Revisiting the Effective W Approximation (at muon colliders!)

Portoroz 2021

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[1] w/ A. Costantini, F. Maltoni, O. Mattelaer, et al [2005.10289] + *to appear*

We made it!

Hardships continue but the outlook is encouraging

the big picture

question: what exactly is this talk about?

- An attempt to better outline when weak gauge bosons $(W^\pm/Z)$ can be treated as partons of a beam, e.g., proton $e^\pm, \mu^\pm$

Dawson('84); Kane, et al ('84); Kunszt and Soper ('88)

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why:

- A many-TeV $\mu^+\mu^-$ collider is being discussed seriously as a future opportunity for the field — European Strategy Update [1901.06150], Snowmass (on-going)
- When collisions reach $Q \sim \mathcal{O}(10)$ TeV, vector boson scattering / fusion **(VBS/VBF)** start to act a bit... funny

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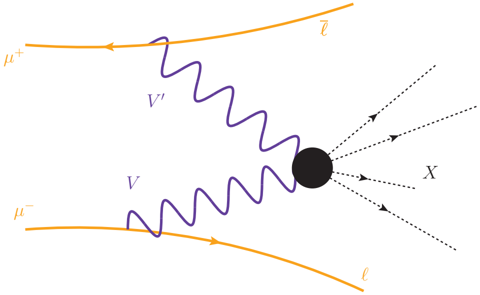
bonus outcome: `MadGraph5` now supports PDFs for **helicity-polarized EW** gauge bosons $W^\pm_\lambda$, Z_λ, γ_λ, where $\lambda \in \{0, \pm 1\}$, from $e^\pm, \mu^\pm$ beams

- Transitioning from α-testing (developers) to β-testing (volunteers)

`bzr branch lp:∼maddevelopers/mg5amcnlo/2.9.x_eva_v2`

why a $\mu^+\mu^-$ collider?



Many motivations for a muon collider

- **Generically,** discovering laws of nature requires larger data sets and higher energies See e.g., Al Ali, et al. [2103.14043]

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  - context: present flavor anomalies are arguably "μ-flavor" anomalies

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- **Timely,** since novel R&D shows $\mathcal{O}(10)$ TeV μ colliders look feasible
  See e.g., Delahaye, et al [1901.06150] and refs therein.

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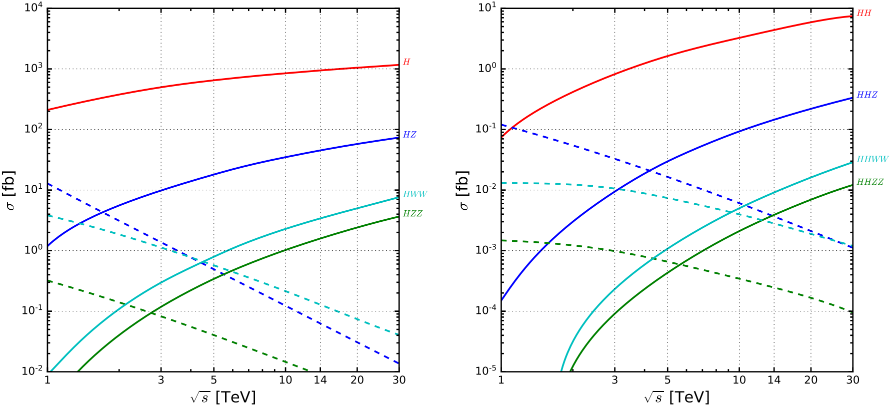
- **Excitingly,** *partonic* collisions at $Q \sim \mathcal{O}(10)$ TeV explore when electroweak (EW) symmetry is nearly restored, i.e., $M^2_{W/Z/H}/Q^2 \to 0$

  See C. Bauer, et al ('16,'17,'18); T. Han, et al ('16,'20,'21); A. Manohar, et al ('14,'18) + others

so what are many-TeV $\mu^+\mu^-$ collisions like anyway?

