Phase transitions with ultra-relativistc bubbles.

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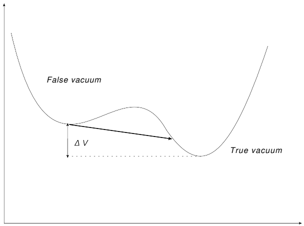
with M.Vanvlasselaer and W. Yin 2010.02590, 2101.05721, 2106.14913

Introduction

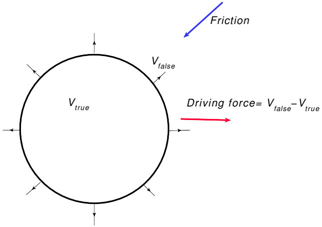


- False and true vacua are separated by the potential barrier
- Transition occurs by bubble nucleation (Coleman 77)

$$\Gamma(T) \sim \max\left[T^4\left(\frac{S_3}{2\pi T}\right)^{3/2} e^{-S_3/T}, R_0^{-4}\left(\frac{S_4}{2\pi}\right)^2 e^{-S_4}\right]$$

Bubbles of true vacua are formed, which later expand

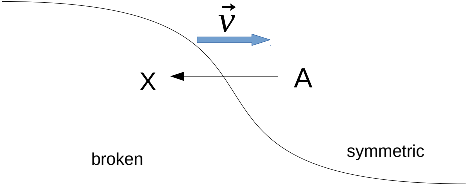
Relativistic bubbles



Forces acting on the bubble

- Driving force $\sim V_{true} - V_{false}$ due to the energy difference between true and false vacuum
- Friction forces due the bubble wall collision with plasma particles. These forces must vanish in the limit of zero temperature $T \to 0$
- **If $T \ll \Delta V^{1/4}$ the friction forces cannot prevent bubbles from reaching relativistic velocities**
- in the regime of supercooling i.e. $T \ll \Delta V^{1/4}$ bubble must be relativistic

Pressure on the wall



- We will assume that mean-free-path of the particles is much larger than the width of the wall $\Rightarrow$ we can consider individual particle collisions with the wall
- Pressure = Flux× (Probability of transition)×(Loss of momenta)

$$\boxed{\mathcal{P}_{A\to X} = \int \frac{p_z d^3p}{p_0(2\pi)^3} f_A(p) \times \sum_X \int dP_{A\to X}(p_A^Z - \sum_X p_X^Z)}$$

$1 \to 1$ transition *Bodeker-Moore 0903.4099(see also hep-ph/9203203)*

- Particle hits the wall and in the broken phase becomes massive

$$i\bar{\chi}\partial\!\!\!/\chi + y\bar{\chi}\chi\phi + V(\phi)$$

- Once the energy is sufficient to pass through the wall the probability of transition is $= 1$, then the pressure for the relativistic particles is equal to:

$$\mathcal{P}_{1\to 1} = \int \frac{d^3p}{(2\pi)^3} f_p(p_s^z - p_h^z) \simeq \int \frac{d^3p}{(2\pi)^3} f_p \times \frac{\Delta m^2}{2p_0}\theta(p_0 - m_{\rm broken})$$

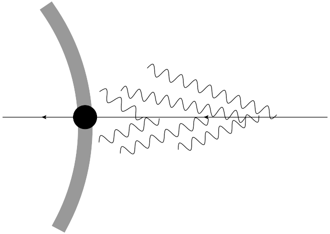
$$\mathcal{P}_{1\to 1} \sim m^2 T^2 \exp[-m/(2\gamma T)]$$

pressure is independent of $\gamma \Rightarrow$ permanently accelerating bubbles are possible if $P_{1\to 1} < \Delta V$

Bubble motion @ NLO

LO pressure is independent of γ and the bubbles can reach arbitrarily high velocities. What can stop such a permanent acceleration?

Pressure effects at NLO in the presence of vector fields obtaining their mass during the phase transition leads to qualitatively different behaviour (*Bodeker-Moore , 1703.08215*) $P_{NLO} \propto \gamma T^3 m$



Effect is dominated by soft vector emission, since in this case the momentum transfer is most efficient

$$\mathcal{P}^{eq.\gamma}_{1\to 2} = \underbrace{\int \frac{d^3p}{(2\pi)^3} f_p}_{\text{incident fermions}} \int_{m_V/p}^{1} dx f_\gamma(x) \times \underbrace{\frac{m_V^2}{2px}}_{\text{momentum transfer}}$$

$$= \int \frac{d^3p}{(2\pi)^3} f_p \times \left[\frac{e^2}{8\pi^2} m_V\right] \log \frac{m_V^2}{e^2 T^2} \propto \gamma T^3 m_V$$

EPA approximation (Fermi 24, Weizsacker 34, Williams 34, Landau Lifshitz 34)

there are claims $P_{NLO} \propto \gamma^2 T^4$ 2007.10343, ongoing discussion.

