$K \to \pi\bar{\nu}\nu$ and ϵ_K: SM and Beyond

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Based on work with J. Brod, F. Bishara, E. Stamou
U. Moldanazarova 1911.06822, 2104.10930, 2105.02868

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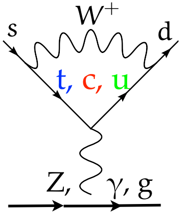
Content

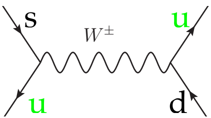
- Updated Standard Model Prediction for
  - $K \to \pi\bar{\nu}\nu$
  - ϵ_K
- New Physics
  - Renormalisation for generic theories
  - Mathematica code for results

$$K \to \pi \bar{\nu} \nu$$

Rare Kaon Decays: CKM Structure



Using the GIM mechanism, we can eliminate either $V_{cs}^* V_{cd}$ or $V_{us}^* V_{ud} \to - V_{cs}^* V_{cd} - V_{ts}^* V_{td}$



Z-Penguin and Boxes (high virtuality): power expansion in: $A_c - A_u \propto 0 + O(m_c^2/M_W^2)$

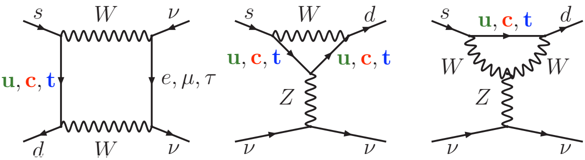
γ/g-Penguin (expand in mom.): $A_c - A_u \propto O(\mathrm{Log}(m_c^2/m_u^2))$

$$\mathrm{Im}V_{ts}^* V_{td} = -\mathrm{Im}V_{cs}^* V_{cd} = \mathcal{O}(\lambda^5) \qquad \mathrm{Im}V_{us}^* V_{ud} = 0$$

$$\mathrm{Re}V_{us}^* V_{ud} = -\mathrm{Re}V_{cs}^* V_{cd} = \mathcal{O}(\lambda^1) \qquad \mathrm{Re}V_{ts}^* V_{td} = \mathcal{O}(\lambda^5)$$

- $K \to \pi\bar{\nu}\nu$ (from Z & Boxes): Clean and suppressed

$K \to \pi\bar{\nu}\nu$ at M_W



$$x_i = \frac{m_i^2}{M_W^2}$$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td}(F(x_t) - F(x_u)) + V_{cs}^* V_{cd}(F(x_c) - F(x_u))$$

Quadratic GIM: $\lambda^5 \frac{m_t^2}{M_W^2}$ $\lambda \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c}$ $\lambda \frac{\Lambda_{QCD}^2}{M_W^2}$

Matching (NLO +EW):

Operator Mixing (RGE)

ChiPT & Lattice

$$Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$$

- Below the charm: Only Q_ν, ME from K_{l3}
- semi-leptonic $(\bar{s}\gamma_\mu u_L)(\bar{\nu}\gamma^\mu \ell_L)$ operator: χ PT gives small contribution (10% of charm contribution)

Leading Effective Hamiltonian for $\mu < m_c$

SM: $\nu\bar{\nu}$ are only invisibles $\Rightarrow$ no γ-Penguin $\Rightarrow$

$$\mathcal{H}_{\text{eff}} = \frac{\sqrt{2}\alpha G_F}{\pi \sin^2\theta_w} \sum_{\ell=e,\mu,\tau} (\lambda_c X^\ell + \lambda_t X_t)(\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_{\ell L}\gamma^\mu \nu_{\ell L}) + \text{h.c.}$$

generated by highly virtual particles + tiny light quark contribution $\Rightarrow$ clean & CKM suppressed ($\lambda_i = V_{is}^* V_{id}$).

