

Model independent analysis of $b s \ell \ell$ data

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Lyon University and CERN

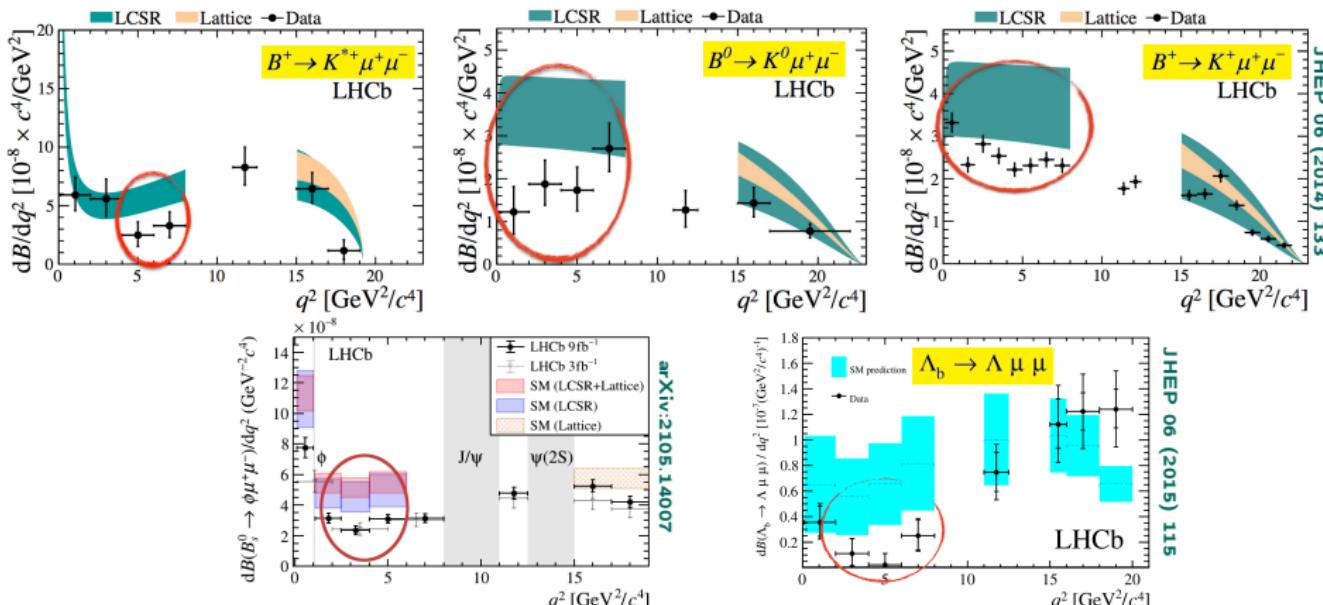
In collaboration with T. Hurth, S. Neshatpour and D. Martinez Santos
(mostly based on arXiv:2104.10058)

Workshop on Anomalies and Precision in the Belle II Era

Vienna, 6-8 September 2021

LHCb anomalies

A consistent deviation pattern with the SM predictions in $b \rightarrow s$ measurements with muons in the final state:

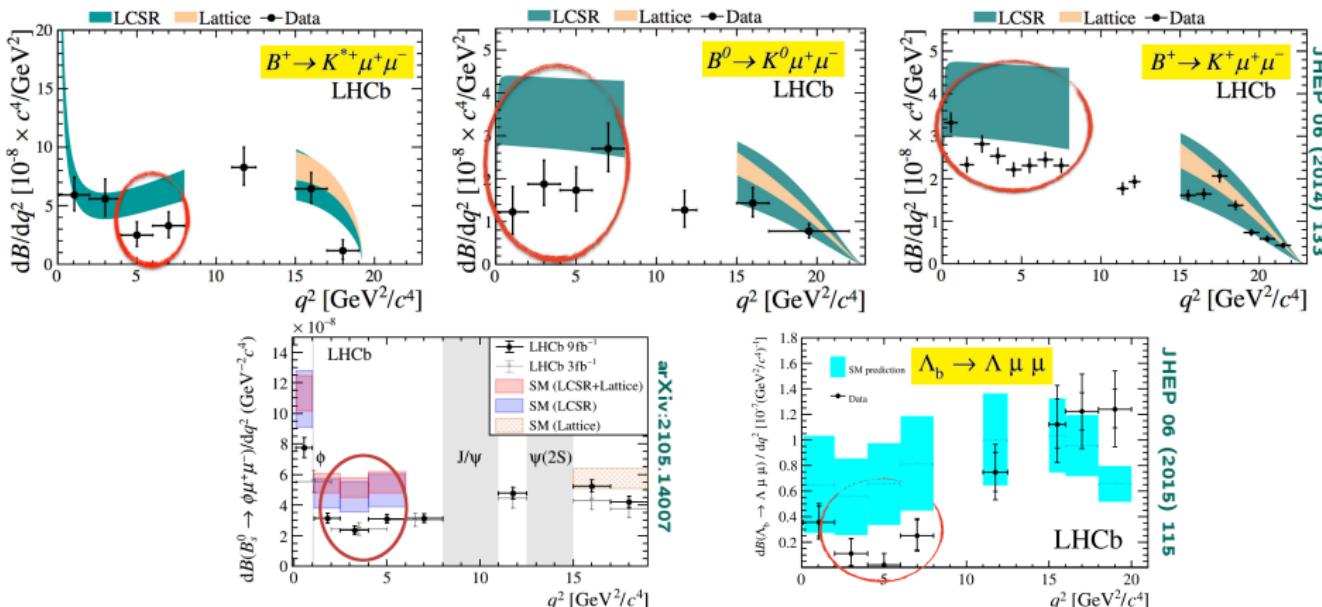


- deviations with the SM predictions between ~ 2 and 3.5σ
- general trend: EXP < SM in low q^2
- ... but the branching ratios have very large theory uncertainties!



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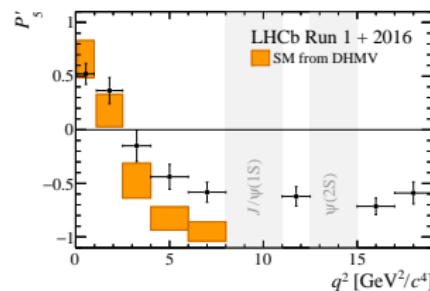
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Tension in the angular observables

$P'_5(B^0 \rightarrow K^{*0} \mu^+ \mu^-)$: Long standing tension since 2013

2020 LHCb update with 4.7 fb^{-1} : $\sim 3\sigma$ local tension



By construction cleaner than branching ratios

Still residual uncertainties from non-local effects

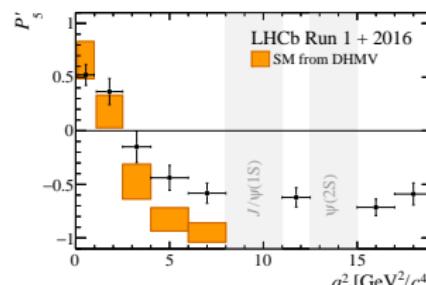
[Phys. Rev. Lett. 125, 011802 \(2020\)](#)



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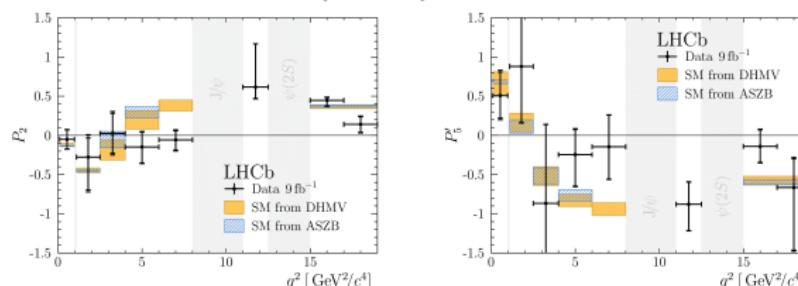


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[Phys. Rev. Lett. 125, 011802 \(2020\)](#)

First measurement of $B^+ \rightarrow K^{*+} \mu^+ \mu^-$ angular observables
using the full Run 1 and Run 2 dataset (9 fb^{-1}):



[Phys. Rev. Lett. 126, 161802 \(2021\)](#)

The results confirm the global tension with respect to the SM!



