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# The story of $V_{cb}$ - continued -

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Keri Vos

in collaboration with M. Fael, Th. Mannel, K. Olschewsky and M. Rahimi  
in collaboration with F. Bernlochner, M. Welsch, R. van Tonder, E. Persson

JHEP 1902 (2019) 177 and work in progress  
[arXiv:1812.07472](#) and [arXiv:2105.02163](#)

# Why $V_{cb}$ ?

## $V_{cb}$ plays important role in CKM unitarity triangle

Kaon CP violation via  $\epsilon_K$

In Flavour-Changing-Neutral-Currents (FCNC)

Aiming at the highest precision possible

- SM test or NP probe?

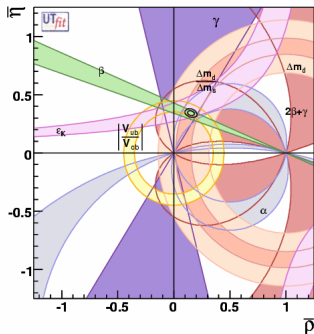
## Inclusive versus Exclusive

$B \rightarrow X_c \ell^+ \ell^-$  versus  $B \rightarrow D^{(*)} \ell^+ \ell^-$

use Heavy Quark Expansion

$V_{cb}$  puzzle

- Discrepancy between both determinations
- Recently a lot of attention for the  $V_{cb}$  puzzle! Bigi, Schacht, Gambino, Jung, Straub, Bernlochner, Bordone, van Dyk, Gubernari



Focus on new ideas for more precise inclusive  $|V_{cb}|$

# Inclusive decays and the Heavy Quark Expansion

Optical Theorem

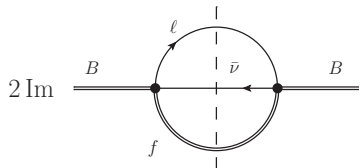
Heavy Quark Expansion (HQE)

- Split the momentum of  $b$  quark:  $p_b = m_b v + k$ , expand in  $k$   $iD$

Operator Product Expansion (OPE)

$$\begin{aligned}
 d\Gamma = & d\Gamma_0 + d\Gamma_2 \frac{\Lambda_{\text{QCD}}^2}{m_b} + d\Gamma_3 \frac{\Lambda_{\text{QCD}}^3}{m_b} + d\Gamma_4 \frac{\Lambda_{\text{QCD}}^4}{m_b} \\
 & + d\Gamma_5 a_0 \frac{\Lambda_{\text{QCD}}^5}{m_b} + a_1 \frac{\Lambda_{\text{QCD}}^3}{m_b} \frac{\Lambda_{\text{QCD}}^2}{m_c} + \dots
 \end{aligned}$$

## Operator Product Expansion (OPE)



$$2 \text{Im} \left[ \text{Diagram} \right] = \mathcal{P} \sum_{n,i} \frac{C_i^{(n)}(\mu, s)}{m_b^{i_b}} \langle h B j O_i^{(n)} j B i \rangle$$

$C_i(\mu)$ : short distance, perturbative coefficients

$\langle h B j O_i j B i \rangle$ : non-perturbative forward matrix elements of local operators  
 operators contain chains of covariant derivatives

$$\langle h B j O_i^{(n)} j B i \rangle = \langle h B j \bar{b}_v (i D \cdot \bar{v}) \dots (i D \cdot v)_n b_v j B i \rangle$$

# Decay rate

$\Gamma_i$  are power series in  $O(\alpha_s)$

$$\Gamma = \Gamma_0 + \frac{1}{m_b} \Gamma_1 + \frac{1}{m_b^2} \Gamma_2 + \frac{1}{m_b^3} \Gamma_3$$

