Perturbative unitarity constraints on generic Yukawa interactions

PERE ARNAN VENDRELL

Based on L. Allwicher, PA, D. Barducci, M. Nardecchia 2108.00013
(accepted for publication in JHEP)

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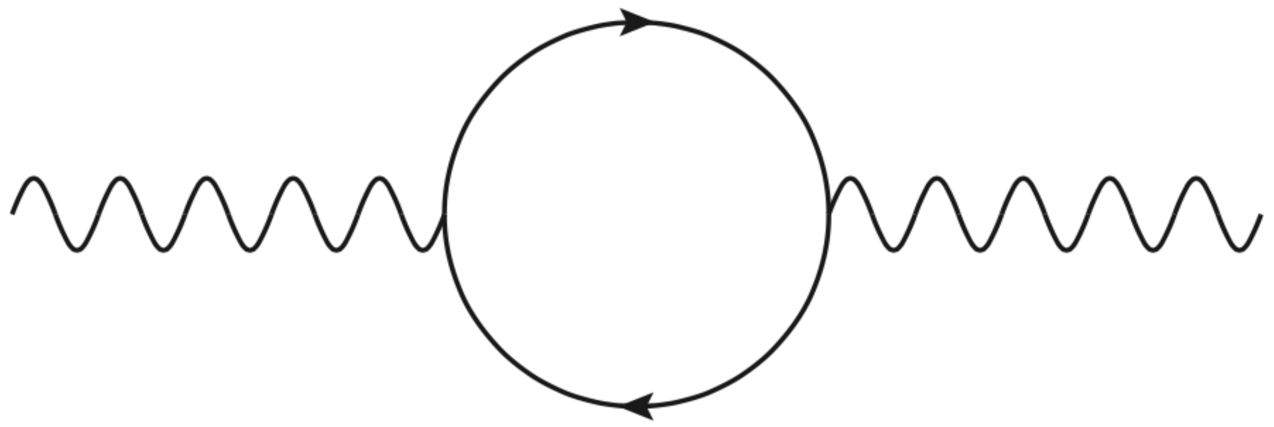
PRIN 2017L5W2PT "The consequences of flavor"

***Portorož* 22/09/2021**

Perturbative Unitarity

Assess Perturbativity of a given model



$\sim \frac{g^2}{16\pi^2} N_f$

$g < 4\pi$

$g < \sqrt{4\pi}$

?

What about Nf

Perturbative Unitarity relies on partial waves

Definite angular momentum J basis

$$a^J_{fi} = \frac{1}{32\pi}\int_{-1}^{1} \mathrm{d}\cos\theta\, d^J_{\mu_i\mu_f}(\theta)\,\mathcal{T}_{fi}(\sqrt{s},\cos\theta)$$

→ Amplitude from Feynman rules

Diagonalize PW and apply Optical Theorem

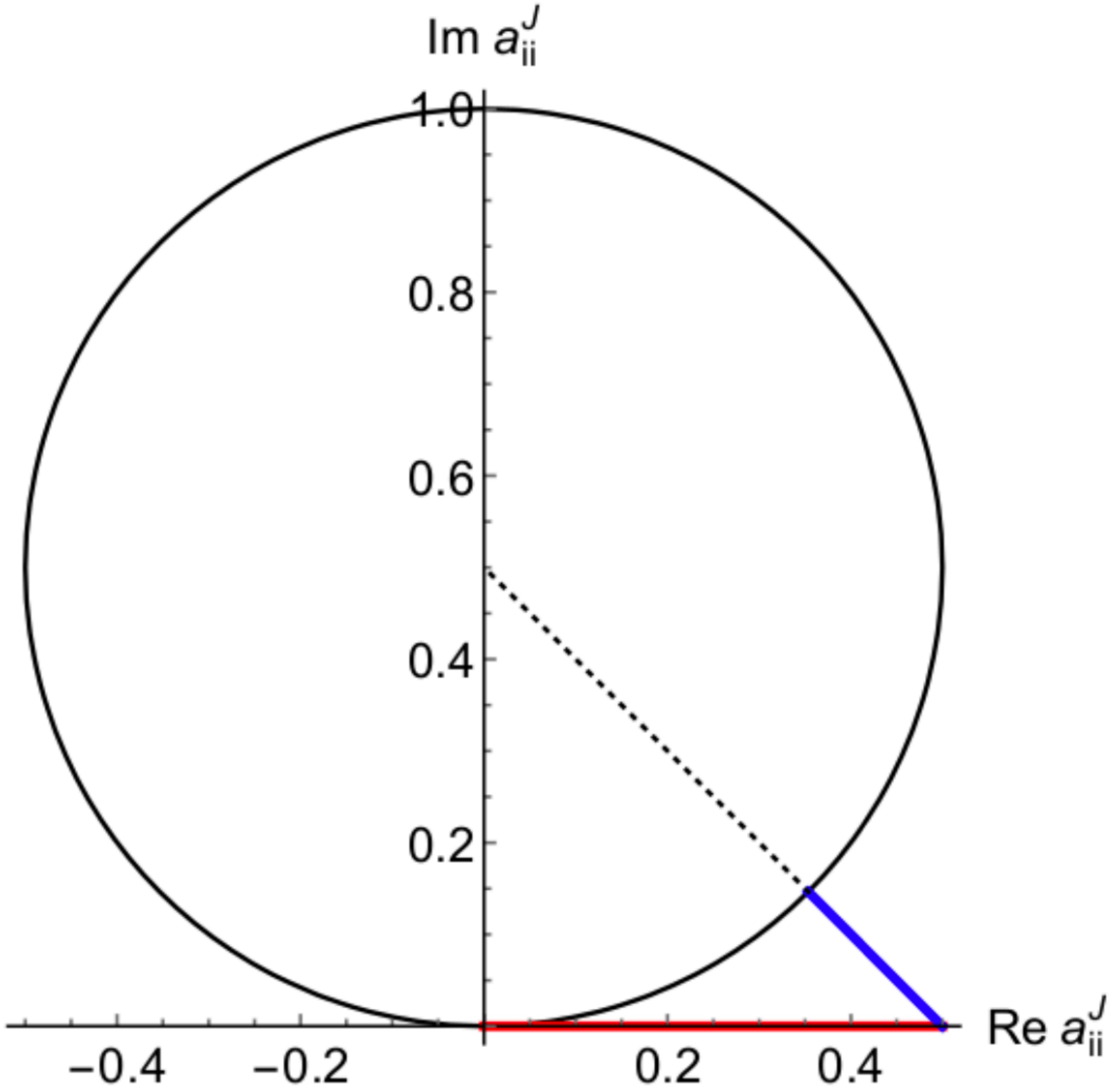
$$\mathrm{Re}^2[a^J_{ii}] + \left(\mathrm{Im}[a^J_{ii}] - \frac{1}{2}\right)^2 \leq \frac{1}{4}$$

Argand Circle

To all perturbation orders

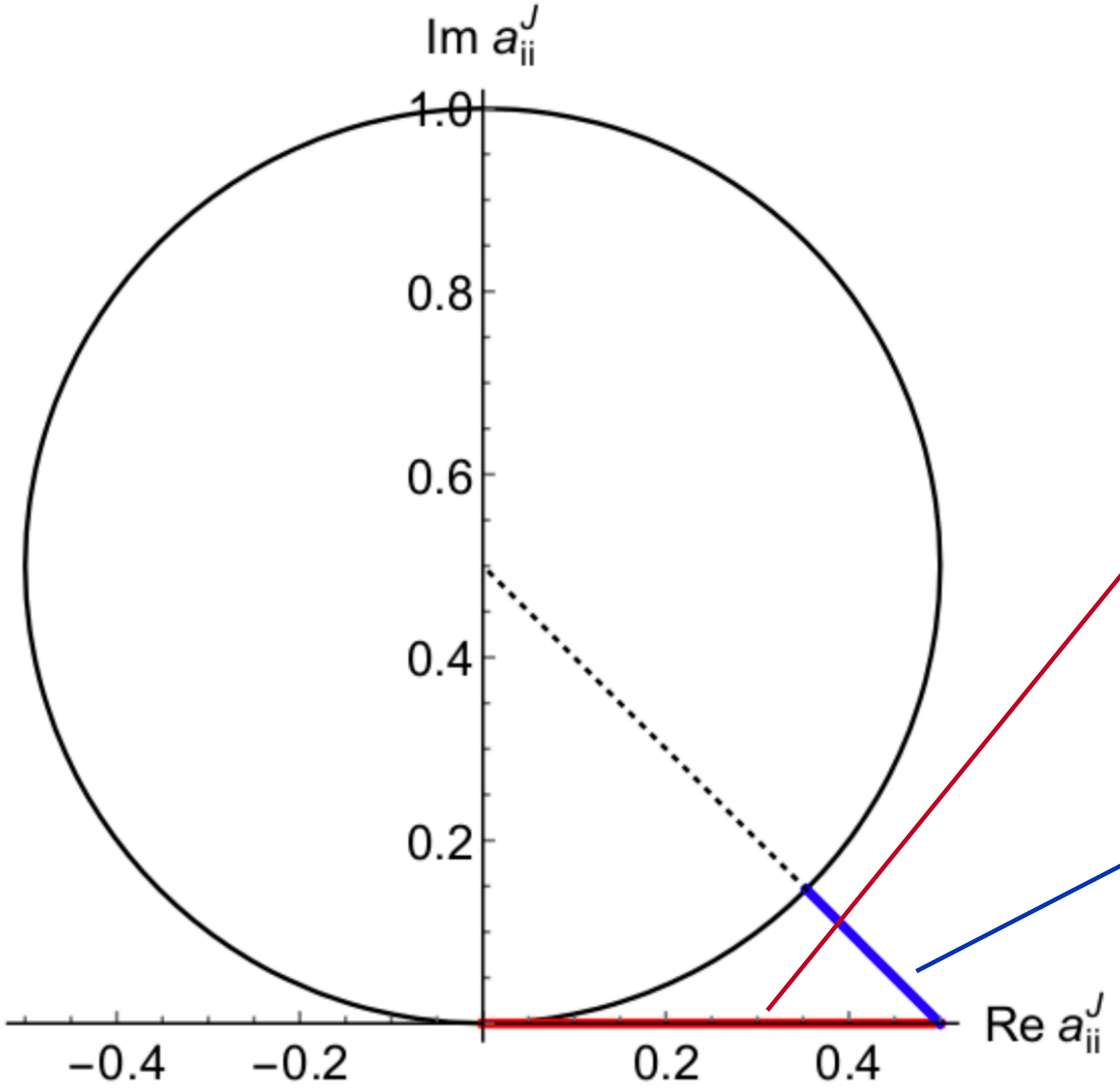
Perturbative Unitarity

$$\mathrm{Re}^2[a_{ii}^J] + \left(\mathrm{Im}[a_{ii}^J] - \frac{1}{2}\right)^2 \leq \frac{1}{4}$$



Perturbative Unitarity

$$\mathrm{Re}^2[a_{ii}^J] + \left(\mathrm{Im}[a_{ii}^J] - \frac{1}{2}\right)^2 \leq \frac{1}{4}$$



$$\mathrm{Re}(a_{ii}^{J,\mathrm{tree}}) \leq \frac{1}{2}$$

Largest Eigenvalue

In the massless limit all the tree-level 2 → 2 amplitudes are real.

Loop-correction has to be of order 40% to restore unitarity

Perturbative Unitarity: examples

$$\mathrm{Re}(a_{ii}^{J,\mathrm{tree}}) \leq \frac{1}{2}$$

Assessing Scales on EFTs

$$\mathrm{Re}(a_{ii}^{J,\mathrm{tree}}) \propto C s$$

Electroweak scale Fermi Theory

$$\sqrt{s} \leq \Lambda_{\mathrm{EW}}^{U} \simeq 900 \text{ GeV}$$

New Resonances

Di Luzio, Kamenik, Nardecchia *Eur.Phys.J.C* 77 (2017) 1, 30

WET RK anomaly C9=-C10

$$\Lambda_{R_K}^{U} \simeq 100 \text{ TeV}$$

SMEFT RK anomaly singlet

$$\Lambda_{R_K}^{U} \simeq 80 \text{ TeV}$$

Di Luzio, Nardecchia Eur.Phys.J.C 77 (2017) 8, 536 (update Allwicher)

New particle mass

$$\mathrm{Re}(a_{ii}^{J,\mathrm{tree}}) \propto M^2 \qquad \sqrt{s} \gg M$$

SU(2)xU(1) breaking Higgs mass

$$m_H \leq 1000 \text{ GeV}$$

Heavy fermions ***Dark Matter***

Assessing perturbativity of couplings

$$\mathrm{Re}(a_{ii}^{J,\mathrm{tree}}) \propto \frac{g^2}{M^2}$$

NP mass (LQ, Z', Heavier NP)

test NP couplings

Assess the perturbative unitarity of the Yukawa couplings

Our purpose: Yukawa Models

Given a Yukawa interaction between a SCALAR and 2 (Weyl) FERMIONS with "generic" quantum numbers under $\mathcal{G} = \prod_i SU(N_i) \otimes U(1)$, what is the maximum allowed value for the coupling requiring PU?

in 2->2 scattering processes

$$-\mathcal{L} = \frac{1}{2}\mathcal{Y}_{\alpha ij}\phi_\alpha \bar{\psi}^i_L \psi^{c,j}_L + h.c.$$

$$\mathcal{Y}_{\alpha ij} = \mathcal{Y}_{\alpha ji}$$

Yukawa coupling containing all possible SU(N) or flavor indices

$$i, j = 1, \ldots N_\psi$$

$$\alpha = 1, \ldots N_\phi$$

ψ^i_L ***Left-handed fermion*** $\psi^c_{L,i} = C\bar{\psi}^T_{L,i}$

ϕ_α ***Real Scalar***

Given a model one has to compute the partial waves and extract the highest bound

taking the high energy limit $\sqrt{s} \gg M$

Partial Waves & Amplitudes *T* $2 \to 2$

$$a^J_{fi} = \frac{1}{32\pi}\int_{-1}^{1} \mathrm{d}\cos\theta\, d^J_{\mu_i\mu_f}(\theta)\,\mathcal{T}_{fi}(\sqrt{s},\cos\theta)$$

massless limit: Helicity=Chirality **Classify amplitudes by helicity states.**

$\mathcal{T}_{fi} =$

	$\mu_i = 0$ $++$	$\mu_i = 0$ $--$	$\mu_i = 0$ 00	$\mu_i = +1$ $+-$	$\mu_i = +1/2$ $+0$	$\mu_i = -1/2$ -0
$\mu_f = 0$ $++$	$\mathcal{T}^{++++}$	$\mathcal{T}^{++--}$	⊗	⊗	⊖	
$\mu_f = 0$ $--$	$\mathcal{T}^{--++}$	$\mathcal{T}^{----}$				
$\mu_f = 0$ 00	⊗		☒	$\mathcal{T}^{00+-}$	⊖	
$\mu_f = +1$ $+-$	⊗		$\mathcal{T}^{+-00}$	$\mathcal{T}^{+-+-}$	⊖	
$\mu_f = +1/2$ $+0$	⊖		⊖	⊖	$\mathcal{T}^{+0+0}$	⊗
$\mu_f = -1/2$ $+0$					⊗	$\mathcal{T}^{-0-0}$