Ultrarelativistic bubbles: terminal velocity vs acceleration

Bubbles can reach relativistic velocities $v \to 1$, why it is important to differentiate between permanent acceleration or terminal velocity?**Phenomenological consequences?**

- If the bubbles are accelerating at the instance of collision, significant part of the energy is stored in the bubble shell

$$E_{shell} \sim 4\pi\sigma R^2, \ \sigma \propto \gamma \propto \frac{R}{R_0}, E_{shell} \propto R^3 \Rightarrow \frac{E_{shell}}{E_{total}} = \mathcal{O}(1)$$

- If the steady velocity is reached most of the energy is in the plasma motion/sound waves $\gamma = const \Rightarrow E_{shell} \propto R^2 \Rightarrow \dfrac{E_{shell}}{E_{total}} \to 0$
- **different predictions for the stochastic gravitational wave background**

Ultrarelativistic bubbles : terminal velocity vs acceleration

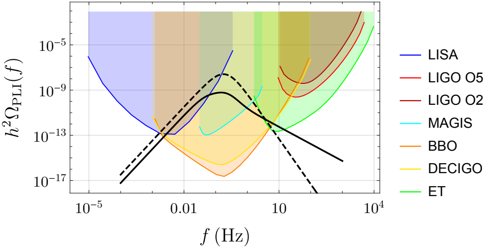


Figure: solid- runaway, dashed fixed velocity

Bubble wall can reach ultra-relativistic velocities

Collision energy between the bubble wall and the plasma particle can be much larger than the transition scale

$$E \sim \sqrt{\gamma T v} \gg v \quad \text{if} \quad \gamma \gg 1$$

- Is it consistent to ignore all other degrees of freedom which are decoupled at the phase transition?
- What effect these heavy fields can have?

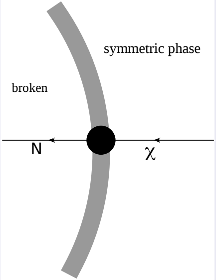
$1 \to 1$ transition, with mixing *2010.02590*

Consider the following lagrangian,

$$\mathcal{L}_{\text{fermion}} = i\bar{\chi}\partial\!\!\!/\chi + i\bar{N}\partial\!\!\!/N + M\bar{N}N + Y_{\text{mixing}}\phi\bar{\chi}N$$
$$M \gg \langle\phi\rangle$$

N-field is decoupled at PT and its density is suppressed by $\exp(-M/T)$

Will N field during χ - wall scattering?



Momentum is not conserved along z direction, $\chi \to N$ conversion is allowed

$1 \to 1$ transition, with mixing *2010.02590*

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Will N field during χ - wall scattering?

$$n_N \sim \underbrace{\int \frac{d^3p}{(2\pi)^3} f_p}_{\text{Incident } \psi \text{ density}} \underbrace{P(\chi \to N)}_{\text{Probability of transition}} \sim T^3 P(\chi \to N)$$

$$P(\psi \to N) \sim (\text{mixing angle})^2 \sim \frac{Y^2_{\text{mixing}}\langle\phi\rangle^2}{M^2}$$

$\boxed{T^3 \frac{Y^2_{\text{mixing}}\langle\phi\rangle^2}{M^2} \gg (MT)^{3/2} e^{-M/T}}$ This extra density will be much larger than the equilibrium value.

$1 \to 1$ transition, with mixing

Wall width is finite, $L \neq 0$!

momentum transfers with $\Delta p_z L \gg 1$ must be suppressed, since L^{-1} is a typical energy scale of the interaction with the wall.

Situation is similar to the neutrino oscillations in matter. If the $\Delta p_z L \gg 1$ is satisfied the evolution is "adiabatic", so the state remains in the lightest flavour:

$$\chi \to \chi_{\langle\phi\rangle\neq 0}$$

$\psi_{\langle\phi\rangle\neq 0}$ is the lightest eigenstate in the broken phase (inside the bubble)

We need to be in the "anti-adiabatic" regime

$$\Delta p_z L \lesssim 1 \to \frac{M^2}{E} \lesssim L^{-1}$$

B-M transitions always satisfy $\Delta p \sim \frac{\Delta m^2}{E} \ll L^{-1}$ since $L \sim m^{-1}$, but for the transitions with light $\to$ heavy this constraint is very important.