- X_t known at NLO QCD and two-loop EW: $X_t = 1.462 \pm 0.017_{\text{QCD}} \pm 0.002_{\text{EW}}$
- $P_c = \lambda^{-4}(\frac{2}{3}X^e + \frac{1}{3}X^\tau)$ at NNLO QCD + NLO EW is
$$P_c = \left(\frac{0.2255}{\lambda}\right)^4 \times (0.3604 \pm 0.0087)$$

$\lambda \simeq V_{us}$ and updated values from [2105.02868]

$K \to \pi\nu\bar{\nu}$ Branching Ratios

- Matrix elements from $K_{\ell 3}$ including strong and em iso-spin breaking [0705.2025] $\kappa_+, \kappa_L, \Delta_{EM}$

$$\kappa_+ = \frac{s_w^{-2}\lambda^8\alpha(M_Z)^2}{7.5248 \cdot 10^{-9}} \times 0.5173(25) \times 10^{-10},\ \Delta_{EM} = -0.003$$

- indirect CP violation contribution given by r_{ϵ_K}

$$\mathrm{Br}_{K^+} = \kappa_+(1+\Delta_{\mathrm{EM}})\left[\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5}X_t\right)^2 + \left(\frac{\mathrm{Re}\lambda_c}{\lambda}\left(P_c+\delta P_{c,u}\right) + \frac{\mathrm{Re}\lambda_t}{\lambda^5}X_t\right)^2\right].$$

$$\mathrm{Br}_{K_L} = \kappa_L r_{\epsilon_K}\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5}X_t\right)^2, \quad \kappa_L = \frac{s_w^{-2}\lambda^8\alpha(M_Z)^2}{7.5248 \cdot 10^{-9}} \times 2.231(13) \times 10^{-10}$$

$K \to \pi\nu\bar{\nu}$ in the Standard Model

- 2105.02868 Standard Model Prediction

$$\begin{aligned} \mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) &= 7.73(16)_{SD}(25)_{LD}(54)_{para.} \times 10^{-11}\,, \\ \mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) &= 2.59(6)_{SD}(2)_{LD}(28)_{para.} \times 10^{-11}\,. \end{aligned}$$

$$\begin{aligned} 10^{11} \times \mathcal{B}_+ = {}& 7.73 \pm 0.12_{X_t^{\rm QCD}} \pm 0.01_{X_t^{\rm EW}} \pm 0.11_{P_c} \pm 0.24_{\delta P_{cu}} \pm 0.04_{\kappa_+} \\ & \pm 0.13_{\lambda} \pm 0.46_{A} \pm 0.18_{\bar{\rho}} \pm 0.03_{\bar{\eta}} \pm 0.05_{m_t} \pm 0.15_{m_c} \pm 0.05_{\alpha_s}\,. \\ 10^{11} \times \mathcal{B}_L = {}& 2.59 \pm 0.06_{X_t^{\rm QCD}} \pm 0.01_{X_t^{\rm EW}} \pm 0.02_{\kappa_L} \\ & \pm 0.16_{\bar{\eta}} \pm 0.22_{A} \pm 0.04_{\lambda} \pm 0.02_{m_t} \end{aligned}$$

- NA62 collaboration
  $\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) = (10.6^{+3.4}_{-3.4}|_{\rm stat} \pm 0.9_{\rm syst}) \times 10^{-11}$
- JPARC-KOTO has $\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) \leq 3.0 \times 10^{-9}$

$$\epsilon_K$$

CP violation in $K \to \pi\pi$

- Experimental definition using $\eta_{ij} = \dfrac{\langle \pi^i \pi^j | K_L \rangle}{\langle \pi^i \pi^j | K_S \rangle}$

$$\epsilon_K = (2\eta_{+-} + \eta_{00})/3\,, \qquad \epsilon' = (\eta_{+-} - \eta_{00})/3$$

- ϵ_K theory expression $\epsilon_K \simeq \dfrac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} =$

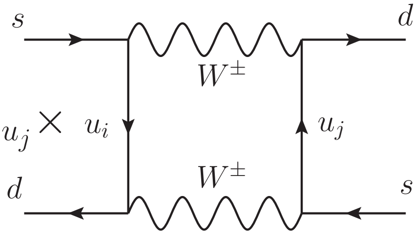
$$e^{i\phi_\epsilon} \sin\phi_\epsilon \frac{1}{2} \arg\left(\frac{-M_{12}}{\Gamma_{12}} \right) = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12})^{Dis}}{\Delta M_K} + \xi \right)$$

$$\langle K^0 | H^{|\Delta S|=2} | \bar{K}^0 \rangle \to \mathrm{Im}(M_{12})^{Dis}\,, \ \frac{\mathrm{Im}\langle (\pi\pi)_{I=0} | K^0 \rangle}{\mathrm{Re}\langle (\pi\pi)_{I=0} | K^0 \rangle} \to \xi \ \ \phi_\epsilon \equiv \arctan \frac{\Delta M_K}{\Delta\Gamma_K/2}$$