$\Gamma_0$ : decay of the free quark (partonic contributions),  $\Gamma_1 = 0$

$\Gamma_2$ :  $^2$  kinetic term and the  $^2_G$  chromomagnetic moment

$$2M_B \ ^2 = B j \bar{b}_v i D \ i D \ b_v j B$$

$$2M_B \ ^2_G = B j \bar{b}_v ( \ i \ ) i D \ i D \ b_v j B$$

$\Gamma_3$ :  $^3_D$  Darwin term and  $^3_{LS}$  spin-orbit term

$$2M_B \ ^3_D = \frac{1}{2} B j \bar{b}_v [ i D \ ; [ i v D ; i D \ ] ] b_v j B$$

$$2M_B \ ^3_{LS} = \frac{1}{2} B j \bar{b}_v \ i D \ ; \ i v D ; i D \ \ ( \ i \ ) b_v j B$$

$\Gamma_4$ : 9 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

$\Gamma_5$ : 18 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

# Inclusive $V_{cb}$ determination

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# Moments of the spectrum

BABAR, PRD 68 (2004) 111104; BABAR, PRD 81 (2010) 032003; Belle, PRD 75 (2007) 032005

Non-perturbative matrix elements obtained from moments of differential rate

## Charged lepton energy

$$\langle E^n \rangle_{\text{cut}} = \frac{\int_{E > E_{\text{cut}}}^R dE \cdot E^n \frac{d\Gamma}{dE}}{\int_{E > E_{\text{cut}}}^R dE \cdot \frac{d\Gamma}{dE}}$$

## Hadronic invariant mass

$$\langle M_X^2 \rangle_{\text{cut}}^n = \frac{\int_{E > E_{\text{cut}}}^R dM_X^2 (M_X^2)^n \frac{d\Gamma}{dM_X^2}}{\int_{E > E_{\text{cut}}}^R dM_X^2 \frac{d\Gamma}{dM_X^2}}$$

$$R(E_{\text{cut}}) = \frac{\int_{E > E_{\text{cut}}}^R dE \cdot \frac{d\Gamma}{dE}}{\int_0^R dE \cdot \frac{d\Gamma}{dE}}$$

Moments up to  $n = 3; 4$  and with several energy cuts available

Experimentally necessary to use lepton energy cut



$$\begin{array}{c}
 R(E_{\text{cut}}) \quad \hbar E^n i_{\text{cut}} \quad \hbar (M_X^2)^n i_{\text{cut}} \\
 \downarrow \\
 2; \frac{2}{G}; \frac{3}{D}; \frac{3}{LS}; m_b; (m_c) \\
 \downarrow \\
 \text{Br}(\bar{B} \rightarrow X_c \gamma) \sim \frac{|V_{cb}|^2}{B} \left[ \Gamma_0 + \Gamma \frac{2}{m_b^2} + \Gamma_G \frac{2}{m_b^2} + \Gamma_D \frac{3}{m_b^3} \right] \\
 \downarrow \\
 V_{cb} = (42:21 \quad 0:78) \quad 10^{-3}
 \end{array}$$

Gambino, Schwanda, PRD 89 (2014) 014022;  
 Alberti, Gambino et al, PRL 114 (2015) 061802

# State-of-the-art

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290.

$$\Gamma \sim \frac{1}{jV_{cb}^2 m_b^5} \left[ \Gamma_0 + \Gamma_0^{(1)} \frac{-s}{m_b} + \Gamma_0^{(2)} \frac{-s^2}{m_b^2} + \frac{2}{m_b^2} \Gamma^{(G;0)} + \frac{-s}{m_b^2} \Gamma^{(G;1)} + \frac{3}{m_b^3} \Gamma^{(D;0)} + \mathcal{O}\left(\frac{1}{m_b^4}\right) \right]$$

Includes all known  $s$  and  $s^2$  corrections

Kinetic mass scheme [1411.6560,1107.3100](https://arxiv.org/abs/1411.6560); [hep-ph/0401063](https://arxiv.org/abs/hep-ph/0401063)

