

# Topological Gravity as the Early Phase of Our Universe

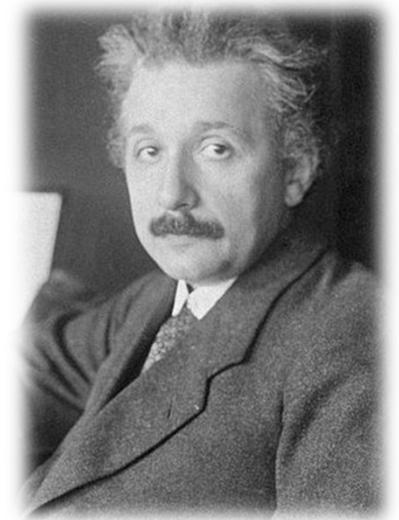
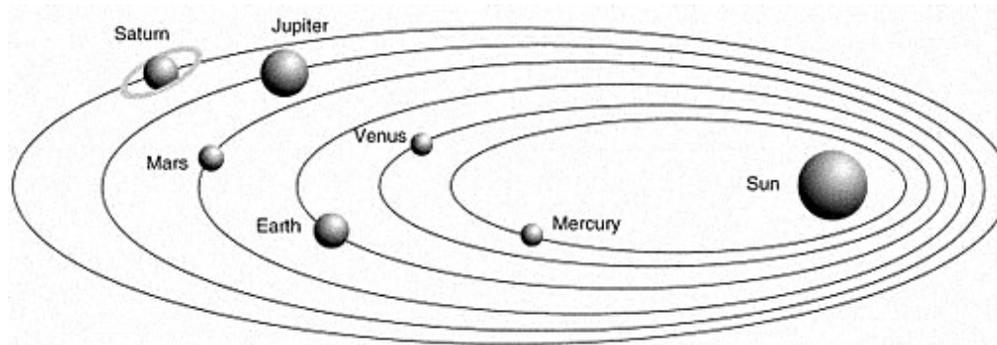
Prateek Agrawal<sup>1</sup>, Sergei Gukov<sup>2</sup>, Georges Obied<sup>3</sup> and Cumrun Vafa<sup>3</sup>

arXiv:2009.10077v2



see also:  
R.Blumenhagen, C.Kneissl, A.Makridou  
A.Kehagias, A.Riotto

talk by Ralph Blumenhagen



“His life work was possible only when he succeeded in freeing himself to a large extent from the intellectual tradition in which he was born.”

