Glueball masses from the lattice: a (partial) review of recent results

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Motivations

- The existence of gauge-invariant bound states of gluons implied by confinement
- Glueballs are being investigated in current and future experiments
- The recent announcements about the odderon open new perspectives for understanding glueballs
- A calculation from first principles using lattice techniques can serve as a guidance to theoretical models and experimental searches

Outline

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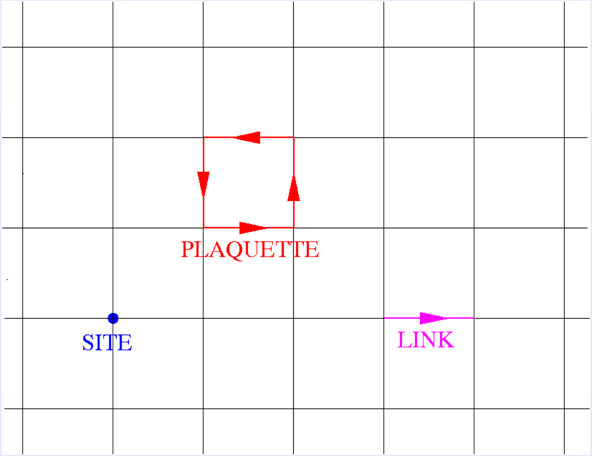
The Lattice



Lattice action for full QCD

Path integral

$$Z = \int \left(\mathcal{D}U_\mu(i)\right) \left(\det M(U_\mu)\right)^{N_f} e^{-S_g(U_{\mu\nu}(i))}$$

with

$$U_\mu(i) = Pexp\left(ig \int_i^{i+a\hat{\mu}} A_\mu(x) \mathrm{d}x \right)$$

and

$$U_{\mu\nu}(i) = U_\mu(i) U_\nu(i+\hat{\mu}) U^\dagger_\mu(i+\hat{\nu}) U^\dagger_\nu(i)$$

Gauge part

$$S_g = \beta \sum_{i,\mu} \left(1 - \frac{1}{N} \mathcal{R}\mathrm{e}\, \mathrm{Tr}(\mathrm{U}_{\mu\nu}(\mathrm{i})) \right) \quad , \quad with\ \beta = 2N/g_0^2$$

Wilson fermions

Take the naive Dirac fermions and add an irrelevant term that goes like the Laplacian

$$M_{\alpha\beta}(ij) = (M + 4r)\delta_{ij}\delta_{\alpha\beta} - \frac{1}{2}\left[(r - \gamma_\mu)_{\alpha\beta} U_\mu(i)\delta_{i,j+\mu} + (r + \gamma_\mu)_{\alpha\beta} U^\dagger_\mu(j)\delta_{i,i-\mu}\right]$$

This formulation breaks explicitly chiral symmetry

Define the hopping parameter

$$\kappa = \frac{1}{2(m + 4r)}$$

Chiral symmetry recovered in the limit $\kappa \to \kappa_c$ (κ_c to be determined numerically)

Quenched approximation

For an observable $\mathcal{O}$

$$\langle \mathcal{O} \rangle = \frac{\int (\mathcal{D}U_\mu(i)) (\det M(U_\mu))^{N_f} f(M) e^{-S_g(U_{\mu\nu}(i))}}{\int (\mathcal{D}U_\mu(i)) (\det M(U_\mu))^{N_f} e^{-S_g(U_{\mu\nu}(i))}}$$

Assume $\det M(U_\mu) \simeq 1$ i.e. fermions loops are removed from the action

The approximation is exact in the $m \to \infty$ and $N \to \infty$ limit ($g^2 N$ is fixed)
$\hookrightarrow$ the large N spectrum is quenched for $m \neq 0$

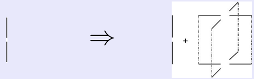
As N increases, unquenching effects are expected for smaller quark masses

Outline

Masses of states from correlators

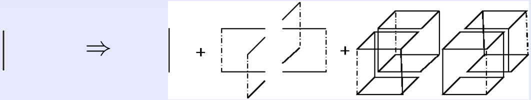
Masses of states extracted from two-point functions (*correlators*) of operators with the right quantum numbers
Starting from links, we can built those operators via

