String Phenomenology 2021,
Northeastern U., July 12–16, 2021

Towards exact matter spectra of F-theory MSSMs

Mirjam Cvetič

UNIVERSITY of PENNSYLVANIA

Univerza v Ljubljani
Fakulteta za matematiko in fiziko

CAMTP
University of Maribor

I. Outline:

- Program: Globally consistent F-theory compactifications with the gauge symmetry and matter spectrum of the Standard Model

- Focus: Quadrillion three-family Standard Models with gauge coupling unification and no chiral exotics (QSM)

[M.C., Jim Halverson, Ling Lin, Muyang Liu, Jiahua Tian, 1903.0009, PRL]

- New results: Determination of charged matter vector-pairs → toward MSSMs (no vector-like exotics, one pair of Higgs doublets)

[Martin Bies, M.C., Ron Donagi, Muyang Liu, Marielle Ong, 2102.10115]
[M. Bies, M.C., M. Liu, 2102.08297 PRD]
[M. Bies, M.C., R. Donagi, M. Ong, work in progress]

Earlier work:
[M. Bies, M.C., R. Donagi, L. Lin, M. Liu, F. Rühle, 2007.00009 JHEP]

II. F-theory particle physics constructions

Globally consistent models via toric techniques

Construction of elliptically fibered Calabi-Yau manifold

i. Elliptic curve E

Examples of constructions via toric techniques:

E_{F_i} as a hypersurface in the two-dimensional toric variety $\mathbb{P}_{F_i}$

(generalized weighted projective spaces, associated with16 reflexive polytopes F_i):

c.f., [Klevers, Pena, Oehlmann, Piragua, Reuter '14]

$$E_{F_i} = \{p_{F_i} = 0\} \text{ in } \mathbb{P}_{F_i}$$

ii. Elliptically fibered Calabi-Yau four-fold: X_{F_i}

Impose Calabi-Yau condition:

coordinates in $\mathbb{P}_{F_i}$ and coeffs. of E_{F_i} lifted to sections of specific line-bundles on B_3

$$\begin{array}{ccc} E_{F_i} \subset \mathbb{P}_{F_i} & \longrightarrow & X_{F_i} \\ & & \downarrow \\ & & B_3 \end{array}$$

Fibration depends only on the anti-canonical divisor $\overline{\mathcal{K}}$ & two additional S_7 and S_9 divisor classes

iii. Chiral index for D=4 matter:

$$\chi(\mathbf{R}) = \int_{\mathcal{C}^w_{\mathbf{R}}} G_4$$

$$\begin{array}{ccc} c_{mat} & \longrightarrow & \boxed{\mathcal{C}_{\mathbf{R}}} \\ & & \downarrow \\ & & \Sigma_{\mathbf{R}} \end{array}$$

a) construct G_4 (=dC_3) flux by computing $H_V^{(2,2)}(\hat{X})$
[so-called vertical fluxes – do not induce Gukov-Vafa-Witten potential]
b) determine matter surface $\mathcal{C}_{\mathbf{R}}$ (via resultant techniques)

iv. Global consistency – D3 tadpole cancellation:

$$\frac{\chi(X)}{24} = n_{\mathrm{D3}} + \frac{1}{2}\int_X G_4 \wedge G_4$$

a) satisfied for integer and positive n_{D3}
b) constraint on integer valued flux G_4

$$G_4 + \frac{1}{2}c_2(X) \in H^4(\mathbb{Z}, \hat{X})$$

C.f., Standard Model building blocks initiated in

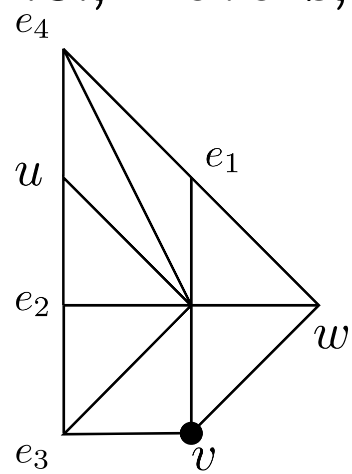
[Lin, Weigand'14] ; SM x U(1) [1604.04292]

Standard Model

[M.C., Klevers, Peña, Oehlmann, Reuter,1503.02068]

P_{F11}

F_{11} polytope



$\mathbb{P}^2$ [u:v:w] with four non-generic blow-ups [e_1:e_2:e_3:e_4]

Elliptic curve:

$$p_{F_{11}} = s_1 e_1^2 e_2^2 e_3 e_4^4 u^3 + s_2 e_1 e_2^2 e_3^2 e_4^2 u^2 v + s_3 e_2^2 e_3^2 u v^2 + s_5 e_1^2 e_2 e_4^3 u^2 w + s_6 e_1 e_2 e_3 e_4 u v w + s_9 e_1 v w^2$$

Hypersurface constraint in P_{F11}

Construction of Calabi-Yau four-fold

Construction of Calabi-Yau four-fold

$$\begin{array}{ccc} E_{F_{11}} \subset \mathbb{P}_{F_{11}} & \longrightarrow & X_{F_{11}} \\ & & \downarrow \\ & & B_3 \end{array}$$

Coordinates and s_i → sections of line-bundles of the base B_3

[Toric techniques via Stanley-Reisner ideal] →

Section	Line Bundle
u	$\mathcal{O}(H - E_1 - E_2 - E_4 + \mathcal{S}_9 + [K_B])$
v	$\mathcal{O}(H - E_2 - E_3 + \mathcal{S}_9 - \mathcal{S}_7)$
w	$\mathcal{O}(H - E_1)$
e_1	$\mathcal{O}(E_1 - E_4)$
e_2	$\mathcal{O}(E_2 - E_3)$
e_3	$\mathcal{O}(E_3)$
e_4	$\mathcal{O}(E_4)$

section	Line Bundle
s_1	$\mathcal{O}_B(3[K_B^{-1}] - \mathcal{S}_7 - \mathcal{S}_9)$
s_2	$\mathcal{O}_B(2[K_B^{-1}] - \mathcal{S}_9)$
s_3	$\mathcal{O}_B([K_B^{-1}] + \mathcal{S}_7 - \mathcal{S}_9)$
s_4	$\mathcal{O}_B(2\mathcal{S}_7 - \mathcal{S}_9)$
s_5	$\mathcal{O}_B(2[K_B^{-1}] - \mathcal{S}_7)$
s_6	K_B^{-1}
s_7	$\mathcal{O}_B(\mathcal{S}_7)$
s_8	$\mathcal{O}_B([K_B^{-1}] + \mathcal{S}_9 - \mathcal{S}_7)$
s_9	$\mathcal{O}_B(\mathcal{S}_9)$
s_{10}	$\mathcal{O}_B(2\mathcal{S}_9 - \mathcal{S}_7)$

H - hyperplane divisor;

K_B^{-1} - anti-canonical divisor $\overline{\mathcal{K}}$

Fibration depends only on additional S_7 and S_9 divisor classes

Gauge Symmetry: Co-dimension one Divisors

$$\begin{array}{ccc} E_{F_{11}} \subset \mathbb{P}_{F_{11}} & \longrightarrow & X_{F_{11}} \\ & & \downarrow \\ & & B \end{array}$$

Over the locus $s_3 = 0$ → fiber degenerates to I_2 - fiber → SU(2)

Over the locus $s_9 = 0$ → fiber degenerates to I_3 - fiber → SU(3)

Cartan divisors of these gauge groups:

$$E_1^{\mathrm{SU}(2)} = [e_1]\,, \quad E_1^{\mathrm{SU}(3)} = [e_2] \quad E_2^{\mathrm{SU}(3)} = [u]$$

Two rational sections:

$[u : v : w : e_1 : e_2 : e_3 : e_4]$

$\hat{s}_0 = X_{F_{11}} \cap \{v = 0\}$: $[1 : 0 : s_1 : 1 : 1 : -s_5 : 1]$ - zero section

$\hat{s}_1 = X_{F_{11}} \cap \{e_4 = 0\}$: $[s_9 : 1 : 1 : -s_3 : 1 : 1 : 0]$ - section associated with U(1)

Shoida divisor

Standard Model gauge symmetry: SU(3) x SU(2) x U(1)

Global Structure of Standard Model Gauge Symmetry

gauge algebra $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$

[M.C., Lin, 1706.08521]

Shioda map (U(1) divisor):

C-central element

$$\sigma(\hat{s}_1) = S_1 - S_0 + \frac{1}{2} E_1^{\mathfrak{su}(2)} + \frac{1}{3}(2 E_1^{\mathfrak{su}(3)} + E_2^{\mathfrak{su}(3)}) \Rightarrow C^6 = 1,$$

$$G_{\text{global}} = [SU(3) \times SU(2) \times U(1)]/\langle C \rangle \cong \boxed{[SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_6.}$$

Matter curves C_R at co-dim 2 singularities:

$$C_{(\mathbf{3},\mathbf{2})_{1/6}} = V(s_3, s_9)$$

$$C_{(\mathbf{1},\mathbf{2})_{-1/2}} = V\left(s_3, s_2 s_5^2 + s_1(s_1 s_9 - s_5 s_6)\right)$$

$$C_{(\overline{\mathbf{3}},\mathbf{1})_{-2/3}} = V(s_5, s_9)$$

$$C_{(\overline{\mathbf{3}},\mathbf{1})_{1/3}} = V\left(s_9, s_3 s_5^2 + s_6(s_1 s_6 - s_2 s_5)\right)$$

$$C_{(\mathbf{1},\mathbf{1})_{1}} = V(s_1, s_5)$$

$s_3=s_9=0$

.

