

# Beam Dynamics Meeting:

“Modelling and evidence of intra-beam scattering  
in high brightness electron linacs”



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# Beam Dynamics Meeting:

“Modelling and evidence of intra-beam scattering  
in high brightness electron linacs”

## Prologue:

- ❑ Theory and experimental data on collective phenomena at FERMI (Elettra Sincrotrone Trieste)

## Acknowledgement:

*Contributions and support from: A. Brynes , P. H. Williams, A. Wolski, I. Setija, E. Roussel, N. Mirian and the FERMI team.*

*Special thanks to: S. Di Mitri (Elettra Sincrotrone Trieste, Univ. Trieste)*



## Introduction

1. Intra-Beam Scattering
2. Microbunching Instability

## Modelling IBS

1. Analytical model

## Model in action

1. Effect of IBS on MBI
2. Experimental evidence of IBS in SES
3. Indications of IBS effect

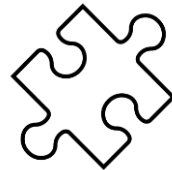
## Conclusions

## What is next...

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# Intra-beam Scattering

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# Intra-beam Scattering

Multiple small-angle  
Coulomb scattering of  
charged particles

Proportional to beam  
density and inversely  
proportional to  $\gamma^4$

Main consequences in SR

- Larger equilibrium emittances
- Limiting the luminosity
- Tendency to relax the momenta distribution to a spherical shape (in beam reference frame)

Emittances and energy  
spread growth

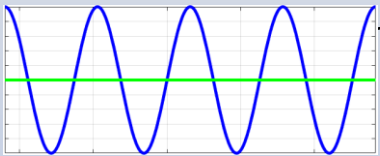
~~SO~~  
~~Why IBS in linac ?~~

**Why now?** Charge density of high brightness linacs driving short wavelength free-electron lasers (FELs) is large enough for IBS to become a significant effect.

# Microbunching instability

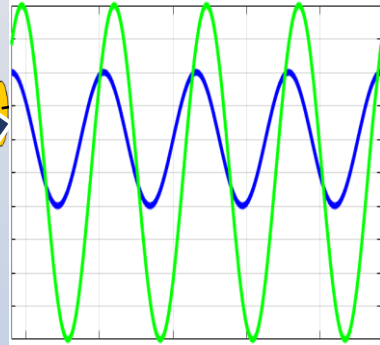
From shot noise disomogeneity, modulation wavelength in the range from 1 to hundreds of  $\mu\text{m}$

density mod.



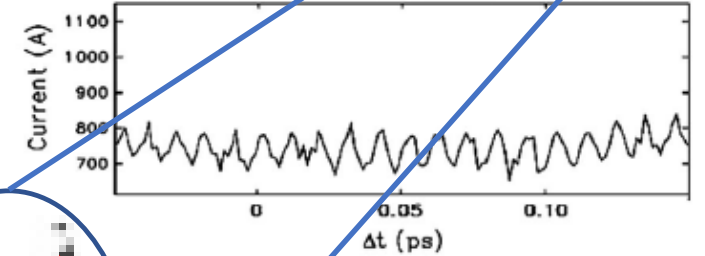
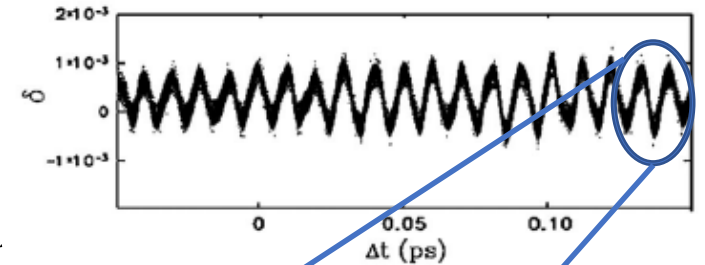
**LSC**  
acceleration

energy mod.



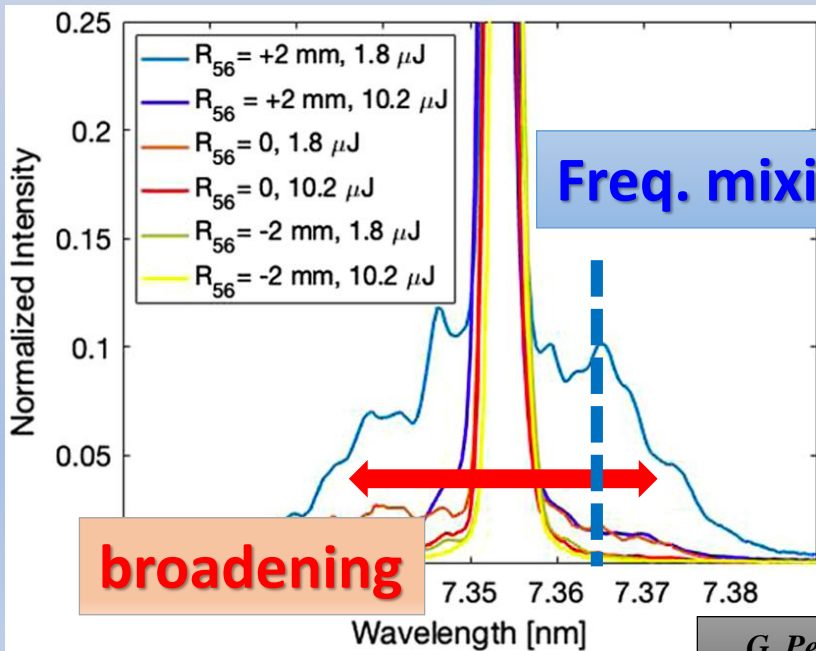
**$R_{56}$ , CSR**

dispersive region



Simulated 30  $\mu\text{m}$  wavelength, 1% amplitude at 100 MeV

*S. Di Mitri S. Spampinati, PRL 112 (2014)*



*G. Perosa et al., PRAB 23 (2020)*

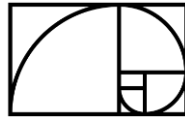
Spurious harmonic content

Increased Energy Spread



# Modelling IBS

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# Analytical model

## Relative energy spread growth rate

$$\frac{1}{\sigma_\delta} \frac{d\sigma_\delta}{ds} = \frac{r_e^2 N_e [\log]}{8\gamma^2 \epsilon_n \sigma_x \sigma_z \sigma_\delta^2}$$

$$\left. \frac{1}{\sigma_\delta} \frac{d\sigma_\delta}{ds} \right|_{disp} = \frac{r_e^2 N_e [\log]}{8\gamma^2 \epsilon_n \sigma_x \sigma_z \sigma_\delta^3} \sigma_H$$

$$\frac{1}{\sigma_H^2} = \frac{1}{\sigma_\delta^2} + \frac{H_x}{\epsilon_x} + \frac{H_y}{\epsilon_y}$$

**Assumptions**

- Ultra-relativistic beam
- Round beam ( $\beta_x = \beta_y \dots$ )
- Use of mean values for optics

**Applications**

- ✓ Straight section with constant energy
- ✓ Straight accelerating section
- ✓ Dispersive region

## Microbunching instability gain

$$G(k) \cong \frac{4\pi I_0}{Z_0 I_A} Ck |R_{56}| \left| \int ds \frac{Z_{LSC}(k; s)}{\gamma(s)} \right| \exp \left[ -\frac{1}{2} (Ck R_{56} \sigma_\delta)^2 \right]$$

**Landau damping**

**How do they couple?**



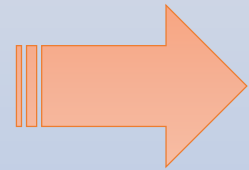
# Analytical model

Integrating the growth rate

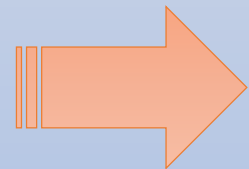
$$\int_0^L \frac{d\sigma_\delta}{ds} ds \longrightarrow \sigma_\delta(L)^2 - \sigma_\delta(0)^2 = f(L) \approx AL$$

Sometimes approximated to first order

Directly inserted in the gain expression



Huang-Kim MBI model



Bosch-Klemann MBI model

$$G(k) \cong \frac{4\pi I_0}{Z_0 I_A} Ck |R_{56}| \left| \int ds \frac{Z_{LSC}(k; s)}{\gamma(s)} \right| \exp \left[ -\frac{1}{2} (Ck R_{56} \sigma_\delta)^2 \right]$$

Can it be done directly?

$$\lambda_{crit} \propto AL_b C^2 R_{56}^2$$

The critical wavelength can be expressed as a function of A and the chicane's parameters and it is  $\ll \mu\text{m}$ .

MBI and IBS dynamics can be decoupled for modulations' wavelength longer than a threshold value!

# Analytical model

Important ingredient

$$[\log] = \log \left( \frac{b_{max}}{b_{min}} \right) \approx \log \left( \frac{\theta_{max}}{\theta_{min}} \right) \approx \ln \left( \frac{\epsilon_n q_{max}}{2\sqrt{2}r_e} \right)$$

Piwinski prescription

$$\theta_{min} \approx \frac{4r_e}{b_{max}\gamma^2\sigma_{x'}^2} \quad \text{with} \quad b_{max} \approx \sigma_x$$

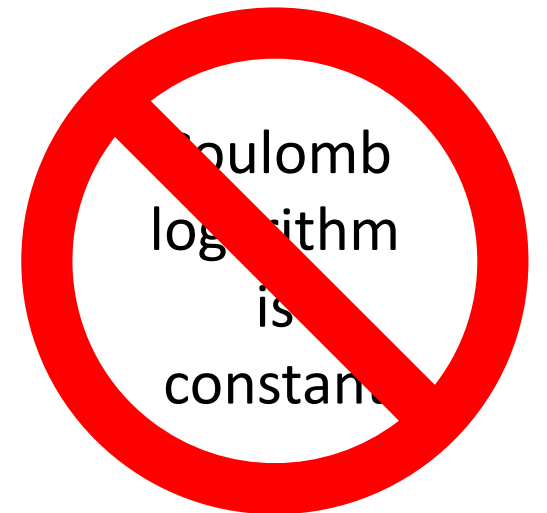
Raubenheimer prescription

$$\theta_{max} \approx \frac{\sqrt{2}q_{max}}{\gamma\sigma_{x'}}$$

$$\text{with} \quad q_{max} = \left[ \frac{cLN_e r_e^2}{2\gamma^{3/2}\epsilon^{3/2}\sigma_z\sqrt{\beta_x}} \frac{\sigma_H}{\sigma_\delta} + \mathcal{O}(\sigma_\delta) \right]^{1/2}$$

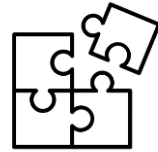
**Limit maximum scattering angle** and **discard** single scattering effects to exclude **tail contribution** from the calculation.