⊖ ***Angular momentum conservation***

⊗ ***Massless limit***

☒ ***No scalar potential***

First Contributing **J=0**

$$\begin{pmatrix} \mathcal{T}^{++++} & \mathcal{T}^{++--} \\ \mathcal{T}^{--++} & \mathcal{T}^{----} \end{pmatrix}$$

First Contributing **J=1/2**

$$\mathcal{T}^{+0+0}$$

First Contributing **J=1**

$$\begin{pmatrix} 0 & \mathcal{T}^{00+-} \\ \mathcal{T}^{+-00} & \mathcal{T}^{+-+-} \end{pmatrix}$$

Toy Models $SU(N) \times U(1)$

$$-\mathcal{L}_{\text{Dirac}} = y S \bar{\chi} \eta + h.c.$$

Real or Complex scalar S
(working with S S)*

LH fermion χ

RH fermion η

$$-\mathcal{L}_{\text{Majorana}} = \frac{1}{2} y S \bar{\chi} \chi^c + h.c.$$

Real or Complex scalar S
(working with S S)*

LH fermion χ

Charging fields under a single SU(N) and U(1)

$$\mathcal{T}^{\lambda_{f_1}\lambda_{f_2}\lambda_{i_1}\lambda_{i_2}}_{f_1 f_2 i_1 i_2}(\sqrt{s},\theta) = \bigoplus_{\mathbf{r}} \sum_{m=s,t,u} \mathcal{T}_m^{\lambda_{f_1}\lambda_{f_2}\lambda_{i_1}\lambda_{i_2}}(\sqrt{s},\theta) \mathcal{F}^{m,\mathbf{r}}_{f_1 f_2 i_1 i_2}(N) \mathbb{1}_{d_{\mathbf{r}}}$$

Lorentz Structure

Group Structure

r representation of 2 particle states

Toy Models: Model 1 Real Scalar

$$-\mathcal{L}_{\rm Dirac} = yS\bar{\chi}\eta + h.c.$$

Singlet Scalar S

fundamental LH fermion χ

fundamental RH fermion η

$$\chi \sim \square_q,\ \eta \sim \square_{q'}\quad S \sim \mathbf{1}_{q-q'}$$

REAL SCALAR q=q'

Consider all the possible 2 particle states

J=0

$\bar{\chi}\eta \sim \mathbf{1} + \text{Adj}$

$\eta\eta \sim \mathbf{S} + \mathbf{AS}$

$\chi\chi \sim \mathbf{S} + \mathbf{AS}$

$$y^2 < \frac{8\pi}{2N+1}$$

J=1/2

$\bar{\chi}S \sim \bar{\square}$

$\eta S \sim \square$

$$y^2 < \frac{16\pi}{3}$$

J=1

$\bar{\chi}\chi \sim \mathbf{1} + \text{Adj}$

$\eta\chi \sim \mathbf{S} + \mathbf{AS}$

$\eta\bar{\eta} \sim \mathbf{1} + \text{Adj}$

$SS \sim \mathbf{1}$

$$y^2 < 16\pi$$

Toy Models: Model 1 Complex Scalar

$$-\mathcal{L}_{\text{Dirac}} = yS\bar{\chi}\eta + h.c.$$

Singlet Scalar S

fundamental LH fermion χ

fundamental RH fermion η

$$\chi \sim \square_q, \eta \sim \square_{q'} \quad S \sim \mathbf{1}_{q-q'}$$

COMPLEX SCALAR

Consider all the possible 2 particle states

J=0

$$\bar{\chi}\eta \sim \mathbf{1} + \text{Adj}$$
$$\eta\eta \sim \mathbf{S} + \mathbf{AS}$$
$$\chi\chi \sim \mathbf{S} + \mathbf{AS}$$

$$y^2 < \frac{8\pi}{N}$$

J=1/2

$$\bar{\chi}S^{(*)} \sim \bar{\square}$$
$$\eta S^{(*)} \sim \square$$

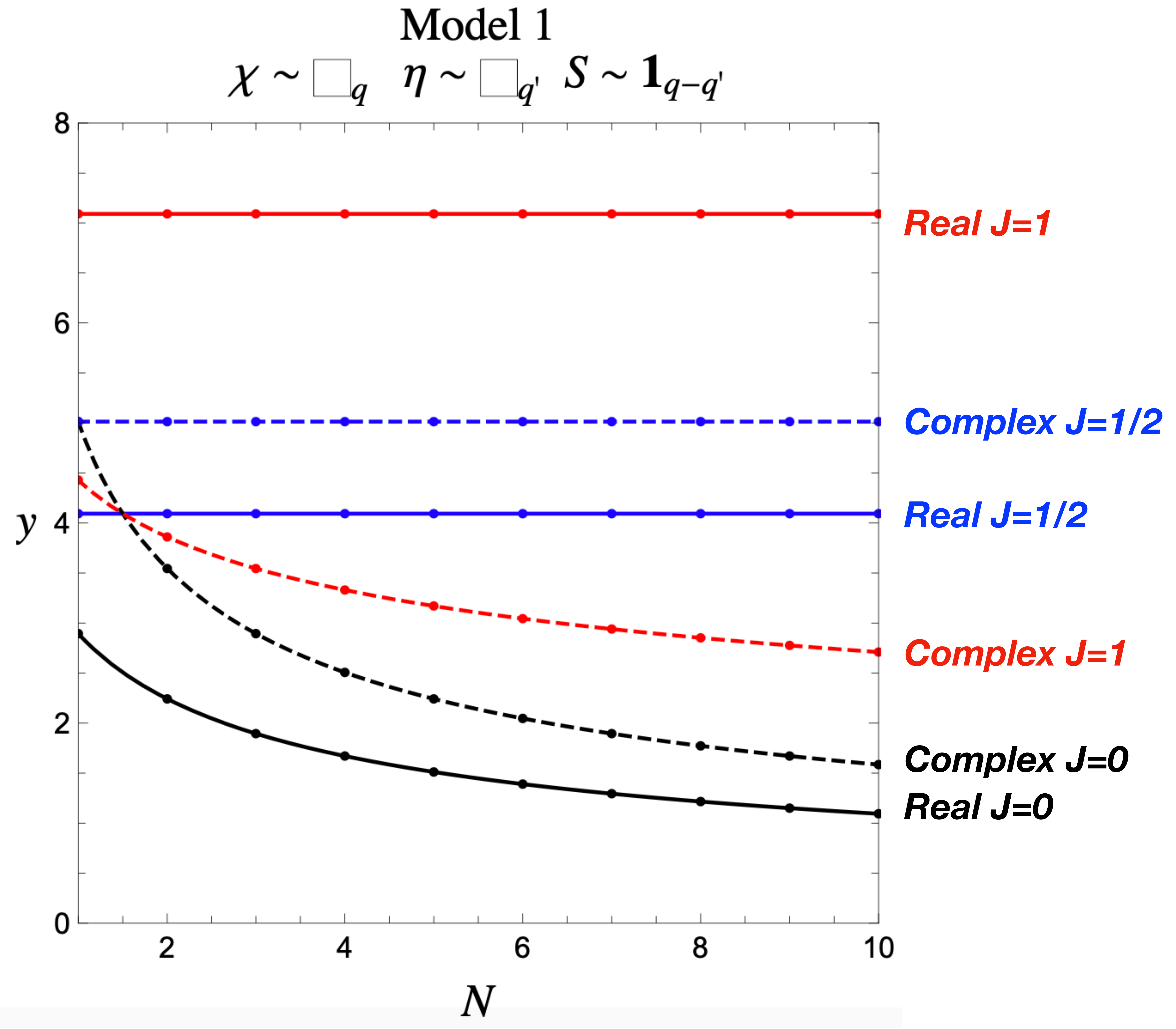
$$y^2 < 8\pi$$

J=1

$$\bar{\chi}\chi \sim \mathbf{1} + \text{Adj}$$
$$\eta\chi \sim \mathbf{S} + \mathbf{AS}$$
$$\eta\bar{\eta} \sim \mathbf{1} + \text{Adj}$$
$$S^{(*)}S^{(*)} \sim \mathbf{1}$$

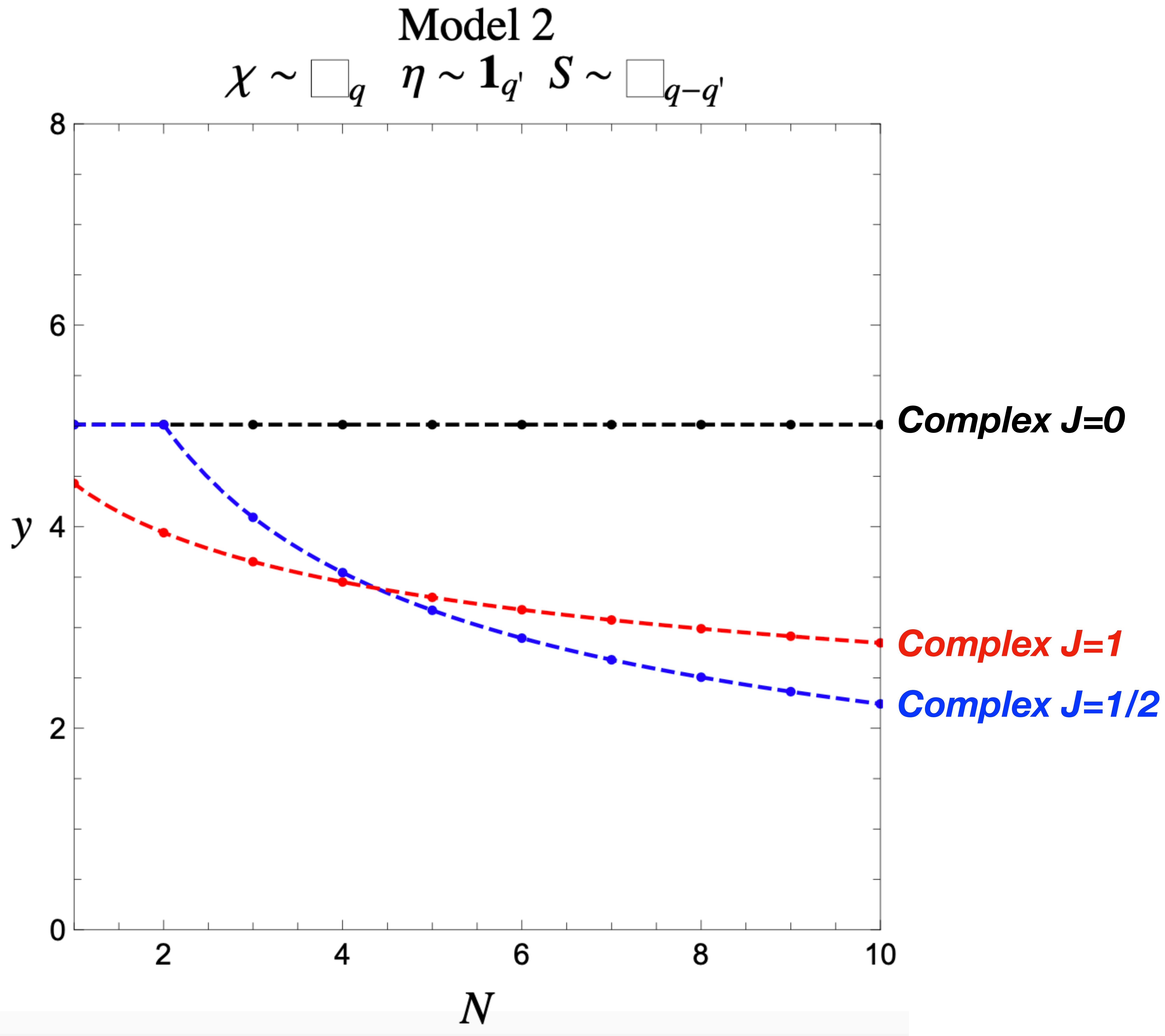
$$y^2 < \frac{32\pi}{1 + \sqrt{1 + 16N}}$$

