

Semileptonic $B_{(s)}$ decays

Oliver Witzel



Physics of the flavourful Universe
Portorož, Slovenia · September 22, 2021

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in collaboration with

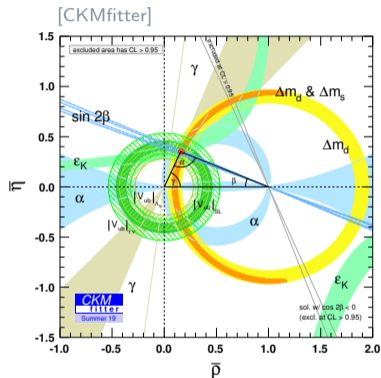
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Andreas Jüttner, Amarjit Soni, J. Tobias Tsang

RBC-UKQCD collaborations



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Motivation



Motivation



V_{cb} and V_{ub}

from FLAG 2019 arXiv:1902.08191

$$|V_{cb}| \text{ (excl)} = (39.09 \pm 0.68) 10^{-3}$$

$$|V_{cb}| \text{ (incl)} = (42.16 \pm 0.50) 10^{-3}$$

from Bordone et al.
arXiv:2107.00604

$\sim 2.8\sigma$ discrepancy

from FLAG 2019 arXiv:1902.08191

$$|V_{ub}| \text{ (excl)} = (3.73 \pm 0.14) 10^{-3}$$

$$|V_{ub}| \text{ (incl)} = (4.19 \pm 0.17 \pm 0.18 \text{ [flat]}) 10^{-3}$$

from GGOU HFLAV 2021
adding a flat uncertainty
covering the spread
of central values

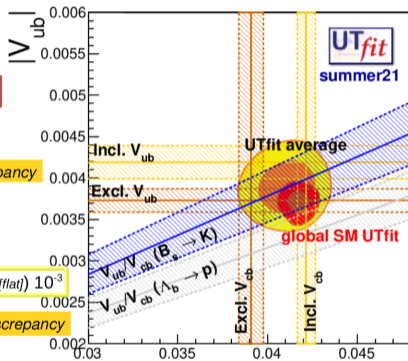
$\sim 1.5\sigma$ discrepancy

$$|V_{ub} / V_{cb}| \text{ (LHCb)} = (9.46 \pm 0.79) 10^{-2}$$

From B_s to K at high q^2

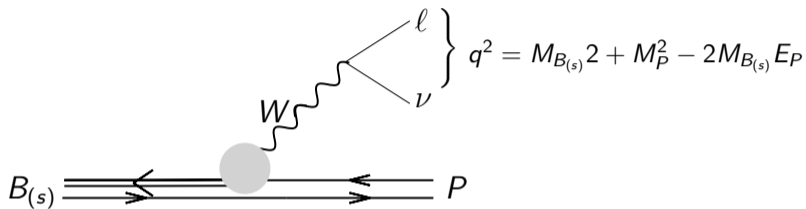
$$|V_{ub} / V_{cb}| \text{ (LHCb)} = (7.9 \pm 0.6) 10^{-2}$$

From Λ_b , excluded following FLAG guidelines


 $|V_{cb}|$

► Marcella Bona
[EPS-HEP 2021]

