The story of V_{cb}
- continued -

Keri Vos

in collaboration with M. Fael, Th. Mannel, K. Olschewsky and M. Rahimi

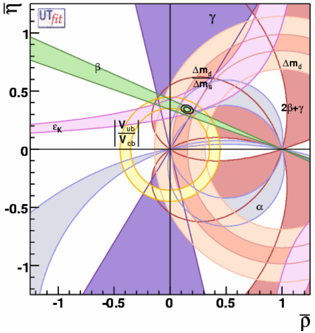
in collaboration with F. Bernlochner, M. Welsch, R. van Tonder, E. Persson

JHEP 1902 (2019) 177 and work in progress
arXiv:1812.07472 and arXiv:2105.02163

Why V_{cb}?

V_{cb} plays important role in CKM unitarity triangle

- Kaon CP violation via ϵ_K
- In Flavour-Changing-Neutral-Currents (FCNC)
- Aiming at the highest precision possible
  - SM test or NP probe?



Inclusive versus Exclusive

- $B \to X_c \ell\nu$ versus $B \to D^{(*)}\ell\nu$
- use Heavy Quark Expansion
- V_{cb} puzzle
  - Discrepancy between both determinations
  - Recently a lot of attention for the V_{cb} puzzle! Bigi, Schacht, Gambino, Jung, Straub, Bernlochner, Bordone, van Dyk, Gubernari

Focus on new ideas for more precise inclusive $|V_{cb}|$

Inclusive decays and the Heavy Quark Expansion

Inclusive B Decays

- Optical Theorem
- Heavy Quark Expansion (HQE)
  - Split the momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Operator Product Expansion (OPE)

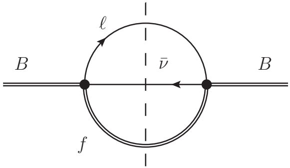
$$d\Gamma = d\Gamma_0 + d\Gamma_2 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 + d\Gamma_3 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 + d\Gamma_4 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^4 + d\Gamma_5 \left[a_0 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^5 + a_1 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2\right] + \cdots$$

Setting up the OPE

Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel, $\cdots$

Operator Product Expansion (OPE)

$2\,\mathrm{Im}$



$$= \sum_{n,i} \frac{C_i^{(n)}(\mu,\alpha_s)}{m_b^i} \langle B|\mathcal{O}_i^{(n)}|B\rangle_\mu$$

- $\mathcal{C}_i(\mu)$: short distance, perturbative coeficients
- $\langle B|\mathcal{O}_i|B\rangle_\mu$: non-perturbative forward matrix elements of local operators
- operators contain chains of covariant derivatives

$$\langle B|\mathcal{O}_i^{(n)}|B\rangle = \langle B|\bar{b}_v(iD_\mu)\ldots(iD_{\mu_n})b_v|B\rangle$$

Decay rate

Γ_i are power series in $\mathcal{O}(\alpha_s)$

$$\Gamma = \Gamma_0 + \frac{1}{m_b}\Gamma_1 + \frac{1}{m_b^2}\Gamma_2 + \frac{1}{m_b^3}\Gamma_3 \cdots$$

- Γ_0: decay of the free quark (partonic contributions), $\Gamma_1 = 0$
- Γ_2: μ_π^2 kinetic term and the μ_G^2 chromomagnetic moment

$$2M_B\mu_\pi^2 = -\langle B|\bar{b}_v iD_\mu iD^\mu b_v|B\rangle$$
$$2M_B\mu_G^2 = \langle B|\bar{b}_v(-i\sigma^{\mu\nu})iD_\mu iD_\nu b_v|B\rangle$$