The cutoff timescale  $\rightarrow$  the time the bunch takes to travel along the section.



# IBS in action

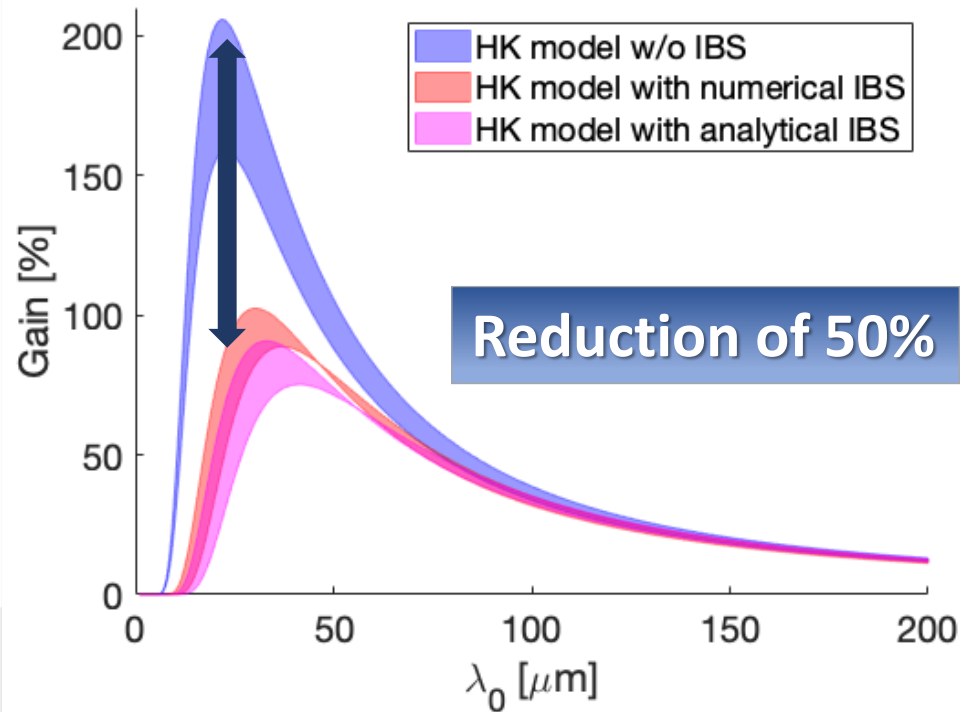
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# Effect of IBS on MBI

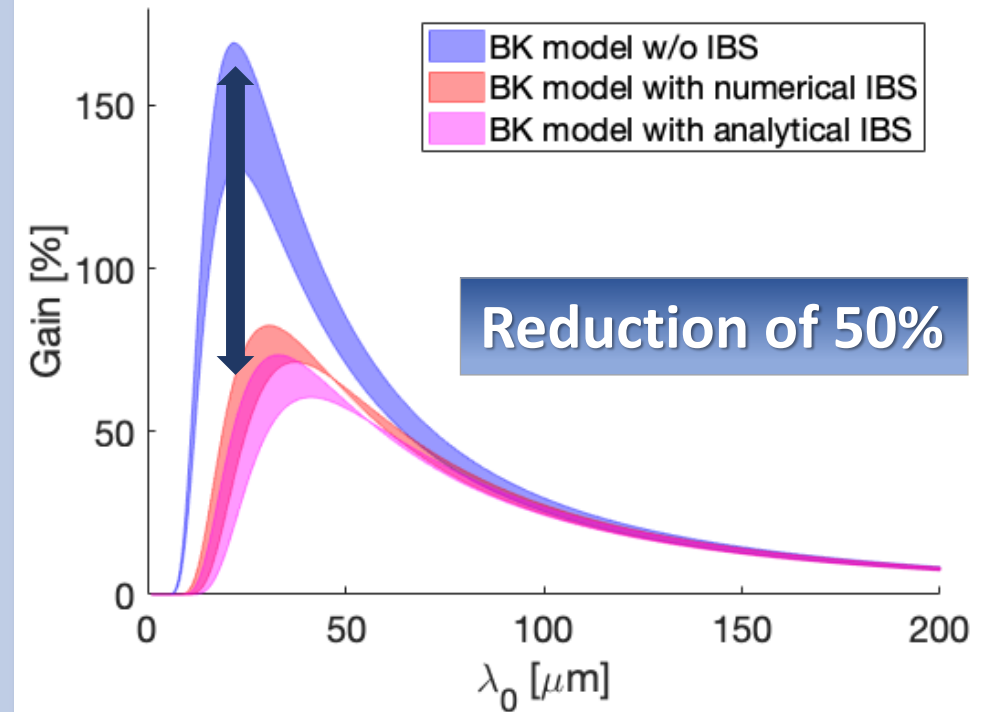
**HK model: linearized Vlasov equation  
in integral form**

**Collective effects: LSC and CSR.**



**BK model: matrix model for  
longitudinal modulations**

**Collective effects: LSC, CSR and CER.**



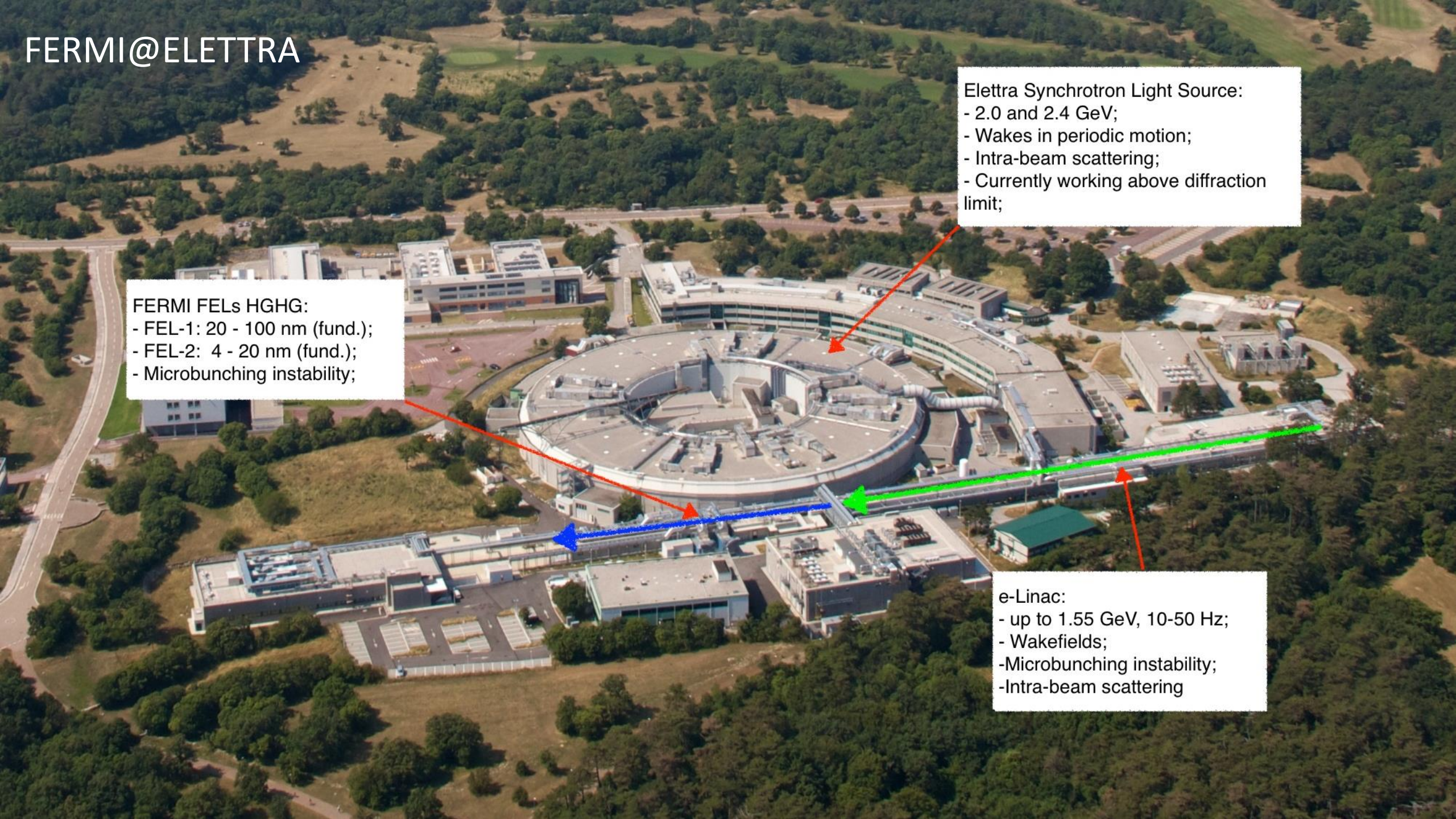


# FERMI@ELETTRA

Elettra Synchrotron Light Source:  
- 2.0 and 2.4 GeV;  
- Wakes in periodic motion;  
- Intra-beam scattering;  
- Currently working above diffraction limit;