Exclusive semileptonic $B_{(s)}$ decays



pseudoscalar initial state

charged vector current

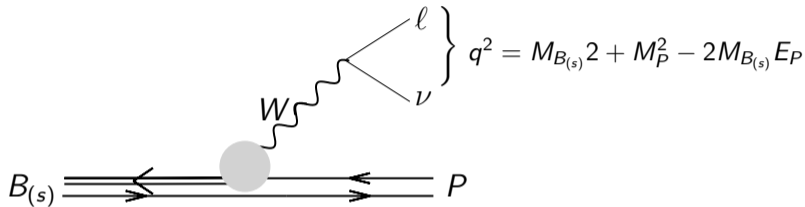
pseudoscalar final state

$$B \rightarrow \pi\ell\nu$$

$$B_s \rightarrow K\ell\nu$$

$$B_s \rightarrow D_s\ell\nu$$

Exclusive semileptonic $B_{(s)}$ decays

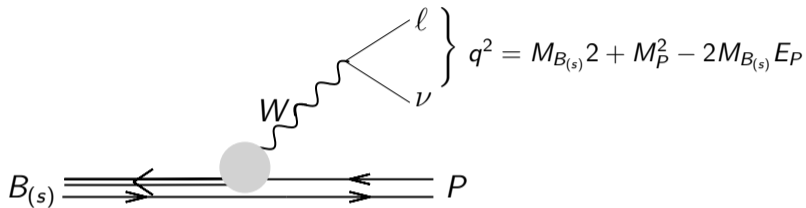


- Conventionally parametrized ($B_{(s)}$ meson at rest)

$$\frac{d\Gamma(B_{(s)} \rightarrow P\ell\nu)}{dq^2} = \frac{\eta_{EW} G_F^2 |V_{xb}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_P^2 - M_P^2}}{q^4 M_{B(s)}^2} \times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M_{B(s)}^2 (E_P^2 - M_P^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (M_{B(s)}^2 - M_P^2)^2 |f_0(q^2)|^2 \right]$$

experiment CKM known nonperturbative input

Exclusive semileptonic $B_{(s)}$ decays



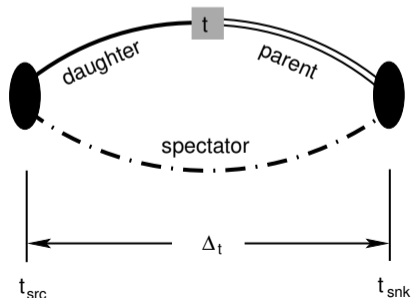
- ▶ Nonperturbative form factors $f_+(q^2)$ and $f_0(q^2)$
 - Parametrizes interactions due to (nonperturbative) strong force
 - Use operator product expansion (OPE) to identify short distance contributions
 - Calculate matrix element of flavor changing currents as point-like operators using lattice QCD

Lattice determination of form factors

- ▶ Parametrize the hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle P | V^\mu | B_{(s)} \rangle = f_+(q^2) \left(p_{B_{(s)}}^\mu + p_P^\mu - \frac{M_{B_{(s)}}^2 - M_P^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_{B_{(s)}}^2 - M_P^2}{q^2} q^\mu$$

- ▶ Calculate the corresponding 3-point function
 - Inserting source for spectator quark at t_{src}
 - Allow it to propagate to t_{sink}
 - Turn it into a sequential source for b quark
 - Propagate light daughter quark from t_{src}
 - Contract with b quark at t with $t_{\text{src}} \leq t \leq t_{\text{sink}}$



Lattice determination of form factors

- ▶ Parametrize the hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle P | V^\mu | B_{(s)} \rangle = f_+(q^2) \left(p_{B_{(s)}}^\mu + p_P^\mu - \frac{M_{B_{(s)}}^2 - M_P^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_{B_{(s)}}^2 - M_P^2}{q^2} q^\mu$$

- ▶ Prefer to compute

$$f_{\parallel}(E_P) = \langle P | V^0 | B_{(s)} \rangle / \sqrt{2M_{B_{(s)}}} \quad \text{and} \quad f_{\perp}(E_P) p_P^i = \langle P | V^i | B_{(s)} \rangle / \sqrt{2M_{B_{(s)}}}$$

which are directly proportional to 3-point functions

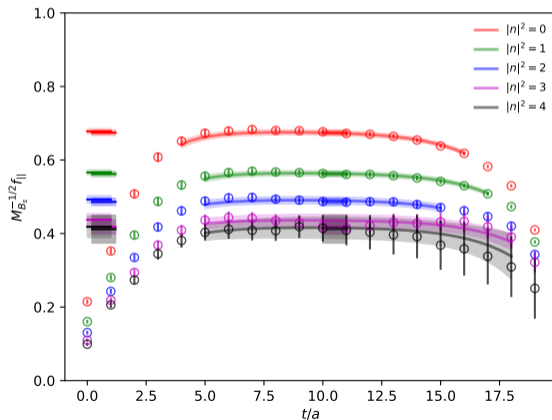
- ▶ Both are related by

$$f_0(q^2) = \frac{\sqrt{2M_{B_{(s)}}}}{M_{B_{(s)}}^2 - M_P^2} \left[(M_{B_{(s)}} - E_P) f_{\parallel}(E_P) + (E_P^2 - M_P^2) f_{\perp}(E_P) \right]$$

