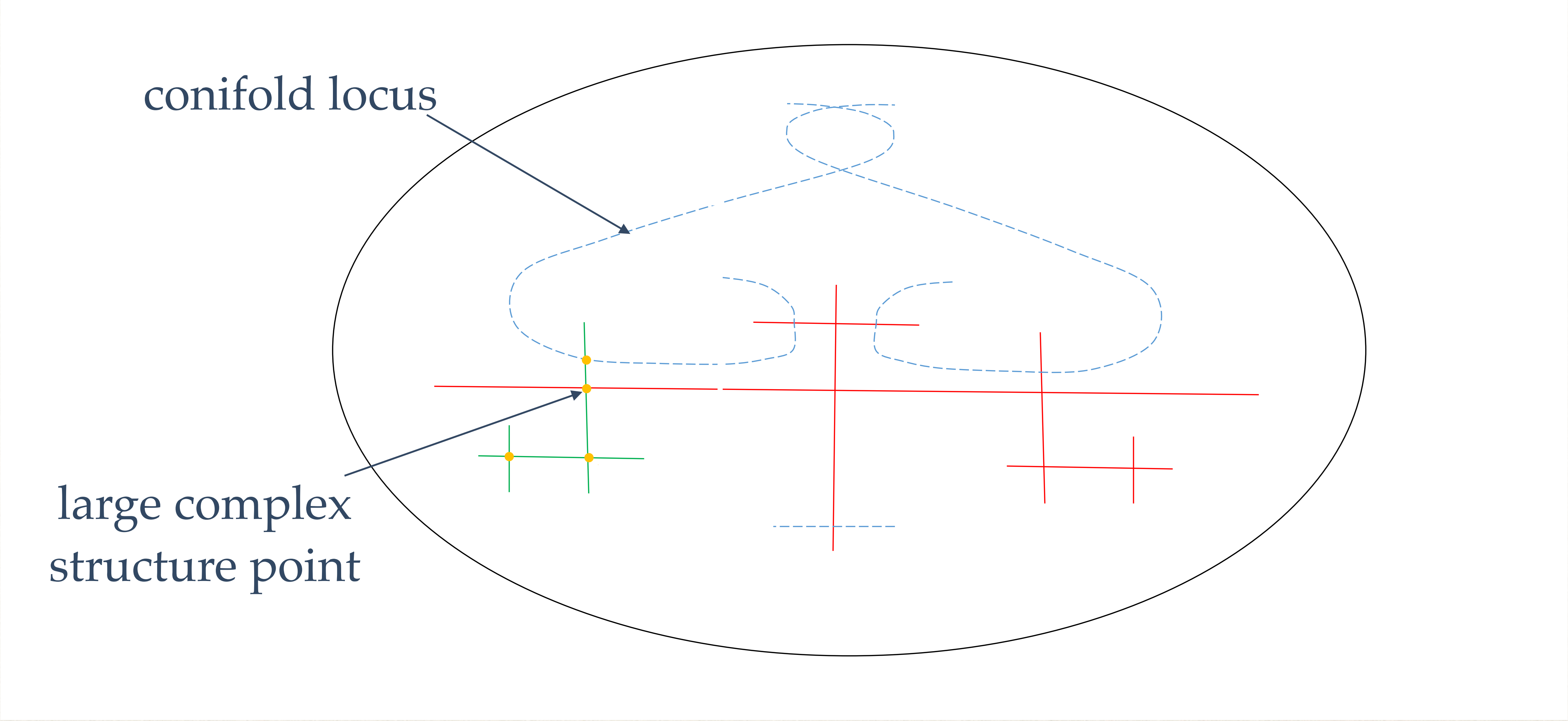


Complex structure moduli space

- Note: geometry of boundaries + asymptotic regions can be very involved for higher-dimensional moduli spaces

Example: mirror $\mathbb{P}^{1,1,1,6,9}[18]$ [Candelas,Font,Katz,Morrison]



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"Conjecture 1:" In order for a swampland conjecture to be true, it has to be true in this Type IIB setting.

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→ develop tools in asymptotic Hodge theory and apply them to test conjectures [TG,Palti,Valenzuela] [TG,Li,Palti] [TG,Li,Valenzuela]

full power starts to become apparent in our more recent works
[TG,Ruehle,vd Heisteeg],[TG],[TG,Monnee,vd Heisteeg][Bastian,TG,vd Heisteeg]

Lesson 1: Classification of boundaries

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  Middle cohomology $H^D(Y_D,\mathbb{C})$ admits boundary (p,q)-decomposition and decomposition into representations of $\mathrm{sl}(2,\mathbb{C})^n$

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Example: Y_3 sending one parameter to boundary: $\mathrm{sl}(2,\mathbb{C})$

$$H^3(Y_3,\mathbb{C}) = H^{3,0}_\infty \oplus H^{2,1}_\infty \oplus H^{2,1}_\infty \oplus H^{0,3}_\infty$$

$$H^{q,3-q}_\infty = \mathrm{span}_{\mathbb{C}}\Big\{|d,l\rangle,\ d=0,...,3;\ l=-3,...,3\Big\}$$

see, e.g., [TG '20] for details

sl(2)-highest-spin sl(2)-spin

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In general: multiple sl(2)-spins

