The Scalar Glueball in Lattice QCD

Mixing with Quarks and Mesons

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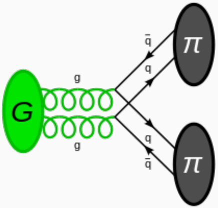
2nd May 2021

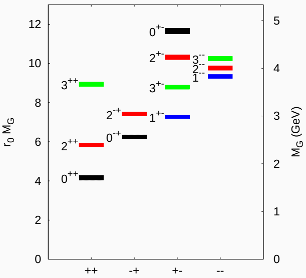
Mini-workshop on glueball hunting

Hadron Spectroscopy and Glueballs

Hadron spectroscopy

- many puzzles remain in hadron spectroscopy
    - e.g. tetra/pentaquarks, glueballs, hybrids, ...
- theoretical descriptions from QCD even more elusive
- glueballs on the lattice: early successes in pure Yang-Mills
    - next step to full QCD “challenging”
- can Lattice QCD complement experiment?
    - eg. $\pi\pi$ hard to (directly) probe experimentally

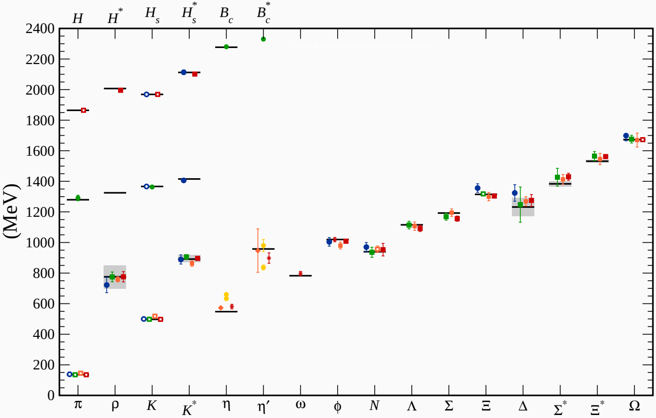




Historically: (QCD) stationary states in finite-volume

- stable hadron masses → precision: isospin breaking relevant

- no decays → no resonance information



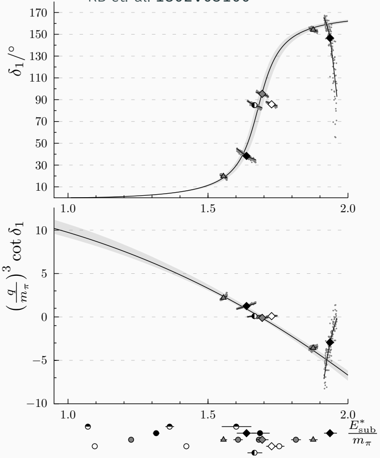
Kronfeld 1203.1204

Spectroscopy on the lattice

Historically: (QCD) stationary states in finite-volume

Last 10 years/today: $2 \to 2$ scattering from finite-vol. energies

RB et. al. `1802.03100`



- meson-meson:
  $\pi\pi$, $K\pi$, $\pi\eta$, $D\pi$, $\dots$
- meson-baryon accessible:
  review: Bulava `1909.13097`
- on the horizon:
  baryon-baryon at light m_π
  e.g. Green et. al. `2103.01054`

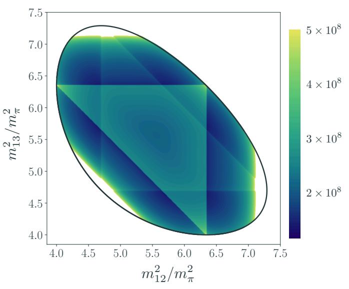
Historically: (QCD) stationary states in finite-volume

Last 10 years/today: $2 \to 2$ scattering from finite-vol. energies

Today: $3 \to 3$ the frontier

- formalisms for 3-body scattering rapidly developing
- first applications so far to $3\pi^+$, $3K^-$
  e.g. from GWU:
  Culver et. al. `1911.09047`
  Alexandru et. al. `2009.12358`
  RB et. al. `2101.06144`

Hansen et. al. (HadSpec) `2009.04931`



Historically: (QCD) stationary states in finite-volume

Last 10 years/today: $2 \to 2$ scattering from finite-vol. energies

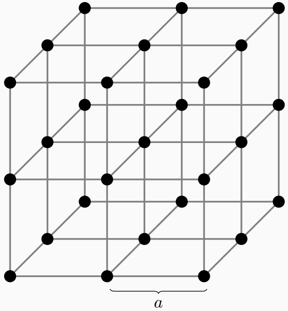
Today: $3 \to 3$ the frontier

Near future: what about scattering producing glueballs?