Toy Models: Model 1



For both complex and real scalar cases the higher bound is in the J=0 channel

Toy Models: Model 2

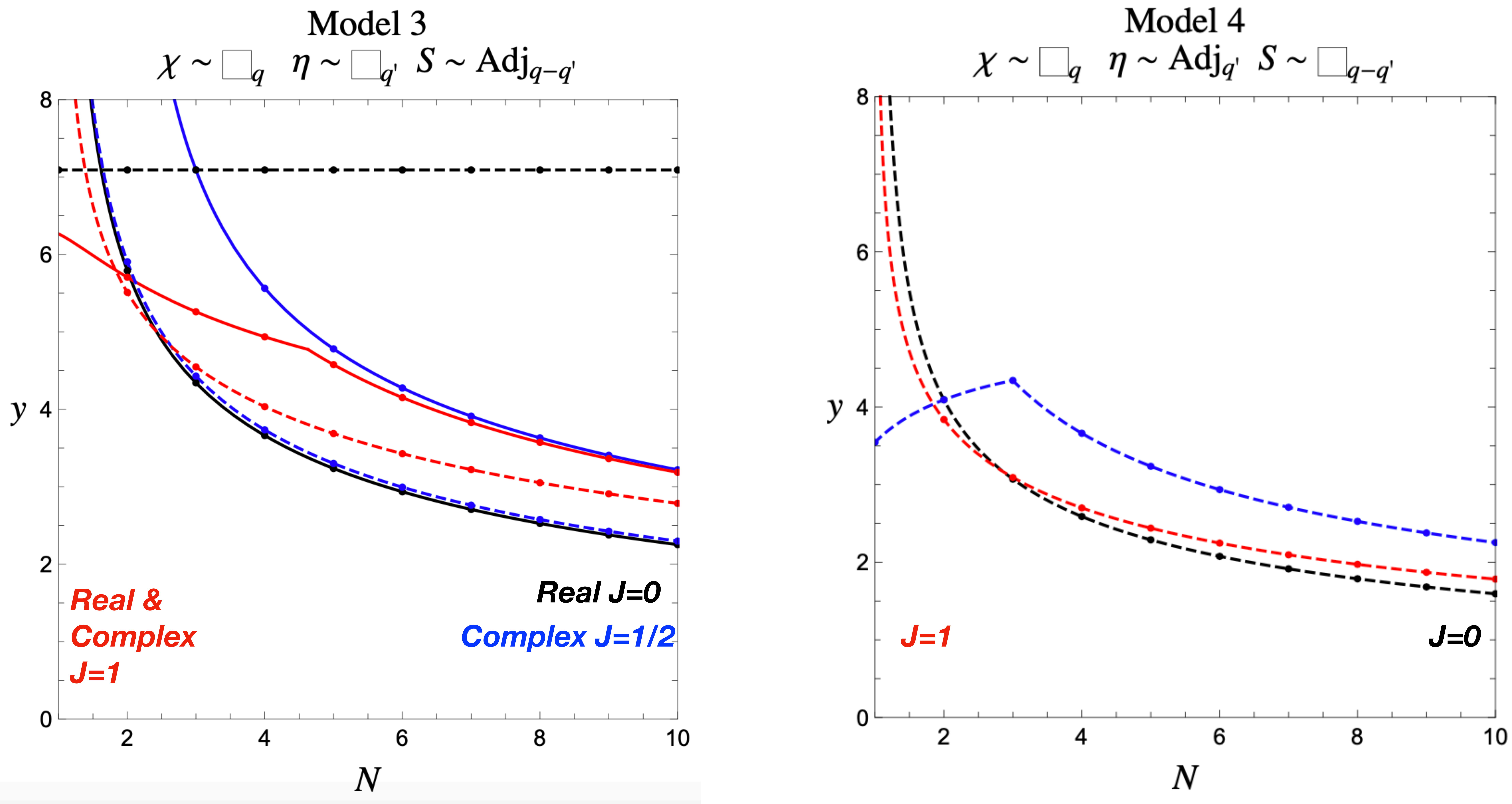


Only complex scalar.

Bound dominated by J=1/2 at large N

Bound dominated by J=1 for N<4

Toy Models: Model 3&4



2 SU(N) symmetries SM Higgs

$$-\mathcal{L} = y_d^{ij} H \bar{q}^i d^j + h.c.$$

q $\sim \square$ **d** $\sim \square$ **H** ~ 1 *SU(3) Model 1* $y_d \lesssim 2.9$

Complex scalar

q $\sim \square$ **H** $\sim \square$ **d** ~ 1 *SU(2) Model 2* $y_d \lesssim 3.9$

Compute factors for all the possible 2 particle states

$$\mathcal{T}^{\lambda_{f_1}\lambda_{f_2}\lambda_{i_1}\lambda_{i_2}}_{f_1 f_2 i_1 i_2}(\sqrt{s},\theta) = \text{Lorentz} \times SU(3) \times SU(2)$$