- Blocking



Fast increase of the size of the operators

- Smearing



Finer resolution

More modern approach: Wilson flow

Correlation matrix

Numerical signal improves considerably using the full correlation matrix

$$\begin{aligned} C_{ij}(t) &= \langle 0| \left(\Phi_i(0)\right)^\dagger \Phi_j(t)|0\rangle = \langle 0| \left(\Phi_i(0)\right)^\dagger e^{-Ht}\Phi_j(0)e^{Ht}|0\rangle \\ &= \sum_n \langle 0| \left(\Phi_i(0)\right)^\dagger |n\rangle\langle n|e^{-Ht}\Phi_j(0)e^{Ht}|0\rangle \\ &= \sum_n e^{-\Delta E_n t}\langle 0| \left(\Phi_i(0)\right)^\dagger |n\rangle\langle n|\Phi_j(0)|0\rangle \\ &= \sum_n c^*_{in} c_{jn} e^{-\Delta E_n t} \end{aligned}$$

After diagonalisation

$$C_{ij}(t) = \delta_{ij} \sum_n |c_{in}|^2 e^{-am_n t} \underset{t\to\infty}{\to} \delta_{ij}|c_{i1}|^2 e^{-am_1 t}$$

Variational principle

1. Find the eigenvector v that minimises
$$am_1(t_d) = -\frac{1}{t_d} \log \frac{v_i^* C_{ij}(t_d) v_j}{v_i^* C_{ij}(0) v_j}$$
for some t_d
2. Fit $v(t)$ with the law $Ae^{-m_1 t}$ to extract m_1
3. Find the complement to the space generated by $v(t)$
4. Repeat 1-3 to extract $m_2, \ldots, m_n$

Sources of systematics

- Need a good overlap of the eigenvectors with the state of interest
- Need a large variational basis including all possible states overlapping with the required one
- Need to keep under control finite size and lattice artefacts
- Care should be taken in assigning the spin

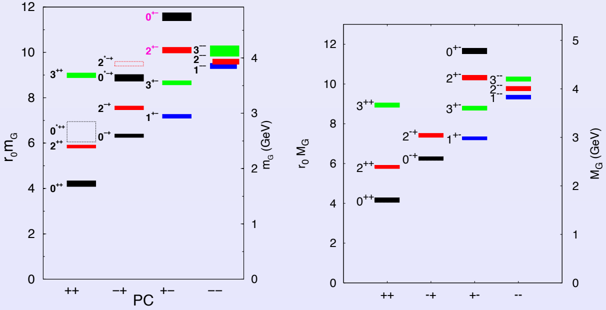
Lattice symmetries and spin

- On the lattice, continuous rotational symmetry broken to the symmetry of the octahedral group
- Irreducible representations are A_1, A_2, E, T_1, T_2
- Operators in irreps of the octahedral group
- Near the continuum limit, full rotational symmetry recovered
- Continuous spin obtained from the subduced representations of the rotation group SO(3) restricted to the octahedral irreps

J	A_1	A_2	E	T_1	T_2
0	1	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	1	0	1	1
4	1	0	1	1	1

Outline

Glueballs in the quenched approximation



Left: C. J. Morningstar and M. J. Peardon, Phys. Rev. D60 (1999) 034509, [hep-lat/9901004]
Right: Y. Chen et al., Phys. Rev. D73 (2006) 014516, [hep-lat/0510074]

QCD at large N

Generalisation of QCD: SU(N) gauge theory (possibly enlarged with N_f fermions in the fundamental representation)

Taking the limit $g^2 \to 0$, $N \to \infty$, $\lambda = g^2 N$ fixed simplifies the theory and one can see that:

- Quark loop effects $\propto 1/N \Rightarrow$ The $N = \infty$ limit is quenched
- Mixing glueballs-mesons $\propto 1/\sqrt{N} \Rightarrow$ No mixing between glueballs and mesons at $N = \infty$
- Meson decay widths $\propto 1/N \Rightarrow$ mesons do not decay at $N = \infty$
- OZI rule $\propto 1/N \Rightarrow$ OZI rule exact at $N = \infty$

$\hookrightarrow$ The simpler large N phenomenology can explain features of QCD phenomenology in a *quenched* setup that removes most of the practical computational difficulties for QCD (and SU(3))

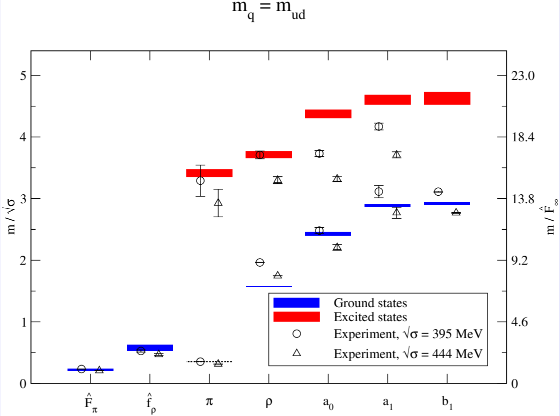
Large N limit on the lattice

The lattice approach allows us to go beyond perturbative and diagrammatic arguments. For a given observable