- Γ_3: ρ_D^3 Darwin term and ρ_{LS}^3 spin-orbit term

$$2M_B\rho_D^3 = \frac{1}{2}\langle B|\bar{b}_v\,[iD_\mu,[ivD,iD^\mu]]\,b_v|B\rangle$$
$$2M_B\rho_{LS}^3 = \frac{1}{2}\langle B|\bar{b}_v\,\{iD_\mu,[ivD,iD_\nu]\}\,(-i\sigma^{\mu\nu})b_v|B\rangle$$

- Γ_4: 9 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109
- Γ_5: 18 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

Inclusive V_{cb} determination

Moments of the spectrum

BABAR, PRD 68 (2004) 111104; BABAR, PRD 81 (2010) 032003; Belle, PRD 75 (2007) 032005

Non-perturbative matrix elements obtained from moments of differential rate

Charged lepton energy

$$\langle E^n \rangle_{\rm cut} = \frac{\int_{E_\ell > E_{\rm cut}} dE_\ell \, E_\ell^n \, \frac{d\Gamma}{dE_\ell}}{\int_{E_\ell > E_{\rm cut}} dE_\ell \, \frac{d\Gamma}{dE_\ell}}$$

Hadronic invariant mass

$$\left\langle (M_X^2)^n \right\rangle_{\rm cut} = \frac{\int_{E_\ell > E_{\rm cut}} dM_X^2 \, (M_X^2)^n \, \frac{d\Gamma}{dM_X^2}}{\int_{E_\ell > E_{\rm cut}} dM_X^2 \, \frac{d\Gamma}{dM_X^2}}$$

$$R^*(E_{\rm cut}) = \frac{\int_{E_\ell > E_{\rm cut}} dE_\ell \, \frac{d\Gamma}{dE_\ell}}{\int_0 dE_\ell \, \frac{d\Gamma}{dE_\ell}}$$

- Moments up to $n = 3, 4$ and with several energy cuts available
- Experimentally necessary to use lepton energy cut

$$R^*(E_{\rm cut}) \quad \langle E^n\rangle_{\rm cut} \quad \langle (M_X^2)^n\rangle_{\rm cut}$$

$$\downarrow$$

$$\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, m_b, (m_c)$$

$$\downarrow$$

$$\mathrm{Br}(\bar{B} \to X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B}\left[\Gamma_0 + \Gamma_{\mu_\pi}\frac{\mu_\pi^2}{m_b^2} + \Gamma_{\mu_G}\frac{\mu_G^2}{m_b^2} + \Gamma_{\rho_D}\frac{\rho_D^3}{m_b^3}\right]$$

$$\downarrow$$

$$V_{cb} = (42.21 \pm 0.78) \times 10^{-3}$$

Gambino, Schwanda, PRD 89 (2014) 014022;
Alberti, Gambino et al, PRL 114 (2015) 061802

State-of-the-art

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290.

$$\Gamma \propto |V_{cb}|^2 m_b^5 \Bigg[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \frac{\mu_\pi^2}{m_b^2}\left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi}\Gamma^{(\pi,1)}\right)$$
$$+ \frac{\mu_G^2}{m_b^2}\left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi}\Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3}\Gamma^{(D,0)} + \mathcal{O}\left(\frac{1}{m_b^4}\right) \cdots \Bigg)$$

- Includes all known α_s and α_s^2 corrections
- Kinetic mass scheme 1411.6560,1107.3100; hep-ph/0401063
- Only uses mild external constraints
- Include terms up to $1/m_b^3$
- Assigned 1.4% theo. error due to missing higher orders

Towards the ulitmate precision in inclusive V_{cb}

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290.