.

.

.

Representations compatible with the Z_6 global constraint

Construction of G_4 for chiral index & D3-tadpole cancellation

More later

Simplest choice $B_3 = \mathbb{P}^3 \rightarrow$ First 3-family Standard Model

[M.C., Klevers, Peña, Oehlmann, Reuter, 1503.02068]

Tip of the Iceberg

II. Landscape of Standard Models

Toric Analysis

[M.C., J. Halverson, L. Lin, M. Liu and J. Tian, 1903.0009]

a) Take the same toric elliptic fibration as before:

hyperplane constraint in 2D reflexive polytope F_{11}

Gauge symmetry: $$\frac{\mathbf{SU(3)\times SU(2)\times U(1)}}{\mathbb{Z}_6}$$

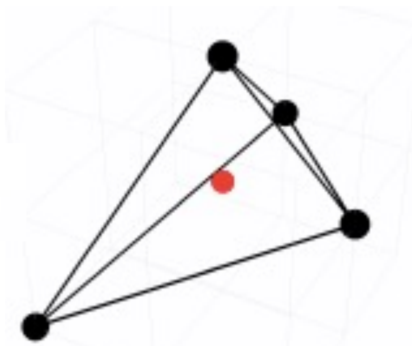
Global gauge symmetry

[M.C., Lin, 1706.08521]

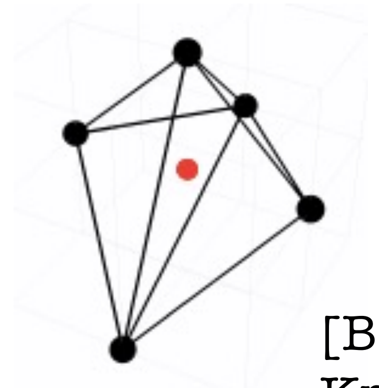
b) Take bases B_3, associated with 3D reflexive polytopes

E.g.,

$\mathbb{P}^3$



$\mathbb{P}^2 \times \mathbb{P}^1$



[Batyrev; Kreuzer-Skarke]

For each reflexive polytope, different bases B_3 are associated with different fine-star-regular triangulations of a chosen polytope

[Triangulations determine intersections of divisors]

Triangulations grow exponentially with the complexity of a polytope

c) Specific choice of divisors: $S_{7,9} = \overline{\mathcal{K}}$

[$\overline{\mathcal{K}}$ - anti-canonical divisor of the base B_3 – fixed by the polytope]

SU(3) an SU (2) divisors S_9 and S_3 with class $\overline{\mathcal{K}}$

$$g_{3,2}^2 = 2/\mathrm{vol}(\overline{\mathcal{K}})$$

U(1) - (height-pairing) divisor volume $5\overline{\mathcal{K}}/6$

$$\frac{5}{3}\, g_Y^2 = \frac{2}{\mathrm{vol}(\overline{\mathcal{K}})}$$

⬇

Standard Model with gauge coupling unification!

$$g_3^2 = g_2^2 = 5/3 g_Y^2$$

d) Remaining conditions (due to G_4)

iii. Chirality: n_F=3 families of quarks and leptons

iv. D3-tadpole cancellation

c.f., [Lin, Mayrhofer, Till,Weigand, 1508.00162]
[M.C., Grassi, Klevers, Piragua, 1306.3987]

G_4 in terms of (1,1)-forms, Poincaré dual to divisor classes:

$$G_4(a,\,\omega) = a\,G_4^a + \pi^*\omega\wedge\sigma$$

$$G_4^a = [e_4]\wedge([e_4]+\pi^*[s_6]) + \frac{[e_1]\wedge\pi^*[s_9]}{2} + \frac{\pi^*[s_3]\wedge([e_2]+2[u])}{3}$$

$$\omega \in H^{1,1}(B_3)$$

Chirality, D3 tadpole and G_4 integrality expressed in terms of intersection numbers of divisors in the base B_3 → Geometric

For $S_{7,9}=\overline{\mathcal{K}}$ → $n_F=\chi(\mathbf{R}) = -\frac{a}{5}\int_{B_3}\overline{\mathcal{K}}\wedge\overline{\mathcal{K}}\wedge\overline{\mathcal{K}} =: -\frac{a}{5}\overline{\mathcal{K}}^3$ → $a=5n_F/\overline{K}^3$

fractional no!

$$\boxed{n_{D3}(n_F,\overline{K}^3) = 12+\frac{5\overline{K}^3}{8}-\frac{5n_F^2}{2\overline{K}^3}\in\mathbb{Z}_{\geq 0}}$$

Independent of triangulation → Universality of the Standard Model

Landscape count for n_F=3 families:

$$12 + \frac{5}{8}\overline{\mathcal{K}}^3 - \frac{45}{2\overline{\mathcal{K}}^3} \in \mathbb{Z}_{\geq 0}$$ satisfied for $\overline{\mathcal{K}}^3 \in \{2, \overset{\text{toric}}{\boxed{6, 10, 18, 30}}, 90\}$

- Out of 4319 3D reflective polytopes → 708 satisfy the constraint (many of them with a large number of lattice points).

- Triangulation of polytopes can be handled combinatorially (each corresponds to a different basis B).
  It can be implemented on computer, e.g., in SageMath:
  i) for 237 polytopes w/ < 15 lattice points →414310 MSSM models.
  ii) for 471 polytopes w/ ≥ 15 lattice points – exp. growing comp. time → counting via fine-regular triangulation of facets & estimate regular fine-star triang.

c.f., [Halverson, Tian, 1610.08864]

- Provide a bound: $7.6 \times 10^{13} \lesssim N_{\rm SM}^{\rm toric} \lesssim 1.6 \times 10^{16}$

Quadrillion Standard Models (QSMs)

III. Matter vector-like pairs → MSSMs

Counting of matter vector-pairs

Depends on C_3 potential encoded in the intermediate Jacobian of Calabi-Yau four-fold $\hat{Y}_4$ via Deligne cohomology

[Diaconescu, Moore, Freed '03]...
[Intriligator, Jockers, Mayr, Morrison, Plesser '12]

$$0 \to J^2(\widehat{Y}_4) \hookrightarrow H^4_D(\widehat{Y}_4, \mathbb{Z}(2)) \twoheadrightarrow H^{(2,2)}_{\mathbb{Z}}(\widehat{Y}_4) \to 0$$

...[Bies, Mayrhofer, Pehle, Weigand '14]...[Bies '18]

When restricted to the matter curve $\mathcal{C}_R$, C_3 potential defines a line bundle $\mathcal{L}_{\mathbf{R}}$

massless chiral modes $\subset H^0(\mathcal{C}_{\mathbf{R}}; \mathcal{L}_{\mathbf{R}})$ in rep **R**

massless chiral modes $\subset H^1(\mathcal{C}_{\mathbf{R}}; \mathcal{L}_{\mathbf{R}})$ in rep $\bar{\mathbf{R}}$

[Recall, chiral index $\chi(R) = h^0 - h^1$ depends on $G_4 = dC_3$, only (topological)]

$H^i(\mathcal{C}_{\mathbf{R}}; \mathcal{L}_{\mathbf{R}})$ – computation (in principle) via algorithm implemented in computer algebra system CA [Bies '17; Bies, Posur '19]

Counting of matter vector-pairs

For the quadrillion Standard Models (QMS) the analysis complex due to the complexity of the flux construction and high genera of matter curves.

