

# New Physics from Oscillations at DUNE Near Detector

Jacobo López-Pavón

*Portorož 2021 – Physics from the Flavourful Universe*  
*28 September 2021*

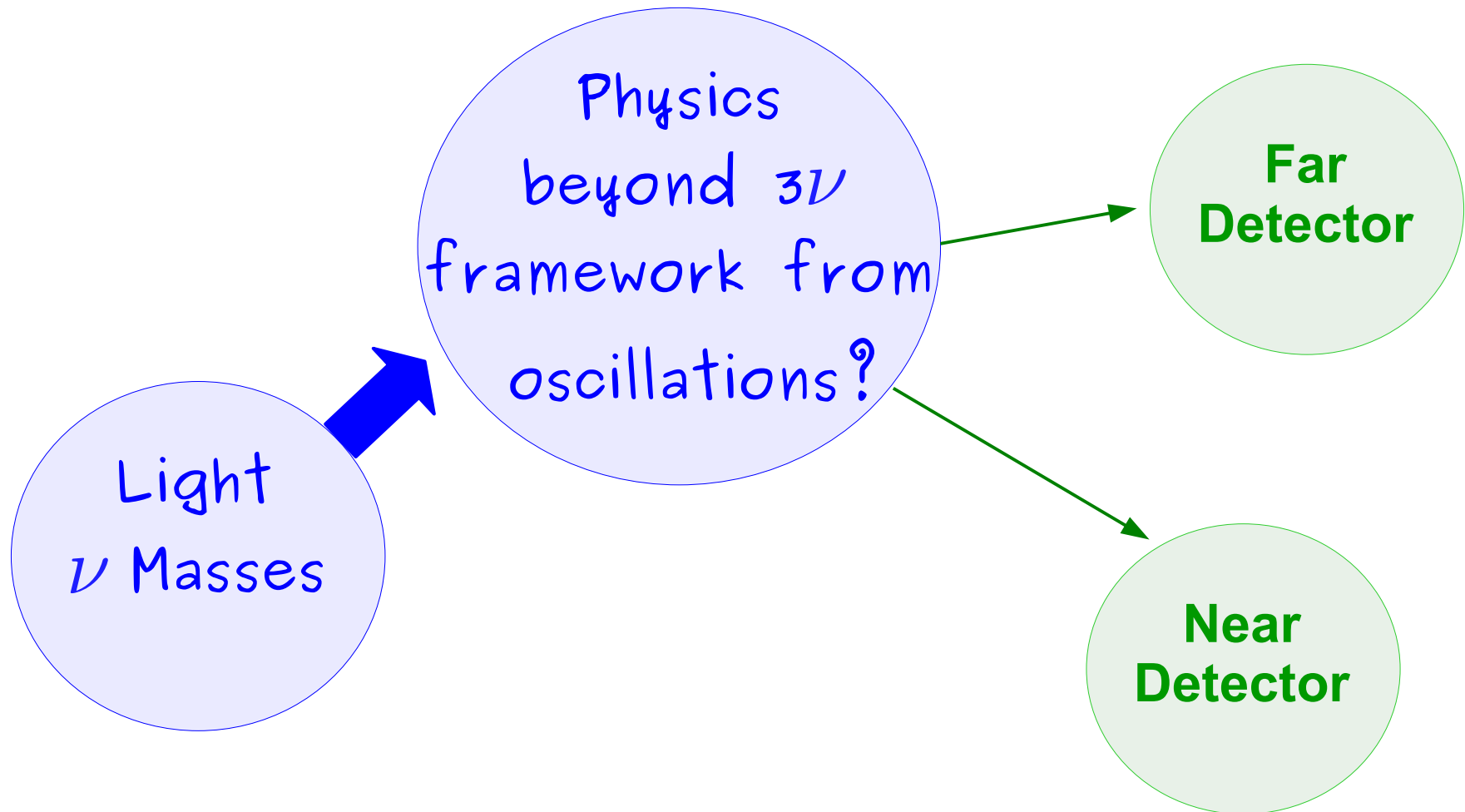


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# Outline



# Minimal model: Seesaw Model

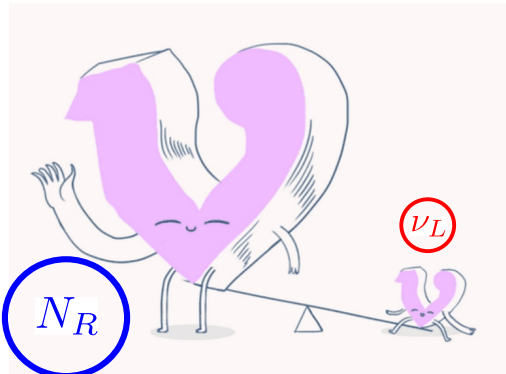
- Simplest extension of SM able to account for neutrino masses. Consists in the addition of **heavy fermion singlets** ( $N_R$ ) to the SM field content:

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{\mathcal{K}} - \frac{1}{2} \overline{N_i^c} M_{ij} N_j - Y_{i\alpha} \overline{N_i} \tilde{H}^\dagger L_\alpha + h.c.$$



Light  
Neutrino  
Masses

$$m_\nu = \frac{v^2}{2} Y^T M^{-1} Y$$



# Minimal model: Seesaw Model

- Simplest extension of SM able to account for neutrino masses. Consists in the addition of **heavy fermion singlets** ( $N_R$ ) to the SM field content:

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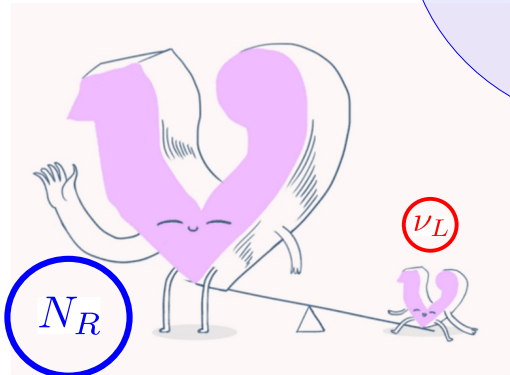


New  
Physics  
Scale

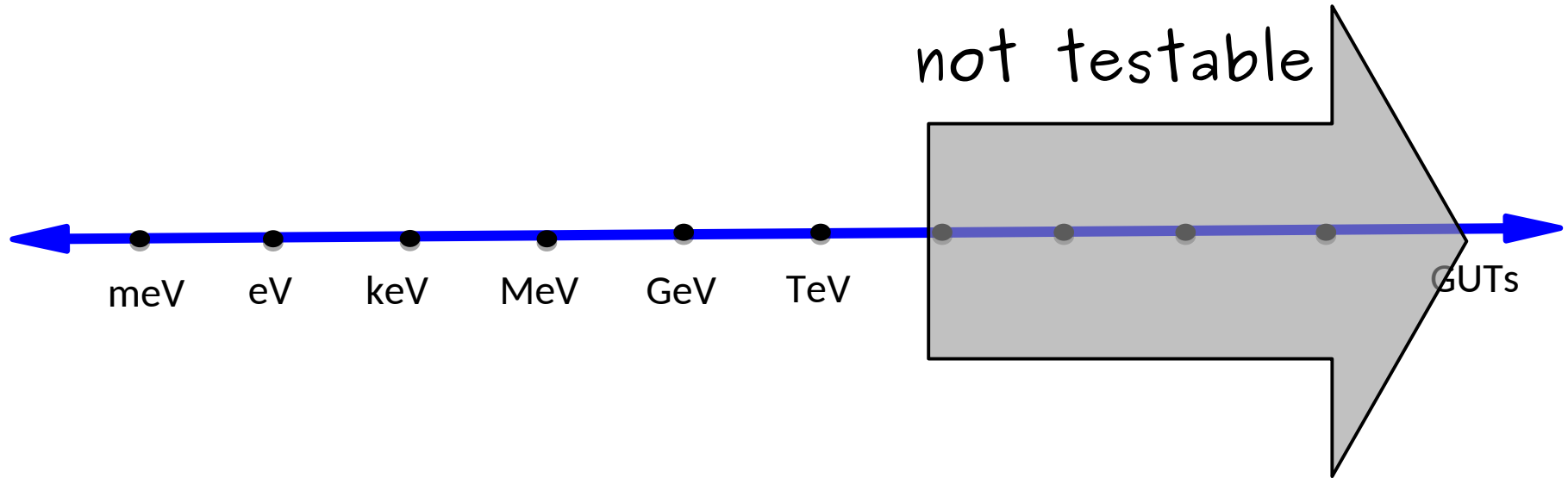
Lepton  
Number  
Violation

$0\nu\beta\beta$   
decay!

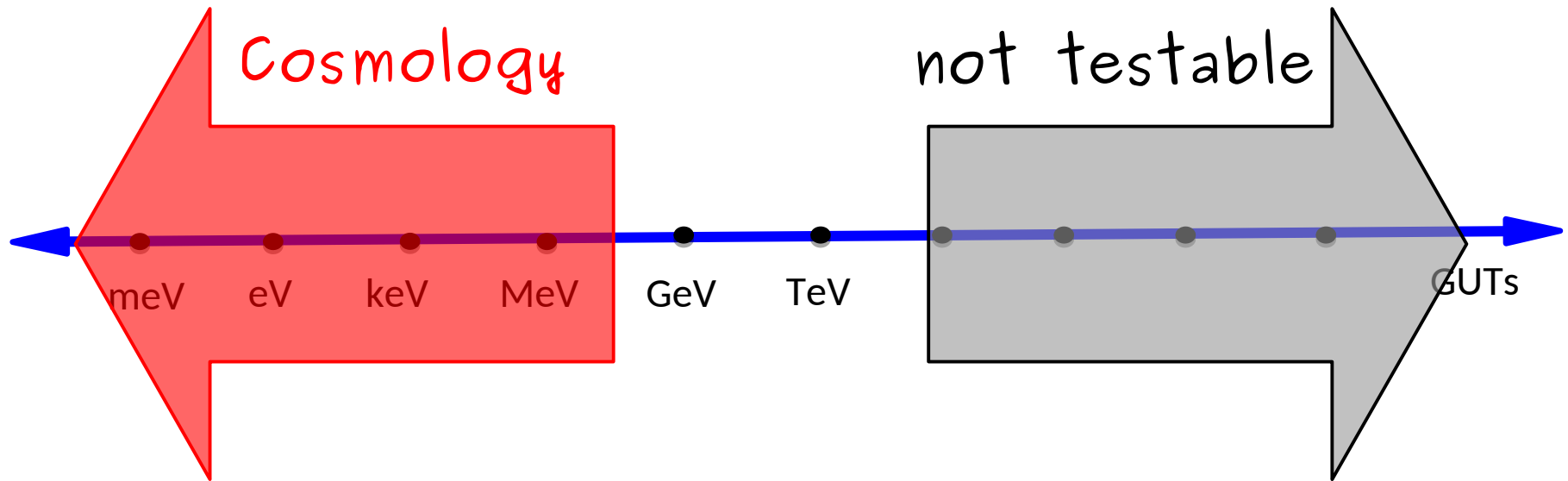
Leptogenesis!