Extracting the finite-volume spectrum

Lattice QCD

- Euclidean space-time
- finite volume L
- lattice spacing a
- (often) unphysical light quark masses



$\rightarrow$ amenable to numerical calculations:

$$\langle A \rangle = \frac{1}{Z} \int \mathcal{D}[U, \overline{\psi}, \psi] \, A[U, \overline{\psi}, \psi] \, e^{-\overline{\psi} M \psi - S_G[U]}$$

- Monte Carlo for gluon integration

$$= \frac{1}{Z} \int \mathcal{D}[U] \, A[U, M^{-1}] \, \det M[U] \, e^{-S_G[U]}$$

- Dirac inverse *expensive*: $\dim M \sim 10^8 - 10^9$

Lattice QCD

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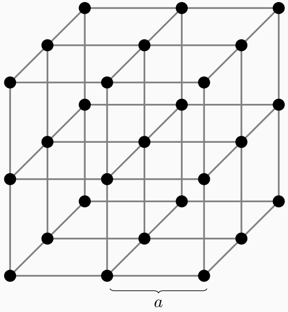
→ correlation functions:



$$C_{ij}(t) = \langle \mathcal{O}_i(t)\bar{\mathcal{O}}_j(0)\rangle = \sum_n \langle 0|\mathcal{O}_i|n\rangle\langle n|\bar{\mathcal{O}}_j|0\rangle\, e^{-E_n t}$$

- (n-)hadron interpolation operators
- discrete energies from 2-pt. functions

Extracting the finite-vol. spectrum

- temporal correlation *matrix*:

$$C_{ij}(t) \equiv \langle 0|\mathcal{O}_i(t+t_0)\overline{\mathcal{O}}_j(t_0)|0\rangle$$
$$= \sum_n \langle 0|\mathcal{O}_i|n\rangle\langle n|\overline{\mathcal{O}}_j|0\rangle\, e^{-E_n t}$$

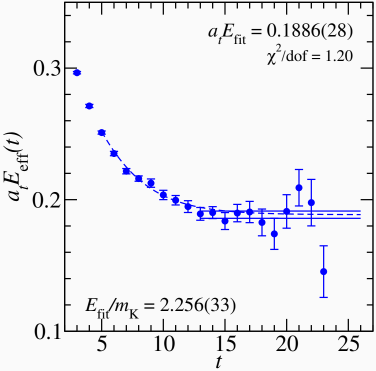
- solve generalized eigenvalue problem

$$C(\tau_0)^{-1/2}C(t)C(\tau_0)^{1/2}v_n(t,\tau_0)$$
$$= \lambda_n(t,\tau_0)v_n(t,\tau_0)$$

- eigenvalues tend to lowest N energies

$$\lim_{t\to\infty}\lambda_n(t) = b_n e^{-E_n t}[1+\mathcal{O}(e^{\Delta_n t})]$$

$$\Rightarrow E^n_{\text{eff}}(t) = \frac{1}{\Delta t}\ln\left(\frac{\lambda_n(t)}{\lambda_n(t+\Delta t)}\right)$$



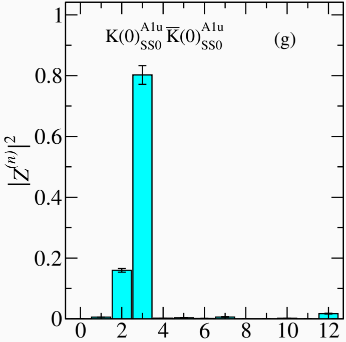
Extracting the finite-volume spectrum

- correlation matrix:

$$C_{ij}(t) = \sum_n \langle 0|\mathcal{O}_i|n\rangle\langle n|\overline{\mathcal{O}}_j|0\rangle\, e^{-E_n t}$$
$$= \sum_n Z_i^{(n)} Z_j^{(n)*} e^{-E_n t}$$

- level ID inferred from Z overlaps with *probe* operators:

$$|\Phi_j\rangle \equiv \mathcal{O}_j|0\rangle \quad \Rightarrow \quad Z_j^{(n)} = \langle \Phi_j|n\rangle$$