Only uses mild external constraints

Include terms up to  $1/m_b^3$

Assigned 1.4% theo. error due to missing higher orders

# Towards the ultimate precision in inclusive $V_{cb}$

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290.

$$\Gamma \sim j V_{cb}^2 m_b^5 \left[ \Gamma_0 + \Gamma_0^{(1)} \frac{s}{m_b} + \Gamma_0^{(2)} \frac{s^2}{m_b^2} + \frac{2}{m_b^2} \Gamma^{(G;0)} + \frac{s}{m_b^2} \Gamma^{(G;1)} + \frac{3}{m_b^3} \Gamma^{(D;0)} + \mathcal{O}\left(\frac{1}{m_b^4}\right) \right]$$

Proliferation of non-perturbative matrix elements

- 4 up to  $1/m_b^3$
- 13 up to  $1/m_b^4$  Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
- 31 up to  $1/m_b^5$  Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109

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$$\Gamma / jV_{cb}^2 m_b^5 = \Gamma_0 + \Gamma_0^{(1)} \frac{s}{m_b} + \Gamma_0^{(2)} \frac{s^2}{m_b^2} + \frac{2}{m_b^2} \Gamma^{(G;0)} + \frac{s}{m_b^2} \Gamma^{(G;1)} + \frac{3}{m_b^3} \Gamma^{(D;0)} + \mathcal{O}\left(\frac{1}{m_b^4}\right)$$

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## The story of $jV_{cb}$ continued:

Include  $s$  corrections to for  $\frac{3}{D}$  Mannel, Pivovarov [2020]

Full determination up to  $1/m_b^4$  from data Fael, KKV, Bernlochner et al. [in progress]

Reconsider how to deal with backgrounds Mannel, Rahimi, KKV [2021]

# Reparametrization Invariance

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# Reparametrization invariance

Dugan, Golden, Grinstein, Chen, Luke, Manohar, Hill, Solon, Heinonen, Mannel  
Mannel, KKV, JHEP 1806 (2018) 115

Choice of  $v$  not unique

Reparametrization Invariant (RPI) under an infinitesimal change

$$V \rightarrow V + \delta V$$
$${}_{RP} V = V \quad \text{and} \quad {}_{RP} iD = m_b V$$

Reparametrization invariance links different orders in  $1=m_b$

- Gives exact relations between different orders
- Resums towers of operators
- Reduces the number of independent parameters

Up to  $1=m_b^4$ : 8 parameters versus previous 13

Ratio between the rate with and without a cut

$$R(q_{\text{cut}}^2) = \frac{\int_{q^2 > q_{\text{cut}}^2} dq^2 \frac{d\Gamma}{dq^2}}{\int_0 dq^2 \frac{d\Gamma}{dq^2}}$$

$q^2$  moments

$$(q^2)^n_{\text{cut}} = \frac{\int_{q^2 > q_{\text{cut}}^2} dq^2 (q^2)^n \frac{d\Gamma}{dq^2}}{\int_{q^2 > q_{\text{cut}}^2} dq^2 \frac{d\Gamma}{dq^2}}$$

Hadronic mass and lepton energy moments are NOT RPI

Energy cut is not RPI, but  $q_{\text{cut}}^2$  is RPI and can be superimposed

# Alternative $V_{cb}$ determination

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# Alternative $V_{cb}$ Method

$$\begin{array}{c}
 R(q_{\text{cut}}^2) \quad h(q^2)^n i_{\text{cut}} \\
 \downarrow \\
 3; \quad G; \quad \tilde{D}, r_E; r_G; S_E; S_B; S_{qB}, m_b; m_c \\
 \downarrow \\
 \text{Br}(\bar{B} \rightarrow X_c \gamma) \sim \frac{jV_{cb}j^2}{B} \left[ \Gamma_3 + \Gamma_G \frac{2}{m_b^2} + \Gamma_{\tilde{D}} \frac{\sim 3}{m_b^3} + \Gamma_{r_E} \frac{r_E^4}{m_b^4} + \Gamma_{r_G} \frac{r_G^4}{m_b^4} + \Gamma_{S_B} \frac{S_B^4}{m_b^4} + \Gamma_{S_E} \frac{S_E^4}{m_b^4} + \Gamma_{S_{qB}} \frac{S_{qB}^4}{m_b^4} \right] \# \\
 \downarrow \\
 V_{cb} = ?
 \end{array}$$