1. Continuum extrapolation
   - Determine its value at fixed a and N
   - Extrapolate to the continuum limit
   - Extrapolate to $N \to \infty$ using a power series in $1/N^2$
2. Fixed lattice spacing
   - Choose a in such a way that its value in physical units is common to the various N
   - Determine the value of the observable for that a at any N
   - Extrapolate to $N \to \infty$ using a power series in $1/N^2$

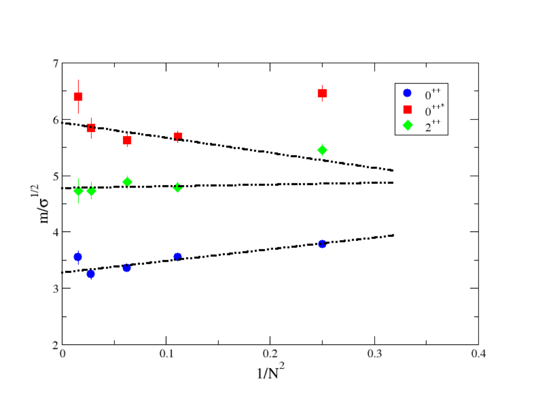
Study performed for various observables both at zero and finite temperature for $2 \leq N \leq 8$

Large N vs. experiments



[Bali *et al.*, JHEP 06 (2013) 071]

Glueball masses at large N



[BL, Teper and Wenger, JHEP 0406 (2004) 012]

Masses at $N = \infty$

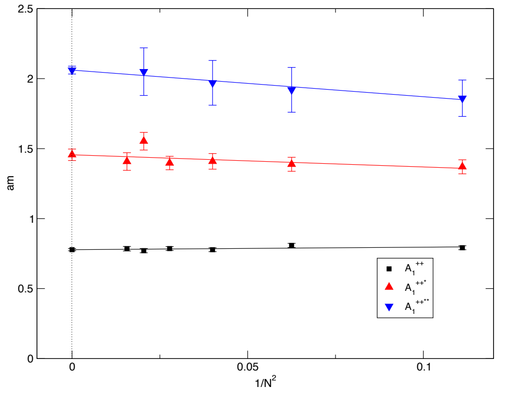
0^{++}
$$\frac{m}{\sqrt{\sigma}} = 3.28(8) + \frac{2.1(1.1)}{N^2}$$

0^{++*}
$$\frac{m}{\sqrt{\sigma}} = 5.93(17) - \frac{2.7(2.0)}{N^2}$$

2^{++}
$$\frac{m}{\sqrt{\sigma}} = 4.78(14) + \frac{0.3(1.7)}{N^2}$$

Accurate $N = \infty$ value, normal $\mathcal{O}(1/N^2)$ correction

0^{++} excitations



Lattice spacing fixed by requiring $aT_c = 1/6$
(BL, Rago and Rinaldi, JHEP 1008 (2010) 119)

Glueballs from the lattice

B. Lucini

Lattice setup

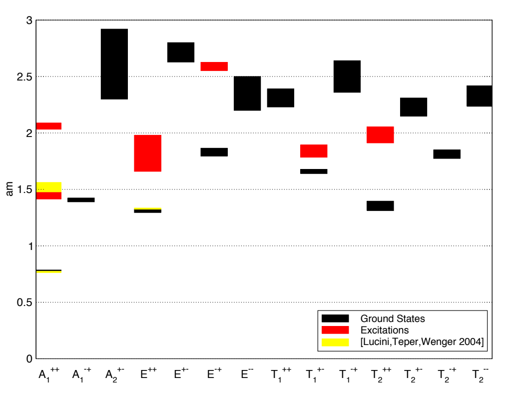
Extracting glueball masses from correlators