Highest Bound **J=0** SU(3) *singlet* SU(2) *fundamental*

$$y_d \lesssim 2.9$$

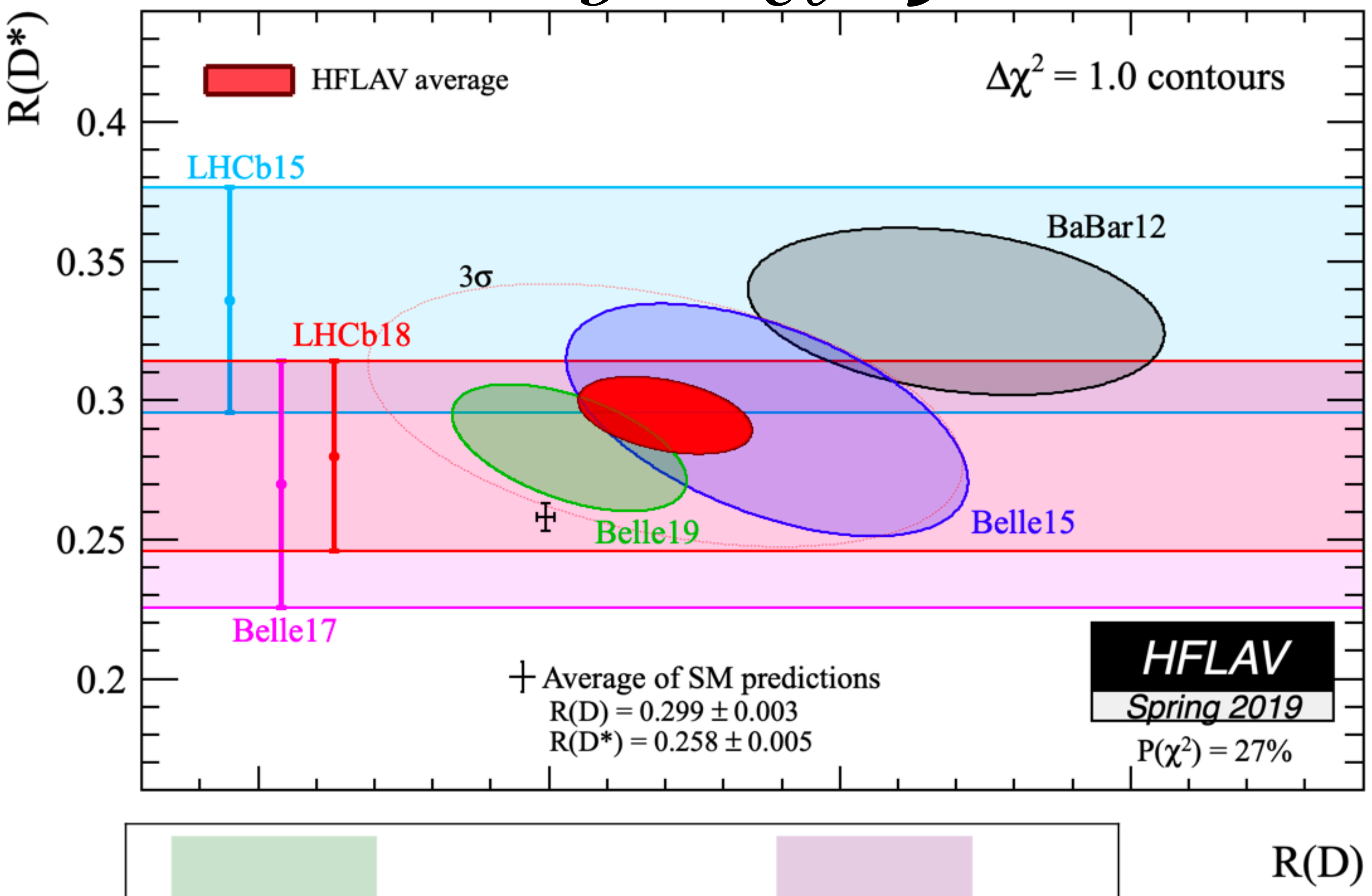
EWSB → *Mq* $\lesssim 500$ GeV

$$-\mathcal{L} = y_\ell H \bar{\ell} e$$

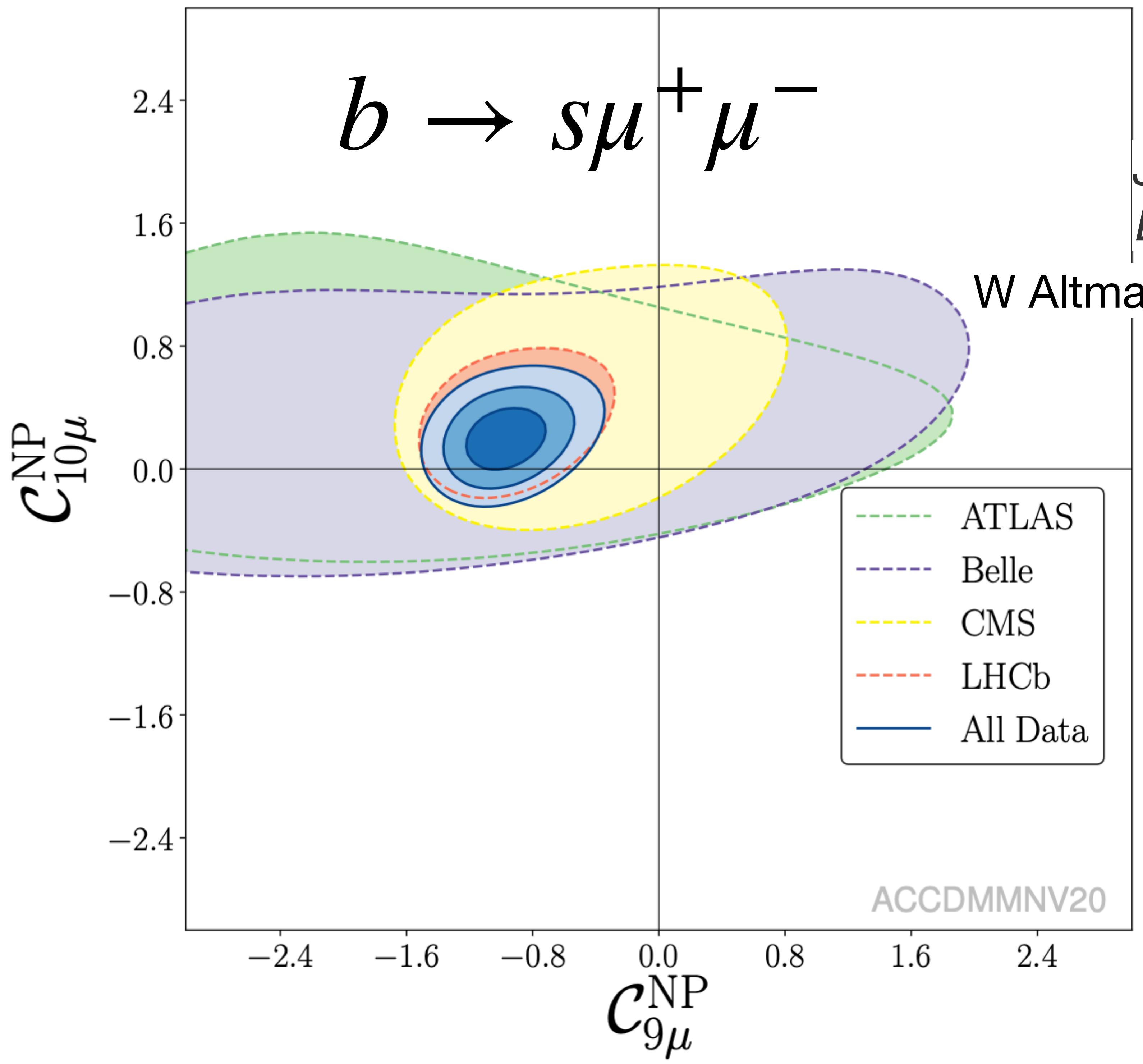
Mlepton $\lesssim 700$ GeV

Pheno Models: Flavor Anomalies

$b \to c\ell^+\nu$



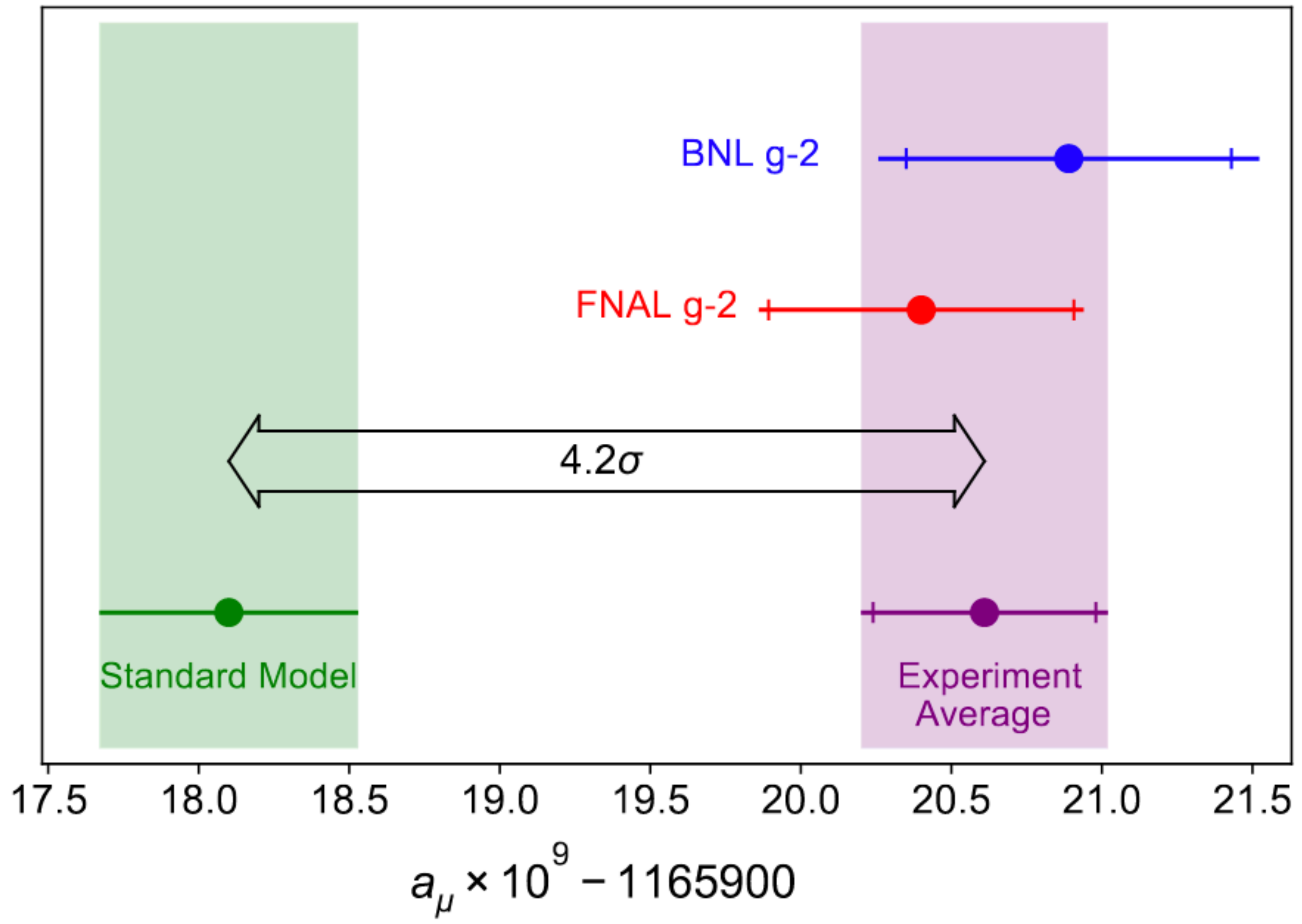
$b \to s\mu^+\mu^-$



M. Ciuchini et. al.
Eur.Phys.J.C 79 (2019) 8, 719

J. Matias et. al.
Eur.Phys.J.C 80 (2020) 6, 511

W Altmannshofer, P Stangl 2103.13370



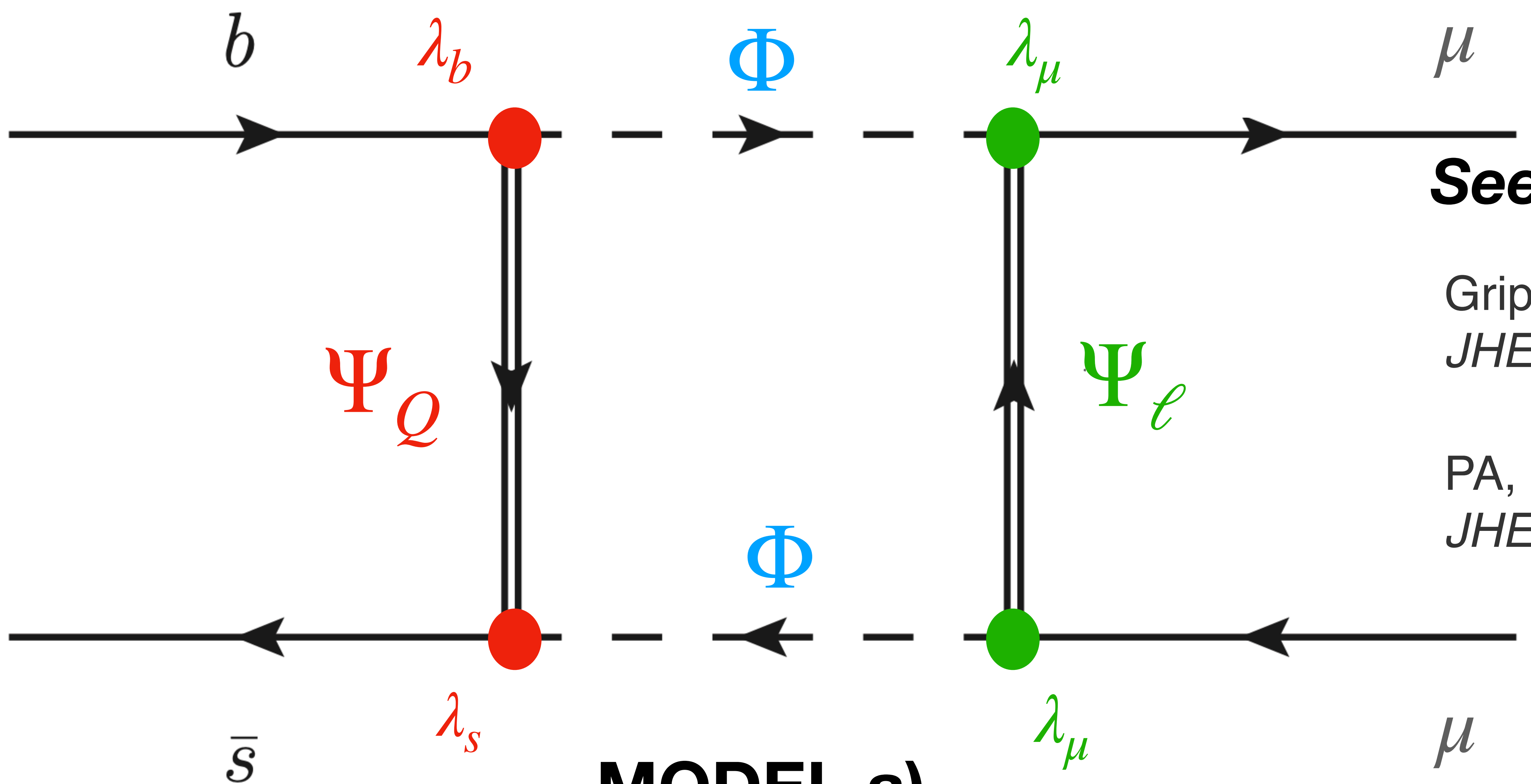
$(g-2)_\mu$

$$\delta \mathrm{Obs}^{NP} = g^2/M_{NP}^2$$

lower mass -> lower coupling-> direct searches

larger mass-> larger coupling-> PU

Heavy Scalars and Fermions $b \to s\mu^+\mu^-$

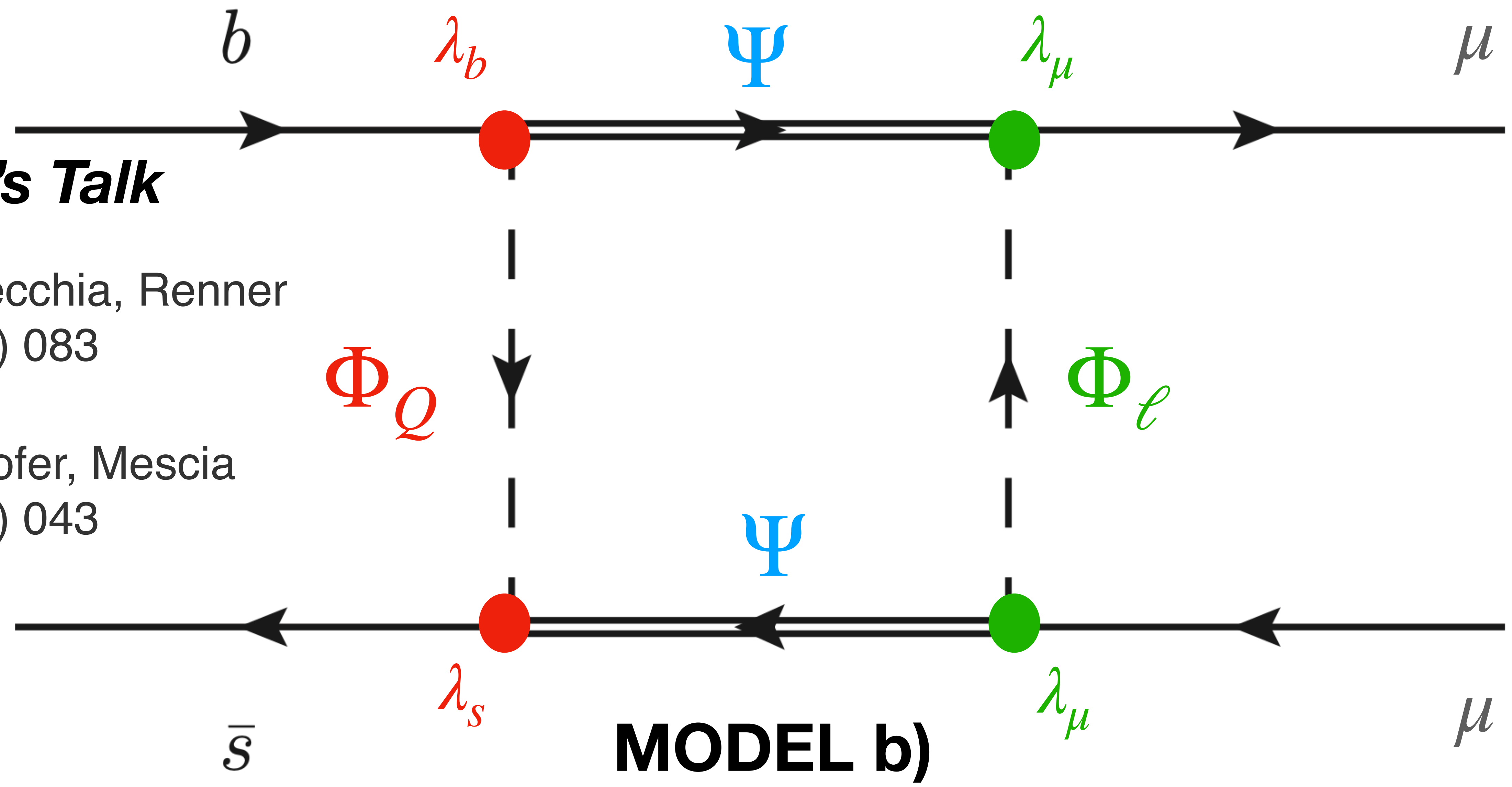


MODEL a)

See Fedele's Talk

Gripaios, Nardecchia, Renner
JHEP 06 (2016) 083

PA, Crivellin, Hofer, Mescia
JHEP 04 (2017) 043



MODEL b)

- 1 scalar Φ coupling to quarks and muons
- 1 fermion Ψ_Q coupling to quarks
- 1 fermion Ψ_ℓ coupling to muons

$$\mathscr{L}^{a)} = \lambda_i^Q \bar{\Psi}_Q P_L Q_i \Phi + \lambda_i^L \bar{\Psi}_\ell P_L L_i \Phi$$

- 1 fermion Ψ coupling to quarks and muons
- 1 scalar Φ_Q coupling to quarks
- 1 scalar Φ_ℓ coupling to muons

$$\mathscr{L}^{b)} = \lambda_i^Q \bar{\Psi} P_L Q_i \Phi_Q + \lambda_i^L \bar{\Psi} P_L L_i \Phi_\ell$$

equal masses $m_{NP} = 1$ TeV

3 LH couplings λ_μ λ_s λ_b

Heavy Scalars and Fermions

PA, Crivellin, Hofer, Mescia
JHEP 04 (2017) 043

model a) $\Phi \sim (\mathbf{1}, \mathbf{1}, X)$ $\Psi_\ell \sim (\mathbf{1}, \mathbf{1}, -\frac{1}{2} + X)$ $\Psi_q \sim (\mathbf{3}, \mathbf{1}, \frac{1}{6} + X)$

model b) $\Psi \sim (\mathbf{1}, \mathbf{2}, X)$ $\Phi_\ell \sim (\mathbf{1}, \mathbf{1}, -\frac{1}{2} + X)$ $\Phi_q \sim (\mathbf{3}, \mathbf{1}, \frac{1}{6} + X)$

