

Marco Drewes, Université catholique de Louvain

**Flavour effects in low scale
leptogenesis and their
experimental tests**

24. 09. 2021

**Physics of the flavourful
Universe Workshop**

Portoroz, Slovenia

Overview

Low Scale Seesaw Predictions for Neutrino Yukawas

Leptogenesis Predictions for Neutrino Yukawas

Other tests: LNV and CPV

The Seesaw Model (type I)

$$\mathcal{L} = \mathcal{L}_{SM} + i\bar{\nu}_R \not{\partial} \nu_R - \bar{L}_L F \nu_R \tilde{H} - \tilde{H}^\dagger \bar{\nu}_R F^\dagger L - \frac{1}{2} (\bar{\nu}_R^c M_M \nu_R + \bar{\nu}_R M_M^\dagger \nu_R^c)$$

Three Generations of Matter (Fermions) spin 1/2

	I	II	III	
mass →	2.4 MeV	1.27 GeV	171.2 GeV	0
charge →	2/3	2/3	2/3	0
name →	u Left up Right	c Left charm Right	t Left top Right	g gluon
Quarks	d Left down Right	s Left strange Right	b Left bottom Right	γ photon
	0 eV	0 eV	0 eV	91.2 GeV
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	Z⁰ weak force
	0.511 MeV	105.7 MeV	1.777 GeV	125 GeV
Leptons	e Left electron Right	μ Left muon Right	τ Left tau Right	H Higgs boson
	-1	-1	-1	spin 0
				80.4 GeV
				W[±] weak force
				Bosons (Forces) spin 1

three light neutrinos mostly "active" SU(2) doublet

$$\nu \simeq U_\nu (\nu_L + \theta \nu_R^c)$$

$$\text{with masses } m_\nu \simeq \theta M_M \theta^T = v^2 F M_M^{-1} F^T$$

three heavy mostly singlet neutrinos

$$N \simeq \nu_R + \theta^T \nu_L^c$$

$$\text{with masses } M_N \simeq M_M$$



Minkowski 79, Gell-Mann/Ramond/Slansky 79,
Mohapatra/Senjanovic 79,
Yanagida 80, Schechter/Valle 80

The Seesaw Model (type I)

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massive SM neutrinos \leq #RH neutrino flavours

- minimal model with $m_{\text{lightest}} = 0$ has two RHN
- if all SM neutrinos are massive, three RHN flavours are needed

Three Generations of Matter (Fermions) spin 1/2					
mass	charge	name	mass	charge	name
2.4 MeV	2/3	u (up)	1.27 GeV	2/3	c (charm)
4.8 MeV	-1/3	d (down)	171.2 GeV	2/3	t (top)
0 eV	0	ν_e (electron neutrino)	104 MeV	-1/3	s (strange)
0 eV	0	ν_μ (muon neutrino)	4.2 GeV	-1/3	b (bottom)
0 eV	0	ν_τ (tau neutrino)	91.2 GeV	0	Z (weak force)
0.511 MeV	-1	e (electron)	80.4 GeV	± 1	W (weak force)
105.7 MeV	-1	μ (muon)	125 GeV	0	H (Higgs boson)
1.777 GeV	-1	τ (tau)			

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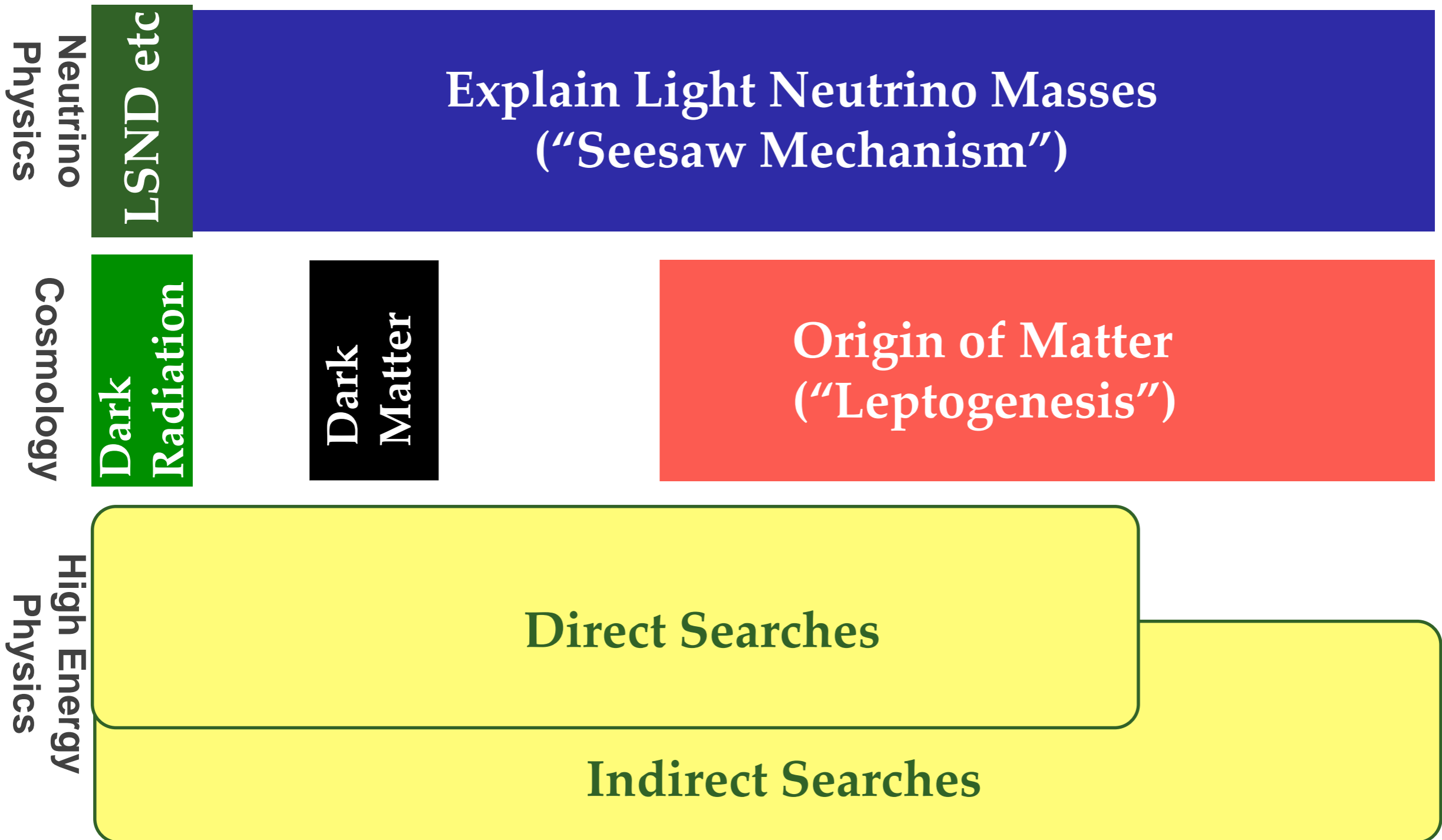
$$\text{with masses } M_N \simeq M_M$$



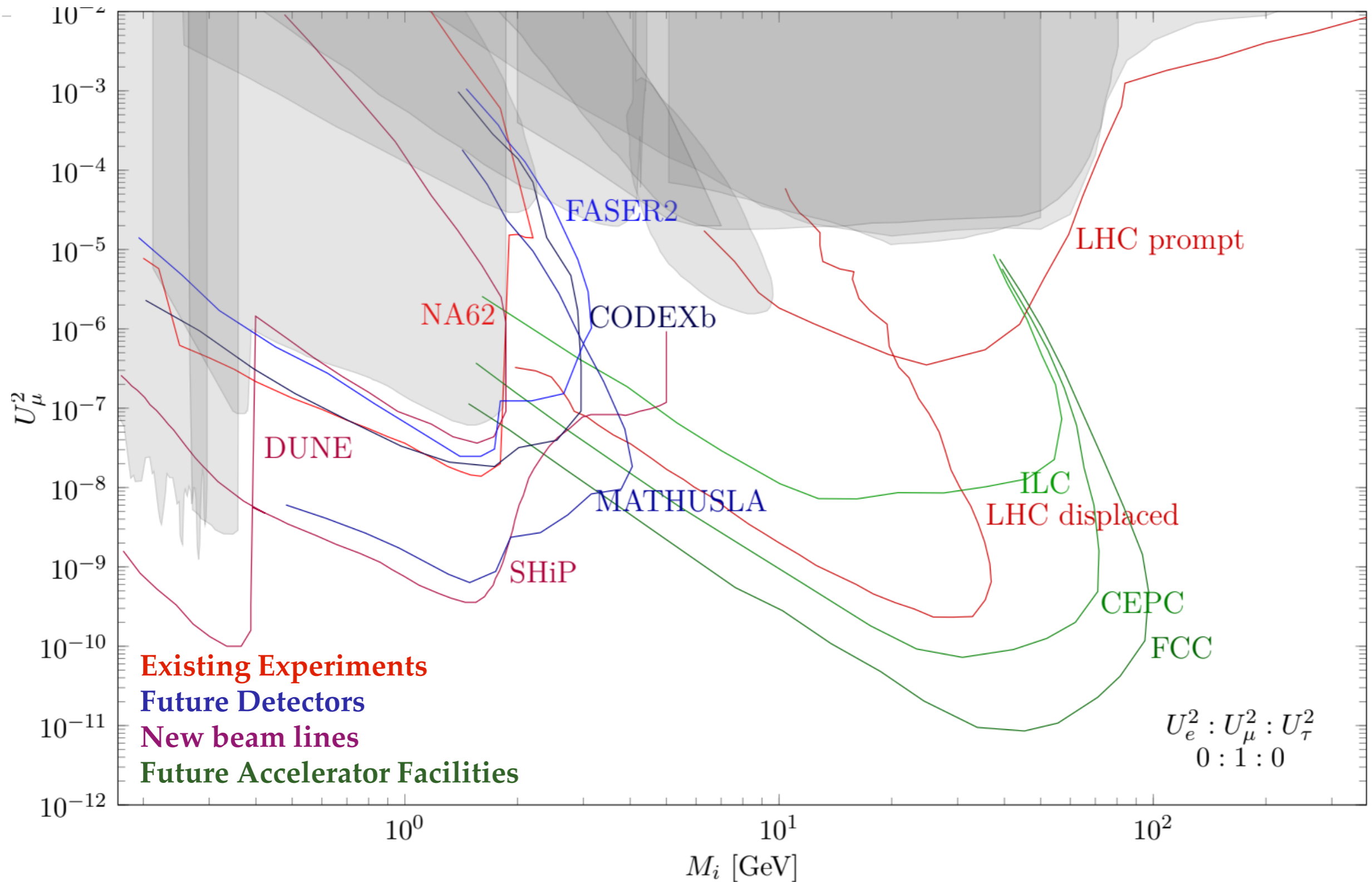
Minkowski 79, Gell-Mann/Ramond/Slansky 79, Mohapatra/Senjanovic 79, Yanagida 80, Schechter/Valle 80