- $\dfrac{2}{3} f_K^2 M_K^2 \hat{B}_K = \langle \bar{K}^0 | Q^{|\Delta S=2|} | K^0 \rangle u^{-1}(\mu_{\rm had})$
- $Q_{S2} = (\bar{s}_L \gamma_\mu d_L) \otimes (\bar{s}_L \gamma^\mu d_L)$

Kaon Mixing: CKM Structure

$$\sum_{u_i,u_j}^{u,c,t} \lambda_{u_i}\lambda_{u_j} \times$$



	Im	Re	$\mathcal{O}$
λ_t^2	$\sim \lambda^{10}$	$\sim \lambda^{10}$	m_t^2/M_W^2
$\lambda_c\ \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$	$m_c^2/M_W^2 \ln(m_t/m_c)$
λ_c^2	$\sim \lambda^6$	$\sim \lambda^2$	m_c^2/M_W^2
$\lambda_u\ \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$	$m_c^2/M_W^2 \ln(m_t/m_c)$
λ_u^2	0	$\sim \lambda^2$	m_c^2/M_W^2

Where $\lambda_i = V_{id}V_{is}^*$, $\lambda \equiv |V_{us}| \sim 0.2$ and we eliminated either: $\lambda_u = -\lambda_c - \lambda_t$ or $\lambda_c = -\lambda_u - \lambda_t$.

$\Delta S = 2$ Hamiltonian - Phase (In)Dependence

- Recall $\epsilon_K \propto \arg(-M_{12}/\Gamma_{12})$
- Trick: pull out λ_u^* and $(\lambda_u^*)^2$ from $H^{\Delta S=1}$ and $H^{\Delta S=2}$:
- Rephaseing invariant: $\lambda_i \lambda_j^* = V_{id} V_{is}^* V_{jd}^* V_{js}$
- $\Gamma_{12} \simeq A_0^* \bar{A}_0$ where $A_0 = \langle (\pi\pi)_{I=0} | K^0 \rangle$

$$\mathcal{H}_{f=3}^{\Delta S=2} = \frac{G_F^2 M_W^2}{4\pi^2 (\lambda_u^*)^2} Q_{S2} \Big\{ f_1 C_1(\mu) + iJ \left[f_2 C_2(\mu) + f_3 C_3(\mu) \right] \Big\} + \text{h.c.}$$

- $J = \mathrm{Im}(V_{us} V_{cb} V_{ub}^* V_{cs}^*)$, f_1, f_2 and f_3 are rephasing invariant
- Real part $f_1 = |\lambda_u|^4$ is unique
- Splitting of $f_2 = 2\mathrm{Re}(\lambda_t \lambda_u^*)$ and $f_3 = |\lambda_u|^2$ not, but expect good convergence for C_2 and C_3.

Traditional Form

Traditionally the effective Hamiltonian is written as:

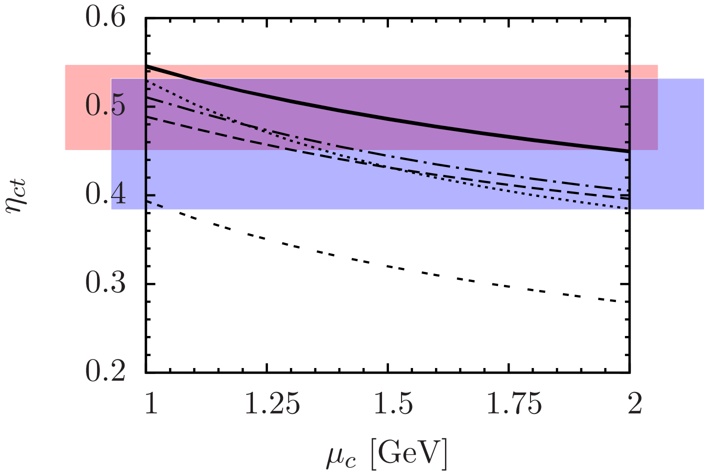
$$\mathcal{H}^{\Delta=2}_{f=3} = \frac{G_F^2 M_W^2}{4\pi^2}\Big[\lambda_c^2 C_{S2}^{cc}(\mu)+\lambda_t^2 C_{S2}^{tt}(\mu)+\lambda_c\lambda_t C_{S2}^{ct}(\mu)\Big]Q_{S2}+\mathrm{h.c.}$$