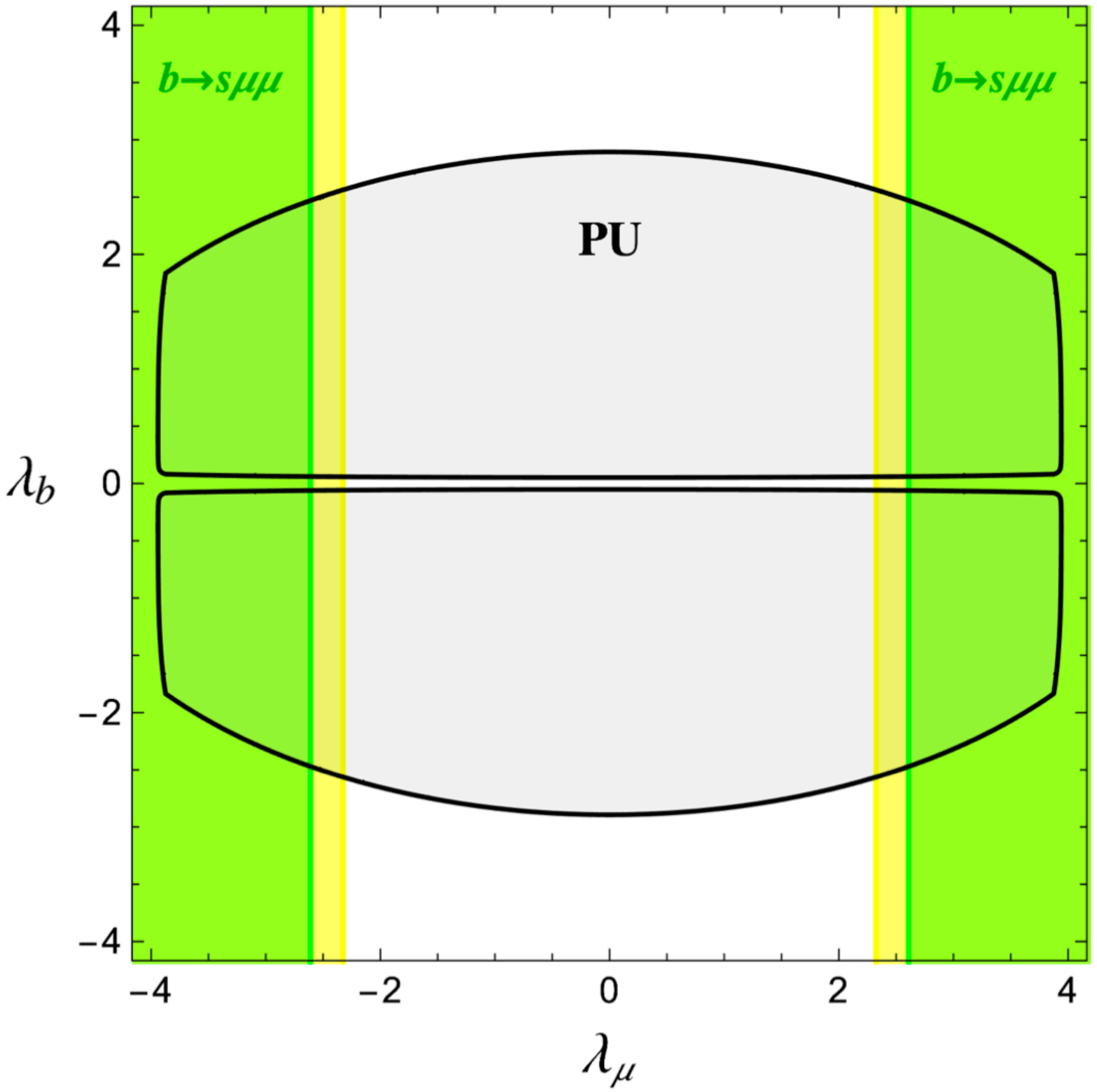
m= 1 TeV

$X = \frac{1}{2}$ **real scalar**

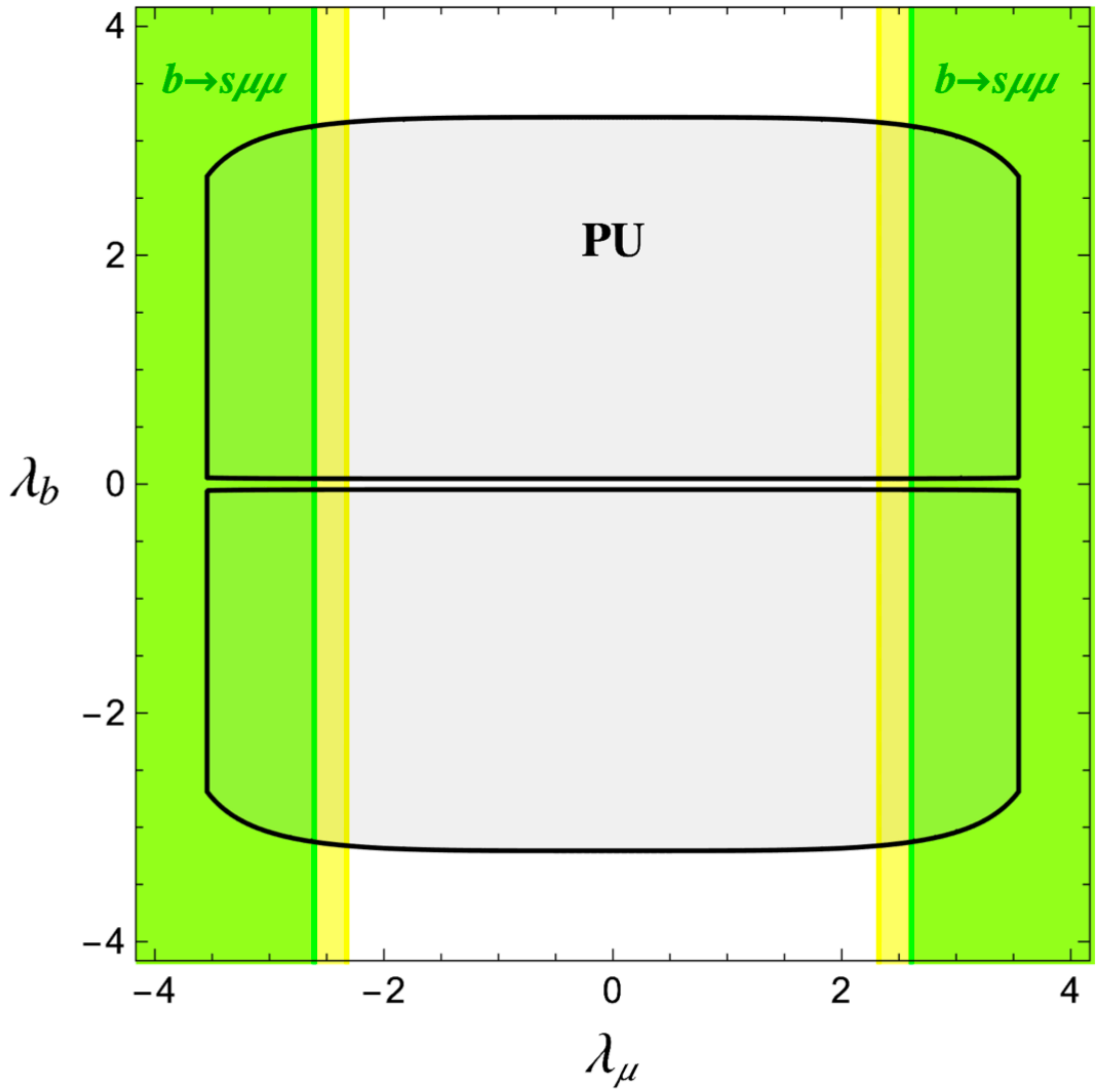
$$|C_9^{box}| = \frac{1}{3} |\lambda_s^* \lambda_b| \, |\lambda_\mu|^2$$

$|\lambda_s^* \lambda_b| = 0.15$ From $B_s - \bar{B}_s$

a) Complex Φ

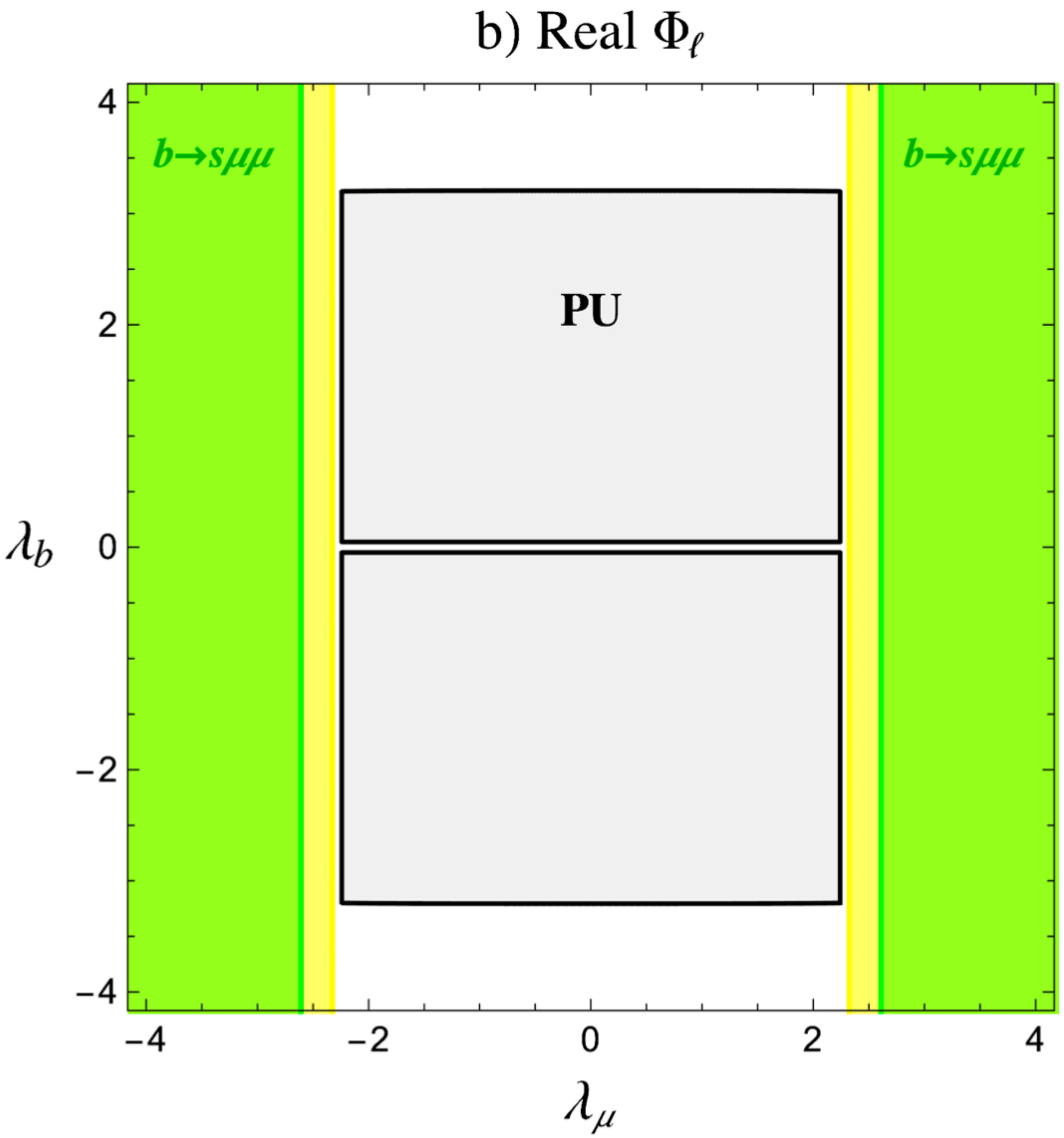


b) Complex Φ_ℓ



model 1
complex scalar
<3.5

Heavy Scalars and Fermions



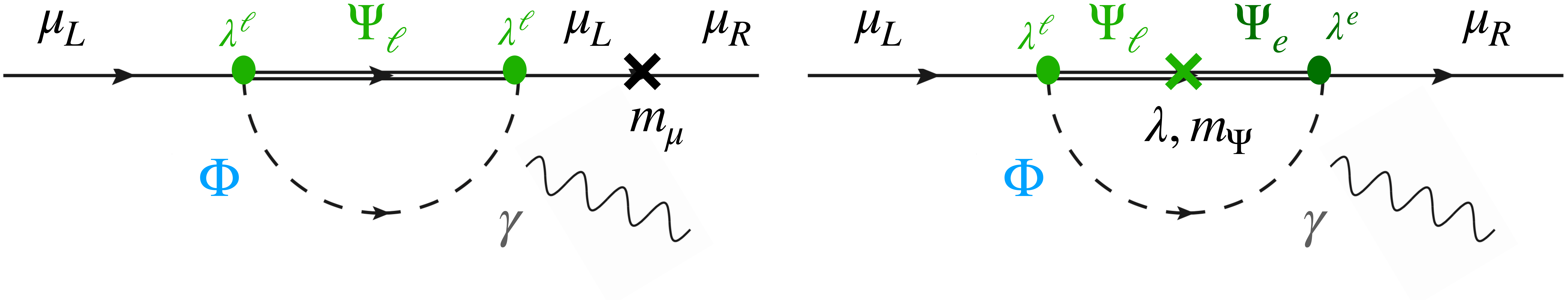
model 1
Real scalar
<2.2

Impossible to explain g-2 with perturbative couplings

RH couplings?

PA, Crivellin, Fedele, Mescia
JHEP 06 (2019) 118

Heavy Scalars and Fermions



$$-\mathscr{L} = \lambda_i^q \bar{q}_L^i \Psi_R \Phi_q + \lambda_i^\ell \bar{\ell}_L^i \Psi_R \Phi_\ell + \lambda_i^e \bar{e}_R^i \Psi'_L \Phi_\ell + \lambda^H (\bar{\Psi}_L \Psi'_R H + \bar{\Psi}_R \Psi'_L H) + h.c.\,,$$

Contribution enchancing g-2

$$\Psi \sim (\mathbf{1}, \mathbf{2}, -\tfrac{1}{2}) \qquad \Psi' \sim (\mathbf{1}, \mathbf{1}, -1) \qquad \Phi_\ell \sim (\mathbf{1}, \mathbf{1}, 0) \qquad \Phi_q \sim (\mathbf{3}, \mathbf{1}, \tfrac{2}{3})\,.$$

$b \to s\mu\mu$, **g-2, Dark matter**

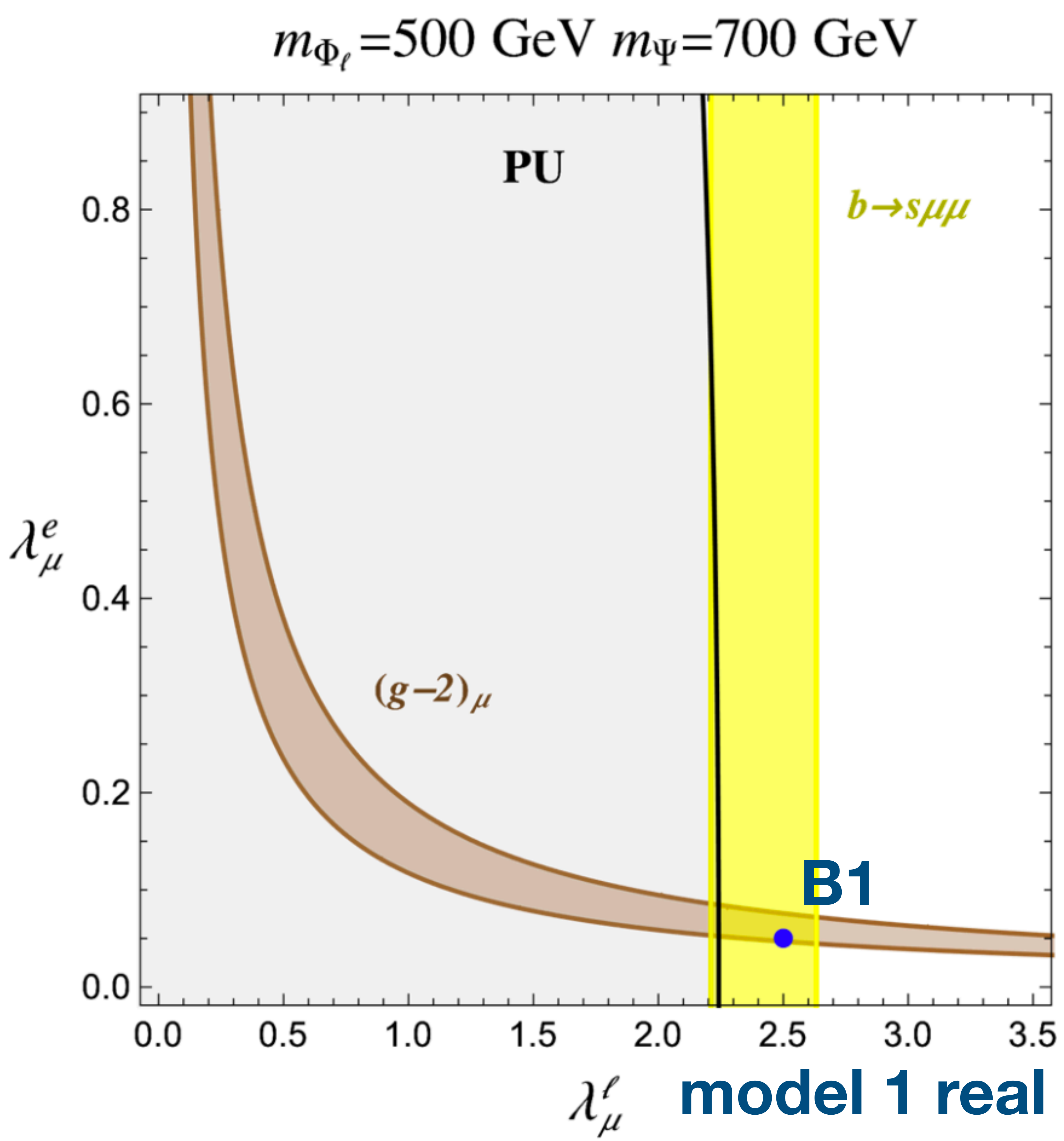
Arcadi, Calibbi, Fedele, Mescia, *Phys.Rev.Lett.* 127 (2021) 6, 061802