Fael, Mannel, KKV, JHEP 02 (2019) 177

# $V_{cb}$ from $q^2$ moments

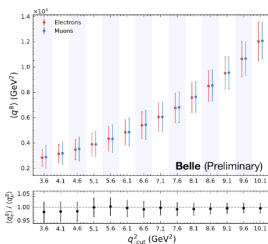
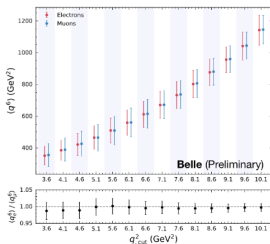
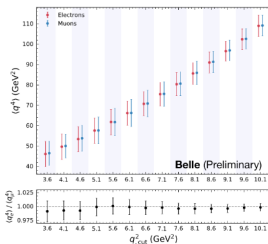
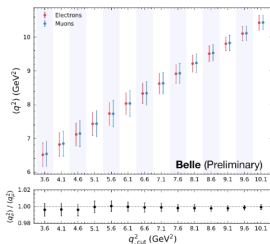
in collaboration with

F. Bernlochner, M. Welsch, M. Fael, K. Olschewsky, R. van Tonder

in progress

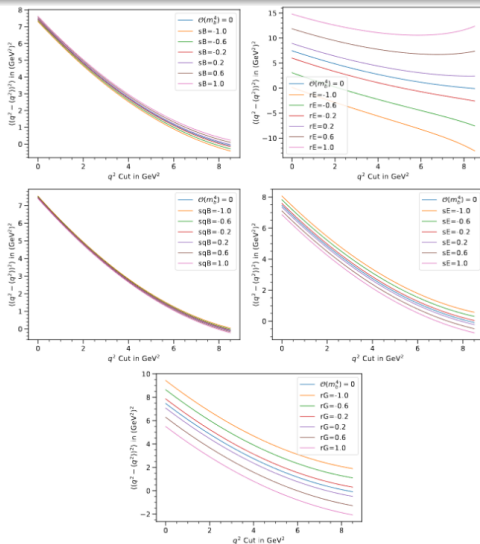
# New Belle $q^2$ measurements!

[2109.01685] and proceedings R. van Tonder [ArXiv:2105.08001]



Total statistical and systematic errors scaled by a factor of 10

# Sensitivity to higher orders



Sensitive to  $r_E$  and  $r_G$

Extracting both  $V_{cb}$  and HQE parameters up to  $1=m_b^4$  from data

## In progress: Software package

Moments and centralized moments

- Theoretical precision versus experimental precision

$s$  corrections  $1=m_b^2$  not yet known in progress

$s^3$  to partonic rate included Fael, Schoenwald, Steinhauser [2020, 2021]

Flexible theoretical covariance matrix

## Preliminary:

$V_{cb}$  rather insensitive to theory covariance, dominated by  $B \rightarrow X_c \ell \bar{\nu}$  branching ratio!

Higher order corrections have small influence

First determinations of  $V_{cb}$  very promising

- HQE parameters sensitive to theory covariance

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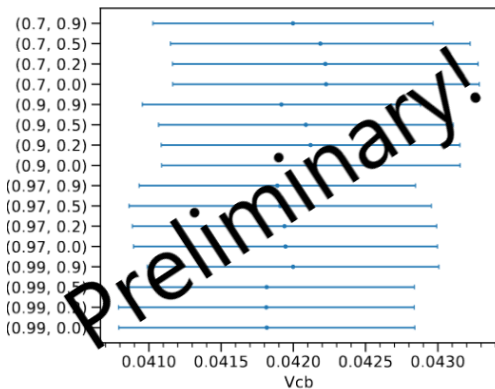
Higher order corrections have small influence

First determinations of  $V_{cb}$  very promising

- HQE parameters sensitive to theory covariance

Stay Tuned!