Goal: to determine the range of complex structure moduli of the F-theory compactification, for which we have the Minimal Supersymmetric Standard Models (MSSMs) [no other vector-pair exotics; one Higgs doublet pair]

No time

``Warm-up'': [Bies, M.C., Donagi, Lin, Liu, Rühle 2007.00009]

Analytic counting of vector matter pairs for toy SU(5)xU(1) model:

Matter curves C - hyperplane constraints in dP_3.

Line bundles $\mathcal{L}$ - pull-backs of bundles in dP_3 [$\mathcal{L} = O_{dP3}(D_L)|_C$]

Employing Brill-Noether Theory and Machine Learning

Counting matter vector-pairs of QSMs

[Bies, M.C., Donagi, Liu, Ong, 2102.10115

Recall:

toric

$$\overline{\mathcal{K}}^3 \in \{2, \boxed{6, 10, 18, 30}, 90\}$$

$$G_4(a,\,\omega) = a\,G_4^a + \pi^*\omega \wedge \sigma$$

$$G_4^a = [e_4] \wedge ([e_4] + \pi^*[s_6]) + \frac{[e_1] \wedge \pi^*[s_9]}{2} + \frac{\pi^*[s_3] \wedge ([e_2] + 2[u])}{3}$$

$$\omega \in H^{1,1}(B_3)$$

$a = 5n_F/\overline{\mathcal{K}}^3$ $\quad n_F=3$

Fractional numbers!

Lead to ``fractional power'' bundles on matter curves $C_{\mathbf{R}}$

→ Root bundles (generalization of spin bundles)

Appearance of Root Bundles

1. Consider $\boxed{G_4' = \overline{K}_{B_3}^3 \cdot G_4.}$

   G'$_4$ - integer valued coeffs.

2. Lift G_4' to gauge potential $A' \in H_D^4(\widehat{Y}_4, \mathbb{Z}(2)) \to$ induces line bundle $L(A, \mathbf{R}) \in \mathrm{Pic}(C_{\mathbf{R}})$.

   standard

3. Consider a $\boxed{\overline{K}_{B_3}^3\text{-th root } A \text{ of } A'}$ in $H_D^4(\widehat{Y}_4, \mathbb{Z}(2))$:
   - If A exists, there are $\left(\overline{K}_{B_3}^3\right)^{2\dim_{\mathbb{C}}(J^2(\widehat{Y}_4))}$ inequivalent choices.
   - Each A specified by data in intermediate Jacobian $J^2(\widehat{Y}_4)$, i.e., strictly beyond G_4-data.
   - $\boxed{A \text{ induces } M \in \mathrm{Pic}(C_{\mathbf{R}})}$ on $C_{\mathbf{R}}$, which is a $\overline{K}_{B_3}^3$ root of $L(A, \mathbf{R})$: $\boxed{M^{\otimes KB^3} = L(A, \mathbf{R}).}$

Goals:

- Find the root bundles which are induced from F-theory gauge potentials C_3 in the Deligne cohomology and have cohomologies that result in the matter spectrum of the MSSM.

- As a first step, a "bottom-up" analysis does not identify the precise root bundles on $C_{\mathbf{R}}$, induced from F-theory gauge potentials C_3. Rather, it provides a systematic study of all admissible root bundles with cohomologies of the MSSM.

 1. Focus first on $C_{\mathbf{R}}$ with no quark and lepton singlet vector-pair exotics. [Bies, M.C., Donagi, Liu, Ong, 2102.10115]

 2. One Higgs doublet pair.

[Bies, M.C., Donagi, Ong, work in progress]

Line bundle ($\mathcal{P}_{\mathbf{R}}$) data for QSMs

- Matter spectra are counted by bundles $\mathcal{P}_{\mathbf{R}} = \mathcal{M}_{\mathbf{R}} \otimes \mathcal{O}_{\text{spin}}$ which is a $2\overline{K}^3_{B_3}$ - root of a line bundle $\mathcal{L}_{\mathbf{R}}$ on $C_{\mathbf{R}}$
- Most bundles $\mathcal{P}_{\mathbf{R}}$ are specific roots of a power of $\overline{K}_{B3}$, restricted to $C_{\mathbf{R}}$.
- Additional contributions from certain triple intersections (Yukawa points)

Example $\overline{K}^3_{B_3}$=18:

curve	genus	constraint	degree
$C_{(\mathbf{3},\mathbf{2})_{\mathbf{1}/\mathbf{6}}} = V(s_3, s_9)$	10	$P^{\otimes 36}_{(\mathbf{3},\mathbf{2})_{\mathbf{1}/\mathbf{6}}} = \overline{K}_{B_3}\vert^{\otimes 24}_{C_{(\mathbf{3},\mathbf{2})_{\mathbf{1}/\mathbf{6}}}}$	12
$C_{(\mathbf{1},\mathbf{2})_{-\mathbf{1}/\mathbf{2}}} = V\left(s_3, s_2 s_5^2 + s_1(s_1 s_9 - s_5 s_6)\right)$	82	$P^{\otimes 36}_{(\mathbf{1},\mathbf{2})_{-\mathbf{1}/\mathbf{2}}} = \overline{K}_{B_3}\vert^{\otimes 66}_{C_{(\mathbf{1},\mathbf{2})_{-\mathbf{1}/\mathbf{2}}}} \otimes \mathcal{O}_{C_{(\mathbf{1},\mathbf{2})_{-\mathbf{1}/\mathbf{2}}}}(-30 \cdot Y_1)$	84
$C_{(\overline{\mathbf{3}},\mathbf{1})_{-\mathbf{2}/\mathbf{3}}} = V(s_5, s_9)$	10	$P^{\otimes 36}_{(\overline{\mathbf{3}},\mathbf{1})_{-\mathbf{2}/\mathbf{3}}} = \overline{K}_{B_3}\vert^{\otimes 24}_{C_{(\overline{\mathbf{3}},\mathbf{1})_{-\mathbf{2}/\mathbf{3}}}}$	12
$C_{(\overline{\mathbf{3}},\mathbf{1})_{\mathbf{1}/\mathbf{3}}} = V\left(s_9, s_3 s_5^2 + s_6(s_1 s_6 - s_2 s_5)\right)$	82	$P^{\otimes 36}_{(\overline{\mathbf{3}},\mathbf{1})_{\mathbf{1}/\mathbf{3}}} = \overline{K}_{B_3}\vert^{\otimes 66}_{C_{(\overline{\mathbf{3}},\mathbf{1})_{\mathbf{1}/\mathbf{3}}}} \otimes \mathcal{O}_{C_{(\overline{\mathbf{3}},\mathbf{1})_{\mathbf{1}/\mathbf{3}}}}(-30 \cdot Y_3)$	84
$C_{(\mathbf{1},\mathbf{1})_{\mathbf{1}}} = V(s_1, s_5)$	10	$P^{\otimes 36}_{(\mathbf{1},\mathbf{1})_{\mathbf{1}}} = \overline{K}_{B_3}\vert^{\otimes 24}_{C_{(\mathbf{1},\mathbf{1})_{\mathbf{1}}}}$	12

- Similar results (w/different root powers) for all toric B_3 w/ $\overline{K}^3_{B_3} \in \{6, 10, 18, 30\}$

Root bundles

- For generic (smooth) curve $C_{\mathbf{R}}$ of genus g & line bundle $\mathcal{L}_{\mathbf{R}} \in \mathbf{Pic}(C_{\mathbf{R}})$
- $\mathcal{L}_{\mathbf{R}}$ admits k-th roots iff $deg(\mathcal{L}_{\mathbf{R}})$=d is divisible by k
- There are k^{2g} such roots
  [Jacobian $J^1(C_{\mathbf{R}}) \cong \mathbb{C}^g / \Lambda$, where Λ is a full-dimensional lattice]

Example: Spin bundles are *2nd* roots of $\bar{K}_{B3}$, restricted to $C_{\mathbf{R}}$
Root bundles naturally generalize this concept

- On generic curves w/ $g \geq 2$, root bundles are hard to construct
  [Abel-Jacobi map hard be compute explicitly]

Alternative approach:

- Access roots on nodal (singular) curves
- Employ deformation theory to access smooth curves

→ Our approach

Root bundles on nodal curves

studied by mathematicians

[Jarvis '98],[Caporaso, Casagrande, Cornalba '04]

- Nodal curve $C^{\bullet}_{\mathbf{R}}$ w/singularities at most nodes: $V(xy) = \{x \cdot y = 0\}$
- $C^{\bullet}_{\mathbf{R}}$ consist of many components, summarized in a dual graph:

  bullet ←→ component ; connecting line ←→ node

- Weighted diagrams, associated to blow-ups $C^{\circ}_{\mathbf{R}}$ of $C^{\bullet}_{\mathbf{R}}$ describe all root bundles [Caporaso, Casagrande, Cornalba '04]