Higgs production

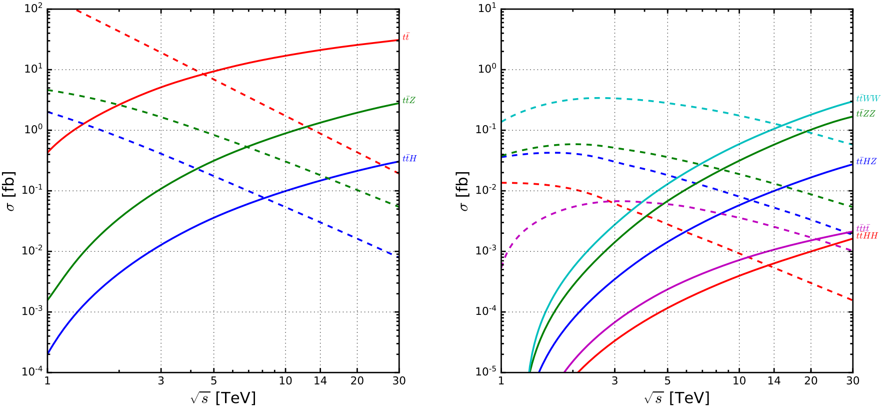
cross sections (σ) vs $\sqrt{s}$ for
s-channel annihilation (dash) vs VBF (solid)



$\sigma^{VBF} > \sigma^{s-channel}$ since

- $\sigma^{s-channel} \sim 1/s$
- $\sigma^{VBF} \sim \log^4(M_{VV}^2/M_V^2)/M_{VV}^2$ due to soft/fwd emission of $V = W/Z$

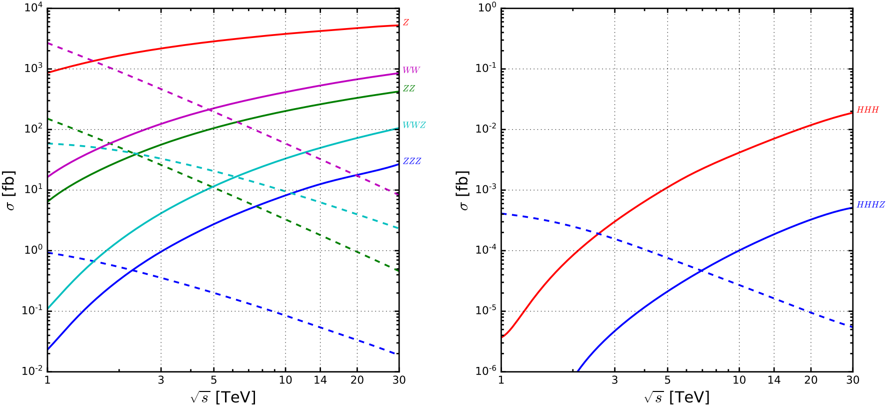
Top production



- Do you notice a pattern?

Many-boson production[2]

[2] My favorite! I find these processes really neat!



When annihilation and VBS channels are driven by same physics, evidence that **dominance of VBS is universal** and occurs at $\sqrt{s}$ for

w/ A. Costantini, et al [2005.10289]

When $(M^2_{W/Z/H}/M^2_{VV}) \to 0$, **qualitatively new behavior emerges**

VBF/S becomes the dominant scattering mechanism

However, in practice, numerical computations of VBF/S are difficult:

- Final states with **many legs and diagrams**
- Larger $\sqrt{s}$ exacerbates **large gauge cancellations**
- onset of **large soft and collinear logarithms**, e.g.,

$$d\mathcal{P}(\mu \to W\nu \text{ splitting}) \propto \underbrace{\frac{g_W^2}{(4\pi)^2}}_{\approx 2.5\cdot 10^{-3}} \times \underbrace{\log^2(M^2_{WW}/M^2_W)}_{\approx 350-500 \text{ for } M_{WW}=1-5 \text{ TeV}} \approx 0.9 - 1.3$$

per leg

Historically, **one approach** to studying the EW theory at high energies is to treat it like massless QCD

- Electroweak PDFs (← rich literature!)
- + EW DGLAP evolution
- Electroweak parton showers
- Electroweak Sudakov resummation (← just super cool !)
- ...