Lepton flavour universality tests

Lepton flavour universality in $B^0 \rightarrow K^{*0} \ell^+ \ell^-$

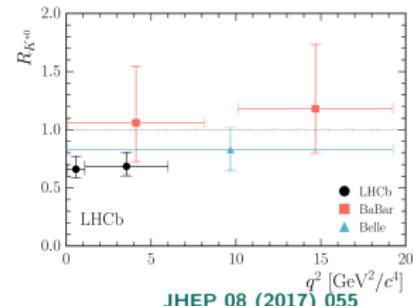
$$R_{K^*} = BR(B^0 \rightarrow K^{*0} \mu^+ \mu^-) / BR(B^0 \rightarrow K^{*0} e^+ e^-)$$

- LHCb measurement from April 2017 using 3 fb^{-1}
- Two q^2 regions: [0.045-1.1] and [1.1-6.0] GeV^2

$$R_{K^*}^{\text{exp,bin1}} = 0.66^{+0.11}_{-0.07} (\text{stat}) \pm 0.03 (\text{syst})$$

$$R_{K^*}^{\text{exp,bin2}} = 0.69^{+0.11}_{-0.07} (\text{stat}) \pm 0.05 (\text{syst})$$

- $2.2\text{-}2.5\sigma$ tension in each bin



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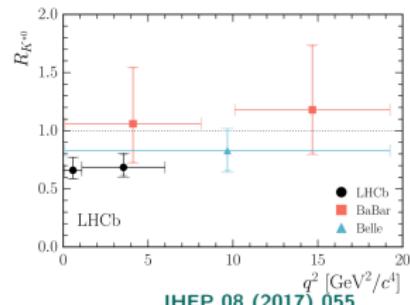
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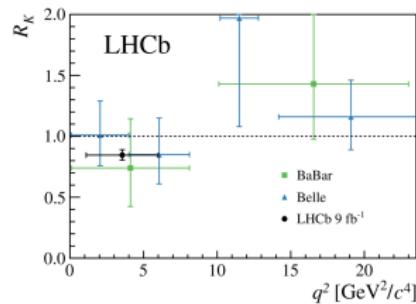
Lepton flavour universality in $B^+ \rightarrow K^+ \ell^+ \ell^-$

$$R_K = BR(B^+ \rightarrow K^+ \mu^+ \mu^-) / BR(B^+ \rightarrow K^+ e^+ e^-)$$

- Theoretical description similar to $B \rightarrow K^* \mu^+ \mu^-$, but different since K is scalar
- SM prediction very accurate: $R_K^{\text{SM}} = 1.0006 \pm 0.0004$
- Latest update: March 2021 using 9 fb^{-1}

$$R_K^{\text{exp}} = 0.846^{+0.042}_{-0.039} (\text{stat})^{+0.013}_{-0.012} (\text{syst})$$

- 3.1σ tension in the [1.1-6] GeV^2 bin



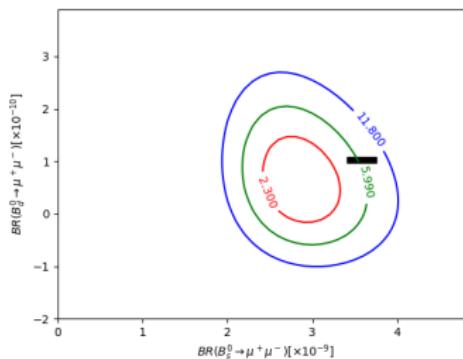
arXiv:2103.11769



$$B_s \rightarrow \mu^+ \mu^-$$

SM prediction: $\text{BR}(B_s \rightarrow \mu^+ \mu^-)^{\text{SM}} = (3.58 \pm 0.17) \times 10^{-9}$