# Preliminary!!



Conservative first attempt, more details will follow. Stay tuned!

# Backgrounds in $B \rightarrow X_c$

in collaboration with

M. Rahimi and T. Mannel

JHEP 09 (2021) 051 [arXiv: 2105.02163]



Avoid background subtraction by calculating the full inclusive width:

$$\Gamma(B \rightarrow X_c \ell) = \Gamma(B \rightarrow X_c \ell) + \Gamma(B \rightarrow X_u \ell) + \Gamma(B \rightarrow X_c \ell + \text{hadrons})$$

$b \rightarrow u \ell$  contribution: suppressed by  $V_{ub}=V_{cb}$

$b \rightarrow c \ell + \text{hadrons}$  contribution: phase space suppressed

- can be calculated exactly in the OPE

QED effects

Quark-hadron duality violation?

Goal:

provide theoretical description and compare with Monte-Carlo data used by Belle (II)

Challenge:

estimate how much this description would improve  $V_{cb}$  determination

Can be analyzed in local OPE as  $B \rightarrow X_c \bar{c}$  by taking  $m_c \rightarrow 0$  limit

For  $V_{ub}$  determination

- large charm background requires experimental cuts
- reduces the inclusivity and local OPE no longer converges
- spectrum described by non-local OPE
- convolution of pert. coefficients with shape function

## Goal:

provide theoretical description and compare with Monte-Carlo data used by Belle (II)

$$\text{NLO} + 1 = m_b^2 + 1 = m_b^3$$

In agreement with partonic calc of DFN De Fazio, Neubert (1999); Gambino, Ossola, Uraltsev (2005)

First study: no  $\delta_s$  for  $1 = m_b^2$ , no additional uncert. due to missing higher orders

Inputs HQE parameters from  $B \rightarrow X_c \bar{c}$  study Gambino, Schwanda [2014]; Gambino, Healey, Turczy [2016]

# Monte Carlo versus HQE

Rahimi, Mannel, KKV [arXiv: 2105.02163]; De Fazio, Neubert 1999; Bosch, Lange, Neubert, Paz 2005

Compare local OPE with generator level Monte-Carlo data provided by Cao, Bernlochner

## Monte Carlo:

BLNP: specific shape function input parameters shape function parameters  $b = 3.95$  and  $\Lambda = 0.72$

DFN:  $s$  corrections convoluted with the exponential shape function model

- Inputs from  $B \rightarrow X_c^-$  and  $B \rightarrow X_s$  data using KN-scheme Kagan, Neubert 1998
- $(\frac{+}{1}; \frac{+}{2}; \frac{-}{1}; \frac{-}{2})$  are obtained by varying  $\alpha$  and  $\beta^2$  within 1 Buchmuller, Flacher, 2006

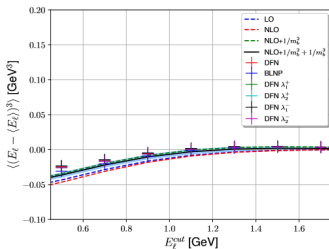
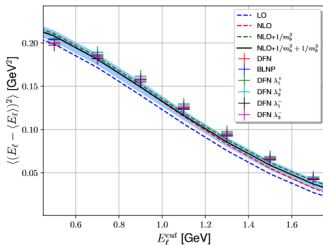
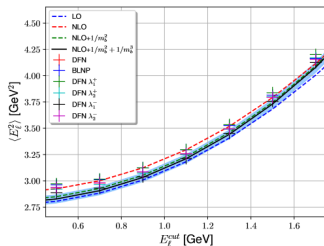
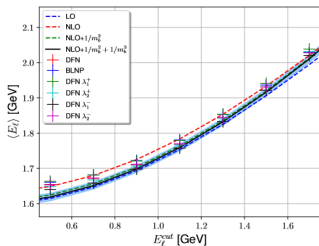
Hadronic contributions: "hybrid Monte Carlo" Belle Collaboration [arXiv:2102.00020.]