Right Handed Neutrino Mass Scale

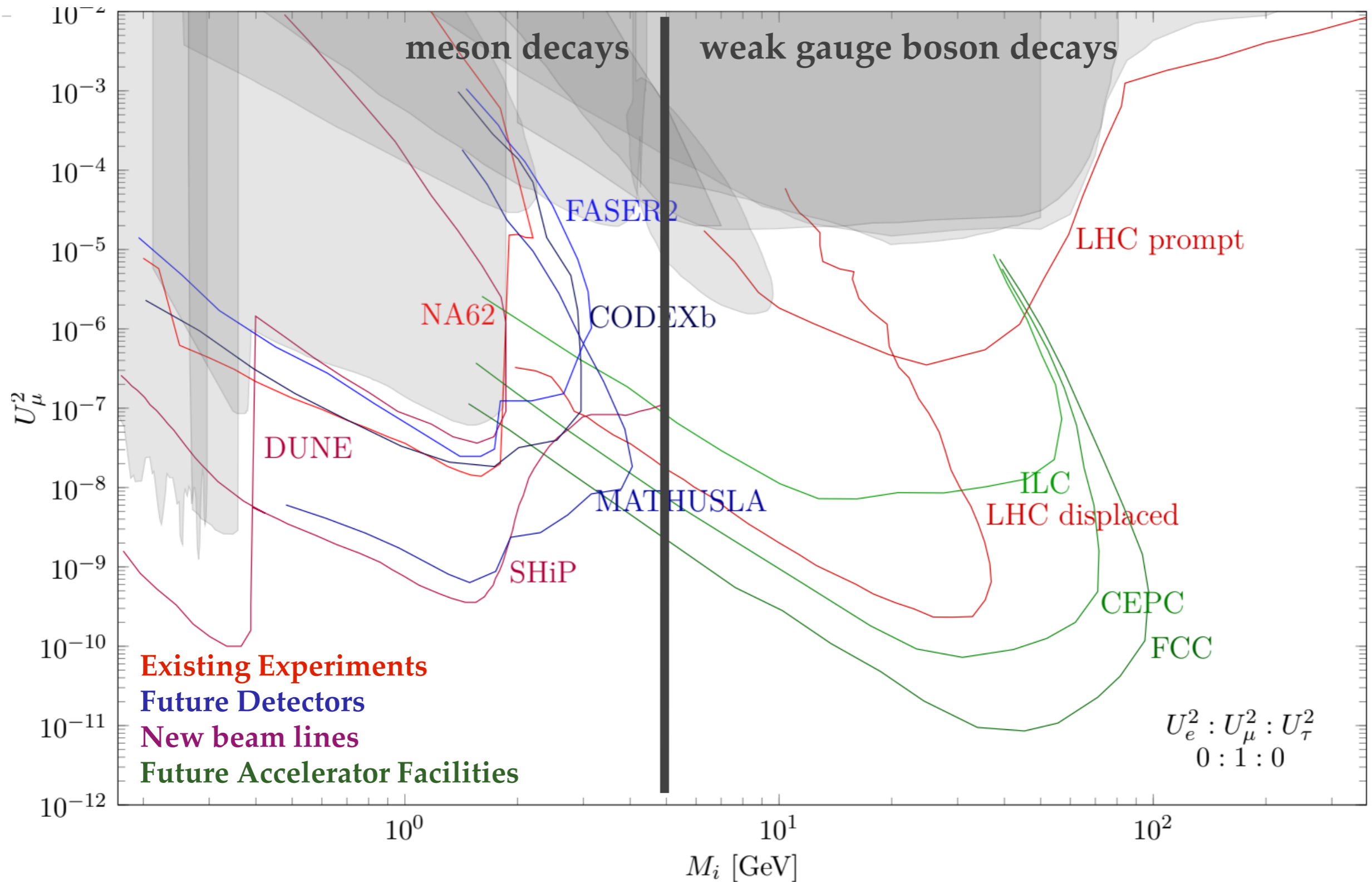
eV keV MeV GeV TeV 10^{14} GeV



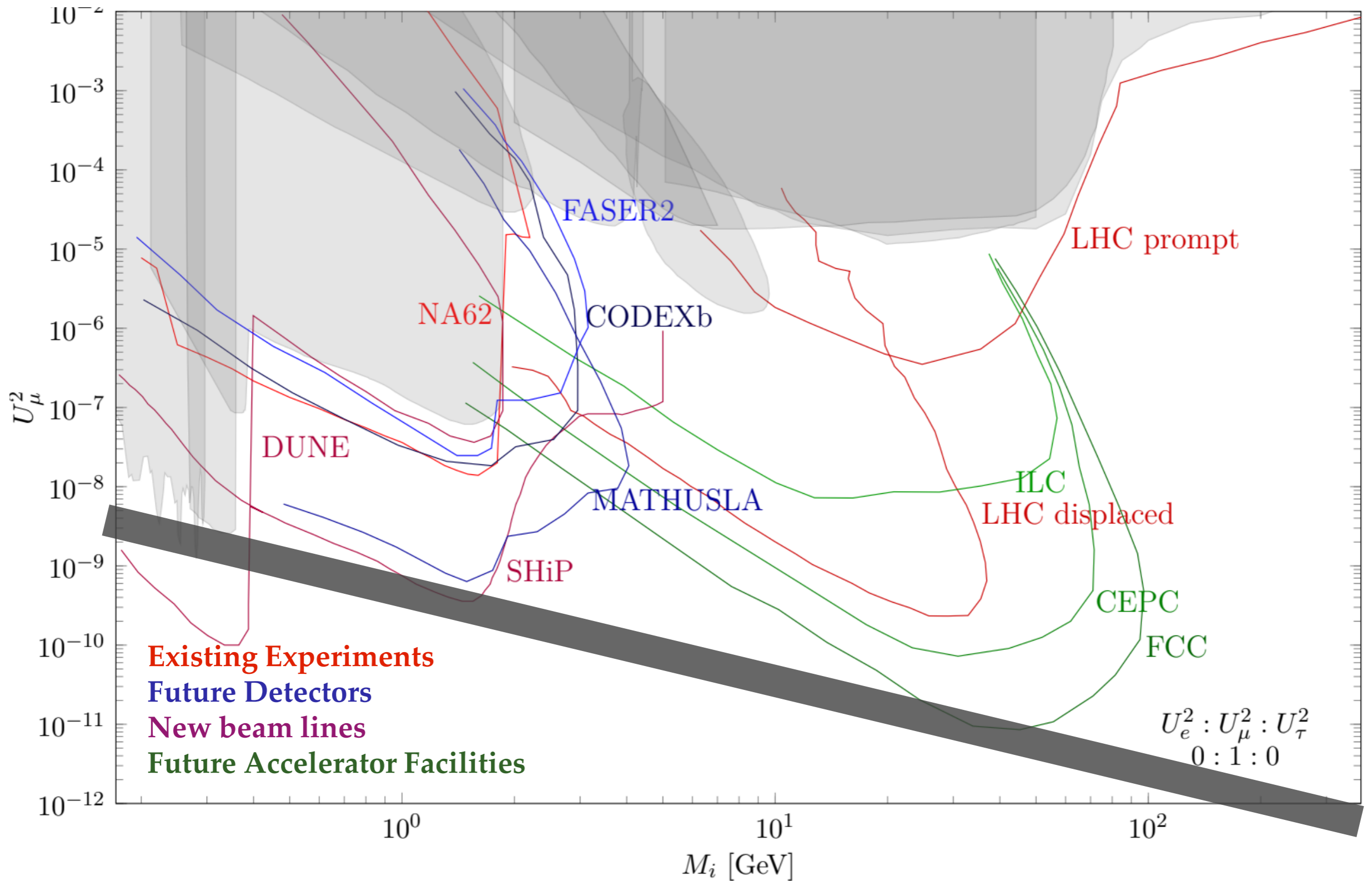
Heavy Neutrino Searches



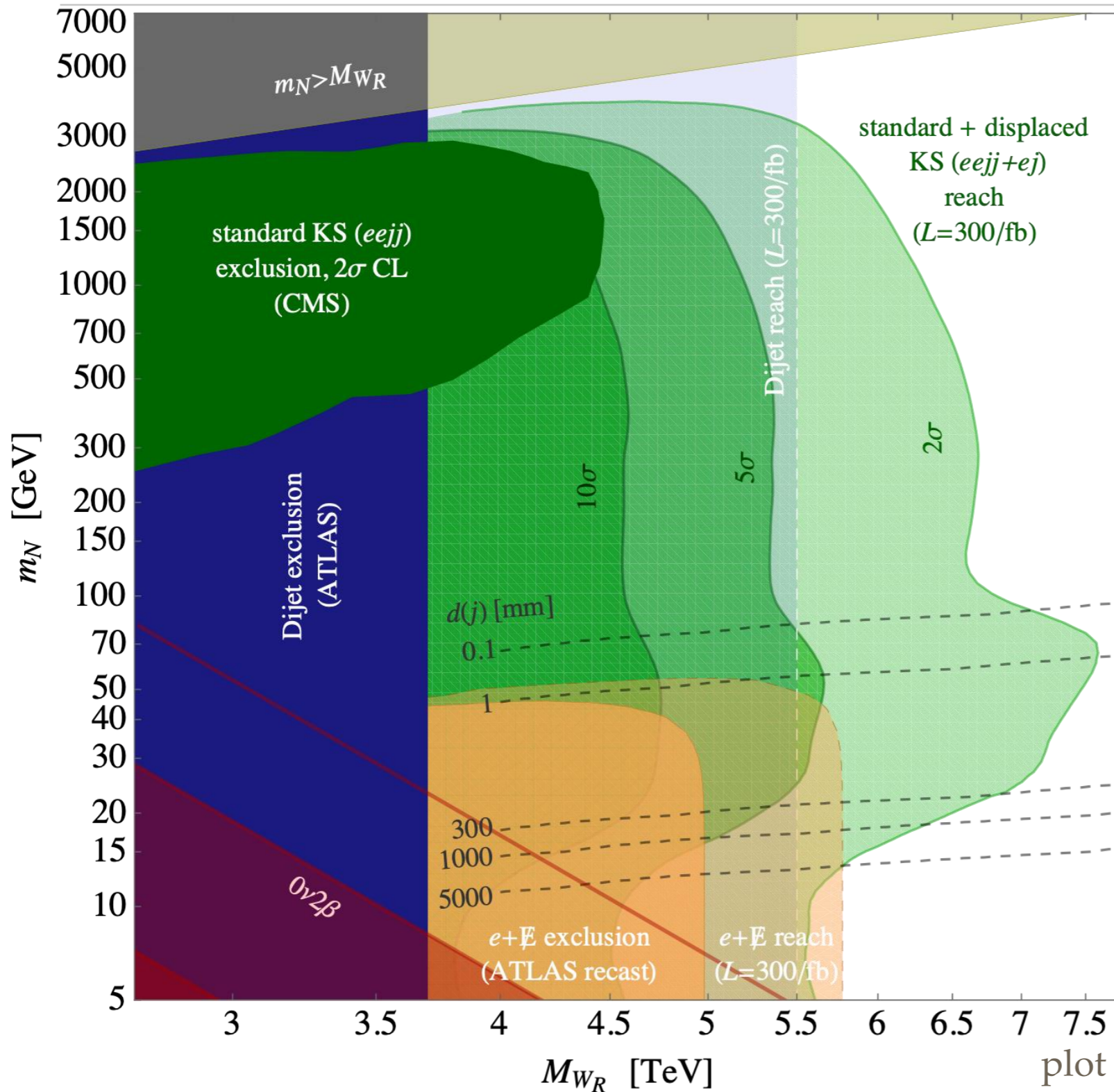
Heavy Neutrino Searches



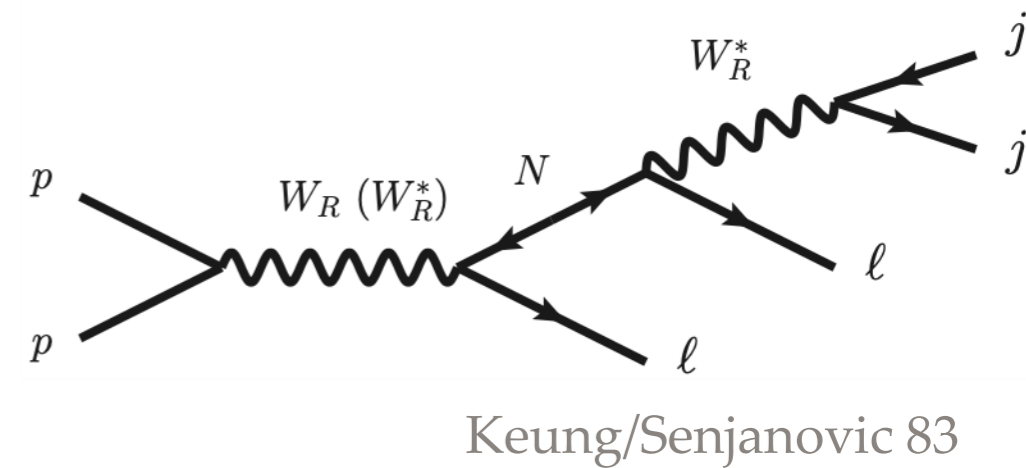
$$m_\nu \simeq \theta M_M \theta^T = v^2 F M_M^{-1} F^T$$



Non-minimal Scenarios



- discovery potential is much better in models with extra gauge interaction
- example: L-R symmetric model
Pati, Salam, Mohapatra, Senjanovic



B-L Symmetric Limit

charge assignment in Lagrangian

spinor

\bar{L} -charge

$$\nu_{Rs} \equiv \frac{1}{\sqrt{2}}(\nu_{R1} + i\nu_{R2})$$

+1

$$\nu_{Rw} \equiv \frac{1}{\sqrt{2}}(\nu_{R1} - i\nu_{R2})$$

-1

ν_{R3}

0

approximately conserved
charges in leptogenesis

spinors

\tilde{L} -charge

$$P_+ N_i, \quad \bar{N}_i P_+$$

+1

$$P_- N_i, \quad \bar{N}_i P_-$$

-1

$$\psi_N = (\nu_{Rs} + \nu_{Rw}^c)$$

B-L violating parameters

μ, ϵ, ϵ'

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{SM}} + \overline{\psi}_N (i\not{\partial} - \bar{M}) \psi_N + \overline{\nu}_{R3} i\not{\partial} \nu_{R3} - F_a^* \overline{\psi}_N \phi^T \epsilon^\dagger \ell_{La} - F_a \bar{\ell}_{La} \epsilon \phi^* \psi_N \\ & - \epsilon_a^* F_a^* \overline{\psi}_N^c \phi^T \epsilon^\dagger \ell_{La} - \epsilon_a F_a \bar{\ell}_{La} \epsilon \phi^* \psi_N^c - \epsilon'_a F_a \bar{\ell}_{La} \epsilon \phi^* \nu_{R3} - \epsilon_a'^* F_a^* \overline{\nu}_{R3} \phi^T \epsilon^\dagger \ell_{La} \\ & - \mu \bar{M} \frac{1}{2} (\overline{\psi}_N^c \psi_N + \overline{\psi}_N \psi_N^c) - \mu' \bar{M} \overline{\nu}_{R3}^c \nu_{R3}, \end{aligned}$$

B-L Symmetric Limit

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+1

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ν_{R3}

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approximately conserved
charges in leptogenesis

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+1

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-1

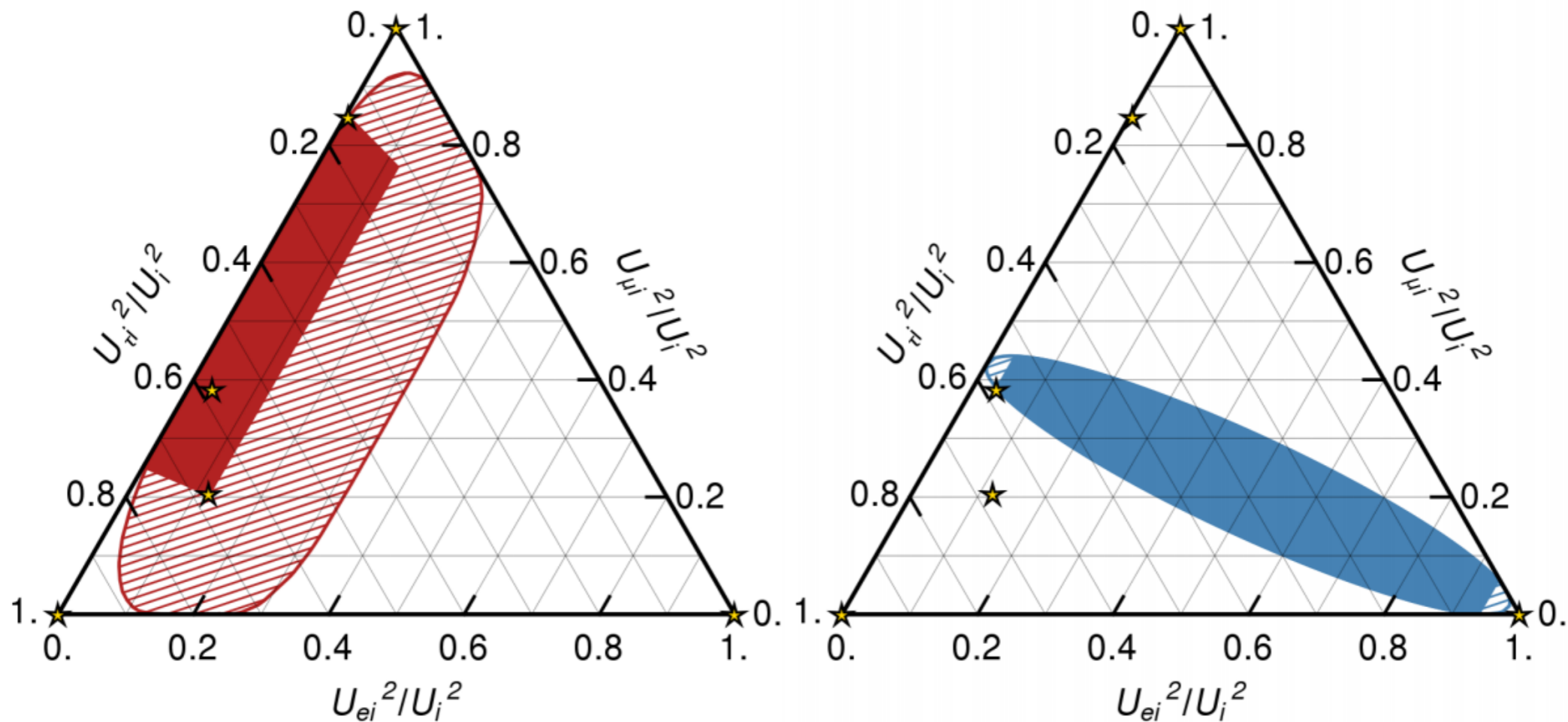
$$M_M = \begin{pmatrix} \bar{M}(1 - \mu) & 0 & 0 \\ 0 & \bar{M}(1 + \mu) & 0 \\ 0 & 0 & M' \end{pmatrix} \quad \begin{array}{l} \text{B-L violating parameters} \\ \mu, \epsilon, \epsilon' \end{array}$$

B-L symmetry dictates structure in sterile flavours

$$F = \begin{pmatrix} F_e(1 + \epsilon_e) & iF_e(1 - \epsilon_e) & F_e\epsilon'_e \\ F_\mu(1 + \epsilon_\mu) & iF_\mu(1 - \epsilon_\mu) & F_\mu\epsilon'_\mu \\ F_\tau(1 + \epsilon_\tau) & iF_\tau(1 - \epsilon_\tau) & F_\tau\epsilon'_\tau \end{pmatrix}, \quad \begin{array}{l} \nu \text{ oscillation} \\ \text{data constrains} \\ \text{structure in SM} \\ \text{flavours} \end{array}$$

Constraints from ν -Oscillation Data in Model with 2 Heavy Neutrinos

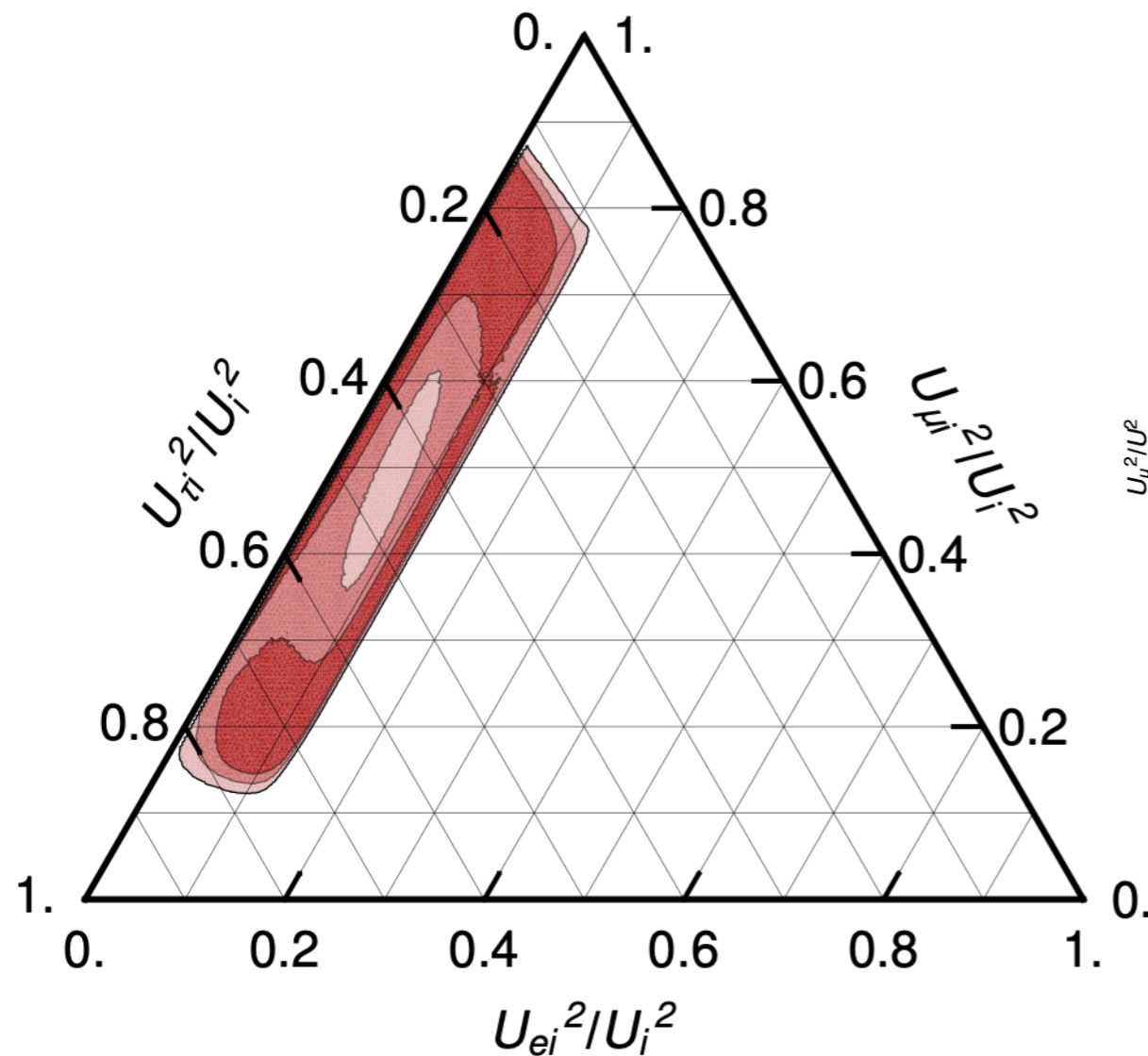
- This ν MSM-like scenario is very predictive
- In particular, the flavour mixing pattern is strongly constrained:
important for experimental sensitivity 1606.06719, 1609.09069, 1704.08721, 1801.04207