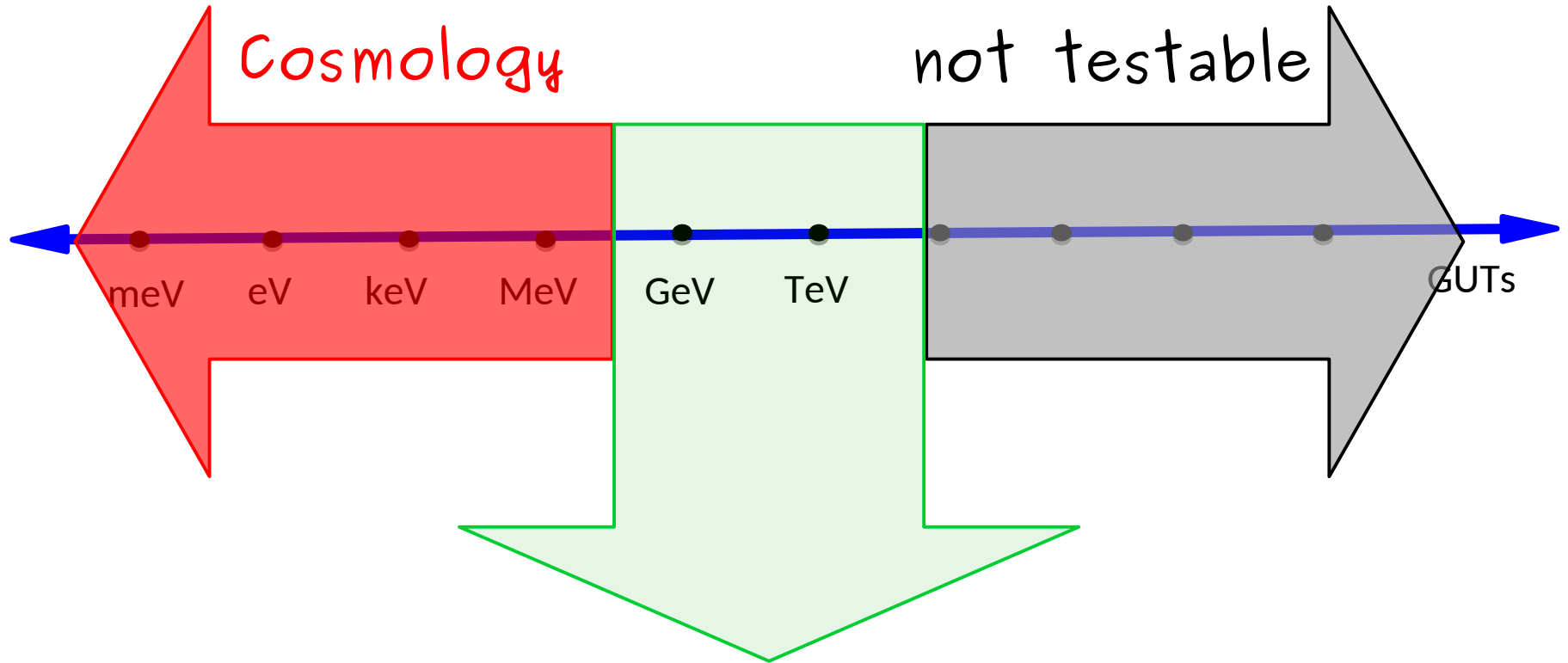
# The New Physics Scale



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# The New Physics Scale



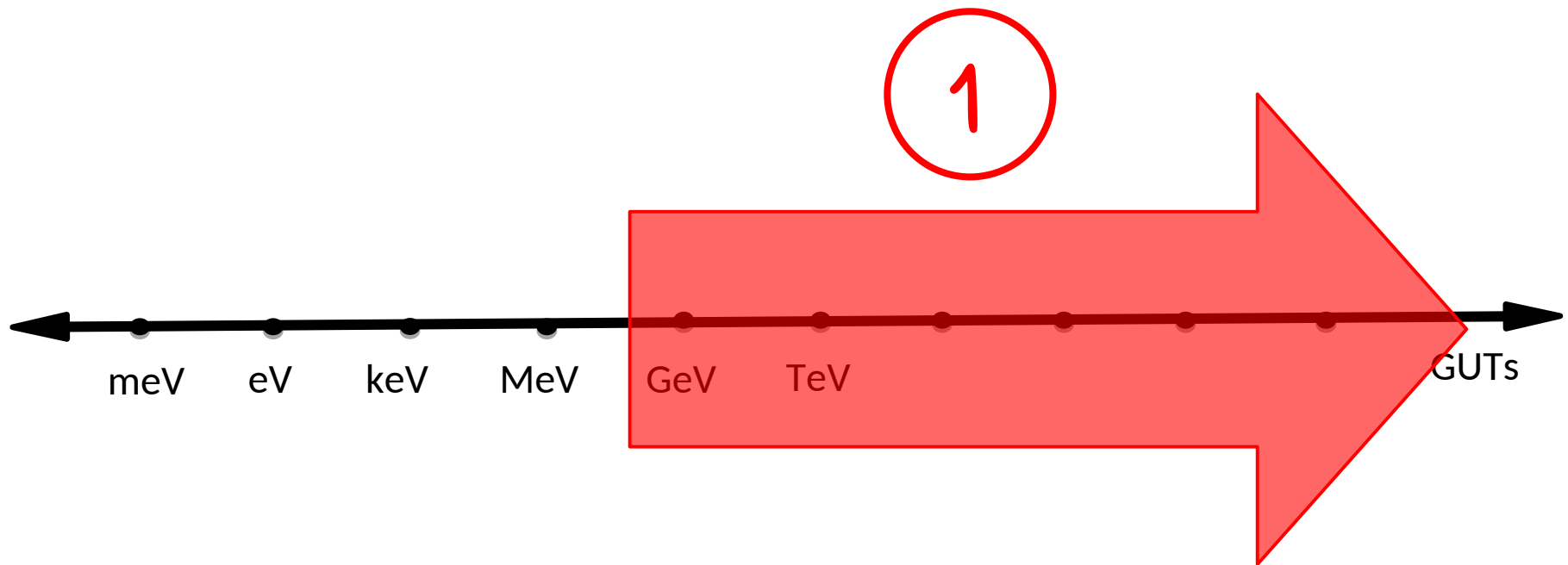
$0\nu\beta\beta$  decay, CLFV, Colliders, direct searches...

Are Long Baseline  
Neutrino Oscillation experiments  
sensitive to  
New Physics  
beyond  $3\nu$  framework



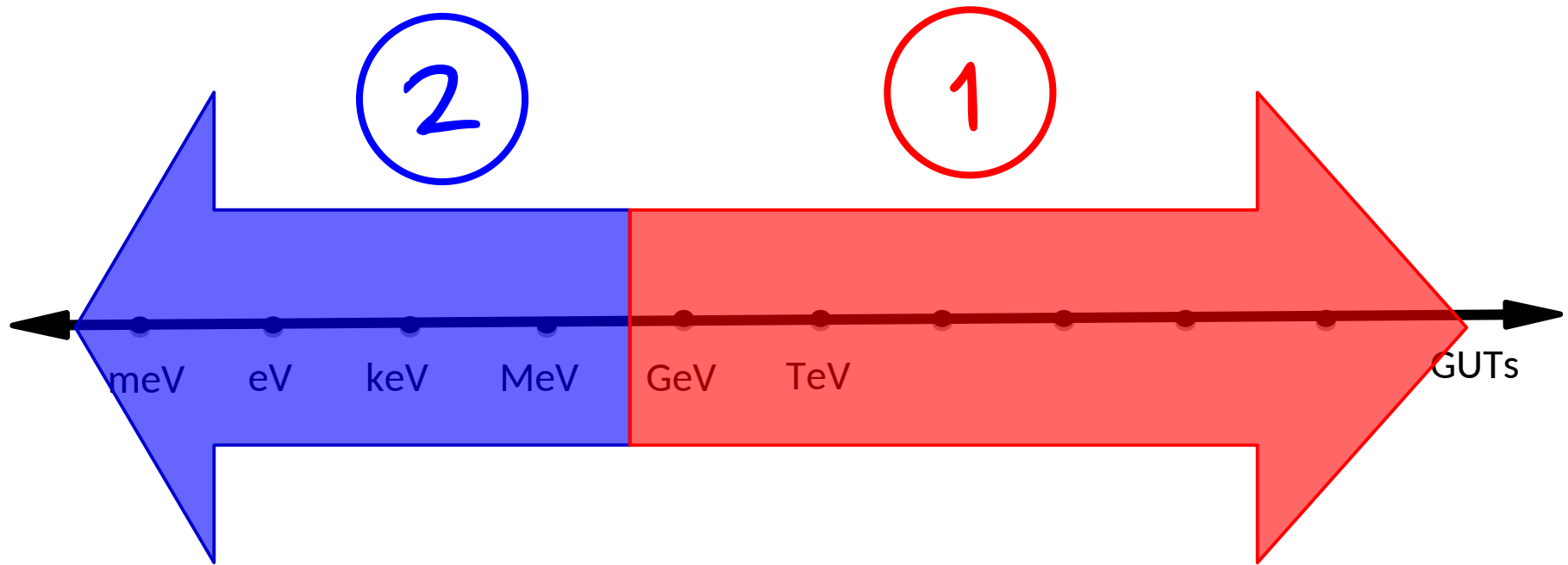


# Neutrino Oscillations vs NP scale



Non-Unitary  
mixing  
(sterile states  
integrated out)

# Neutrino Oscillations vs NP scale



Kinematically  
accessible sterile  
neutrinos

Non-Unitary  
mixing  
(sterile states  
integrated out)

Both limits can be studied  
in a  
unified & model independent way

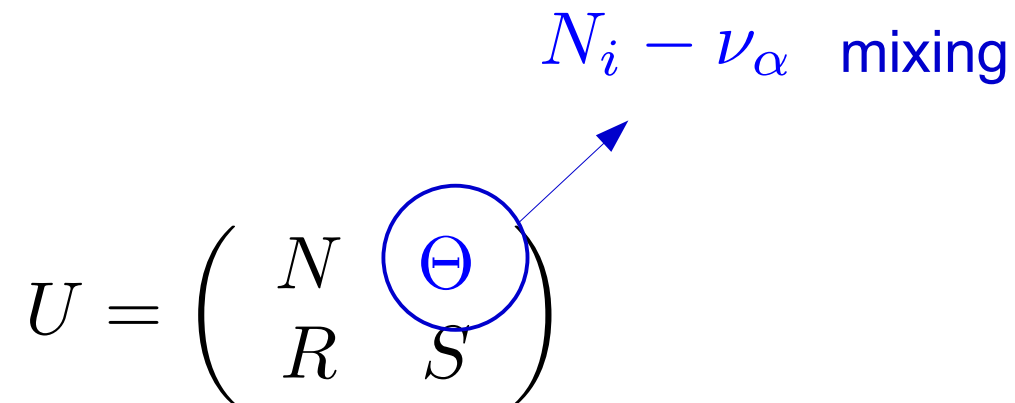
# Model Independent Approach

$$U = \begin{pmatrix} N & \Theta \\ R & S \end{pmatrix}$$

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$$U = \begin{pmatrix} N & \Theta \\ R & S \end{pmatrix}$$

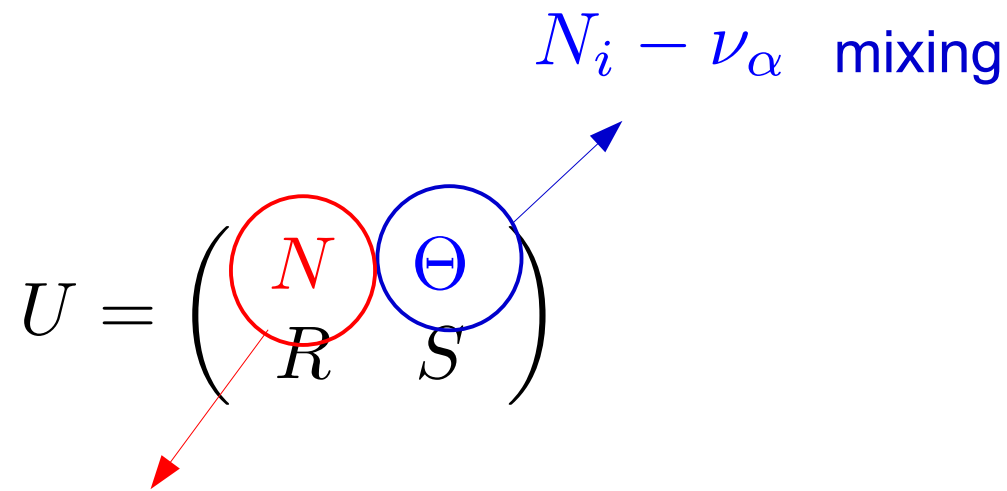
$N_i - \nu_\alpha$  mixing

The diagram shows a 2x2 matrix U with elements N, R, S, and Theta. The element Theta is circled in blue. A blue arrow points from the circled Theta to the text 'Ni - nu\_alpha mixing' located above and to the right of the matrix.

# Model Independent Approach

$$U = \begin{pmatrix} \textcircled{N} & \textcircled{\Theta} \\ R & S \end{pmatrix}$$

$N_i - \nu_\alpha$  mixing



Deviation from unitarity of the PMNS matrix

Langacker, London 1988

Antusch, Biggio, Fernandez-Martinez, Gavela, JLP 2006

# General Parameterizations

- Triangular parameterization

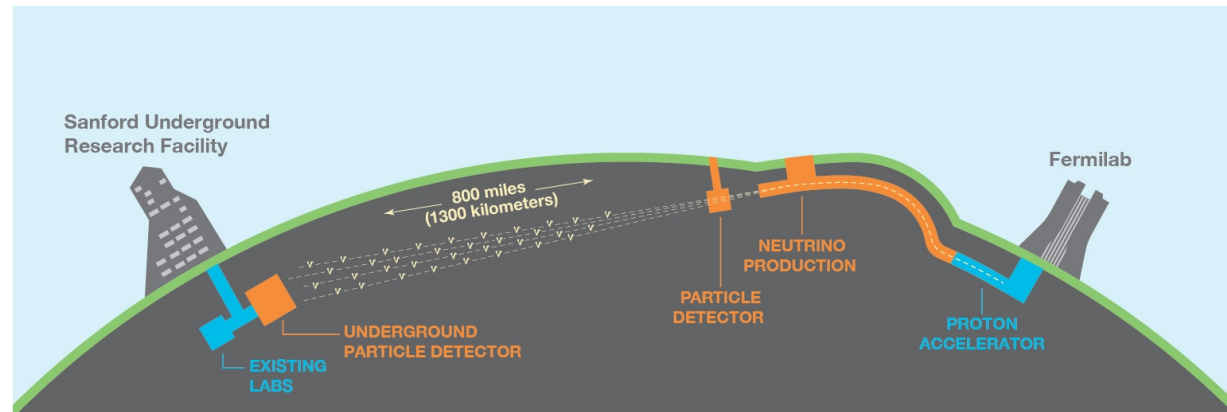
$$N = (I - T)U$$

Deviation from unitarity

$$T = \begin{pmatrix} \alpha_{ee} & 0 & 0 \\ \alpha_{\mu e} & \alpha_{\mu\mu} & 0 \\ \alpha_{\tau e} & \alpha_{\tau\mu} & \alpha_{\tau\tau} \end{pmatrix}$$

Unitary matrix  
(standard unitary PMNS  
matrix  
up to small corrections)

# Far Detector vs Near detector



$$N_{\nu_{\alpha} \rightarrow \nu_{\beta}} \sim \frac{\Phi_{\alpha}(E)}{L^2} P_{\alpha\beta}(L/E) \sigma_{\beta}(E) \epsilon_{\beta}(E)$$

- **Sources of systematics**
  - Cross sections
  - Neutrino flux
- **Near detector measurements reduce far detector systematic uncertainties**
- **New Physics at near detector (strongly affected by systematic uncertainties)**