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$$\left. + \frac{\mu_G^2}{m_b^2} \left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi} \Gamma^{(G,1)} \right) + \frac{\rho_D^3}{m_b^3} \Gamma^{(D,0)} + \mathcal{O}\left(\frac{1}{m_b^4}\right) \cdots \right)$$

- Proliferation of non-perturbative matrix elements
  - 4 up to $1/m_b^3$
  - 13 up to $1/m_b^4$ Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
  - 31 up to $1/m_b^5$ Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109

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The story of $|V_{cb}|$ continued:

- Include α_s corrections to for ρ_D^3 Mannel, Pivovarov [2020]
- Full determination up to $1/m_b^4$ from data Fael, KKV, Bernlochner et al. [in progress]
- Reconsider how to deal with backgrounds Mannel, Rahimi, KKV [2021]

Reparametrization Invariance

Reparametrization invariance

Dugan, Golden, Grinstein, Chen, Luke, Manohar, Hill, Solon, Heinonen, Mannel
Mannel, KKV, JHEP 1806 (2018) 115

- Choice of v not unique
- Reparametrization Invariant (RPI) under an infinitesimal change

$$v_\mu \to v_\mu + \delta v_\mu$$

$$\delta_{RP}\, v_\mu = \delta v_\mu \;\text{ and }\; \delta_{RP}\, iD_\mu = -m_b \delta v_\mu$$

- Reparametrization invariance links different orders in $1/m_b$
  - Gives exact relations between different orders
  - Resums towers of operators
  - Reduces the number of independent parameters
- Up to $1/m_b^4$: 8 parameters   versus previous 13

RPI Observables

Mannel, KKV, JHEP 1806 (2018) 115; Fael, Mannel, KKV, JHEP 02 (2019) 177

- Ratio between the rate with and without a cut

$$R^*(q^2_{\rm cut}) = \int_{q^2>q^2_{\rm cut}} dq^2 \, \frac{d\Gamma}{dq^2} \Bigg/ \int_0 dq^2 \, \frac{d\Gamma}{dq^2}$$

- q^2 moments

$$\langle (q^2)^n \rangle_{\rm cut} = \int_{q^2>q^2_{\rm cut}} dq^2 \, (q^2)^n \, \frac{d\Gamma}{dq^2} \Bigg/ \int_{q^2>q^2_{\rm cut}} dq^2 \, \frac{d\Gamma}{dq^2}$$

- Hadronic mass and lepton energy moments are NOT RPI
- Energy cut is not RPI, but $q^2_{\rm cut}$ is RPI and can be superimposed

Alternative V_{cb} determination

Alternative V_{cb} Method

$$R^*(q^2_{\rm cut}) \qquad \langle (q^2)^n \rangle_{\rm cut}$$

$$\downarrow$$

$$\mu_3, \mu_G, \tilde{\rho}_D, r_E, r_G, s_E, s_B, s_{qB}, m_b, m_c$$

$$\downarrow$$

$$\mathrm{Br}(\bar{B} \to X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \left[\Gamma_{\mu_3} \mu_3 + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \Gamma_{\tilde{\rho}_D} \frac{\tilde{\rho}_D^3}{m_b^3} + \Gamma_{r_E} \frac{r_E^4}{m_b^4} + \Gamma_{r_G} \frac{r_G^4}{m_b^4} + \Gamma_{s_B} \frac{s_B^4}{m_b^4} + \Gamma_{s_E} \frac{s_E^4}{m_b^4} + \Gamma_{s_{qB}} \frac{s_{qB}^4}{m_b^4} \right]$$