- Focus on full blow-ups $C^{\circ}_{\mathbf{R}}$ of $C^{\bullet}_{\mathbf{R}}$

Example: Base three-fold ∈ $B_3(\boldsymbol{\Delta}_{40})$
& quark-doublet matter curve:

$$C_{(\mathbf{3},\mathbf{2})_{1/6}} = V(s_3, s_9) \quad \rightarrow \quad C^{\bullet}_{(\mathbf{3},\mathbf{2})_{1/6}} = \bigcup_{i=1}^{11} V(x_i, s_9) \longrightarrow C^{\circ}_{(\mathbf{3},\mathbf{2})_{1/6}}$$

generic nodal blow-up

Solving the root bundle constraint – limits roots

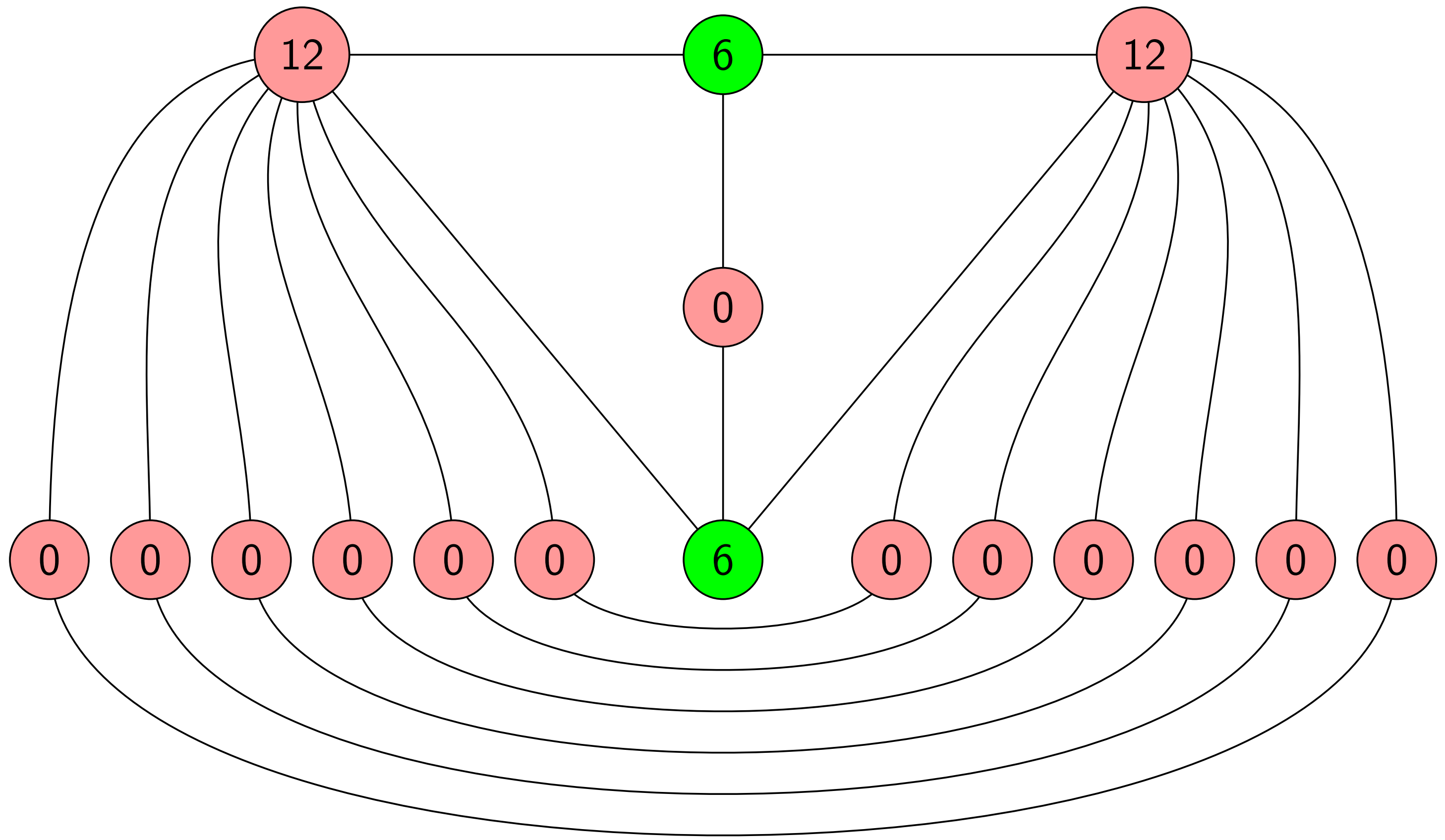
[Caporaso,Casagrande, Cornalba '04]

- Have to solve $P_{(\mathbf{3},\mathbf{2})_{\mathbf{1/6}}}^{\otimes 36} = \overline{K}_{B_3}\big|_{C_{(\mathbf{3},\mathbf{2})_{\mathbf{1/6}}}}^{\otimes 24}$
  Sufficient to solve $P_{(\mathbf{3},\mathbf{2})_{\mathbf{1/6}}}^{\otimes 3} = \overline{K}_{B_3}\big|_{C_{(\mathbf{3},\mathbf{2})_{\mathbf{1/6}}}}^{\otimes 2}$
- Required input:
  1. Dual graph of nodal curve $C^{\bullet}_{(3,2)1/6}$
  2. Degree of restriction of $\bar{K}_{B3}$ to all components of $C^{\bullet}_{(3,2)1/6}$