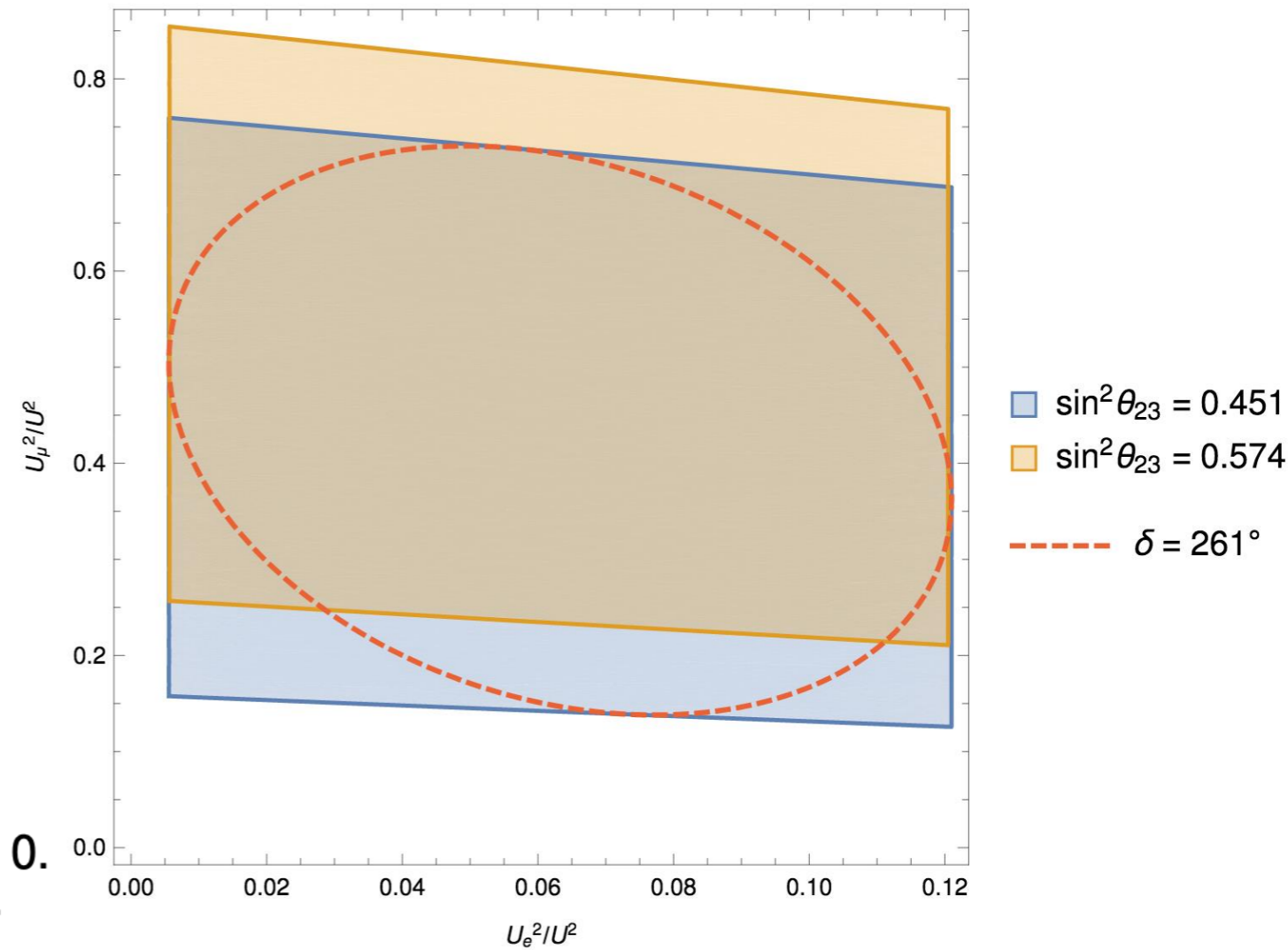
(a) Normal ordering.

(b) Inverted ordering.

Constraints from ν -Oscillation Data in Model with 2 Heavy Neutrinos



normal neutrino mass ordering



normal neutrino mass ordering

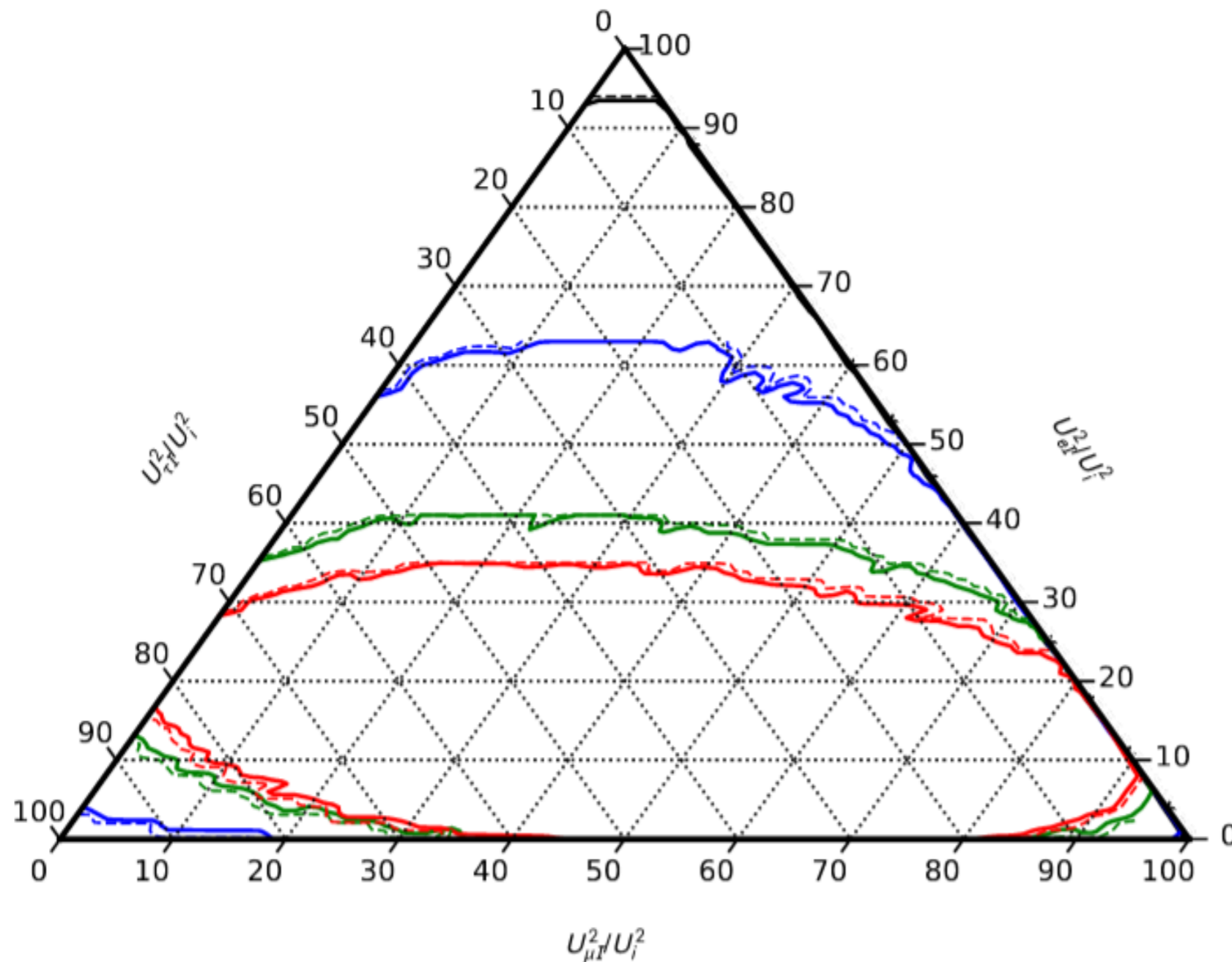
can measure Majorana phase α !

MaD et al [1609.09069](#), Caputo et al [1611.05000](#)

coloured areas: consistent with ν -oscillation data at 1σ , 2σ and 3σ

MaD/Hajer/Klaric/Lafranchi [1801.04207](#)

Constraints from ν -Oscillation Data in Model with 3 Heavy Neutrinos



normal ordering

$m_{\text{lightest}} < 10 \text{ meV}$

$m_{\text{lightest}} < 1 \text{ meV}$

$m_{\text{lightest}} < 0.1 \text{ meV}$

$m_{\text{lightest}} < 0.01 \text{ meV}$

Chrzaszcz et al [1908.02302](#)

Overview

Low Scale Seesaw Predictions for Neutrino Yukawas

Leptogenesis Predictions for Neutrino Yukawas

Other tests: LNV and CPV

Thermal Leptogenesis

Basic idea Fukugita/Yanagida

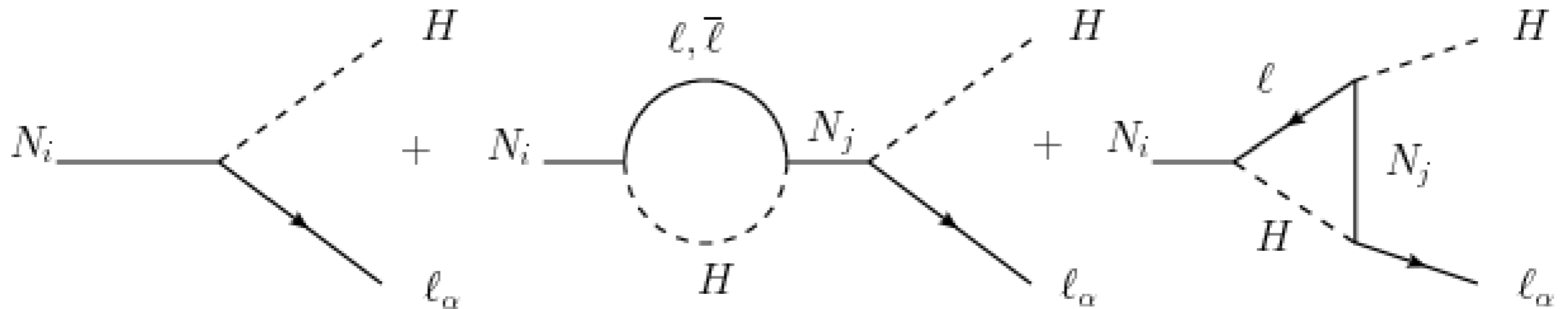
- N are around in the early universe
- N interactions are CP violating
- N may preferably decay into matter

CP violating parameter ϵ

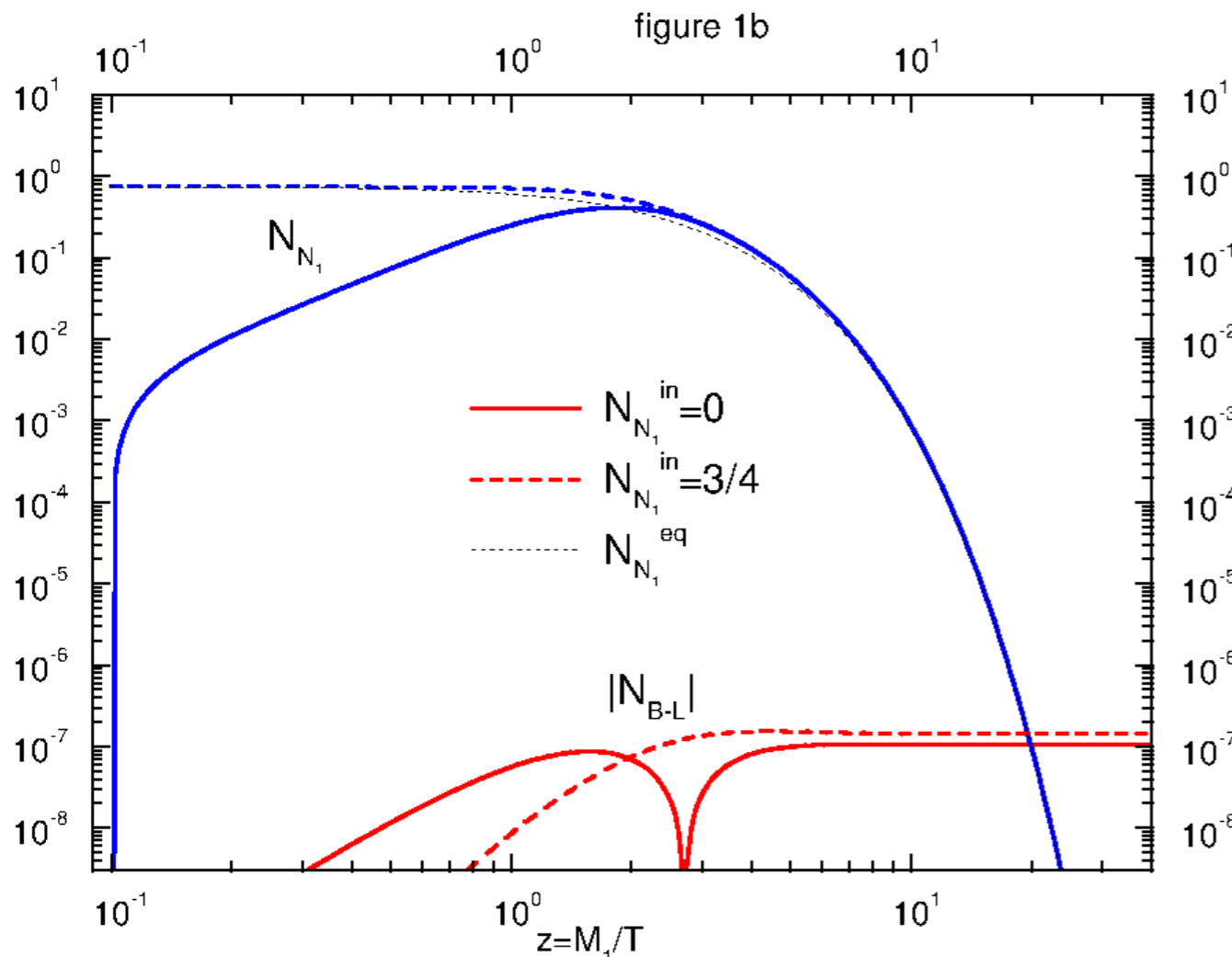
$$\epsilon = \frac{\Gamma_{N \rightarrow \ell H} - \Gamma_{N \rightarrow \bar{\ell} H^*}}{\Gamma_{N \rightarrow \ell H} + \Gamma_{N \rightarrow \bar{\ell} H^*}}$$

final asymmetry

$$Y_{B-L} \propto \epsilon/g_*$$



Leptogenesis with sub-TeV M ?



