Life Without Holomorphy

Jonathan J. Heckman

University of Pennsylvania

Thank You

Many thanks to the organizers;

Thank You

Many thanks to the organizers;

It is great that there is still a big annual conference where topics closely connected to Strings are discussed

Thank You

Many thanks to the organizers;

It is great that there is still a big annual conference where topics closely connected to Strings are discussed (as opposed to other conferences with the word "Strings" or "String" in the title...)

Facts of Life

- No superpartners have been seen (yet)

  "...before the rooster crows, you will deny me three times..."

- Exact computations w/o SUSY are difficult

  (Usually in tandem with power of holomorphy)

Main Idea of Talk

Use anomalies to constrain candidate string vacua
(an old idea)

Compute in situations with little or no SUSY
and no holomorphic structure
(comparatively little studied)

Why Do This?

How well is the "landscape" understood?
(In diverse dimensions)

Little is known about string compactification without holomorphy

Interesting for its own sake

Plan of the Talk

- Example I: 3D $\mathcal{N} = 1$ via M / $Spin(7)$

  (two real Q's)

- Example IIB: Duality Anomalies

  (Independent of # of Q's)

- Conclusions / Future Directions

Plan of the Talk

- Example I: 3D $\mathcal{N} = 1$ via M / $Spin(7)$

  (two real Q's)

  hep-th/2107.00025 w/ Cvetič Torres Zoccarato

  See also hep-th/2009.10090 w/ Cvetič Rochais Torres Zoccarato

- Example IIB: Duality Anomalies

  (Independent of # of Q's)

  To Appear Soon w/ Debray Dierigl Montero

  To Appear A Little Later w/ Debray Dierigl Montero

  See also hep-th/2012.00013 w/ Dierigl

- Conclusions / Future Directions

Example I:

3D $\mathcal{N} = 1$ via M / $Spin(7)$

Cvetič JJH Torres Zoccarato '21

Special Holonomy and SUSY

Some Compactifications of M-theory:

$CY_1 \Rightarrow 32Q$'s (9D Vacua)

$CY_2 \Rightarrow 16Q$'s (7D Vacua)

$CY_3 \Rightarrow 8Q$'s (5D Vacua)

$CY_4 \Rightarrow 4Q$'s (3D Vacua)

$CY_5 \Rightarrow 2Q$'s (1D Vacua)

$G_2 \Rightarrow 4Q$'s (4D Vacua)

$Spin(7) \Rightarrow 2Q$'s (3D Vacua)

Special Holonomy and SUSY

Some Compactifications of M-theory:

$CY_1 \Rightarrow 32Q$'s (9D Vacua)

$CY_2 \Rightarrow 16Q$'s (7D Vacua)

$CY_3 \Rightarrow 8Q$'s (5D Vacua)

$G_2 \Rightarrow 4Q$'s (4D Vacua)

$CY_4 \Rightarrow 4Q$'s (3D Vacua)

$Spin(7) \Rightarrow 2Q$'s (3D Vacua)

$CY_5 \Rightarrow 2Q$'s (1D Vacua)

3D $\mathcal{N} = 1$?

It's SUSY, but non-holomorphic:

Q_a is a *real* spinor of $\mathfrak{so}(2,1)$

Closely related to 4D "$\mathcal{N} = 1/2$" bkgnds of

F-th on $Spin(7)$ space

JJH Lawrie Lin (Sakstein) Zoccarato '19, '20

11D M-th $\rightarrow$ 3D QFTs?

See also Becker '00; Gukov Sparks '01; Cvetič Lu Pope '01 + ...

Braun Schäfer-Nameki '18; Cvetič JJH Rochais Torres Zoccarato '20 + ...