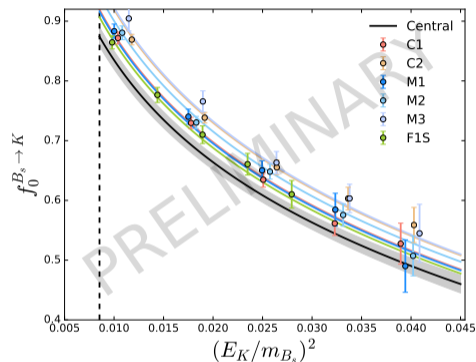
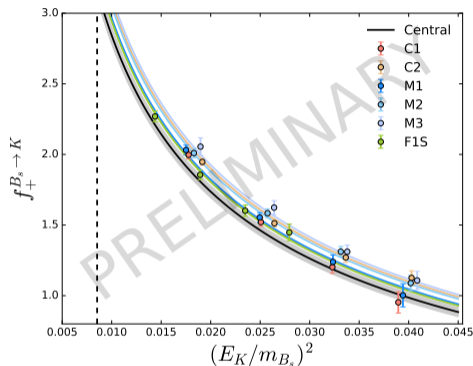
$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_{(s)}}}} \left[f_{\parallel}(E_P) + (M_{B_{(s)}} - E_P) f_{\perp}(E_P) \right]$$

$B_S \rightarrow K\ell\nu$: form factor

▶ Example: form factor $f_{||}$ on coarse ensemble C1



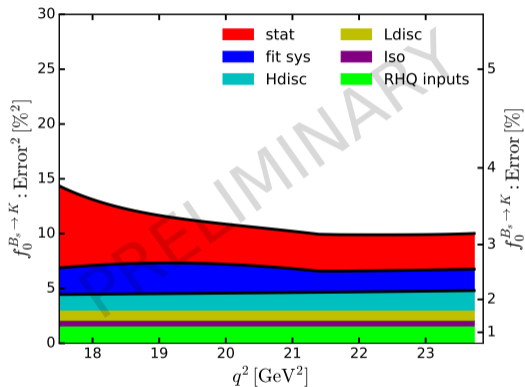
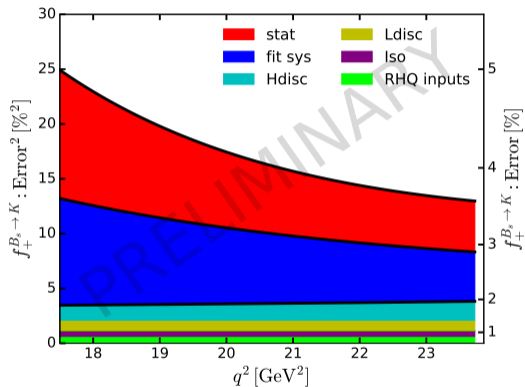
$B_s \rightarrow K\ell\nu$: chiral-continuum extrapolation