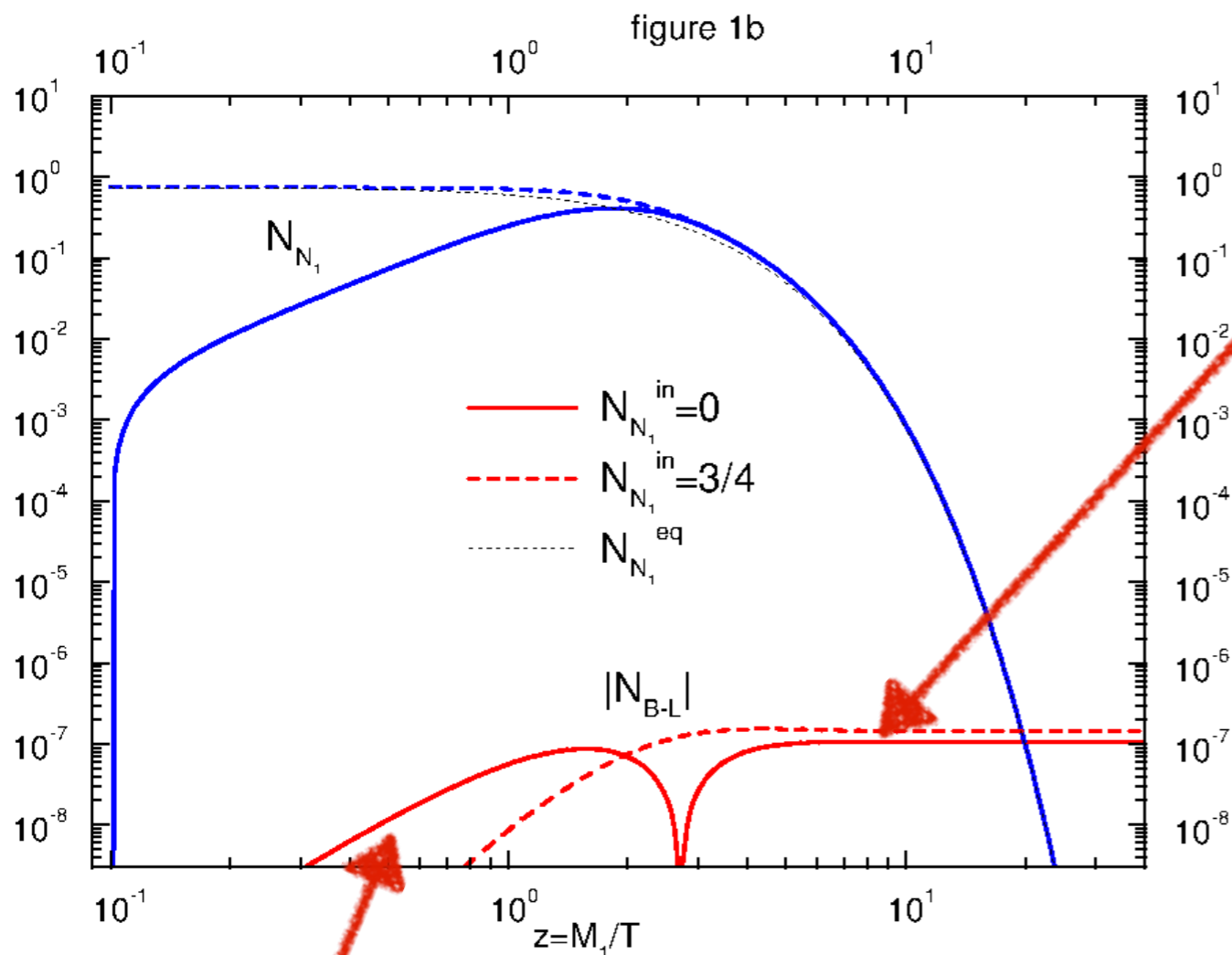
Buchmuller/Di Bari/Plumacher [0205349](#)

$$xH \frac{dY_N}{dx} = -\Gamma_N (Y_N - Y_N^{eq}) \quad x = M/T$$

$$xH \frac{dY_{B-L}}{dx} = \underbrace{\epsilon \Gamma_N (Y_N - Y_N^{eq})}_{\text{“source”}} - \underbrace{c_W \Gamma_N Y_{B-L}}_{\text{“washout”}}$$

- In “vanilla leptogenesis” flavour plays no role, asymmetry is independent of PMNS phases
- M should be larger than 10^9 GeV (Davidson/Ibarra 02)
- Flavour effects reduce lower bound on M a bit Abada/Davidson/Josse-Michaux/Losada/Riotto 06, Nardi/Nir/Roulet/Racker 06, Blanchet/Di Bari 07, Pascoli/Petcov/Riotto 07, De Simone/Riotto 07
- But how to go below TeV?

Leptogenesis with sub-TeV M ?



asymmetry generated during N decay ("freeze-out scenario")

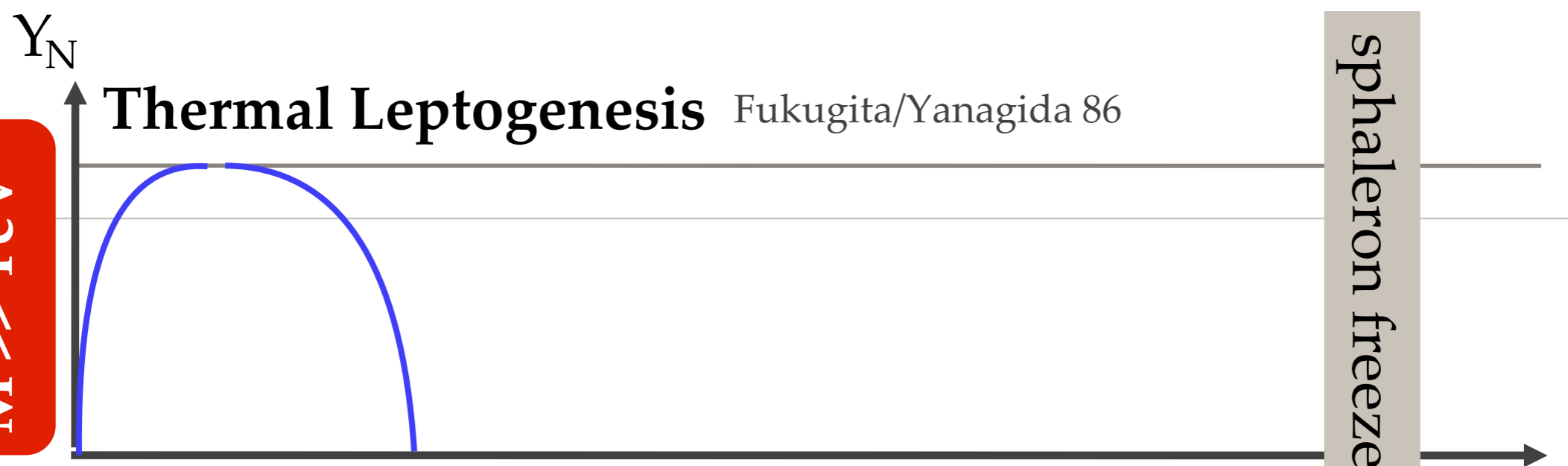
Sakharov's nonequilibrium condition can be fulfilled in two ways.

asymmetry generated during N production ("freeze-in scenario")

$$xH \frac{dY_N}{dx} = -\Gamma_N (Y_N - Y_N^{eq}) \quad x = M/T$$

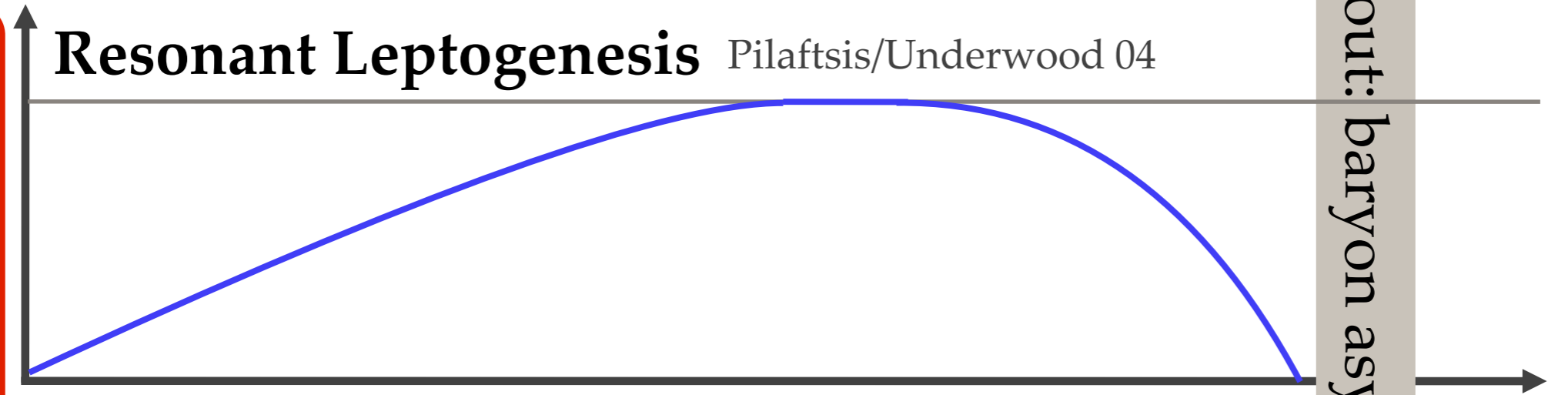
$$xH \frac{dY_{B-L}}{dx} = \underbrace{\epsilon \Gamma_N (Y_N - Y_N^{eq})}_{\text{"source"}} - \underbrace{c_W \Gamma_N Y_{B-L}}_{\text{"washout"}}$$

high scale
 $M \gg \text{TeV}$

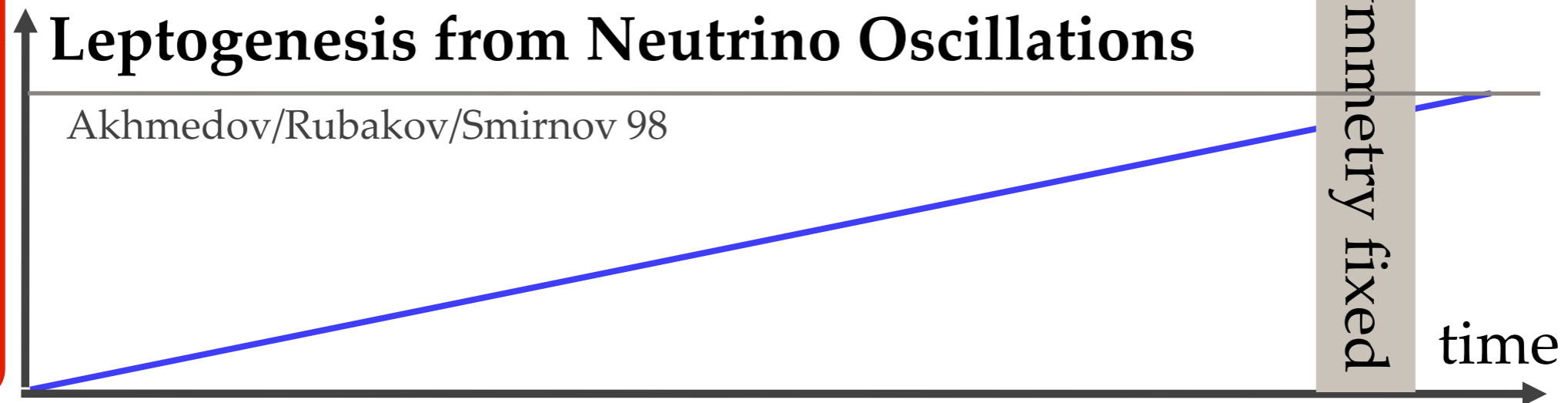


asymmetry generated in
freeze-out and decay

low scale
 $M < \text{TeV}$



asymmetry
generated in
freeze-in



"big bang"

$T = 130 \text{ GeV}$

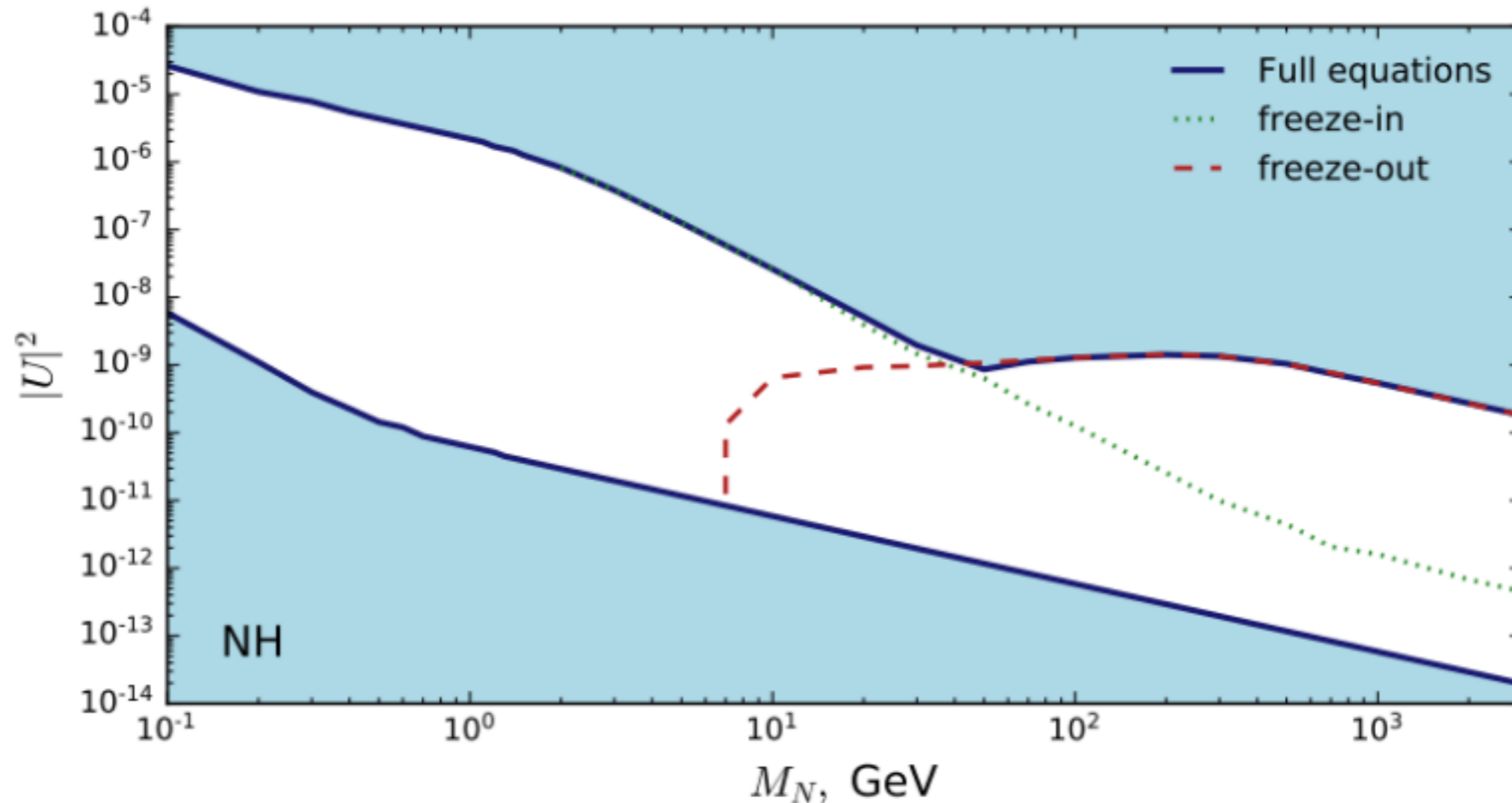
Quantitative Description

- Need to track three SM chemical potentials
- Track coherences for heavy neutrinos (“density matrix equations”)

$$\begin{aligned}
 i \frac{dn_{\Delta_\alpha}}{dt} &= -2i \frac{\mu_\alpha}{T} \int \frac{d^3 k}{(2\pi)^3} \text{Tr}[\Gamma_\alpha] f_N (1 - f_N) + i \int \frac{d^3 k}{(2\pi)^3} \text{Tr}[\tilde{\Gamma}_\alpha (\delta\bar{\rho}_N - \delta\rho_N)], \\
 i \frac{d\delta\rho_N}{dt} &= -i \frac{d\rho_N^{eq}}{dt} + [H_N, \rho_N] - \frac{i}{2} \{\Gamma, \delta\rho_N\} - \frac{i}{2} \sum_\alpha \tilde{\Gamma}_\alpha \left[2 \frac{\mu_\alpha}{T} f_N (1 - f_N) \right], \\
 i \frac{d\delta\bar{\rho}_N}{dt} &= -i \frac{d\rho_N^{eq}}{dt} - [H_N, \bar{\rho}_N] - \frac{i}{2} \{\Gamma, \delta\bar{\rho}_N\} + \frac{i}{2} \sum_\alpha \tilde{\Gamma}_\alpha \left[2 \frac{\mu_\alpha}{T} f_N (1 - f_N) \right].
 \end{aligned}$$

