Glueball spectra from gauge-gravity duality

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Light scalars from a compact fifth dimension
(DE, Maurizio Piai, arXiv:1010.1964 [hep-th])

Holographic glueballs from the circle reduction of Romans supergravity
(DE, Maurizio Piai, John Roughley, arXiv:1811.01010 [hep-th])
Dilatonic states near holographic phase transitions
(DE, Maurizio Piai, John Roughley, arXiv:2010.04100 [hep-th])

Glueballs on the Baryonic Branch of Klebanov-Strassler: dimensional deconstruction and a light scalar particle
(DE, Maurizio Piai, arXiv:1703.10158 [hep-th])
Calculable mass hierarchies and a light dilaton from gravity duals
(DE, Maurizio Piai, arXiv:1703.09205 [hep-th])

Gauge-gravity duality:

- Strongly coupled dynamics from holography
- Correlation functions and a toy example
- Gauge-invariant formalism for calculating spectra

Applications:

- Confining theories dual to Romans supergravity
- Baryonic branch of Klebanov-Strassler and a light dilaton

Gauge-gravity duality:

- Duality between certain QFTs and gravitational theories
- Most explicit examples from string theory/supergravities
- Can be used to obtain strong coupling results in QFT by performing calculations at weak coupling in gravity
- *Holographic*: QFT in d dimensions is dual to gravity in $d+1$ dimensions (+ extra compact ones)

Bottom-up models:

- Phenomenologically motivated 5d models, where the action and field content is chosen by hand to get the desired dynamics
- Flexible for model building, qualitatively captures strongly coupled dynamics
- Given a gravitational model in 5d, there does not necessarily exist a corresponding dual 4d field theory

Top-down models:

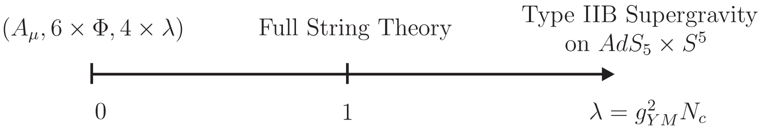
- Gravity side of the duality is described by a higher-dimensional theory: 10d type-IIA/IIB supergravity, 11d supergravity/M-theory
- More precise: in many cases, the exact form of the field theory dual is known

Applications include both holographic models of QCD, as well as strongly coupled physics beyond the Standard Model (composite Higgs models, walking technicolor)

Gauge-gravity duality: strongly coupled dynamics from holography

Canonical example:

- Duality between $\mathcal{N}=4$ super Yang-Mills with gauge group $SU(N_c)$ and Type IIB String Theory on $\mathrm{AdS}_5 \times S^5$:
  (Maldacena, 1997; Gubser, Klebanov, Polyakov, 1998; Witten, 1998)



- Global symmetries match:
  Isometry group of $S^5 \longleftrightarrow SO(6)_R$ symmetry,
  Isometry group of AdS$_5 \longleftrightarrow$ conformal group $SO(2,4)$

Two limits:

- Large N_c corresponds to the classical limit on the string theory side ($\lambda/N_c = 4\pi g_s$)
- Large 't Hooft coupling λ corresponds to the low energy limit of string theory ($\lambda = R^4/l_s^4$)
- *Strongly coupled field theory* can be described by *classical supergravity*

What is the meaning of the holographic radial direction?

- The radial coordinate r is related to energy scale in the field theory, and thus the bulk is in a sense a geometrical representation of the RG flow of the dual theory
- Example: AdS geometry with metric $\mathrm{d}s^2 = \mathrm{d}r^2 + e^{2r}\mathrm{d}x_{1,3}^2$ is invariant under scale transformations $x^\mu \to \lambda x^\mu$, $r \to r - \log\lambda$

Many of the dynamics we know from strongly coupled field theories can be captured using gauge-gravity duality:

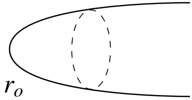
- $\mathcal{N} = 4$ super Yang-Mills (CFT)
- Witten model, Maldacena-Nunez, Klebanov-Strassler (confinement)
- Sakai-Sugimoto model (chiral symmetry breaking)
- Baryonic branch of Klebanov-Strassler (moduli space)

Limitations:

- Only *specific* QFTs are known to have dual gravity descriptions
- Difficult to obtain results at finite number of colours N_c and/or finite 't Hooft coupling λ

What is the dual description of confinement?