FERMI FELs HGHG:  
- FEL-1: 20 - 100 nm (fund.);  
- FEL-2: 4 - 20 nm (fund.);  
- Microbunching instability;

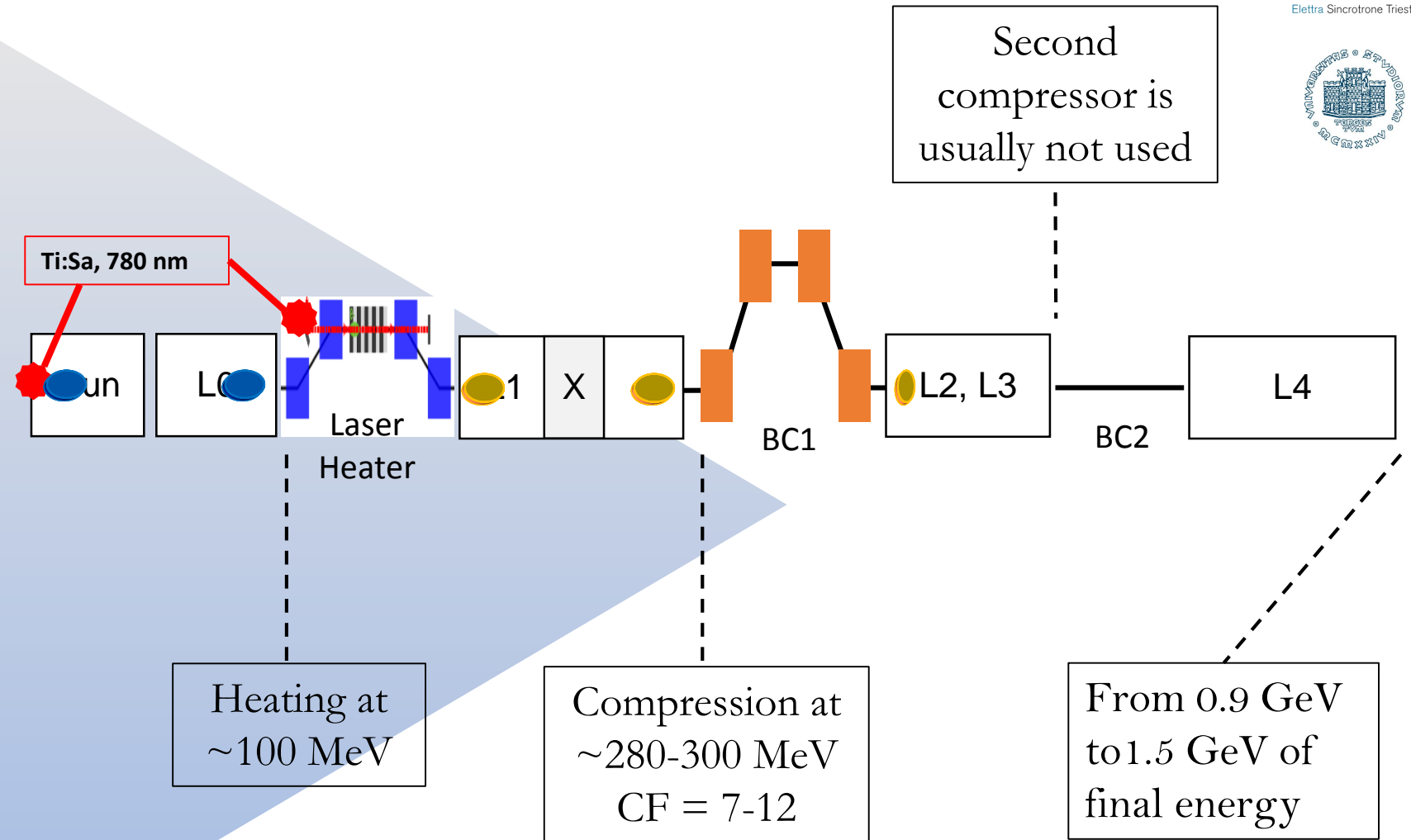
e-Linac:  
- up to 1.55 GeV, 10-50 Hz;  
- Wakefields;  
- Microbunching instability;  
- Intra-beam scattering



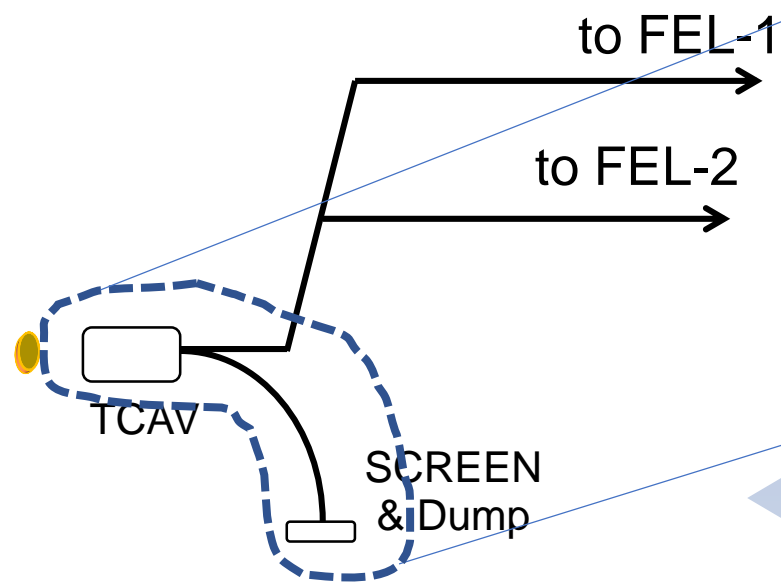
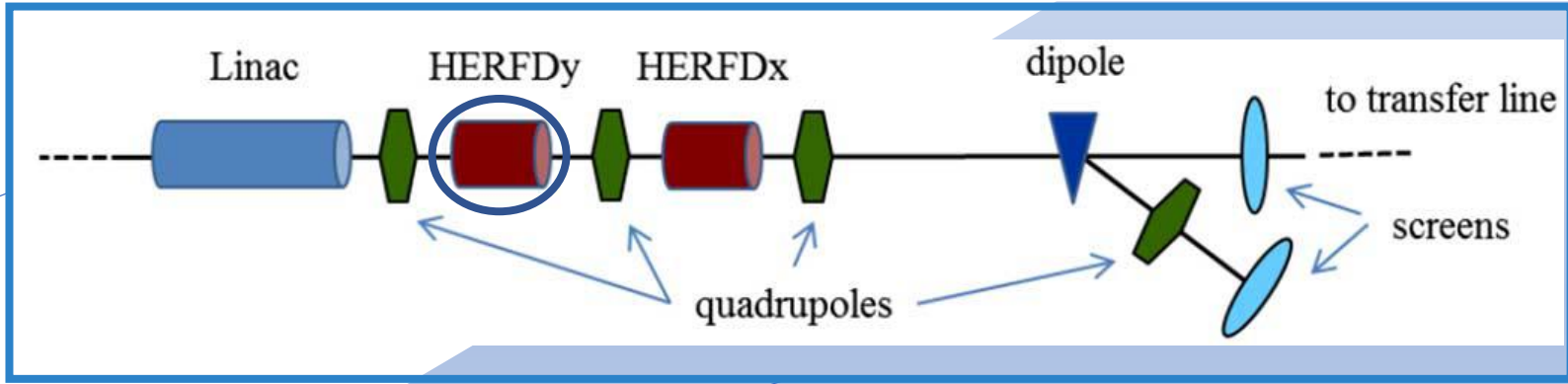


# FERMI linac

Parameters	Values
Charge	500-700 pC
Emittances	1-2 mm mrad
Energy spread	100-170 keV
Peak current	500-700 A
Beam Energy	0.9 - 1.5 GeV

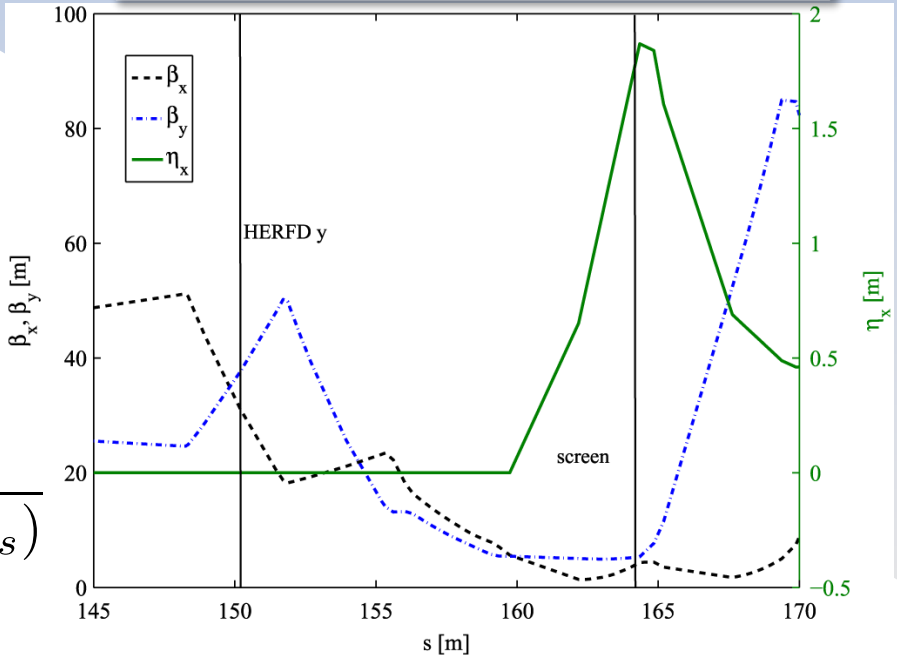
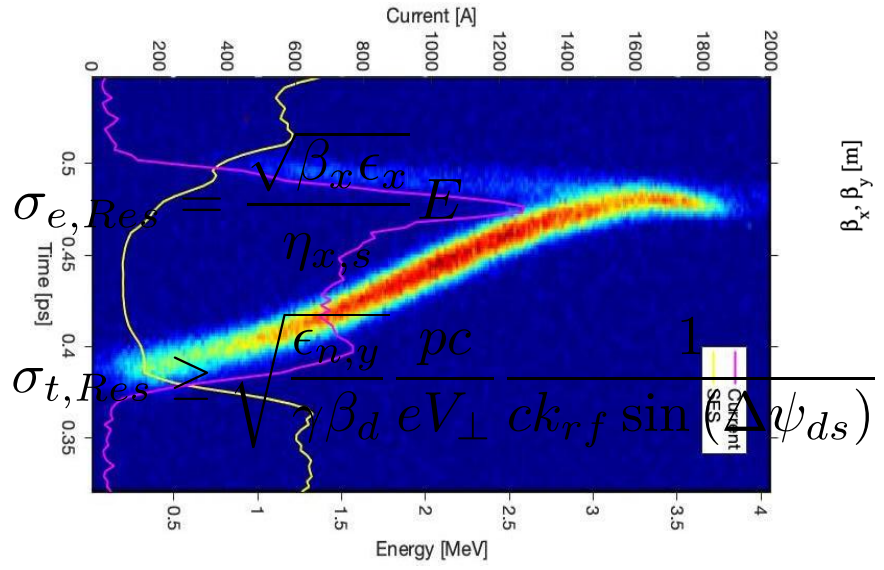