↑ Heavy neutrino density matrix
↑ SM chemical potentials
↑ Heavy neutrino effective Hamiltonian
↑ LNC rate $\sim F^2 T$
↑ LNV rate $\sim (M/T)^2 F^2 T$

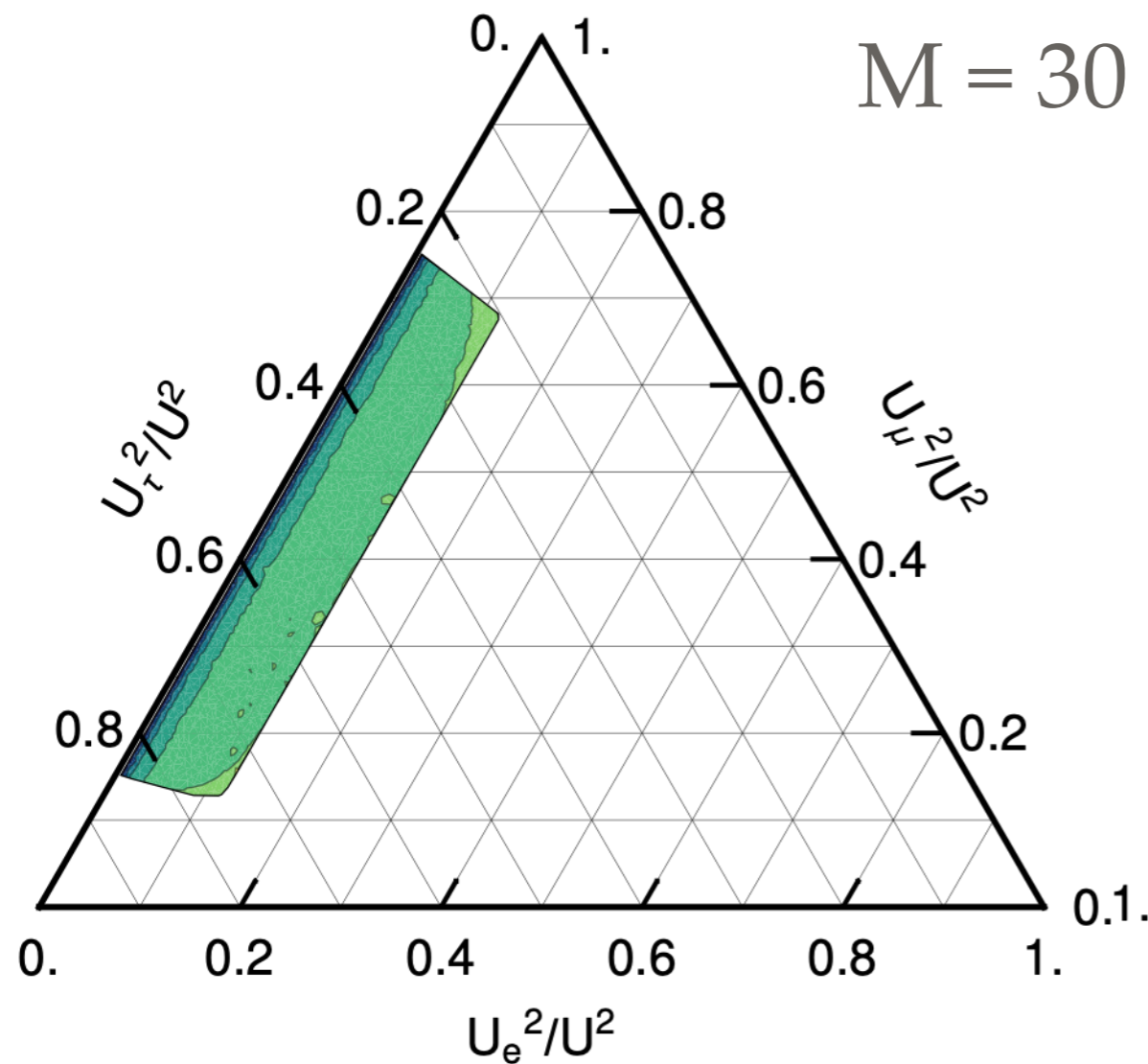
Leptogenesis with 2 RH Neutrinos



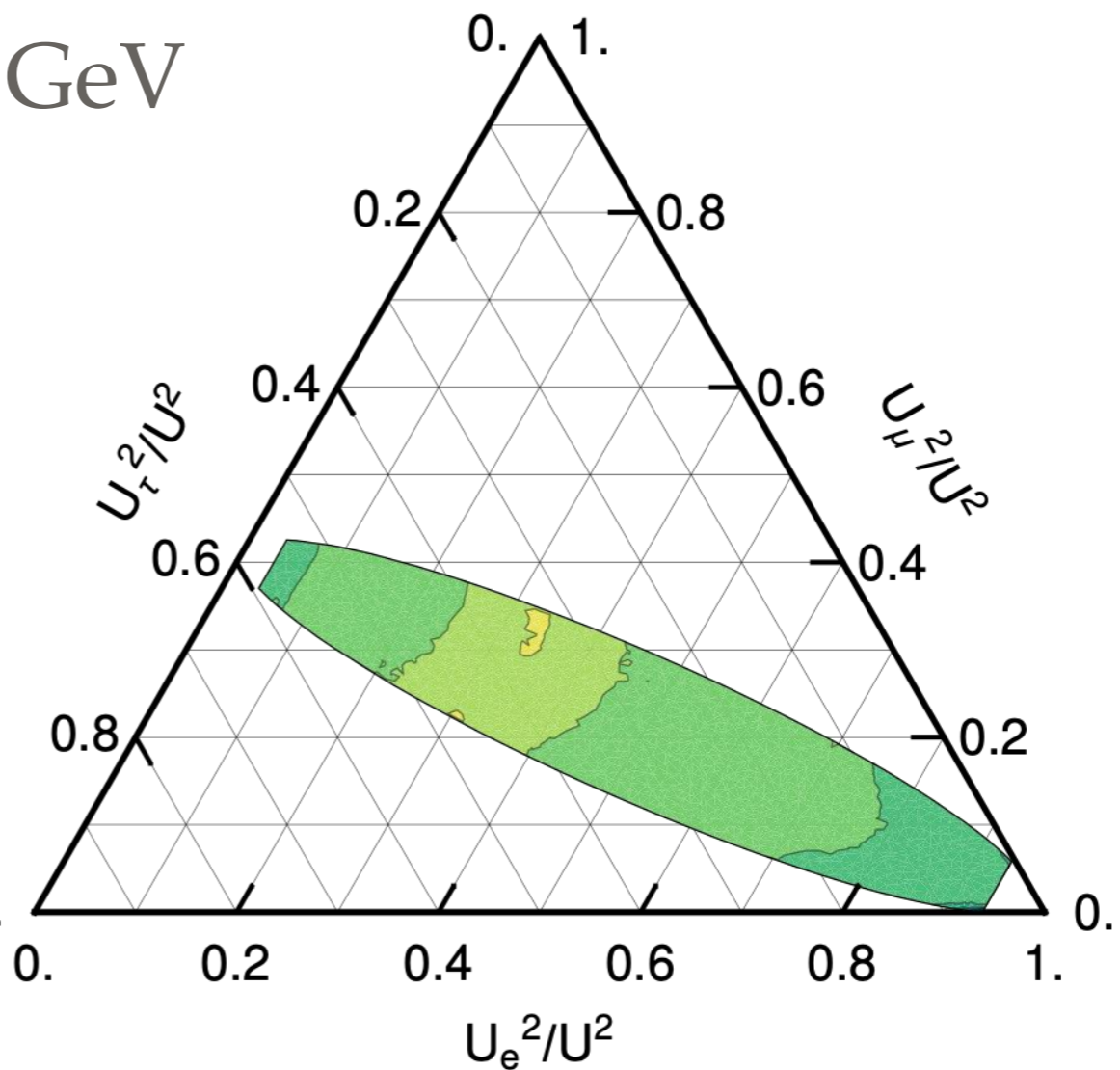
The region in which the freeze-out scenario (“resonant leptogenesis”) and freeze-in scenario (“ARS leptogenesis”) work overlap!

Constraints from Leptogenesis in Model with 2 Heavy Neutrinos

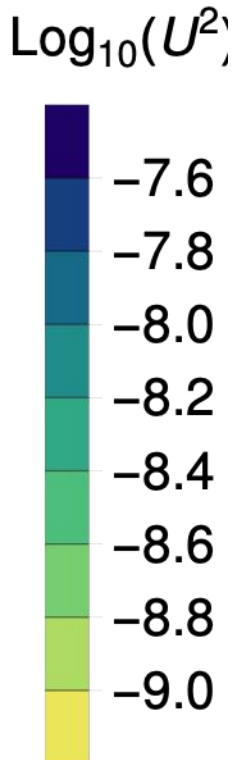
$M = 30 \text{ GeV}$



normal neutrino mass ordering



inverted neutrino mass ordering



Large U^2 require strong hierarchies in couplings to SM generations

Constraints from Leptogenesis in Model with 2 Heavy Neutrinos

- Mass basis at $T=0$ is the one where M is diagonal
- B-L limit: ν_{Rs} and ν_{Rw} define “interaction basis”
- $T \gg M$: thermal masses dominate, interaction basis is mass basis

$$F = \begin{pmatrix} F_e(1 + \epsilon_e) & iF_e(1 - \epsilon_e) \\ F_\mu(1 + \epsilon_\mu) & iF_\mu(1 - \epsilon_\mu) \\ F_\tau(1 + \epsilon_\tau) & iF_\tau(1 - \epsilon_\tau) \end{pmatrix}$$

“mass basis”

$$F \sim \begin{pmatrix} F_e & F_e \epsilon_e \\ F_\mu & F_\mu \epsilon_\mu \\ F_\tau & F_\tau \epsilon_\tau \end{pmatrix}$$

“interaction basis”

spinor	\bar{L} -charge
$\nu_{Rs} \equiv \frac{1}{\sqrt{2}}(\nu_{R1} + i\nu_{R2})$	+1
$\nu_{Rw} \equiv \frac{1}{\sqrt{2}}(\nu_{R1} - i\nu_{R2})$	-1

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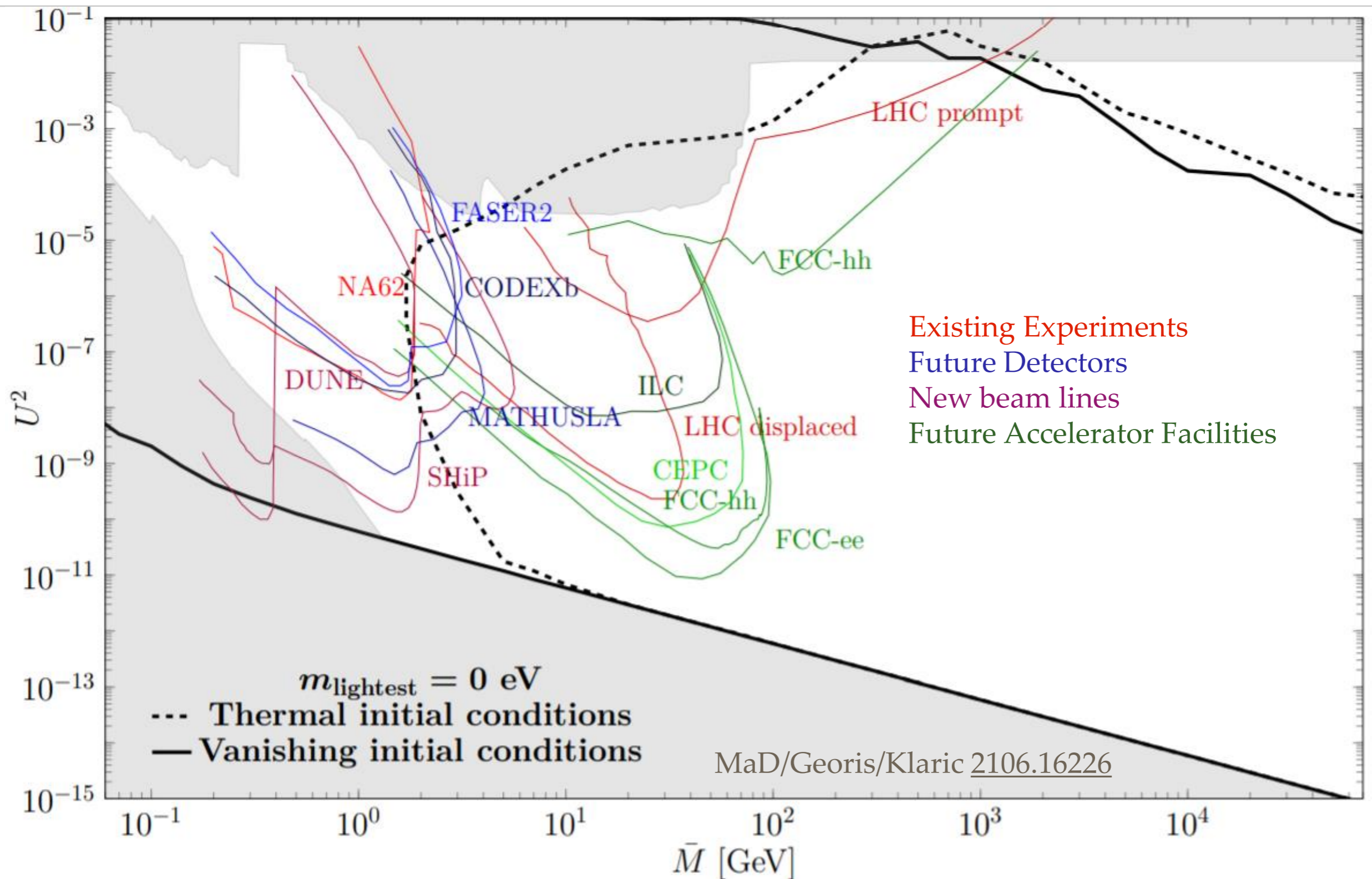
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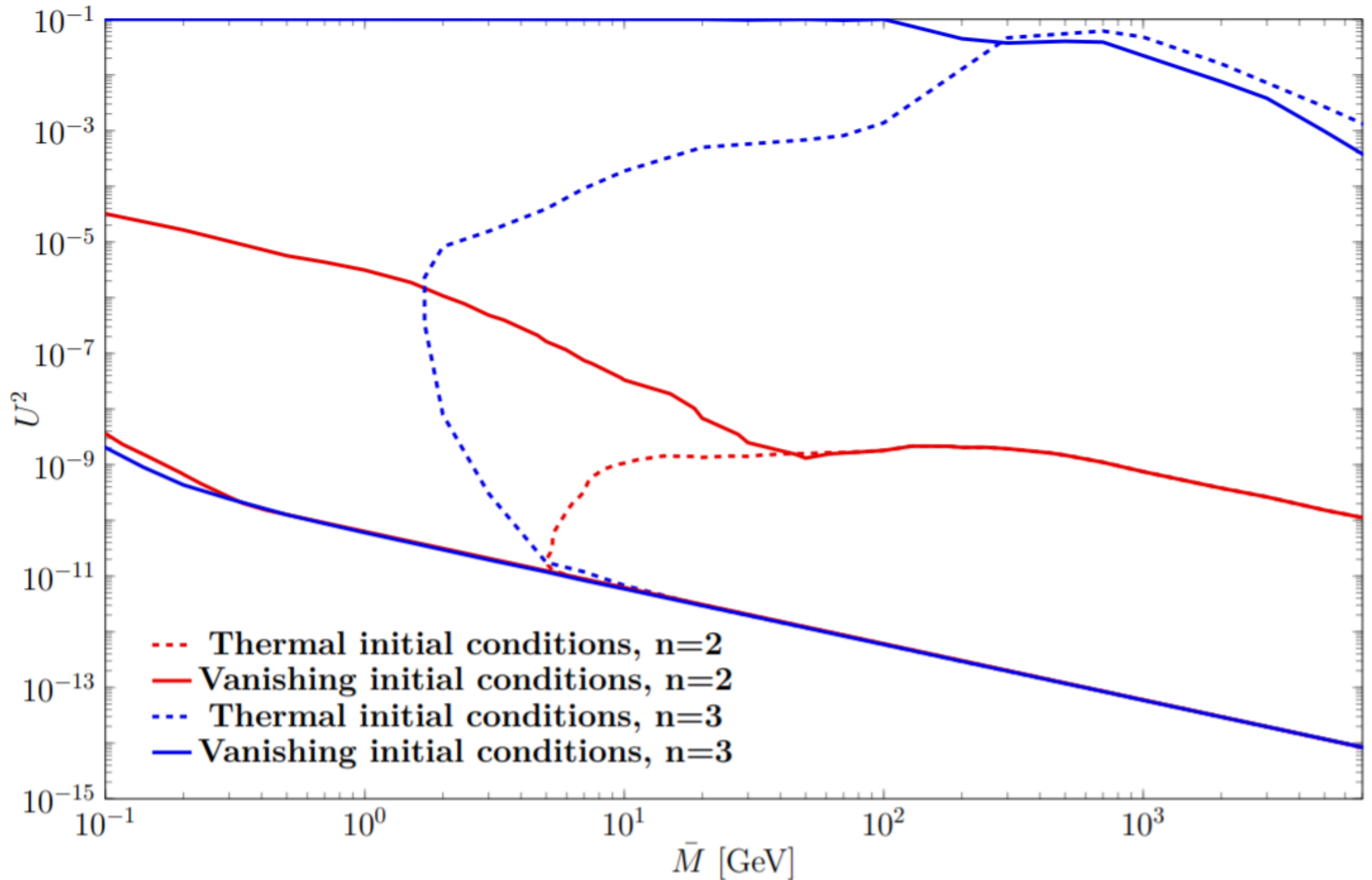
- For large U^2 , ν_{Rs} comes into equilibrium quickly, deviation from equilibrium necessary for baryogenesis comes from ν_{Rw}
- For $T \sim M$ both states become “strongly” coupled (LNV rates)
- Only way to prevent washout: Have one SM flavour feebly coupled

Structure in sterile flavours enforces hierarchy in SM flavour!