convolution with hadronization simulation based on PYTHIA

plus explicit resonances:  $B \rightarrow \rho^-$  and  $B \rightarrow \rho^0$

# Monte Carlo versus HQE

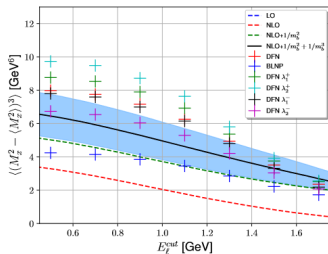
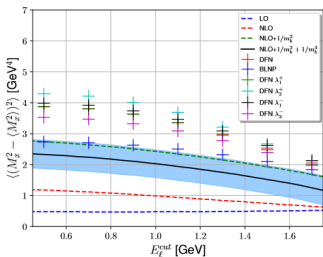
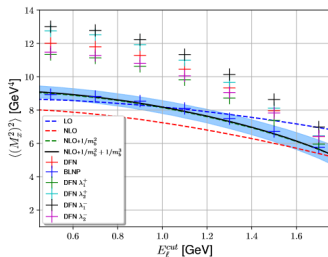
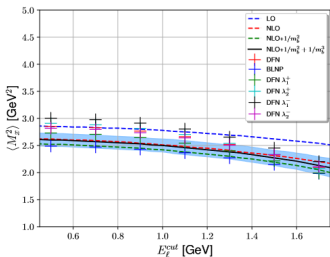
Rahimi, Mannel, KKV [arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner



MC-results are in good agreement with the HQE results

# Monte Carlo versus HQE

Rahimi, Mannel, KKV[arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner



Wide spread between MC for higher moments

Rahimi, Mannel, KKV[arXiv: 2105.02163];

## Remarks:

DFN: Smearing corresponding to a shape function, mimicking some non-perturbative effects; may not capture all

BLNP: should reproduce the HQE, with parameters adjusted to local HQE prediction

- should include higher moments of the shape-function model?
- include subleading shape functions?
- Update in progress

our HQE: interesting to include  $\alpha_s$  to HQE parameters,  $\frac{2}{s}$ ?

# Outlook

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## The story of $V_{cb}$ continues:

RPI reduces number of non-perturbative matrix elements

Total rate and  $q^2$  moments are RPI: 8 instead of 13 up to  $1=m_b^4$

Extract  $jV_{cbj}$  up to  $1=m_b^4$ , completely data driven

**NEW!  $q^2$  moments available!**

## In progress:

$V_{cb}$  from  $q^2$  moments

Detailed analysis of theory correlations



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**NEW!  $q^2$  moments available!**

## In progress:

$V_{cb}$  from  $q^2$  moments

Detailed analysis of theory correlations

Close collaboration between theory and experiment necessary!

# Backup

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# The $V_{cb}$ puzzle: Inclusive versus Exclusive decays

## Exclusive $B \rightarrow D^{(*)}$

Form factor required (only for  $B \rightarrow D$  available at different kinematic points)

Different parametrizations for form factors: CLN [Caprini, Lellouch, Neubert [1997]] and BGL

Boyd, Grinstein, Lebed [1995]

- BGL: model independent based on unitarity and analyticity
- CLN: Simple parametrization using HQE relations

Some inconsistencies in the Belle data were pointed out see e.g. van Dyk, Jung, Bordone,

Gubernari [2104.02094]