Finite wall with effects: brute force calculation

$$\mathcal{L}_{\text{fermion}} = i\bar{\chi}\partial\!\!\!/\chi + i\bar{N}\partial\!\!\!/N + M\bar{N}N + Y_{\text{mixing}}\phi\bar{\chi}N$$

- We need to calculate the probability of $\psi \to N$ transition in the presence of the wall
- Focus on the energies of the incident particles much larger than $E \gg \langle\phi\rangle$, use $\frac{\langle\phi\rangle}{E}$ as expansion parameter

$$\langle 0|T\{\bar{\chi}(x_1)N(x_2)\}|0\rangle = \int d^4x Y\langle\phi(x)\rangle S_\psi(x_1 - x)S_N(x - x_2) + \mathcal{O}\left(\frac{Y\langle\phi\rangle}{M}\right)$$

$$\Downarrow$$

$$P_{\psi\to N} \simeq \frac{Y^2\langle\phi\rangle^2}{M^2}\Theta(k_0 - M^2 L_w),$$

Exact function, suppressing the transitions for momentum transfers larger than $\Delta p \sim \frac{M^2}{k_0} \gg L$ depends on the wall shape.

Heavy particle production, modification of the bubble expansion dynamics?

- Production of these heavy particles will induce additional pressure on the wall

$$\mathcal{P}_{\text{mixing}} \sim \frac{T^2}{48} Y^2_{\text{mixing}} \langle \phi \rangle^2 \Theta(\gamma T - M^2 L)$$

  which is not suppressed by the mass of the heavy fields and can potentially modify the motion of the bubbles

- If we are in the regime

$$\mathcal{P}_{\text{No mixing}} + \mathcal{P}_{\text{mixing}} > \Delta V > \mathcal{P}_{\text{No mixing}}$$

  These new contribution to the friction can prevent accelerated motion of the bubbles $\Rightarrow$ modifies stochastic GW signal .

Maximal mass which can be produced?

- $$M_{\rm max} = \sqrt{\gamma_{\rm max}\frac{T_{\rm nuc}}{L}} \sim \sqrt{\gamma_{\rm max}\, T_{\rm nuc}\langle\phi\rangle}$$
- If there are no gauge fields, Lorentz expansion factor for runaway bubbles can reach $\gamma_{\rm max} \sim \frac{R_*}{R_0}$

$$R_0 \sim \frac{1}{T_{\rm nuc}}, \quad R_* \sim H^{-1} \sim \frac{M_{\rm pl}}{{\rm scale}^2},$$

- The maximal mass which can be probed is :

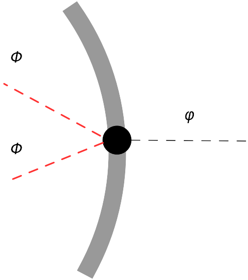
$$M^{MAX} \sim {\rm Min}\left[\frac{4\pi}{g_{gauge}^{3/2}}\frac{\langle\phi\rangle^2}{T_{\rm nuc}}, \frac{M_p^{1/2}\, T_{\rm nuc}}{\langle\phi\rangle^{1/2}}\right]$$

Heavy scalar production

Similar effect happens also with bosons even without mixing

$$\lambda\phi^2\Phi^2_{\rm heavy} + M^2_{\rm heavy}\Phi^2_{\rm heavy}$$

there will be $\phi \to \Phi_{\rm heavy}\Phi_{\rm heavy}$ production during the transition through the wall. Since the trilinear vertex $\phi\Phi\Phi$ is position dependent and momentum is not conserved. Apart from some numerical pre-factor difference effect is very similar to the heavy fermion production.

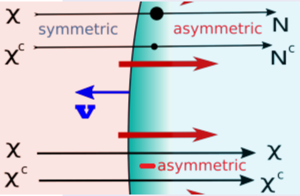


Phenomenological relevance of heavy states production

- At FOPT states with the masses much larger than the typical scale can be produced with denisties much larger than the equilibrium ones. Can this be important?
- New mechanism for DM non-thermal production, very different different parameter dependence compared to the usual freeze-out scenarios *2101.05721*
- Baryon asymmetry generation, the process of heavy particle production is out-of equilibrium so if accompanied with CP violation and baryon number violating interactions can lead to BAU *2106.14913*

Baryon asymmetry generation *2106.14913*

asymmetry generation by passage through the wall



- The process of the heavy N production is out of equilibrium
- In the presence of the CP violation $\Gamma(\chi \to N) \neq \Gamma(\chi^c \to N^c)$ asymmetry between N, N^c and χ, χ^c will be generated.