Historically, success of **approach** unclear since computations are difficult to produce and prescriptions varied

This is not necessarily the case today due to new technology

The Effective W Approximation (EWA)[3]

(Electroweak Parton Distribution Functions)

[3] Dawson('84); Kane, et al ('84); Kunszt and Soper ('88)

Idea: Treat EW bosons in (VV')-scattering as partons when $M_{VV'} \gg M_W, M_Z$,

- PDFs for V_T identical to gluons in QCD
- PDFs for V_0 is "novel" complication ("scalar" PDFs)
- To derive PDFs, one expands matrix elements in powers of

$$\mathcal{O}\left(\frac{p_T^2}{M_{VV}^2}\right) \text{ for } V_T \quad \text{or} \quad \mathcal{O}\left(\frac{p_T^2}{M_{VV}^2}\right) \text{ and } \mathcal{O}\left(\frac{M_V^2}{M_{VV}^2}\right) \text{ for } V_0$$

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Result: one can write a type of factorization theorem

$$\sigma(\mu^+\mu^- \to \mathcal{F} + X) = \sum_{V_{\lambda_A}, V'_{\lambda_B}} \int_{\tau_0}^1 d\xi_1 \int_{\tau_0/\xi_1}^1 d\xi_2 \int dPS_{\mathcal{F}} \times$$

$$\left[\underbrace{f_{V_{\lambda_A}/\mu^+}(\xi_1, \mu_f) f_{V'_{\lambda_B/\mu^-}}(\xi_2, \mu_f) + f_{V'_{\lambda_B}/\mu^+}(\xi_1, \mu_f) f_{V_{\lambda_A/\mu^-}}(\xi_2, \mu_f)}_{W^+_\lambda / W^-_\lambda / Z_\lambda / \gamma_\lambda \ \text{PDFs}} \right] \times$$

$$\underbrace{\frac{d\hat{\sigma}(V_{\lambda_A} V'_{\lambda_B} \to \mathcal{F})}{dPS_n}}_{\text{"hard scattering"}} + \underbrace{\mathcal{O}\left(\frac{M_{V_k}^2}{M_{VV'}^2}\right) + \mathcal{O}\left(\frac{p_{T,V_k}^2}{M_{VV'}^2}\right)}_{\text{power corrections}}$$

EWA / EVA in MadGraph

- **NEW: (Polarized) Effective Vector Boson Approx. (EVA)**
  - Helicity-polarized $W_\lambda, Z_\lambda, \gamma_\lambda$ PDFs at leading twist
  - Unpolarized W/Z PDFs at leading twist **(EWA)**
- **OLD: Improved Weizsäcker-Williams approximation (iWWA)**
  - Unpolairzed γ PDF + power corrections for $\ell^\pm$ (Frixione, et al [hep-ph/9310350])
- Support for arbitrary μ_f, μ_f variation, evo. by p_T and q

https://code.launchpad.net/~maddevelopers/mg5amcnlo/2.9.x_eva_v2

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Implementing all this was a multi-step process:

- **Automation** of matrix elements and cross sections for external partons with fixed helicity polarizations

  D. Buarque Franzosi, O. Mattelaer, RR, S. Shil [1912.01725]

  - Essentially enable $A_\lambda + B_{\lambda_B} \to C_{\lambda_C} + D_{\lambda_D} + \ldots$ (λ_k=helicity)
  - Theoretically easy (after reorganizing Collinear Fact. Thm and defining polarized PDF/parton shower)
  - Dev. tricky since Lorentz invariance is lost (a ref. frame must be specified)
- **Improvement** of dPS integration routine (`sde2`) for t-channel mom.

  K. Ostrolenk and O. Mattelaer [2102.00773]

some results on $V_\lambda V'_{\lambda'} \to X$ in $\mu^+\mu^-$ collisions

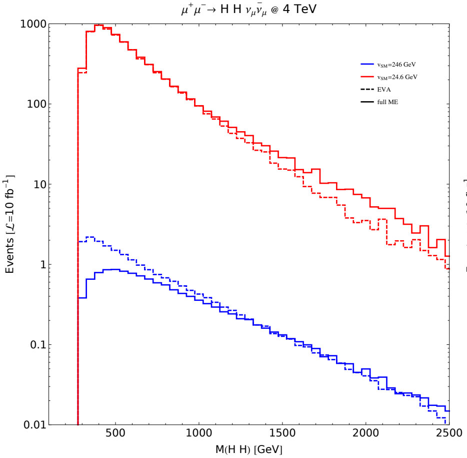
PRELIMINARY[4]