-- Albert Einstein on work of Johannes Kepler

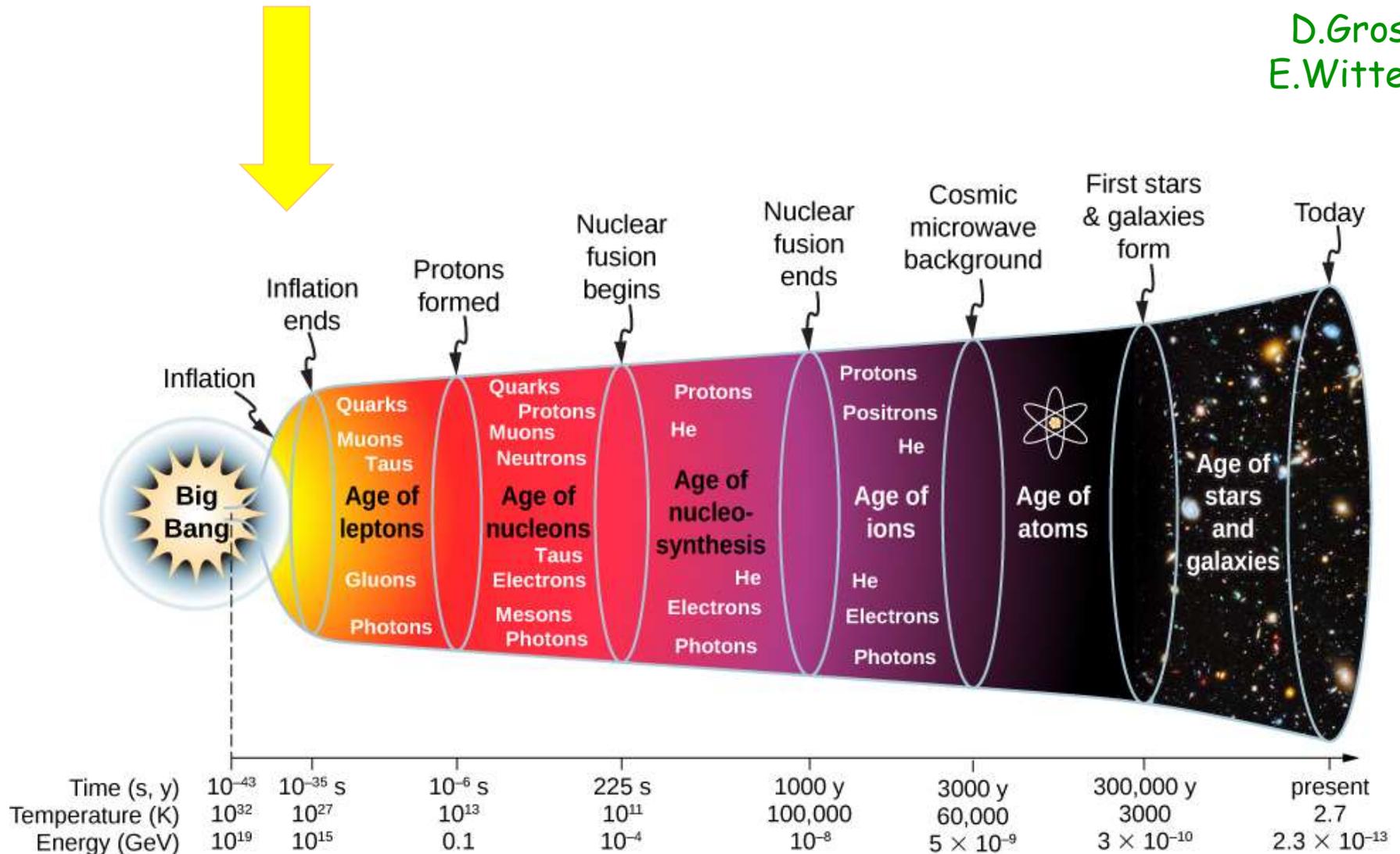
## Motivation:

- New mathematical descriptions of cosmological physics?
- Do exotic smooth structures play a role in gravity? cosmology?
- Is physics of the early universe well described by the same degrees of freedom (fields) we see now?
- Is Einstein gravity the right framework to describe the early universe?

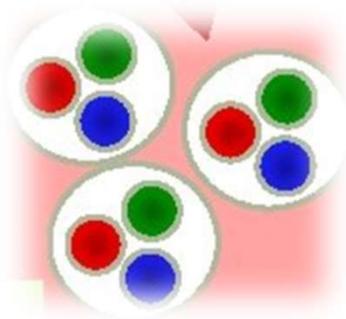
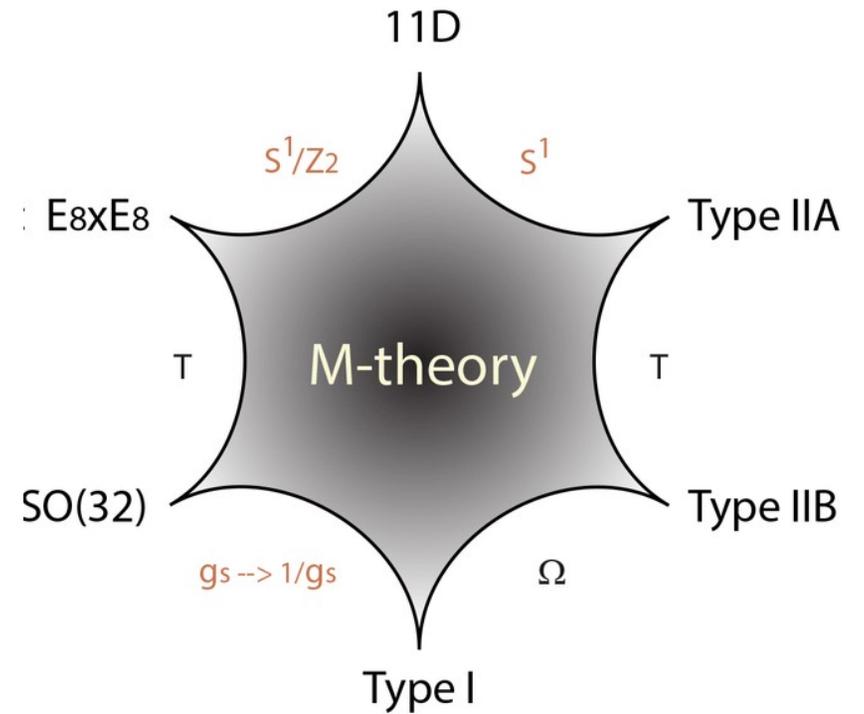
Strong gravitational interactions,  
 Strong time-dependence  
 (derivatives of all orders)

→ Non-local physics  
 (no local excitations?)