# Far Detector

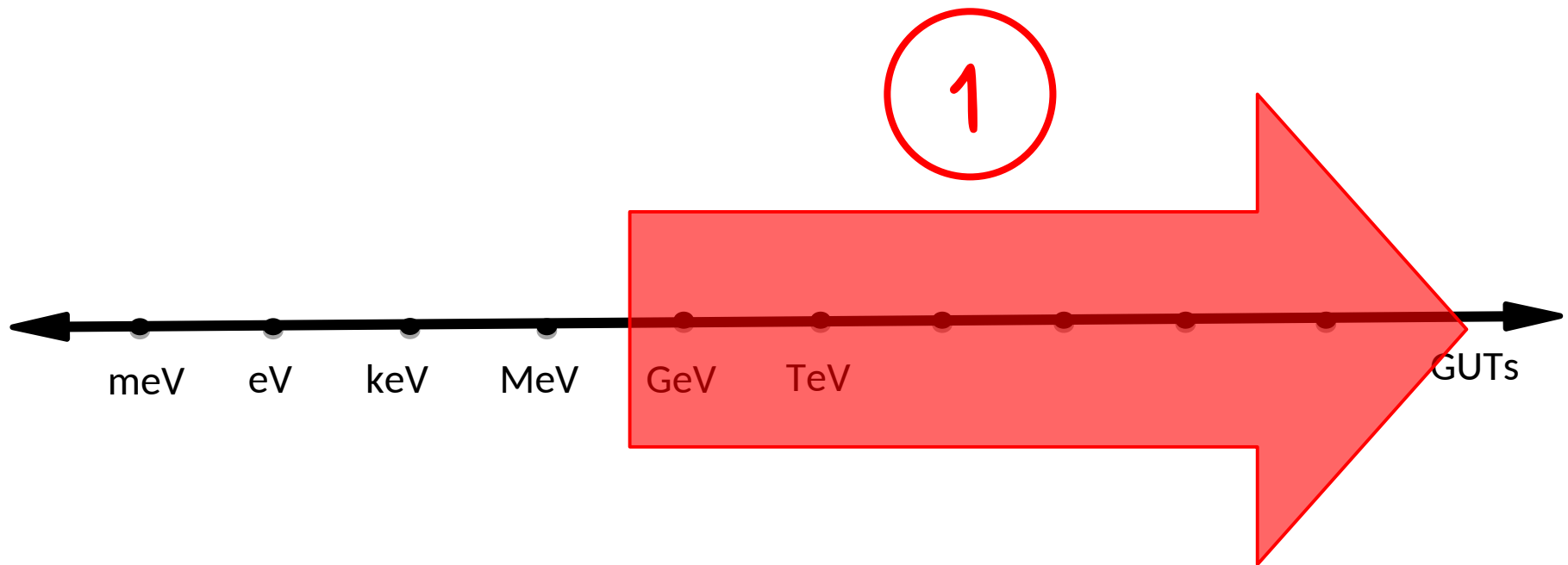
# Far Detector

- What is measured in neutrino oscillation experiments

$$\mathcal{P}_{\alpha\beta} = \frac{R_{\beta}}{R_{\alpha}}$$

The diagram shows the equation  $\mathcal{P}_{\alpha\beta} = \frac{R_{\beta}}{R_{\alpha}}$ . The numerator  $R_{\beta}$  is enclosed in a blue circle, with a blue arrow pointing to the text "Event rate Far Detector". The denominator  $R_{\alpha}$  is enclosed in a green circle, with a green arrow pointing to the text "Extrapolation of Near Detector".

# ① Non-Unitary Mixing



Non-Unitary  
mixing  
(sterile states  
integrated out)

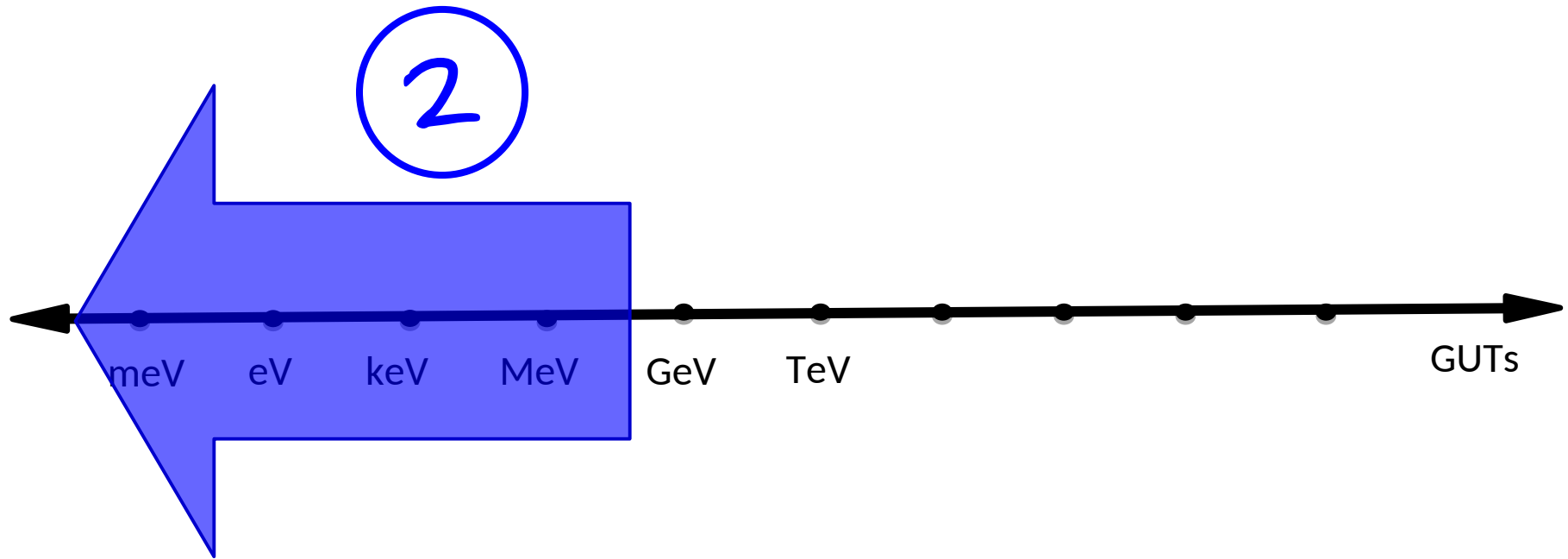
# ① Non-Unitary Mixing

- What is measured in neutrino oscillation experiments

$$\mathcal{P}_{\alpha\beta} = \frac{|(N \exp(-iHL)N^\dagger)_{\beta\alpha}|^2}{[(NN^\dagger)_{\alpha\alpha}]^2}.$$

- When  $NN^\dagger = I \implies \mathcal{P}_{\alpha\beta} = P_{\alpha\beta}$  (SM limit recovered)

## ② Kinematically accessible sterile $\nu$



Kinematically  
accessible sterile  
neutrinos

## ② Kinematically accessible sterile $\nu$

1. The light-heavy oscillations averaged out at the near detector.  
Identical to the heavy non-unitarity case

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1. The light-heavy oscillations averaged out at the near detector.

Identical to the heavy non-unitarity case

2. The light-heavy oscillations have not yet developed at the near detector.

No normalization factor

$$\text{DUNE: } 0.1 \text{ eV}^2 \lesssim \Delta m^2 \lesssim 1 \text{ eV}^2$$

## ② Kinematically accessible sterile $\nu$

1. The light-heavy oscillations averaged out at the near detector.

Identical to the heavy non-unitarity case

2. The light-heavy oscillations have not yet developed at the near detector.

No normalization factor

3. The oscillation frequency dictated by the light-heavy frequency matches the near detector distance.

Oscillations could be observed at the near detector



## ② Kinematically accessible sterile $\nu$

1. The light-heavy oscillations averaged out at the near detector.

Identical to the heavy non-unitarity case

2. The light-heavy oscillations have not yet developed at the near detector.

No normalization factor

Low Scale  
Non-Unitarity

# Present Bounds

|                      | High-scale Non-Unitarity<br>( $m > \text{EW}$ ) |   |
|----------------------|---|---|
| $\alpha_{ee}$        | $1.3 \cdot 10^{-3}$                             | <p>EW &amp; CLFV<br/>precision<br/>data</p> |
| $\alpha_{\mu\mu}$    | $2.2 \cdot 10^{-4}$                             |   |
| $\alpha_{\tau\tau}$  | $2.8 \cdot 10^{-3}$                             |   |
| $ \alpha_{\mu e} $   | $6.8 \cdot 10^{-4}$ ( $2.4 \cdot 10^{-5}$ )     |   |
| $ \alpha_{\tau e} $  | $2.7 \cdot 10^{-3}$                             |   |
| $ \alpha_{\tau\mu} $ | $1.2 \cdot 10^{-3}$                             |   |

Fernandez-Martinez, Hernandez-Garcia, JLP  
 1605.08774  
 Blenow, Coloma, Fernandez-Martinez,  
 Hernandez-Garcia, JLP  
 1609.08637

# Present Bounds

|                      | High-scale Non-Unitarity<br>( $m > \text{EW}$ ) | Low-scale Non-Unitarity<br>$\Delta m^2 \gtrsim 100 \text{ eV}^2$ $\Delta m^2 \sim 0.1 - 1 \text{ eV}^2$ |                                  |
|----------------------|---|---|----------------------------------|
| $\alpha_{ee}$        | $1.3 \cdot 10^{-3}$                             | $2.4 \cdot 10^{-2}$ <b>BUGEY</b>  | $1.0 \cdot 10^{-2}$ <b>BUGEY</b> |
| $\alpha_{\mu\mu}$    | $2.2 \cdot 10^{-4}$                             | $2.2 \cdot 10^{-2}$ <b>SK</b>   | $1.4 \cdot 10^{-2}$ <b>MINOS</b> |
| $\alpha_{\tau\tau}$  | $2.8 \cdot 10^{-3}$                             | $1.0 \cdot 10^{-1}$ <b>SK</b>   | $1.0 \cdot 10^{-1}$ <b>SK</b>    |
| $ \alpha_{\mu e} $   | $6.8 \cdot 10^{-4}$ ( $2.4 \cdot 10^{-5}$ )     | $2.5 \cdot 10^{-2}$ <b>NOMAD</b>  | $1.7 \cdot 10^{-2}$              |
| $ \alpha_{\tau e} $  | $2.7 \cdot 10^{-3}$                             | $6.9 \cdot 10^{-2}$   | $4.5 \cdot 10^{-2}$              |
| $ \alpha_{\tau\mu} $ | $1.2 \cdot 10^{-3}$                             | $1.2 \cdot 10^{-2}$ <b>NOMAD</b>  | $5.3 \cdot 10^{-2}$              |

Fernandez-Martinez, Hernandez-Garcia, JLP  
 1605.08774  
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 Hernandez-Garcia, JLP  
 1609.08637

$$\alpha_{\alpha\beta} \leq 2\sqrt{\alpha_{\alpha\alpha}\alpha_{\beta\beta}}$$

# Present Bounds

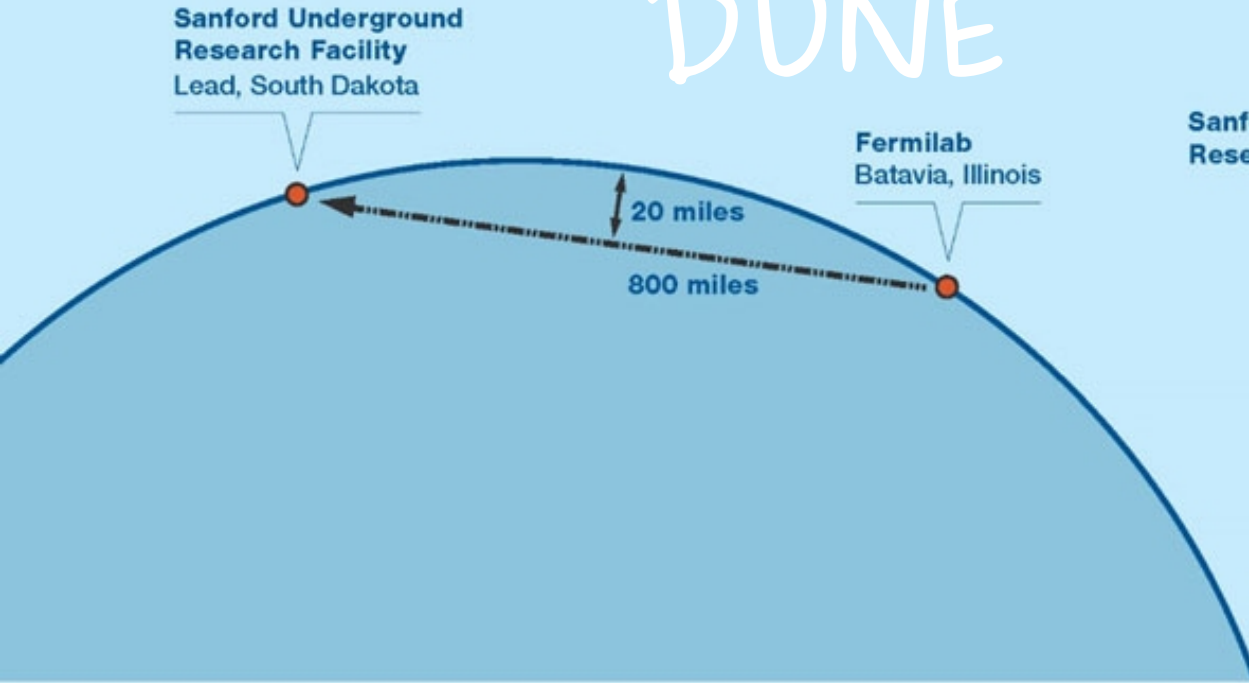
|                      | High-scale Non-Unitarity<br>( $m > \text{EW}$ ) | Low-scale Non-Unitarity<br>$\Delta m^2 \gtrsim 100 \text{ eV}^2$ $\Delta m^2 \sim 0.1 - 1 \text{ eV}^2$ |                     |
|----------------------|---|---|---------------------|
| $\alpha_{ee}$        | $1.3 \cdot 10^{-3}$                             | $2.4 \cdot 10^{-2}$   | $1.0 \cdot 10^{-2}$ |
| $\alpha_{\mu\mu}$    | <b>0.1-0.001%<br/>level</b>                     | $2.2 \cdot 10^{-2}$   | <b>10-1%</b>        |
| $\alpha_{\tau\tau}$  |   | $1.0 \cdot 10^{-1}$   | <b>level</b>        |
| $ \alpha_{\mu e} $   |   | $2.5 \cdot 10^{-2}$   | $0^{-2}$            |
| $ \alpha_{\tau e} $  | $2.7 \cdot 10^{-3}$                             | $6.9 \cdot 10^{-2}$   | $4.5 \cdot 10^{-2}$ |
| $ \alpha_{\tau\mu} $ | $1.2 \cdot 10^{-3}$                             | $1.2 \cdot 10^{-2}$   | $5.3 \cdot 10^{-2}$ |

