Chiral Indices and Resolution Independent Structure in 4D F-theory Vacua

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Introduction

Recent work on analyzing F-theory compactifications on elliptic CY4 with flux, described at low energies by 4D $\mathcal{N} = 1$ supergravity.

Nontrivial flux background permits chiral excess transforming in representation $\mathcal{R}$ of 4D gauge group [Beasley-Heckman-Vafa; Donagi-Wijnholt '08]:

$$\chi_{\mathcal{R}} \equiv N_{\mathcal{R}} - N_{\mathcal{R}^*} \neq 0$$

where $N_{\mathcal{R}}$ ($N_{\mathcal{R}^*}$) is the number of 4D chiral (antichiral) multiplets.

Some motivations

Chiral spectra of the $(\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1))/\mathbb{Z}_6$ model [Taylor-Turner, Raghuram-Taylor-Turner '19]. Broadest class of F-theory models with *generic* [Taylor-Turner '19] matter and tuned SM gauge symmetry. Special case: F_{11} model [Klevers-Mayorga Pena-Oehlmann-Piragua-Reuter '15] admitting $\mathcal{O}(10^{15})$ SM chiral spectra [Cvetič-Grassi-Klevers-Piragua '14]

Landscape versus swampland, i.e. what set of 4D $\mathcal{N}=1$ anomaly free theories can be UV completed in F-theory?

Overview of results

I discuss features of a new approach to flux compactifications that combines various techniques appearing throughout the literature with a less explored mathematical characterization of the problem.

I hope to convince you that there is something to be gained by revisiting what is by all appearances a thoroughly studied problem.

Overview of results

Combination of computational methods: topological intersection numbers via pushforward formulas [Aluffi '10; Esole-PJ-Kang '17], matching with one-loop exact Chern-Simons couplings [Grimm-Hayashi '11, Cvetič-Klevers-Grimm '13].

We compute vertical fluxes and chiral indices for numerous F-theory models over arbitrary threefold bases B, collecting evidence that an F-theory model $(\mathcal{G}, \mathcal{R})$ realizes all anomaly free 4D chiral matter spectra.

Overview of results

Mathematical characterization: vertical flux backgrounds belong to a lattice Λ equipped with a symmetric bilinear pairing

$$M : \Lambda \times \Lambda \to \mathbb{Z}$$

We find evidence (and conjecture) that Λ (whose non-degenerate part is isomorphic to a particular homology subgroup of smooth elliptic CY4s) is **resolution independent**.

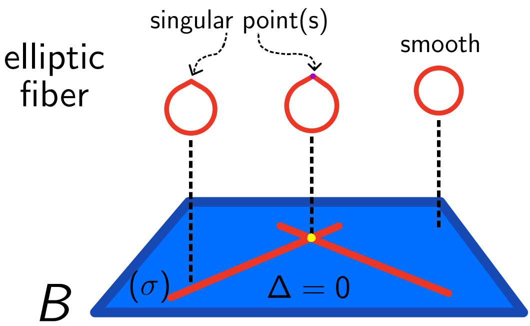
Resolution independence of Λ may facilitate more complete explorations of F-theory flux vacua

Basic setup

Consider F-theory on singular elliptic CY4

$$X_0 \to B, \qquad y^2 = x^3 + fx + g$$

with gauge group $\mathcal{G}$, charged matter $\mathcal{R}$, and flux background.



Resolution of singularities

Compactify on a circle and switch on gauge holonomies $\varphi^i = \int_{S^1} A^i$ to move onto the Coulomb branch and resolve the singularities,

$$X \to X_0.$$

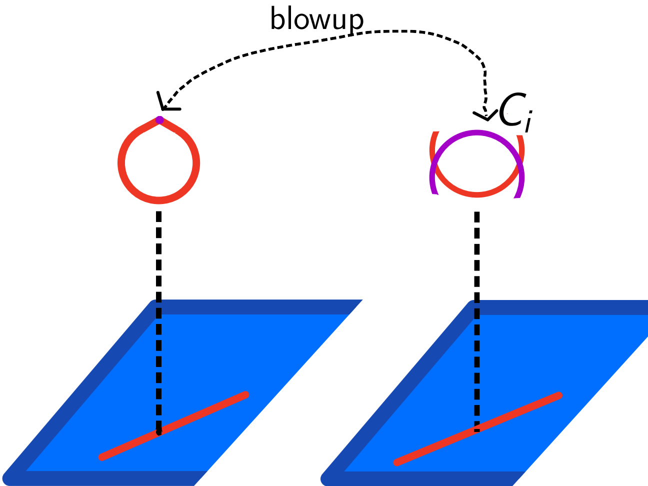
Resolution can be viewed as moving to a generic point of the Kähler moduli space of X.