Classical Backgrounds I/II

Local $Spin(7)$

$$\begin{array}{ccc} \mathbb{R}^4/\Gamma_{ADE} \rightarrow & X_8 \\ & \downarrow \\ & M_4 \end{array}$$

$\Rightarrow$ 7D SYM w/ gp G_{ADE} on $\mathbb{R}^{2,1} \times M_4$

Six-brane wrapping M_4

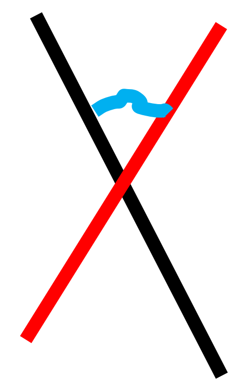
Local G_2

IIA analog: D6-branes on $\Lambda^2_{SD} \rightarrow M_4$

Classical Backgrounds II/II

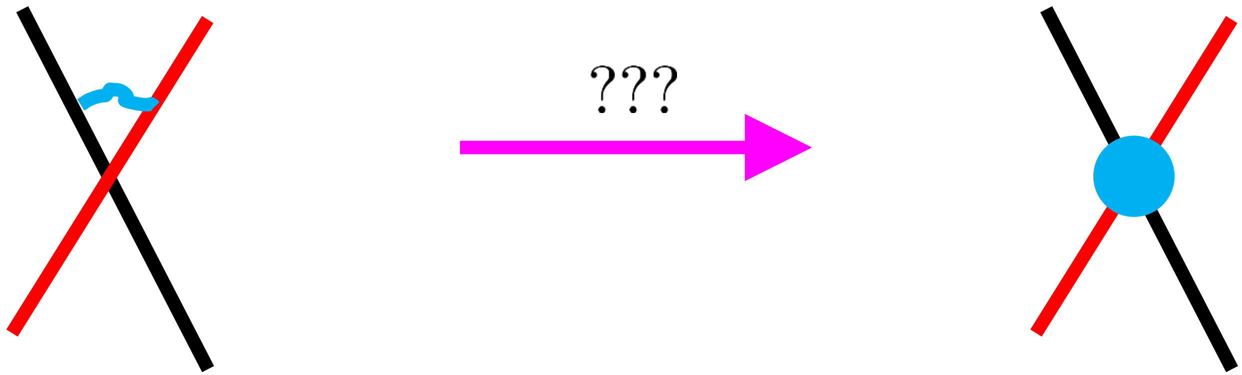
Intersecting 6-branes $\Rightarrow$ Localized Matter

	$\mathbb{R}^{2,1}$			M_4				Λ^2_{SD}		
	0	1	2	3	4	5	6	7	8	9
6	×	×	×	×	×	×	×			
6′	×	×	×	×				×	×	×



Matter is on one-dimensional subspace

¿Quantum Backgrounds?



¿Quantum Backgrounds?

Study this via Anomalies

Involving Spacetime Reflection Symmetries

Reflections

$$\mathsf{R}_1^{3D} : (x^0, x^1, x^2) \to (x^0, -x^1, x^2)$$

$a(x) \to a(\mathsf{R}_1 x)$ (scalar)

$\varphi(x) \to -\varphi(\mathsf{R}_1 x)$ (pseudo-scalar)

$\psi(x) \to \gamma_1 \psi(\mathsf{R}_1 x)$ (+ fermion)

$\chi(x) \to -\gamma_1 \chi(\mathsf{R}_1 x)$ (− fermion)

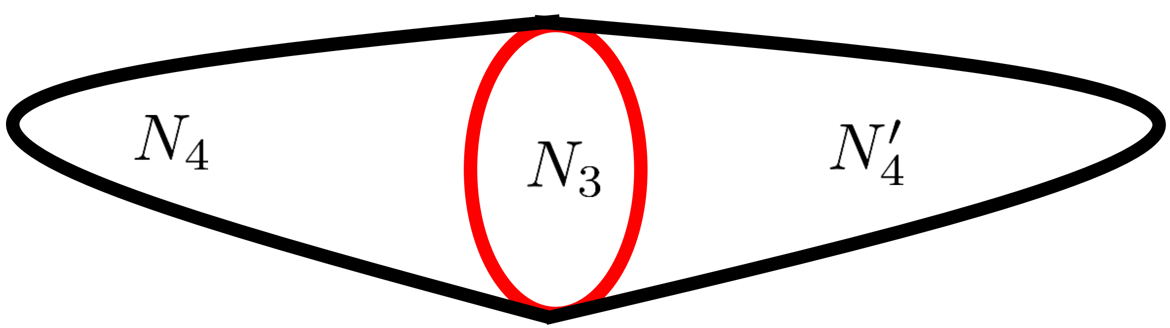
Tuning M_4 Moduli, 7D reflections descend to 3D reflections