5 couplings $\lambda_\mu^\ell \quad \lambda_\mu^e \quad \lambda_s \, \lambda_b \, \lambda^H$

Heavy Scalars and Fermions

Arcadi, Calibbi, Fedele, Mescia,
Phys.Rev.Lett. 127 (2021) 6, 061802

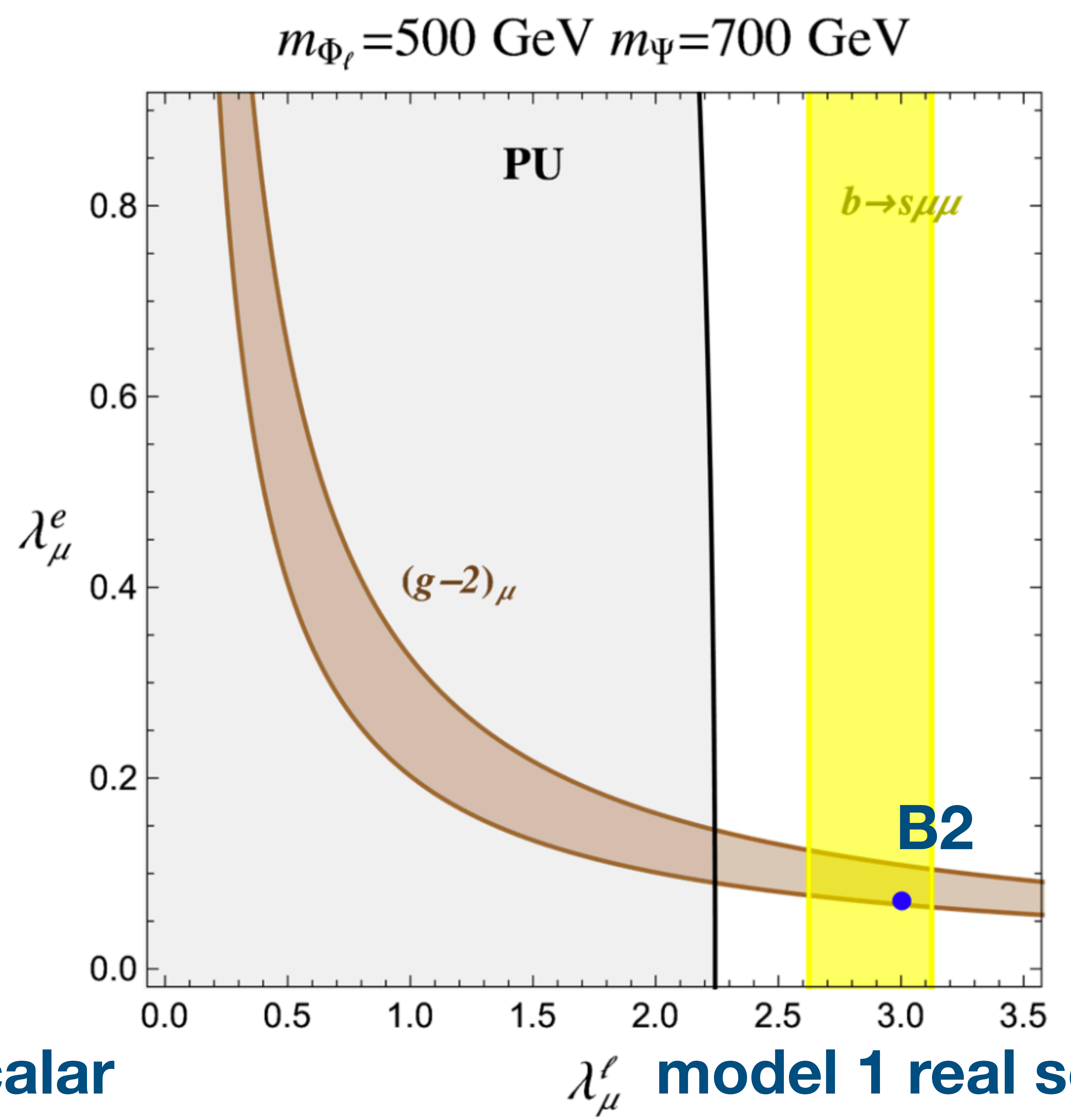
Benchmark 1



model 1 real scalar

<2.2

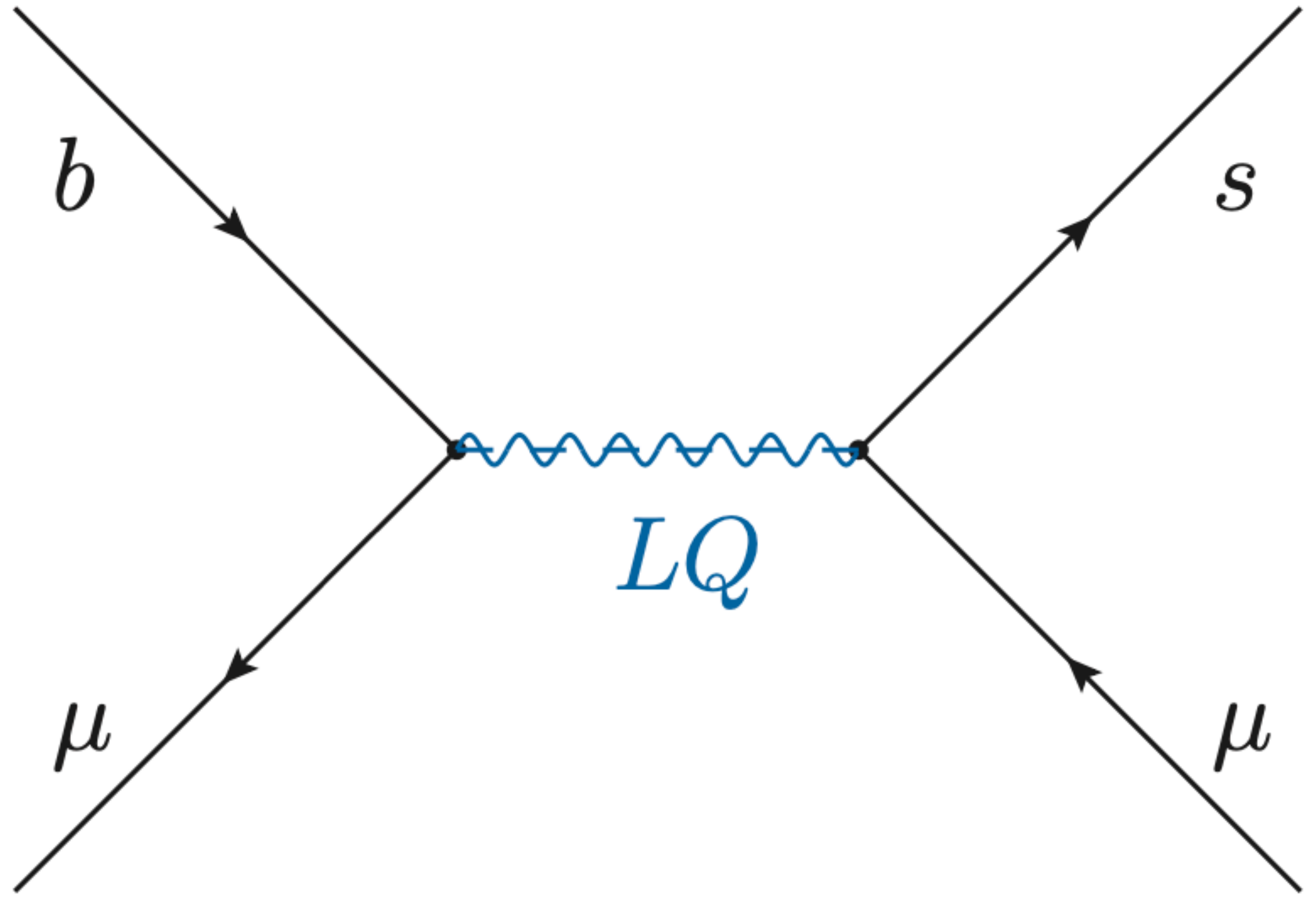
Benchmark 2

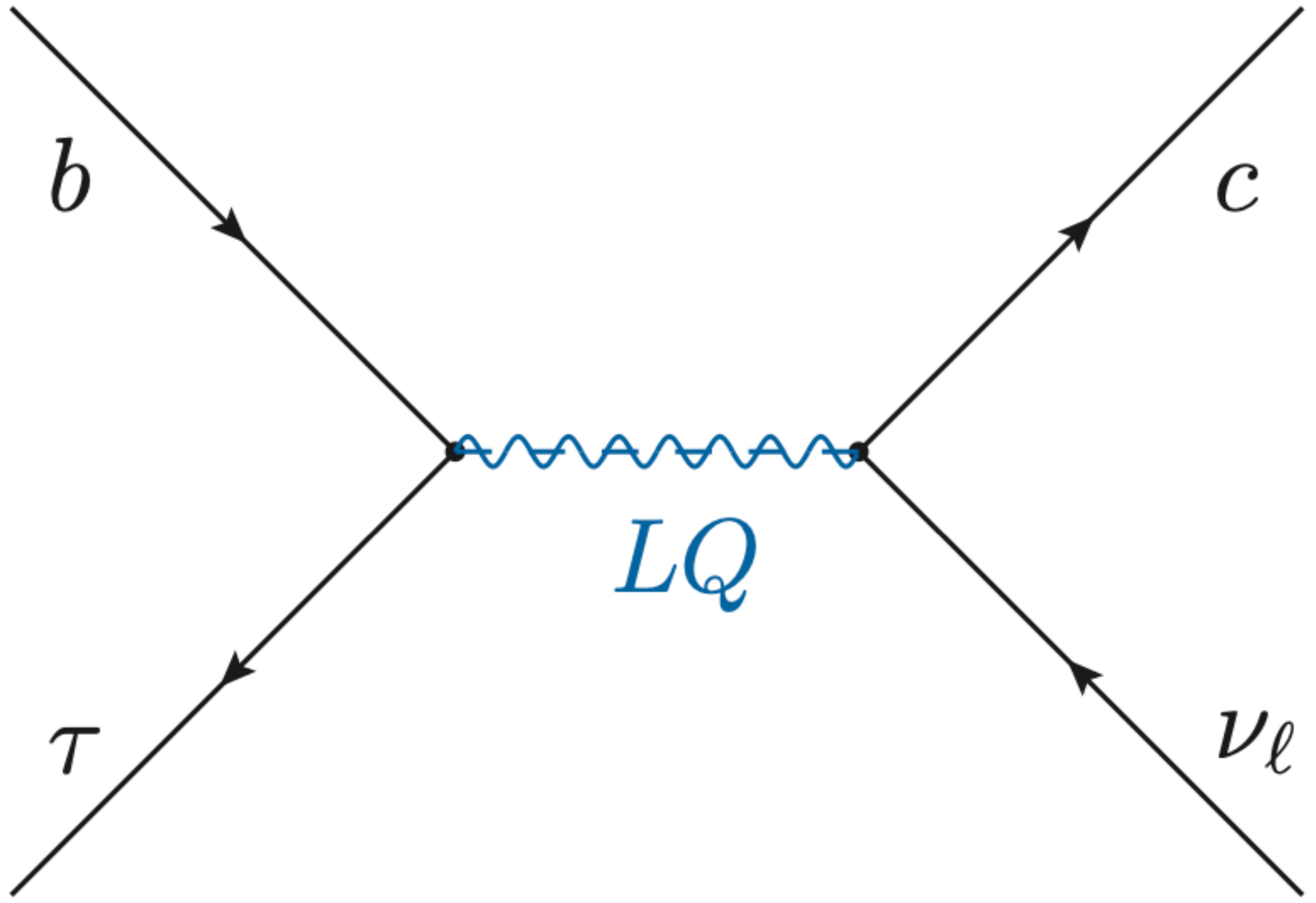


model 1 real scalar

<2.2

Leptoquarks





$S_1 + S_3$ **Everything perturbative mLQ~1 TeV**

See Becirevic and Marzocca's Talks

Buttazzo, Greljo, Isidori, Marzocca, *JHEP* 11 (2017) 044

Marzocca, JHEP 07 (2018) 121

Crivellin, Müller, Saturnino *JHEP* 06 (2020) 020

Bigaran, Gargalionis, Volkas *JHEP* 10 (2019) 106

Han Yan, Ya-Dong Yang, Xing-Bo Yuan *Chin.Phys.C* 43 (2019) 8, 083105

PA, Becirevic, Mescia, Sumensari *JHEP* 02 (2019) 109

Gherardi, Marzocca, Venturini *JHEP* 01 (2021) 006

Crivellin, Müller, Toshihiko Ota *JHEP* 09 (2017) 040

Di Luzio, Fuentes Martín, Greljo, Nardecchia, Renner *JHEP* 11 (2018) 081

U_1 **Yukawa sector**

$$-\mathcal{L}_{\mathrm{SM-like}} = \bar{q}'_L Y_d H d'_R + \bar{q}'_L Y_u \tilde{H} u'_R + \bar{\ell}'_L Y_e H e'_R + \mathrm{h.c.}\ ,$$

$$-\mathcal{L}_{\mathrm{mix}} = \bar{q}'_L \lambda_q \Omega_3^T \Psi_R + \bar{\ell}'_L \lambda_\ell \Omega_1^T \Psi_R + \bar{\Psi}_L (M + \lambda_{15}\Omega_{15}) \Psi_R + \mathrm{h.c.}\,,$$