- The geometry pinches off at some finite value of the radial direction $r = r_o$



- This leads to a dynamically generated IR scale Λ_{IR} (like Λ_{QCD}) and linear confinement (quark-antiquark potential $V_{q\bar{q}} \sim L$ for large separation L)
- Advantage of top-down models (requires a cycle in the internal part of the geometry to shrink)

Correlation functions:

- There is a *dictionary* for translating between field theory and bulk quantities. Fields in the bulk map to composite operators in the field theory: $\phi \longleftrightarrow \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$, $g_{MN} \longleftrightarrow T^{\mu\nu}$, $A_M \longleftrightarrow J^\mu$, $\ldots$
- Correlators of an operator $\mathcal{O}$ dual to a bulk field σ can be computed by using

$$\langle e^{\int \mathrm{d}^4x\, \sigma_0(x^\mu)\mathcal{O}(x^\mu)}\rangle_{\mathrm{QFT}} = Z_{bulk}[\sigma(x^\mu, r)|_{r=\infty} = \sigma_0(x^\mu)]$$

$$W_{\mathrm{QFT}}[\sigma_0] = S^{(bulk)}_{on-shell}[\sigma_0] \qquad (N_c \gg 1)$$

  differentiating with respect to the boundary value of the bulk field
- Mass spectra of composite states (glueballs, mesons, etc), can be extracted from the poles of two-point functions

Toy example: scalar field in a *fixed* AdS background geometry:

- Action for the scalar σ:

$$S = \int_{r_{\mathrm{IR}}}^{r_{\mathrm{UV}}} \mathrm{d}r \int \mathrm{d}^4x \sqrt{-g} \left(-\frac{1}{2} g^{MN} \partial_M \sigma \partial_N \sigma - \frac{m^2}{2} \sigma^2 \right)$$

- Metric for AdS given by $\mathrm{d}s^2 = \mathrm{d}r^2 + e^{2r} \mathrm{d}x_{1,3}^2$
- Hard-wall in the IR: cut off the geometry at finite $r_{\mathrm{IR}} = 0$ (crude model of confinement, leads to a mass gap)
- The scaling dimension of the operator $\mathcal{O}$ dual to σ is given by $[\mathcal{O}] = 2 + \sqrt{4 + m^2}$

Equation of motion for the scalar $\sigma(q, r)$:

$$\partial_r^2 \sigma + 4 \partial_r \sigma - m^2 \sigma - q^2 e^{-2r} \sigma = 0$$

Impose IR boundary condition: $\sigma(q, r_{\mathrm{IR}}) = 0$

In order to compute the two-point function $\langle\mathcal{O}(q)\mathcal{O}(-q)\rangle$, we need the on-shell action:

$$S_{on-shell} = \underbrace{\frac{1}{2}\int \mathrm{d}^4x\sqrt{-\tilde{g}}\,\sigma\partial_r\sigma\Big|_{r_{\mathrm{IR}}}}_{=0\text{ (IR boundary condition)}} - \underbrace{\frac{1}{2}\int \mathrm{d}r\int \mathrm{d}^4x\sqrt{-g}\,(e.o.m.)\,\sigma}_{=0\text{ (equation of motion)}}$$
$$- \frac{1}{2}\int \mathrm{d}^4x\sqrt{-\tilde{g}}\,\sigma\partial_r\sigma\Big|_{r_{\mathrm{UV}}}$$
$$= -\frac{1}{2}\int \mathrm{d}^4q\sqrt{-\tilde{g}}\,\sigma(-q,r)\underbrace{\left(\frac{\partial_r\sigma(q,r)}{\sigma(q,r)}\right)}_{\sim\langle\mathcal{O}(q)\mathcal{O}(-q)\rangle}\sigma(q,r)\Big|_{r_{\mathrm{UV}}}$$