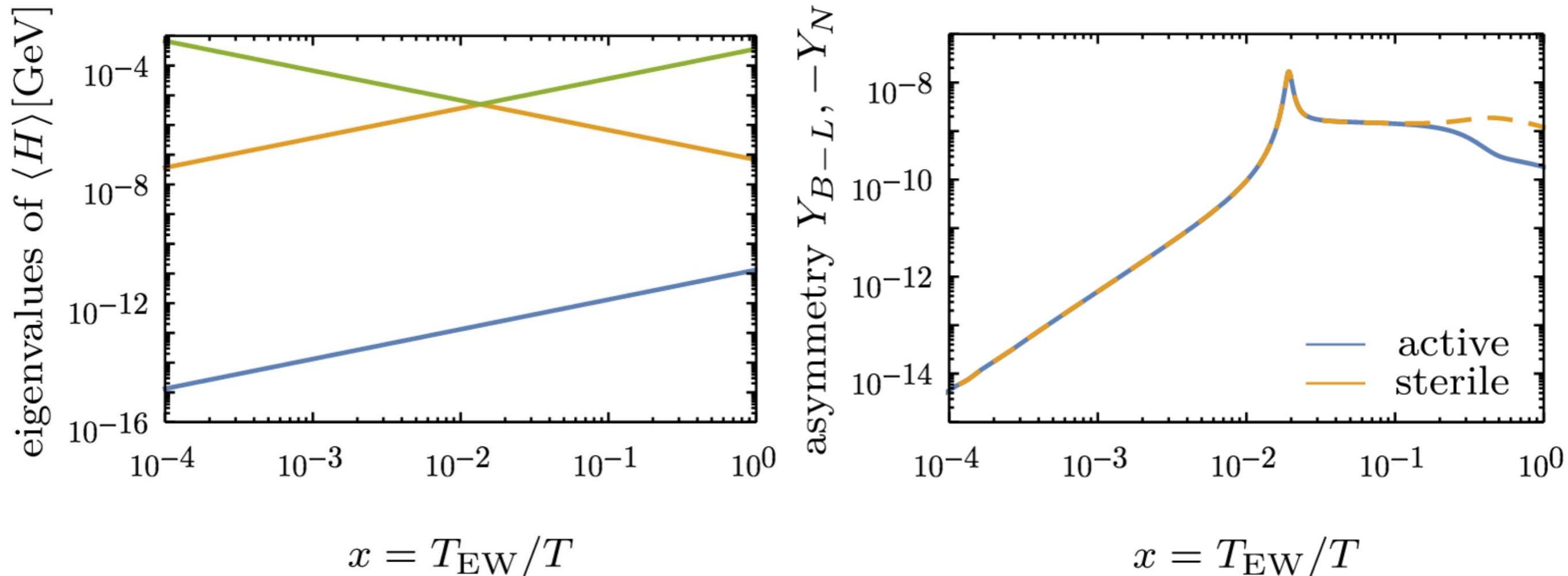
Leptogenesis with 3 RH Neutrinos



Leptogenesis with 3 RH Neutrinos



Dynamical Generation of Resonance



Abada et al [1810.12463](#)

- level crossing between the quasiparticle dispersion relations in the plasma (“thermal masses”) can dynamically generate a resonance
- Strong enhancement of the asymmetry with only moderate degeneracy in the vacuum masses

Maverick Heavy Neutrino

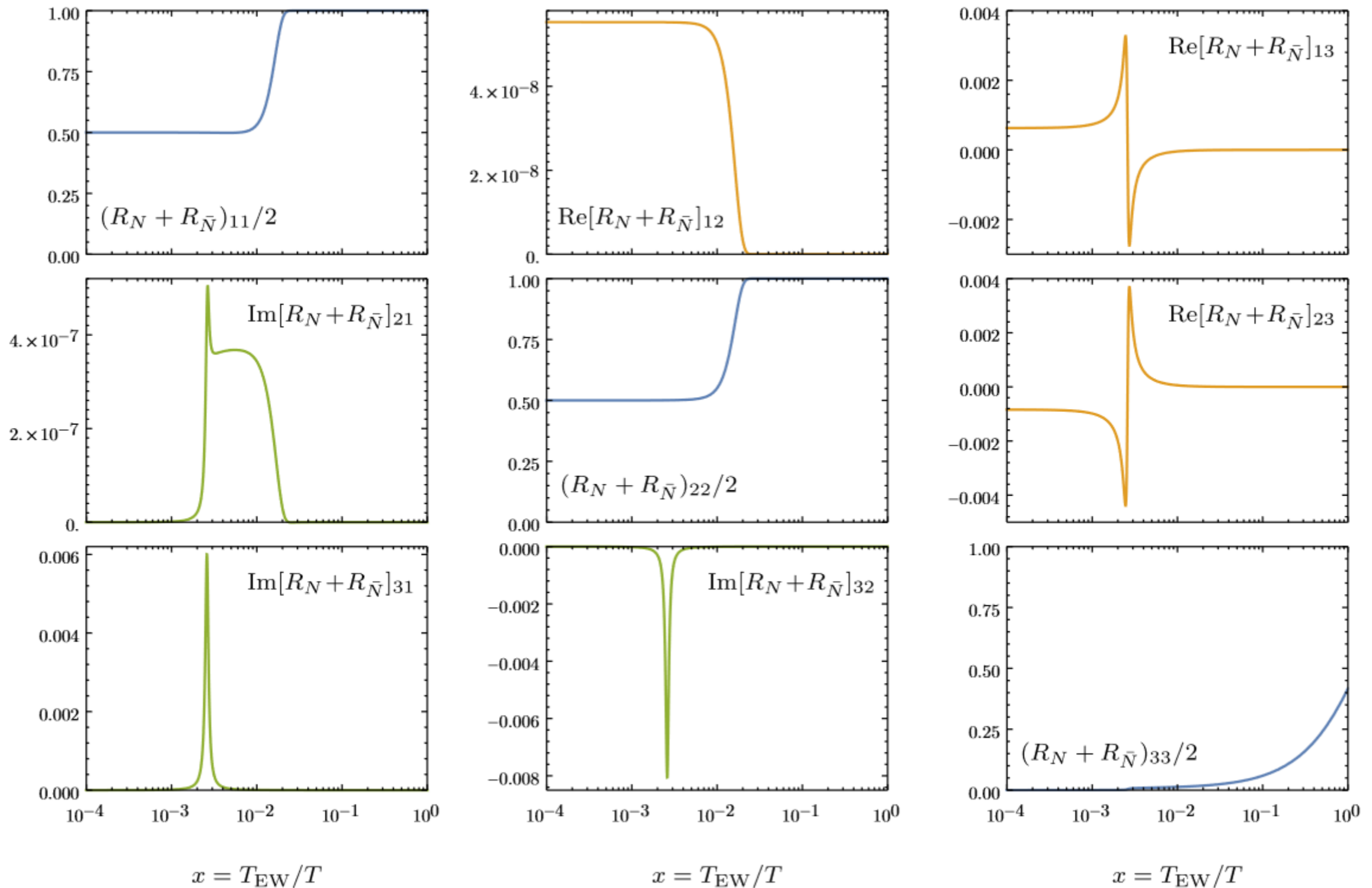
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$\nu_{Rw} \equiv \frac{1}{\sqrt{2}}(\nu_{R1} - i\nu_{R2})$	-1
ν_{R3}	0

$$F = \begin{pmatrix} F_e(1 + \epsilon_e) & iF_e(1 - \epsilon_e) & F_e\epsilon'_e \\ F_\mu(1 + \epsilon_\mu) & iF_\mu(1 - \epsilon_\mu) & F_\mu\epsilon'_\mu \\ F_\tau(1 + \epsilon_\tau) & iF_\tau(1 - \epsilon_\tau) & F_\tau\epsilon'_\tau \end{pmatrix},$$

- Third state ν_{R3} is free of the constraints that relates ν_{R3} and ν_{Rw}
- It can maintain deviation from equilibrium even when LNV rates come into equilibrium
- Can avoid washout even for large couplings of pseudo-Dirac pair
- No need for a hierarchy in SM flavour couplings to prevent washout!

Maverick Heavy Neutrino



Overview

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Leptogenesis Predictions for Neutrino Yukawas

Other tests: LNV and CPV

What can we learn?

Dirac vs Majorana

- **LNV vs LFV decay rates** e.g. Anamiati / Hirsch / Nardi [1607.05641](#)
- **Angular distribution** e.g. Arbelaez et al [1712.08704](#), Balantekin/Gouvea/Kayser [1808.10518](#)
- **Flavour mixing pattern** e.g. Dib / Kim / Wang / Zhang [1605.01123](#)

CP properties

e.g. Cvetič / Dib / Kim / Saa [1503.01358](#)

Mass spectrum

e.g. Antusch / Fischer [1709.03797](#)

Test seesaw mechanism and leptogenesis

- **For 2 RHN all Lagrangian parameters can in principle be constrained**

Hernandez / Kekic / Lopez-Pavon / Racker / Savaldo [1606.06719](#),

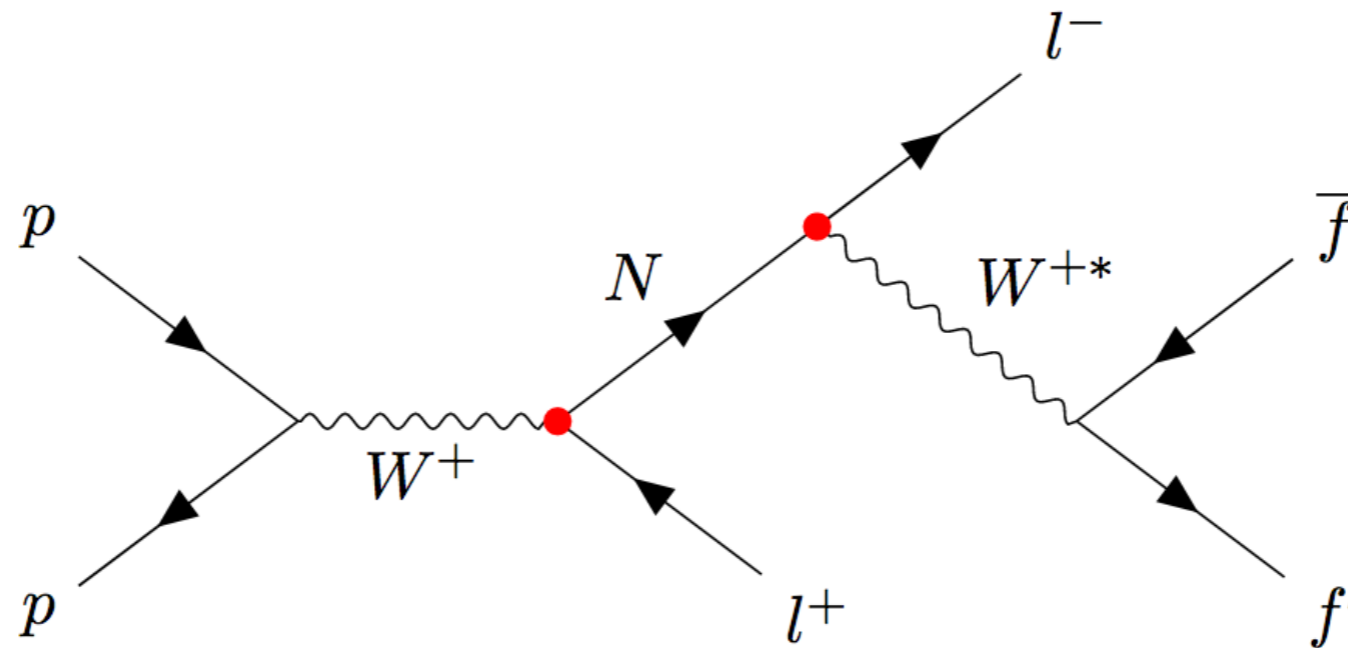
MaD / Garbrecht / Gueter / Klaric [1609.09069](#)

Antusch/Cazzato/MaD/Fischer/Garbrecht/Gueter/Klaric [1710.03744](#)

Can LNV be observed?

B-L symmetry: destructive interference amongst contributions from different HNL flavours

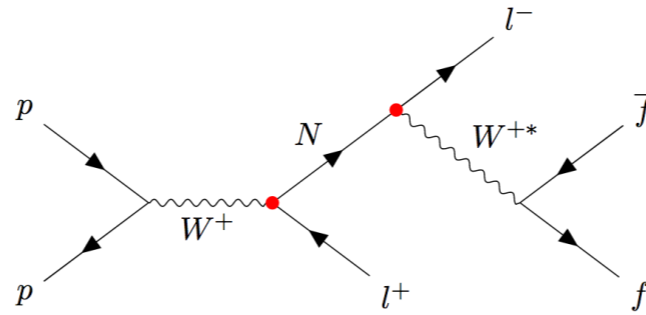
But: B-L is broken to generate neutrino mass. Is this enough???



(a) Charged current decay

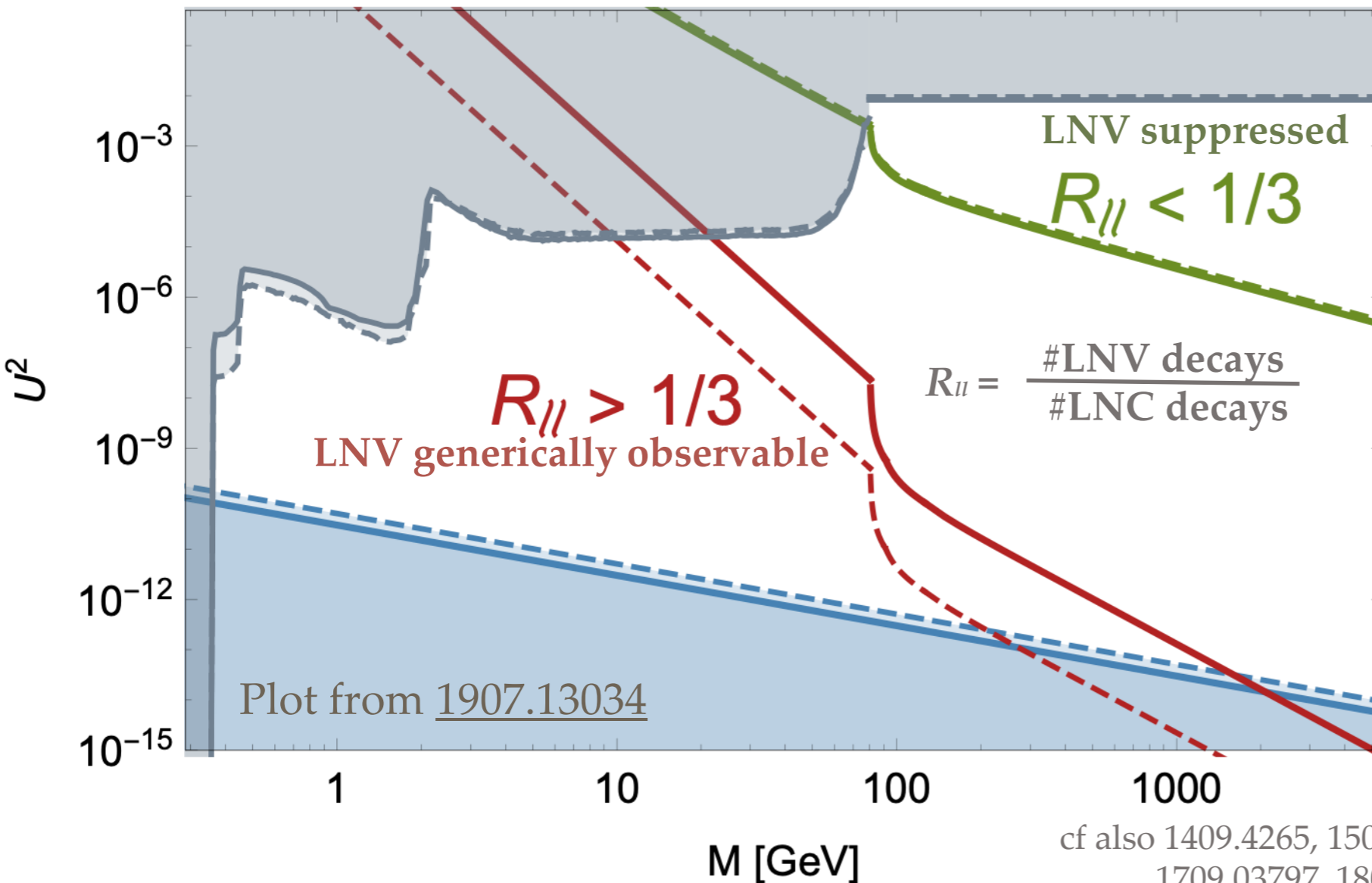
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But: B-L is broken to generate neutrino mass. Is this enough???