$$\downarrow$$

$$V_{cb} = ?$$

Fael, Mannel, KKV, JHEP 02 (2019) 177

V_{cb} from q^2 moments

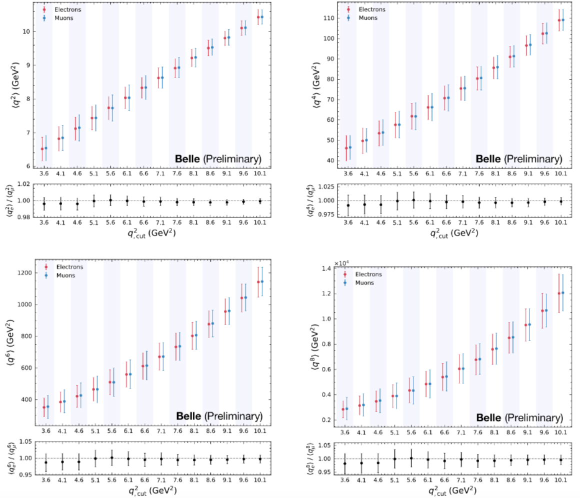
in collaboration with

F. Bernlochner, M. Welsch, M. Fael, K. Olschewsky, R. van Tonder

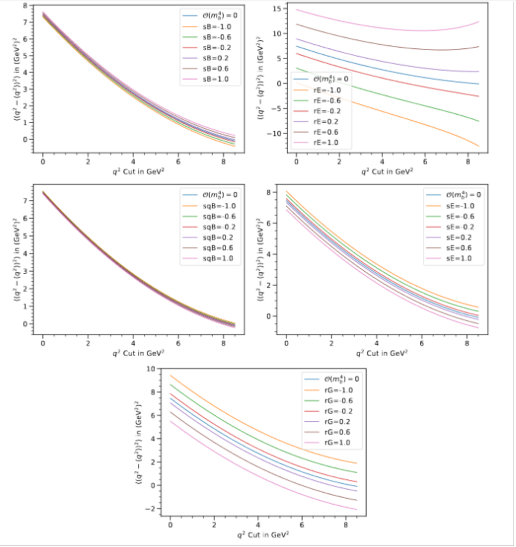
in progress

New Belle q^2 measurements!

[2109.01685] and proceedings R. van Tonder [ArXiV:2105.08001]



Total statistical and systematic errors scaled by a factor of 10



Sensitive to r_E and r_G

Towards a new extraction of V_{cb}

In progress

Extracting both V_{cb} and HQE parameters up to $1/m_b^4$ from data

In progress: Software package

- Moments and centralized moments
  - Theoretical precision versus experimental precision
- α_s corrections $1/m_b^2$ not yet known in progress
- α_s^3 to partonic rate included Fael, Schoenwald, Steinhauser [2020, 2021]
- Flexible theoretical covariance matrix

Preliminary:

- V_{cb} rather insensitive to theory covariance, dominated by $B \to X_c \ell \nu$ branching ratio!
- Higher order corrections have small influence
- First determinations of V_{cb} very promising
  - HQE parameters sensitive to theory covariance

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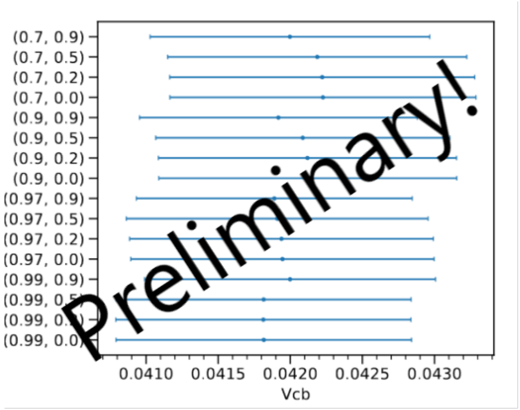
Stay Tuned!

Preliminary!!



Conservative first attempt, more details will follow. Stay tuned!

Backgrounds in $B \to X_c \ell \nu$

in collaboration with

M. Rahimi and T. Mannel

JHEP 09 (2021) 051 [arXiv: 2105.02163]

Contamination of the $B \to X_c \ell \nu$ signal

Rahimi, Mannel, KKV JHEP 09 (2021) 051 [arXiv: 2105.02163]

Avoid background subtraction by calculating the full inclusive width:

$$\mathrm{d}\Gamma(B \to X\ell) = \mathrm{d}\Gamma(B \to X_c \ell \bar{\nu}) + \mathrm{d}\Gamma(B \to X_u \ell \bar{\nu}) + \mathrm{d}\Gamma(B \to X_c(\tau \to \ell \bar{\nu} \nu)\bar{\nu})$$

- $b \to u \ell \nu$ contribution: suppressed by V_{ub}/V_{cb}
- $b \to c(\tau \to \mu \nu \bar{\nu})\bar{\nu}$ contribution: phase space suppressed
  - can be calculated exactly in the OPE
- QED effects
- Quark-hadron duality violation?