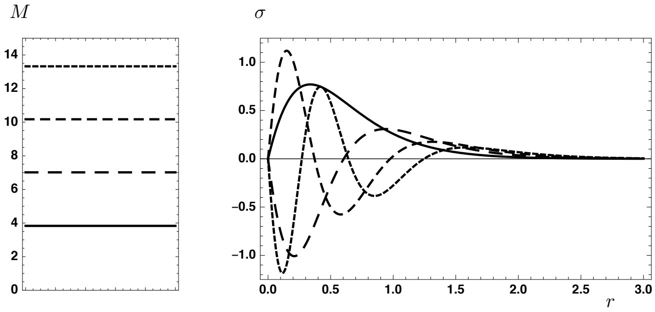
The poles can be extracted by imposing the UV BC: $\sigma(q,r_{\mathrm{UV}})=0$
Spectrum is obtained after removing the UV regulator: $r_{\mathrm{UV}}\to\infty$

(Caveat: the full calculation using holographic renormalization requires introducing counter-terms to cancel divergences. This does not affect the position of the poles.)

Summary of toy model:

1. Solve the equation of motion for the bulk scalar field σ,
2. Impose the appropriate boundary conditions on σ at r_{IR} and r_{UV},
3. Find the four-momenta q_μ for which solutions exist
4. The mass spectrum is given by $M^2 = -q^2$ in the limit $r_{\mathrm{UV}} \to \infty$

Spectrum and wavefunctions for the case $[\mathcal{O}] = 3$:



In general, there is operator mixing:

- There may be many scalars $\Phi^a \longleftrightarrow \mathcal{O}^a$ $(a = 1, \cdots, n)$
- Mixing between $\mathcal{O}^a$ and the stress-energy tensor $T^{\mu\nu}$
  $\Longrightarrow$ bulk scalars Φ^a couple to gravity g_{MN}

Coupled system:

- Background geometry is determined by the backreaction of the scalars on the metric
- Scalar and metric fluctuations around a given background mix with each other
- Treating bulk fields as perturbations on top of a frozen background can only be justified in certain situations (e.g. mesons when $N_f \ll N_c$, quenched approximation)

Gauge-invariant formalism for the spin-0 and spin-2 sectors:

Often, there exists a consistent truncation from 10d/11d to a sigma-model consisting of a number of scalars Φ^a coupled to gravity in 5d:

$$\mathcal{S} = \int \mathrm{d}^4x \mathrm{d}r \sqrt{-g} \left[\frac{R}{4} - \frac{1}{2} G_{ab}(\Phi^c) \partial_M \Phi^a \partial^M \Phi^b - V(\Phi^a) \right]$$

- In top-down models, the scalar potential V and the sigma-model metric G_{ab} are determined by the particular higher-dimensional system under study
- The formalism we will describe also works for bottom-up models when V and G_{ab} are chosen by hand

Background solutions:

- 4d Poincaré invariance implies $\mathrm{d}s^2 = \mathrm{d}r^2 + e^{2A(r)}\mathrm{d}x_{1,3}^2$ with warp factor $A(r)$
- Solve coupled equations of motion for $\Phi^a(r)$ and $A(r)$

Algorithm for computing spectra:

- Study fluctuations of the metric and scalar fields around the background solutions
- For two-point functions, it is sufficient to consider the action to *quadratic* order in the fluctuations $\Longrightarrow$ *linearized* equations of motion for the fluctuations
- Impose appropriate BCs on fluctuations in the IR and UV
- The values of four-momenta $q^2 = -M^2$ for which solutions exist give us the spectrum

Fluctuating around a background solution:

- ADM-formalism: write the metric as (lapse function n and shift vector n^μ)

$$\mathrm{d}s^2 = (n^\mu n_\mu + n^2)\mathrm{d}r^2 + 2n_\mu \mathrm{d}x^\mu dr + g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$$