# Vertical RF Deflecting Cavity



**Beam optics matched to optimize time or energy resolution**

- Important parameters**
- $V_{\perp}$  peak voltage
  - $k_{rf}$  RF wave vector
  - L length of the section
  - $\eta_x$  dispersion
  - $\beta$  at VRFD (d) and screen (s)
  - $\Delta\psi_{ds}$  vertical betatron phase advance from d to s



- Systematic errors**
- Screen pixel size
  - Beam emittance  $\epsilon_y$
  - VRFD induced energy spread

# Experimental evidence of IBS

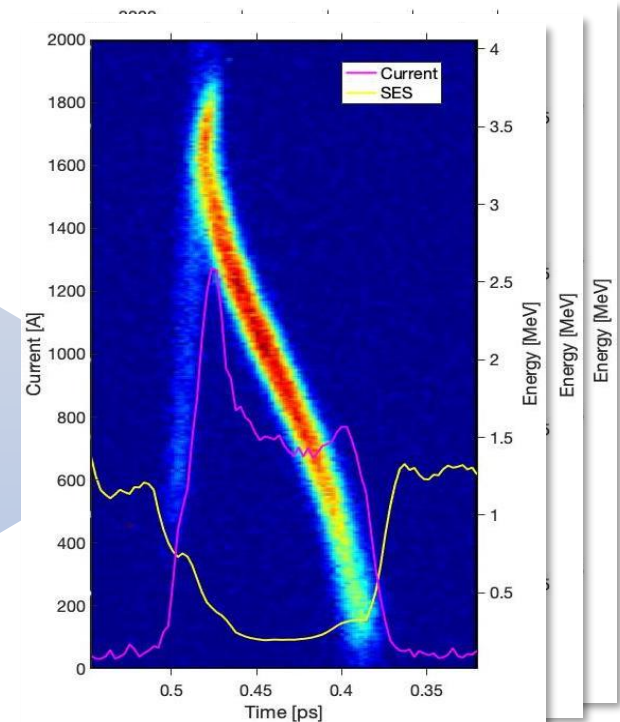
## Predictions of the model:

- Compressed initial energy spread:  $C\sigma_0^2$
- IBS\_induced energy spread:  $\sigma_{IBS}^2$
- Energy spread due to MBI:  $\sigma_\gamma^2$

$$\sigma_E^{(no\ IBS)} = \sqrt{C\sigma_0^2 + \sigma_\gamma^2}$$

$$\sigma_E^{(model)} = \sqrt{C\sigma_0^2 + \sigma_{IBS}^2 + \sigma_\gamma^2}$$

## Processing of measured data:



$$\sigma_E^{(meas)} = \sqrt{\bar{\sigma}^2 - \sigma_{VRFD}^2}$$

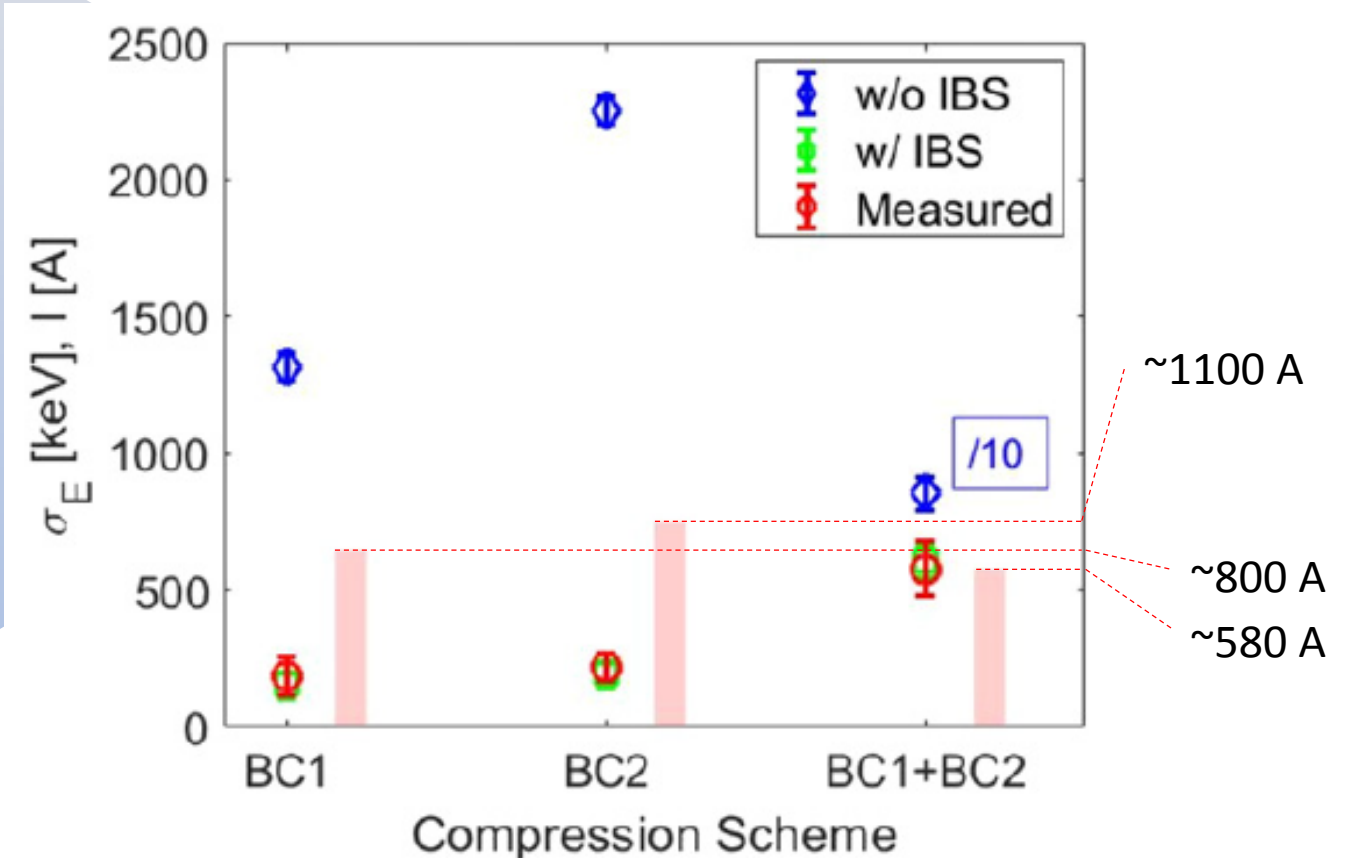
**Comparison**



# Experimental evidence of IBS

	100 pC	650 pC	Units
Initial peak current	18	60	MeV A
Initial beam energy	96	96	
Beam energy at BC1	~290	~290	
Beam energy at BC2	424–740	610	MeV
Final beam energy	713–754	900	MeV
$R_{56}$ of BC1	-35, -42	-42	mm
$R_{56}$ of BC2	0, -35	0	mm
Compression factor	31–45	11–19	
Normalized emittance	0.4	1.3	$\mu\text{m m rad}$
$\langle\beta_{x,y}\rangle$ along the linac	7–30	7–30	
Temporal resolution	8–12	8–12	fs
Energy spread resolution	65–85	65–85	keV

Comparison between **measured SES** and the one **predicted by model** with and without IBS



Measurements for three different compressions (only BC1, only BC2 and both) and without LH