Grav-Refl^n Anomaly

Consider 3D theory on 3-manifold N_3:

Track $Z(N_3) \to e^{2\pi i \nu_{\mathsf{R}}/16} Z(\mathsf{R}N_3)$

see e.g. Fidkowski Chen Vishwanath '13; Wang Senthil '14;
Metlitski Chen Vishwanath '14; Witten '15 + ...



Relevant bordism group: $\Omega_4^{\mathsf{Pin}^+} \simeq \mathbb{Z}_{16}$

Grav-Refl^n Anomaly

Consider $\cap$ 6-branes wrapped on 4-manifolds:

$$\nu_{\mathsf{R}} = \nu_{\mathsf{R}}^{\text{bulk}} + \nu_{\mathsf{R}}^{\text{local}}$$

$$\nu^{\text{bulk}} = \dim_{\mathbb{R}} \mathbf{R}_{Adj} \times \frac{1}{2}(\chi(M_4) + \sigma(M_4)) \mod 16$$

$$\nu^{\text{local}} = \sum_{i \in +} \dim_{\mathbb{R}} \mathbf{R}_i - \sum_{i \in -} \dim_{\mathbb{R}} \mathbf{R}_i \mod 16$$

The Value of ν_{R}

$\nu_{\mathsf{R}} \neq 0 \Rightarrow$ *something* survived in IR

In fact, 7D picture shows "strong coupling"
only happens at $E \ll M_{KK}$
so even $\nu_{\mathsf{R}} = 0$ is still "valuable"

Another $\nu^{\text{bulk}}_{\mathsf{R}\text{Gauge}}$ detects 1/2 integer G_4-flux

Examples

$4\mathrm{D} \to 3\mathrm{D}$

Consider 4D $\mathcal{N} = 1$ theory via M-th on local G_2

"7D SYM + Defects on a 3-manifold M_3"

see e.g. Acharya Witten '01; Pantev Wijnholt '09;

Braun Cizel Hübner Schäfer-Nameki '18;

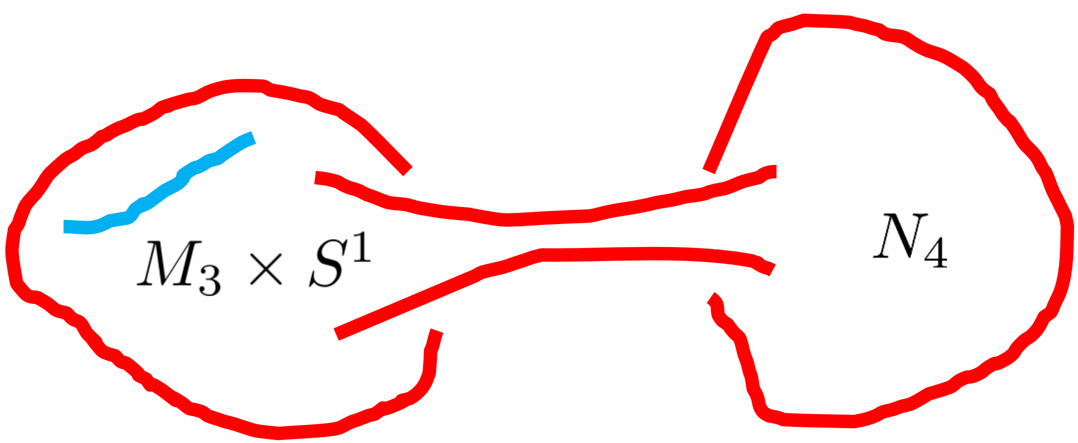
Barbosa Cvetič JJH Lawrie Torres Zoccarato '19;