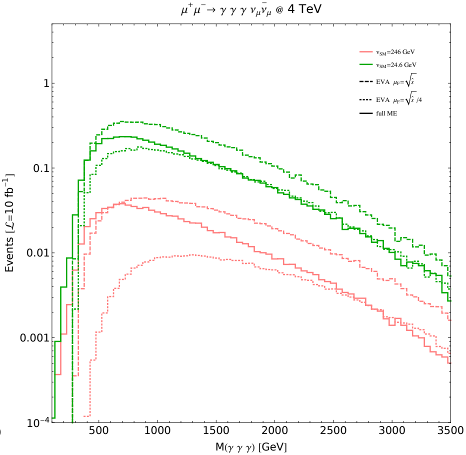
[4] w/ A. Costantini, F. Maltoni, L. Mantani, O. Mattelaer [2108.?????]

the money plot #1

Plot: M_{WW} for (L) $W_0 W_0 \to HH$ (R) $W_T W_T \to \gamma\gamma\gamma$

solid (dashed) = full ME (EVA); lower (upper)= $\sqrt{2}\langle\Phi\rangle = v_{\rm EW}\left(\frac{v_{\rm EW}}{10}\right)$





- EVA works within uncertainties when $(M_V/M_{VV}) \ll 1$. Fact of life: M_V is large $\Longrightarrow$ M_{VV} must be larger!

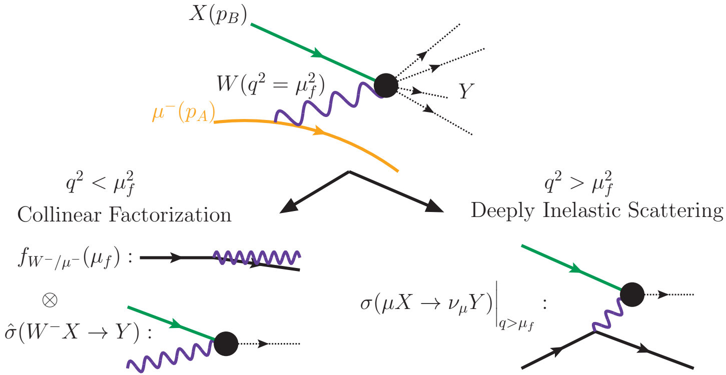
Consistent with Georgi& Politzer ('74)!

matching with EW pdfs and matrix elements

Matching EW PDFs to matrix elements

Idea: Total cross section can be (σ^{total}) into "collinear" ($\sigma^{\text{collinear}}$) and "wide-angle" ($\sigma^{\text{wide-angle}}$) bits. Summing the two *should* recover σ^{total}

$$\sigma^{\text{total}} = \sigma^{\text{collinear}} + \sigma^{\text{wide-angle}} + \mathcal{O}\left(\frac{M_W^2}{M_{WW}^2}\right) + \mathcal{O}\left(\frac{p_T^{\nu 2}}{M_{WW}^2}\right)$$



Consider $\mu^- \to W^-\nu_\mu$ splitting in W^+W^- scattering

$$\sigma^{\text{total}} = \underbrace{\int_0^{\mu_f} dp_T^\nu \frac{d\sigma}{dp_T^\nu}}_{\text{collinear}} + \underbrace{\int_{\mu_f}^{\Lambda} dp_T^\nu \frac{d\sigma}{dp_T^\nu}}_{\text{quasi-collinear}} + \underbrace{\int_{\Lambda}^{M_{WW}} dp_T^\nu \frac{d\sigma}{dp_T^\nu}}_{\text{hard}}$$

The collinear term contains the W PDF:

$$\sigma^{\text{collinear}} = \int_0^{\mu_f} dp_T^\nu \frac{d\sigma}{dp_T^\nu} \sim \underbrace{\log\left(\frac{\mu_f^2}{M_W^2}\right)}_{\text{PDF}} + \underbrace{\text{power corrections (PC1)}}_{\text{neglect}}$$