Combination of LHCb, ATLAS and CMS measurements:



$$\text{BR}(B_s \rightarrow \mu^+ \mu^-)^{\text{exp(comb.)}} = (2.85^{+0.34}_{-0.31}) \times 10^{-9}$$

LHCb (9 fb^{-1}): [arXiv:2108.09283](https://arxiv.org/abs/2108.09283)

ATLAS: JHEP 04 (2019) 098

CMS: JHEP 04 (2020) 188

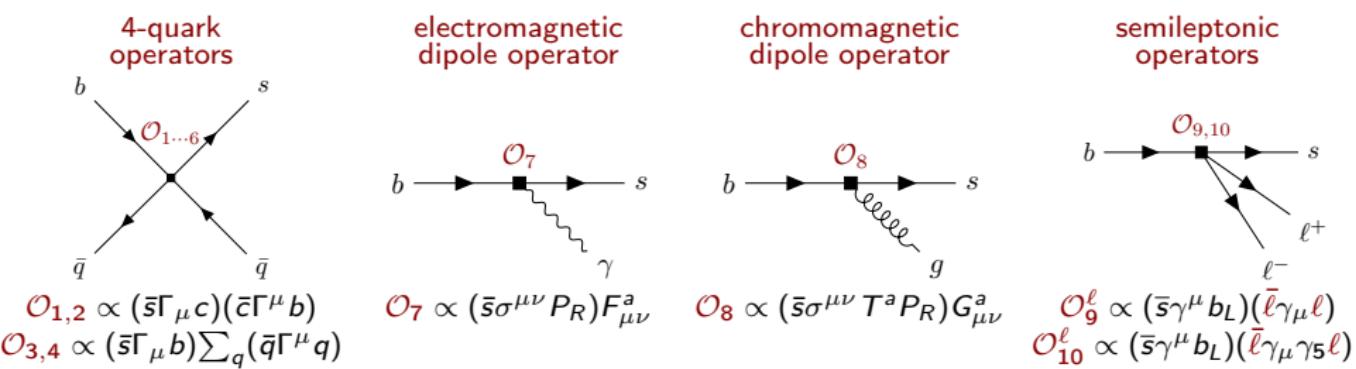


Theoretical framework

Theoretical framework: Effective Hamiltonian

separation between low and high energies using Operator Product Expansion

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(\sum_{i=1 \dots 10, S, P} (C_i(\mu) \mathcal{O}_i(\mu) + C'_i(\mu) \mathcal{O}'_i(\mu)) \right)$$



In the SM: $C_7 = -0.29$ $C_9 = 4.20$ $C_{10} = -4.15$

New physics:

- Corrections to the Wilson coefficients: $C_i \rightarrow C_i^{\text{SM}} + \delta C_i^{\text{NP}}$
- Additional operators: Chirally flipped (\mathcal{O}'_i), (pseudo)scalar (\mathcal{O}_S and \mathcal{O}_P), ...