Confining theories

Glueball states in full QCD

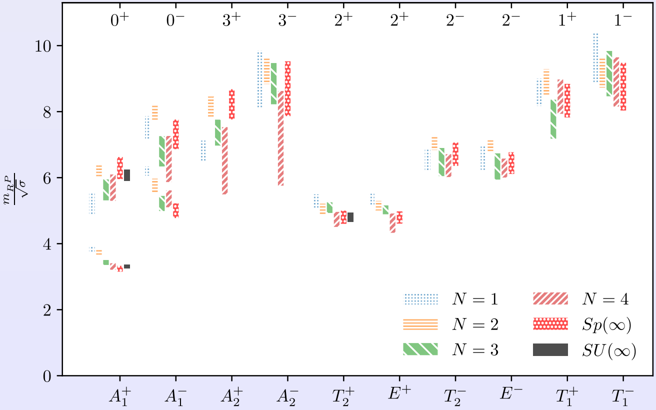
Conclusions and outlook

Large-N glueball spectrum at $aT_c = 1/6$



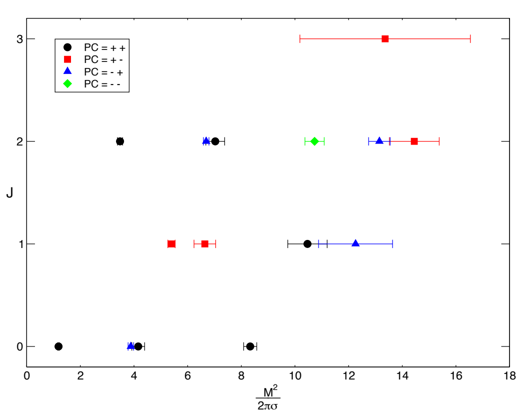
[BL, Rago and Rinaldi, JHEP 1008 (2010) 119]

Large-N spectrum in the continuum



[E. Bennett *et al.*, Phys.Rev.D 103 (2021) 5, 054509, arXiv:2010.15781]
[A. Athenodorou and M. Teper, to appear tomorrow]

Regge trajectories



[B. Lucini, A. Rago and E. Rinaldi, JHEP 1008 (2010) 119]

Outline

Construction of operators

For single states, start with a zero-momentum operator

$$\phi(x,t) = \mathrm{Tr} \prod_{(i;\hat{\mu})\in\mathcal{C}} U_\mu(i) \qquad \mathcal{O}(t) = \phi(t) = \frac{1}{N_L^3} \sum_{x\in\Lambda_s} \phi(x,t)$$

We then build the irreducible representation

$$\begin{aligned}\Phi^{(R)}(t) &= \sum_i c_i^{(R)} \mathcal{R}_i(\phi(t)) - \sum_i c_i^{(R)} \mathcal{R}_i(\langle\phi(t)\rangle) \\ &= \sum_i c_i^{(R)} \mathcal{R}_i(\phi(t)) - \langle\phi(t)\rangle \sum_i c_i^{(R)}\end{aligned}$$

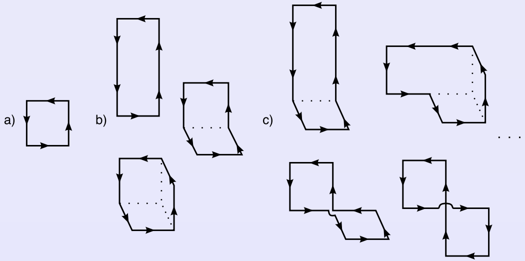
For the scattering states, we square the operator

$$\Phi^{(R)}(t) = \sum_i c_i^{(R)} \mathcal{R}_i \left((\phi(t) - \langle\phi(t)\rangle)^2 \right) - \left(\langle\phi^2(t)\rangle - \langle\phi(t)\rangle^2 \right) \sum_i c_i^{(R)}$$

Torelons are built in a similar way to glueballs, but as operators we use Polyakov loops