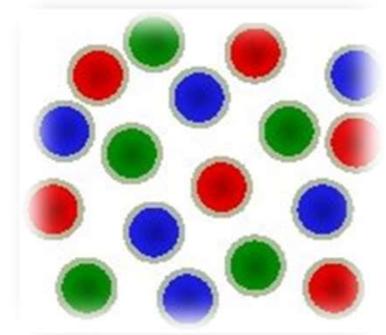
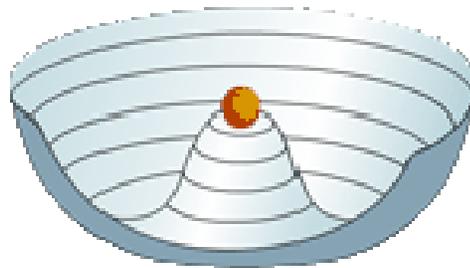
D.Gross  
 E.Witten



# Lessons from dualities:



chiral symmetry  
breaking

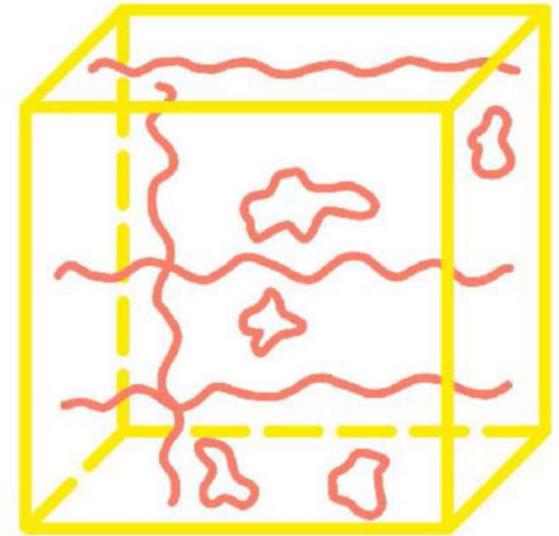


QCD

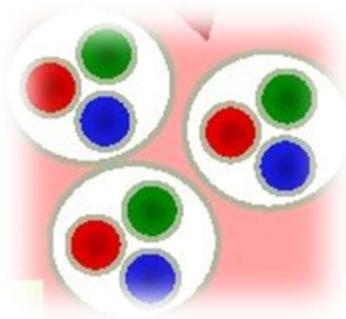
RG time



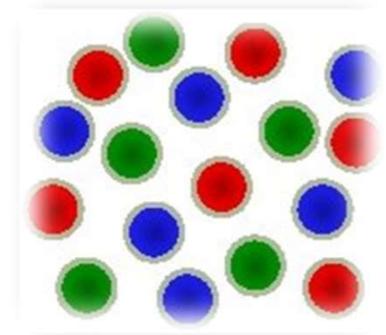
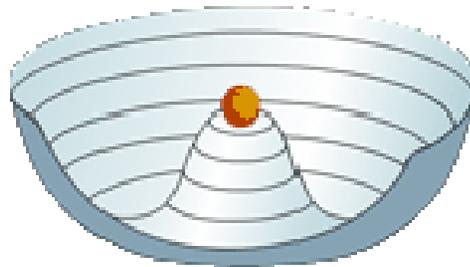
# Lessons from dualities:



R.Brandenberger, C.Vafa  
A.Tseytlin, C.Vafa  
:



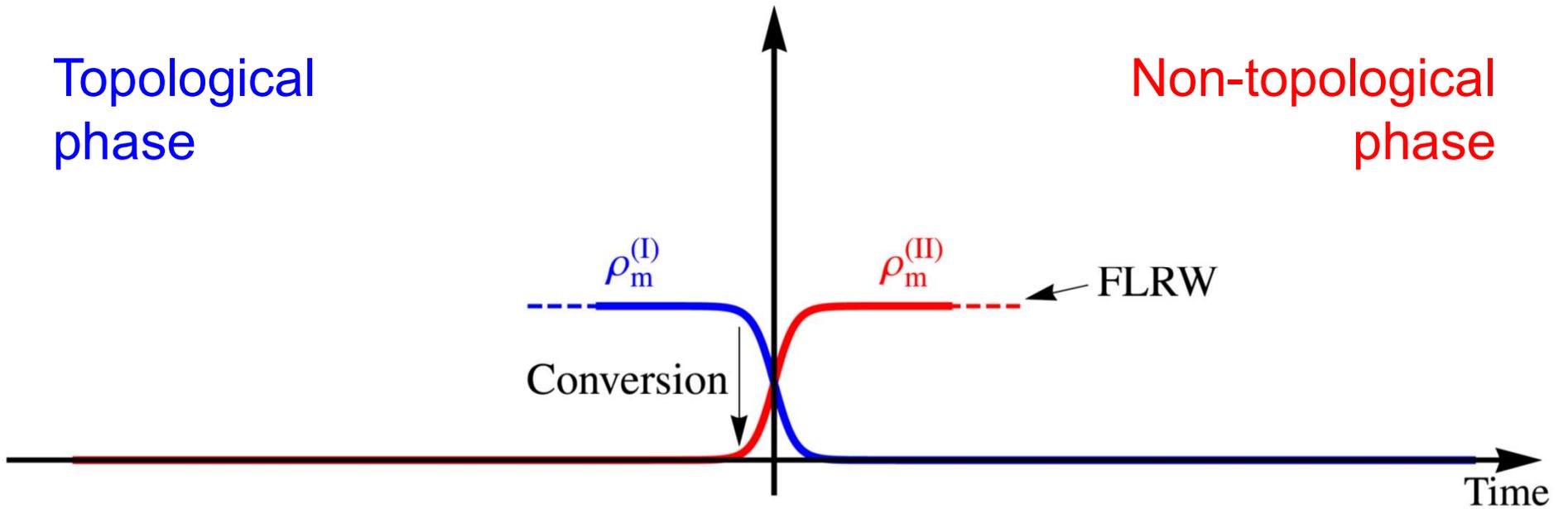
chiral symmetry  
breaking



QCD

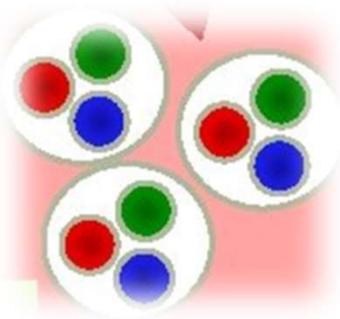
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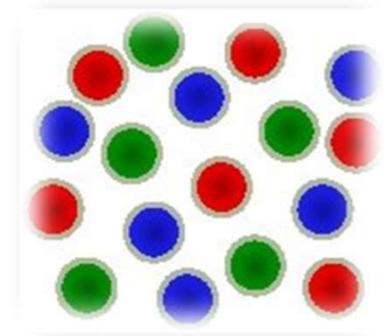
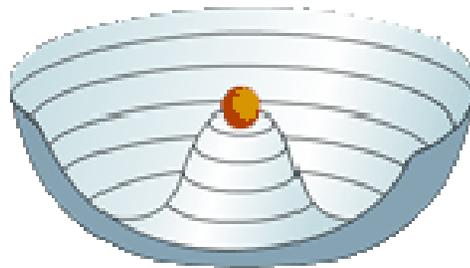