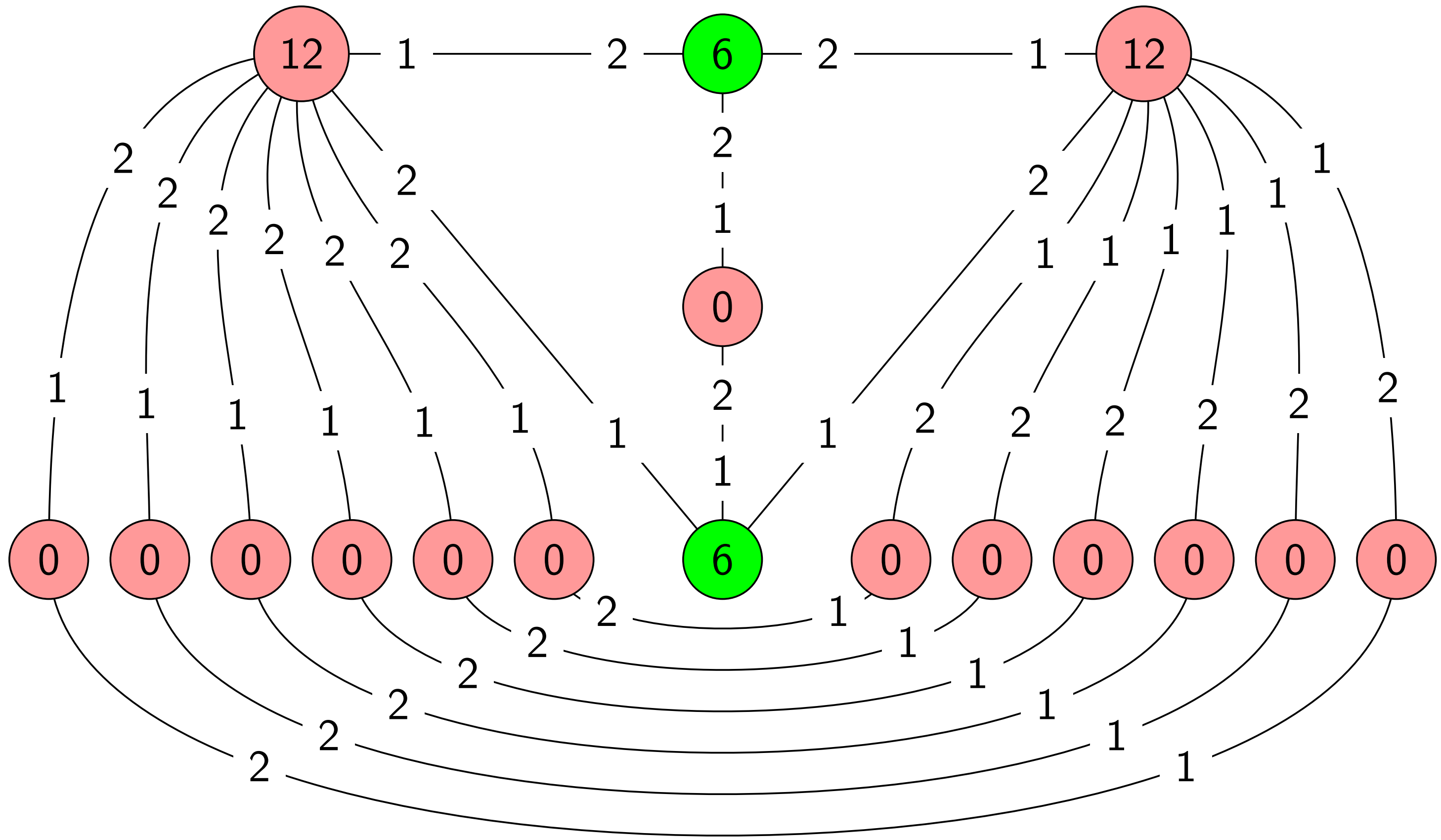
- Strategy

  Place weights (degrees of restriction of $\bar{K}_{B3}$) w ∈ 1, 2 (looking for 3-rd roots) along each connecting line subject to:
  1. Along each connecting line, two weights w_1, w_2; $w_1 + w_2 = 3$
  2. At each node: degree of $2\bar{K}_{B3}$ minus sum of the adjacent weights must be divisible by 3.

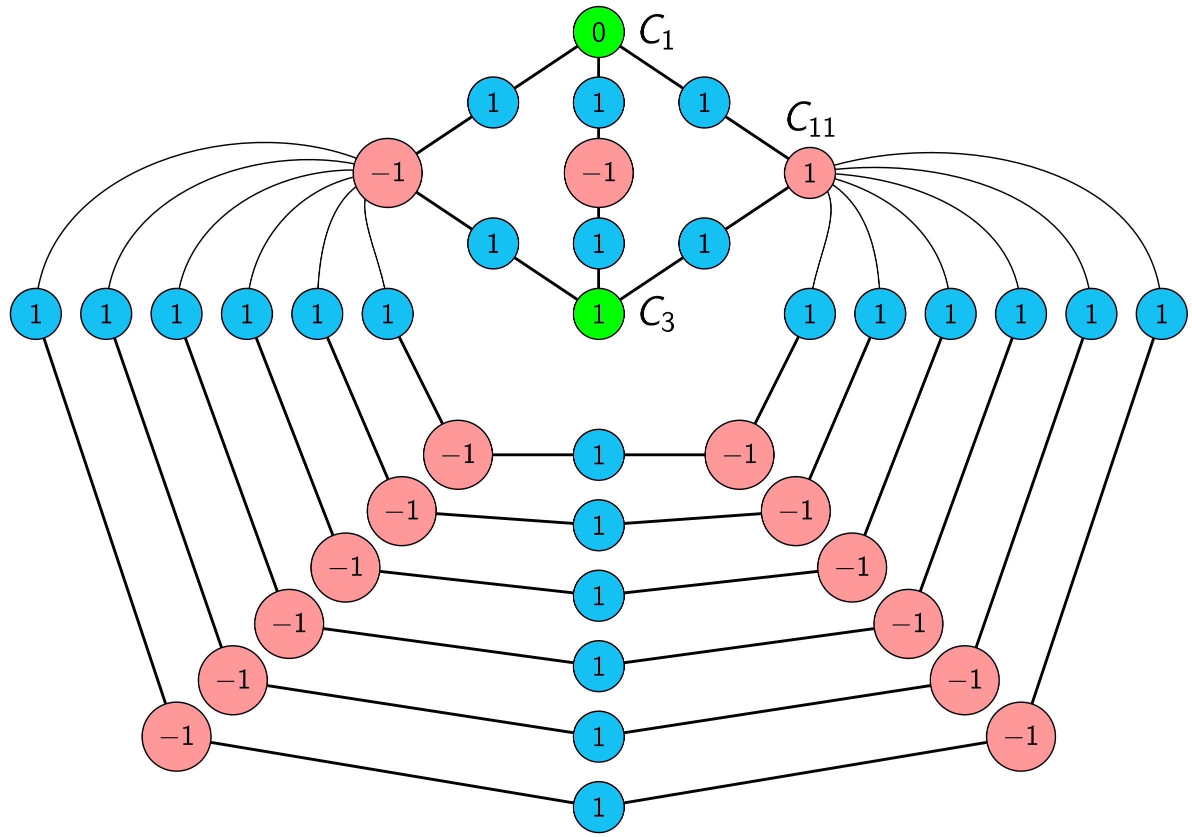
Example – Dual graph of $C^{\bullet}_{(\mathbf{3},\mathbf{2})_{1/6}}$ and degrees of $\overline{K}_{B_3}\big|^{\otimes 2}_{C_{(\mathbf{3},\mathbf{2})_{1/6}}}$



Example – Weighted diagram to solve $P_{(\mathbf{3},\mathbf{2})_{1/6}}^{\otimes 3} = \overline{K}_{B_3}\Big|_{C_{(\mathbf{3},\mathbf{2})_{1/6}}}^{\otimes 2}$

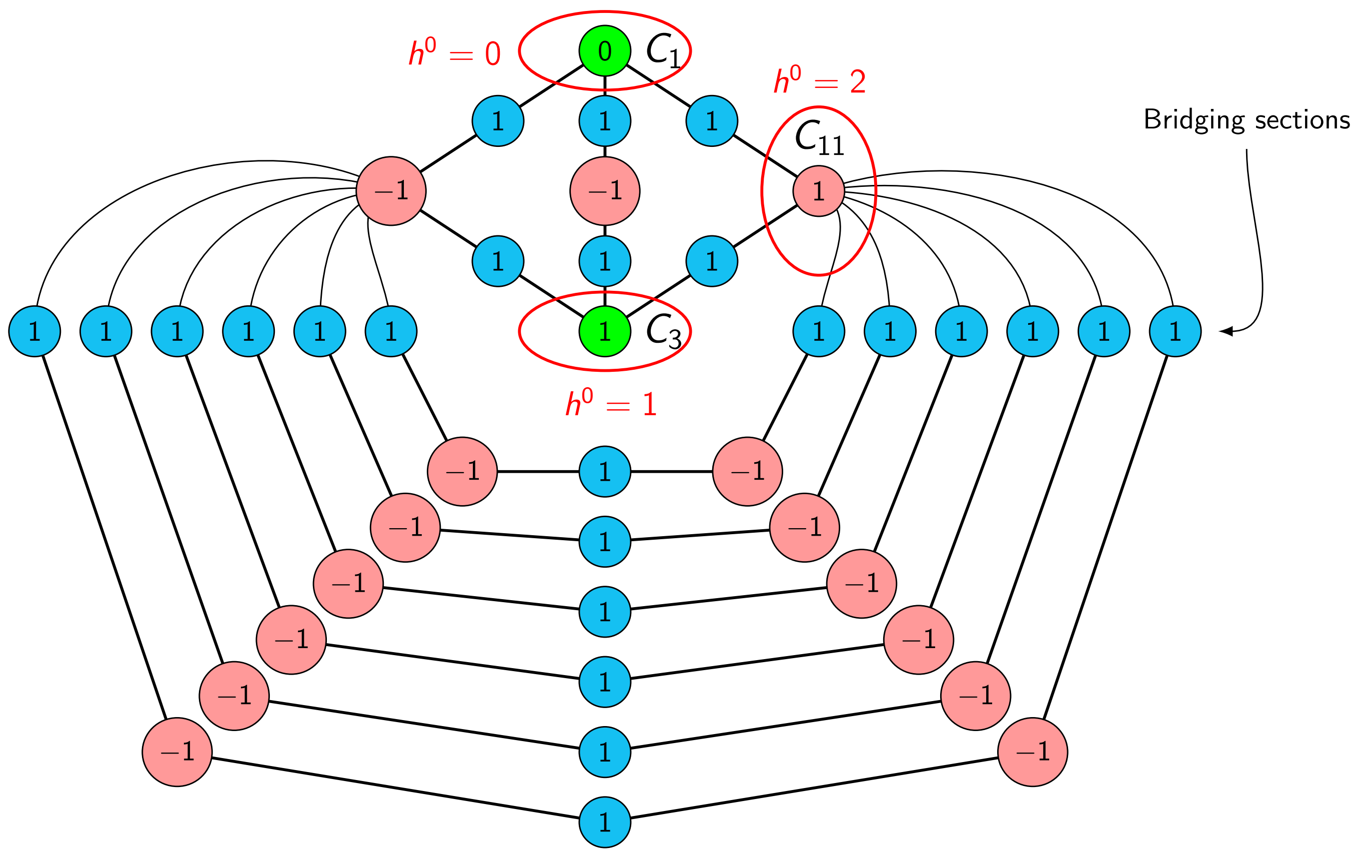


Example – count h^0 for some solutions to $P_{(\mathbf{3},\mathbf{2})_{1/6}}^{\otimes 3} = \overline{K}_{B_3}\big|_{C_{(\mathbf{3},\mathbf{2})_{1/6}}}^{\otimes 2}$