If we add some baryon number violating process all three Sakharov's conditions will be satisfied

see also 2106.15602 for similar ideas

Example model : Phase-transition induced leptogenesis

$$\underbrace{\sum_I \left(Y_I(\phi^\dagger \bar{\chi}) P_L N_I + Y_I^\star \bar{N}_I P_R(\phi\chi) \right) - V(\phi) + \frac{1}{2}\lambda_\chi \phi \bar{\chi}^c \chi + \sum_I M_I \bar{N}_I N_I}_{\text{Toy model of Dark Sector}}$$

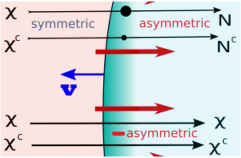
$$+ \underbrace{\sum_{\alpha I} y_{\alpha I} (h \bar{l}_{\alpha,SM}) P_R N_I + h.c.,}_{\text{Connection to SM}}$$

ϕ some field experiencing the phase transition, we will be agnostic about the origin and shape of the potential

Sakharov's conditions

- CP &C violation from complex couplings Y_I, y_{α_I}
- Lepton number is broken by the $\lambda_\chi \phi \chi^c \chi$ interaction. Later lepton number asymmetry is converted to baryon number asymmetry at EW phase transition.
- $\chi \to N$ transition is out of equilibrium.

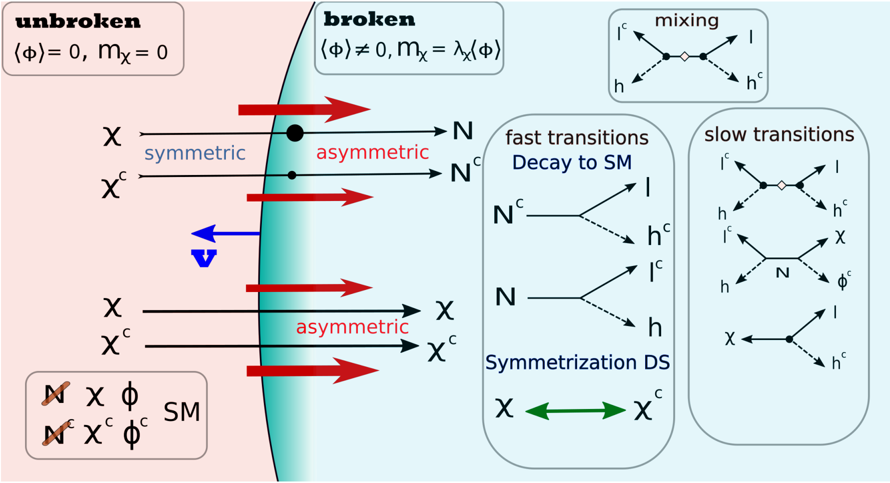
Transferring asymmetry to SM



- χ has Majorana mass inside bubble $\boxed{\lambda_\chi \phi \bar{\chi}^c \chi}$ $\Rightarrow$ any asymmetry in χ disappears. Total lepton number is generated.
- N decays to $N \to hl$ and $N \to \chi\phi$, part of the asymmetry is transferred to SM

Decay rates $\Gamma(N \to hl) \neq \Gamma(N^c \to h^c l^c) \Rightarrow$ additional source of CP violation.

The mechanism at work



Baryon asymmetry

$$\frac{\Delta n_B}{s} \equiv \frac{n_B - n_{\bar{B}}}{s} \simeq \quad -\frac{28}{79} \times \frac{135\zeta(3) g_\chi}{8\pi^4 g_*} \times \sum_I \theta_I^2 \sum_{\alpha,J} \mathrm{Im}(Y_I Y_J^* y_{\alpha J} y_{\alpha I}^*) \mathrm{Im} f_{IJ}^{(hl)}$$
$$\times \left(\frac{2}{|Y_I|^2} - \frac{1}{\sum_\alpha |y_{\alpha I}|^2} \right) \left(\frac{T_{nuc}}{T_{reh}} \right)^3 \frac{\sum_\alpha |y_{\alpha I}|^2}{\sum_\alpha |y_{\alpha I}|^2 + |Y_I|^2}$$