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- Classification of boundaries using sl(2)-representations and positivity

works for all Kähler manifolds

ensure: $\int \alpha \wedge *_\infty \alpha > 0$

Calabi-Yau threefold examples

- All cases with $h^{2,1} = 1$

$$\begin{aligned} \mathrm{I}_1 : &\qquad \text{conifold point}\,, \\ \mathrm{II}_0 : &\qquad \text{Tyurin degeneration}\,, \\ \mathrm{IV}_1 : &\qquad \text{large complex structure point} \end{aligned}$$

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- All cases with $h^{2,1} = 2$

[Kerr,Pearlstein,Robles][TG,Li]

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[Alvarez-Garcia,Blumenhagen et al. '20]
[Demirtas et al. '20]

Seiberg-Witten theory

appear in [Kreuzer,Skarke] (after mirror symmetry)

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  Hodge star can be approximated using $\mathrm{sl}(2,\mathbb{C})^n$ - spins

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regime: $y_1 \gg y_2 \gg \ldots \gg y_n$ ('strict asymptotic')

$$\|\alpha\|^2 \sim \sum_{l_1,\ldots,l_n} (y^1)^{l_1-n}(y^2)^{l_2-l_1}\ldots(y^n)^{l_{n-1}-l_n}\left\|\left(e^{-x^i N_i}\alpha\right)_{l_1\ldots l_n}\right\|_\infty$$

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sl(2)-spins

$\rightarrow$ **leading, most 'crude' approximation, but easy to handle**

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- reconstruction of CY_3 periods - [Bastian,TG,vd Heisteeg]

  combine [Cattani,Kaplan,Schmid],[Fernandez,Cattani],[Brosnan,Pearlstein,Robles]

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- reconstruct periods with polynomial & essential exponential corrections

fits the conjecture of [Palti,Weigand,Vafa]

Modeling two-parameter periods

- Results for two-cubes: I_2 class : $\langle \mathrm{I}_1|\mathrm{I}_2|\mathrm{I}_1\rangle\,,\ \langle \mathrm{I}_2|\mathrm{I}_2|\mathrm{I}_1\rangle\,,\ \langle \mathrm{I}_2|\mathrm{I}_2|\mathrm{I}_2\rangle$

  $\rightarrow$ finite distance

$$\Pi = \begin{pmatrix} 1 - \frac{a^2}{8\pi k_2} z_1^{2k_1} z_2^{2k_2} - \frac{b^2}{8\pi m_1} z_1^{2m_1} z_2^{2m_2} \\ a z_1^{k_1} z_2^{k_2} \\ b z_1^{m_1} z_2^{m_2} \\ i + \frac{ia^2}{8\pi k_2} z_1^{2k_1} z_2^{2k_2} + \frac{ib^2}{8\pi m_1} z_1^{2m_1} z_2^{2m_2} \\ -\frac{a}{2\pi i} z_1^{k_1} z_2^{k_2} \big(n_1 \log[z_1] + \log(z_2) - 1/k_1\big) + ib\delta_1 z_1^{m_1} z_2^{m_2} \\ -\frac{b}{2\pi i} z_1^{m_1} z_2^{m_2} \big(\log(z_1) + n_2 \log[z_2] - 1/m_2\big) + ia\delta_1 z_1^{k_1} z_2^{k_2} \end{pmatrix}$$

parameters	$\langle \mathrm{I}_1\|\mathrm{I}_2\|\mathrm{I}_1\rangle$	$\langle \mathrm{I}_2\|\mathrm{I}_2\|\mathrm{I}_1\rangle$	$\langle \mathrm{I}_2\|\mathrm{I}_2\|\mathrm{I}_2\rangle$
log-monodromies n_1, n_2	$n_1 = n_2 = 0$	$n_1 \in \mathbb{Q}_{>0},\ n_2 = 0$	$n_1, n_2 \in \mathbb{Q}_{>0},\ n_1 n_2 \neq 1$
instanton orders k_1, k_2	$k_1 = 0, k_2 = 1$	$k_1 = n_1 k_2$	$k_1 = n_1 k_2$
instanton orders m_1, m_2	$m_1 = 1, m_2 = 0$	$m_1 = 1, m_2 = 0$	$m_2 = n_2 m_1$
instanton coefficients a, b	$a, b \in \mathbb{R} - \{0\}$		
phase operator δ	$\delta_1 \in \mathbb{R}$		

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$$\Pi = \begin{pmatrix} 1 \\ a z_1 \\ \frac{\log[z_2]}{2\pi i} \\ -\frac{i\log[z_2]^3}{48\pi^3} - \frac{i a^2 n z_1^2 \log[z_2]}{4\pi} + \frac{a^2}{4\pi i} z_1^2 + i\delta_2 + i\delta_1 a z_1 \\ -a z_1 \frac{\log[z_1] + n\log[z_2]}{2\pi i} + i\delta_1 \\ -\frac{\log[z_2]^2}{8\pi^2} - \frac{1}{2} a^2 n z_1^2 \end{pmatrix}$$

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Note: flat directions in sl(2)-approx. might be stabilized in successive steps
e.g. linear scenario [Palti,Tasinato,Ward] [Marchesano,Prieto,Wiesner]

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    → L. McAllister's talk
  - Possible for large number of moduli + fluxes → tadpole conjecture?