where $f_2 = 2\mathrm{Re}(\lambda_t\lambda_u^*)$, $f_3 = |\lambda_u|^2$ and, using PDG convention and CKM unitarity,

$$C_{S2}^{cc} \equiv \mathcal{C}_1, \quad C_{S2}^{ct} \equiv 2\mathcal{C}_1 - \mathcal{C}_3, \quad C_{S2}^{tt} \equiv \mathcal{C}_1 + \mathcal{C}_2 - \mathcal{C}_3$$

- A_{cu} denotes amplitude with internal charm and up
- $\mathcal{C}_1 \leftarrow A_{uu} - 2A_{cu} + A_{cc}$ bad short distance behaviour
- $\mathcal{C}_1$ determines ΔM_K via $\mathrm{Re}M_{12}$
- But $\mathcal{C}_1$ contributes to $\mathrm{Im}M_{12}$ and hence ϵ_K

Residual scale dependence



- QCD corrections to $C_{S2}^{ct} \rightarrow \eta_{ct} = 0.497(47)$
- QCD corrections to $C_{S2}^{cc} \rightarrow \eta_{cc} = 1.87(76)$

$\mathrm{Im}M_{12}$ without ΔM_K pollution

- Using CKM unitarity and the PDG convention we can also write (as used in Lattice [Christ et.al.]):

$$\mathcal{H}^{\Delta=2}_{f=3} = \frac{G_F^2 M_W^2}{4\pi^2}\Big[\lambda_u^2 C_{S2}^{uu}(\mu)+\lambda_t^2 C_{S2}^{tt}(\mu)+\lambda_u\lambda_t C_{S2}^{ut}(\mu)\Big]Q_{S2}+\mathrm{h.c.}$$

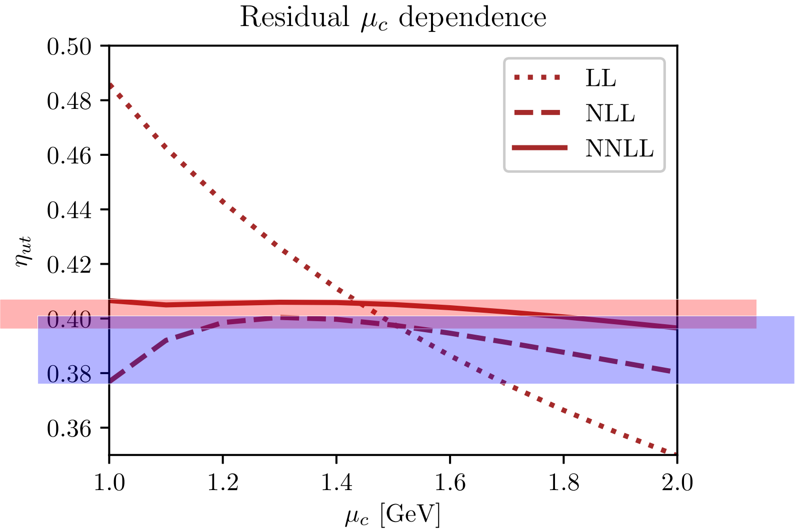
- Now real $\mathrm{Re}M_{12}$ and $\mathrm{Im}M_{12}$ are disentangled

$$C_{S2}^{uu} \equiv \mathcal{C}_1, \quad C_{S2}^{tt} \equiv \mathcal{C}_2, \quad C_{S2}^{ut} \equiv \mathcal{C}_3$$

$$\begin{aligned}\mathcal{C}_3 \;&\leftarrow\; (A_{tu} - A_{tc} + A_{cc} - A_{cu}) \leftarrow \\ &\leftarrow\; (A_{uu} - 2A_{cu} + A_{cc}) - (A_{tc} - A_{tu} + A_{uu} - A_{cu})\end{aligned}$$