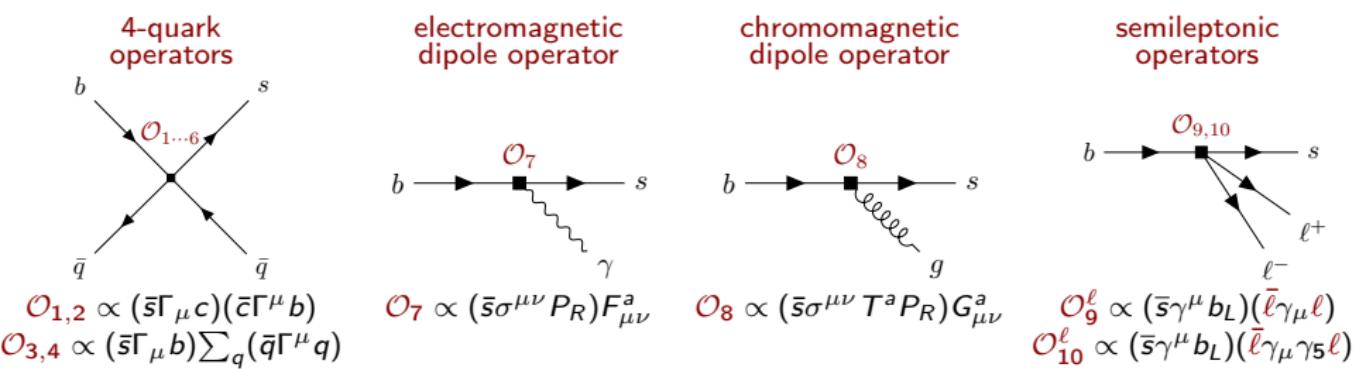


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Issue of the hadronic power corrections

Effective Hamiltonian has two parts:

$$\mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{eff}}^{\text{had}} + \mathcal{H}_{\text{eff}}^{\text{sl}}$$

$$\mathcal{H}_{\text{eff}}^{\text{sl}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[\sum_{i=7,9,10} c_i^{(\prime)} o_i^{(\prime)} \right]$$

$\langle \bar{K}^* | \mathcal{H}_{\text{eff}}^{\text{sl}} | \bar{B} \rangle$: $B \rightarrow K^*$ form factors $V, A_{0,1,2}, T_{1,2,3}$

Transversity amplitudes:

$$A_{\perp}^{L,R} \simeq N_{\perp} \left\{ (c_9^+ \mp c_{10}^+) \frac{V(q^2)}{m_B + m_{K^*}} + \frac{2m_b}{q^2} c_7^+ T_1(q^2) \right\}$$

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$$A_S = N_S (c_S - c'_S) A_0(q^2)$$

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$$\mathcal{H}_{\text{eff}}^{\text{had}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[\sum_{i=1\dots 6} c_i o_i + c_8 o_8 \right]$$

$$\begin{aligned} \mathcal{A}_{\lambda}^{(\text{had})} &= -i \frac{e^2}{q^2} \int d^4 x e^{-iq \cdot x} (\ell^+ \ell^-) j_{\mu}^{\text{em, lept}}(x) |0\rangle \\ &\times \int d^4 y e^{iq \cdot y} \langle \bar{K}_\lambda^* | T \{ j^{\text{em, had}, \mu}(y) \mathcal{H}_{\text{eff}}^{\text{had}}(0) \} | \bar{B} \rangle \\ &\equiv \frac{e^2}{q^2} \epsilon \mu \underbrace{L_V^\mu}_{\substack{\text{LO in } \mathcal{O}\left(\frac{\Lambda}{m_b}, \frac{\Lambda}{E_{K^*}}\right)}} \underbrace{\text{Non-Fact., QCDF}}_{\substack{\text{power corrections}}} \\ &+ \underbrace{h_{\lambda}(q^2)}_{\substack{\text{power corrections}}} \end{aligned}$$



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Recent progress show that these corrections should be very small (2011.09813)



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Significance of the anomalies depends on the assumptions on the power corrections
 This does not affect R_K and R_{K^*}
 → Separate R_K and R_{K^*} (and $B_s \rightarrow \mu^+ \mu^-$) from the less clean observables

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Global fits



How to make sense of the data?

Many observables → **Global fits**

NP manifests itself in shifts of individual coefficients with respect to SM values:

$$C_i(\mu) = C_i^{\text{SM}}(\mu) + \delta C_i$$

- Scans over the values of δC_i
- Calculation of flavour observables

Theoretical uncertainties and correlations

- variation of the “standard” input parameters: masses, scales, CKM, ...
- decay constants taken from the latest lattice results
- $B \rightarrow K^{(*)}$ and $B_s \rightarrow \phi$ form factors are obtained from the lattice+LCSR combinations, including all the correlations
- Parameterisation of uncertainties from power corrections:

$$A_k \rightarrow A_k \left(1 + a_k \exp(i\phi_k) + \frac{q^2}{6 \text{ GeV}^2} b_k \exp(i\theta_k) \right)$$

$|a_k|$ between 10 to 60%, $b_k \sim 2.5 a_k$

⇒ Computation of a (theory + exp) correlation matrix



Global fits

Global fits of the observables obtained by minimisation of

$$\chi^2 = (\vec{O}^{\text{th}} - \vec{O}^{\text{exp}}) \cdot (\Sigma_{\text{th}} + \Sigma_{\text{exp}})^{-1} \cdot (\vec{O}^{\text{th}} - \vec{O}^{\text{exp}})$$

$(\Sigma_{\text{th}} + \Sigma_{\text{exp}})^{-1}$ is the inverse covariance matrix.