    → M. Graña's talk

Finiteness of self-dual flux vacua

Self-dual flux vacua

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- Important note: fix Y_4 in this discussion (finitely many CY)

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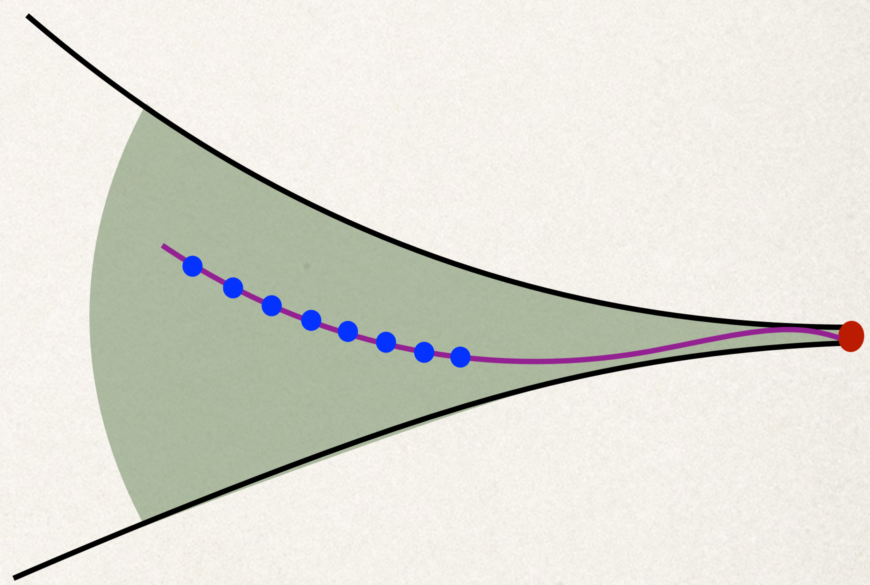
- First idea: show that on every path to every boundary only finitely many vacua can arise



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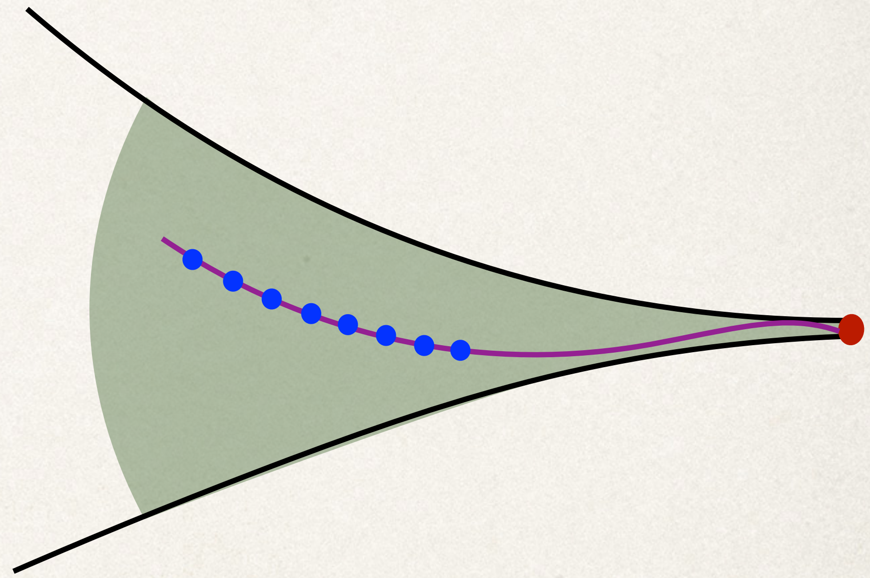
- Important note: fix Y_4 in this discussion (finitely many CY)
- Evidence for finiteness of flux choices:
  [Ashoke,Douglas],[Douglas,Shiffman,Zelditch],[Douglas,Lu] using vacuum density
- First idea: show that on every path to every boundary only finitely many vacua can arise



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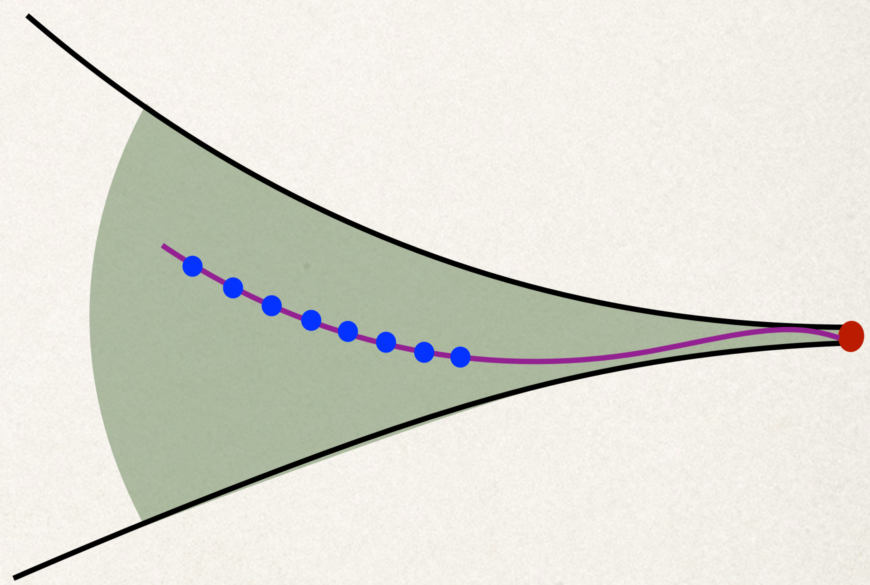

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- general story is orders of magnitude more complicated

General proof - What is behind this?