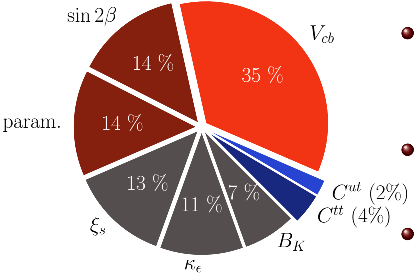
- Extract anomalous dimensions and matching from old calculation and incorporate matching from η_{cc}

Residual scale dependence



SM prediction (1911.06822) using PDG input

$$|\epsilon_K| = \kappa_\epsilon C_\epsilon \widehat{B}_K |V_{cb}|^2 \lambda^2 \bar{\eta} \times \left[|V_{cb}|^2 (1-\bar{\rho}) \eta_{tt}(x_t) - \eta_{ut}(x_c, x_t) \right]$$



- $\hat{B}_K = 0.7625(97)$
  [FLAG 2019, 1902.08191]
- $|\epsilon_K^{\rm SM}| = 2.16(18) \times 10^{-3}$
- $|\epsilon_K^{\rm exp}| = 2.228(11) \times 10^{-3}$

- Improvements 2108.00017 2-loop EW, NNLO $\mu_{\rm t}$ [Brod, Gorbahn, Stamou, Yu] and $\mu_{\rm Lattice}$ [Gorbahn, Jager, Kvedaraitė] matching in progress. Lattice κ_ϵ

New Physics

Heavy New Physics

$$\mathcal{H}_{\text{eff}} \supset \frac{C_{lq}^{(1),sd}}{(100\text{TeV})^2} \sum_{\ell=e,\mu,\tau} (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_{\ell L} \gamma^\mu \nu_{\ell L}) + \text{h.c.}$$

- Currently: $1/\sqrt{C_{lq}^{(1),sd}} \simeq 2$ in units of 100TeV @ 2σ
- 10% measurement: $1/\sqrt{C_{lq}^{(1),sd}} \simeq 4$ in units of 100TeV
- Same light ($m \leq v_{ew}$) particle content
  - Match onto $\Delta S = 1$ and $\Delta S = 2$ to find correlations in UV models
  - E.g. $K \to \pi\nu\bar{\nu}$, ΔM_K, ...
  - General one-loop result involves effects of symmetry breaking
- Could be extended to extra light degrees of freedom

Field content

Result should depend on Field content and minimal set of couplings

- In the SM:

Field	Mass	U(1)
W	m_W	1
Z	m_Z	0
e	0	-1
{u,t}	$\{0, m_t\}$	2/3

- CKM mixing and W couplings of Bosons to fermions

https://wellput.github.io/

Consider SM particle content & arbitrary couplings

```
In[1]:= AppendTo[$Path, NotebookDirectory[]]; << WellPut`

This Package is based on the work [2104.10930]
Type "wellPutInfo[]" for a description of all available functions.

In[2]:= SetOptions[getc, Externals → {{s, 0, -1/3}, {d, 0, -1/3}, {ν, 0, 0}},
  Leptons → {{μ, 0, -1}}, Quarks → {{u, 0, 2/3}, {t, mt, 2/3}},
  Scalars → {{h, mh, 0}}, ZBosons → {{Z, mz, 0}}, Vectors → {{W, mw, 1}}];

In[3]:= getc[{L, L}, {d, s, ν}, ReplaceCouplings → False, ReplaceCharges → False] // TraditionalForm
```

Out[3]//TraditionalForm=

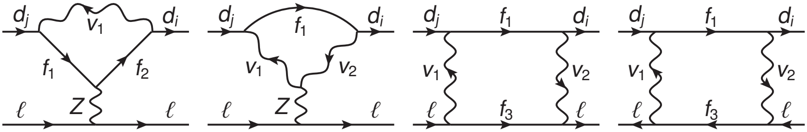
$$\frac{\mathrm{el}^2\, \mathrm{Q}_\nu\, g^L_{W\bar{d}t}\, g^L_{W\bar{t}s}\, \mathrm{F}^{\gamma Z}_{\mathrm{V}}(0, \mathrm{x}^t_W)}{\mathrm{m}_W{}^2} + \frac{g^L_{W\bar{d}t}\, g^L_{W\bar{t}s}\, g^L_{W\bar{\nu}\mu}\, g^L_{W\bar{\mu}\nu}\, \mathrm{F}^{\mathrm{L,B'Z}}_{\mathrm{V}}(0, \mathrm{x}^t_W, 1, 0)}{\mathrm{m}_W{}^2}$$

```
In[4]:= replaceFunctions[F["VB'Z", L, 0, x, 1, 0]] // TraditionalForm
```