$\Omega_{15} \sim (\mathbf{15}, \mathbf{1}, \mathbf{1}, 0)$ $\quad \Psi_L \sim (\mathbf{4}, \mathbf{1}, \mathbf{2}, 0)$ $\quad \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{2}, 0)$

model 1
model 3

$\lambda_{15} \lesssim 2.1$

Leptoquark S1+lepton scalar: RK+RD+g-2+CAA

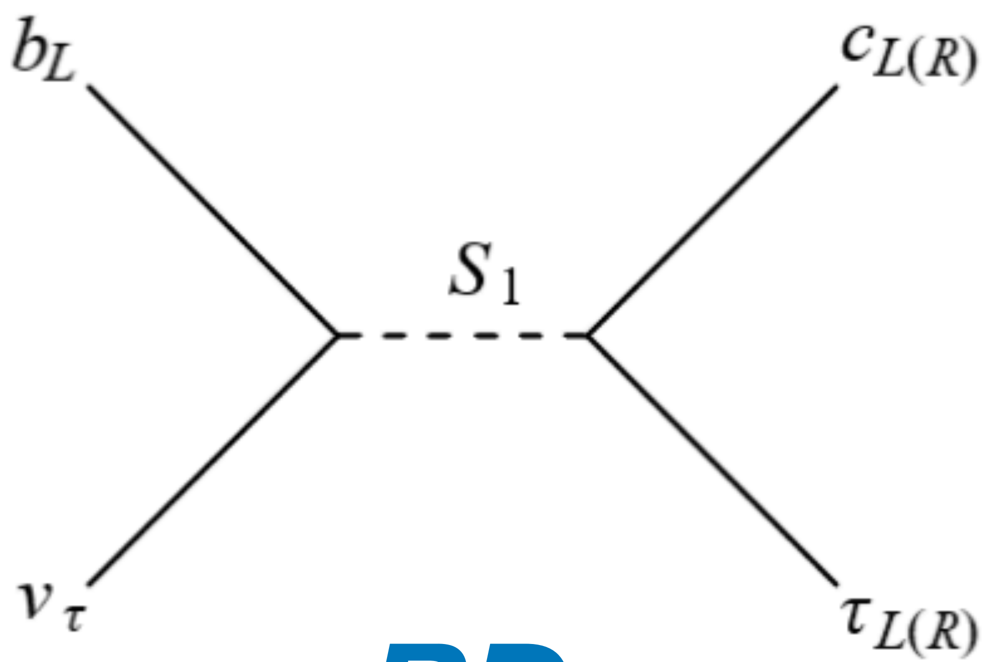
$$-\mathcal{L} = \frac{1}{2}\lambda^{\ell}_{\alpha\beta}\bar{\ell}^{c,\alpha}_{L}\varepsilon\ell^{\beta}_{L}\phi^{+} + \lambda^{u}_{i\alpha}\bar{u}^{c,i}_{R}e^{\alpha}_{R}S_1 + \lambda^{q}_{i\alpha}\bar{q}^{c,i}_{L}\varepsilon\ell^{\alpha}_{L}S_1 + h.c.$$

$m_\phi = m_{S_1} = 5.5$ TeV

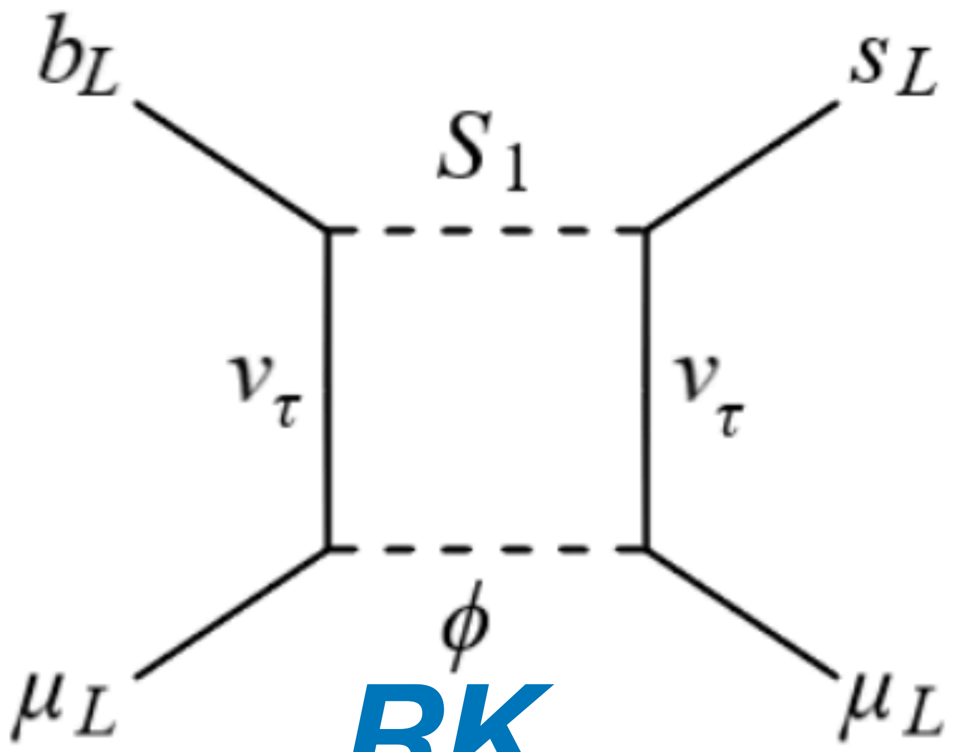
$$\lambda^q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \lambda^q_{s\tau} \\ 0 & \lambda^q_{b\mu} & \lambda^q_{b\tau} \end{pmatrix}, \quad \lambda^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda^u_{c\mu} & \lambda^u_{c\tau} \\ 0 & 0 & \lambda^u_{t\tau} \end{pmatrix}, \quad \lambda = \begin{pmatrix} 0 & \lambda_{e\mu} & 0 \\ -\lambda_{e\mu} & 0 & \lambda_{\mu\tau} \\ 0 & -\lambda_{\mu\tau} & 0 \end{pmatrix}$$

8 couplings

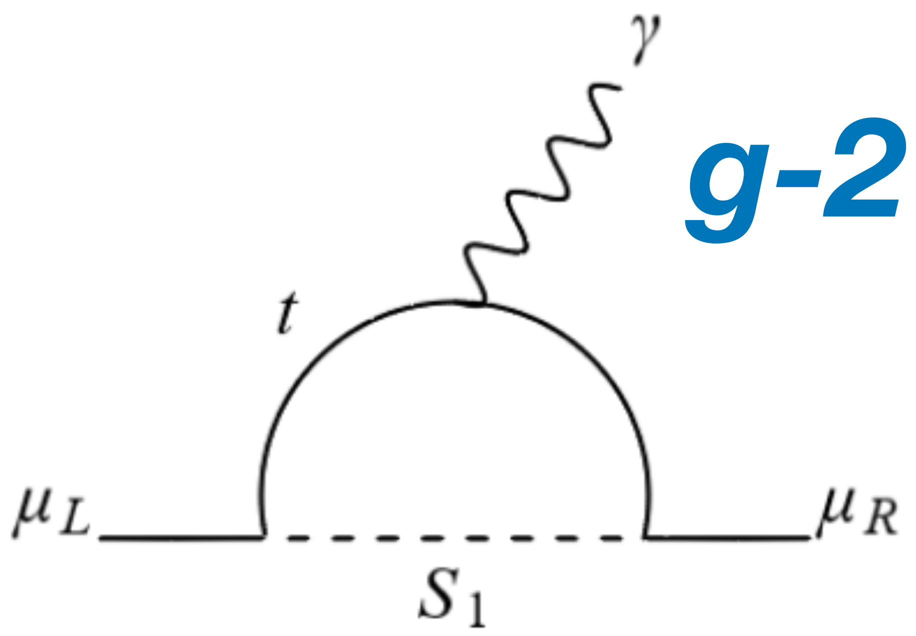
Marzocca, Trifinopoulos
Phys.Rev.Lett. 127 (2021) 6, 2021