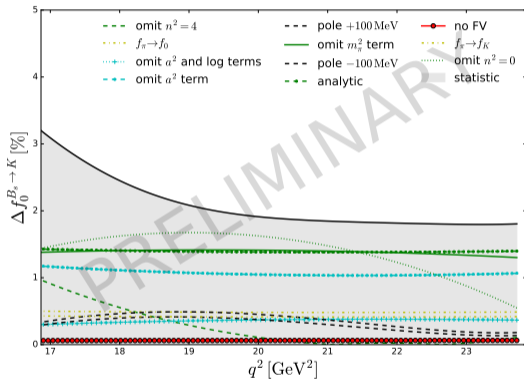
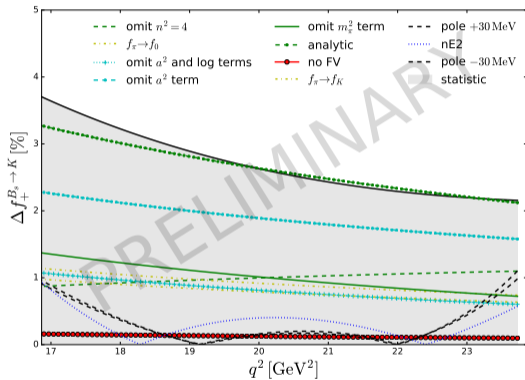


- Chiral-continuum extrapolation using SU(2) hard-kaon χ PT

$$\rightarrow f_{pole}(M_K, E_K, a^2) = \frac{c_0 \Lambda}{E_K + \Delta} \times \left[1 + \frac{\delta f}{(4\pi f)^2} + c_1 \frac{M_K^2}{\Lambda^2} + c_2 \frac{E_K}{\Lambda} + c_3 \frac{E_K^2}{\Lambda^2} + c_4 (a\Lambda)^2 \right]$$

- δf non-analytic logs of the kaon mass and hard-kaon limit is taken by $M_K/E_K \rightarrow 0$

$B_s \rightarrow K\ell\nu$: error budget

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Kinematical extrapolation (z-expansion)

[Boyd, Grinstein, Lebed, PRL 74 (1995) 4603] [Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]

- ▶ Map complex q^2 plane with cut $q^2 > t_*$ onto the unit disk in z

$$z(q^2, t_*, t_0) = \frac{\sqrt{t_* - q^2} - \sqrt{t_* - t_0}}{\sqrt{t_* - q^2} + \sqrt{t_* - t_0}}$$

with

$$t_* = (M_B + M_\pi)^2 \quad (\text{two-particle production threshold})$$

$$t_\pm = (M_{B_s} \pm M_K)^2 \quad (\text{with } t_- = q_{max}^2)$$

$$t_0 \equiv t_{opt} = t_* - \sqrt{t_*(t_* - t_-)} \quad (\text{symmetrize range of } z)$$

- ▶ BCL express form factor f_+ for $B \rightarrow \pi\ell\nu$

$$f_+(q^2) = \frac{1}{1 - q^2/M_{pole}^2} \sum_{k=0}^{K-1} b_k^+(t_0) z^k$$

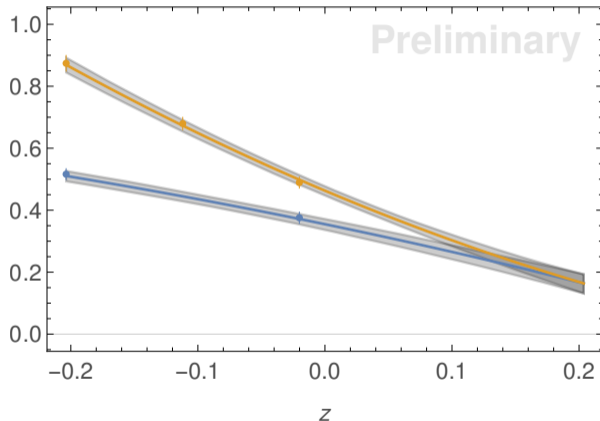
- ▶ For other decays use product of factors for subthreshold poles for both f_+ and f_0 paralleling the Blaschke factors for a BGL fit to the same decay

Kinematical extrapolation (z-expansion)

- ▶ Perform fit in z-space with K parameters
- ▶ Then convert back to physical q^2
- ▶ BCL with pole $M_+ = B^* = 5.33$ GeV for f_+
- ▶ Exploit kinematic constraint $f_+ = f_0 \Big|_{q^2=0}$