We then use duality with M-theory compactified on X. The F-theory flux background maps to a nontrivial background $G_4 = \mathrm{d}C_3$. We determine the chiral spectrum from the geometry of X.

Resolution of singularities

Resolution of SU(2) model

Cartan divisor $C_i \hookrightarrow D_i \to \Sigma$ behaves like simple coroot



M-theory on CY4 + flux

G_4 consistency conditions studied extensively

[Becker-Becker; Dasgupta-Mukhi; Dasgupta-Rajesh-Sethi; Sethi-Vafa-Witten; Witten 96; Gukov-Vafa-Witten; Freed-Witten 99; Haack-Lewis 01; others]. Preserving SUSY requires minimizing $W(\Omega) + \tilde{W}(J)$, which implies

$$G_4 \in H^{2,2}(X, \mathbb{R}) \cap H^4(X, \mathbb{Z}/2)$$

I further assume G_4 lives in the subgroup

$$H^{2,2}_{\text{vert}}(X, \mathbb{Z}) \subset \cdots \oplus H^{2,2}_{\text{vert}}(X, \mathbb{C}) \oplus H^{2,2}_{\text{hor}}(X, \mathbb{C}) \oplus H^{2,2}_{\text{rem}}(X, \mathbb{C}) \oplus \cdots$$

generated by $\text{PD}(D_I) \wedge \text{PD}(D_J)$ where D_I are divisors of X. Vertical fluxes are then given by

$$\Theta_{IJ} = \int_{[D_I \cap D_J]} G_4, \qquad [D_I \cap D_J] \in H_{2,2}^{\text{vert}}(X, \mathbb{Z}).$$

Lifting to F-theory

To lift to F-theory, take X to be elliptically fibered. We then have canonical basis of divisors $D_I = D_A, D_\alpha, D_i \subset X$ where D_α are pullbacks from B [Shioda-Tate, Wazir].

Conditions to preserve local Lorentz and gauge symmetry:

$$\int_{[D_I \cap D_\alpha]} G_4 = 0.$$

Chiral indices given by integrals over homology classes (matter surfaces) $S_{\mathcal{R}} = x_{\mathcal{R}}^{IJ}[D_I \cap D_J]$ [Donagi-Wijnholt 08; Marsano-Schafer-Nameki; Braun-Collinucci-Valandro; Krause-Mayrhofer-Weigand; Grimm-Hayashi 11]:

$$\chi_{\mathcal{R}} = N_{\mathcal{R}} - N_{\mathcal{R}^*} = \int_{S_{\mathcal{R}}} G_4.$$

Computing chiral indices

We can use intersection theory to evaluate intersection pairing $\langle , \rangle$:

$$\int_{[D_I \cap D_J]} G_4 = \langle \mathrm{PD}(G_4), [D_I \cap D_J] \rangle = \mathbb{G}^{KL} D_K \cdot D_L \cdot D_I \cdot D_J$$

This can be computed via pushforward, $\pi : X \to B$ [Aluffi '10; Esole-PJ-Kang '17]:

$$\pi_*(D_K \cdot D_L \cdot D_I \cdot D_J) = W_{KLIJ}^{\alpha\beta\gamma} D_\alpha \cdot D_\beta \cdot D_\gamma .$$

If we know $S_{\mathcal{R}}^{IJ}[D_I \cap D_J]$ then we are done. Or, we can compare to low energy 3D physics [Grimm-Hayashi '11, Cvetič-Klevers-Grimm '13]:

$$\int_{[D_i \cap D_j]} G_4 = \Theta_{ij}^{3\mathrm{D}} = x_{ij}^{\mathcal{R}} \chi_{\mathcal{R}}, \qquad \mathcal{L}^{3\mathrm{D}} \supset \Theta_{ij}^{3\mathrm{D}} A^i \wedge F_j$$

Example: Simple Tate models

$$y^2z + a_1xyz + a_3yz^2 - (x^3 + a_2x^2z + a_4xz^2 + a_6z^3) = 0.$$

Tune gauge symmetry on 7-branes wrapping $\sigma = 0$ in arbitrary B by enforcing $a_s = a_{s,m_s}\sigma^{m_s}$.