HNL oscillations destroy coherence and make LNV observable!

Anamiati et al [1607.05641](#)

$$\mathcal{R}_{ll} = \frac{\Delta M_{\text{phys}}^2}{2\Gamma_N^2 + \Delta M_{\text{phys}}^2}$$

Does neutrino osc. data allow for this without fine tuning? It depends

MaD/Klaric/Klose [1907.13034](#)

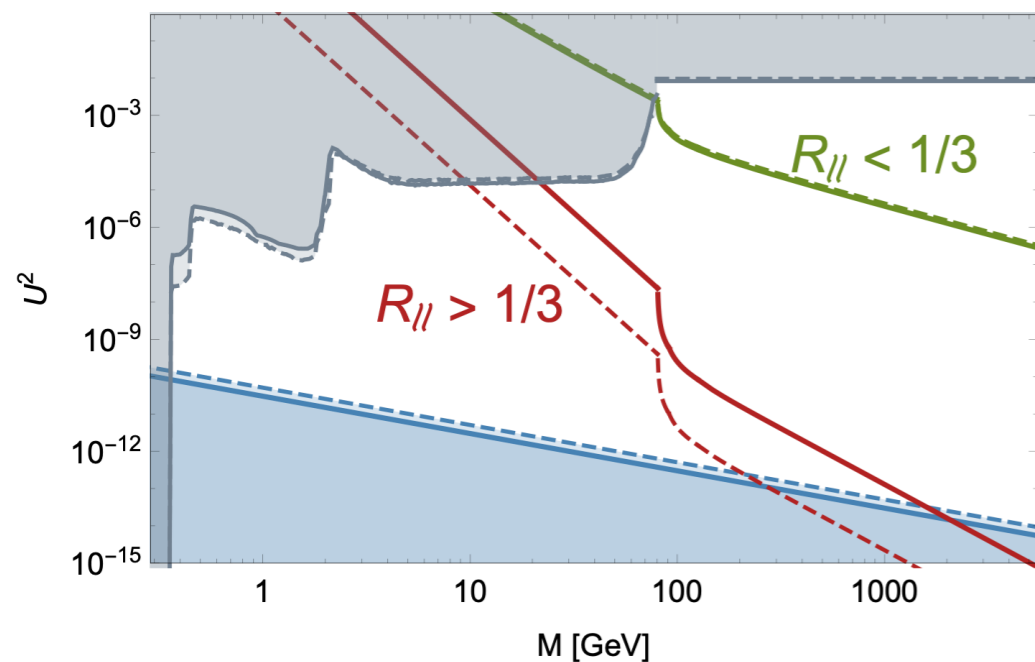
cf also [1409.4265](#), [1505.04749](#), [1605.01123](#), [1709.06553](#), [1703.01934](#), [1709.03797](#), [1805.00070](#), [1810.07210](#), [1905.03097](#), [1904.05367](#)

How to measure ΔM ?

ratio of LNV to LNC decays
is sensitive to ΔM

$$\mathcal{R}_{ll} = \frac{\Delta M_{\text{phys}}^2}{2\Gamma_N^2 + \Delta M_{\text{phys}}^2}$$

Anamiati et al [1607.05641](#)



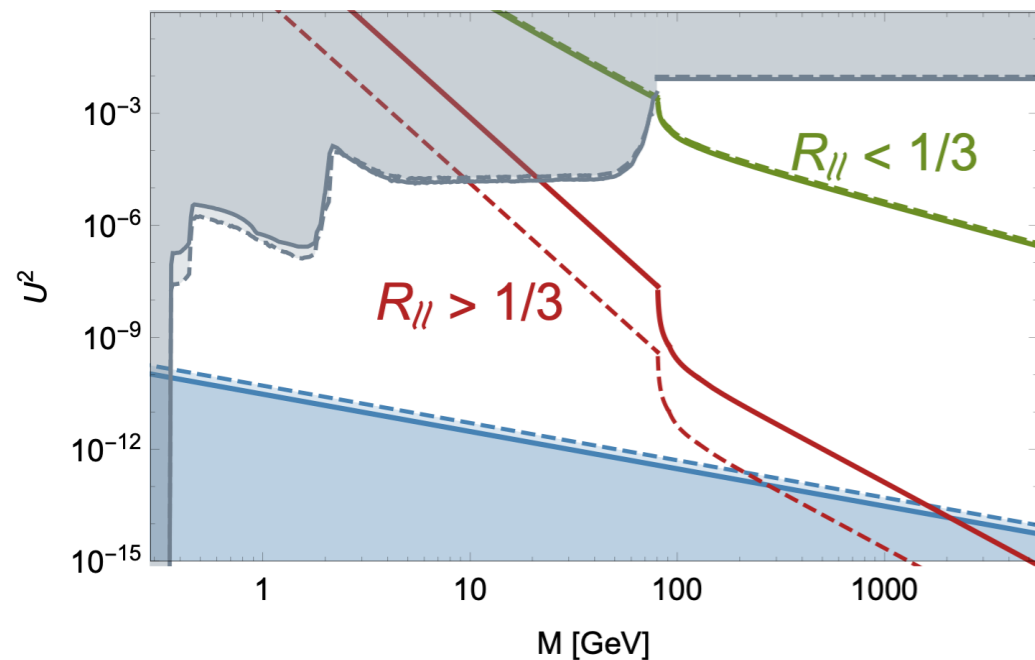
MaD/Klaric/Klose [1907.13034](#)

How to measure ΔM ?

ratio of LNV to LNC decays is sensitive to ΔM

$$\mathcal{R}_{ll} = \frac{\Delta M_{\text{phys}}^2}{2\Gamma_N^2 + \Delta M_{\text{phys}}^2}$$

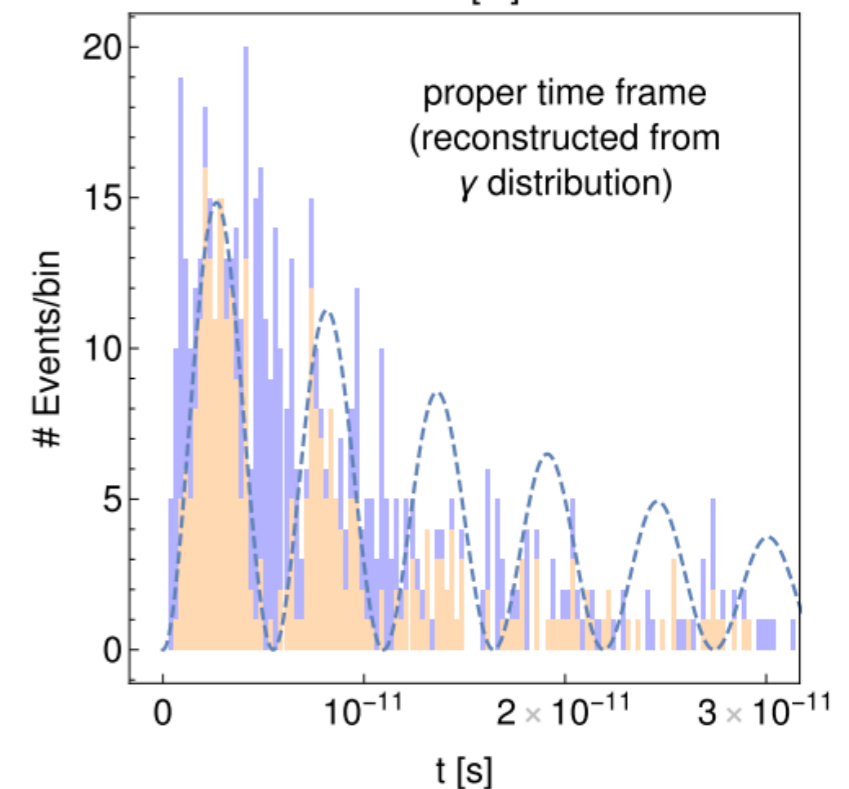
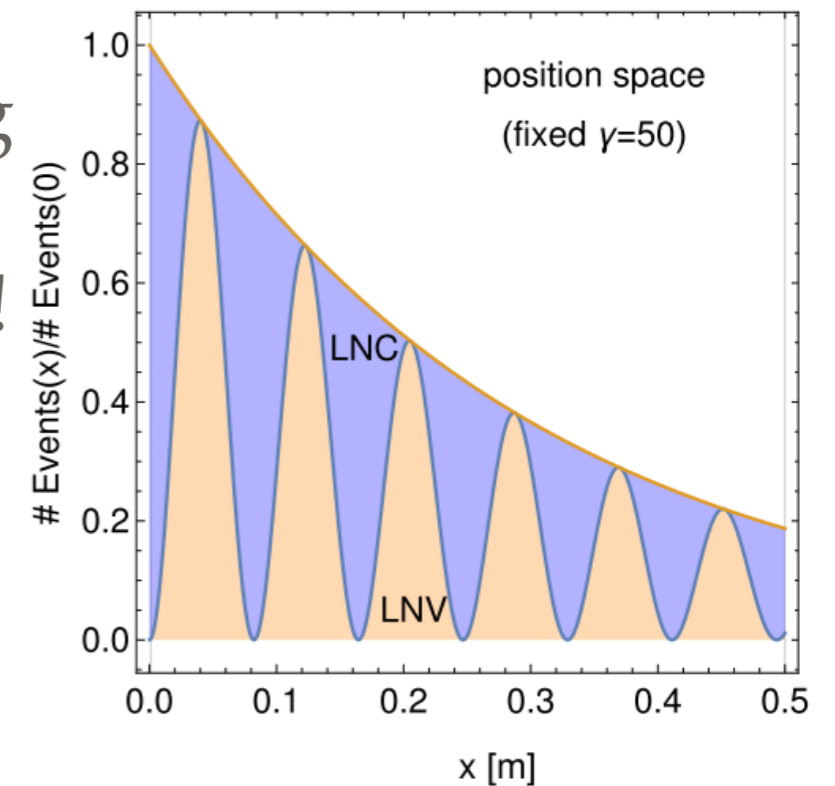
Anamiati et al [1607.05641](#)



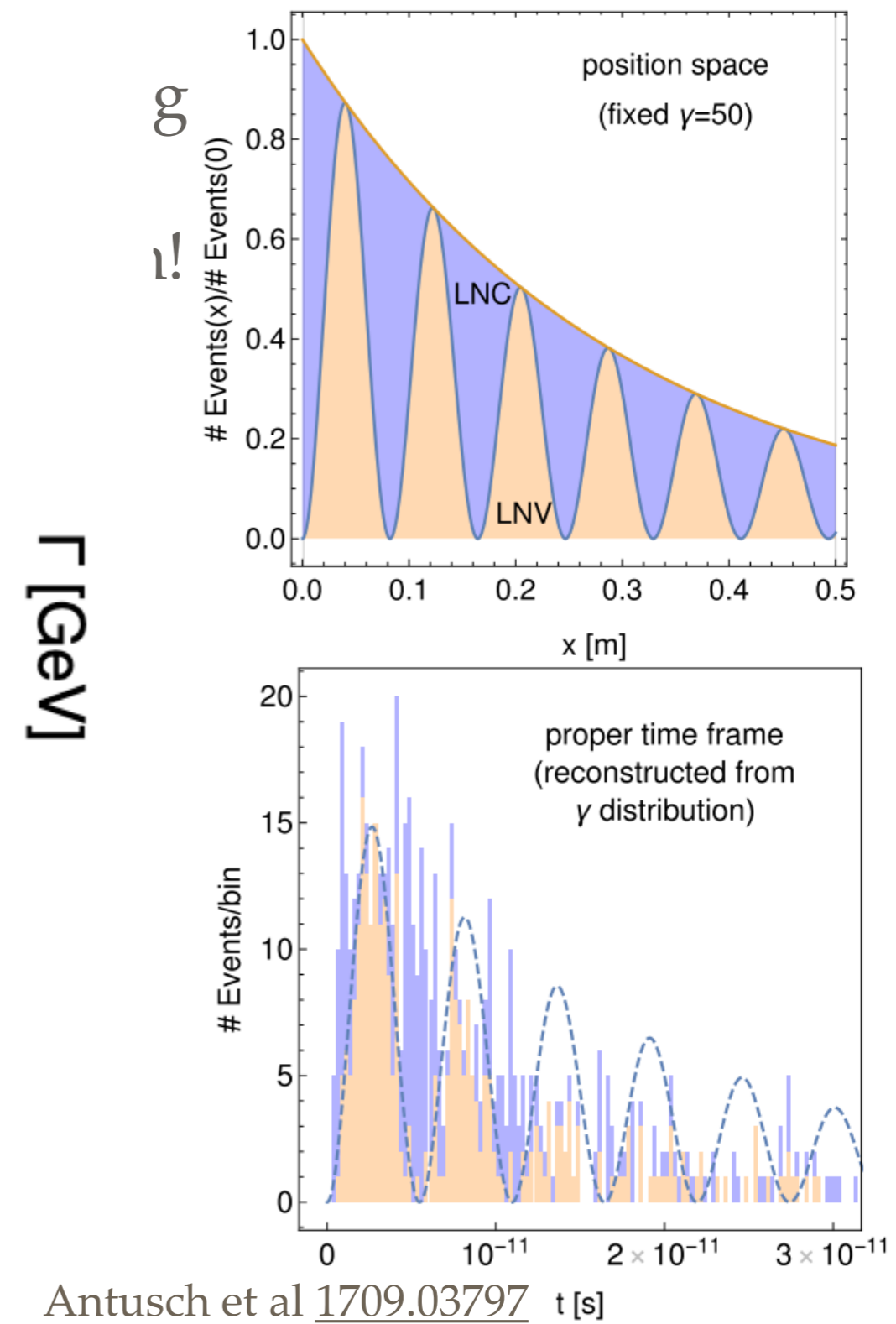
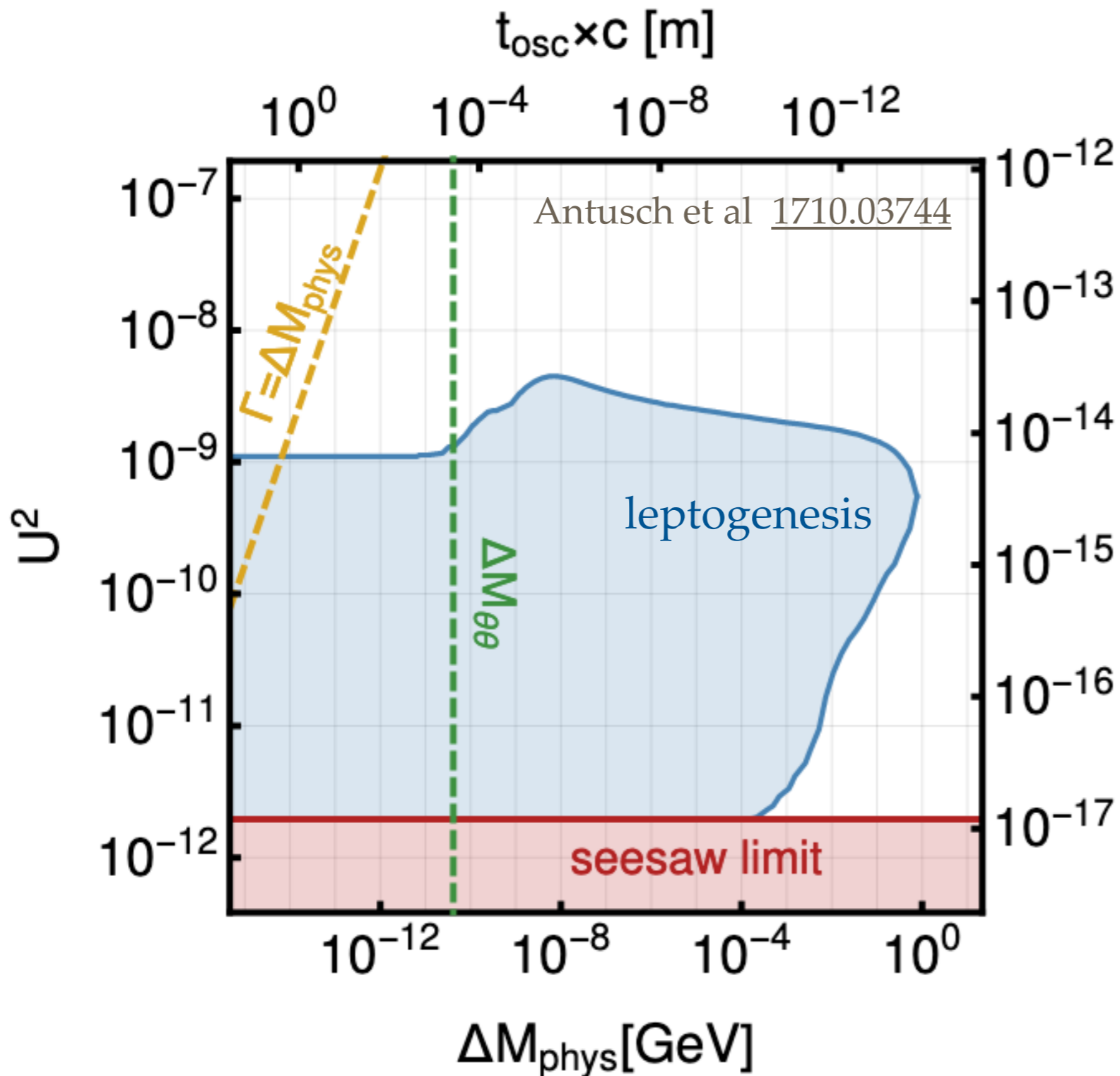
MaD/Klaric/Klose [1907.13034](#)