- Susy (2,2)-fluxes → Hodge classes $H^4(Y,\mathbb{Z}) \cap H^{2,2}(Y,\mathbb{C})$
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use sl(2)-techniques to control every path to every boundary
however: they use holomorphicity ('Susy vaccum')

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  → main results:

  (1) coset spaces $\Gamma\backslash G/K$ have a certain tame topology

  (2) period map is special map that is 'tame' near boundaries

  shown by using sl(2)-techniques

  (3) alternative proof to the theorem of [Cattani,Deligne,Kaplan]

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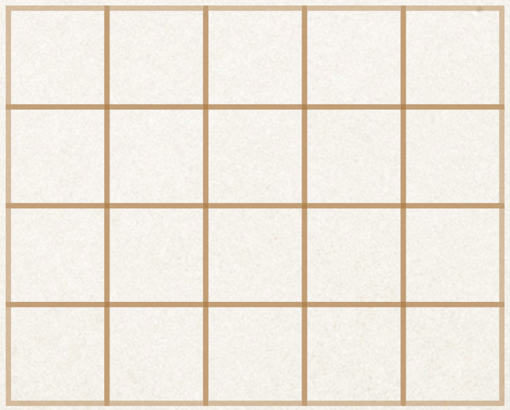

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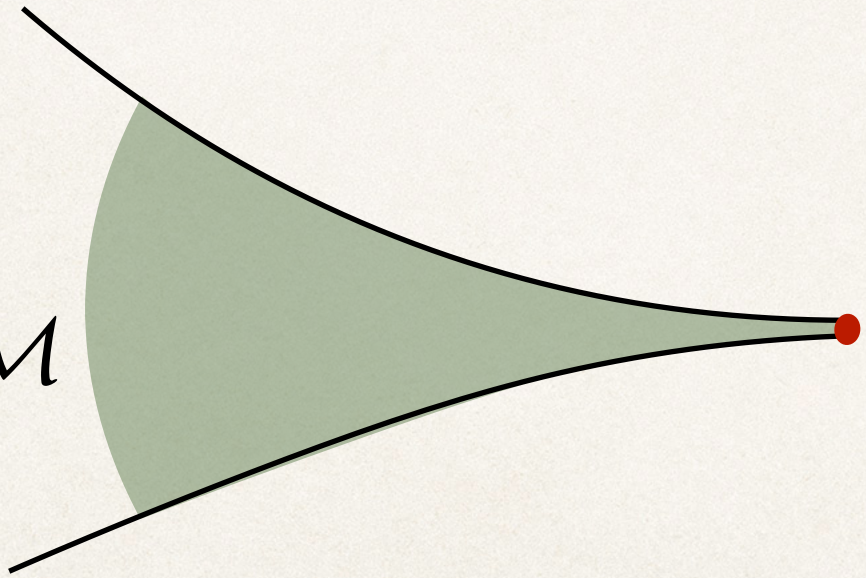

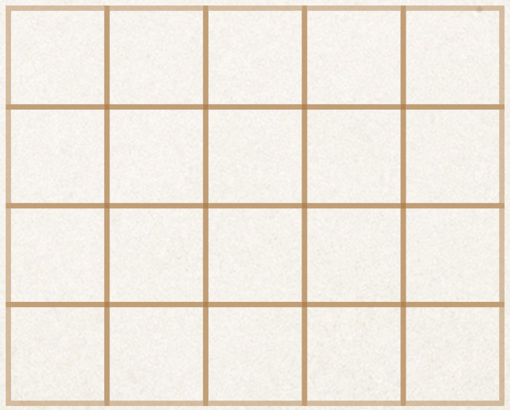

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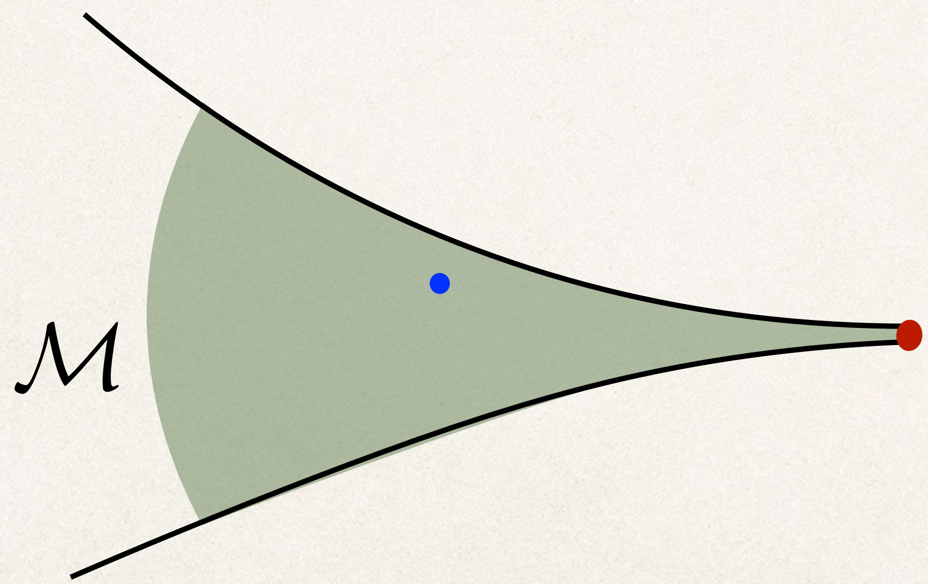