Goal:
provide theoretical description and compare with Monte-Carlo data used by Belle (II)

Challenge:
estimate how much this description would improve V_{cb} determination

$b \to u\ell\nu$ contribution: Local OPE

Neubert (1994); Bosch, Paz, Lange, Neubert (2004,2005)

- Can be analyzed in local OPE as $B \to X_c \ell \nu$ by taking $m_c \to 0$ limit
- For V_{ub} determination
  - large charm background requires experimental cuts
  - reduces the inclusivity and local OPE no longer converges
  - spectrum described by non-local OPE
  - convolution of pert. coefficients with shape function

Goal:

provide theoretical description and compare with Monte-Carlo data used by Belle (II)

- NLO + $1/m_b^2$+$1/m_b^3$
- In agreement with partonic calc of DFN De Fazio, Neubert (1999); Gambino, Ossola, Uraltsev (2005)
- First study: no α_s for $1/m_b^2$, no additional uncert. due to missing higher orders
- Inputs HQE parameters from $B \to X_c \ell \nu$ study Gambino, Schwanda [2014]; Gambino, Healey, Turczy [2016]

Monte Carlo versus HQE

Rahimi, Mannel, KKV [arXiv: 2105.02163]; De Fazio, Neubert 1999; Bosch, Lange, Neubert, Paz 2005

Compare local OPE with generator level Monte-Carlo data provided by Cao, Bernlochner

Monte Carlo:

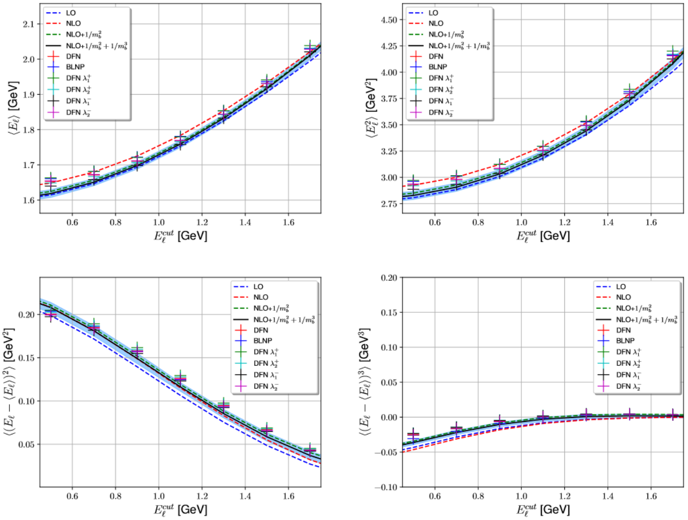
- BLNP: specific shape function input parameters shape function parameters $b = 3.95$ and $\Lambda = 0.72$
- DFN: α_s corrections convoluted with the exponential shape function model
  - Inputs from $B \to X_c \ell\nu$ and $B \to X_s\gamma$ data using KN-scheme Kagan, Neubert 1998
  - $(\lambda_1^+, \lambda_2^+, \lambda_1^-, \lambda_2^-)$ are obtained by varying $\bar{\Lambda}$ and μ_π^2 within 1σ Buchmuller, Flacher, 2006

Hadronic contributions: "hybrid Monte Carlo" Belle Collabroation [arXiv:2102.00020.]