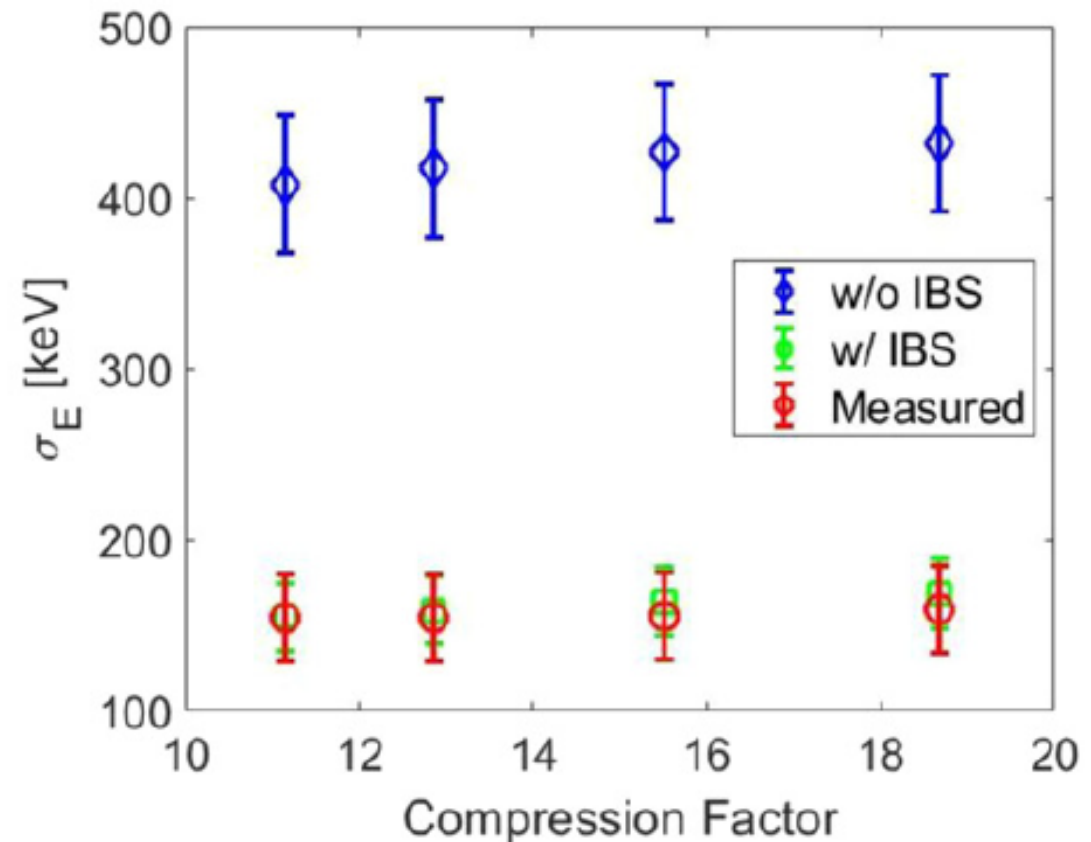
- ❖ Error bars in the data: reproducibility of the measurements in the same experimental session.
- ❖ Error bars for the model are related to optics uncertainties

# Experimental evidence of IBS



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Comparison between **measured SES** and the one **predicted by model** with and without IBS



Measurements for four different single

co To summarize: only with the **inclusion of the integrated IBS** along the whole linac, it becomes possible to **reproduce the measured SES** at the end of accelerator.

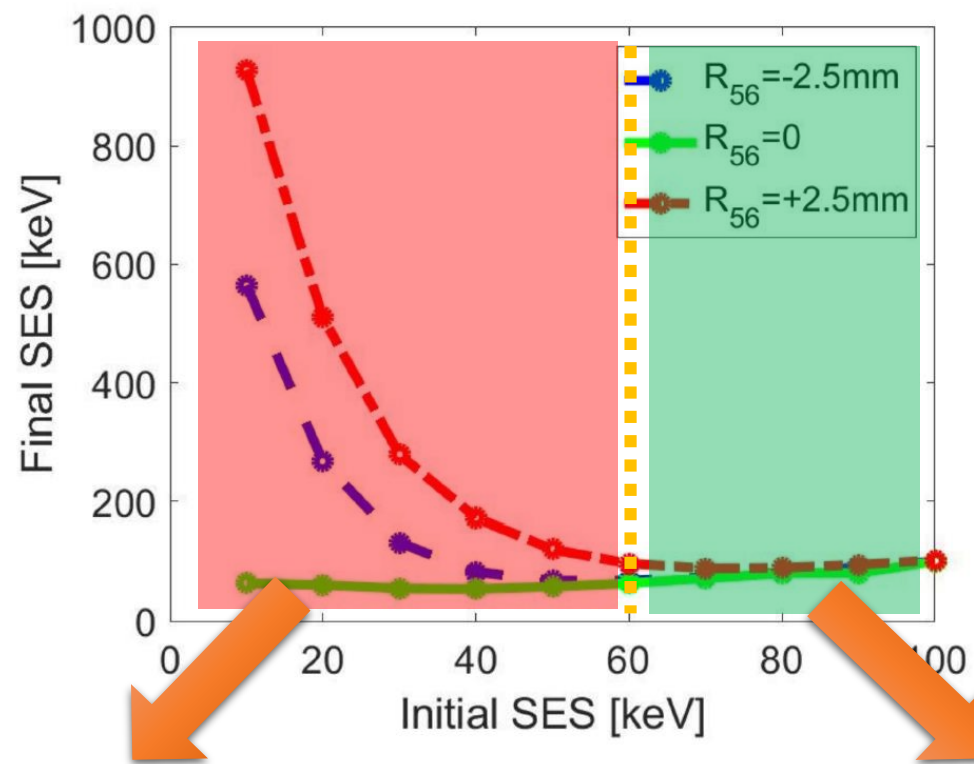
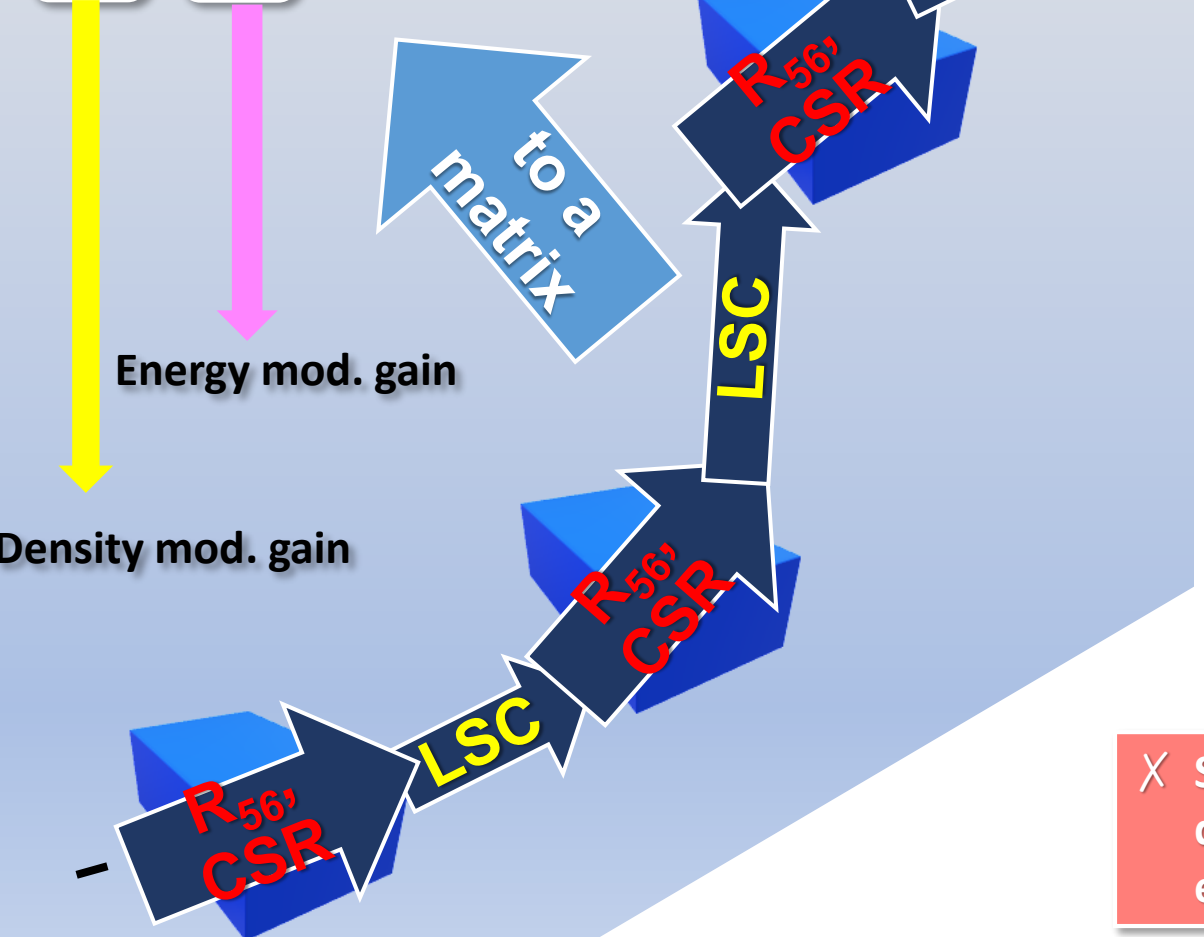
❖ Error bars for the model are related to optics uncertainties

# Design and predictions

S. Di Mitri G. Perosa, Physics 2 (2020)

$$T_{ij} = M_{ia}^{(n)} M_{ab}^{(n-1)} \dots M_{zj}^{(1)}$$

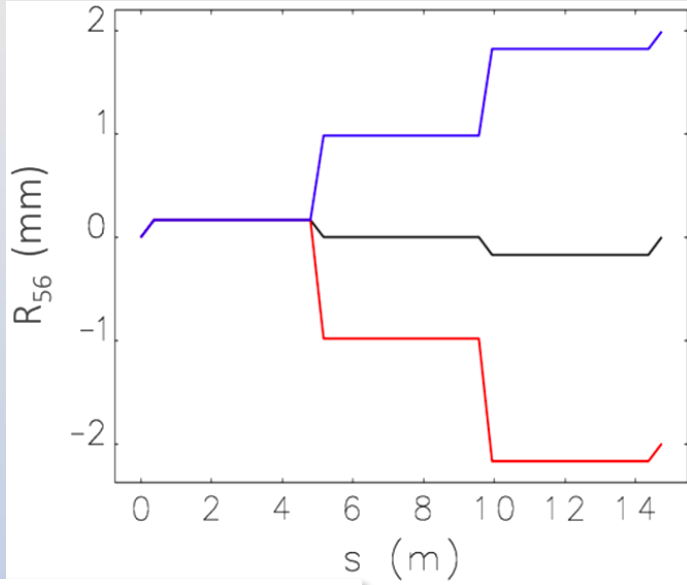
$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} \Delta I_0 \\ \Delta E_0 \end{pmatrix} = \begin{pmatrix} \Delta I_1 \\ \Delta E_1 \end{pmatrix}$$