- θ_I suppression from mixing between $\chi - N$
- $\left(\frac{T_{nuc}}{T_{reh}}\right)^3$ - suppression sinve the universe heats up to T_{reh} after completion of the PT.
- $\frac{2}{|Y_I|^2}$ from CP violation in production and $\frac{1}{\sum_\alpha |y_{\alpha I}|^2}$ CP violation in decay.

$$\boxed{\mathrm{Max}[\theta^2 y^2] \left(\frac{T_{nuc}}{T_{reh}} \right)^3 \sim 10^{-6}}$$

Upper bound on the scale

The model leads to the generation of the neutrino masses

$$\sum_{I,\alpha,\beta} \theta_I^2 \frac{y_{\alpha I} y^*_{\beta I} (\bar{l}^c_\alpha h)(l_\beta h)}{m_\chi}$$

which induces a mass for the active neutrinos (for the heaviest light neutrino)

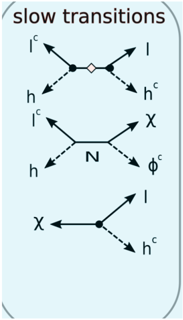
$$\mathrm{Max}[m_\nu] \sim \mathrm{Max}\left[\sum_I |y_{\alpha I}|^2 \theta_I^2\right] \frac{v_{EW}^2}{m_\chi}$$

BAU requires $\mathrm{Max}[\theta_I^2] \gtrsim 10^{-5}, y \sim \mathcal{O}(1)$ combining with neutrino masses leads to

$$m_\chi \gtrsim 5 \times 10^9 \mathrm{GeV} \qquad \Rightarrow \qquad \langle \phi \rangle \gtrsim 10^9 \mathrm{GeV}$$

Avoiding wash out

We need to make sure that the processes washing out asymmetry are slow



- Weinberg's operator $\boxed{\frac{(\bar{l}^c h)(lh)}{\Lambda}} \Rightarrow$ $T_{reh} < 10^{12}$ GeV
- $lh \to \chi$ will be suppressed if $\frac{m_\chi}{T_{reh}} \gtrsim 15$

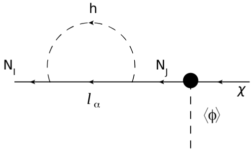
need mild hierarchy between m_χ and T_{reh}

Summary

- First order phase transitions with ultra relativistic bubbles in the early universe lead to very interesting scenarios.
- Particles seemingly decoupled are playing an important role and can be produced abundantly. Important phenomenological consequences.
  - Modification of the bubble expansion velocity.
  - DM production
  - Models of baryogenesis
- **all of these must be accompanied with strong stochastic GW signal observable at current/future experiments.**

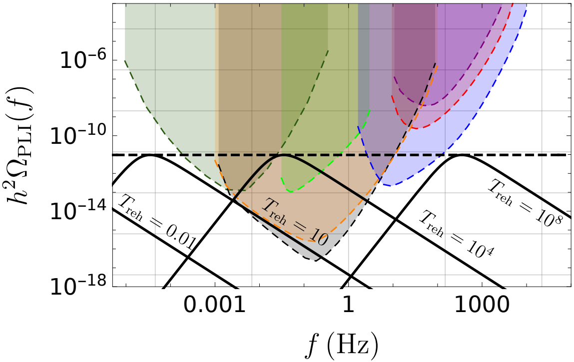
CP violation in passage through the wall

- We need to calculate at least one loop corrections to $\langle 0|\bar{\chi} N|0\rangle$ in the presence of the wall
- If the energy of the incident particle is much larger than the $\langle\phi\rangle$ we can expand in $\frac{\langle\phi\rangle}{E}$, even at one loop.
- The only diagram contributing will be



$$\epsilon_I = \frac{2\sum_{\alpha,J,i} \mathrm{Im}(Y_{iI} Y^*_{iJ} y_{\alpha J} y^*_{\alpha I}) \mathrm{Im} f^{(hl)}_{IJ}}{\sum_i |Y_{iI}|^2}, \ \mathrm{Im}[f^{(hl)}_{IJ}(x)] = \frac{1}{16\pi}\frac{\sqrt{x}}{1-x}, \ x = \frac{M_J^2}{M_I^2}$$

$\alpha = 1, \quad \beta = 100$



Can such a scenario be realized during the EW phase transition?