Fernandez-Martinez, Hernandez-Garcia, JLP  
1605.08774  
Blennow, Coloma, Fernandez-Martinez,  
Hernandez-Garcia, JLP  
1609.08637

See also  
Park, Ross-Lonergan 1508.05095  
Ellis, Kelly, Weishi Li 2004.13719  
Ellis, Kelly, Weishi Li 2008.01088

# Deep Underground Neutrino Experiment

# DUNE



Sanford Underground Research Facility

Fermilab



UNDERGROUND PARTICLE DETECTOR

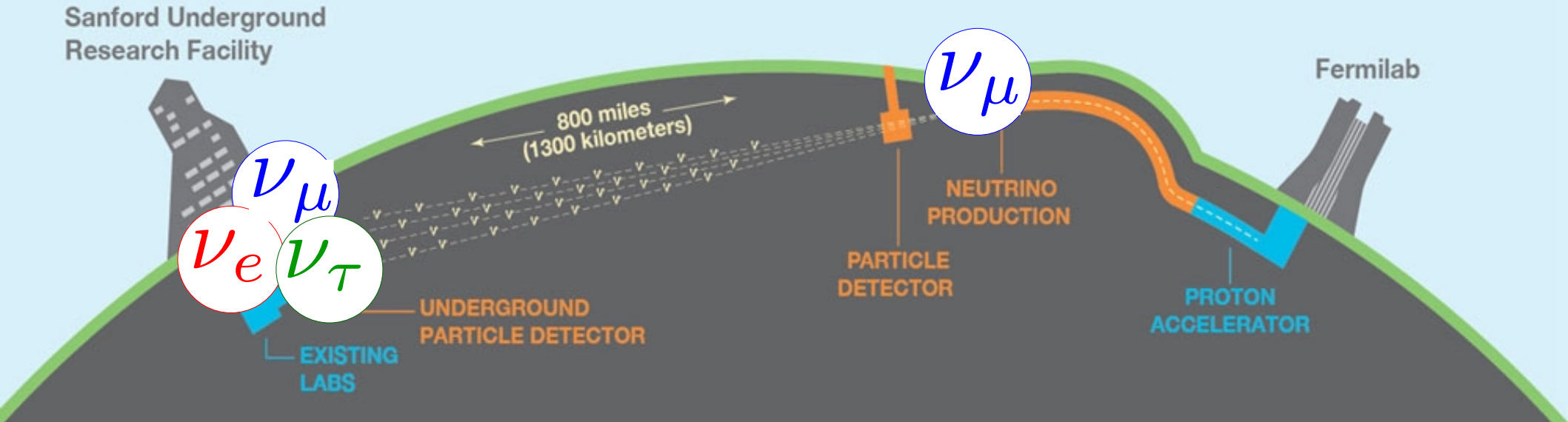
PARTICLE DETECTOR

NEUTRINO PRODUCTION

PROTON ACCELERATOR

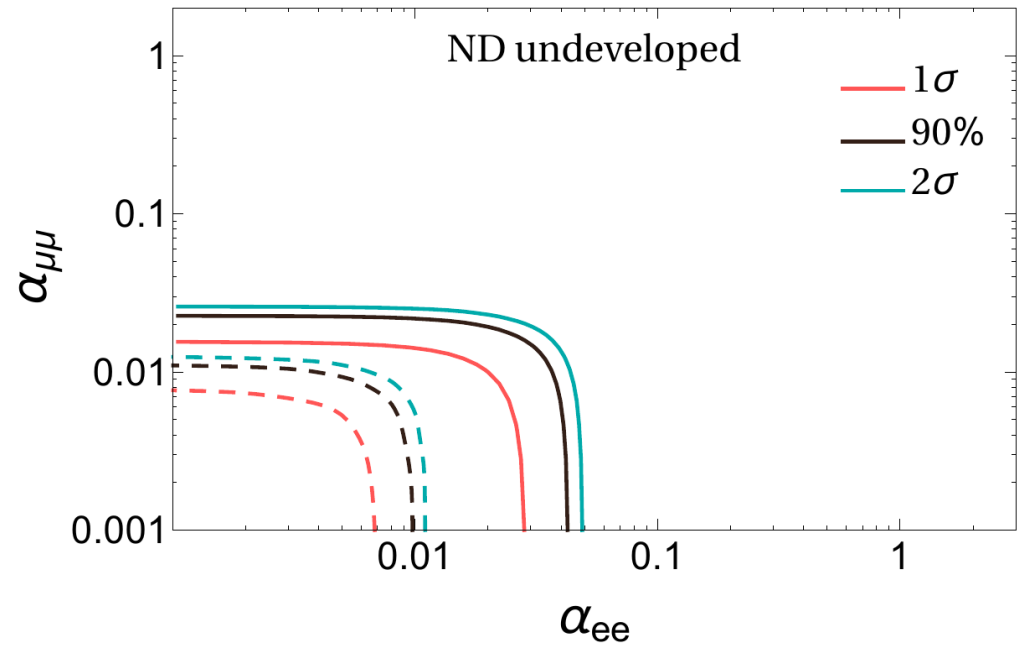
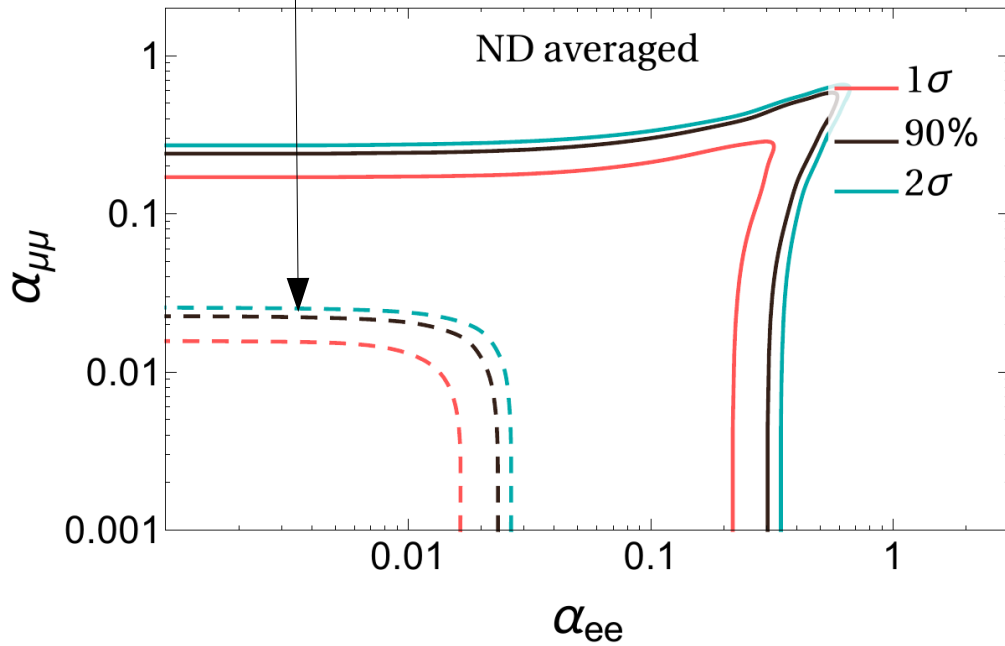
EXISTING LABS

800 miles  
(1300 kilometers)



# Far Detector

Prior  
(present bounds)



$$\mathcal{P}_{\alpha\beta} = \frac{|(N \exp(-iHL)N^\dagger)_{\beta\alpha}|^2}{[(NN^\dagger)_{\alpha\alpha}]^2}$$

$$\mathcal{P}_{\alpha\beta} = |(N \exp(-iHL)N^\dagger)_{\beta\alpha}|^2$$

# Near Detector

Coloma, JLP, Rosauero-Alcaraz, **Urrea** 2105.11466.

See also Escrihuela, Forero, Miranda, Tortola, Valle arXiv:1503.08879 for other Near Detector configurations (without including tau detection).

# High Scale Non-Unitarity

- What is measured in Near Detector

$$\mathcal{P}_{\alpha\beta} = |(NN^\dagger)_{\beta\alpha}|^2 = |\alpha_{\alpha\beta}|^2$$

zero  
distance  
effect!

$$\mathcal{P}_{\alpha\alpha} = |(NN^\dagger)_{\alpha\alpha}|^2 = 1 - 4\alpha_{\alpha\alpha}$$



# sterile Neutrinos: 3+1

- What is measured in Near Detector

$$\mathcal{P}_{\alpha\beta} = 4|U_{\alpha 4}||U_{\beta 4}| \sin^2 \frac{\Delta m_{41}^2 L}{4E}$$

$$\mathcal{P}_{\alpha\alpha} = 1 - 4|U_{\alpha 4}|^2 \sin^2 \frac{\Delta m_{41}^2 L}{4E}$$

# Averaged-out regime

- What is measured in Near Detector

$$\Delta m_{41}^2 \gtrsim 100 \text{ eV}^2$$

$$\langle \mathcal{P}_{\alpha\beta} \rangle = 2|U_{\alpha 4}| |U_{\beta 4}|$$

zero  
distance  
effect!

$$\langle \mathcal{P}_{\alpha\alpha} \rangle = 1 - 2|U_{\alpha 4}|^2$$

# Averaged-out regime

- What is measured in Near Detector

$$\Delta m_{41}^2 \gtrsim 100 \text{ eV}^2$$

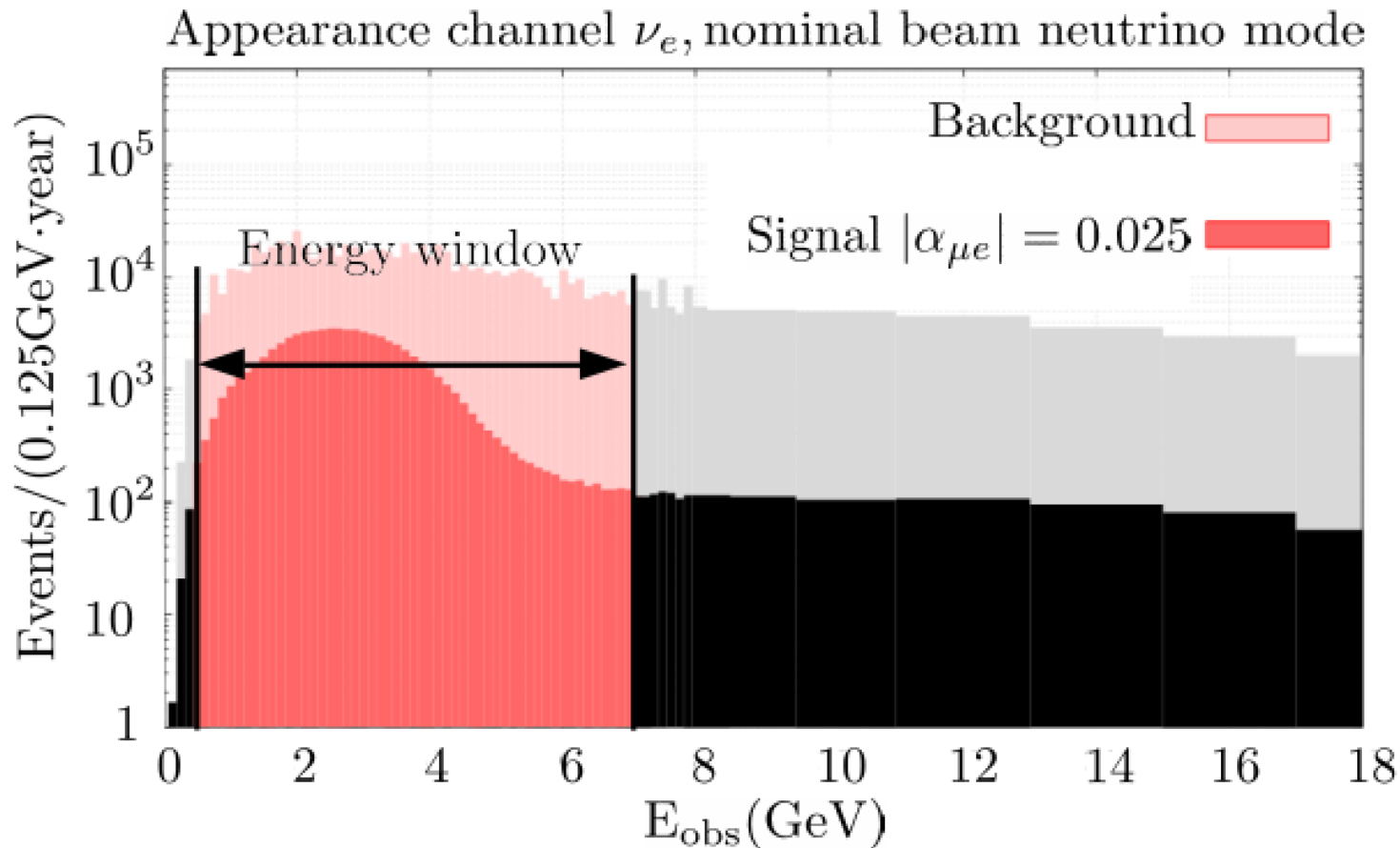
$$\langle \mathcal{P}_{\alpha\beta} \rangle = 2|\alpha_{\alpha\beta}|^2$$

zero  
distance  
effect!