- Expand to linear order in fluctuations $\{\varphi^a, \nu, \nu^\mu, h^{TT\,\mu}{}_\nu, h, H, \epsilon^\mu\}$ around the background:

$$\begin{aligned}
\Phi^a &= \bar{\Phi}^a + \varphi^a, \\
n &= 1 + \nu, \\
n^\mu &= \nu^\mu, \\
g_{\mu\nu} &= e^{2A}(\eta_{\mu\nu} + h_{\mu\nu}),
\end{aligned}$$

with

$$h^\mu{}_\nu = h^{TT\,\mu}{}_\nu + \partial^\mu \epsilon_\nu + \partial_\nu \epsilon^\mu + \frac{\partial^\mu \partial_\nu}{\Box} H + \frac{1}{3}\delta^\mu{}_\nu h$$

In order to decouple the equations, we form the gauge invariant variables (invariant under diffeomorphisms):

(Berg, Haack, Mück, 2005)

$$
\begin{aligned}
\mathfrak{a}^a &= \varphi^a - \frac{\bar{\Phi}'^{\,a}}{6A'} h, \\
\mathfrak{b} &= \nu - \frac{\partial_r (h/A')}{6}, \\
\mathfrak{c} &= e^{-2A} \partial_\mu \nu^\mu - \frac{e^{-2A} \Box h}{6A'} - \frac{1}{2} \partial_r H, \\
\mathfrak{d}^\mu &= e^{-2A} \Pi^\mu{}_\nu \nu^\nu - \partial_r \epsilon^\mu, \\
\mathfrak{e}^\mu{}_\nu &= h^{TT\,\mu}{}_\nu
\end{aligned}
$$

Linearized equations of motion (spin-0 sector):

- from the equations of motion for the scalars:

$$\left[D_r^2 + 4A' D_r + e^{-2A}\Box\right]\mathfrak{a}^a - \left(V^a{}_{|c} - \mathcal{R}^a{}_{bcd}\bar{\Phi}'^b\bar{\Phi}'^d\right)\mathfrak{a}^c - \bar{\Phi}'^a(\mathfrak{c} + \partial_r\mathfrak{b}) - 2V^a\mathfrak{b} = 0$$

$$\left(D_r\varphi^a \equiv \partial_r + \mathcal{G}^a{}_{bc}\bar{\Phi}'^b\varphi^c,\ V_a \equiv \frac{\partial V}{\partial\Phi^a},\ V^a{}_{|b} \equiv \partial_b V^a + \mathcal{G}^a{}_{bc}V^c\right)$$

- from Einstein's equations:

$$6A'\mathfrak{c} + 4\bar{\Phi}'_a(D_r\mathfrak{a}^a) - 4V_a\mathfrak{a}^a - 8V\mathfrak{b} = 0,$$
$$-\frac{1}{2}\Box\mathfrak{d}_\mu + 3A'\partial_\mu\mathfrak{b} - 2\bar{\Phi}'_a\partial_\mu\mathfrak{a}^a = 0$$

- We can solve *algebraically* for $\mathfrak{b}$ and $\mathfrak{c}$ in terms of $\mathfrak{a}^a = \varphi^a - \frac{\bar{\Phi}'^a}{6A'}h$

- Plugging back into the linearized equations of motion for the scalars leads to:

$$\left[D_r^2 + 4A'D_r + e^{-2A}M^2\right]\mathfrak{a}^a -$$
$$\left[V^a{}_{|c} - \mathcal{R}^a{}_{bcd}\bar{\Phi}'^b\bar{\Phi}'^d + \frac{4(\bar{\Phi}'^a V_c + V^a\bar{\Phi}'_c)}{3A'} + \frac{16V\bar{\Phi}'^a\bar{\Phi}'_c}{9A'^2}\right]\mathfrak{a}^c = 0$$

- Computing the mass spectrum is hence reduced to the problem of finding solutions to a second order linear differential equation for n scalar fluctuations $\mathfrak{a}^a$, for different values of M^2, while imposing correct boundary conditions in the IR and UV

What boundary conditions should we impose?