✗ Slice energy spread and LPS dominated by density and energy modulations

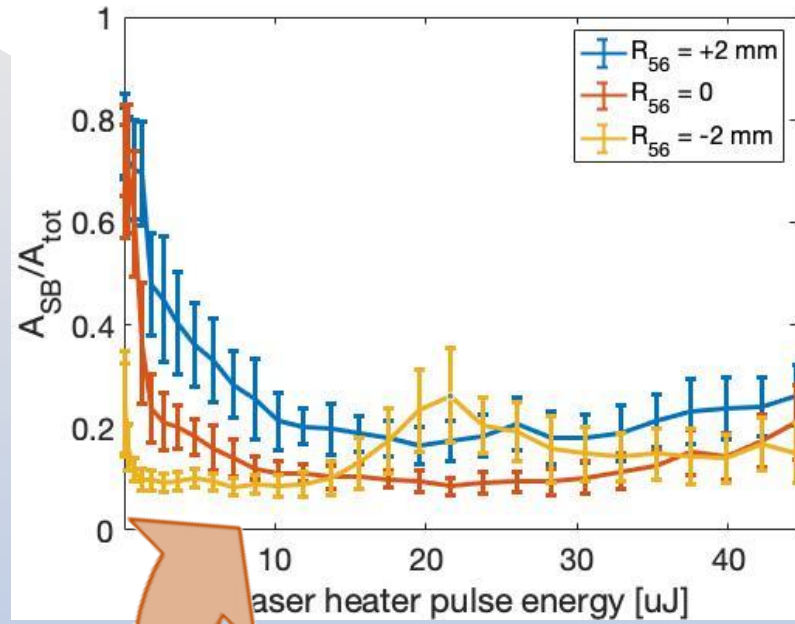
✓ Suppression of non-linear behaviour, region of Liouvillian behaviour

# Indication of IBS



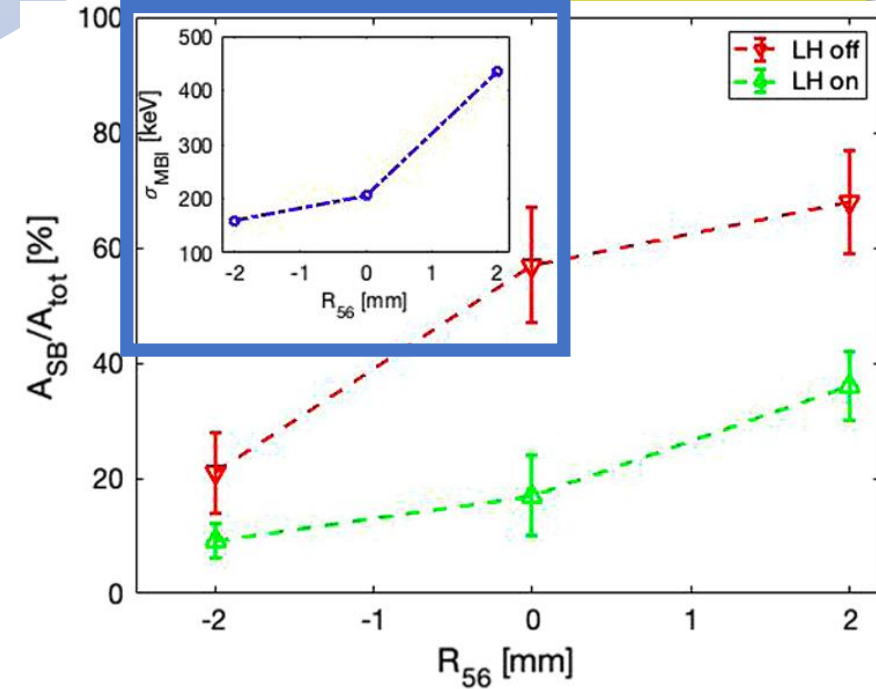
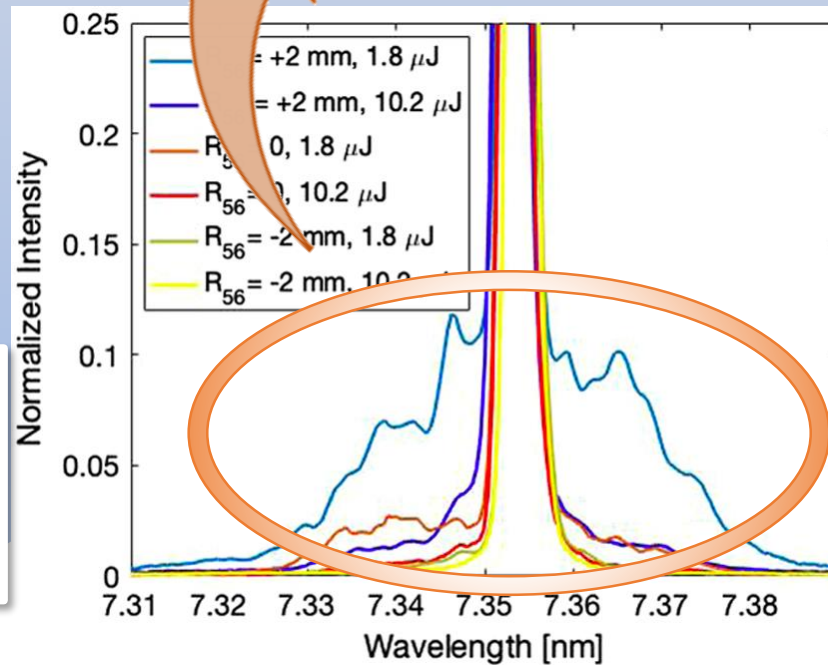
3 different optics with different values of  $R_{56}$

Mean spectra of EEHG scheme for each optics and different LH values



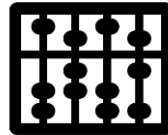
Sidebands for the 3 different optics as a function of LH  $\mu\text{J}$

Behavior of the MBI given by the model



# Conclusions

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# In conclusion...



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- **Intra-beam scattering** is no longer a negligible effect in linacs, when dealing with a faithful characterization of microbunching instability and phase space dynamics.
- A **new derivation** of the **Coulomb logarithm** is proposed, in order to properly normalize the contribution coming from hard scattering.
- The **combination of IBS and MBI** models results in a fast and comprehensive semi-analytical tool to predict final energy spread and modulations in linacs and multi-bend transfer lines.
- This model has been **tested and benchmarked**, showing good agreement with measurements in a vast set of machine configurations.

**What is next...**





# What is next...



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- IBS in linac (part 2):** the model MBI + IBS has been tested at the end of the linac  
→ we want to benchmark it also in the first region of the accelerator
- MBI+IBS:** other authors are considering second order effect of IBS in MBI dynamics.
- Effect of IBS in EEHG scheme:** at very high harmonics, a simplified theory predicts an impact on EEHG performances (rigid diffusion of the bunching factor) → we want to explore deeply the theoretical aspects of IBS diffusion in EEHG.
- Application of IBS:** control of IBS in the accelerating sections before compression is, in theory, an additional knob to reduce MBI → is there a useful way to use IBS?





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**Thank you for your attention**

# Back-up slides

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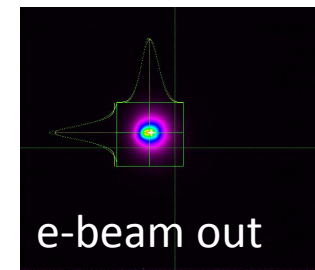
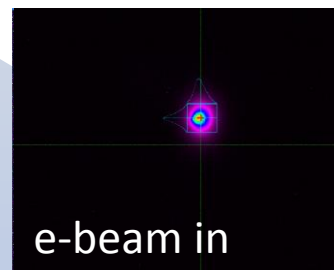
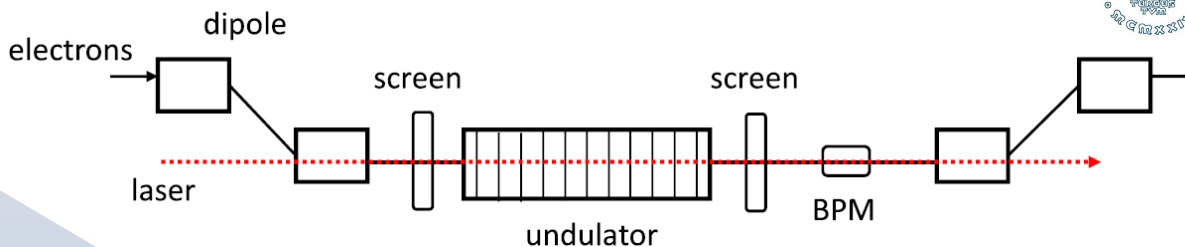
# Laser Heater