$$\langle \mathcal{P}_{\alpha\alpha} \rangle = 1 - 4|\alpha_{\alpha\alpha}|^2$$

Low Scale  
Non-Unitarity

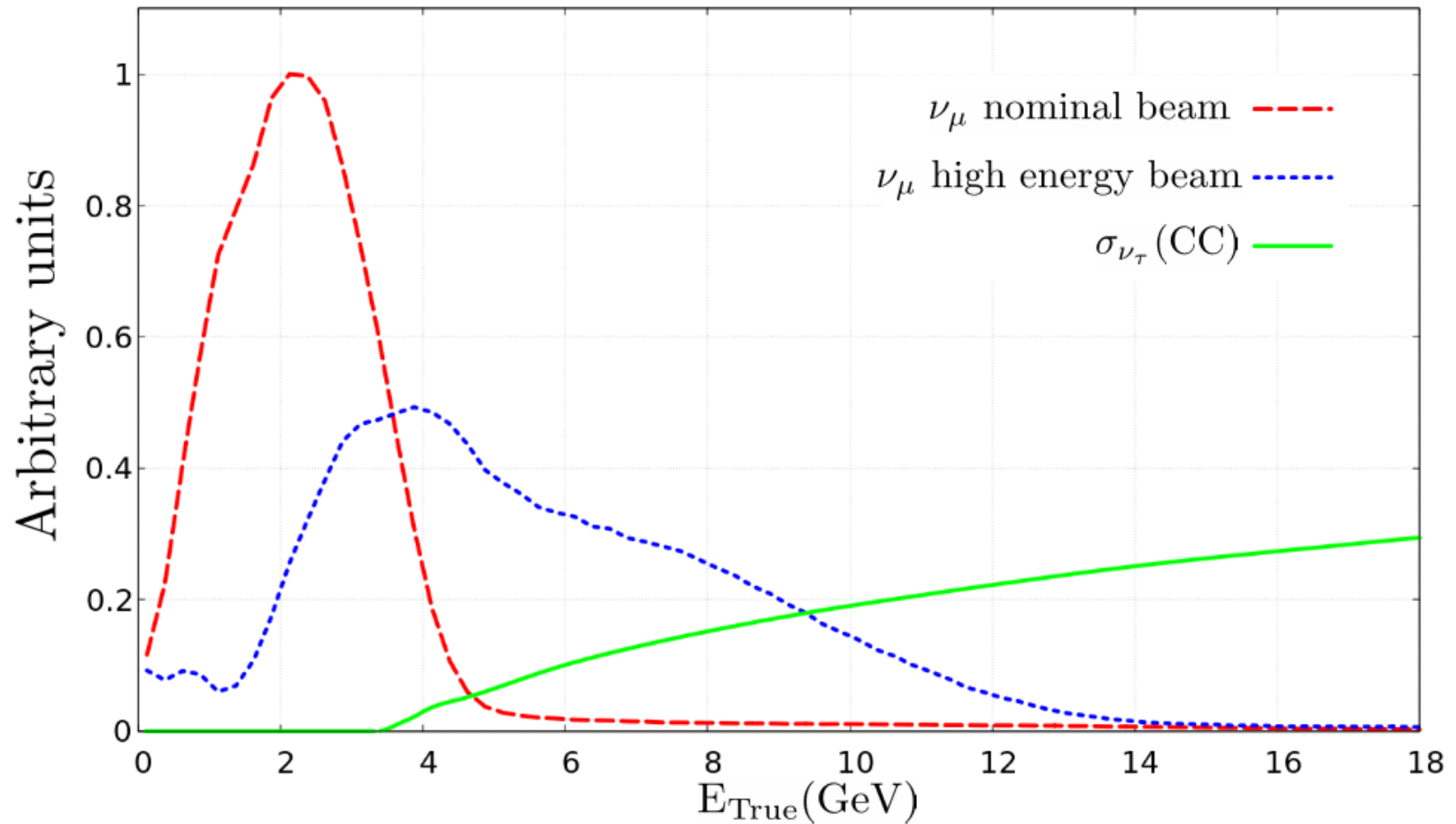
# Role of shape uncertainty



- Sensitivity driven by spectral information.
- Marginal impact of global normalization error.

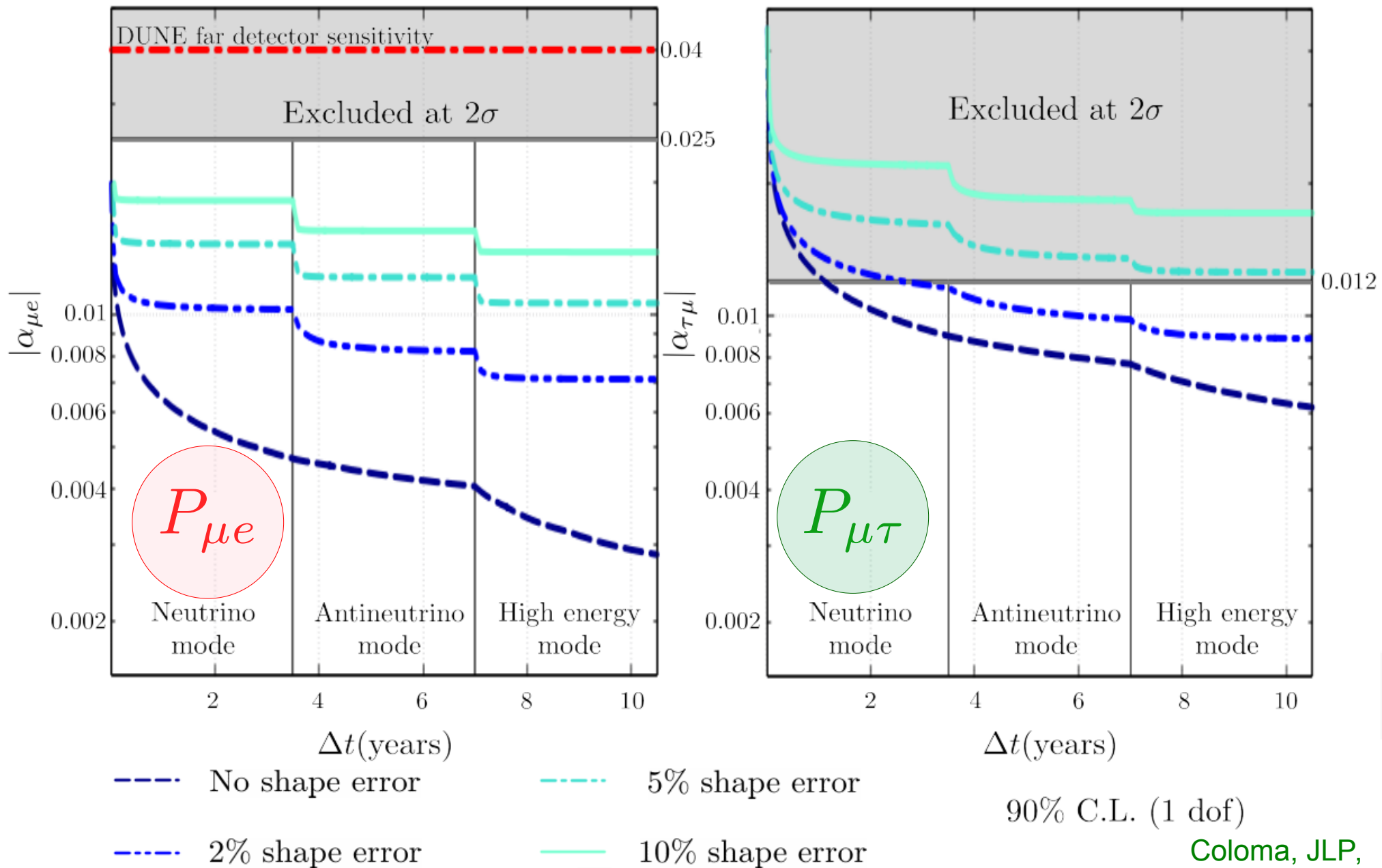
# $\nu_\tau$ appearance channel

- Energy threshold of  $\tau$  production 3.2 GeV.

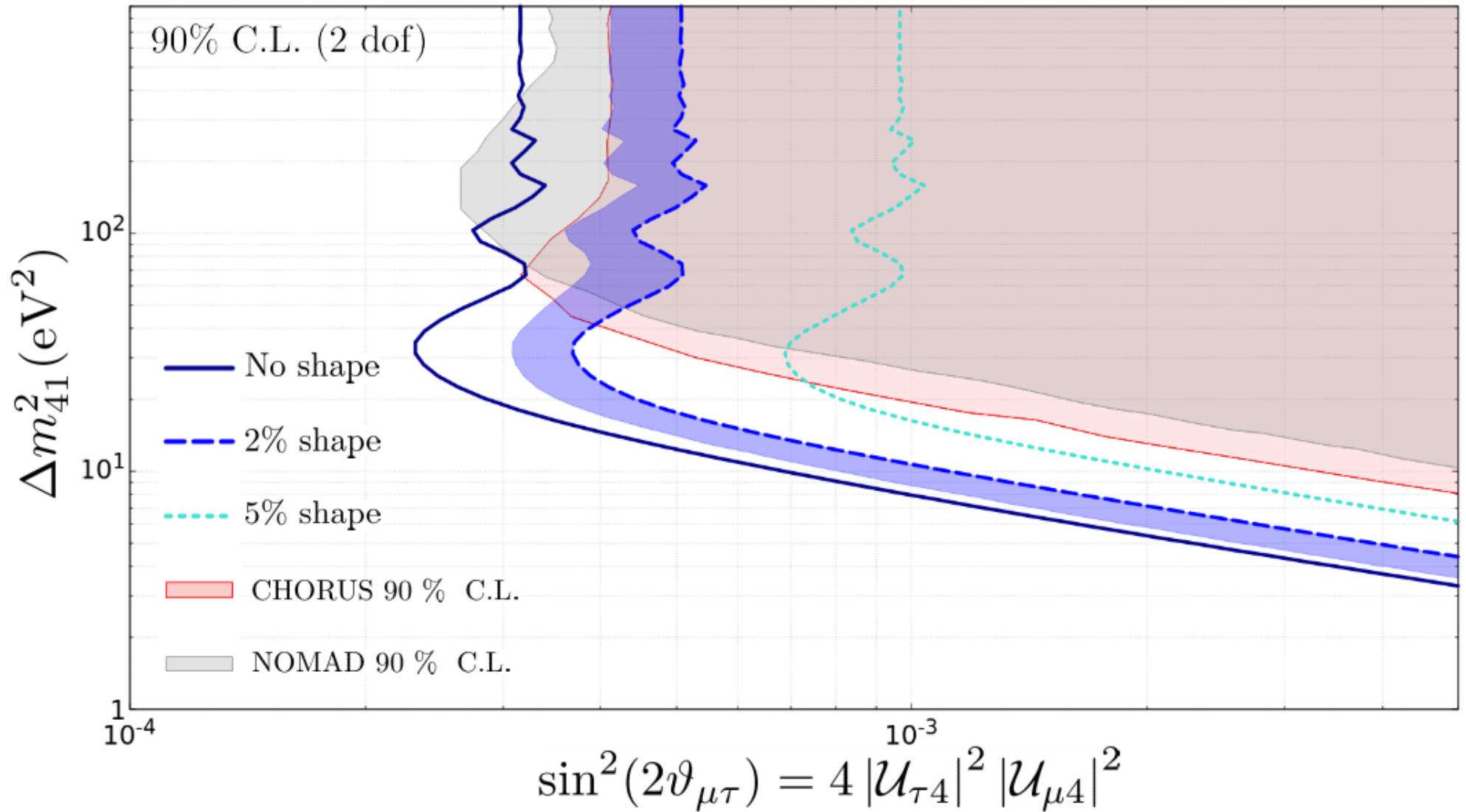
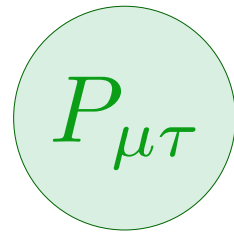


- $\nu_\tau$  **detection:** we follow de Gouvêa, Kelly, Stenico, Pasquini 1904.07265