Hübner '20

$4D \to 3D$

Consider 4D $\mathcal{N} = 1$ theory via M-th on local G_2
"7D SYM + Defects on a 3-manifold M_3"

3D $\mathcal{N} = 2$: Take $M_4 = M_3 \times S^1$
3D $\mathcal{N} = 1$: Take $M_4 = (M_3 \times S^1) \# N_4$ (gluing)



E.G.: $S(10)$ "GUT"

Matter from $E_8 \to SO(10) \times S(U(1)^4)$

$\mathbf{16}$'s and $\mathbf{10}$'s on lines, $\mathbf{45}$'s in bulk

(see also Tong's talk)

$$\nu_{\mathsf{R}} = 4N_{\mathsf{10}}^{\mathrm{net}} - \frac{3}{2}(\chi(M_4) + \sigma(M_4)) \mod 16$$

Example IIB: Duality Anomalies

Debray Dierigl JJH Montero:

To Appear Soon and *To Appear A Little Later*

IIB Duality Group

IIB SUGRA has $SL(2,\mathbb{R})$

"Broken to $SL(2,\mathbb{Z})$ by branes / flux quantiz^n, etc."

IIB Duality Group

IIB SUGRA has $SL(2,\mathbb{R})$

"Broken to $SL(2,\mathbb{Z})$ by branes / flux quantiz^n, etc."

Including Fermions: $Mp(2,\mathbb{Z}) \to SL(2,\mathbb{Z})$
(dilatini + gravitini)

Pantev Sharpe '16

IIB Duality Group

IIB SUGRA has $SL(2,\mathbb{R})$

"Broken to $SL(2,\mathbb{Z})$ by branes / flux quantiz^n, etc."

Including Fermions: $Mp(2,\mathbb{Z}) \to SL(2,\mathbb{Z})$
(dilatini + gravitini)

Pantev Sharpe '16

Including Reflections: Pin^+ Cover of $GL(2,\mathbb{Z}) \equiv GL^+(2,\mathbb{Z})$

Tachikawa Yonekura '18

IIB Duality Group

IIB SUGRA has $SL(2,\mathbb{R})$

"Broken to $SL(2,\mathbb{Z})$ by branes / flux quantiz^n, etc."

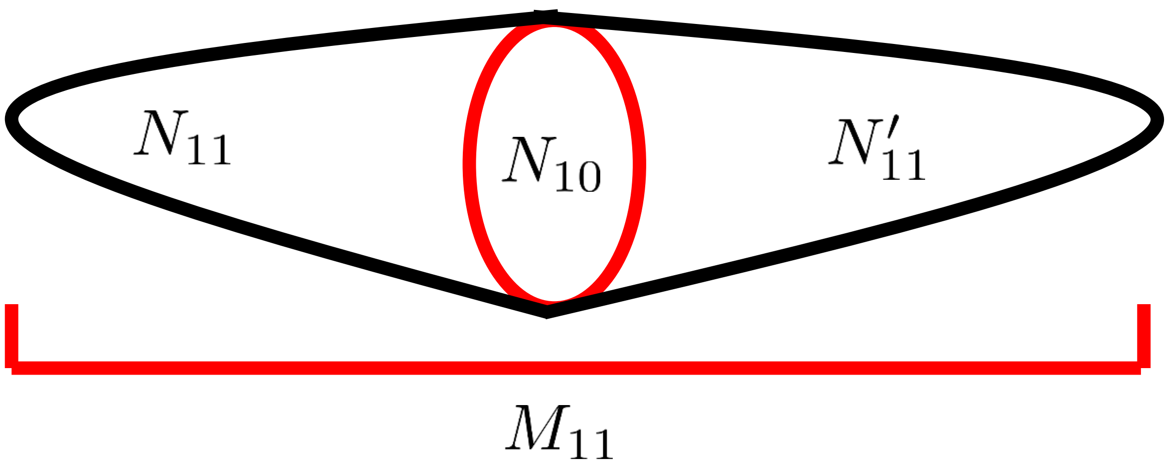
Including Fermions: $Mp(2,\mathbb{Z}) \to SL(2,\mathbb{Z})$
(dilatini + gravitini)