→ overlaps give **qualitative** measure of mixing between states

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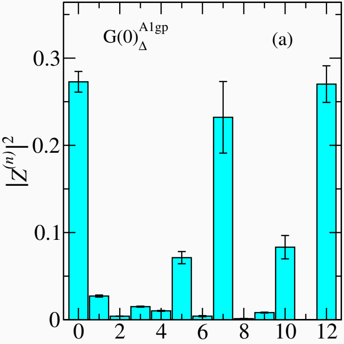


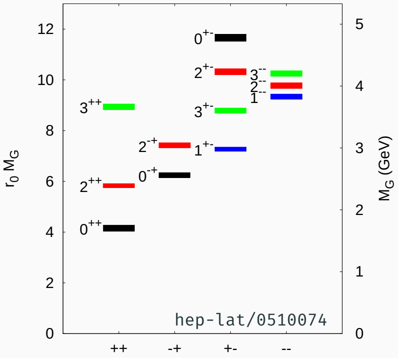
→ overlaps give **qualitative** measure of mixing between states

→ large operator bases crucial for reliably probing the spectrum

From Pure Glue to QCD

The scalar glueball

- Glueball: bound state of gluons
- experimental evidence elusive, light scalar candidates:
    - $f_0(1370)$, $f_0(1500)$, $f_0(1710)$
- lattice studies to date:
    - only glueball operators
    - light scalar ≈ 1.7 GeV
    - most in pure $SU_c(3)$/quenched QCD



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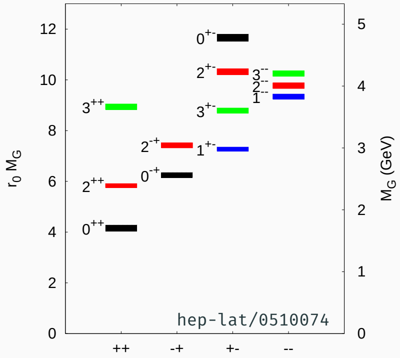


here: low-lying A_{1g}^{+} ($J^{PG} = 0^{++}$) spectrum up to ~ 2 GeV

- **first study** including glueball, $\overline{q}q$, and two-hadron operators
- stepping stone to scattering studies

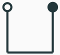
RB, PhD. Thesis, 1909.07306

(Multi) Meson operators

single-site

singly-displaced

doubly-displaced-L



triply-displaced-U

triply-displaced-O

q's = smeared, displaced quark fields

$$\overline{\Phi}^{AB}_{\alpha\beta;ij}(\boldsymbol{p},t) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot(\boldsymbol{x}+\frac{1}{2}(\boldsymbol{d}_\alpha+\boldsymbol{d}_\beta))}\delta_{ab}\,\overline{q}^{B}_{b\beta j}(\boldsymbol{x},t)\,q^{A}_{a\alpha i}(\boldsymbol{x},t)$$

group-theoretic projections for definite “spin”, parity, etc.

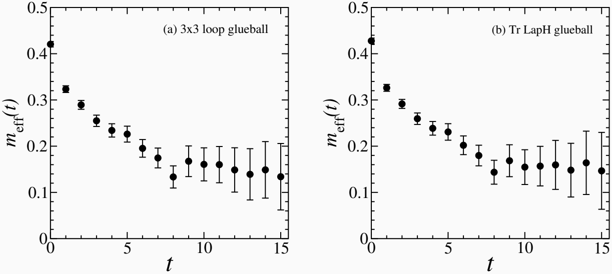
$$\overline{M}_l(\boldsymbol{p},t) = c^{(l)*}_{\alpha\beta}\overline{\Phi}^{AB}_{\alpha\beta}(\boldsymbol{p},t)$$

multi-hadron operators from single-hadron building blocks
i.e.

$$\mathcal{O}_{\pi(\boldsymbol{p}_1)\pi(\boldsymbol{p}_2)} \sim \sum_{i,j} c_{ij}\pi(\boldsymbol{p}_i)\pi(\boldsymbol{p}_j)$$

Morningstar et. al. `1303.6816`

A different glueball operator



any gauge invariant, purely gluonic object with the correct transformation properties is valid as a glueball operator[1]

scalar $\mathrm{Tr\,LapH}$ operator:

$$G_\Delta = -\,\mathrm{Tr}[\Theta(\sigma_s^2 + \tilde{\Delta})\tilde{\Delta}], \qquad \tilde{\Delta} \equiv \text{covariant laplacian}$$