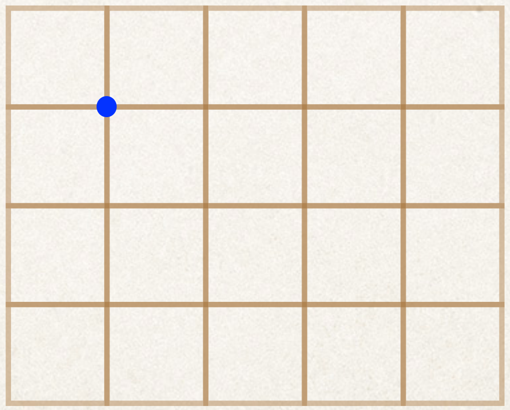

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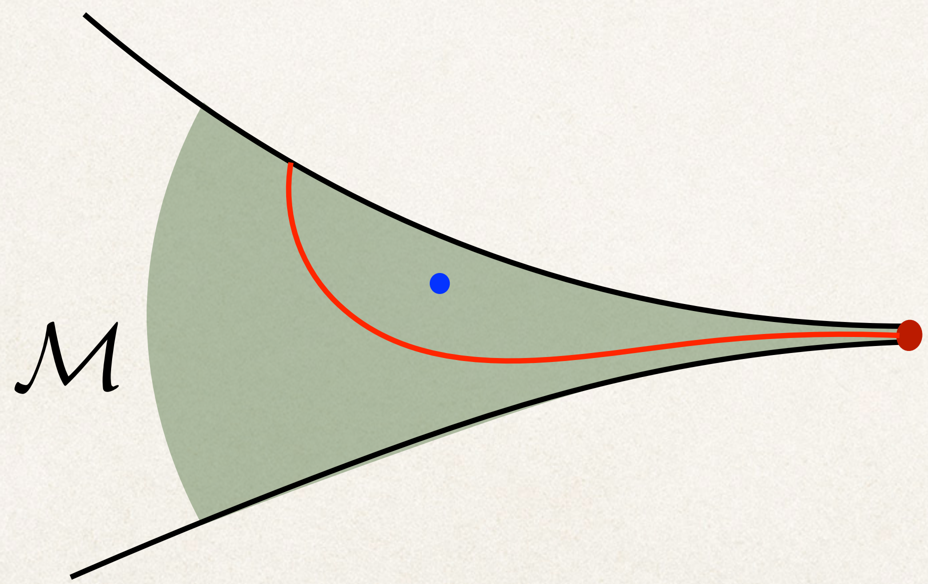

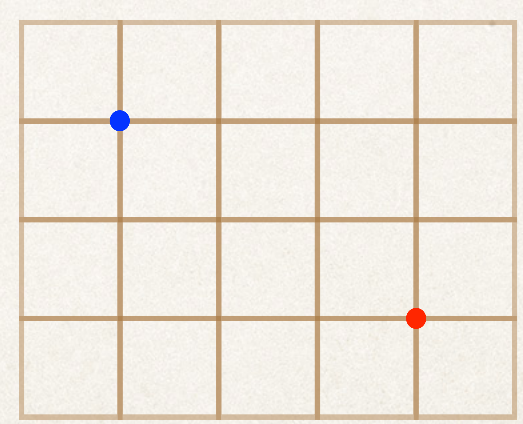

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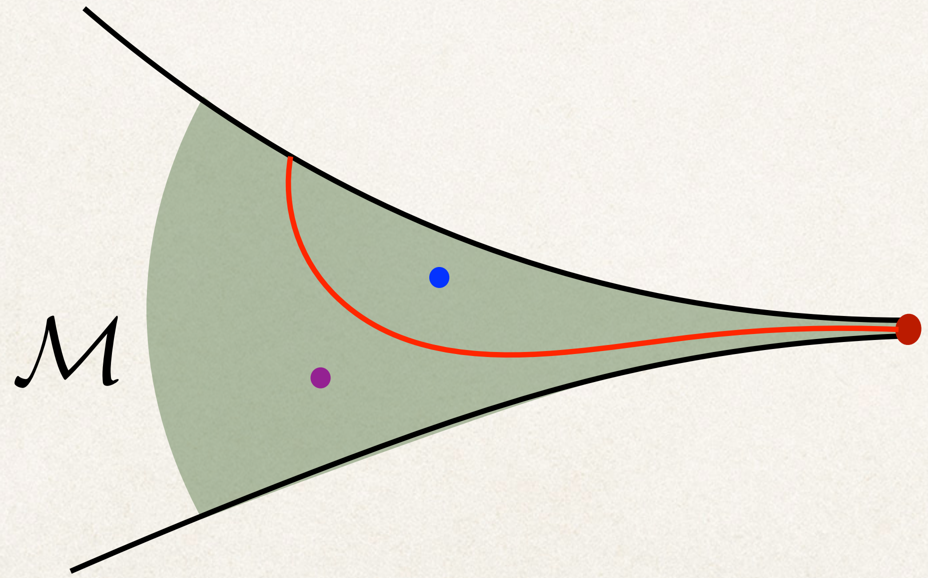

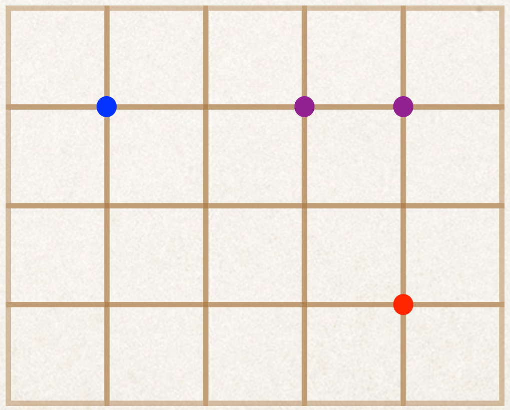

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Minimal structure for the landscape