Kodaira fiber	gauge group	min. chiral index	constraints
I_5^s	SU(5)	$n(\sigma)\cdot(a_1)\cdot(a_{6,5})$	$\chi_{\mathbf{5}} + \chi_{\mathbf{10}} = 0$
I_6^s	SU(6)	$n(\sigma)\cdot(a_1)\cdot(a_{4,3}^2)$	$\chi_{\mathbf{6}} + 2\chi_{\mathbf{15}} = 0$
I_6^s	SU(6)$^\circ$	$n(\sigma)\cdot(a_1)\cdot(a_{3,2}^3)$	$\chi_{\mathbf{6}} = 0$
I_7^s	SU(7)	$n(\sigma)\cdot(a_1)\cdot(a_{6,7})$	$\chi_{\mathbf{7}} + 3\chi_{\mathbf{21}} = 0$
I_6^{ns}	Sp(6)	0	—
I_1^{*s}	SO(10)	$n(\sigma)\cdot(a_{2,1})\cdot(a_{6,5})$	any $\chi_{\mathbf{16}}$
I_2^{*ns}	SO(11)	$n(\sigma)\cdot(a_{2,1})\cdot(a_{6,5})$	—
I_2^{*s}	SO(12)	$n(\sigma)\cdot(a_{2,1})\cdot(a_{4,3}^2)$	—
IV^{*s}	E_6	$n(\sigma)\cdot(a_{3,2})\cdot(a_{6,5})$	any $\chi_{\mathbf{27}}$
III^*	E_7	$n(\sigma)\cdot(a_{4,3})\cdot(a_{6,5})$	—
IV^{*ns}	F_4	0	—
I_0^{*ns}	G_2	0	—

Constraints agree with 4D anomaly cancellation.

Example: $(\mathbf{SU}(3) \times \mathbf{SU}(2) \times \mathbf{U}(1))/\mathbb{Z}_6$

This model contains the MSSM and three exotic representations. Realizable as a general cubic in $\mathbb{CP}^2$ [Raghuram-Taylor-Turner '19]:

$$b_1 W(d_0 V^2 + d_1 VW + d_2 W^2) + U(s_1 U^2 + s_2 UV + s_5 UW + s_6 VW + s_8 W^2) = 0.$$

4D anomaly cancellation permits

	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$	$(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$(\mathbf{1},\mathbf{1})_{1}$	$(\mathbf{3},\mathbf{1})_{-\frac{4}{3}}$	$(\mathbf{1},\mathbf{2})_{\frac{3}{2}}$	$(\mathbf{1},\mathbf{1})_{2}$
MSSM	1	-1	-1	-1	1	0	0	0
family 2	2	-1	-4	-2	0	1	0	1
family 3	-2	2	2	-1	0	0	1	-1

F-theory realizes all three families over generic B
[PJ-Taylor-Turner (in progress)]

Resolution independent structure?

Is there a clear geometric interpretation independent of choice of X? A motivating set of examples are the Tate models:

$$\chi_{\mathcal{R}} = n \times (\sigma) \cdot (p(a_{s,m_s})) \cdot (p'(a_{s,m_s}))$$
$$= \# \text{ of special points in } B.$$

Compare to the numbers of $\mathcal{R}$ hypermultiplets in 6D compactifications [Grassi-Morrison '12]:

$$N_{\mathcal{R}}^{6\text{D}} = (\sigma) \cdot p((a_{s,m_s}))$$
$$= \# \text{ of points in codim two component of } \Delta = 0.$$

$\chi_{\mathcal{R}}$ more complicated for product groups, U(1)s, but still linear combinations of collections of points

Resolution independent lattice?

Consider the lattice Λ of 4-cycles $D_I \cap D_J$ with integral pairing

$$\langle D_I \cap D_J, D_K \cap D_L \rangle \equiv M_{(IJ)(KL)} = D_I \cdot D_J \cdot D_K \cdot D_L .$$

Background PD(G_4) is a vector $\mathbb{G} \in \Lambda$.

Conjecture: Λ is resolution independent

That is, given resolutions X, X' and choosing bases $\Lambda = \Lambda_{\text{null}} \oplus \Lambda_{\text{nd}}$, $M = 0 \oplus M_{\text{nd}}$ there exists an invertible integer matrix U such that

$$M_{\text{nd}}(X) = U^T M_{\text{nd}}(X') U$$

where M_{nd} is the pairing M on $\Lambda_{\text{nd}} \cong H^{\text{vert}}_{2,2}(X, \mathbb{Z})$.

Evidence

For Tate models we can come close to a proof as $M_{\text{nd}} \subset M$ is almost entirely specified, since

$$\Lambda_{\text{null}} \cong \tilde{\Lambda}_{\text{null}} \oplus \Lambda_{\text{sym,null}}, \qquad \Lambda_{\text{sym,null}} = \text{nullspace}(M_{\text{sym}})$$

where M_{sym} is the restriction of M to the sublattice $\Lambda_{\text{sym}} \subset \Lambda$ of $\mathbb{G}$ lifting to F-theory preserving $\mathcal{G}$:

$$M_{\text{sym}(IJ)(KL)} = W_{IJKL} + \kappa^{mn} W_{IJ|m} \cdot W_{KL|n} \cdot \Sigma + K_B \cdot W_{IJ} \cdot W_{KL}.$$

The pushforwards W_{IJKL} carry structure related to low energy gauge theory [Cvetič-Grimm-Klevers '13, others], e.g. $W_{ijk} = \rho^{\mathcal{R}}_{ijk} C_{\mathcal{R}}$ and $W_{ij} = -\kappa_{ij}(\sigma)$, enough to determine a rational U.