- ▶ Include HQ power counting to constrain size of f_+ coefficients (work in progress)

- ▶ Compare form factors to other determinations
 - FNAL/MILC [Bazavov et al. arXiv:1901.02561]
 - ALPHA [Bahr et al. PLB757(2016)473]
 - RBC/UKQCD [Flynn et al. PRD 91 (2015) 074510]
 - HPQCD [Bouchard et al. PRD 90 (2014) 054506][Monahan et al. PRD 98 (2014) 114509]



Kinematical extrapolation (z-expansion)

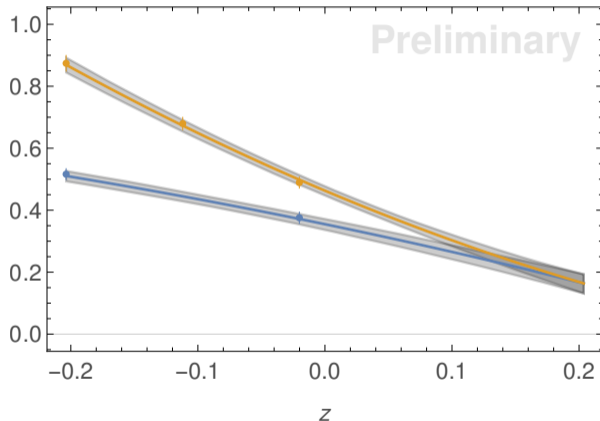
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- ▶ Compare form factors to other determinations
 - Analytic predictions at $q^2 = 0$

[Duplancic et al. PRD78 (2008) 054015]

[Faustov et al. PRD87 (2013) 094028]

[Wang et al. PRD86 (2012) 114025]

[Khodjamirian et al. JHEP08 (2017) 112]



Ratios testing lepton flavor universality

Traditional R -ratio

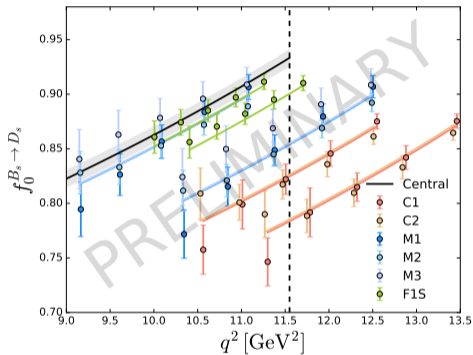
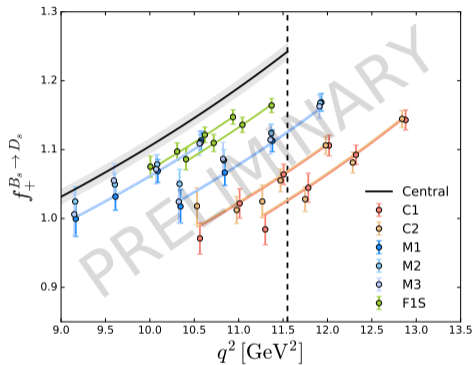
$$R_P = \frac{\int_{m_\tau^2}^{q_{\max}^2} dq^2 \frac{d\Gamma(B_{(s)} \rightarrow P\tau\bar{\nu}_\tau)}{dq^2}}{\int_{m_\ell^2}^{q_{\max}^2} dq^2 \frac{d\Gamma(B_{(s)} \rightarrow P\ell\bar{\nu}_\ell)}{dq^2}}$$

Alternative R -ratio

$$R_P^{\text{imp}} = \frac{\int_{q_{\min}^2}^{q_{\max}^2} dq^2 \frac{d\Gamma(B_{(s)} \rightarrow P\tau\bar{\nu}_\tau)}{dq^2}}{\int_{q_{\min}^2}^{q_{\max}^2} dq^2 \frac{\omega_\tau(q^2)}{\omega_\ell(q^2)} \frac{d\Gamma(B_{(s)} \rightarrow P\ell\bar{\nu}_\ell)}{dq^2}}$$