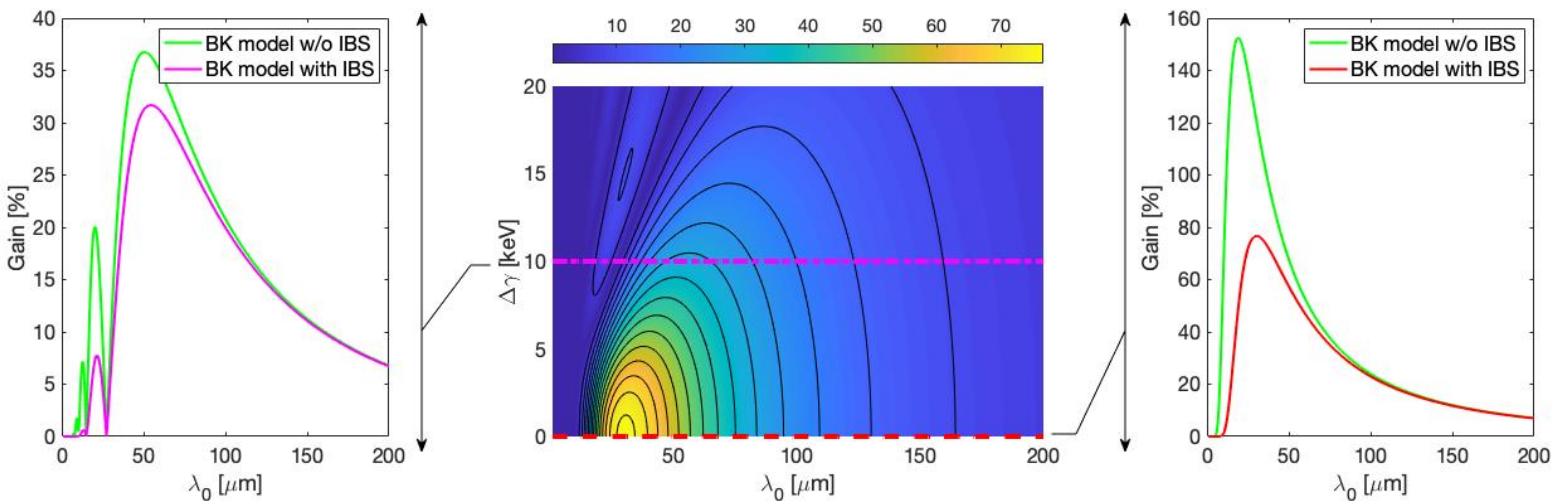
## What is it?

Laser Heater is composed of a laser and a short undulator embedded in a 4-dipole chicane:

- UV laser-electrons collinear superposition
- Smearing of the laser-induced modulations in the second half of the chicane



G. Perosa Simone Di Mitri, *Sci Rep* 11 (2021)



## Why?

To induce uncorrelated energy spread, for suppression of MBI via energy Landau Damping

# Experimental evidence of IBS

**Processing of measured data:** for each row of an image, the width of the energy profile is calculated with different processing  $\rightarrow W$

For each image, we pick the minimum value of the width function along the bunch  $\rightarrow \min(W) = \sigma$

For each set of images (usually 20 images), we determine the mean value and the standard deviation of the minimum of the «width slice function»  $\rightarrow \bar{\sigma}$

We subtract from this value the induced energy spread of the cavity

$$\sigma_E^{(meas)} = \sqrt{\bar{\sigma}^2 - \sigma_{VRFD}^2}$$

**Predictions of the model:** the slice energy spread at the end of the linac is determined computing the following terms:

❑ Compressed initial energy spread:  $C\sigma_0^2$

❑ Energy spread induced by IBS in each section:  $\sigma_{IBS}^2$

❑ Energy spread due to MBI:  $\sigma_\gamma^2$

$$\sigma_E^{(model)} = \sqrt{C\sigma_0^2 + \sigma_{IBS}^2 + \sigma_\gamma^2}$$

# Experimental evidence of IBS

**Predictions of the model:** the slice energy spread at the end of the linac is determined computing the following terms:

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$$\sigma_E^{(model)} = \sqrt{C\sigma_0^2 + \sigma_{IBS}^2 + \sigma_\gamma^2}$$

- ✓ **The initial energy spread** is «gauged» and verified with GPT simulations
- ✓ **IBS-induced SES** is estimated with our model (next chapter)
- ✓ **MBI-related SES** is estimated using the expression

$I_0$  is the initial current

$G$  is the gain after the compressor

$Z$  is the LSC impedance integrated along a section length

$$\sigma_\gamma^2 = \frac{2ec}{I_0} \int d\lambda \frac{|G(\lambda) Z_{LSC}^{int}(\lambda)|^2}{\lambda^2}$$

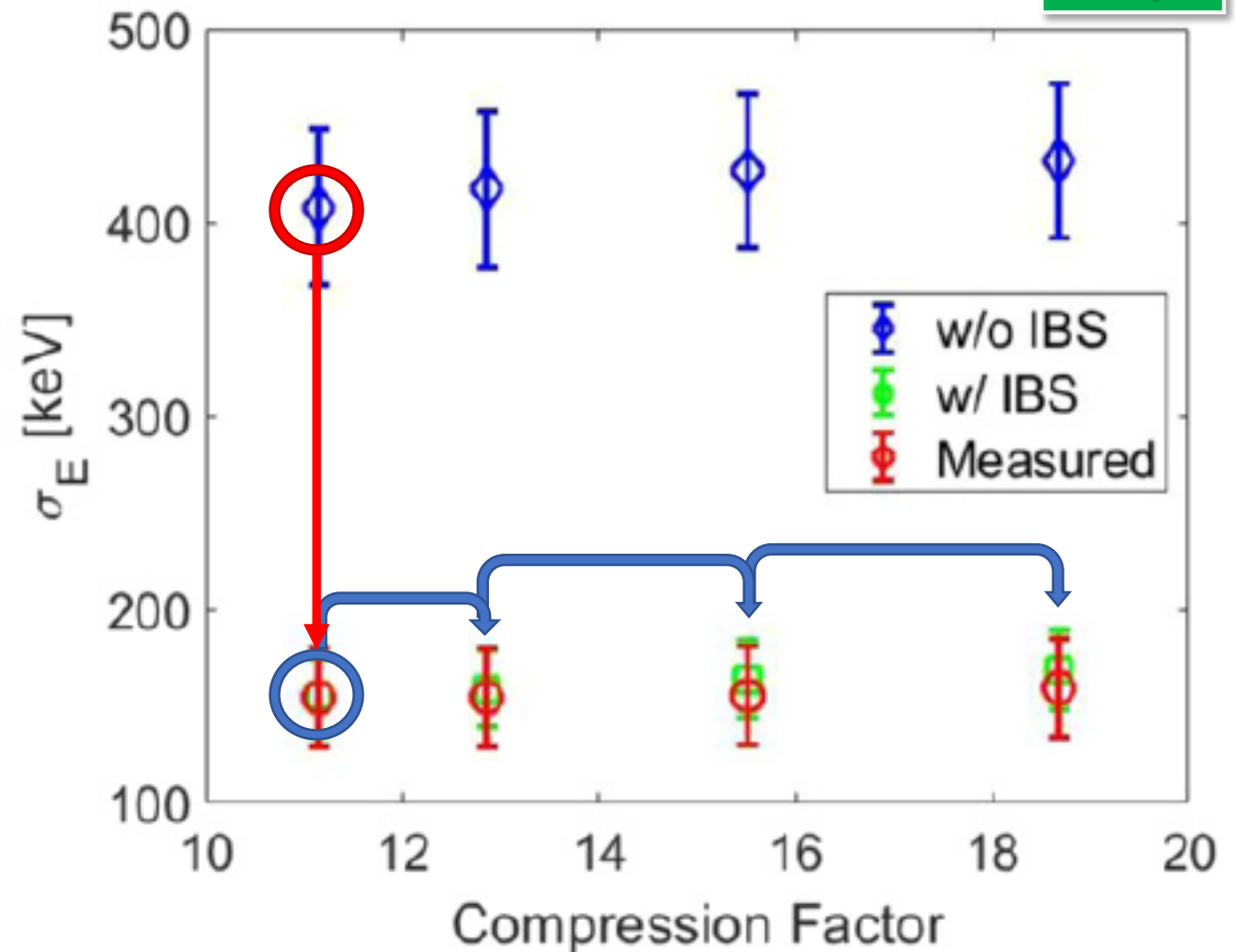
# Experimental evidence of IBS

$$\sigma_E^{(model)} = \sqrt{C\sigma_0^2 + \sigma_{IBS}^2 + \sigma_\gamma^2}$$

- We “gauge” the initial energy spread in order to match the measured data for a single point
- We use the same value to check the other data
- We repeat the procedure without IBS
- We compare the found values with a simulation of GPT

**N.B:** in order to match measured and predicted data w/o IBS,  $\sigma_0$  must be increased, reaching «unphysical» level