Degrees of root-bundle P_R in each component of $C_{\mathbf{R}}^{\circ}$

Example – count h^0 for some solutions to $P_{(\mathbf{3},\mathbf{2})_{1/6}}^{\otimes 3} = \overline{K}_{B_3}\big|_{C_{(\mathbf{3},\mathbf{2})_{1/6}}}^{\otimes 2}$

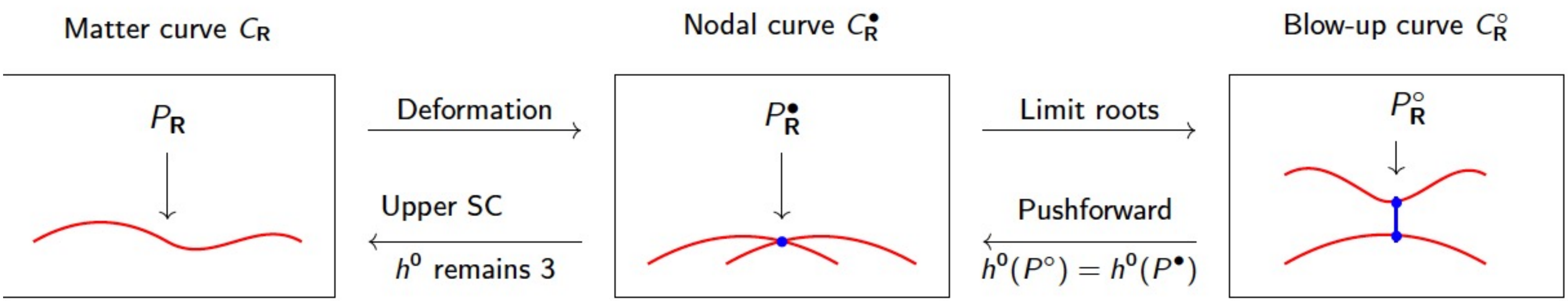


Root bundle construction of $\mathcal{P}_{\mathbf{R}}$ with $h^0 = 3$

From $C^{\bullet}_{(\mathbf{3},\mathbf{2})_{1/6}}$ to $C_{(\mathbf{3},\mathbf{2})_{1/6}}$: Upper Semi-Continuity

- $h^0 = 3$ cannot increase as we make the curve $C^{\bullet}_{(3,2)1/6}$ more generic
- $h^0 = 3$ is the lower bound ($n_F = h^0 - h^1 = 3$)

  → no quark-doublet vector-pair exotics



- Related construction w/ h^0 applies to quark-singlet and lepton-singlet curves → no vector-pair exotics

Counting root bundles w/ $h^0=3$ for all promising B_3

[Bies, M.C., Liu, 2102.08297]

- Necessary condition for root bundles on $C_{(3,2)1/6}$ to stem from F-theory gauge potentials C_3:

$$h^{2,1}(\widehat{Y}_4) \geq g \quad \rightarrow \quad \text{33 (out of 708) polytopes}$$

- Found $\check{N}_P^{(3)}$ *triangulation independent** root bundles w/ $h^0=3$

  Computed with Gap4-package QSMExplorer
  https://github.com/homalg-project/ToricVarieties_project

Example $\overline{K}^3_{B_3} = 6$:

	$\check{N}_P^{(3)}$
Δ°_8	142560
Δ°_4	11110
Δ°_{134}	10100
Δ°_{128}	8910
Δ°_{130}	8910
Δ°_{136}	8910
Δ°_{236}	8910

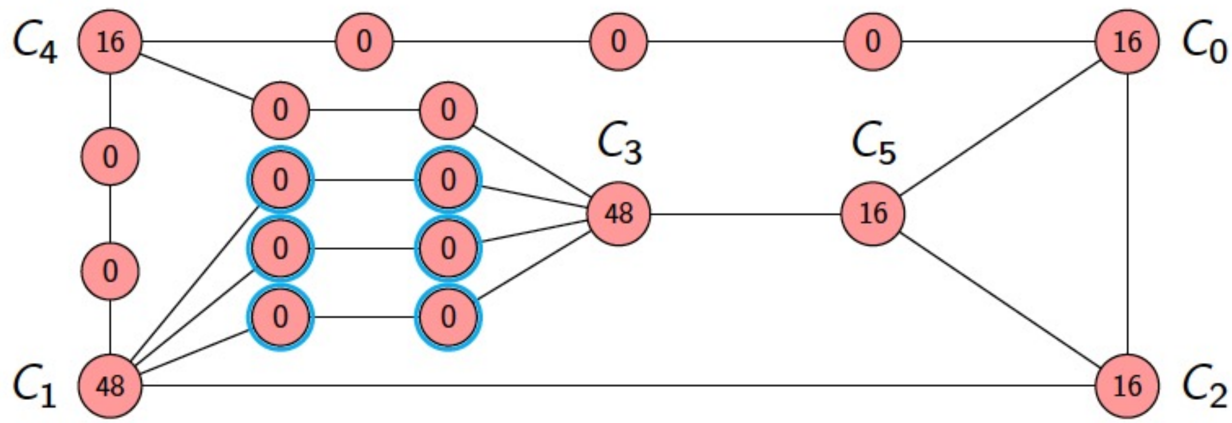
$\boldsymbol{\Delta}_i$ - 3D Polytope list [Kreuzer, Skarke '98]

*$C_{(3,2)1/6}$ components $V(x_i,s_9)$ are divisors in K3 $V(s_9)$. $s_9=\overline{\mathcal{K}}$

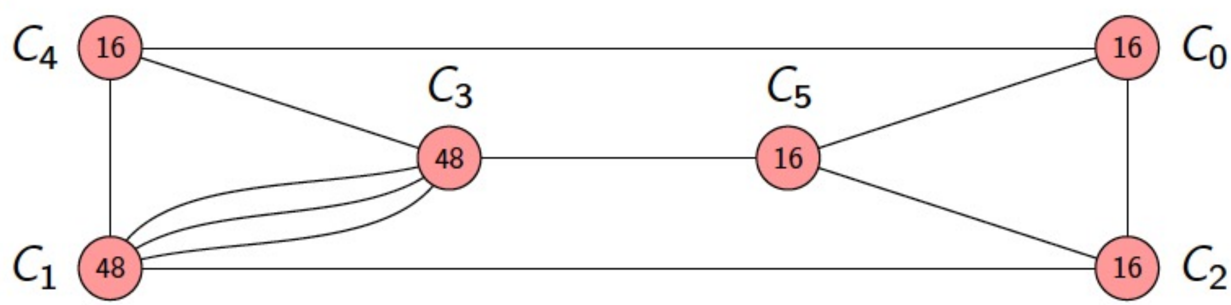
Comment

Computation of $\check{N}_P^{(3)}$ made possible by simplification

- Complicated dual graph:



- Simple dual graph:



Toward one Higgs doublet pair →MSSM

[M. Bies, M.C., R. Donagi, M. Ong, work in progress]

Find limit roots on nodal Higgs curves $C^{\bullet}_{(\mathbf{1},\mathbf{2})_{-1/2}}$ with $h^i = (4, 1)$