We need to have strong first order EW phase transition with relativistic bubbles.

Prototype model

$$\mathcal{L} = \mathcal{L}_{SM} + m_\eta^2|\eta|^2 + \sum_{I=1,2} M_I \bar{B}_I B_I$$
$$+ \left(\sum_{I=1,2} Y_I(\bar{B}_I H) P_L Q + y_I \eta^* \bar{B}_I P_R \chi + \kappa \eta^c du + \frac{1}{2} m_\chi \bar{\chi^c} \chi + h.c. \right)$$

- We will not specify the origin of Higgs potential, need some additional sources which can lead to FOPT.

Prototype model

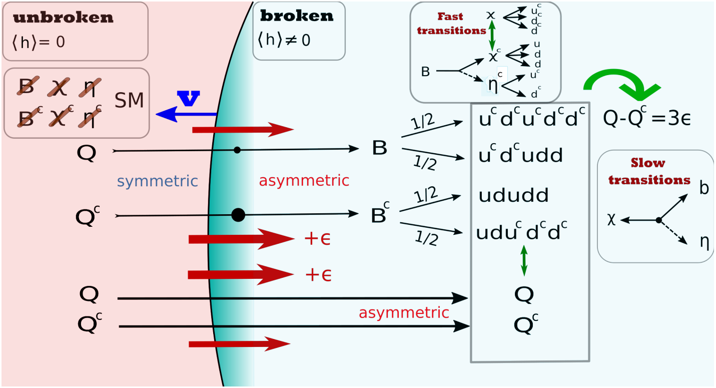
$$\mathcal{L} = \mathcal{L}_{SM} + m_\eta^2|\eta|^2 + \sum_{I=1,2} M_I \bar{B}_I B_I$$

$$+ \left(\sum_{I=1,2} Y_I (\bar{B}_I H) P_L Q + y_I \eta^* \bar{B}_I P_R \chi + \kappa \eta^c du + \frac{1}{2} m_\chi \bar{\chi}^c \chi + h.c. \right)$$

- η scalar with $Q(\eta) = 1/3$, χ - Majorana fermion
- $B(\eta) = 2/3, \quad B(\chi) = 1$
- Baryon number violation is coming from χ mass and it
- **Baryon number violated by 2, proton will be stable, but $n - \bar{n}$ oscillations will be present.**

Model at work

- $B_l \to \chi d^c u^c \to (bdud^c u^c)$ conserves B number
- $B_l \to \chi^c d^c u^c \to (b^c d^c u^c d^c u^c)$ violates by factor of 2



Baryon asymmetry

$$\frac{\Delta n_{Baryon}}{s} \approx \frac{135\zeta(3)}{8\pi^4}\sum_{I,J}\theta_I^2\frac{|y_I|^2}{|y_I|^2+|Y_I|^2}\times\frac{g_b}{g_\star}\left(\frac{T_{\rm nuc}}{T_{reh}}\right)^3$$
$$\times{\rm Im}(Y_I Y_J^* y_I^* y_J)\left(-\frac{2{\rm Im}[f_B^{IJ}]}{|Y_I|^2}+\frac{4{\rm Im}[f_B^{IJ}]|_{m_{\chi,\eta}\to 0}}{|y_I|^2}\right).$$

assuming order phases and requiring $\frac{\Delta n_{Baryon}}{s} \sim 8.8\times 10^{-11}$

$$\boxed{\theta_I^2\left(\frac{T_{\rm nuc}}{T_{reh}}\right)^3 \sim 10^{-(6-7)}}$$

$\theta_I \sim \frac{Yv}{M}$ **cannot be too small, need new physics in the 1-100 TeV range**

Constraints/signatures

- **avoiding wash out**: need to suppress baryon number violating interactions after the phase transition $\boxed{\frac{M_{B,T,\chi}}{T_{reh}} \gtrsim 30}$
- **neutron EDM:** the operator will be $\boxed{\frac{(\sum \kappa\theta_I y_I)^2}{M_\eta^4 m_\chi}\overline{u^c d^c d^c} udd}$ generated, for $\theta \sim 10^{-(1-2)}$ we will get $M_{\eta,m_\chi} \gtrsim 10^5$ GeV. If new physics couples only to the third generation the bound relaxes.
- **flavour violation**: diaquark η leads to the flavour violation, but these can be suppressed if new physics couples only to the third generation.