spatially resolving this ratio gives more information!

Antusch et al [1709.03797](#)



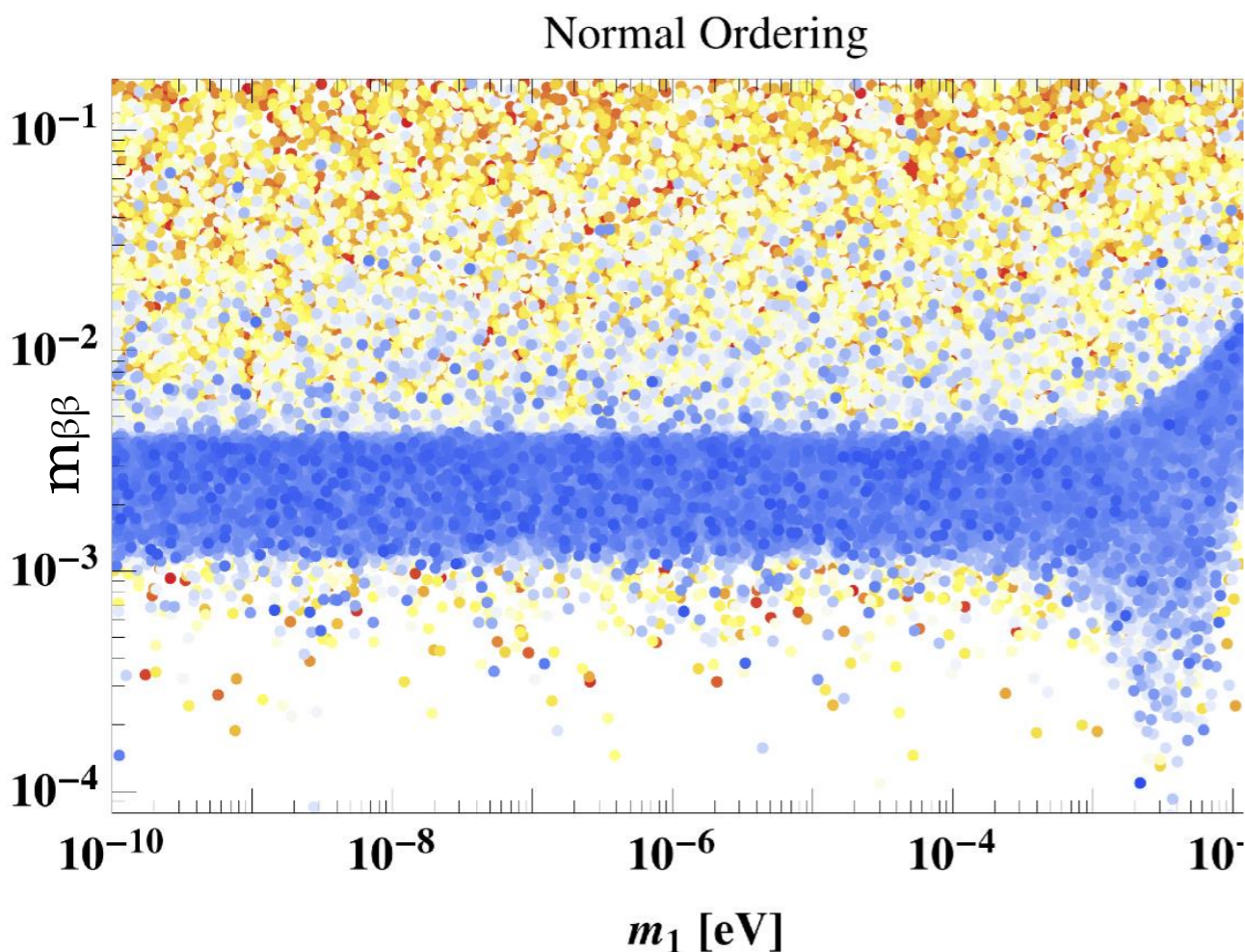
How to measure ΔM ?



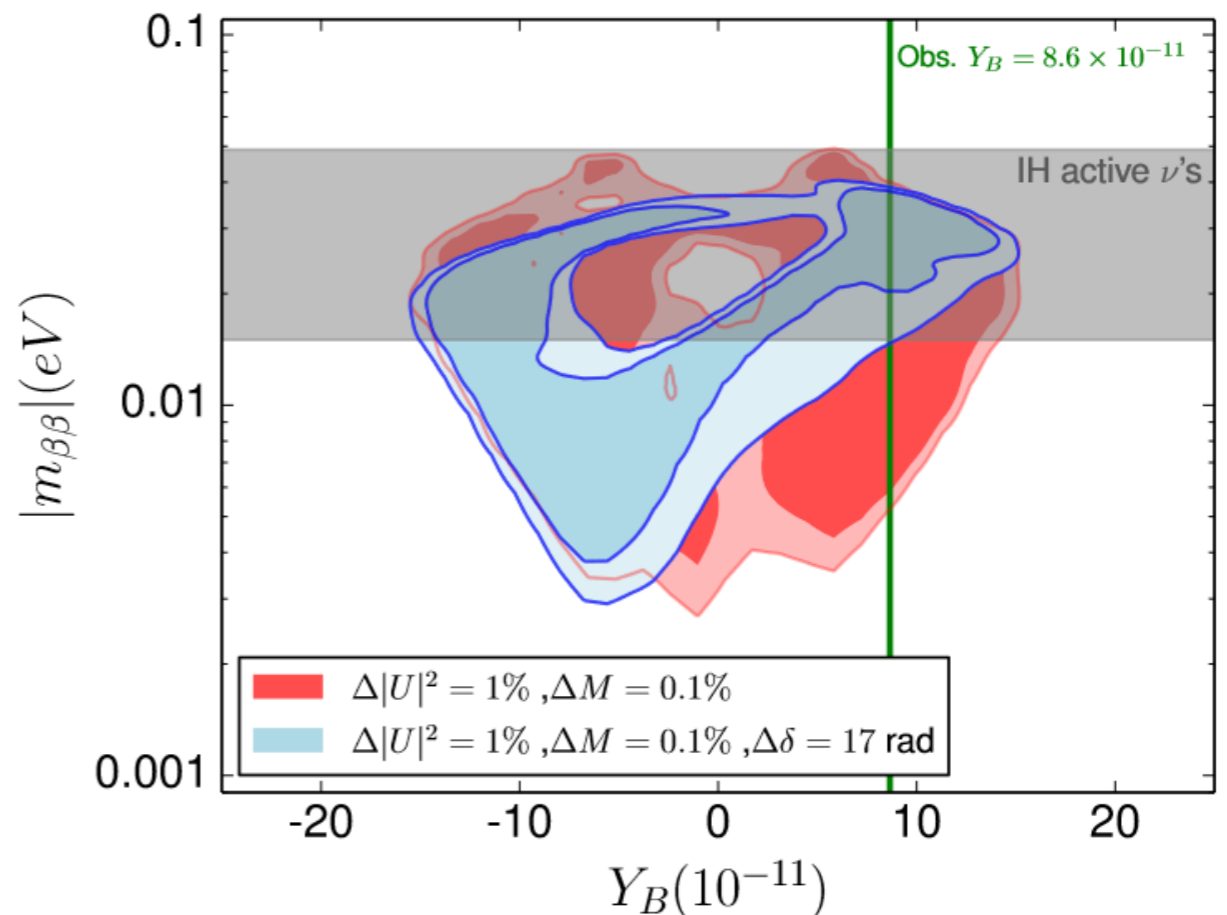
The $0\nu\beta\beta$ Connection

Heavy neutrino exchange can dominate $0\nu\beta\beta$...
...even in the leptogenesis region
 \Rightarrow additional probe of phase in the sterile sector

Bezrukov [0505247](#)
Blennow et al [1005.3240](#)
Lopez Pavon et al [1209.5342](#)
MaD/Eijima [1606.06221](#)
Hernandez et al [1606.06719](#)
Asaka et al [1606.06686](#)
Abada et al [1810.12463](#)



Abada et al [1810.12463](#)



Hernandez et al [1606.06719](#)

Summary

- Leptogenesis during freeze-out is possible for GeV Majorana masses M , for freeze-in even M in the MeV range is feasible
- With two RH neutrinos the model makes clear predictions for the flavour structure of the Yukawas that can be tested
- With three RH neutrinos the viable parameter space is several orders of magnitude larger, but the model is less predictive
- If heavy neutral leptons are discovered, the leptogenesis hypothesis can be tested by combining data from different “frontiers”

A Multi Frontier Problem!

Indirect probes at accelerators
rare decays, EWPD,
lepton universality)

absolute neutrino mass
searches (KATRIN ect.)

non-accelerator
searches
(TRISTAN...)

neutrinoless
double β decay

fixed target experiments
(SHiP, NA62, DUNE,
T2K..)

neutrino oscillation
experiments
DUNE, Hyper-K

new detectors
(FASER, Codex-b,
MATHUSLA, A13X,
ANUBIS

Collider searches for heavy neutrinos

X-ray searches: SRG/eROSITA,
SRG/ART-XC, ATHNEA, XRISM,
Lynx...

CMB and LSS :
absolute neutrino mass

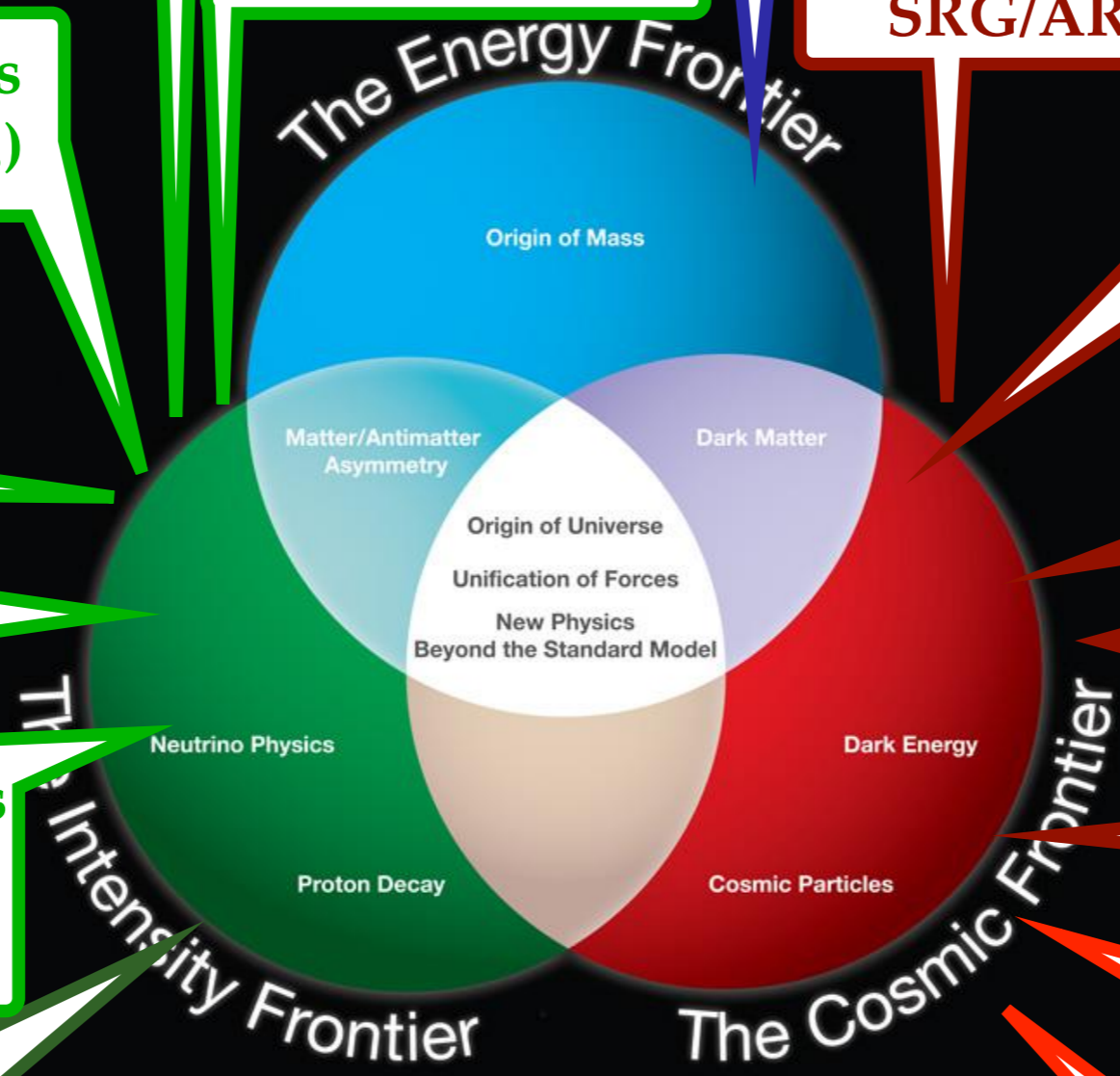
astrophysics:
supernovae etc.

Structure formation:
simulation, observation

IGM temperature:
WDM vs CDM

Theory: leptogenesis
parameter region

Theory: Sterile neutrino
DM production



RF, NF, EF, CF, TF

Full Testability of the ν MSM

In principle all parameters in the Lagrangian can be constrained experimentally...

cf. Hernandez et al [1606.06719](#), MaD et al [1609.09069](#)

$$F = \frac{1}{v} U_\nu \sqrt{m_\nu^{\text{diag}}} \mathcal{R} \sqrt{M^{\text{diag}}} \quad \text{Casas/Ibarra 01}$$

Full Testability of the ν MSSM

In principle all parameters in the Lagrangian can be constrained experimentally...
cf. Hernandez et al [1606.06719](#), MaD et al [1609.09069](#)

Unknown parameters:

M , ΔM , $\text{Re}\omega$, $\text{Im}\omega$, δ , α

Higgs vev v

light neutrino
mixing angles

light neutrino
mass splittings

$$F = \frac{1}{v} U_\nu \sqrt{m_\nu^{\text{diag}}} \mathcal{R} \sqrt{M^{\text{diag}}}$$

Casas/Ibarra 01

Dirac phase δ
Majorana phase α

lightest ν mass
(almost) vanishes

complex
angle ω

N -mass M
and splitting ΔM