More evidence?

More generally, rank and signature of M seem to be resolution independent.

In practice the rank of M_{sym} is equal to the number of independent chiral indices. E.g. in the $(\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1))/\mathbb{Z}_6$ model, rank $M_{\mathrm{sym}} = 3$ and there are three independent chiral indices

$$\chi_{(\mathbf{1},\mathbf{2})_{\frac{3}{2}}}, \qquad \chi_{(\mathbf{3},\mathbf{1})_{-\frac{4}{3}}}, \qquad \chi_{(\mathbf{3},\mathbf{1})_{\frac{2}{3}}}$$

More evidence?

The signature of M may be related to a topological invariant of oriented manifolds with real dimension divisible by 4, namely the signature of the *full* pairing (Hirzebruch signature theorem)

$$H^4(X) \times H^4(X) \to \mathbb{Z}$$

Sylvester's law of inertia implies there exists a *real* change of basis $M_{\text{nd}}(X) = U^T M_{\text{nd}}(X')U$, but not enough to show U is integer and unimodular.

Relation of chiral index to Λ

The nullspace $\Lambda_{\text{null}} \subset \Lambda$ contains all linear relations among $\chi_{\mathcal{R}}$ including anomaly constraints

[Bies-Mayrhofer-Weigand '17].

It follows that $\Lambda_{\text{sym,nd}} \subset \Lambda$ is isomorphic to the lattice of independent $\chi_{\mathcal{R}}$, expected to be resolution independent on physical grounds.

E.g. in the $(\text{SU}(3) \times \text{SU}(2) \times \text{U}(1))/\mathbb{Z}_6$ model

$$\Lambda_{\text{sym,nd}} \cong \text{span}_{\mathbb{Z}}\left(\chi_{(\mathbf{1},\mathbf{2})_{\frac{3}{2}}}, \chi_{(\mathbf{3},\mathbf{1})_{-\frac{4}{3}}}, \chi_{(\mathbf{3},\mathbf{1})_{\frac{2}{3}}}\right)$$

Relation of chiral index to Λ

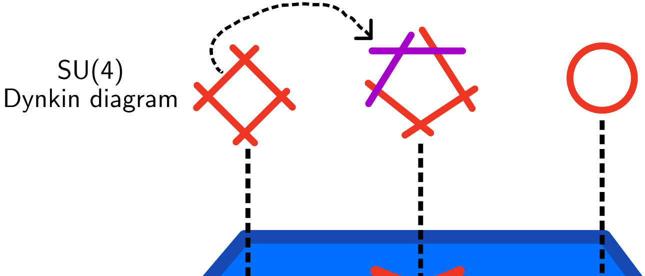
The evidence however points to additional structure for elliptic CY4s that may not require the preservation of 4D gauge symmetry, namely that the resolution independence of $\Lambda_{\text{sym,nd}}$ lifts through the following diagram to Λ:

$$
\begin{array}{ccc}
\Lambda & \xrightarrow{\langle \mathbb{G}, D_I \cap D_\alpha \rangle = 0} & \Lambda_{\text{sym}} \\
{\scriptstyle /\sim}\downarrow & & \downarrow \\
\Lambda_{\text{nd}} & \longrightarrow & \Lambda_{\text{sym,nd}}
\end{array}
$$

An analogy?

Physically, it's alluring to compare Λ (defined by the pairing M) to the Dynkin diagram in F-theory models:

$$D_i \cdot D_j = -(\sigma)\kappa_{ij} \quad \overset{?}{\sim} \quad (D_i \cdot D_j)\cdot(D_k \cdot D_l) = M_{(ij)(kl)}$$



Looking ahead

Rigorously proving resolution independence of Λ could be useful for several future research directions:

- Possible intrinsic definition of $H^{2,2}_{\text{vert}}(X_0, \mathbb{Z})$ in the singular F-theory limit?
- Interpretation of chiral indices as "counting" special points in B, computation of $\chi_{\mathcal{R}}$ directly in X_0 (no resolution)
- Computation of $H^{2,2}_{\text{hor}}(Y, \mathbb{Z})$ via mirror symmetry [Greene-Morrison-Plesser '95]? If $H^{2,2}_{\text{vert}}(X, \mathbb{Z})$ is invariant over subset of Kähler moduli space, is the "mirror" statement also true?

Thank you!