Finiteness in the landscape

- finiteness questions are long part of the string phenomenology program

  e.g. [Douglas '03] [Acharya,Douglas '06]

- 16, 32 supercharges: finite rank of gauge groups and finite number of massless modes

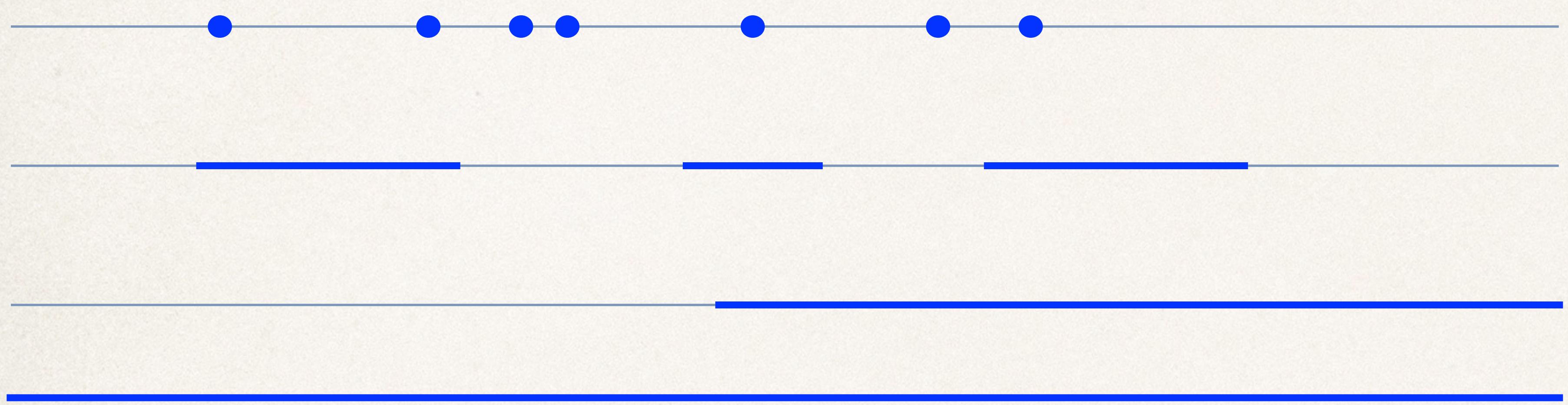
  [Adams,DeWolfe,Taylor] [Kim,Shiu,Vafa] [Kim,Tarazi,Vafa] [Cvetic,Dierigl,Lin,Zang] [Dierigl,Heckman] [Font,Fraiman,Grana,Nunez,DeFreitas] [Hamada,Vafa]

- 8 supercharges: finite rank gauge groups, finite number of massless modes

  [Taylor etal],[Kim,Shiu,Vafa],[Lee,Weigand],[Tarazi,Vafa]

Finite subsets on the real line

- simplest situation: finite subsets of real line



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finitely many intervals

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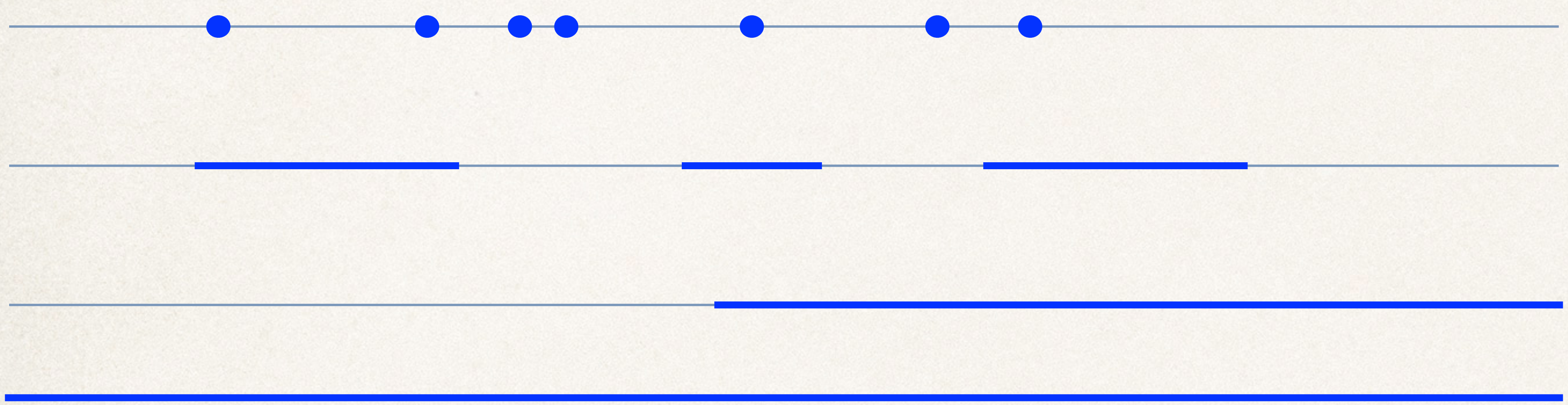