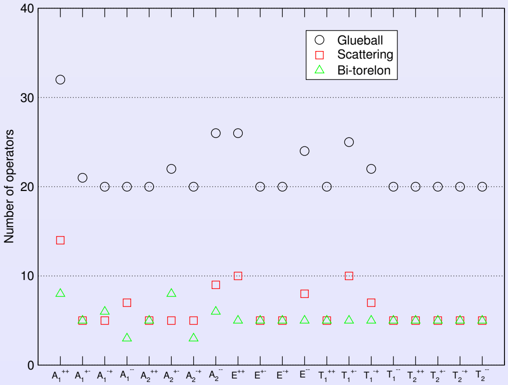
Contours for operators

Glueballs and scattering states

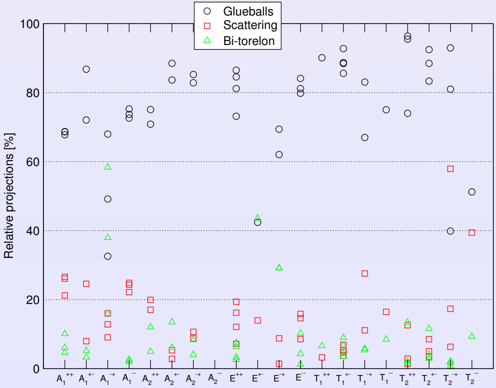


Torelons

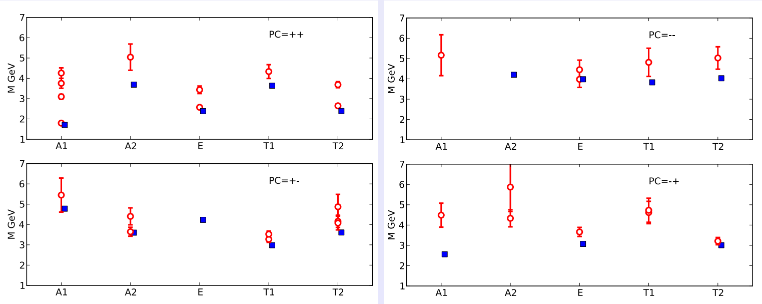
Operators used in calculation



Identification of lattice artefacts

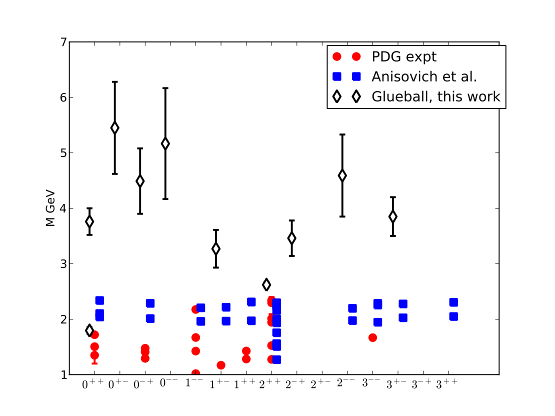


Comparison with quenched results



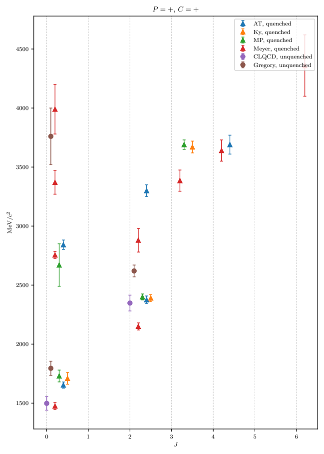
Quenched results (blue points) from Y. Chen et al., Glueball spectrum and matrix elements on anisotropic lattices, Phys. Rev. D73 (2006) 014516, [hep-lat/0510074].

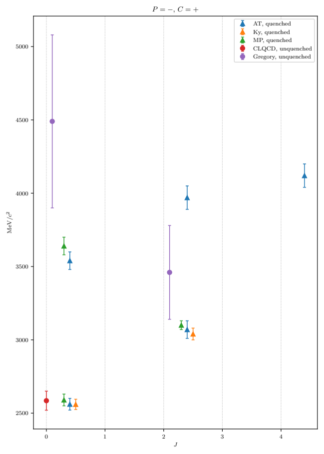
Glueball and lowest-lying hadrons



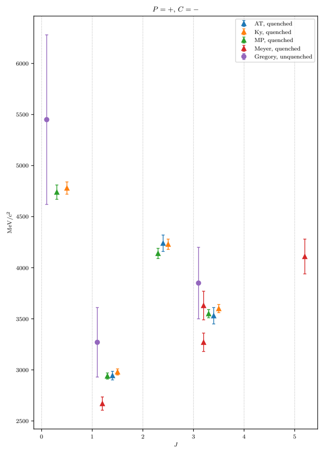
[Gregory *et al.*, JHEP 1210 (2012) 170]

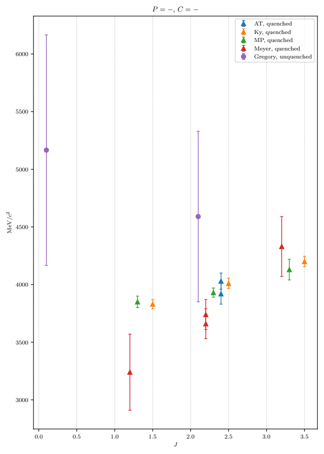
Comparison of glueball calculations - C=+





Comparison of glueball calculations - C=−





Outline

Conclusions and outlook

- Lattice calculations are an (increasingly more) useful tool to understand the fate of glueballs in QCD
- Valuable information can be provided by lattice calculations in the large N limit
- Results of various calculations in broad agreement
- No evidence for noticeable mass shifts between quenched and dynamical calculations
- Need to control better mixing with scattering and meson states
- Need to fully evaluate mixing with fermionic states