- convolution with hadronization simulation based on PYTHIA
- plus explicit resonances: $\bar{B} \to \pi\ell\bar{\nu}$ and $\bar{B} \to \rho\ell\bar{\nu}$

Monte Carlo versus HQE

Rahimi, Mannel, KKV [arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner



MC-results are in good agreement with the HQE results

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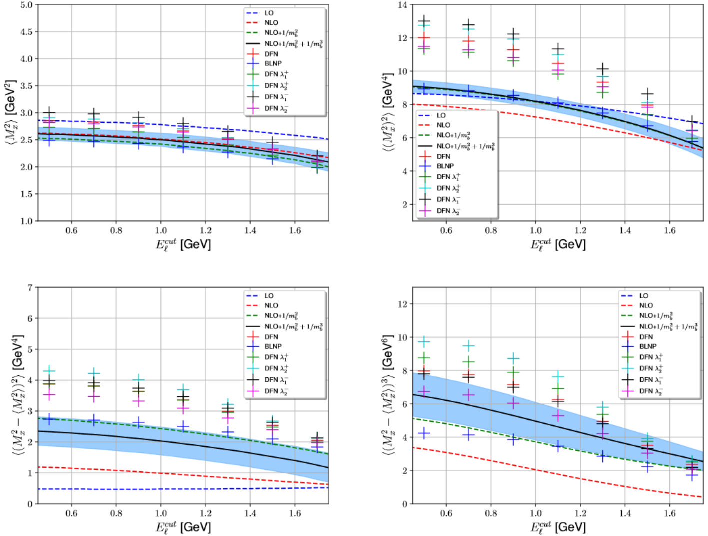

Wide spread between MC for higher moments

Monte Carlo versus local OPE: remarks

Rahimi, Mannel, KKV[arXiv: 2105.02163];

Remarks:

- DFN: Smearing corresponding to a shape function, mimicking some non-perturbative effects; may not capture all
- BLNP: should reproduce the HQE, with parameters adjusted to local HQE prediction
  - should include higher moments of the shape-function model?
  - include subleading shape functions?
  - Update in progress
- our HQE: interesting to include α_s to HQE parameters, α_s^2?

Outlook

Summary & Outlook

The story of V_{cb} continues:

- RPI reduces number of non-perturbative matrix elements
- Total rate and q^2 moments are RPI: 8 instead of 13 up to $1/m_b^4$
- Extract $|V_{cb}|$ up to $1/m_b^4$, completely data driven
- NEW! q^2 moments available!

In progress:

- V_{cb} from q^2 moments
- Detailed analysis of theory correlations

Summary & Outlook

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In progress:

- V_{cb} from q^2 moments
- Detailed analysis of theory correlations

Close collaboration between theory and experiment necessary!

Backup

The V_{cb} puzzle: Inclusive versus Exclusive decays

Exclusive $B \to D^{(*)}\ell\bar{\nu}$

- Form factor required (only for $B \to D$ available at different kinematic points)
- Different parametrizations for form factors: CLN Caprini, Lellouch, Neubert [1997] and BGL Boyd, Grinstein, Lebed [1995]
  - BGL: model independent based on unitarity and analyticity
  - CLN: Simple parametrization using HQE relations
- Some inconsistencies in the Belle data were pointed out see e.g. van Dyk, Jung, Bordone, Gubernari [2104.02094]