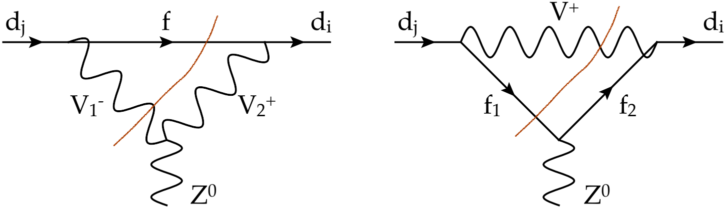
Out[4]//TraditionalForm=

$$\frac{x\,(x^2 + x + 3\,(x-2)\log(x) - 2)}{2\,(x-1)^2}$$

First consider only massive vector case



- Idea: Renormalisation via high energy tree-level properties derived in 1903.05116.



Remnants of gauge symmetry

- Massive vector bosons from a spontaneously broken gauge symmetry [Cornwall et.al. 73/74]
- Fix the gauge for massive vector ($\sigma_{V^\pm} = \pm i$, $\sigma_V = 1$)

$$\mathcal{L}_{\text{fix}} = -\sum_v (2\xi_v)^{-1} F_{\bar{v}} F_v \,, \qquad F_v = \partial_\mu V_v^\mu - \sigma_v \xi_v M_v \phi_v \,,$$

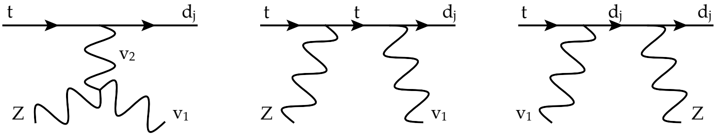
- BRST invariant field combination $s(\ldots)_{\text{ph}} = 0$
- STIs from $s\langle T\{\bar{u}_v(\ldots)_{\text{ph}}\}\rangle = 0$ at required order:

$$\langle T\Big\{k^\mu \underline{V_v^\mu} - i\sigma_{\bar{v}} M_v \underline{\phi_v}\Big\}(\ldots)_{\text{ph}}\rangle \,,$$

- E.g. for $(\ldots)_{\text{ph}} = \bar{f}_1 f_2$ we have

$$y^{L/R}_{\phi_1 \bar{f}_1 f_2} = -i\sigma_{v_1} \frac{1}{M_{v_1}} \left(m_{f_1} g^{L/R}_{v_1 \bar{f}_1 f_2} - g^{R/L}_{v_1 \bar{f}_1 f_2} m_{f_2} \right)$$

Identities for $d > 4$ Green's functions



Setting $v_3 = Z$, $f_2 = d_j$ there are two additional STIs:

$$g^L_{Z\bar{t}t} g^L_{v_1^+\bar{t}d_j} = g^L_{v_1^+\bar{t}d_j} g^L_{Z\bar{d}_j d_j} + \sum_{v_2} g_{Zv_1^+v_2^-} g^L_{v_2^+\bar{t}d_j}$$

$$g^R_{Z\bar{t}t} g^L_{v_1^+\bar{t}d_j} = \frac{1}{2} g^L_{v_1^+\bar{t}d_j} \left(g^L_{Z\bar{t}t} + g^L_{Z\bar{d}_j d_j} \right) + \sum_{v_2} \frac{M^2_{v_1} - M^2_Z}{2M^2_{v_2}} g_{Zv_1^+v_2^-} g^L_{v_2^+\bar{t}d_j}$$

Which can be used to eliminate $g^{L/R}_{Z\bar{t}t}$ from the expression

Generic Vector Interactions

$$\mathcal{L}_3^V = g^{L/R}_{v_1 \bar{f}_1 f_2} V_{v_1,\mu} \bar{\psi}_{f_1} \gamma^\mu P_{L/R} \psi_{f_2} + \frac{i}{6} g^{abc}_{v_1 v_2 v_3} \Big(V_{v_1,\mu} V_{v_2,\nu} \partial^{[\mu} V^{\nu]}_{v_3} + \dots \Big).$$