- We put $\varphi^a = 0$ in the IR and UV, which corresponds to

$$\frac{2\Phi'^a\Phi'_b}{3A'}D_r\mathfrak{a}^b\Big|_{r_{\mathrm{IR,UV}}} = \left[e^{-2A}M^2 - \frac{A'}{2}\partial_r\left(\frac{A''}{A'^2}\right)\right]\mathfrak{a}^a\Big|_{r_{\mathrm{IR,UV}}}$$

- This automatically picks the regular modes in the IR and normalizable modes in the UV (usual prescription)

The spin-2 sector is simpler:

$$\left[\partial_r^2 + 4A'\partial_r + e^{-2A}M^2\right]\mathfrak{e}^\mu{}_\nu = 0\,, \qquad \partial_r\mathfrak{e}^\mu{}_\nu|_{r_{\mathrm{IR,UV}}} = 0$$

In more general models, spectra for pseudo-scalar and vector states can be obtained along similar lines

Applications:

- Confining theories dual to Romans supergravity

  Holographic glueballs from the circle reduction of Romans supergravity
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  Dilatonic states near holographic phase transitions
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- Baryonic branch of Klebanov-Strassler and a light dilaton

  Glueballs on the Baryonic Branch of Klebanov-Strassler: dimensional deconstruction and a light scalar particle
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  Calculable mass hierarchies and a light dilaton from gravity duals
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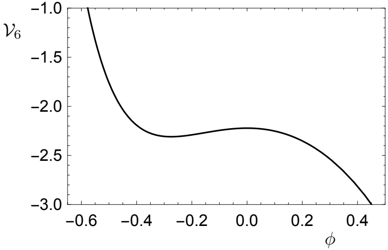
Romans supergravity in 6d can be truncated to a single scalar field ϕ coupled to gravity with action:

$$S_6 = \int d^6x\sqrt{-g}\left(\frac{R}{4} - (\partial_M\phi)^2 - V_6(\phi)\right)$$

$$V_6(\phi) = \frac{1}{9}\left(e^{-6\phi} - 9e^{2\phi} - 12e^{-2\phi}\right)$$



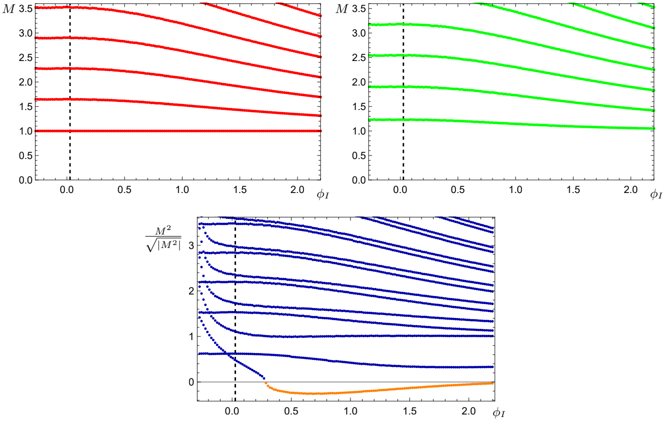
- Compactify on a circle:

$$S_5 = \int \mathrm{d}^5x\sqrt{-g}\left(\frac{R}{4} - (\partial_M\phi)^2 - 3(\partial_M\chi)^2 - \frac{1}{16}e^{8\chi}(F_{MN})^2 - e^{-2\chi}V_6(\phi)\right)$$

  Scalar χ parametrizes the size of the circle, F_{MN} is the field strength for the gravi-photon V_M
- Solutions exist for which the circle shrinks to zero size smoothly in the IR $\implies$ dual to 4d confining field theories
- One-parameter family of such *confining solutions*, corresponding to the size of a relevant deformation (dim-2 operator $\mathcal{O}$ dual to ϕ)

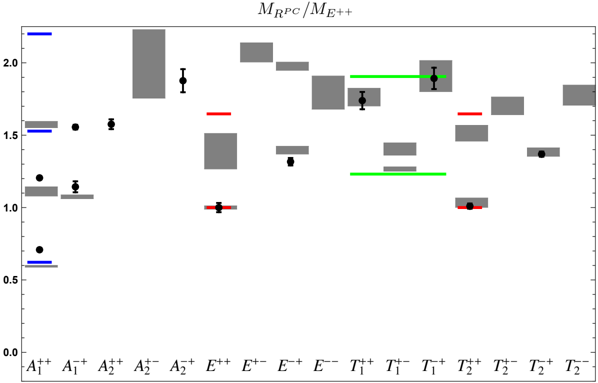
Applications: confining theories dual to Romans supergravity