## Inclusive $B \rightarrow X_c$

Determined fully data driven including  $1/m_b$  power corrections

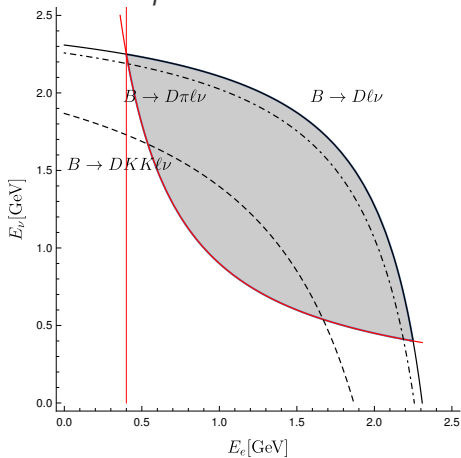
Recently a lot of attention for the  $V_{cb}$  puzzle! Bigi, Schacht, Gambino, Jung, Straub, Bernlochner,

Bordone, van Dyk, Gubernari

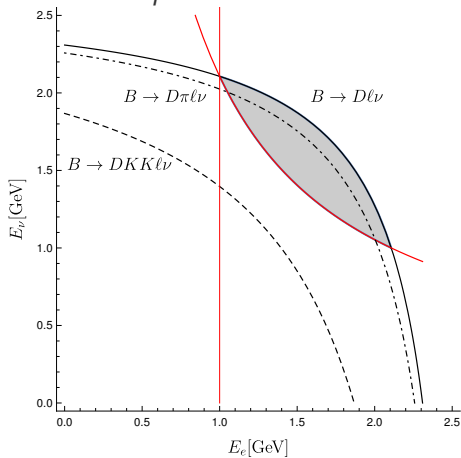
Stay tuned!

# $q^2$ versus energy cut

$$q^2 > 3.6 \text{ GeV}^2$$



$$q^2 > 8.4 \text{ GeV}^2$$



Contribution from five-body charm decay to  $b \rightarrow c \ell \ell$  via

$$B(p_B) \times X_c(p_{X_c}) \int (q_{1+}) \int (q_{1-}) \int (q_{2+}) \int (q_{2-}) \int (q_{3+}) \int (q_{3-})$$

:  
Phase space suppressed:

$$\frac{\Gamma_{\text{tot}}(b \rightarrow c \ell \ell \ell \ell)}{\Gamma_{\text{tot}}(b \rightarrow c \ell \ell)} \approx 4.0\%$$

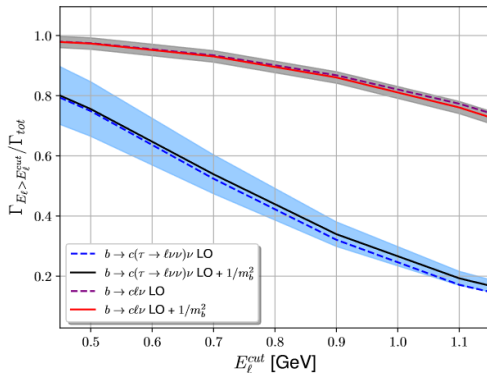
Experimentally effects diminished by cutting on the invariant mass of the  $B$

Can be calculated exactly in the HQE

$$\frac{d^8 \Gamma}{dq^2 d\bar{q}^2 - dp_{X_c}^2 d^2 \Omega d\Omega^* d^2 \Omega^{**}} = \frac{3G_F^2 V_{cb}^2 j^2 P^-(q^2 - m^2)(m^2 - \bar{q}^2) B(\ell \ell)}{2^{17} 5 m^8 m_b^3 q^2} W_L$$

- $L$  five-body leptonic tensor (narrow-width limit for  $\ell$ )
- $W$  standard hadronic tensor including HQE parameters

Interesting to search for new physics! Mannel, Rusov, Shahriaran (2017); Mannel, Rahimi, KKV [in progress]



No MC data available to test with