In SM, for $K \to \pi \nu \bar{\nu}$ we would need the following:

- $g^L_{W^+ \bar{u}_j d_k} = \frac{e}{s_w \sqrt{2}} V_{jk}, \quad y^L_{G^+ \bar{u}_j d_k} = \frac{m_{uj}}{M_W} \frac{e}{s_w \sqrt{2}} V_{jk}$
- $g^L_{Z \bar{f}_j f_k} = \frac{2e}{s_{2w}} \left(T_3^f - Q_f s_w^2 \right) \delta_{jk}, \quad g^R_{Z \bar{f}_j f_k} = -\frac{2e}{s_{2w}} Q_f s_w^2 \delta_{jk}$
- $g_{ZW^+W^-} = \frac{e}{t_w}, \; g_{ZW^+G^-} = -t_w^2 \frac{e}{t_w}, g_{ZG^+G^-} = \left(1 - \frac{1}{2c_W^2}\right) \frac{e}{t_w}$

E.g. we can combine Z/γ-Penguin and Boxes using:

$$\sum_Z g^\sigma_{Z \bar{\ell} \ell} g_{Z v_2 \bar{v}_1} = -\delta_{\bar{v}_1 v_2} g^\sigma_{\gamma \bar{\ell} \ell} g_{\gamma v_2 \bar{v}_1} - \sum_{f_3} \left(g^\sigma_{\bar{v}_1 \bar{\ell} f_3} g^\sigma_{v_2 \bar{f}_3 \ell} - g^\sigma_{v_2 \bar{\ell} f_3} g^\sigma_{\bar{v}_1 \bar{f}_3 \ell} \right)$$

Gauge independent result for $K \to \pi\nu\bar{\nu}$

$$C^{sd\nu}_{L\sigma} = \sum_{v_1 v_2 f_1 f_3} \frac{g^L_{\bar{v}_2 \bar{s} f_1} g^L_{v_1 \bar{f}_1 d}}{M^2_{v_1}} g^\sigma_{v_2 \bar{\nu} f_3} g^\sigma_{\bar{v}_1 \bar{f}_3 \nu} F^{\sigma,B'Z}_V(x^{f_0}_{v_1}, x^{f_1}_{v_1}, x^{v_1}_{v_2}, x^{f_3}_{v_1})$$
$$+ \sum_{Z v_1 v_2 f_1 f_2} \frac{g^\sigma_{Z\bar{\nu}\nu} g^L_{v_1 \bar{f}_1 d} g^L_{\bar{v}_2 \bar{s} f_2}}{M^2_Z} \Bigg\{ \delta_{f_1 f_2} g_{Z \bar{v}_1 v_2} F^Z_{V''}(x^{f_0}_{v_1}, x^{f_1}_{v_1}, x^{v_1}_{v_2})$$
$$+ \delta_{v_1 v_2} \left[g^L_{Z\bar{f}_2 f_1} F^Z_V(x^{f_1}_{v_1}, x^{f_2}_{v_1}) + g^R_{Z\bar{f}_2 f_1} F^Z_{V'}(x^{f_1}_{v_1}, x^{f_2}_{v_1}) \right] \Bigg\},$$

Extends the Penguin Box Coefficients to generic theories ($X_t \leftrightarrow F^{\sigma,B'Z}_V(0, x^t_W, 1, 0)$ & $F^Z_{V^{(\prime)}}(x,x) = F^Z_{V''}(x,y,1) = 0$)

- Full results includes also scalars and fermion flow in opposite direction in 2104.10930 and on `https://wellput.github.io/`.