# Low Scale Non-Unitarity

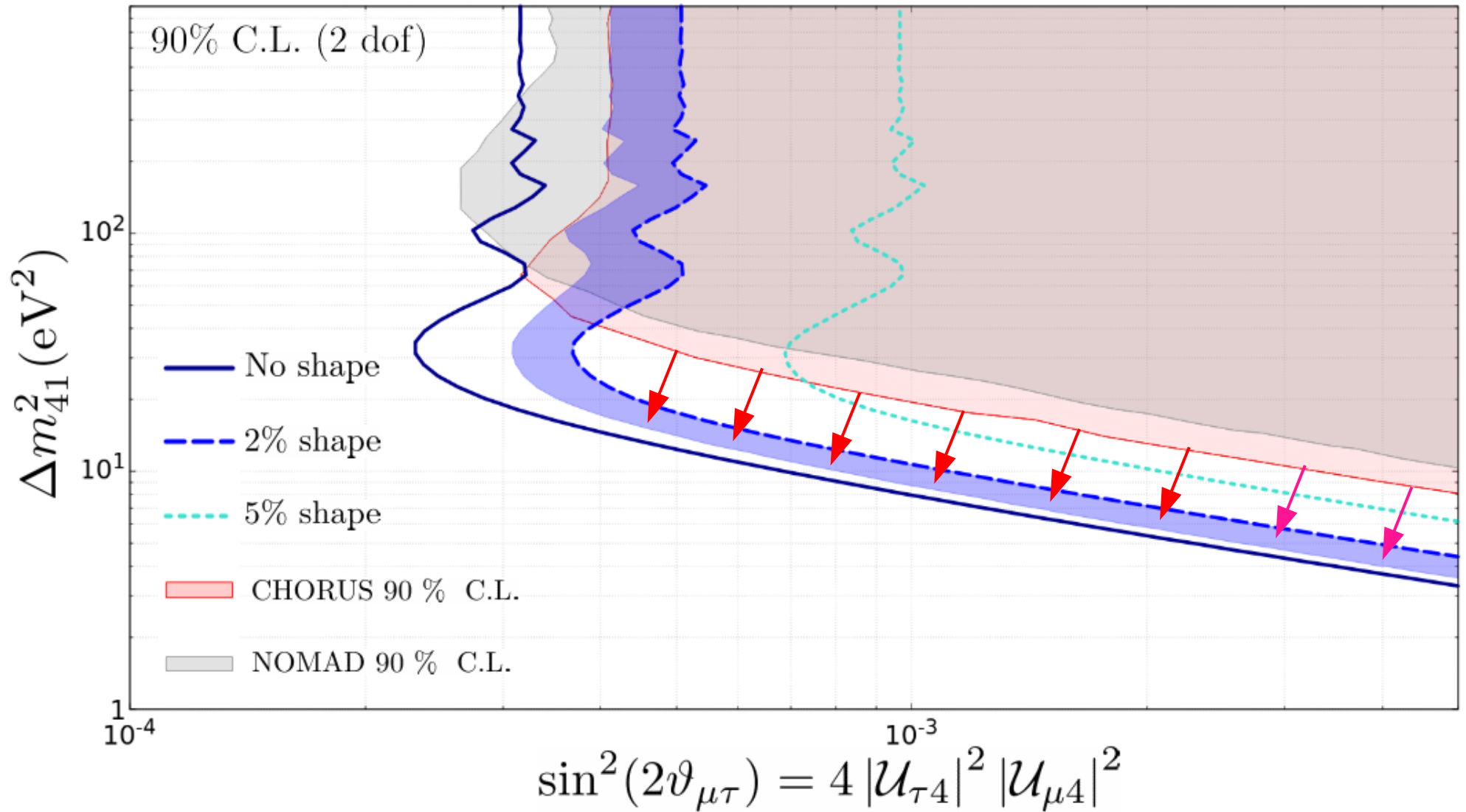


# 3+1 Sterile Neutrinos:



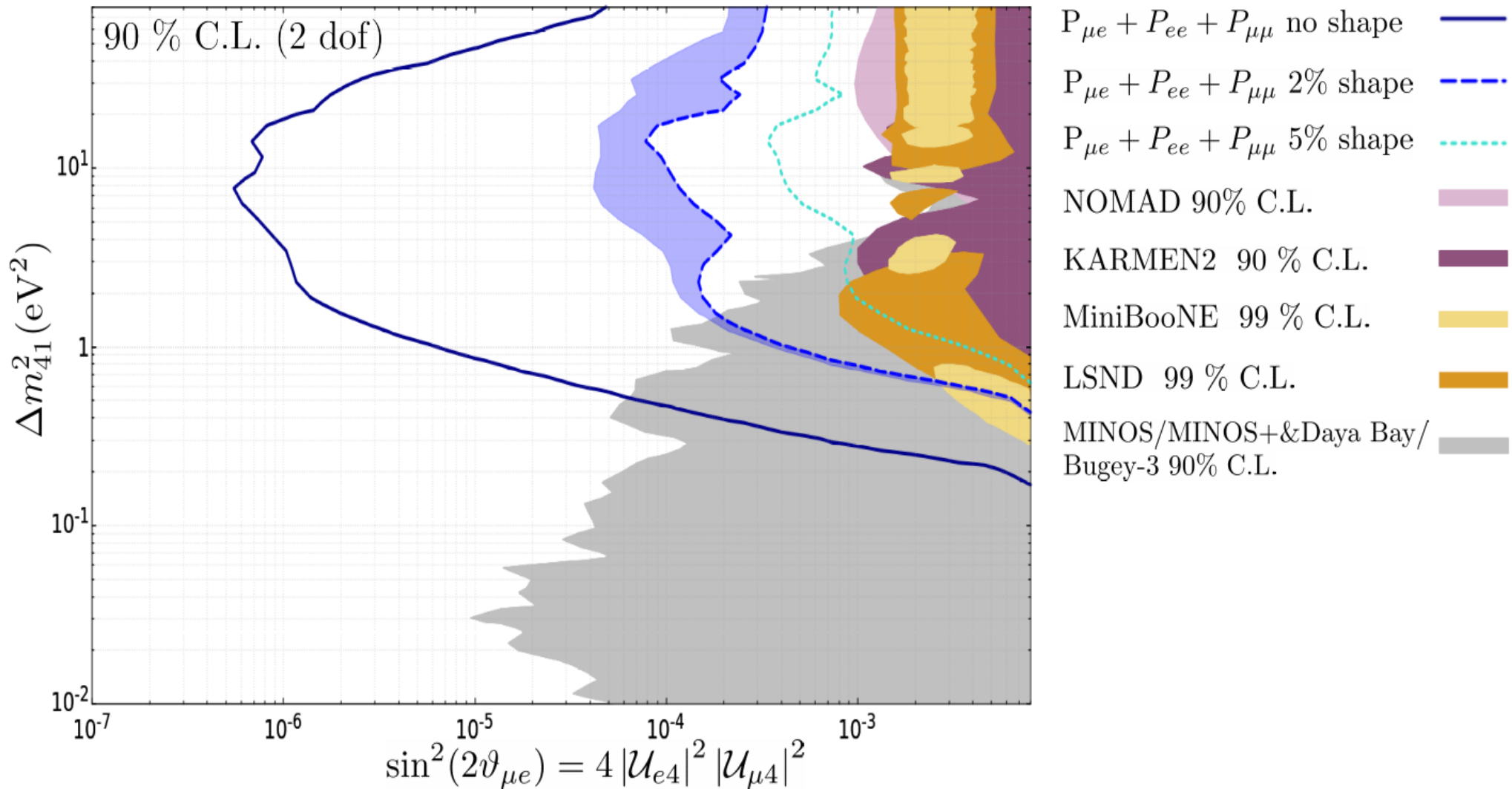
# 3+1 Sterile Neutrinos:

$P_{\mu\tau}$





# 3+1 Sterile Neutrinos: $P_{\mu\mu} + P_{\mu e} + P_{ee}$



# Conclusions

- Near detectors can play a relevant role in testing the robustness of the 3-neutrino picture

Low scale Non-Unitarity, sterile neutrino oscillations, NSI (Non-Unitarity results can be easily mapped to NSI framework, see 2105.11466)

- **Keeping under control shape uncertainties is a key issue.**

- Sensitivity beyond present bounds only when systematics below 5%
- Joint experimental and theoretical effort required to reduce systematics.
- Independent measurements of the cross sections would give very relevant information regarding the energy dependence.

(see for instance  $\nu$ STORM proposal)

- **Neutrino tau detection opens a complementary New Physics channel.**

Hvala!

# $\nu_\tau$ appearance channel

$\nu_\tau$  detection:

- Energy threshold of  $\tau$  production 3.2 GeV.
- Short lifetime of  $\tau$ , indirect measurement via hadronic decays ( $\sim 65\%$  branching ratio).
- NC background. We have considered a sample in which 30% of the hadronic events are identified keeping 0.5% of NC background.

de Gouvêa, Kelly, Stenico, Pasquini 1904.07265

# DUNE set up

## Globes files

DUNE Collaboration, arXiv:2103.04797 [hep] 8 Mar 2021.

## Flux configuration

| Beam configuration | Power  | $E_p$   | PoT/yr               | $t_\nu$ (yr) | $t_{\bar{\nu}}$ (yr) | $M_{\text{det}}$ |
|--------------------|--------|---------|----------------------|--------------|----------------------|------------------|
| Nominal            | 1.2 MW | 120 GeV | $1.1 \times 10^{21}$ | 3.5          | 3.5                  | 67.2 tons        |
| High-Energy        | 1.2 MW | 120 GeV | $1.1 \times 10^{21}$ | 3.5          | –                    | 67.2 tons        |

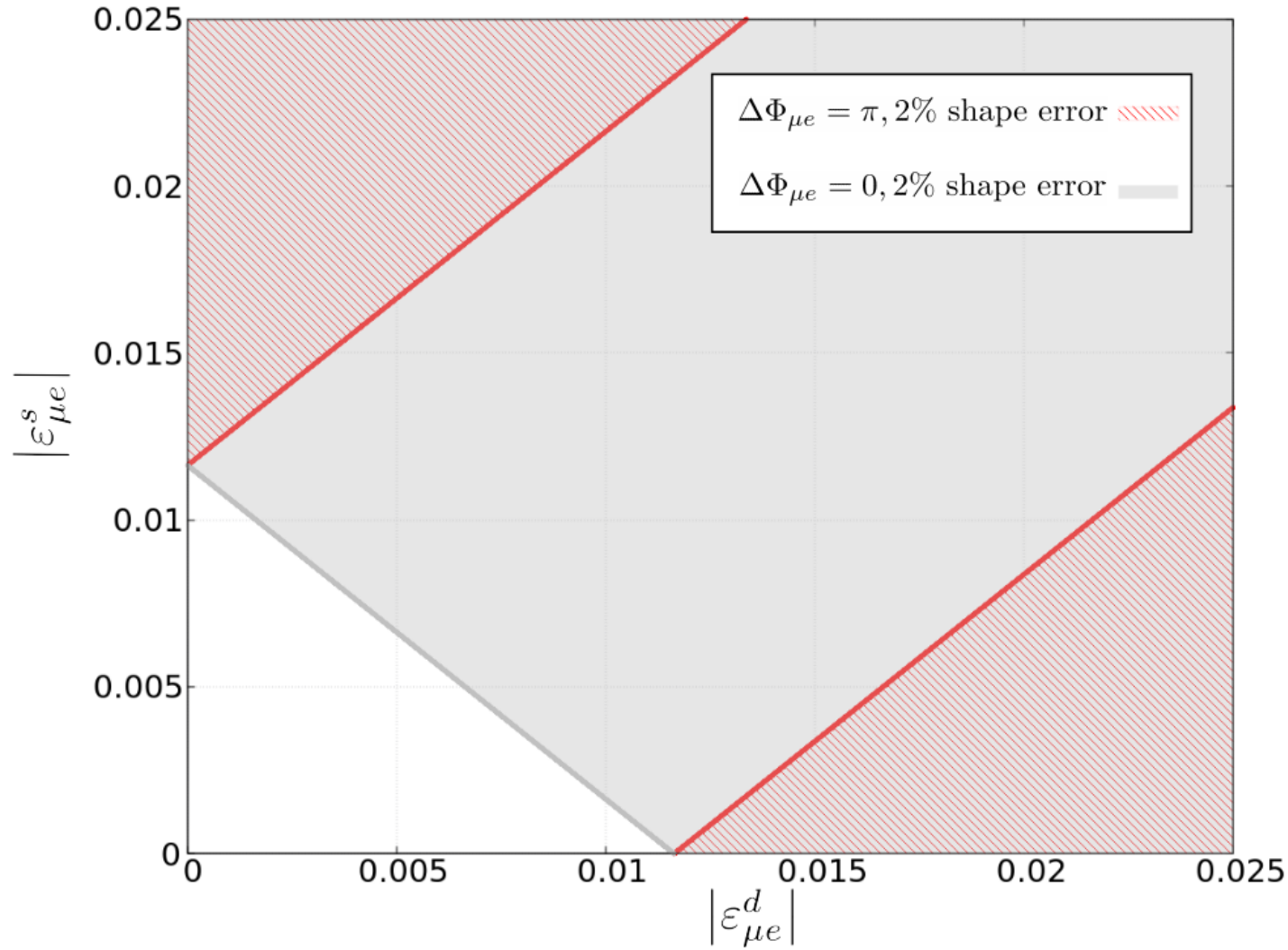
| Running mode               | Sample                 | Contribution  | Event rates ( $\times 10^5$ ) | $E_{obs}^{\max}$ (GeV) |
|----------------------------|------------------------|---|-------------------------------|------------------------|
| $\nu$ mode (nominal)       | $\nu_e$ -like          | Intrinsic cont.                                     | 20.18                         | 7.125                  |
|                            |                        | Flavor mis-ID                                       | 4.61                          |                        |
|                            |                        | NC  | 6.77                          |                        |
|                            | $\nu_\mu$ -like        | $\nu_\mu, \bar{\nu}_\mu$ CC ( $P_{\mu\mu} = 1$ )    | 2,235.72                      | 7.125                  |
|                            |                        | NC  | 17.35                         |                        |
|                            | $\nu_\tau$ -like       | $\nu_\tau, \bar{\nu}_\tau$ CC ( $P_{\mu\tau} = 1$ ) | 39.33                         | 18                     |
| NC                         |                        | 3.23  |                               |                        |
| $\bar{\nu}$ mode (nominal) | $\bar{\nu}_e$ -like    | Intrinsic cont.                                     | 11.18                         | 7.125                  |
|                            |                        | Flavor mis-ID                                       | 1.07                          |                        |
|                            |                        | NC  | 3.89                          |                        |
|                            | $\bar{\nu}_\mu$ -like  | $\nu_\mu, \bar{\nu}_\mu$ CC ( $P_{\mu\mu} = 1$ )    | 1,013.42                      | 7.125                  |
|                            |                        | NC  | 9.76                          |                        |
|                            | $\bar{\nu}_\tau$ -like | $\nu_\tau, \bar{\nu}_\tau$ CC ( $P_{\mu\tau} = 1$ ) | 27.75                         | 18                     |
| NC                         |                        | 1.80  |                               |                        |
| $\nu$ mode (HE)            | $\nu_e$ -like          | Intrinsic cont.                                     | 38.10                         | 18                     |
|                            |                        | Flavor mis-ID                                       | 12.98                         |                        |
|                            |                        | NC  | 30.51                         |                        |
|                            | $\nu_\mu$ -like        | $\nu_\mu, \bar{\nu}_\mu$ CC ( $P_{\mu\mu} = 1$ )    | 5,784.30                      | 18                     |
|                            |                        | NC  | 72.15                         |                        |
|                            | $\nu_\tau$ -like       | $\nu_\tau, \bar{\nu}_\tau$ CC ( $P_{\mu\tau} = 1$ ) | 259.67                        | 18                     |
| NC                         |                        | 9.42  |                               |                        |

| Event sample     | Contribution                         | Benchmark 1     |                  | Benchmark 2     |                  | Benchmark 3     |                  |
|------------------|--------------------------------------|-----------------|------------------|-----------------|------------------|-----------------|------------------|
|                  |                                      | $\sigma_{norm}$ | $\sigma_{shape}$ | $\sigma_{norm}$ | $\sigma_{shape}$ | $\sigma_{norm}$ | $\sigma_{shape}$ |
| $\nu_e$ -like    | Signal                               | 5%              | –                | 5%              | –                | 5%              | –                |
|                  | Intrinsic cont.                      | 10%             | –                | 10%             | 2%               | 10%             | 5%               |
|                  | Flavor mis-ID                        | 5%              | –                | 5%              | 2%               | 5%              | 5%               |
|                  | NC                                   | 10%             | –                | 10%             | 2%               | 10%             | 5%               |
| $\nu_\mu$ -like  | $\nu_\mu, \bar{\nu}_\mu$ CC (signal) | 10%             | –                | 10%             | 2%               | 10%             | 5%               |
|                  | NC                                   | 10%             | –                | 10%             | 2%               | 10%             | 5%               |
| $\nu_\tau$ -like | Signal                               | 20%             | –                | 20%             | –                | 20%             | –                |
|                  | NC                                   | 10%             | –                | 10%             | 2%               | 10%             | 5%               |

# NSI in production/detection:

$P_{\mu e}$

Appearance channel  $\nu_e$  90% C.L.