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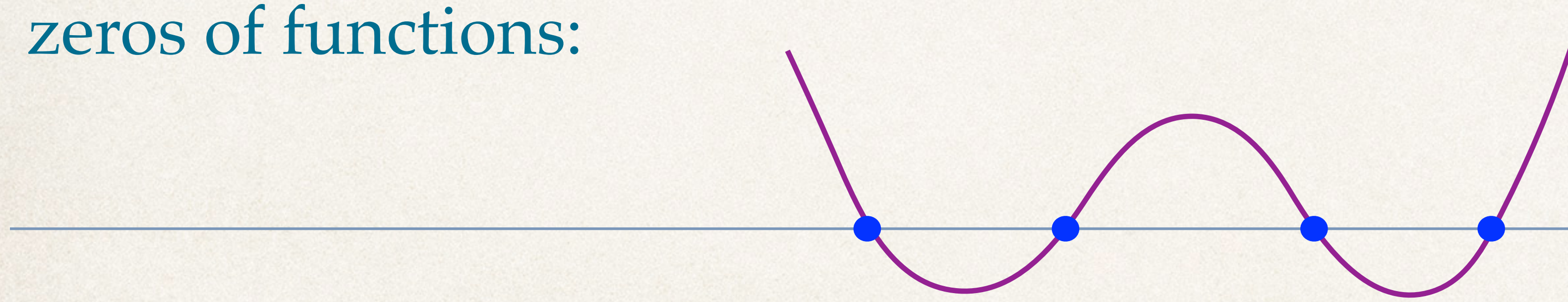

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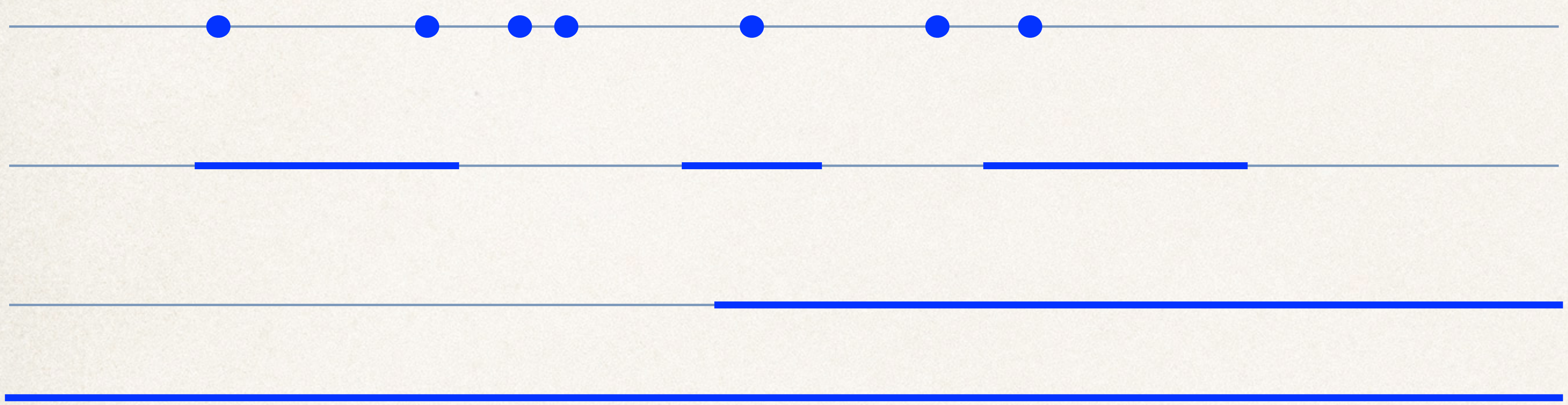

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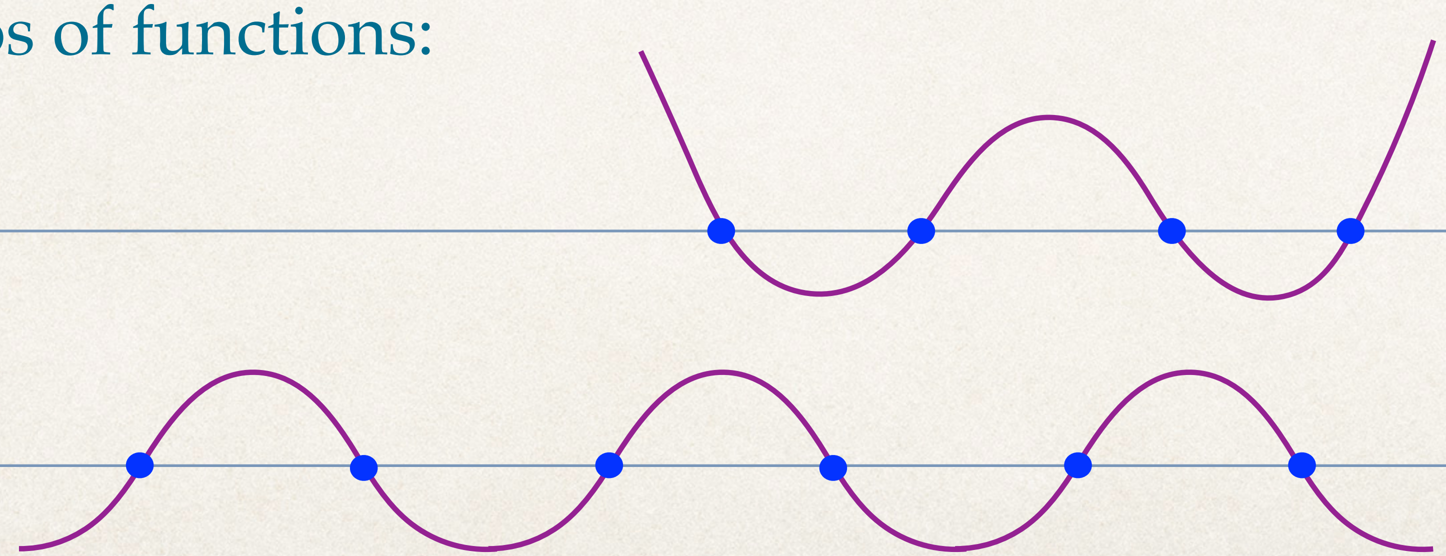