Spin-0 (blue/orange), spin-1 (green), and spin-2 (red) glueball spectra as a function of the size of the relevant deformation corresponding to $\mathcal{O}$:



Tachyonic instability, hidden away by a first-order phase transition
Universality for some of the towers of states in the physical region

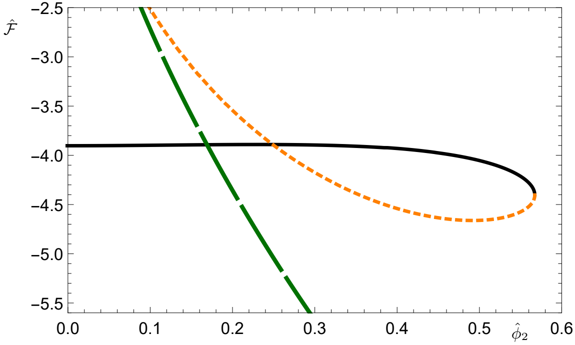
Comparison of *universal* results to lattice simulations:



Shaded gray rectangles:
B. Lucini, A. Rago and E. Rinaldi, "Glueball masses in the large N limit," JHEP 1008, 119 (2010)

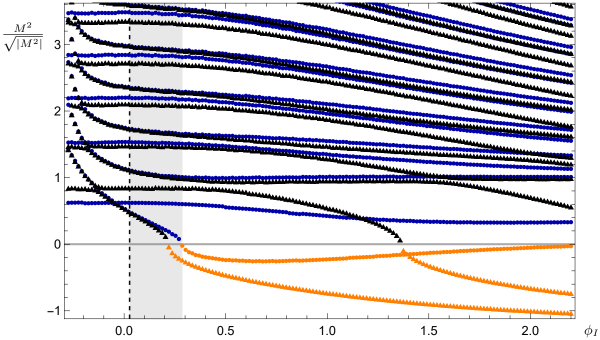
Black dots:
E. Bennett, D. K. Hong, J. W. Lee, C.-J. D. Lin, B. Lucini, M. Piai and D. Vadacchino, "Sp(4) gauge theory on the lattice: towards SU(4)/Sp(4) composite Higgs (and beyond)," JHEP 1803, 185 (2018)

Free energy as a function of the size of the relevant deformation shows a first order phase transition:



Stable, metastable, and unstable phases. Confining solutions in black and orange (tachyonic), while green indicates another branch of (singular) solutions

Spectrum of spin-0 glueballs (blue/orange dots), compared to results in the *probe approximation* (black/orange triangles):



Probe approximation *neglects* mixing with gravity, and hence *misses* states that overlap significantly with the dilatation operator (related to the trace of the bulk metric)

$\Longrightarrow$ light scalar is an *approximate* dilaton (Goldstone boson of spontaneously broken scale invariance, relevant for BSM physics)

Klebanov-Strassler field theory:

(Klebanov, Strassler, 2000)

- 4d theory with $SU(N+M) \times SU(N)$ gauge group, $\mathcal{N}=1$ SUSY, bifundamental matter A_i and B_i $(i=1,2)$ in representations $(N+M, \overline{N})$ and $(\overline{N+M}, N)$, superpotential $W \sim \mathrm{Tr}(A_i B_j A_k B_l)\epsilon^{ik}\epsilon^{jl}$
- Gravity dual is known

Rich dynamics:

- UV duality cascade (of Seiberg dualities): $SU(N+M) \times SU(N) \to SU(N) \times SU(N-M) \to \cdots$
- The theory is confining
- Non-trivial moduli space – there is a baryonic branch parametrized by the VEV of a dim-2 operator
- For large dim-2 VEV, the theory effectively becomes six-dimensional over a range of energies (deconstruction)