- ▶ Follow idea proposed for $B \rightarrow V$ [Isidori and Sumensari EPJC80 (2020)1078]
 - Common integration range: $q_{\min}^2 \geq m_\tau^2$ [Freytsis et al. PRD92(2015)054018] [Bernlochner and Ligeti PRD95(2017)014022] [Flynn et al. PoS ICHEP2020 436]
 - Same weights in integrands for τ and ℓ

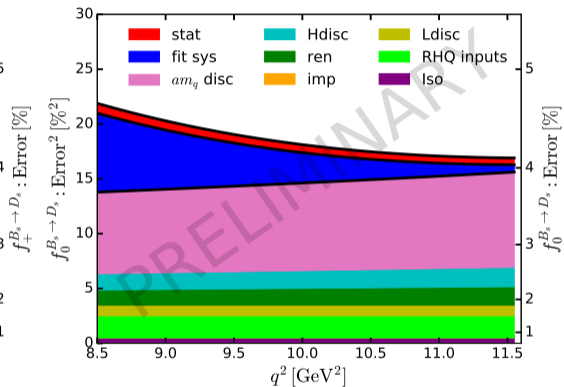
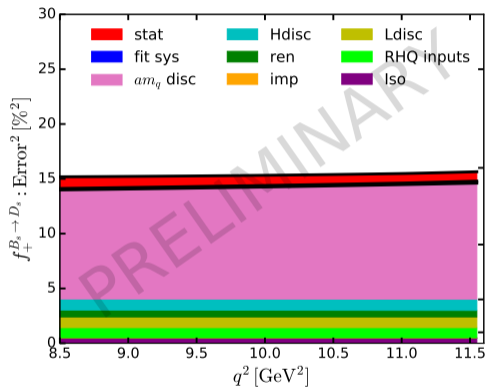
$$\omega_l(q^2) = \left(1 - \frac{m_l^2}{q^2}\right)^2 \left(1 + \frac{m_\ell^2}{2q^2}\right) \quad \text{for } l = e, \mu, \tau$$

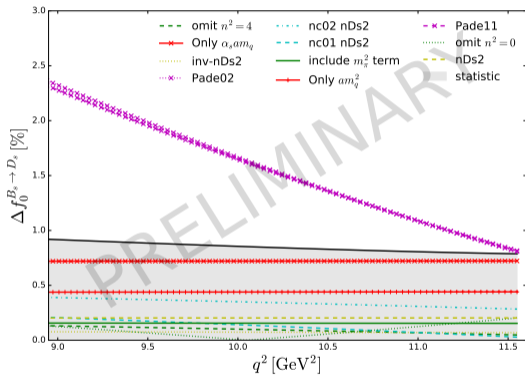
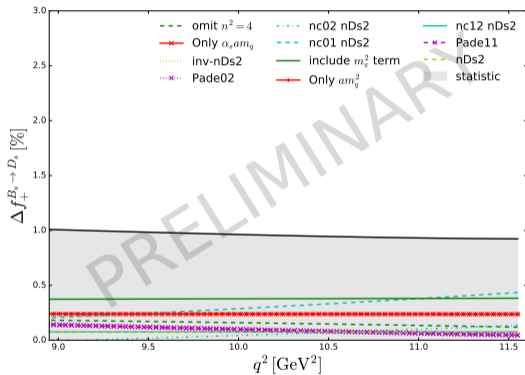
$B_s \rightarrow D_s\ell\nu$: charm inter-/extrapolation + chiral-continuum extrapolation

$$f(q^2, a, M_\pi, M_{D_s}) = \left[c_0 + \sum_{j=1}^{n_{D_s}} c_{1,j} [\Lambda \Delta M_{D_s}^{-1}]^j + c_2 (a\Lambda)^2 \right] P_{a,b} \left(\frac{q^2}{M_{D_s}^2} \right)$$

$$\Delta M_{D_s}^{-1} \equiv \left(\frac{1}{M_{D_s}} - \frac{1}{M_{D_s}^{\text{phys}}} \right)$$