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sin(x)

O-minimal structure as a tame topology

- Idea: build a structure $\mathcal{S}$ of sets $\{S_n\}_{n=0,1,..}$:
  - S_n are subsets of $\mathbb{R}^n$
  - S_n is closed under finite intersections, finite unions and complements
  - collection $\{S_n\}$ closed under finite Cartesian products & coordinate projections
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- theory of o-minimal structures gives a generalization of semi-algebraic geometry and realizes Grothendieck's dream of a tame topology

book [van den Dries]

O-minimal structure as a tame topology

- Some remarkable examples:
  - structure generated by graphs of real polynomials: $\mathbb{R}_{\mathrm{alg}}$
  - $\mathbb{R}_{\mathrm{alg}}$ plus graphs of real analytic functions restricted to ball: $\mathbb{R}_{\mathrm{an}}$
  - $\mathbb{R}_{\mathrm{alg}}$ plus graph of exponential function: $\mathbb{R}_{\mathrm{exp}}$ [Wilikie '96]
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- Note 2: structures forbid accumulation points of zeros near $\phi = 0$

  $V(\phi) = \sin(\phi^{-1})$ $\qquad$ $V(\phi) = \phi^8 \sin(\phi^{-1})$ discussed by [Acharya,Douglas]

A conjecture

Set of effective theories arising from string theory

collect vectors:

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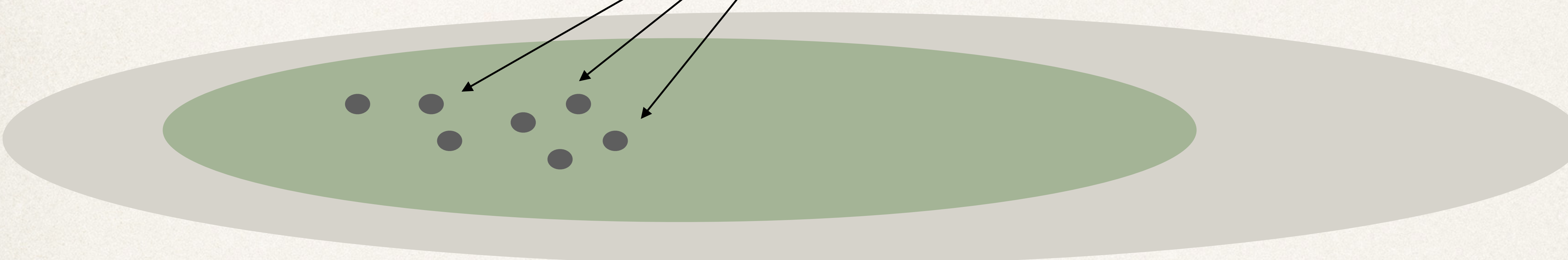

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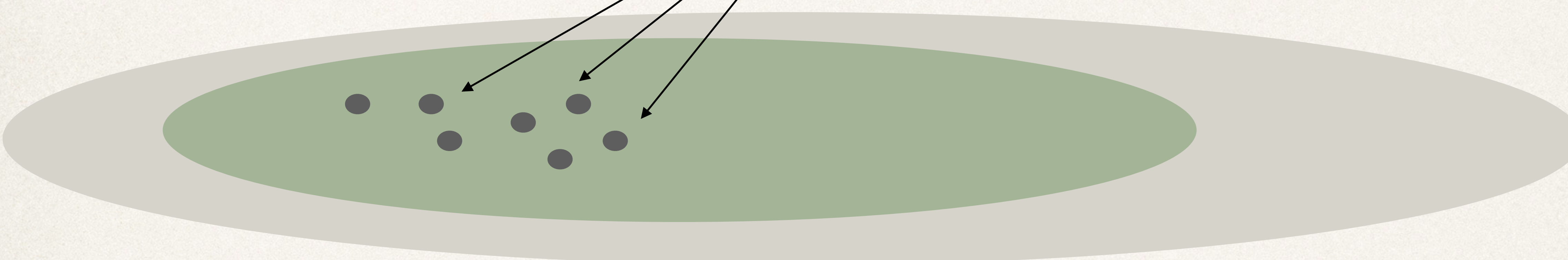

Conjecture 2: The string vacuum landscape is definable in an $\mathbb{R}_{\mathrm{an,exp}}$ o-minimal structure.

can be true (assume Conjecture 1)

Conclusions

- Uncover the structure of complex structure moduli space using asymptotic Hodge theory

  - sl(2) structure allows for a classification of boundaries
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- Highly non-trivial finiteness result for the number of self-dual flux vacua

- Suggested to use o-minimal structure to describe the string theory vacuum landscape

  $\Rightarrow$ build-in finiteness properties
  $\Rightarrow$ general enough for also non-supersymmetric situations