The quasi-collinear term also depends on μ_f:

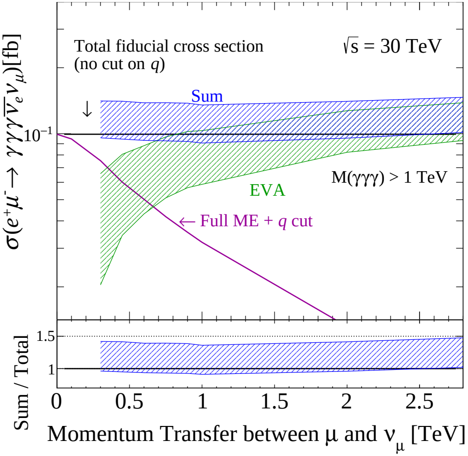
$$\sigma^{\text{quasi-collinear}} = \int_{\mu_f}^{\Lambda} dp_T^\nu \frac{d\sigma}{dp_T^\nu} \sim \underbrace{\log\left(\frac{\Lambda^2}{\mu_f^2}\right)}_{\text{same log as in PDF}} + \underbrace{\text{PC2}}_{\text{keep}}$$

The full matrix element without collinear term (the wide-angle term!) is

$$\sigma^{\text{wide-angle}} \sim \log\left(\frac{\Lambda^2}{\mu_f^2}\right) + \text{PC2} + \sigma^{\text{Hard}}$$

the money plot #2

Plot: $\sigma(e^+\mu^- \to \gamma\gamma\gamma\overline{\nu_e}\nu_\mu)$ vs factorization scale / cutoff (μ_f)



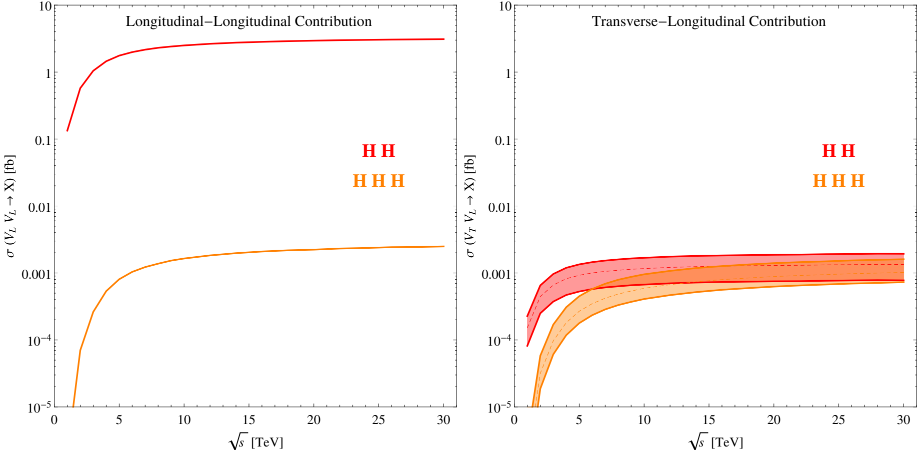
Take away: PDFs run high for large μ_f since $\mathcal{O}(p_T^\nu/M_{WW}^2)$ are too large! (breakdown of coll. limit)

- okay since EW-DGLAP will pull down

See Han, Ma, and Xi [2007.14300; 2103.09844]