$$P_{a,b}(x) = \frac{1 + \sum_{i=1}^a a_i x^i}{1 + \sum_{i=1}^b b_i x^i}$$

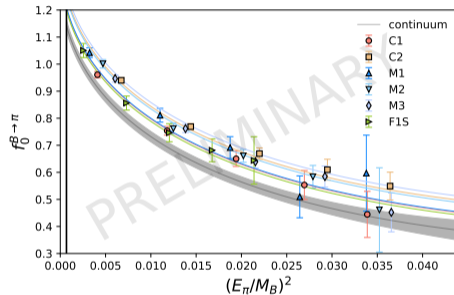
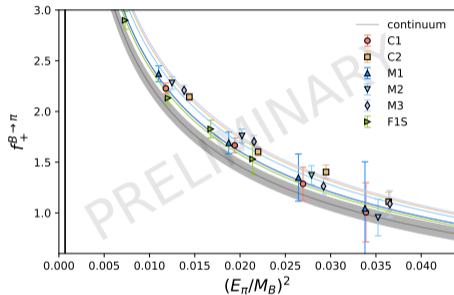
$B_s \rightarrow D_s\ell\nu$: error budget

$B_S \rightarrow D_S\ell\nu$: error budget

Summary

- ▶ Publication on $B_s \rightarrow K\ell\nu$ and $B_s \rightarrow D_s\ell\nu$ under preparation
- ▶ Update on $B \rightarrow \pi\ell\nu$ next
- ▶ Subsequently $B \rightarrow D\ell\nu$, vector final states, rare decays, B_c decays, ...

$B \rightarrow \pi\ell\nu$: chiral-continuum extrapolation

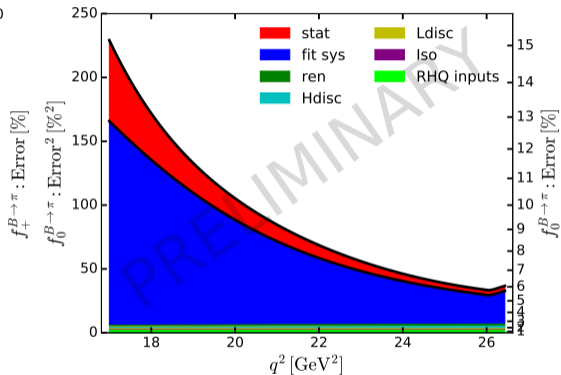
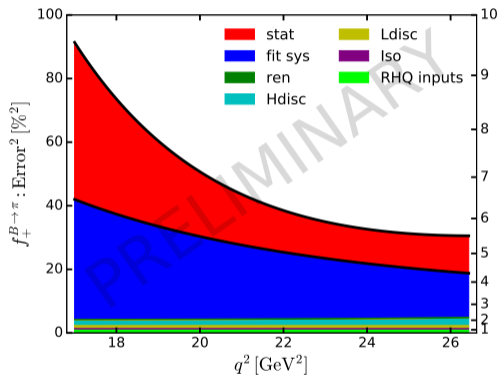


- ▶ Chiral-continuum extrapolation using SU(2) hard-pion χ PT

$$\rightarrow f_{pole}(M_\pi, E_\pi, a^2) = \frac{c_0 \Lambda}{E_\pi + \Delta} \times \left[1 + \frac{\delta f}{(4\pi f)^2} + c_1 \frac{M_\pi^2}{\Lambda^2} + c_2 \frac{E_\pi}{\Lambda} + c_3 \frac{E_\pi^2}{\Lambda^2} + c_4 (a\Lambda)^2 \right]$$

→ δf non-analytic logs of the kaon mass and hard-kaon limit is taken by $M_\pi/E_\pi \rightarrow 0$

- ▶ Substantially reduced statistical errors compared to [Flynn et al. PRD 91 (2015) 074510]

$B \rightarrow \pi\ell\nu$: error budget

$B \rightarrow \pi\ell\nu$: error budget