Including Reflections: Pin^+ Cover of $GL(2,\mathbb{Z}) \equiv GL^+(2,\mathbb{Z})$

Is IIB Duality Anomalous?

View IIB as Boundary of 11D "Anomaly Theory"

Equip M_{11} with a Duality Bundle



11D Anomaly Theory

Consider 11D Bulk Theory on M_{11} w/ Duality Bundle:

$$\mathcal{A}[M_{11}] = \eta_1^{\mathrm{RS}} - 2\eta_1^{\mathrm{D}} - \eta_{-3}^{\mathrm{D}} - \frac{1}{8}\eta_{-}^{\mathrm{Sig}} + \mathrm{Arf}(\widetilde{Q}) - \widetilde{\mathcal{Q}}(c)$$

11D Anomaly Theory

Consider 11D Bulk Theory on M_{11} w/ Duality Bundle:

$$\mathcal{A}[M_{11}] = \eta_1^{\mathrm{RS}} - 2\eta_1^{\mathrm{D}} - \eta_{-3}^{\mathrm{D}} - \frac{1}{8}\eta_-^{\mathrm{Sig}} + \mathrm{Arf}(\widetilde{Q}) - \widetilde{\mathcal{Q}}(c)$$

Gravitino

Dilatino

Chiral 4-form

11D Anomaly Theory

Consider 11D Bulk Theory on M_{11} w/ Duality Bundle:

$$\mathcal{A}[M_{11}] = \eta_1^{\mathrm{RS}} - 2\eta_1^{\mathrm{D}} - \eta_{-3}^{\mathrm{D}} - \frac{1}{8}\eta_{-}^{\mathrm{Sig}} + \mathrm{Arf}(\widetilde{Q}) - \widetilde{\mathcal{Q}}(c)$$

Q: Does $\mathcal{A}[M_{11}]$ Vanish for all M_{11}???

11D Anomaly Theory

Consider 11D Bulk Theory on M_{11} w/ Duality Bundle:

$$\mathcal{A}[M_{11}] = \eta_1^{\mathrm{RS}} - 2\eta_1^{\mathrm{D}} - \eta_{-3}^{\mathrm{D}} - \frac{1}{8}\eta_{-}^{\mathrm{Sig}} + \mathrm{Arf}(\widetilde{Q}) - \widetilde{\mathcal{Q}}(c)$$

Q: Does $\mathcal{A}[M_{11}]$ Vanish for all M_{11}???

A: No!?!?!?

Computing $\mathcal{A}[M_{11}]$:

$$\Omega_{11}^{\text{Duality}} \simeq \mathbb{Z}_8 \oplus \mathbb{Z}_2^{\oplus 9} \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_{27}$$

$\mathcal{A}[S^{11}/\mathbb{Z}_3] = 1/3$ ($S^{11}/\mathbb{Z}_3$ Generates $\mathbb{Z}_{27}$ factor)

$\mathcal{A}[M_{11}] = 0$ (Many $\mathbb{Z}_k$ Factors of $\Omega_{11}^{\text{Duality}}$)

¿Is IIB Duality Anomalous?

¿Is IIB Duality Anomalous? Probably Not (But Why?)