To compute the spectrum, we use the 5d sigma model defined by ($\Phi^a = \{\tilde{g}, p, x, \phi, a, b, h_1, h_2\}$):

$$
\begin{aligned}
G_{ab}\partial_M\Phi^a\partial_N\Phi^b \quad = \quad & \frac{1}{2}\partial_M\tilde{g}\partial_N\tilde{g} \,+\, \partial_M x\partial_N x \,+\, 6\partial_M p\partial_N p \\
& + \frac{1}{4}\partial_M\phi\partial_N\phi + \frac{1}{2}e^{-2\tilde{g}}\partial_M a\partial_N a + \frac{1}{2}N^2 e^{\phi-2x}\partial_M b\partial_N b \\
& + \frac{e^{-\phi-2x}}{e^{2\tilde{g}}+2a^2+e^{-2\tilde{g}}(1-a^2)^2}\Bigg[\frac{1}{2}(e^{2\tilde{g}}+2a^2+e^{-2\tilde{g}}(1+a^2)^2)\partial_M h_2\partial_N h_2 \\
& + (1+2e^{-2\tilde{g}}a^2)\partial_M h_1\partial_N h_1 + 2a(e^{-2\tilde{g}}(a^2+1)+1)\partial_M h_1\partial_N h_2\Bigg] ,
\end{aligned}
$$

$$
\begin{aligned}
V(\Phi^a) \quad = \quad & -\frac{1}{2}e^{2p-2x}(e^{\tilde{g}}+(1+a^2)e^{-g}) \,+\, \frac{1}{8}e^{-4p-4x}(e^{2\tilde{g}}+(a^2-1)^2e^{-2\tilde{g}}+2a^2) \\
& + \frac{1}{4}a^2e^{-2\tilde{g}+8p} \,+\, \frac{1}{8}N^2e^{\phi-2x+8p}\left[e^{2\tilde{g}}+e^{-2\tilde{g}}(a^2-2ab+1)^2+2(a-b)^2\right] \\
& + \frac{1}{4}e^{-\phi-2x+8p}h_2^2 \,+\, \frac{1}{8}e^{8p-4x}(M+2N(h_1+bh_2))^2
\end{aligned}
$$

(Caveat: a rigorous treatment needs to consider a more general truncation that includes also vectors (Cassani, Faedo, 2010) – this is important at the level of the fluctuations. We expect the physics to be qualitatively the same.)

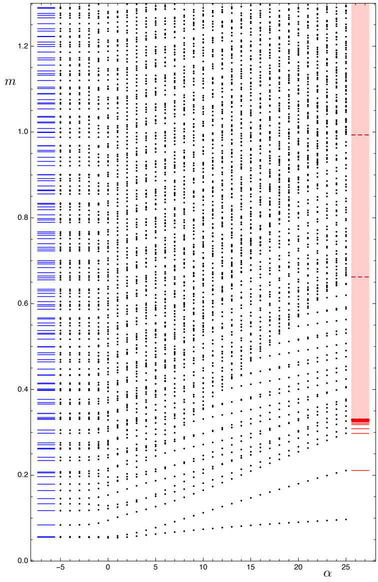
Applications: baryonic branch of Klebanov-Strassler and a light dilaton

Spectrum of spin-0 glueballs on the baryonic branch of Klebanov-Strassler (Elander, Piai, 2017)

Far from the origin of the moduli space, there is a light dilaton

Scale invariance is broken: spontaneous breaking $\gg$ explicit breaking

Densely packed states approaching continua (evidence of deconstruction)



Summary:

- Gauge-gravity duality offers an approach to strongly coupled theories that complements others such as lattice gauge theory
- Applications include holographic models of QCD and strongly coupled physics beyond the Standard Model
- There exists a powerful and general formalism for the computation of glueball spectra
- We provided two illustrations of this formalism:
  1. Romans supergravity dual to confining 4d field theories
  2. Baryonic branch of Klebanov-Strassler dual to confining 4d theories with a moduli space
- In both examples, we found evidence of a light (approximate) dilaton in regions of the parameter space, related to either an instability in the theory or the presence of a moduli space