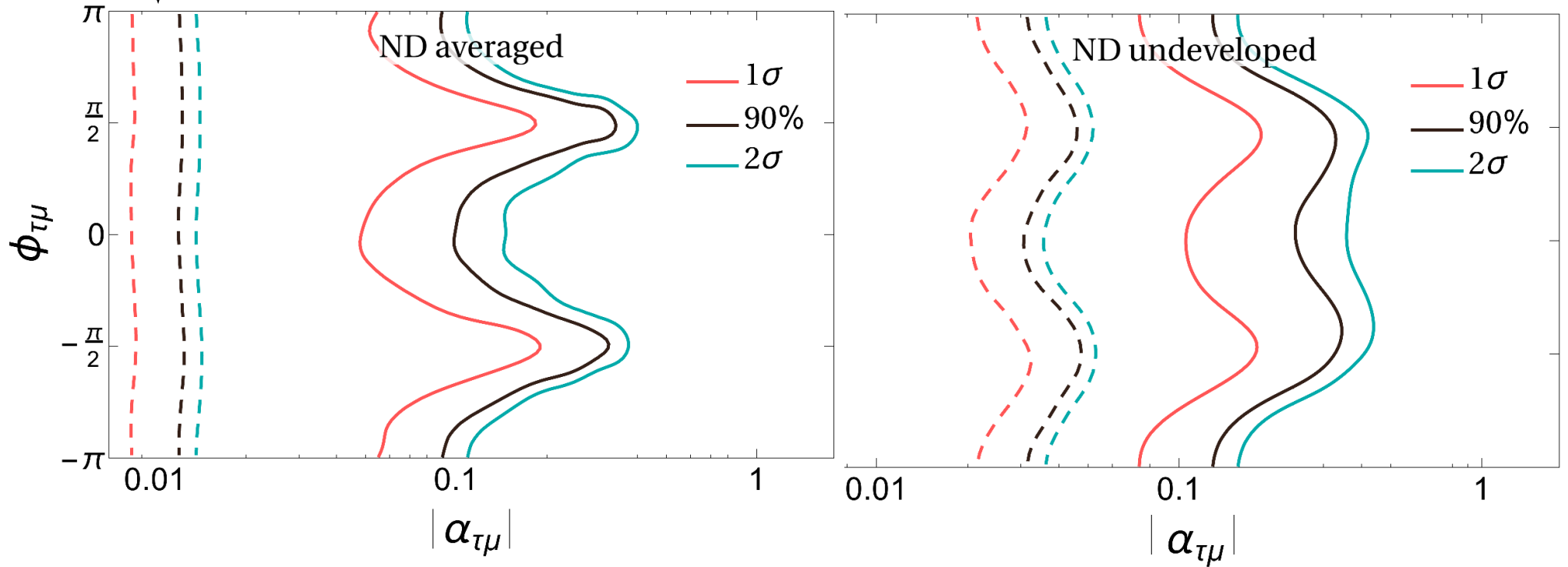
Coloma, JLP,  
Rosauro-Alcaraz,  
Urrea 2105.11466

- Mapping:  $2|\alpha_{\beta\gamma}|^2 = |\epsilon_{\beta\gamma}^d|^2 + |\epsilon_{\beta\gamma}^s|^2 + 2|\epsilon_{\beta\gamma}^d||\epsilon_{\beta\gamma}^s| \cos(\Phi_{\beta\gamma}^s - \Phi_{\beta\gamma}^d)$



Prior  
(present bounds)

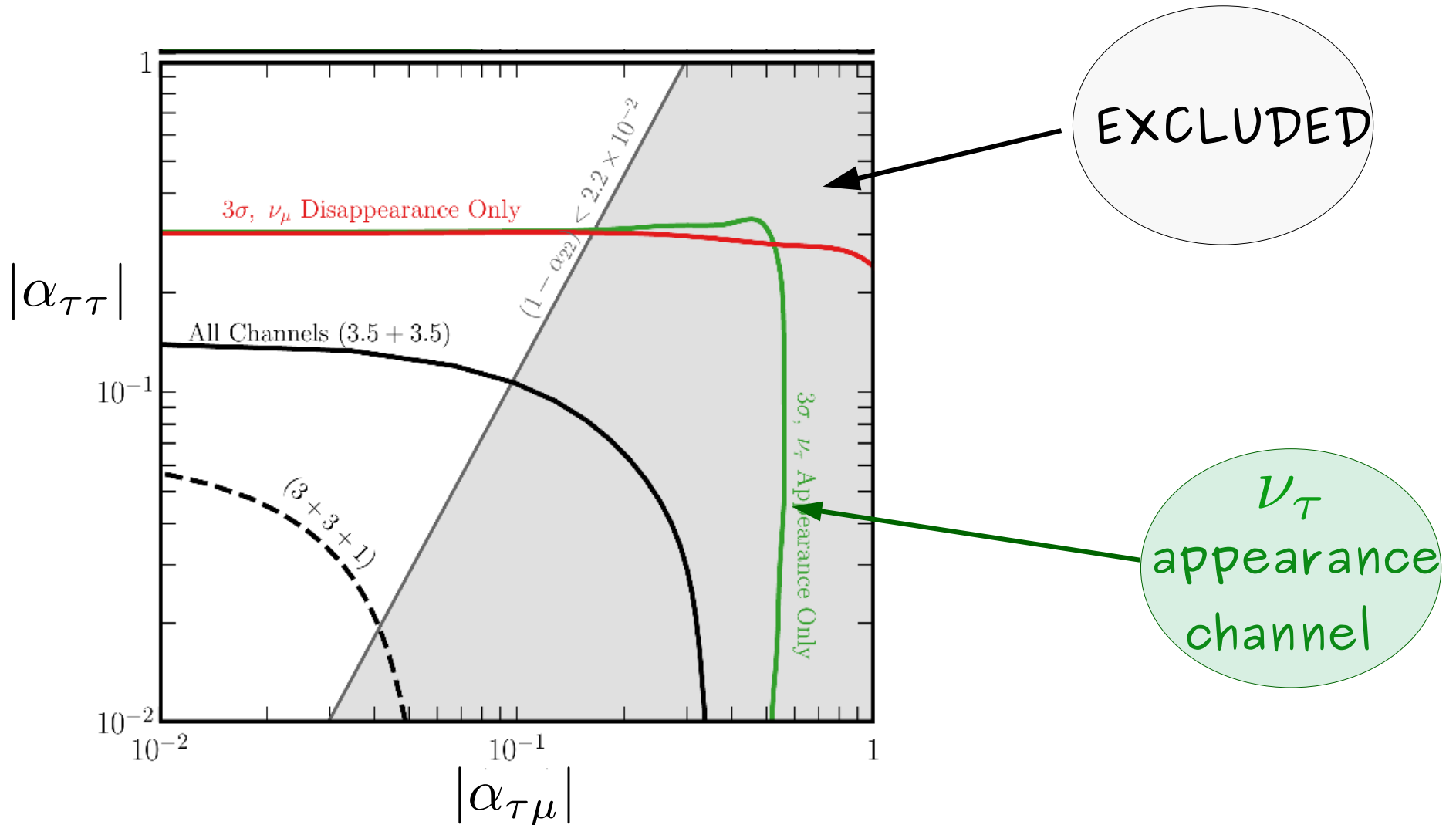
# Far Detector



$$\mathcal{P}_{\alpha\beta} = \frac{|(N \exp(-iHL)N^\dagger)_{\beta\alpha}|^2}{[(NN^\dagger)_{\alpha\alpha}]^2}$$

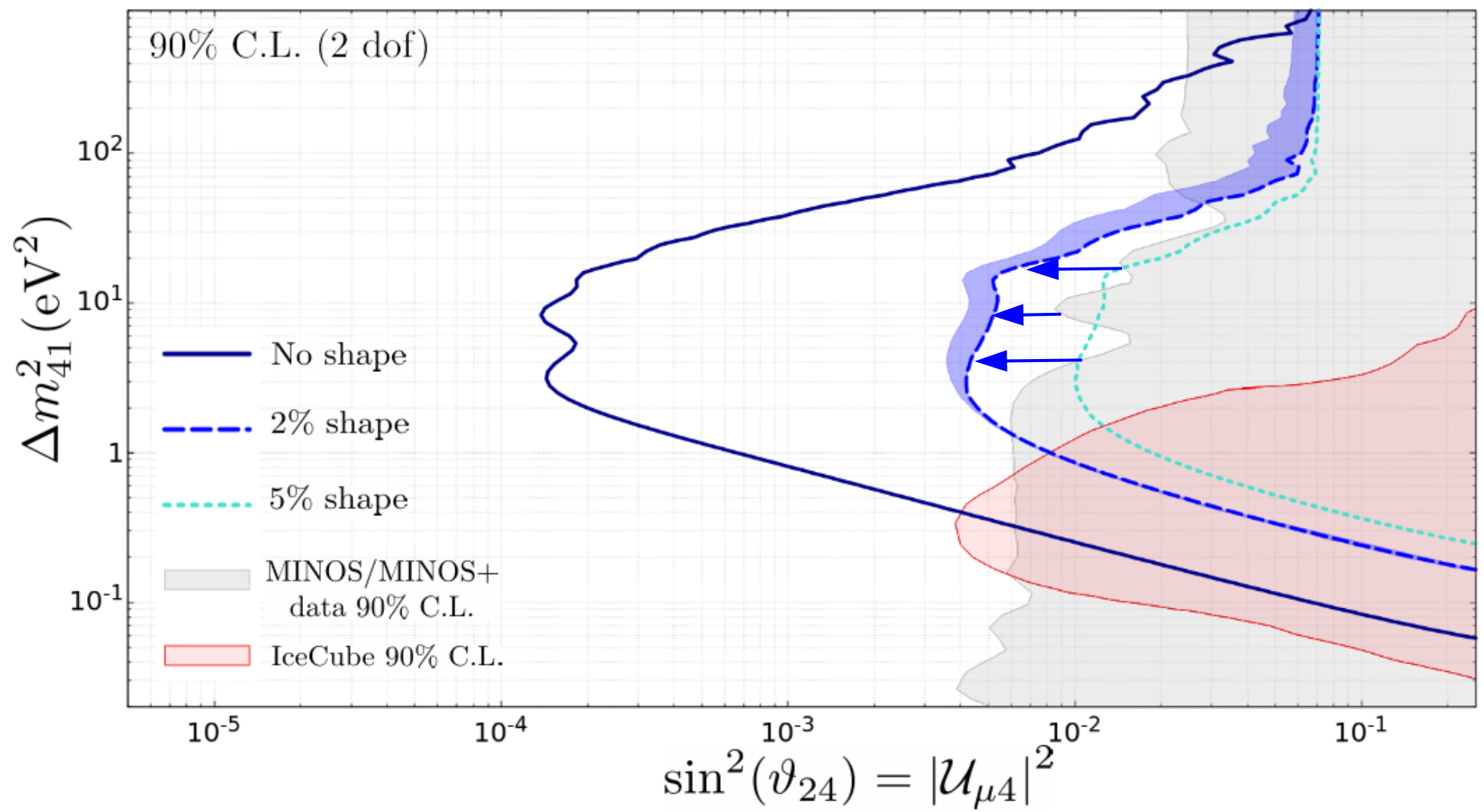
$$\mathcal{P}_{\alpha\beta} = |(N \exp(-iHL)N^\dagger)_{\beta\alpha}|^2$$

# Far Detector

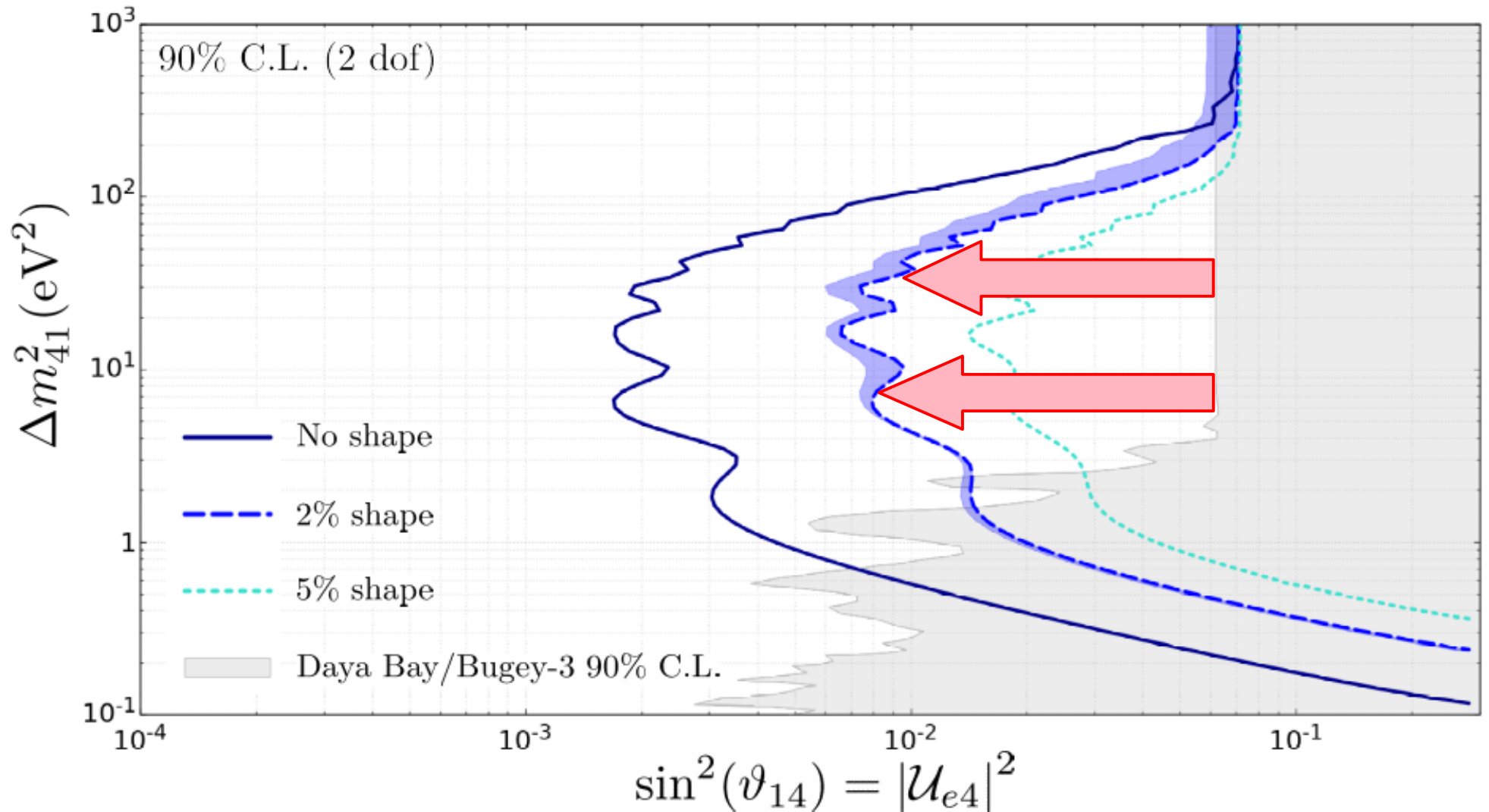


- Including  $\nu_\tau$  appearance channel does not change the picture.