Breaking of topological invariance vs. breaking of chiral symmetry

E. Witten



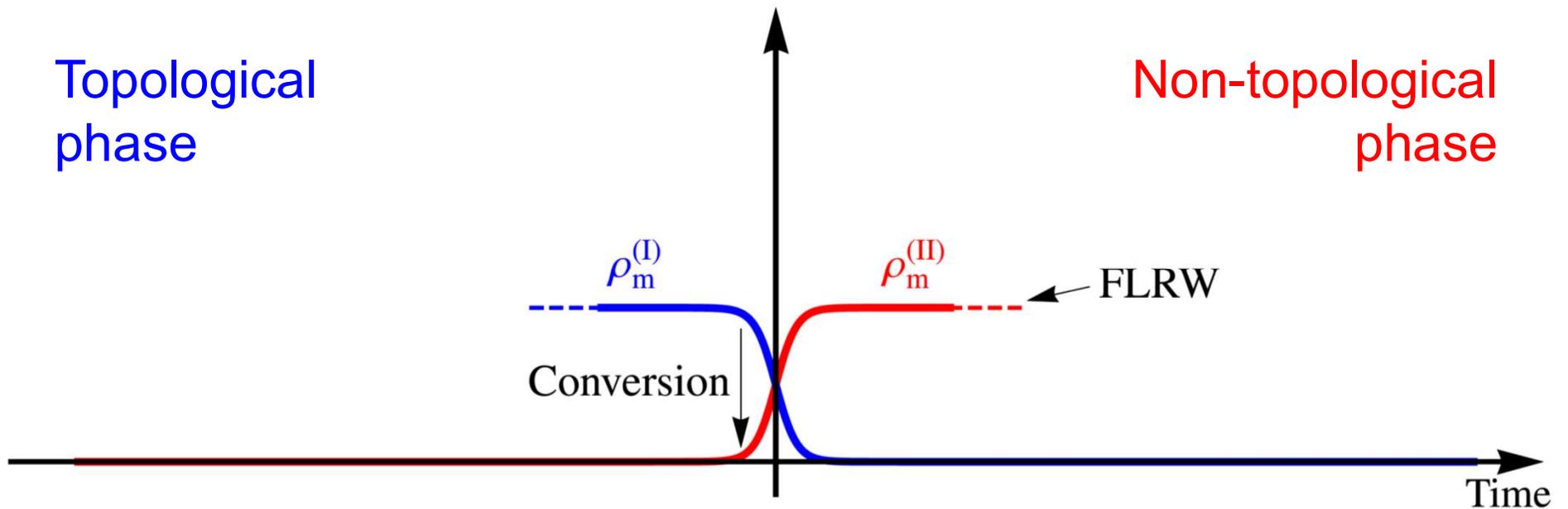
chiral symmetry breaking



RG time

QCD

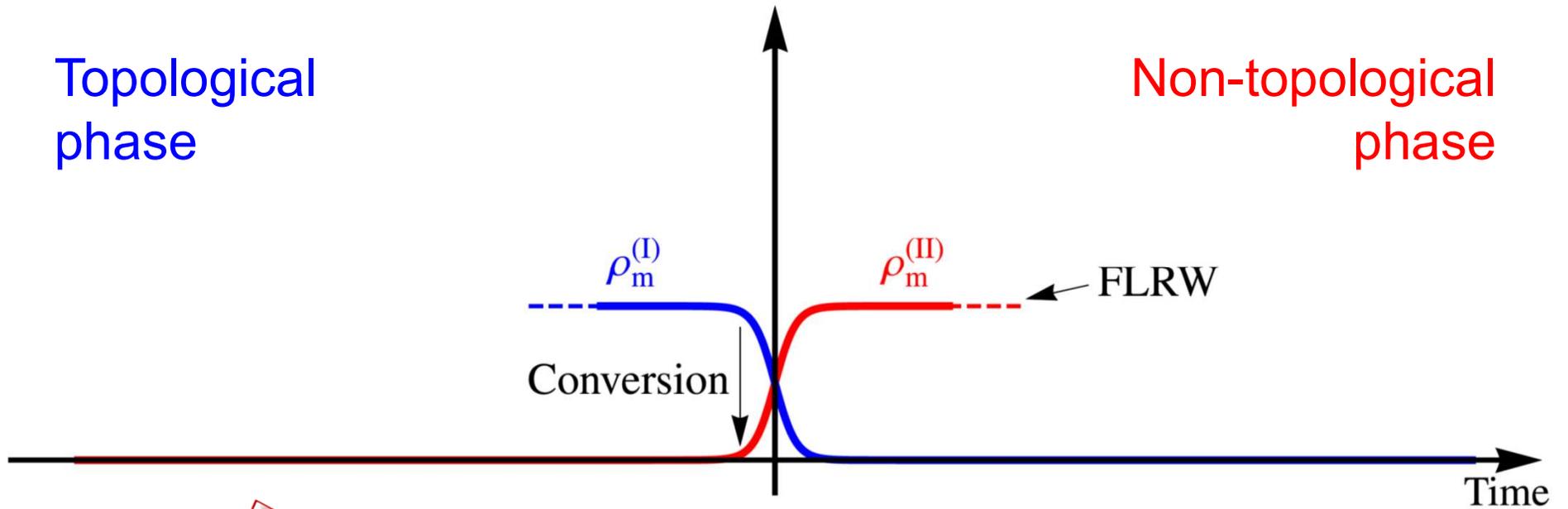




Period of accelerated expansion is replaced by a topological phase to address

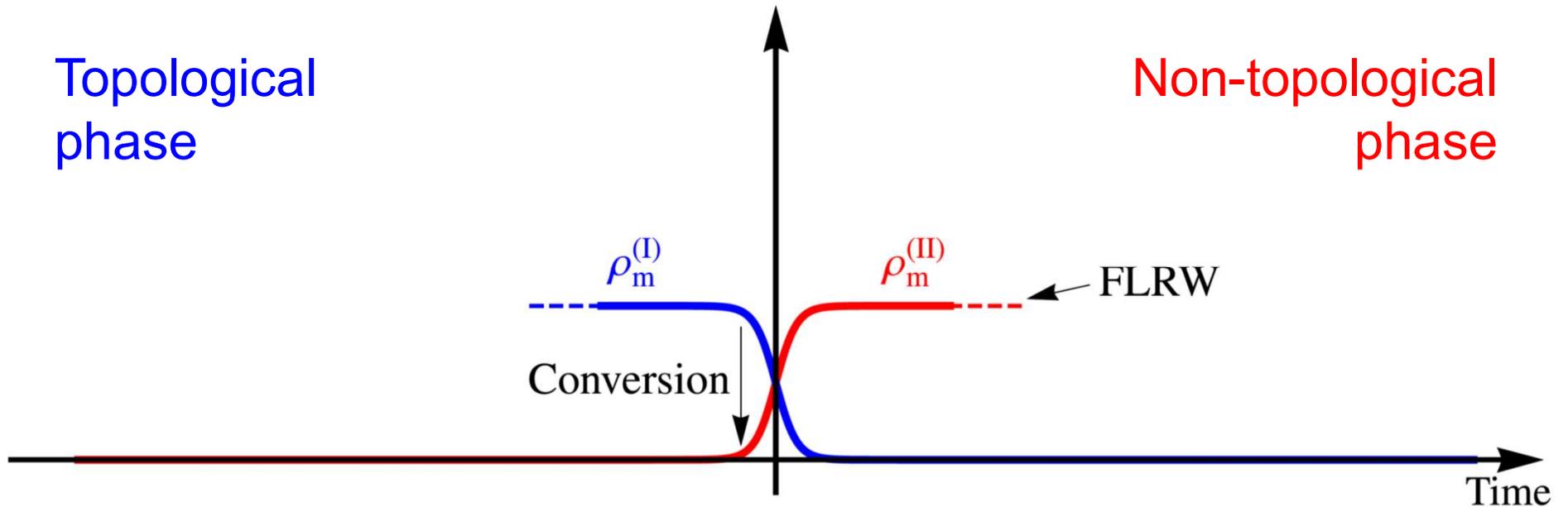
- Horizon problem (homogeneity, isotropy)  $\partial_i \rho(x) = 0$