Z' model with flavour off-diagonal couplings

- [1704.06005] for $b \to s\ell\ell$: vector-like T quark charged under spontaneously broken $U(1)''$

```
In[2]:= SetOptions[getc, Externals → {{s, 0, -1/3}, {d, 0, -1/3}, {ν, 0, 0}},
  Leptons → {{μ, 0, -1}}, Quarks -> {{u, 0, 2 / 3}, {t, Null, 2 / 3}, {T, Null, 2 / 3}},
  ZBosons -> {{Z, , 0}, {"Z'", , 0}}, Vectors → {{W, Null, 1}},
  CouplingRules → {g["vff", R, Z, a_, b_] :> 0 /; a =!= b,
    g["vff", L, _, u, t | T] -> 0, g["vff", L, _, t | T, u] -> 0}];

In[3]:= getc[{L, L}, {d, s, ν}, ReplaceCouplings → False] // TraditionalForm
```

3]//TraditionalForm=

$$\frac{g^L_{W\bar{d}t}\, g^L_{W\bar{t}s}\, g^L_{W\bar{\nu}\mu}\, g^L_{W\bar{\mu}\nu}\, F_V^{L,B'Z}(0, x^t_W, 1, 0)}{m_W{}^2} + \frac{g^L_{W\bar{d}T}\, g^L_{W\bar{T}s}\, g^L_{W\bar{\nu}\mu}\, g^L_{W\bar{\mu}\nu}\, F_V^{L,B'Z}(0, x^T_W, 1, 0)}{m_W{}^2} +$$

$$\frac{g^L_{Z'\bar{\nu}\nu}\, g^L_{W\bar{d}t}\, g^L_{W\bar{T}s}\, g^R_{Z'\bar{t}T}\, F^Z_{V'}(x^T_W, x^t_W)}{m_{Z'}{}^2} + \frac{g^L_{Z'\bar{\nu}\nu}\, g^L_{W\bar{d}T}\, g^L_{W\bar{t}s}\, g^R_{Z'\bar{T}t}\, F^Z_{V'}(x^t_W, x^T_W)}{m_{Z'}{}^2} + \frac{g^L_{Z'\bar{\nu}\nu}\, g^L_{W\bar{d}t}\, g^L_{W\bar{T}s}\, g^L_{Z'\bar{t}T}\, F^Z_V(x^t_W, x^T_W)}{m_{Z'}{}^2} +$$

$$\frac{g^L_{Z'\bar{\nu}\nu}\, g^L_{W\bar{d}T}\, g^L_{W\bar{t}s}\, g^L_{Z'\bar{T}t}\, F^Z_V(x^t_W, x^T_W)}{m_{Z'}{}^2} + \frac{g^L_{Z\bar{\nu}\nu}\, g^L_{W\bar{d}t}\, g^L_{W\bar{T}s}\, g^L_{Z\bar{t}T}\, F^Z_V(x^t_W, x^T_W)}{m_Z{}^2} + \frac{g^L_{Z\bar{\nu}\nu}\, g^L_{W\bar{d}T}\, g^L_{W\bar{t}s}\, g^L_{Z\bar{T}t}\, F^Z_V(x^t_W, x^T_W)}{m_Z{}^2}$$

- Exact result reproduces approximation in [1704.06005]

Conclusions

- Measurement of $K \to \pi \bar{\nu} \nu$ can be compared with precise theory prediction.
- New formula for ϵ_K allows for better theory control.
- Suppression in the Standard Model gives high sensitivity to new physics.
- Generic one-loop results for renormalisable models of new physics available.

Backup

Comparison with Older Work

- We find $\mathcal{B}_+^{BGS} = 7.73(16)_{SD}(25)_{LD}(54)_{para.} \times 10^{-11}$
- Improvements in m_t and α_S and slightly larger scale variation reduce X_t
- Largest difference to $\mathcal{B}_+^{BBGK} = (8.4 \pm 1.0) \cdot 10^{-11}$ from different parametric (CKM) input
- Numerical Update by BBGK used $|V_{ub}| = 3.88(29) \cdot 10^{-3}$, $|V_{cb}| = 40.7(1.4) \cdot 10^{-3}$ and $\gamma = (73.2^{+6.3}_{7.0})^\circ$ as inputs

	$\bar{\rho}$	$\bar{\eta}$
PDG/BGS 2021	0.141(17)	0.357(11)
BBGK 2015	0.119	0.394

- 2-loop EW Calculation by BGS 2010 used CKM fit for Wolfenstein parameters $\mathcal{B}_+^{BGS2010} = 7.81(29)_{SD+LD}(75)_{para.} \times 10^{-11}$