Higgs production in EVA

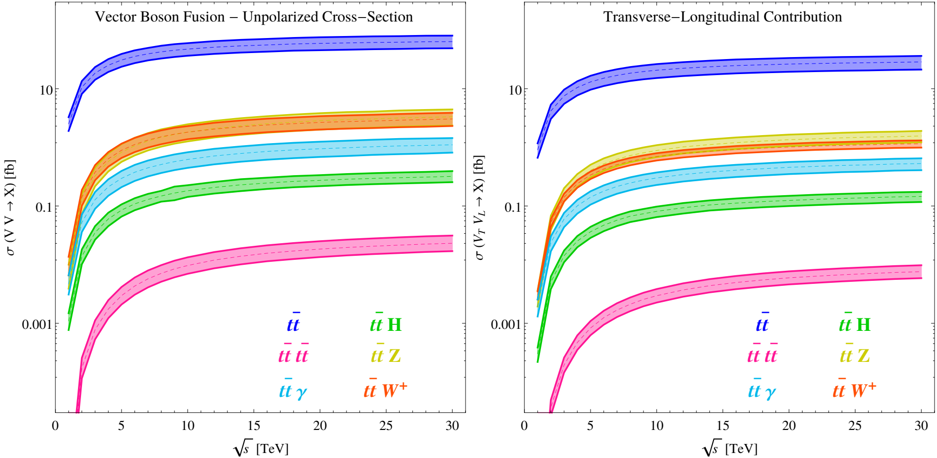
$V_\lambda V'_\lambda \to nH$: (L) $V_0 V_0$ (R) $V_0 V_T + V_T V_0$



- Importance of $V_\lambda V'_\lambda$ depends on number of H
- At $\mathcal{L} = 1\ \mathrm{ab}^{-1}/yr \implies 10^3\ HH/yr$ (incredibly rich physics!)

Tops in EVA

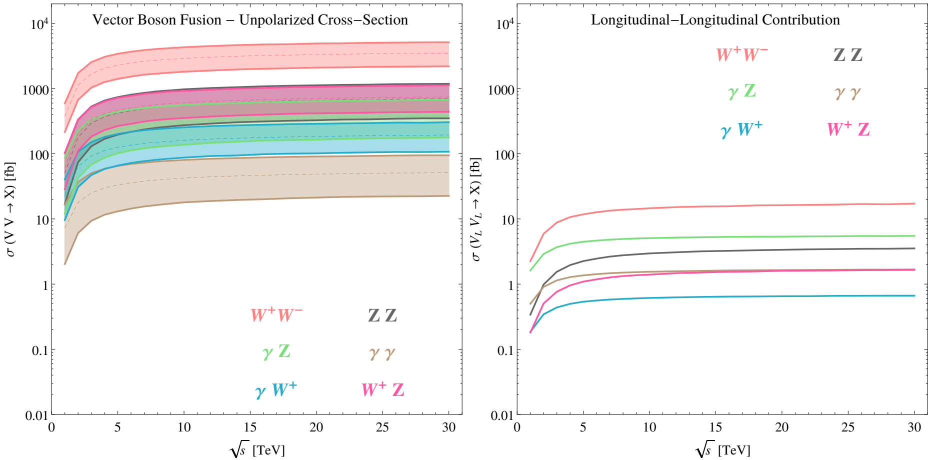
$V_\lambda V'_\lambda \to t\bar{t} + X$: (L) all polarizations (R) $V_T V_0 + V_0 V_T$



- Nontrivial contribution from all $V_\lambda V'_\lambda$ combinations
- Clear hierarchy of weak and EM couplings

Diboson in EVA

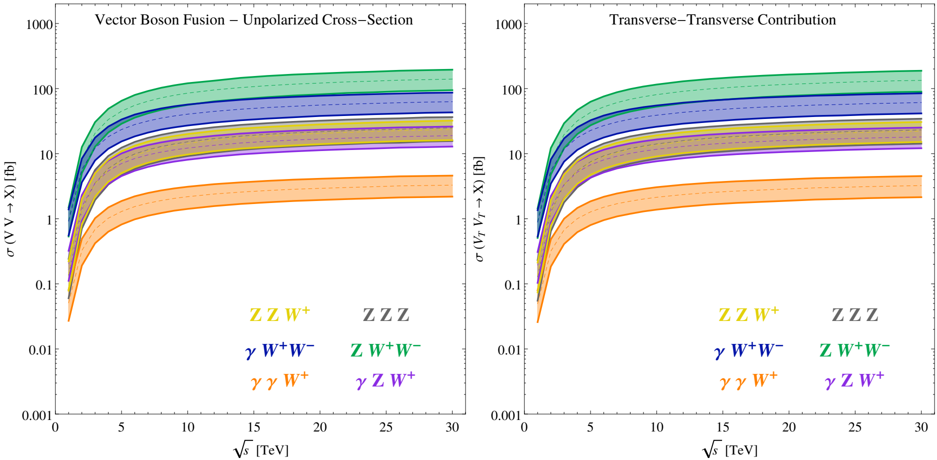
$V_\lambda V'_\lambda \to VV'$: (L) all polarizations (R) $V_0 V_0$