- Flatness problem  $[D_i, D_j] = 0$
- Nearly scale-invariant power-spectrum

$$\langle \delta(x) \delta(y) \rangle \sim 10^{-10} |x - y|^{0.03}$$



$$\langle \mathcal{O}^{i_1}(x_1) \dots \mathcal{O}^{i_n}(x_n) \rangle = A^{i_1, \dots, i_n}$$

independent of  
positions  $\partial_j A^{i_1, \dots, i_n} = 0$



Fits in with the distance conjecture

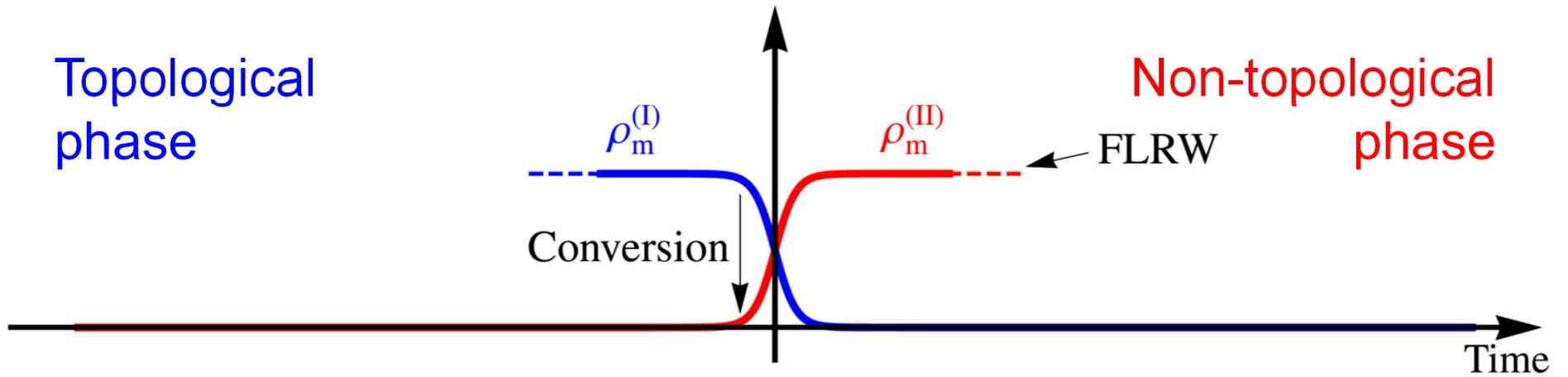
talks by:

Dieter Lust  
 Timo Weigand  
 Ralph Blumenhagen  
 Irene Valenzuela  
 Angel Uranga  
 Anthony Ashmore  
 Tom Rudelius  
 :

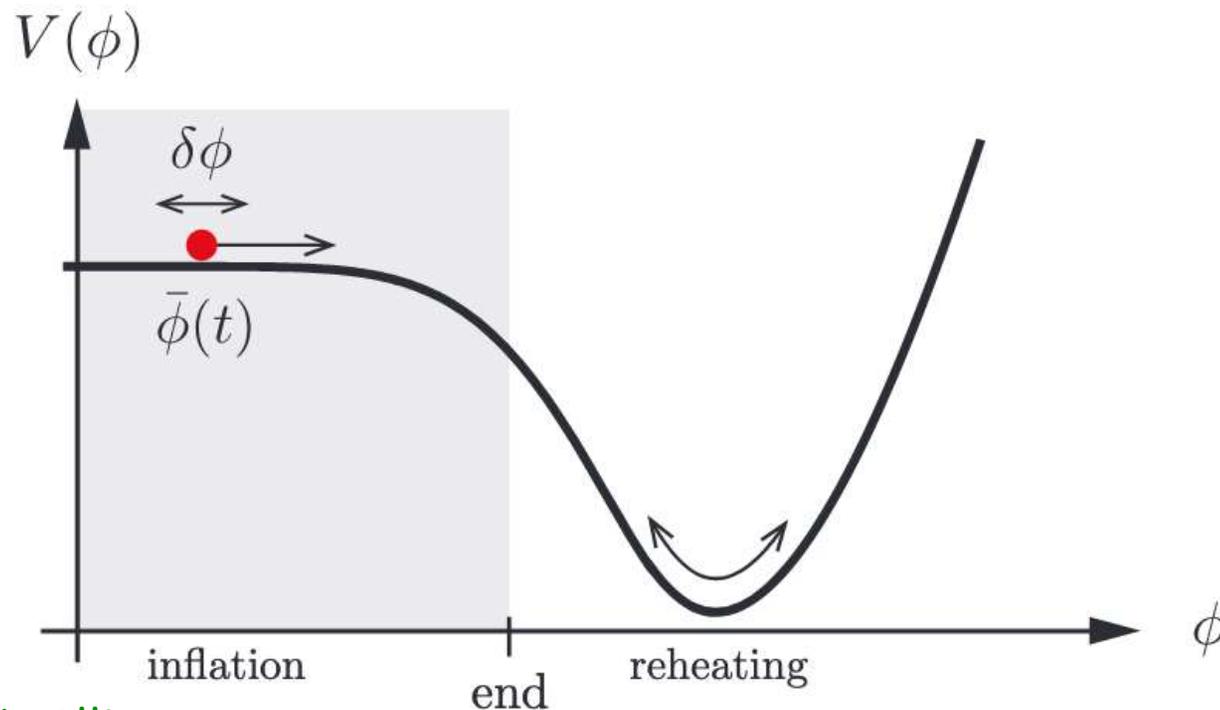
Falsifiable:

- no tensor mode fluctuations (no propagating graviton)
- $O(1)$  non-Gaussianities

cf. A.Kehagias, A.Riotto



Reheating at the end of inflation  $\rightsquigarrow$  phase transition  
 (a lot of complicated physics)