→ one Higgs-doublet pair

$$C_{(\mathbf{1},\mathbf{2})_{-1/2}} = V(s_3, s_2 s_5^2 + s_1^2 s_9 - s_1 s_5 s_6) \to C^{\blacksquare}_{(\mathbf{1},\mathbf{2})_{-1/2}} = \bigcup_i^N V(x_i, s_2 s_5^2 + s_1^2 s_9 - s_1 s_5 s_6)$$

smooth nodal

x_i-homogeneous coordinates of toric B_3

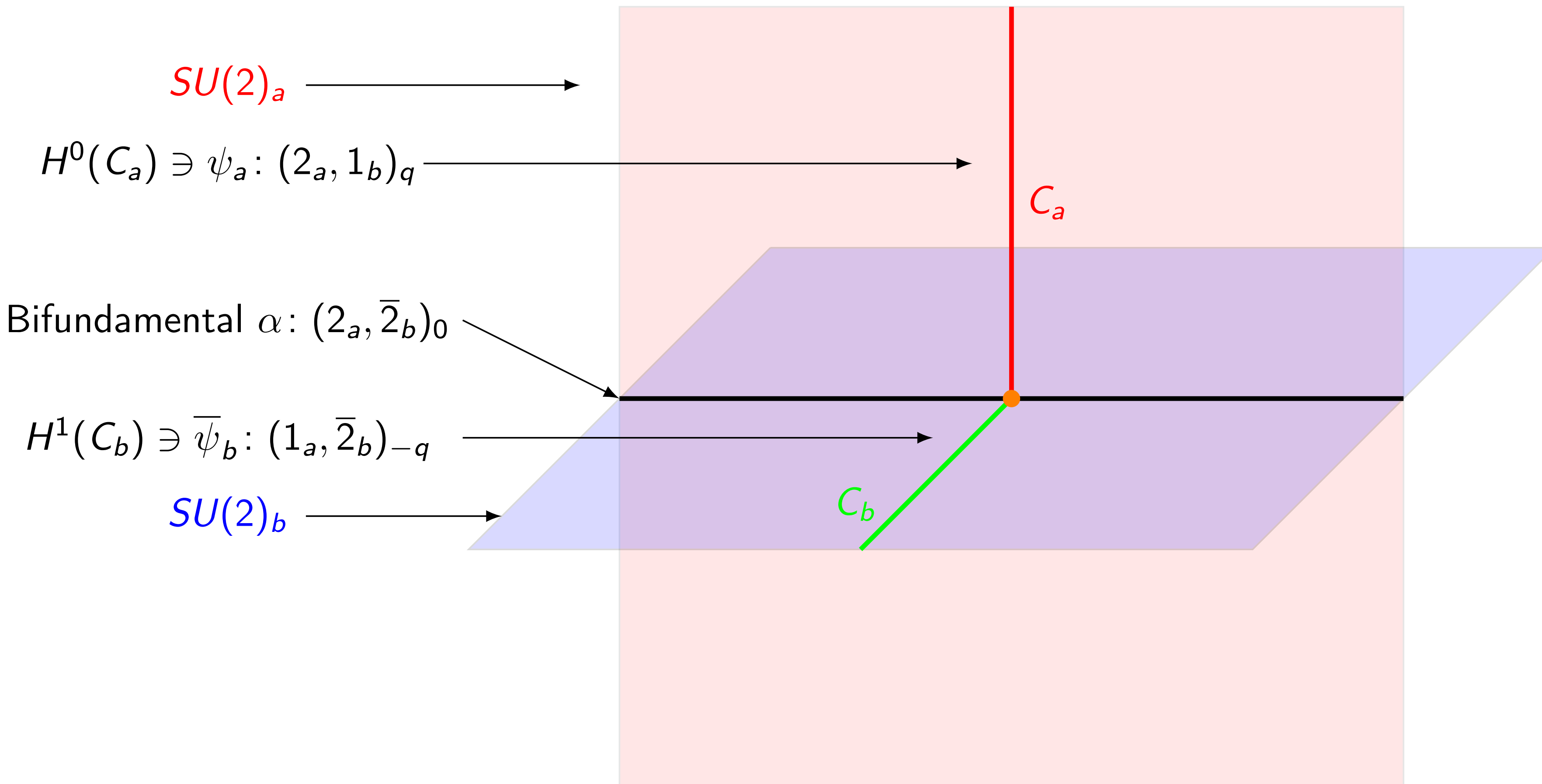
$$V(s_3) \to \bigcup_i^N V(x_i)$$

- $V(s_3)$ - SU(2) gauge divisor → geometric un-Higgsing SU(2) $\rightarrow$ SU(2)^N
- Trace sections of root bundles $C^{\bullet}_{(\mathbf{1},\mathbf{2})_{-1/2}}$ when Higgsing SU(2)$^N \rightarrow$ SU(2)

  Does no of sections remain intact under this transition?

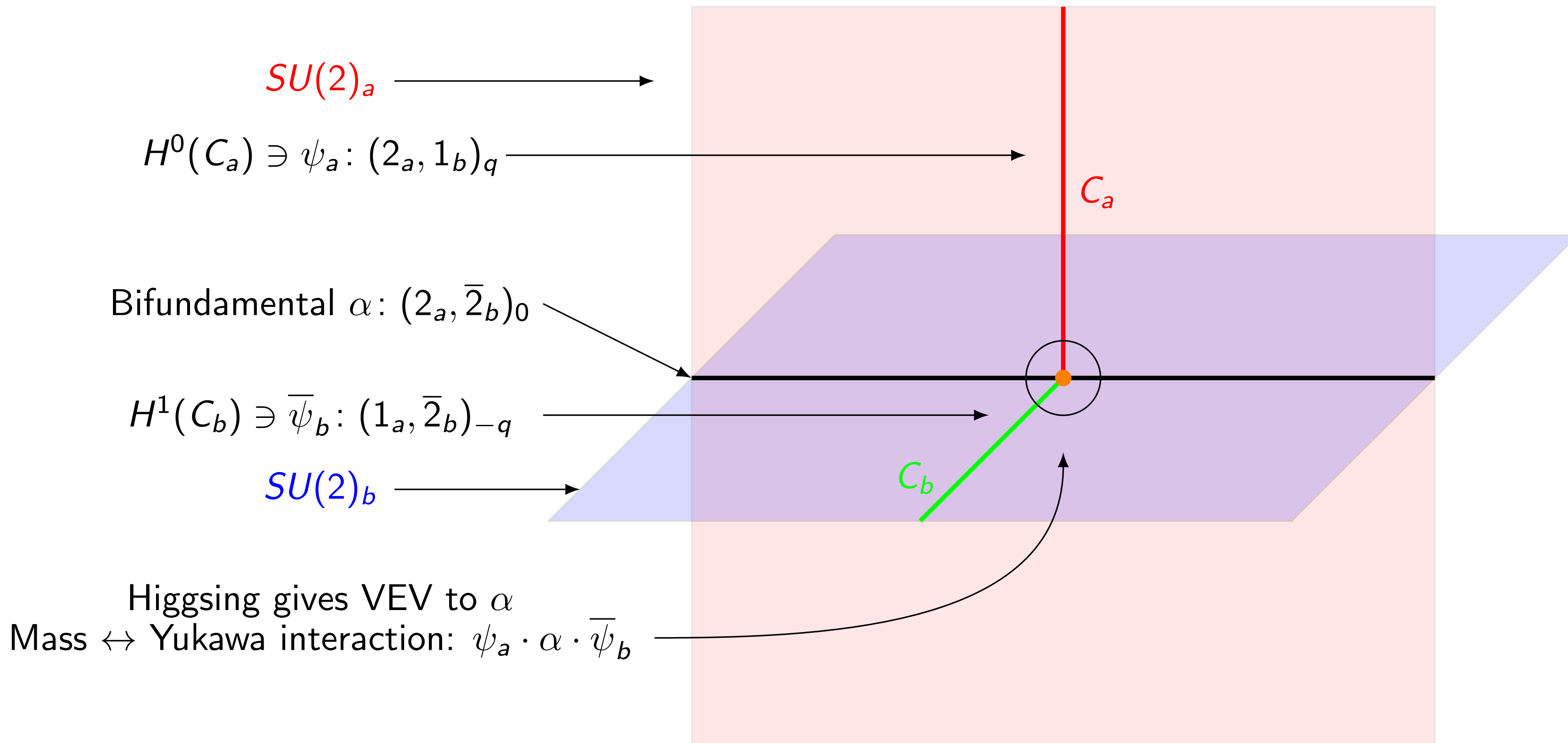
Building blocks:

Higgsing $SU(2)_a$ x $SU(2)_b$ → SU(2) [``Brane recombination'']



Building blocks:

Higgsing $SU(2)_a$ x $SU(2)_b$ → SU(2) [``Brane recombination'']