- Only minor role played by $V_0 V'_0$ scattering
- At $\mathcal{L} = 1\ \mathrm{ab}^{-1}/yr \implies 10^6\ WW/yr$ driven by non-Abelian couplings

Triboson in EVA

$V_\lambda V'_\lambda \to VV'V''$:   (L) all polarizations   (R) $V_T V_T$



- Major role played by $V_T V'_T$ scattering
- At $\mathcal{L} = 1~\mathrm{ab}^{-1}/yr \implies 10^3~VV'V''/yr$

When $M^2_{W/Z/H}/Q^2 \to 0$, qualitatively new behavior emerges

Bluntly, a $\mathcal{O}(10)$ TeV $\mu^+\mu^-$ collider behaves more like
a 14 TeV proton collider (LHC/HL-LHC) than
a 240 GeV e^+e^- collider (FCC-ee/ILC)

Take-away: EWA/EVA works (EW theory is a gauge theory!) but historical disagreements can be tied to size of power corrections

Outlook: EWA/EVA in MadGraph will be released in coming weeks and plans underway to merge parallel Snowmass efforts

see Snowmass 21 LoI: SNOWMASS21-TF7_TF0-EF4_EF0-026

Thank you!

$W_\lambda/Z_\lambda/\gamma_\lambda$ PDFs depend on helicity (λ) of V and $\mu^\pm$

- Subtle but important differences if evolving by q vs p_T

(this can account for some differences between groups!)

$$f_{V_+/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2(1-z)^2}{2z}\log\left[\frac{\mu_f^2}{M_V^2}\right],$$

$$f_{V_-/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2}{2z}\log\left[\frac{\mu_f^2}{M_V^2}\right],$$

$$f_{V_0/f_L}(z,\mu_f^2) = \frac{g_V^2}{4\pi^2}\frac{g_L^2(1-z)}{z},$$

$$f_{V_+/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_-/f_L}(z,\mu_f^2)$$

$$f_{V_-/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_+/f_L}(z,\mu_f^2)$$

$$f_{V_0/f_R}(z,\mu_f^2) = \left(\frac{g_R}{g_L}\right)^2 \times f_{V_0/f_L}(z,\mu_f^2)$$

```
c     /* ********************************************************* *
c     EVA (1/6) for f_L > v_+
      double precision function eva_fL_to_vp(gg2,gL2,mv2,x,mu2,ievo)
      implicit none
      integer ievo             ! evolution by q2 or pT2
      double precision gg2,gL2,mv2,x,mu2
      double precision coup2,split,xxlog,fourPiSq
      data fourPiSq/39.47841760435743d0/ ! = 4pi**2

c      print*,'gg2,gL2,mv2,x,mu2,ievo',gg2 !3,gL2,mv2,x,mu2,ievo
      coup2 = gg2*gL2/fourPiSq
      split = (1.d0-x)**2 / 2.d0 / x
      if(ievo.eq.0) then
         xxlog = dlog(mu2/mv2)
      else
         xxlog = dlog(mu2/mv2/(1.d0-x))
      endif

      eva_fL_to_vp = coup2*split*xxlog
      return
      end
c     /* ********************************************************* *
c     EVA (2/6) for f_L > v_-
      double precision function eva_fL_to_vm(gg2,gL2,mv2,x,mu2,ievo)
      implicit none
      integer ievo             ! evolution by q2 or pT2
      double precision gg2,gL2,mv2,x,mu2
      double precision coup2,split,xxlog,fourPiSq
      data fourPiSq/39.47841760435743d0/ ! = 4pi**2

      coup2 = gg2*gL2/fourPiSq
      split = 1.d0 / 2.d0 / x
      if(ievo.eq.0) then
         xxlog = dlog(mu2/mv2)
      else
         xxlog = dlog(mu2/mv2/(1.d0-x))
      endif

      eva_fL_to_vm = coup2*split*xxlog
      return
      end
```