# Topological gravity:

$$S = \int d^4x \sqrt{g} (W_+)^2 + \dots$$



E.Witten

unitary, BRST, and scale-invariant

field	ghost number	$[Q, \text{field}]$
$C_{A\dot{A}}$	2	$\psi_{AB, \dot{A}\dot{B}} C^{B\dot{B}}$
$\psi_{AB, \dot{A}\dot{B}}$	1	$-\frac{i}{4} (e_{A\dot{A}}^\alpha D_\alpha C_{B\dot{B}} + e_{B\dot{A}}^\alpha D_\alpha C_{A\dot{B}} + e_{A\dot{B}}^\alpha D_\alpha C_{B\dot{A}} + e_{B\dot{B}}^\alpha D_\alpha C_{A\dot{A}})$
$e_{\alpha A\dot{A}}$	0	$e_\alpha^{B\dot{B}} \psi_{AB, \dot{A}\dot{B}}$
$W_{ABCD}$	0	$\frac{1}{6} (\psi_{AB, \dot{A}\dot{B}} R_{CD, \dot{A}\dot{B}} - e_{C\dot{C}}^\alpha e_{D\dot{D}}^\beta D_\alpha D_\beta \psi_{AB, \dot{C}\dot{D}})$ + 5 permutations of $A, B, C, D$
$\chi_{ABCD}$	-1	$-iW_{ABCD}$
$\lambda_{A\dot{A}}$	-1	$-iC^\alpha D_\alpha B_{A\dot{A}} + “(\psi\psi + eDC)B” - \frac{i}{4} B_{A\dot{A}} e_{X\dot{X}}^\beta D_\beta C^{X\dot{X}}$
$B_{A\dot{A}}$	-2	$\lambda_{A\dot{A}}$

Topological gravity:

$$S = \int d^4x \sqrt{g} (W_+)^2 + \dots$$



E.Witten

unitary, BRST, and scale-invariant



- Einstein-Hilbert term not generated
- Conformal anomaly at the quantum level



$S_{\text{anomaly}}$

- Witten adds a massless BRST singlet to produce non-degenerate kinetic terms



Topological gravity:

$$S = \int d^4x \sqrt{g} (W_+)^2 + \dots$$



E.Witten

Critical points: “gravitational instantons”

$$W_+ = 0$$

self-dual (a.k.a. conformally half-flat) metrics

Natural to assume that conformal factor depends only on time (shared by the two phases)

$$\Rightarrow ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{(1 - kr^2)} + r^2 d\Omega^2 \right] \quad \text{with } k = 0$$

The tilt is due to the Weyl anomaly:

$$\langle T_{\mu}^{\mu} \rangle = \frac{1}{16\pi^2} (cW^2 - a \cdot \text{Euler})$$

$$S_{\text{anomaly}} = \frac{1}{16\pi^2} \int \left[ -\tau (cW^2 - aE_4) + a \left( 4(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R) \partial_{\mu}\tau \partial_{\nu}\tau - 4(\partial\tau)^2 \square\tau + 2(\partial\tau)^4 \right) \right]$$

D.Capper, M.Duff  
E.Fradkin, A.Tseytlin  
Z.Komargodski, A.Schwimmer  
:

 dilatation mode (cf. gauge-invariant scalar perturbation in the FLRW phase)

cf. C.Cheung, P.Creminelli, A.Fitzpatrick, J.Kaplan, L.Senatore

$$\Delta^2(k) = \frac{k^3}{2\pi^2} \langle \tau(\vec{k}) \tau(-\vec{k}) \rangle \simeq \frac{1}{a} \left( \frac{k}{k_0} \right)^{-cg^2/16\pi^2}$$

# Primordial power-spectrum and the spectral index:

$$\Delta^2(k) = \frac{k^3}{2\pi^2} \langle \tau(\vec{k}) \tau(-\vec{k}) \rangle \simeq \frac{1}{a} \left( \frac{k}{k_0} \right)^{-cg^2/16\pi^2}$$

PLANCK  
data:

$$a \sim c \sim 10^{10} \quad \text{cf. D.Hofman, J.Maldacena}$$

$$cg^2/16\pi^2 \simeq 0.03 \quad \Rightarrow \quad g \sim 10^{-5}$$

cf. the eta-problem

Unitarity ( $c > 0$ )  $\Rightarrow$  red tilt of the spectrum

## Future directions:

- Exotic relic problem? (monopoles)
- Realizations of the topological scenario?
- String models of topological gravity / cosmology?
- Other topological gravity theories?  
G.Horowitz  
R.Myers
- Brane world realization?  
A.Nakamichi, A.Sugamoto, I.Oda  
H.Kunitomo  
:
- Celestial dual for topological gravity?
- “Spontaneous breaking” of topological invariance?  
(initial conditions for FLRW cosmology)