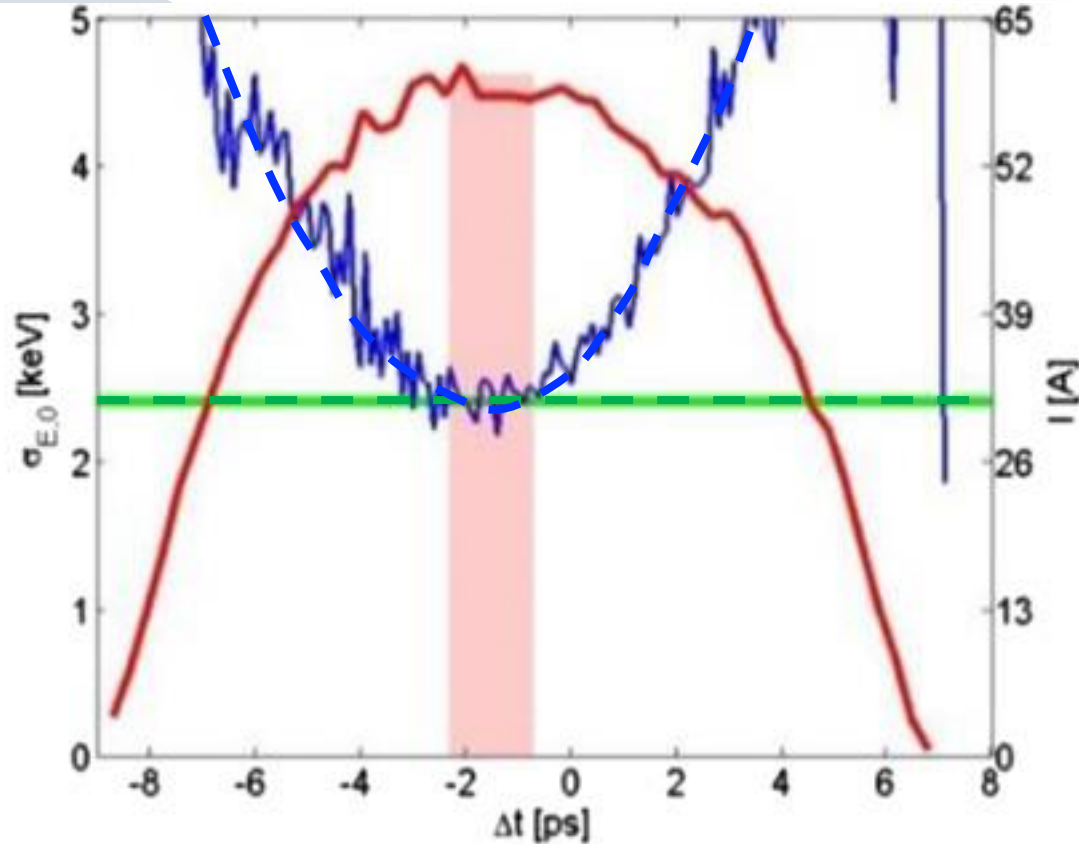
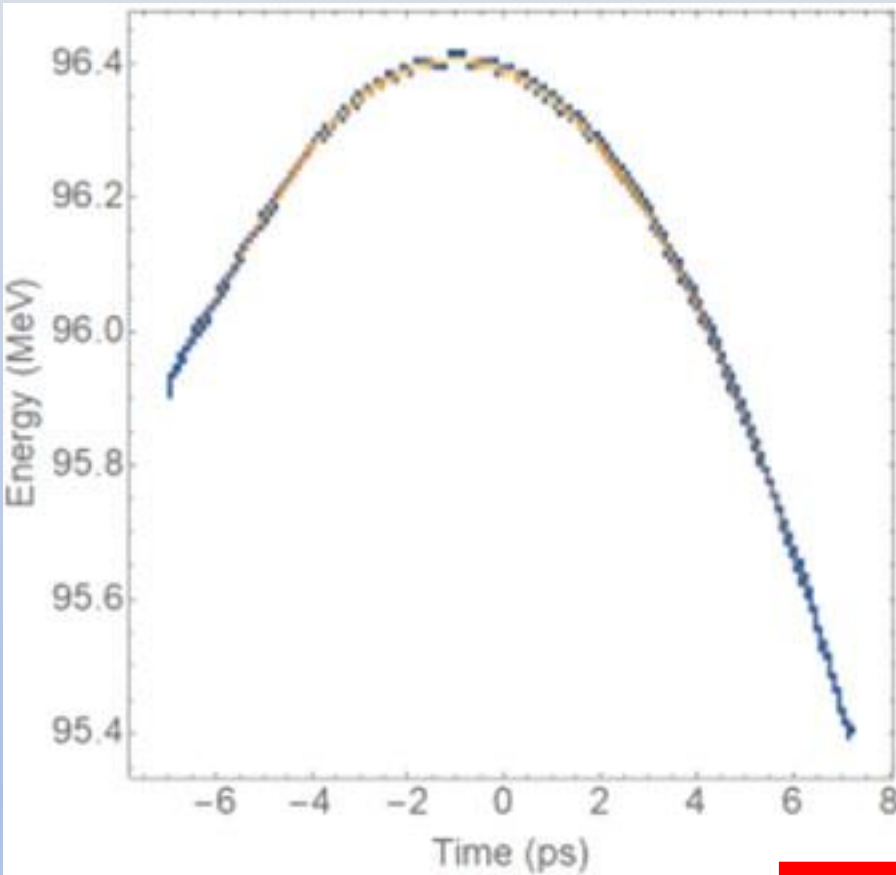
650 pC



# Experimental evidence of IBS

$$\sigma_E^{(model)} = \sqrt{C\sigma_0^2 + \sigma_{IBS}^2 + \sigma_\gamma^2}$$

650 pC



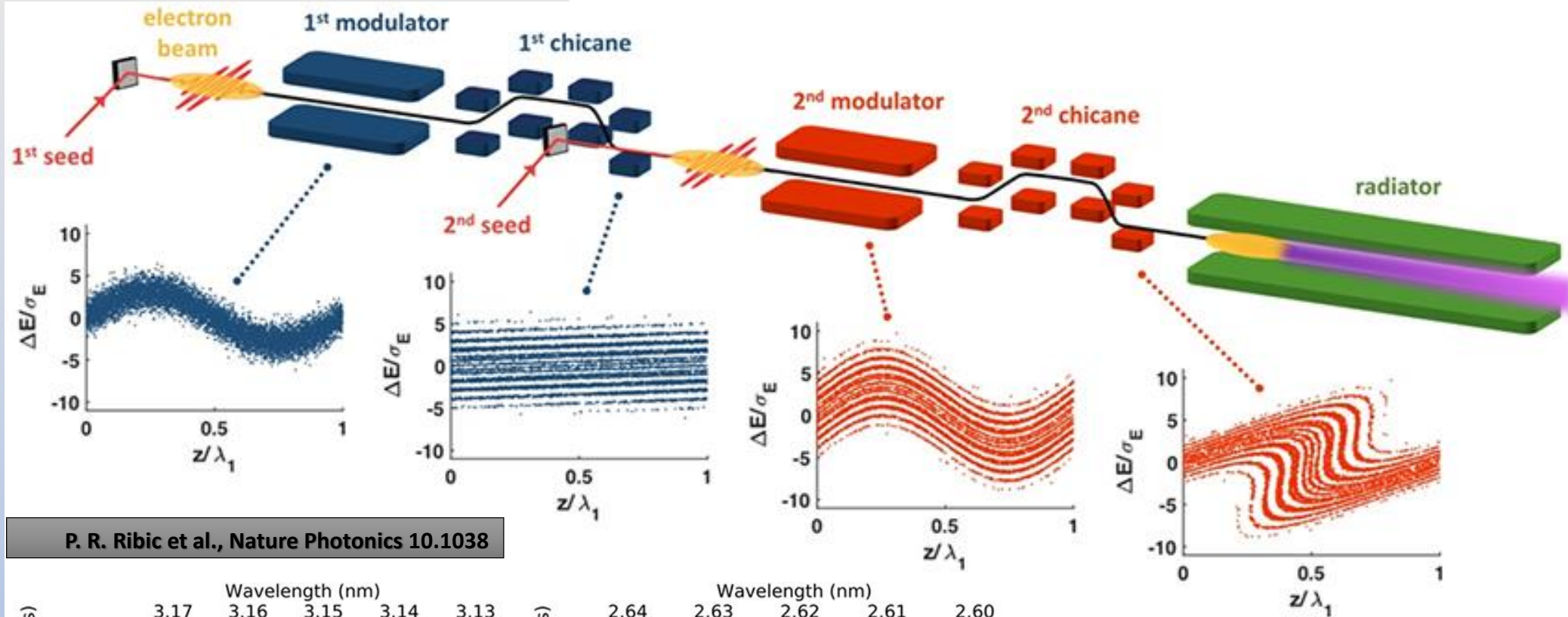
----- initial SES  
used in the model

----- initial SES  
predicted by GPT

The value used in basence of IBS is ~6uJ,  
well above the level shown here!

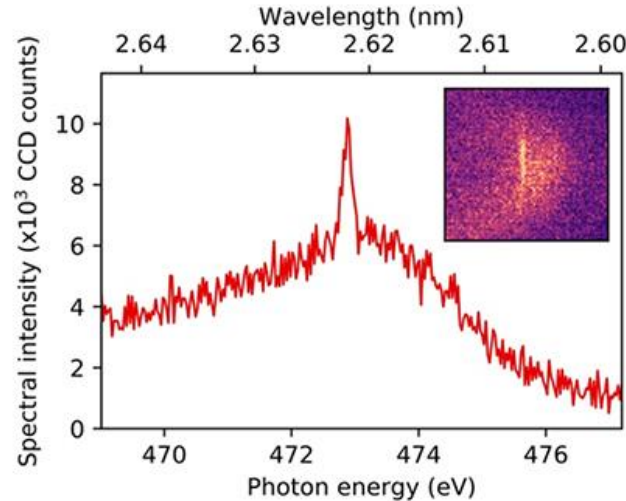
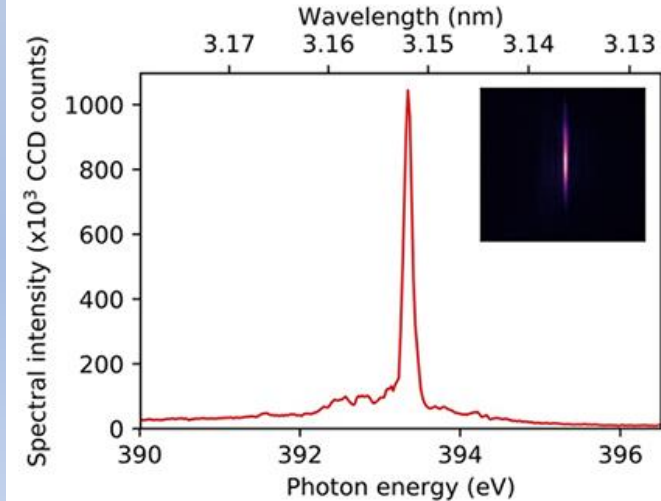


# Echo-enabled harmonic generation



Down to 2.6 nm,  
i.e.  $h = 101$

P. R. Ribic et al., Nature Photonics 10.1038



- ✓ Shaping by 2 seed laser require less seed energy
- ✓ Requires a much weaker energy modulation
- ✓ Is less sensitive to beam's imperfections