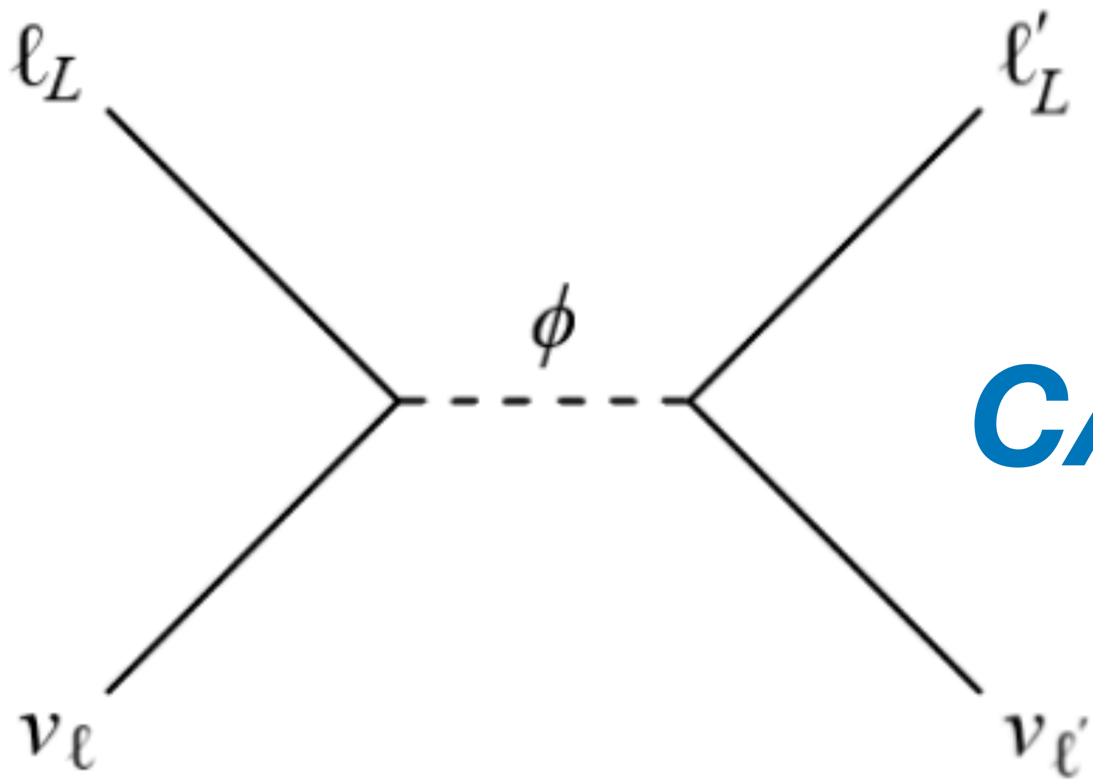
RD



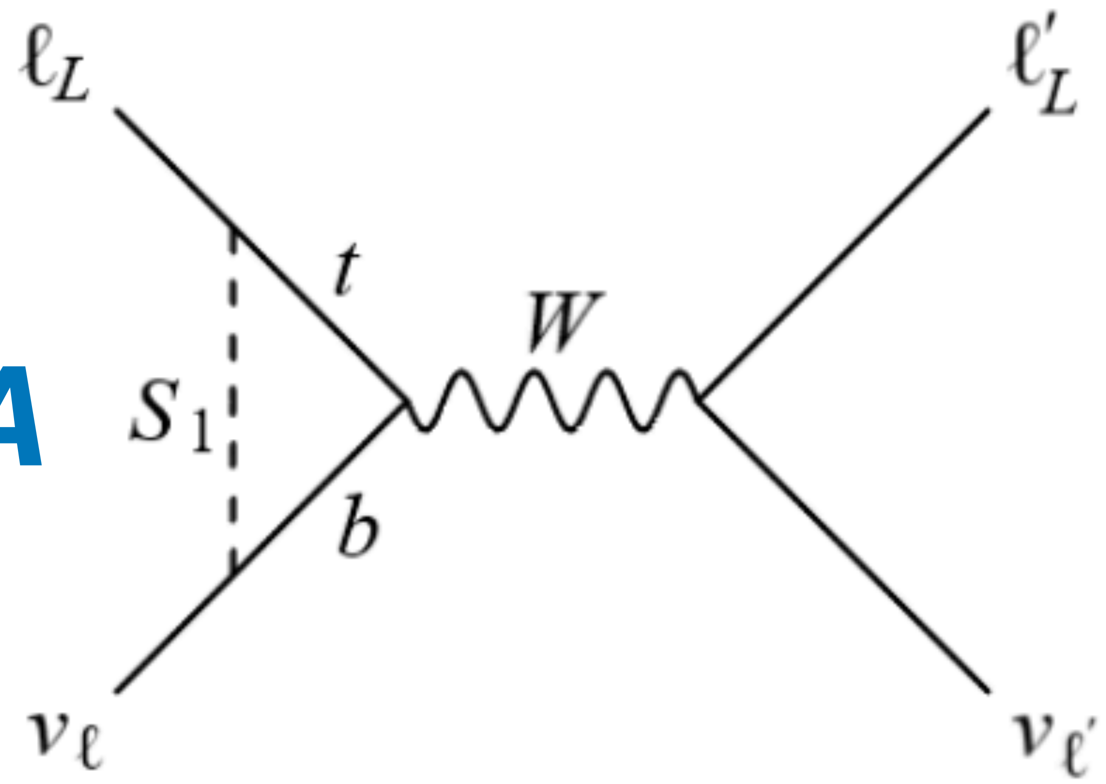
RK



g-2



CAA



Best Fit

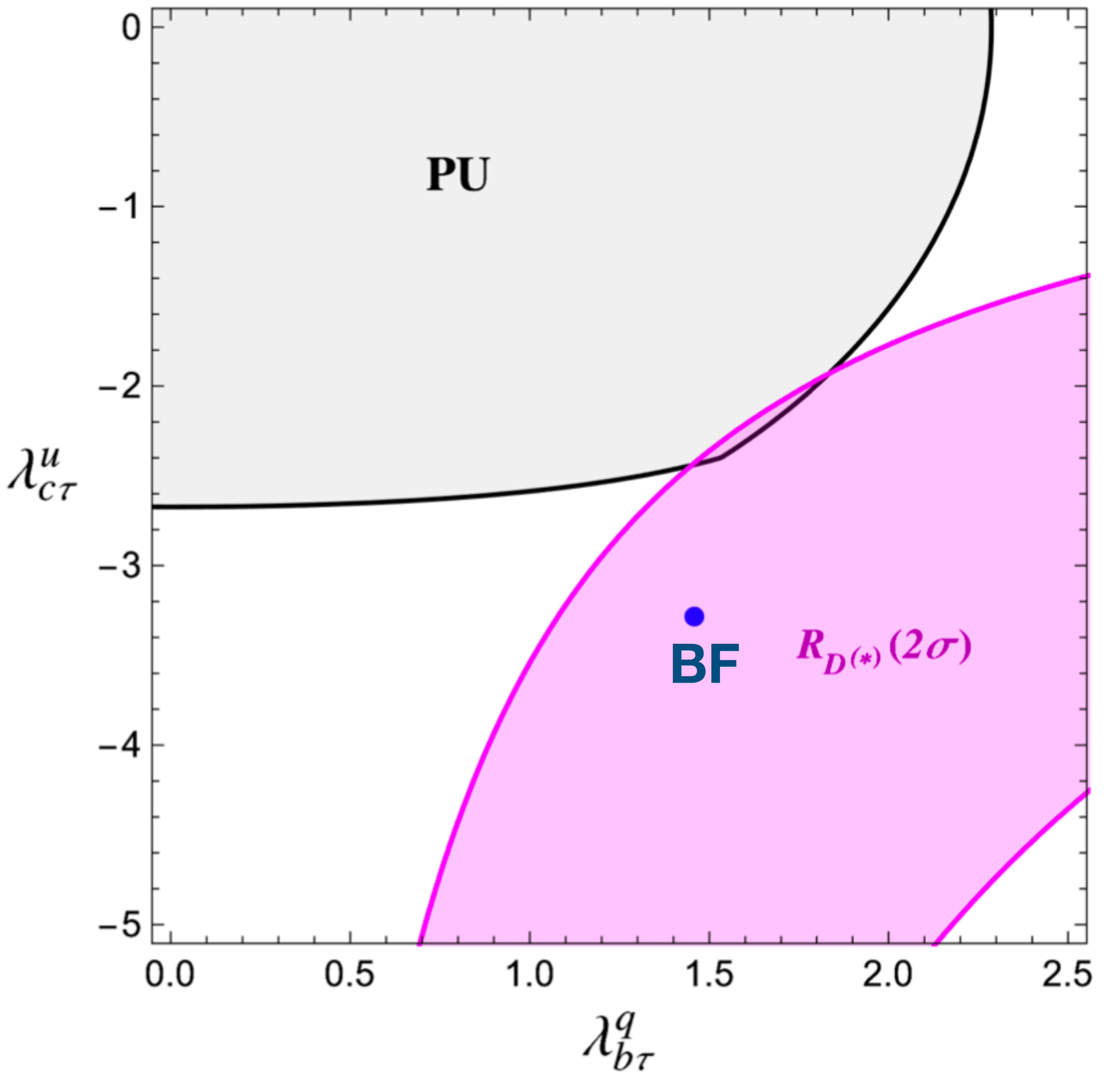
RK

$$\lambda_{e\mu} = 1.35\,, \quad \boxed{\lambda_{\mu\tau} = 3.17}\,,$$
$$\lambda^{1L}_{b\tau} = 1.46\,, \quad \lambda^{1L}_{s\tau} = -0.54\,, \quad \lambda^{1L}_{b\mu} = 2.07$$
$$\boxed{\lambda^{1R}_{c\tau} = -3.28}\,, \quad \lambda^{1R}_{t\mu} = 0.01\,, \quad \lambda^{1R}_{c\mu} = 2.35$$

RD

Leptoquark S1: RK+RD+g-2+CAA

Marzocca, Trifinopoulos
Phys.Rev.Lett. 127 (2021) 6, 2021



Leptoquark S1: RK+RD+g-2+CAA

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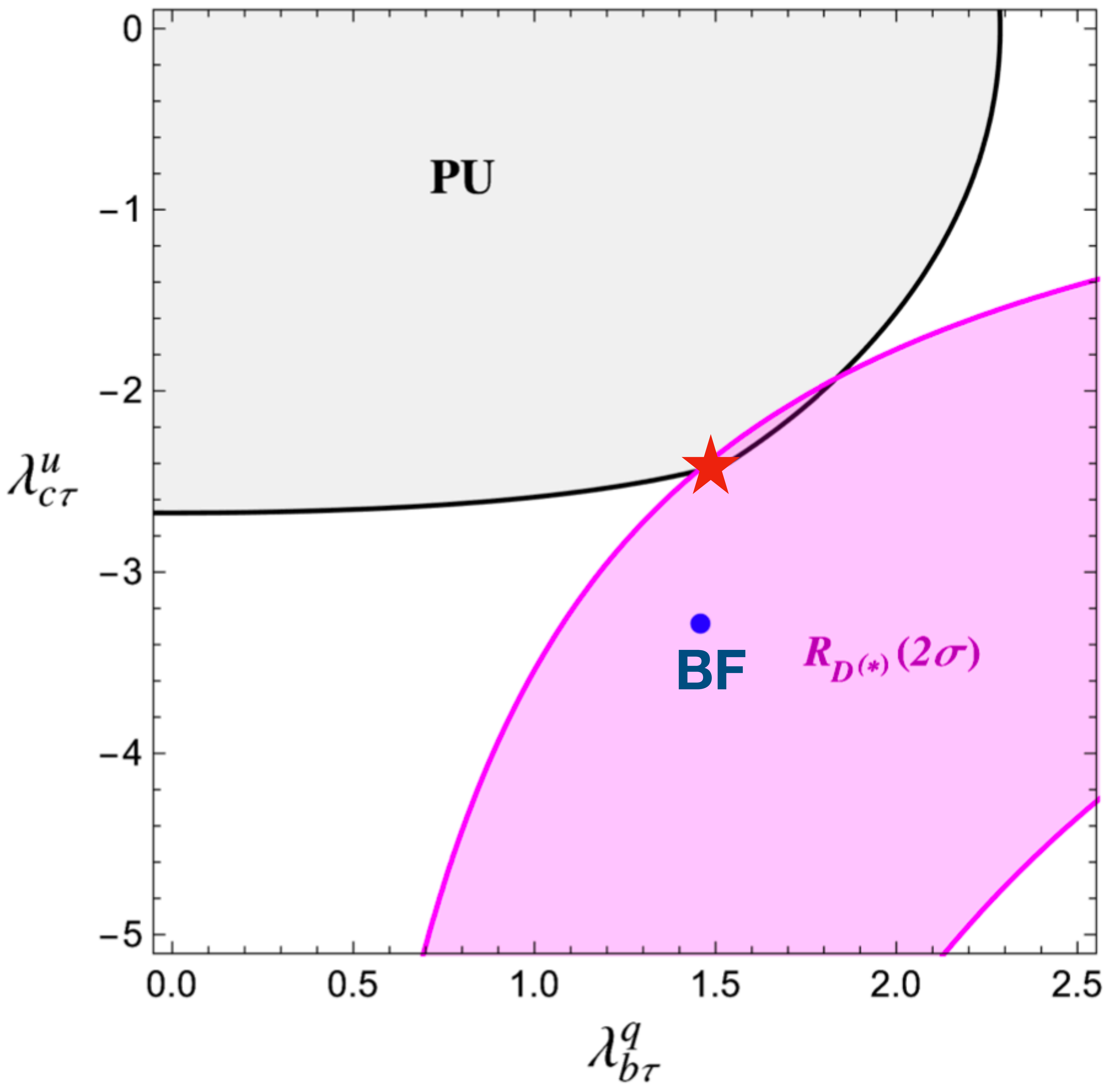
Phys.Rev.Lett. 127 (2021) 6, 2021



Leptoquark S1: RK+RD+g-2+CAA

Marzocca, Trifinopoulos
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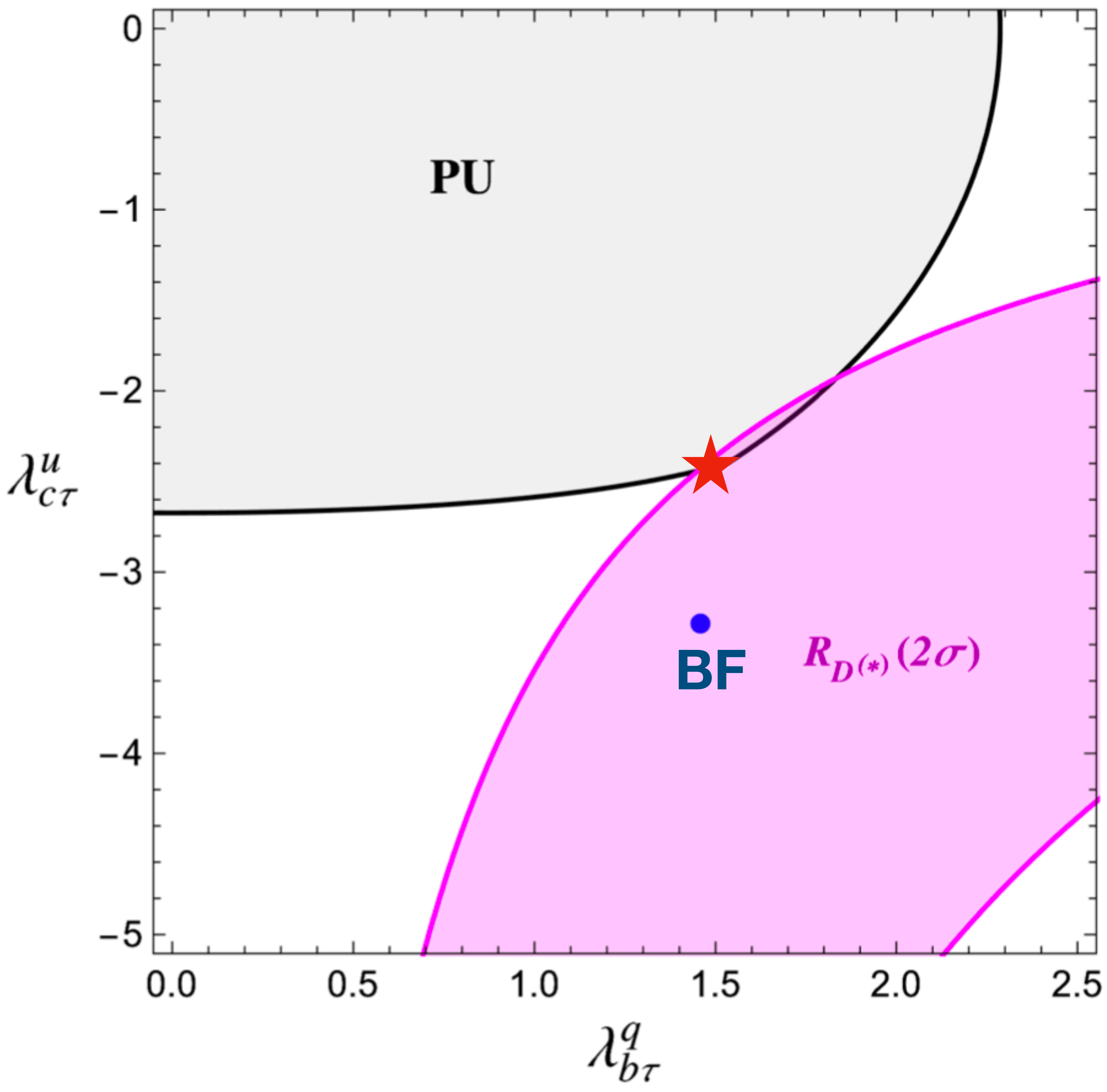

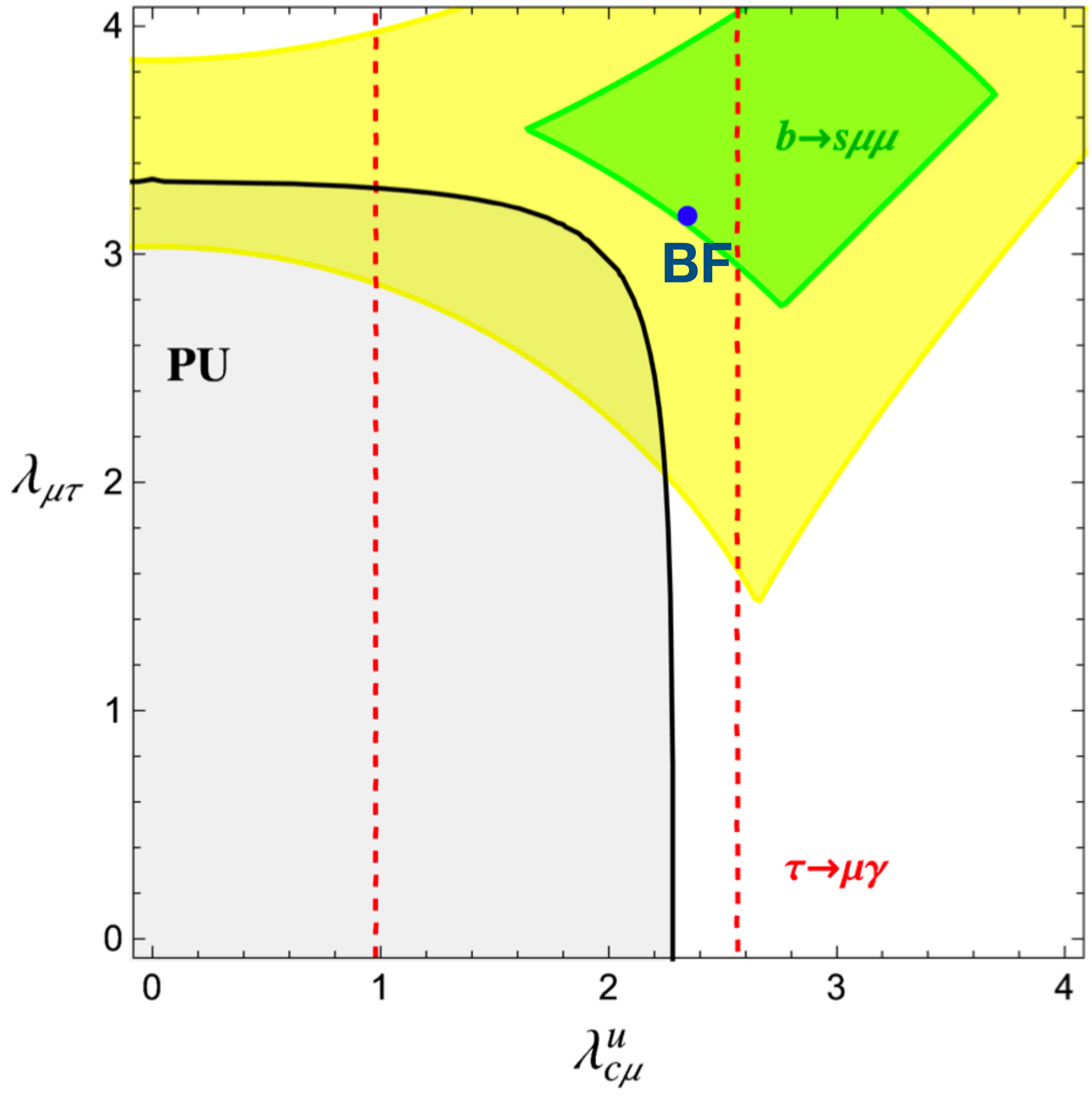

Summary

We can compute the perturbative unitarity bounds for any model with Yukawa couplings

Some Pheno models are at the edge of perturbativity, PU can give important constraints

Important to work in massive case, for scalar and fermions.
Combination with the beta function.

Allow for more channels with Scalar Potential or Vector couplings

Gràcies!
Hvala!