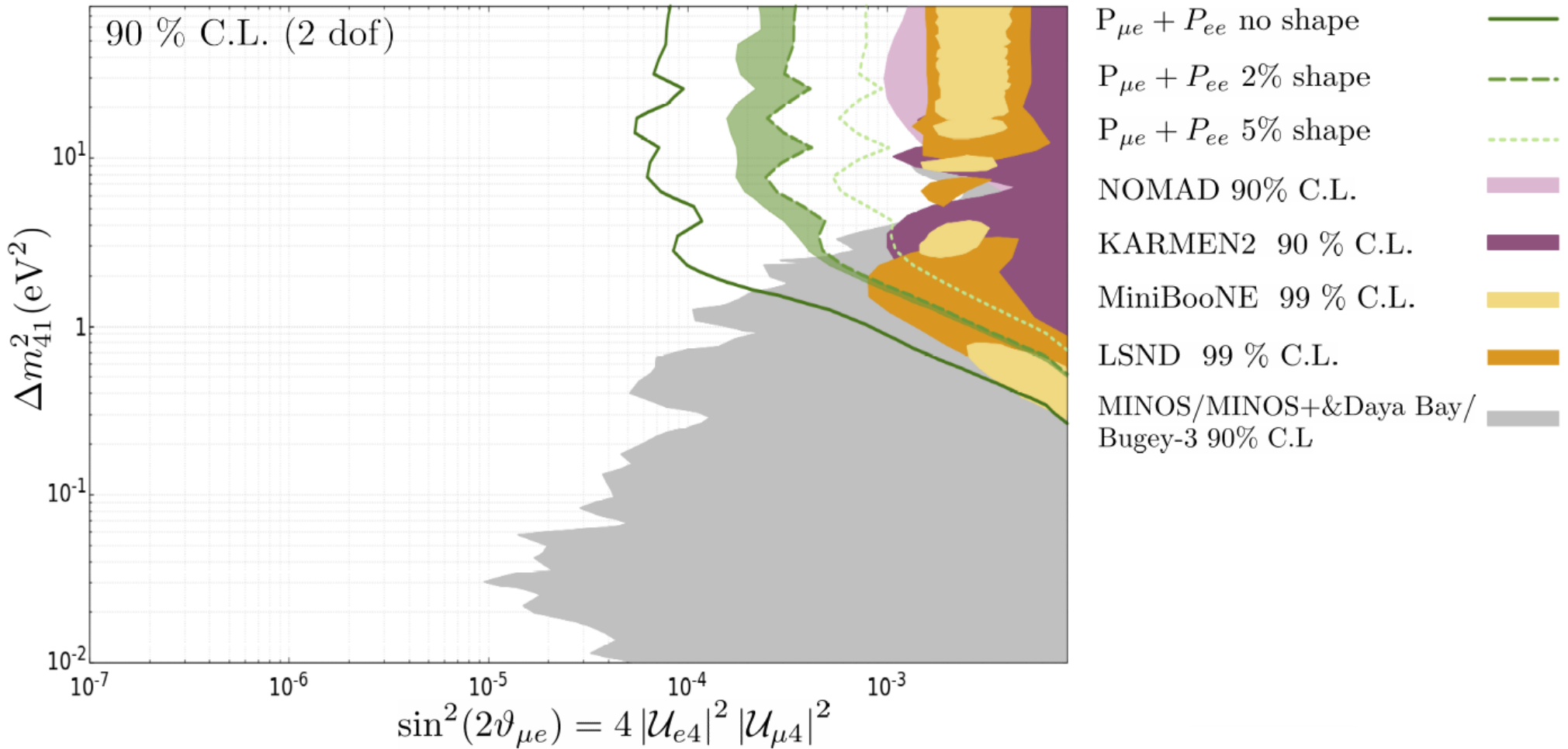
# 3+1 Sterile Neutrinos: $P_{\mu\mu}$



# 3+1 Sterile Neutrinos: $P_{ee}$



# 3+1 Sterile Neutrinos: $P_{\mu e} + P_{ee}$



# Systematics: Disappearance

$$N_{\nu_\alpha \rightarrow \nu_\beta} \sim \frac{\Phi_\alpha(E)}{L^2} P_{\alpha\beta}(L/E) \sigma_\beta(E) \epsilon_\beta(E)$$

- Using **near detectors** is a very effective way of reducing systematics in disappearance experiments (K2K, MINOS, reactors...).

$$\frac{N_{\nu_\alpha}^{\text{FD}}}{N_{\nu_\alpha}^{\text{ND}}} \sim \frac{L_{\text{ND}}^2}{L_{\text{FD}}^2} \frac{\cancel{\Phi_\alpha \sigma_\alpha \epsilon_\alpha}}{\cancel{\Phi_\alpha \sigma_\alpha \epsilon_\alpha}} P_{\alpha\alpha}$$

# Systematics: Appearance (CP violation)

$$N_{\nu_\alpha \rightarrow \nu_\beta} \sim \frac{\Phi_\alpha(E)}{L^2} P_{\alpha\beta}(L/E) \sigma_\beta(E) \epsilon_\beta(E)$$

- For appearance experiments the situation is more complicated

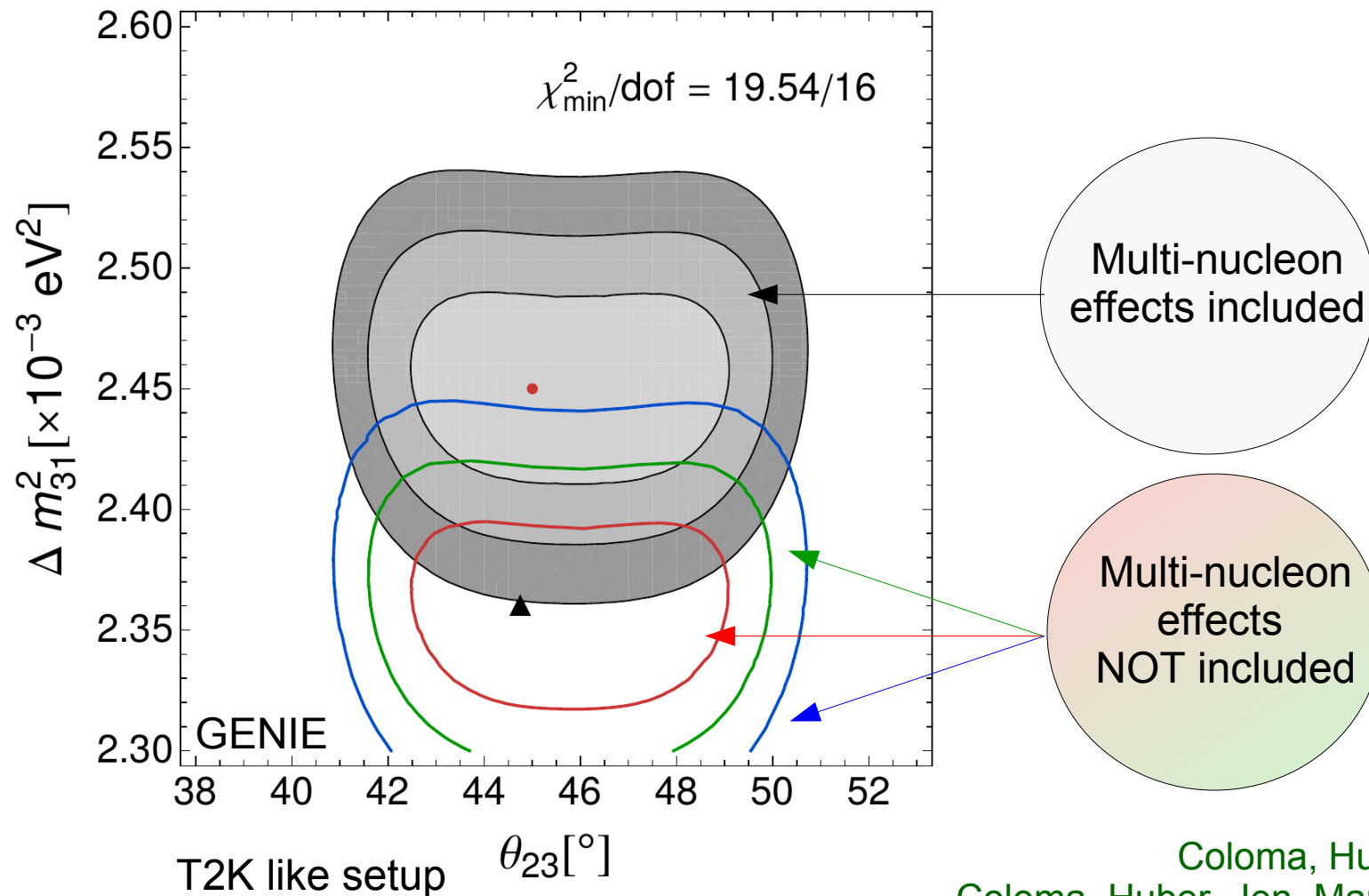
$$\frac{N_{\nu_e}^{\text{FD}}}{N_{\nu_\mu}^{\text{ND}}} \sim \frac{L_{\text{ND}}^2}{L_{\text{FD}}^2} \frac{\sigma_e \epsilon_e}{\sigma_\mu \epsilon_\mu} P_{\mu e}$$

- CP violation requires comparison between neutrino and anti-neutrino signals.

$$\frac{N_{\nu_e}^{\text{Far}}}{N_{\bar{\nu}_e}^{\text{Far}}} \sim \frac{N_{\nu_\mu}^{\text{ND}}}{N_{\bar{\nu}_\mu}^{\text{ND}}} \frac{\sigma_e \epsilon_e}{\sigma_\mu \epsilon_\mu} \frac{\sigma_{\bar{\mu}} \epsilon_{\bar{\mu}}}{\sigma_{\bar{e}} \epsilon_{\bar{e}}} \frac{P_{\mu e}}{P_{\bar{\mu} \bar{e}}}$$

# Nuclear Cross sections

- Neutrino-nucleus cross section missmodeling could lead to unacceptably large systematic uncertainties or biased measurements, even after the inclusion of a near detector.



Coloma, Huber 1307.1243  
Coloma, Huber, Jen, Mariani 1311.4506



$$\chi_{\min}^2(\{\Theta\}) = \min_{\{\xi, \zeta\}} \left[ \chi_{\text{stat}}^2(\{\Theta, \xi, \zeta\}) + \sum_s \left( \frac{\zeta_s}{\sigma_{\text{norm},s}} \right)^2 + \sum_b \left( \frac{\zeta_b}{\sigma_{\text{norm},b}} \right)^2 + \sum_i \left( \frac{\xi_i^{\text{sig}}}{\sigma_{\text{shape},\text{sig}}} \right)^2 + \sum_i \left( \frac{\xi_i^{\text{bg}}}{\sigma_{\text{shape},\text{bg}}} \right)^2 \right],$$

$$\chi_{\text{stat}}^2(\{\Theta, \xi, \zeta\}) = \sum_i 2 \left( N_i(\{\Theta, \xi, \zeta\}) - O_i + O_i \ln \frac{O_i}{N_i(\{\Theta, \xi, \zeta\})} \right)$$

$$N_i(\{\Theta, \xi, \zeta\}) = \sum_s (1 + \xi_i^{\text{sig}} + \zeta_s) s_i(\{\Theta\}) + \sum_b (1 + \xi_i^{\text{bg}} + \zeta_b) b_i(\{\Theta\})$$

# General Parameterizations

- Hermitian parameterization

$$N = (I - \eta) U'$$

Deviation from unitarity

$$\eta = \begin{pmatrix} \eta_{ee} & \eta_{e\mu} & \eta_{e\tau} \\ \eta_{e\mu}^* & \eta_{\mu\mu} & \eta_{\mu\tau} \\ \eta_{e\tau}^* & \eta_{\mu\tau}^* & \eta_{\tau\tau} \end{pmatrix} = \frac{\Theta\Theta^\dagger}{2}$$

Unitary matrix  
(standard unitary PMNS  
matrix  
up to small corrections)

Broncano, Gavela, Jenkins 2003

Fernandez-Martinez, Gavela, JLP, Yasuda 2007