(function of gauge links/gluons)

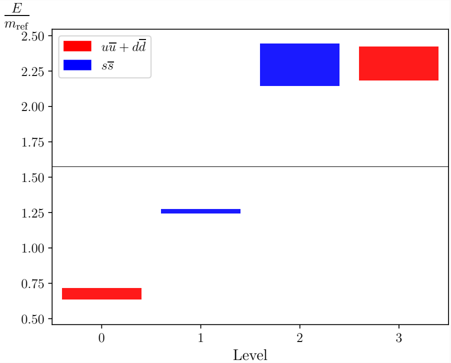
Morningstar et. al. `1303.6816`

[1]though not necessarily useful!

A_{1g}^{+} SPECTRUM - $\overline{q}q$ ONLY

$N_f = 2 + 1$ Wilson-clover, anisotropic lattice, $m_\pi \approx 390$ MeV, $m_K \approx 550$ MeV:

- first: $\overline{q}q$ operators only
- only 2 states below ~ 2 GeV
- σ and $f_0(980)$
- no sign of $\overline{q}q$-dominated states $\sim 1.5 - 1.8$ GeV*



$$m_{\text{ref}} = 1.82 m_K \approx 1 \text{ GeV}$$

RB, PhD. Thesis, 1909.07306

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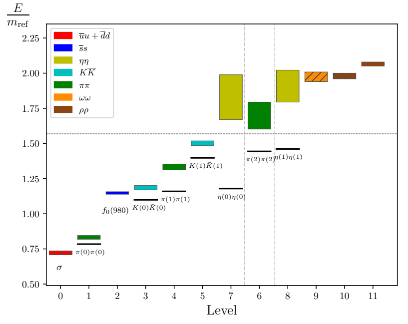

w/o glueball operator

Hatched boxes: significant overlap $\left(|\langle n|\overline{\mathcal{O}}_\alpha|0\rangle|^2\right)$ with multiple operators

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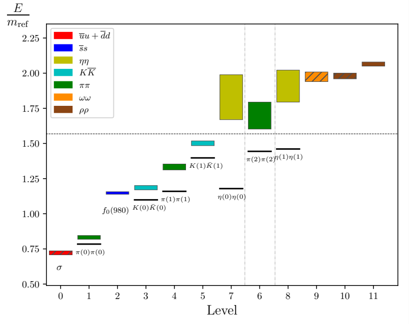

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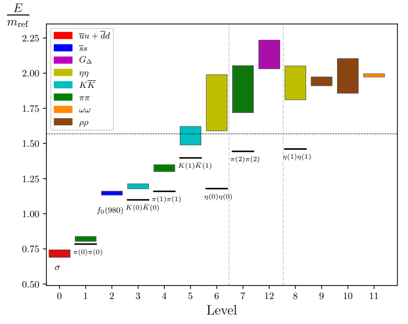

with glueball operator

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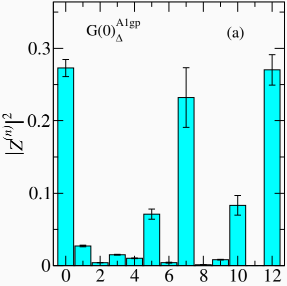

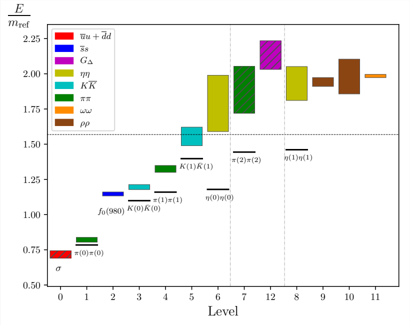

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RB, PhD. Thesis, `1909.07306`

Looking Forward

Interpreting these results

we find only two "$\bar{q}q$ dominated", and no new "glueball dominated" states below 2 GeV

- does this tell us there isn't a "pure glueball" state below 2 GeV?
  (at $m_\pi \approx 390$ MeV, etc.)
- *suggestion* that the f_0's below 2 GeV might not be $\bar{q}q$ either?
  $\rightarrow$ molecular states?
- more importantly, "is it even in principle possible to know?"
- either way, getting a handle on the glueball-meson mixing will be crucial
- what matrix elements will be most insightful (and calculable)?
- 4π looms overhead, but experience with 3π makes it less intimidating

Conclusions

- **first study** in full QCD to include the mixing between glueball, $\bar{q}q$, and meson-meson operators
  - relative dearth of new lattice studies (compared to other hadron scattering) indicates large hurdles that glueballs present

- including/excluding a scalar glueball operator introduces no new states
  $\rightarrow$ but plenty of noise!