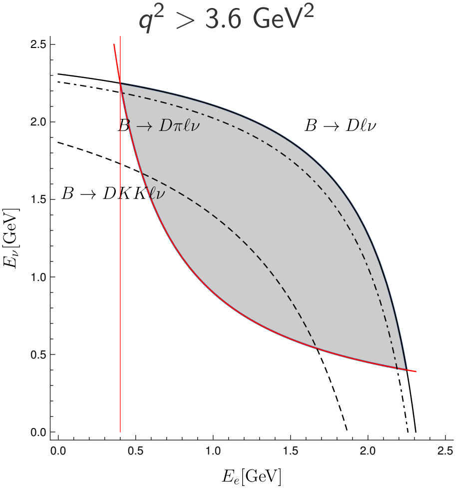
Inclusive $B \to X_c\ell\nu$

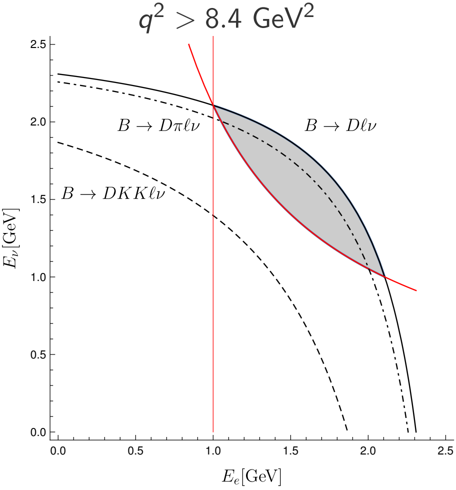
- Determined fully data driven including $1/m_b$ power corrections

Recently a lot of attention for the V_{cb} puzzle! Bigi, Schacht, Gambino, Jung, Straub, Bernlochner, Bordone, van Dyk, Gubernari

Stay tuned!

q^2 versus energy cut





Five-body τ contribution

Rahimi, Mannel, KKV JHEP 09 (2021) 051 [arXiv: 2105.02163];

Contribution from five-body charm decay to $b \to c\ell\nu$ via

$$B(p_B) \to X_c(p_{X_c})(\tau(q_{[\tau]}) \to \mu(q_{[\mu]})\nu_\mu(q_{[\bar{\nu}_\mu]})\nu_\tau(q_{[\nu_\tau]}))\bar{\nu}_\tau(q_{[\bar{\nu}_\tau]})$$

:

- Phase space suppressed:

$$\frac{\Gamma_{\text{tot}}(b \to c\tau(\to \ell\bar{\nu}_\ell\nu_\tau)\bar{\nu}_\tau)}{\Gamma_{\text{tot}}(b \to c\ell\bar{\nu})} \sim 4.0\%$$

- Experimentally effects diminished by cutting on the invariant mass of the B
- Can be calculated exactly in the HQE

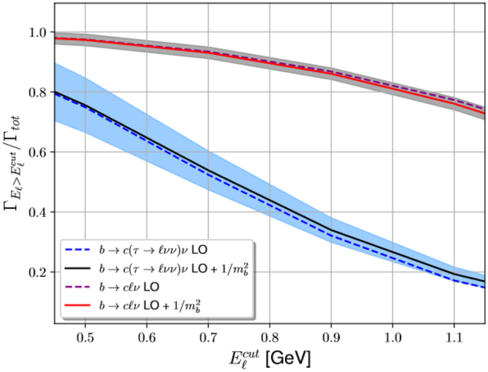
$$\frac{\mathrm{d}^8\Gamma}{\mathrm{d}q^2\,\mathrm{d}q^2_{\nu\bar{\nu}}\,\mathrm{d}p^2_{X_c}\,\mathrm{d}^2\Omega\,\mathrm{d}\Omega^*\,\mathrm{d}^2\Omega^{**}} = -\frac{3G_F^2|V_{cb}|^2\sqrt{\lambda}(q^2-m_\tau^2)(m_\tau^2-q^2_{\nu\bar{\nu}})\mathcal{B}(\tau\to\mu\nu\nu)}{2^{17}\pi^5 m_\tau^8 m_b^3 q^2} W_{\mu\nu}L^{\mu\nu}$$

  - $L_{\mu\nu}$ five-body leptonic tensor (narrow-width limit for τ)
  - $W_{\mu\nu}$ standard hadronic tensor including HQE parameters

- Interesting to search for new physics! Mannel, Rusov, Shahriaran (2017); Mannel, Rahimi, KKV [in progress]

Five-body τ contribution

Rahimi, Mannel, KKV[arXiv: 2105.02163];



No MC data available to test with