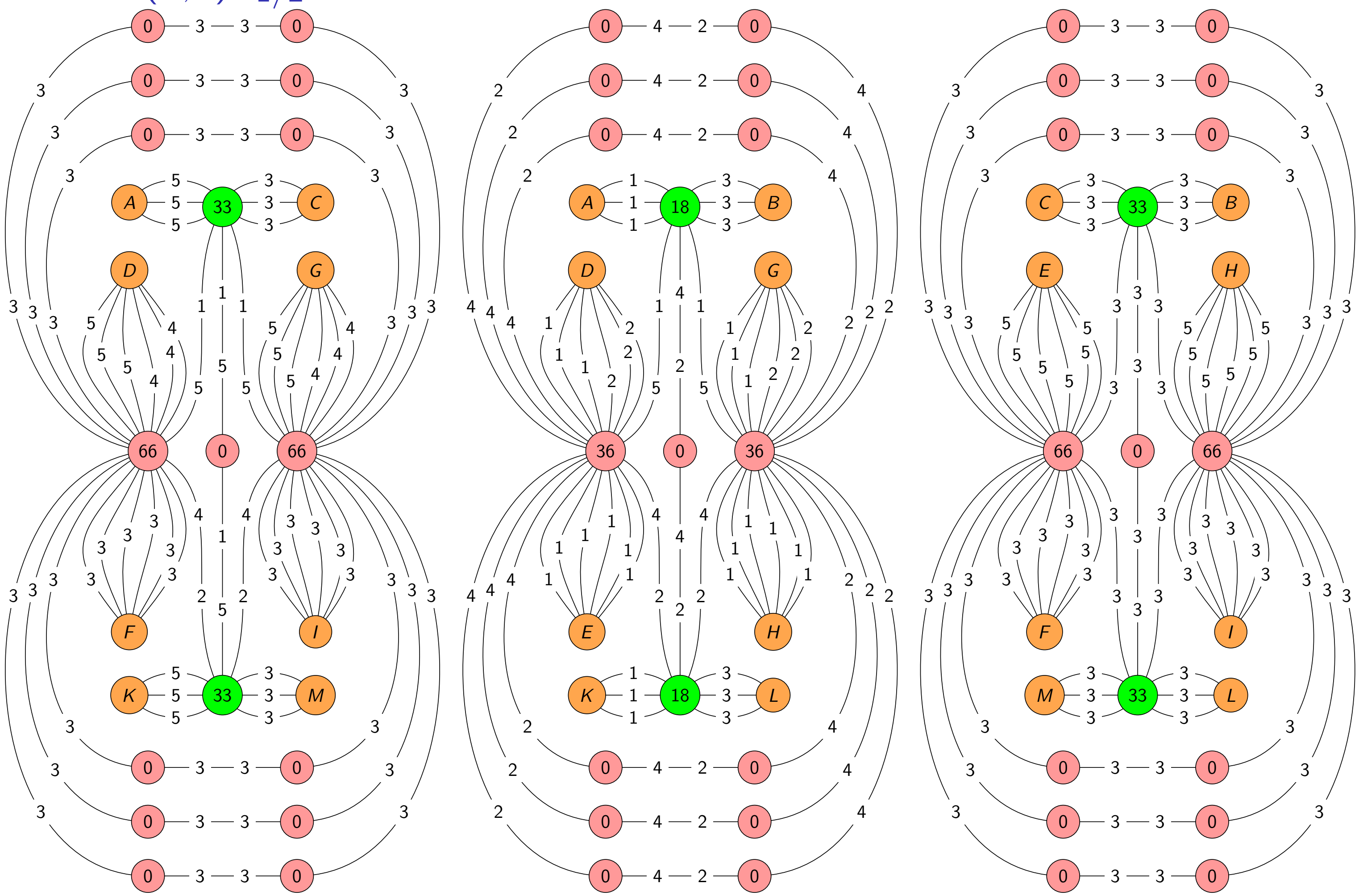


For $h^0(C_a) \cdot h^1(C_b) = 0\,,\qquad \left(h^1(C_a) \cdot h^0(C_b) = 0\right)$ → no mass term

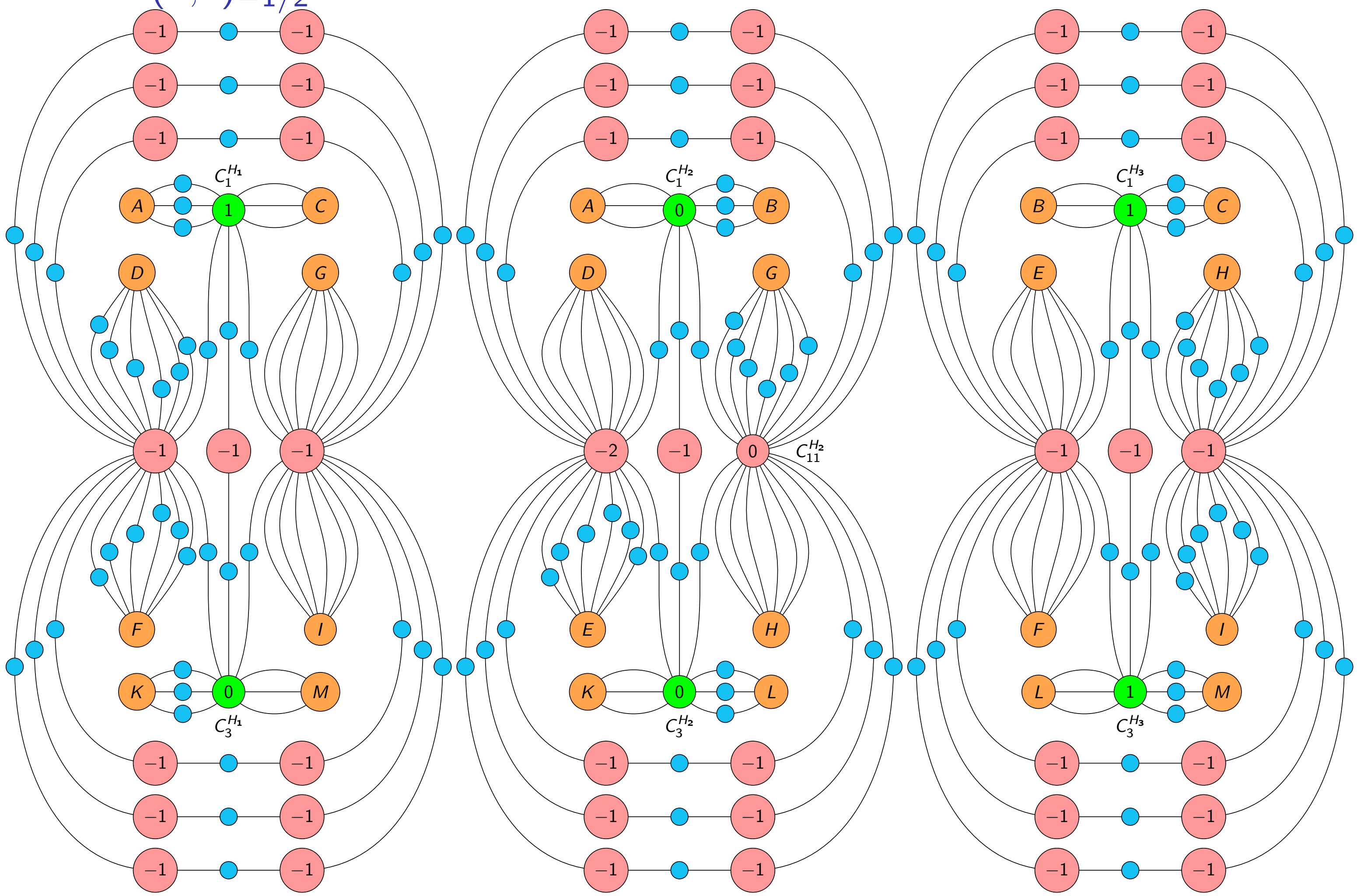
→ do not loose a section

Such algebro-geometric conditions unfamiliar → math conjecture/theorems?

Example: $C^{\bullet}_{(\mathbf{1},\mathbf{2})_{-1/2}} \subset B_3(\Delta^{\circ}_{40})$ – **stable** limit root with $h^i = (4,1)$



Example: $C^{\bullet}_{(\mathbf{1},\mathbf{2})_{-1/2}} \subset B_3(\Delta^{\circ}_{40})$ – **stable** limit root with $h^i = (4, 1)$



Summary and Outlook

Particle physics of globally consistent F-theory compactifications has come a long way:

Focused on QSMs: globally consistent 3-family SMs
without chiral exotics and gauge coupling unification

- ``Bottom-up'' construction of root bundles → exact MSSM spectra

But there is much more to go:

- ``Top-down'' determination of line bundle cohomologies
- Yukawa couplings (some progress for a toy model)
[M.C., Lin, Liu, Zoccarato, Zhang 1906.10119]
- Systematic exploration of other particle physics models
(possibly beyond toric techniques)

.

.

.

Thank you!