How To Fix It

10D Topological Term

Amazingly, everything* cancels with one more term:

$S_{IIB} = S_{\text{old}} + S_{\text{top}}$, with:

$$S_{top} = \int F_5 \cup \left[\left(\mathfrak{F}_a^2 + \frac{(p_1)_3}{2} \right) \cup a + \frac{1}{2}[(p_1)_4 - \mathcal{P}(w)] \cup b \right]$$

Discrete Connections: $a \in H^1(B\mathbb{Z}_3; \mathbb{Z}_3)$ $b \in H^1(B\mathbb{Z}_4; \mathbb{Z}_4)$

Discrete Curvatures: $\mathfrak{F}_a$, $(p_1)_n$ (Pont mod. n)

* (up to small caveats)

Cross-Checks

S-fold Check

S-fold: Non-Pert Generalization of O-plane

Garcia-Etxebarria Regalado '15; Aharony Tachikawa '16 + ...

$$S_{top} = \int F_5 \cup \left[\left(\mathfrak{F}_a^2 + \frac{(p_1)_3}{2} \right) \cup a + \frac{1}{2}[(p_1)_4 - \mathcal{P}(w)] \cup b \right]$$

$\Rightarrow$ predicts fractional D3- charge for S-fold

S-fold charges in units of 1/3 and 1/8

see also Tachikawa Yonekura '18

Holographic Check

Take IIB on $AdS_5 \times S^5$, reduce on S^5

$$S_{5D} = N \int_{AdS_5} \left[\left(\mathfrak{F}_a^2 + \frac{(p_1)_3}{2} \right) \cup a + \frac{1}{2}[(p_1)_4 - \mathcal{P}(w)] \cup b \right]$$

$\Rightarrow$ predicts duality anomaly for $\mathcal{N} = 4$ SYM

see also Seiberg Tachikawa Yonekura '18; Hsieh Tachikawa Yonekura '19

Alternatives...

IIB Variants

We proposed a term:

$$S_{top} = \int F_5 \cup \left[\left(\mathfrak{F}_a^2 + \frac{(p_1)_3}{2} \right) \cup a + \frac{1}{2}[(p_1)_4 - \mathcal{P}(w)] \cup b \right]$$

IIB Variants

We proposed a term:

$$S_{top} = \int F_5 \cup \left[\left(\mathfrak{F}_a^2 + \frac{(p_1)_3}{2} \right) \cup a + \frac{1}{2}[(p_1)_4 - \mathcal{P}(w)] \cup b \right]$$

Alternative: Introduce non-invertible (bosonic) TQFT

"Topological Green-Schwarz Mechanism"