- *conservatively*: a precursor to full blown scattering studies
  $\rightarrow$ not so far away

- *more speculative*: no scalar "pure glueball" state below 2 GeV when quark dynamics fully factored in

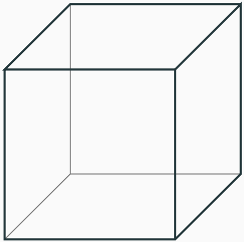
Angular momentum on the lattice

- periodic B.C. in cubic box:

  $\Rightarrow$ J no longer a good quantum number

- label states with irreps of cubic group O_h^D

- continuum spin ID:



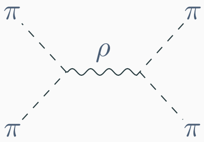
J	A_1	A_2	E	T_1	T_2	J	G_1	G_2	H
0	1	0	0	0	0	$\frac{1}{2}$	1	0	0
1	0	0	0	1	0	$\frac{3}{2}$	0	0	1
2	0	0	1	0	1	$\frac{5}{2}$	0	1	1
3	0	1	0	1	1	$\frac{7}{2}$	1	1	1
4	1	0	1	1	1	$\frac{9}{2}$	1	0	2

boosted frames introduce further mixing
(but allow more sampling of scattering region)

Finite volume spectra

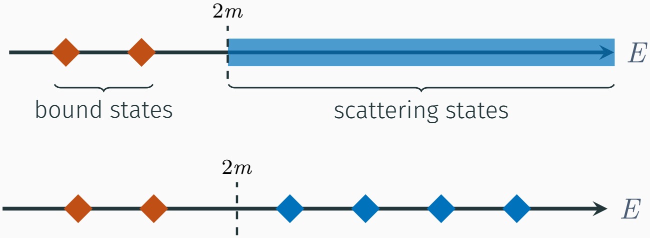
Scattering process: eg.

$I = 1 \quad \pi\pi \to \pi\pi$



∞-volume

Finite volume

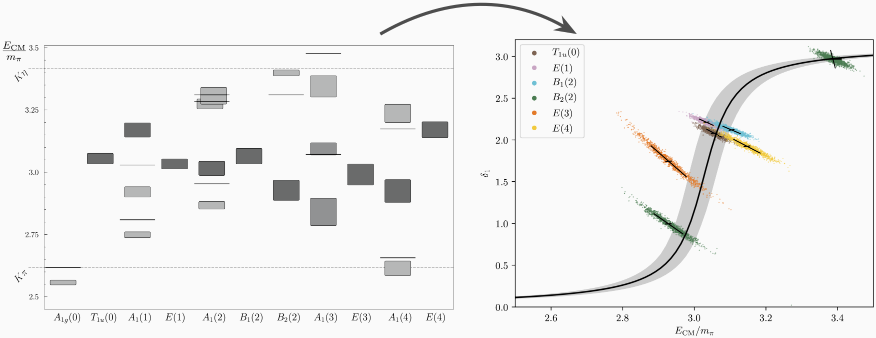


$\to$ how to access ∞-vol. physics?

Two-body quantization condition

- quantisation condition (Lüscher formula):

$$\det[\, \widetilde{K}^{-1}(E_{\mathrm{cm}}) - B(E_{\mathrm{cm}}, L)\,] = 0$$



RB, PhD. Thesis

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$$\det[\,\widetilde{K}^{-1}(E_{\mathrm{cm}}) - B(E_{\mathrm{cm}}, L)\,] = 0$$

$\widetilde{K}_\ell^{-1} \sim \cot\delta_l$ → $\widetilde{K}^{-1}$

B ← known, mixes partial waves

- determinant over decay channel, partial waves (truncation!)

- implementation with group theory for cubic box (& boosted frames):
  `github.com/cjmorningstar10/TwoHadronsInBox`

Morningstar et. al. `1707.05817`