Garcia-Etxebarria Hayashi Ohmori Tachikawa '17

Predicts *Different* Spectrum of Stringy Objects / Bkgnds

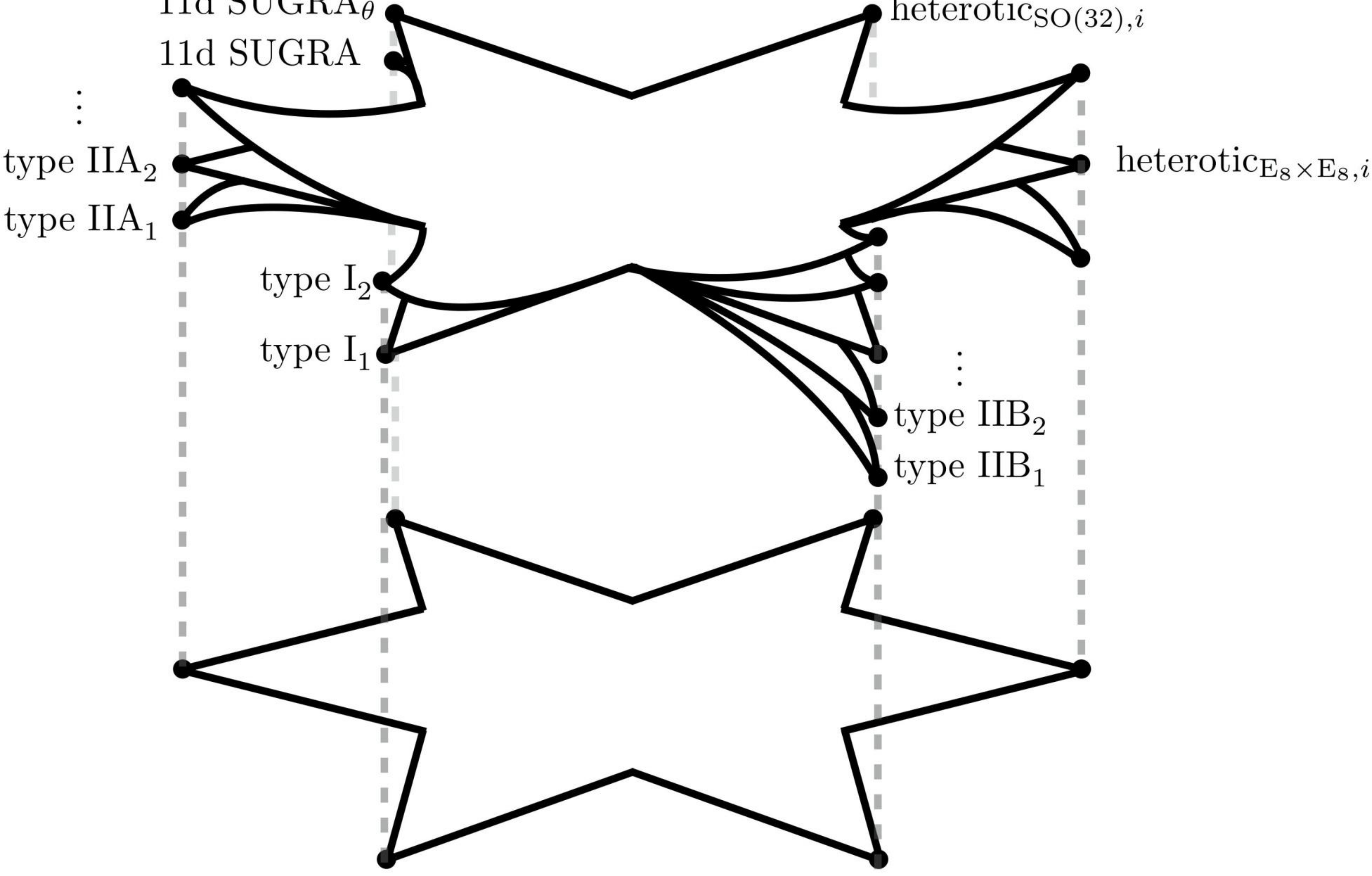
(no orbifolds $\mathbb{C}^2/\mathbb{Z}_3$, no S-folds, ...)

¿¿¿IIB Variants???

- Option 1: Variant IIB theories are in the Swampland
  (but then what is the inconsistency?)

- Option 2: They connect to $\mathrm{IIB}_{\mathrm{standard}}$ via domain walls
  (but then how to build domain walls?)
  c.f. McNamara Vafa '19

- Comment: M-theory "discrete theta angle" Freed Hopkins '19



11d SUGRA$_\theta$
11d SUGRA
⋮
type IIA$_2$
type IIA$_1$
type I$_2$
type I$_1$
heterotic$_{SO(32),i}$
heterotic$_{E_8\times E_8,i}$
⋮
type IIB$_2$
type IIB$_1$

Future and Conclusions

Conclusions

- Topological Constraints (Life Without Holomorphy)

- ν_{R} and M-theory / $Spin(7)$ Vacua

- IIB Duality Anomaly, and Proposed Resolution

Questions for the Future

- Implications of 3D dualities for quantum $Spin(7)$?

- Do variant IIB theories exist?

- How much can topological structures "reconstruct"?

  "Speculations on Physical Discretization and Arithmetic Geometry"

  working paper @ jjheckman.com/research