173 observables relevant for leptonic and semileptonic decays:

- $\text{BR}(B \rightarrow X_s \gamma)$
- $\text{BR}(B \rightarrow X_d \gamma)$
- $\text{BR}(B \rightarrow K^* \gamma)$
- $\Delta_0(B \rightarrow K^* \gamma)$
- $\text{BR}^{\text{low}}(B \rightarrow X_s \mu^+ \mu^-)$
- $\text{BR}^{\text{high}}(B \rightarrow X_s \mu^+ \mu^-)$
- $\text{BR}^{\text{low}}(B \rightarrow X_s e^+ e^-)$
- $\text{BR}^{\text{high}}(B \rightarrow X_s e^+ e^-)$
- $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$
- $\text{BR}(B_s \rightarrow e^+ e^-)$
- $\text{BR}(B_d \rightarrow \mu^+ \mu^-)$
- R_K in the low q^2 bin
- R_{K^*} in 2 low q^2 bins
- $\text{BR}(B \rightarrow K^0 \mu^+ \mu^-)$
- $\text{BR}(B \rightarrow K^+ \mu^+ \mu^-)$
- $\text{BR}(B \rightarrow K^* e^+ e^-)$
- $B \rightarrow K^{*0} \mu^+ \mu^-$: $\text{BR}, F_L, A_{FB}, S_3, S_4, S_5, S_7, S_8, S_9$
in 8 low q^2 and 4 high q^2 bins
- $B^+ \rightarrow K^{*+} \mu^+ \mu^-$: $\text{BR}, F_L, A_{FB}, S_3, S_4, S_5, S_7, S_8, S_9$
in 5 low q^2 and 2 high q^2 bins
- $B_s \rightarrow \phi \mu^+ \mu^-$: $\text{BR}, F_L, S_3, S_4, S_7$
in 3 low q^2 and 2 high q^2 bins
- $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$: $\text{BR}, A_{FB}^\ell, A_{FB}^h, A_{FB}^{\ell h}, F_L$ in the high q^2 bin

Computations performed using **SuperIso** public program



Single operator fits

Comparison of one-operator NP fits:

Only $R_{K^{(*)}}$, $B_{s,d} \rightarrow \mu^+ \mu^-$ ($\chi^2_{\text{SM}} = 28.19$)			
	b.f. value	χ^2_{min}	Pull _{SM}
δC_9	-1.00 ± 6.00	28.1	0.2σ
δC_9^e	0.80 ± 0.21	11.2	4.1σ
δC_9^μ	-0.77 ± 0.21	11.9	4.0σ
δC_{10}	0.43 ± 0.24	24.6	1.9σ
δC_{10}^e	-0.78 ± 0.20	9.5	4.3σ
δC_{10}^μ	0.64 ± 0.15	7.3	4.6σ
δC_{LL}^e	0.41 ± 0.11	10.3	4.2σ
δC_{LL}^μ	-0.38 ± 0.09	7.1	4.6σ



Clean observables

$\delta C_{\text{LL}}^\ell$ basis corresponds to $\delta C_9^\ell = -\delta C_{10}^\ell$.



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δC_9^μ	-0.77 ± 0.21	11.9	4.0σ
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All observables except $R_{K^{(*)}}, B_{s,d} \rightarrow \mu^+ \mu^-$ ($\chi^2_{\text{SM}} = 200.1$)			
	b.f. value	χ^2_{min}	Pull _{SM}
δC_9	-1.01 ± 0.13	158.2	6.5σ
δC_9^e	0.70 ± 0.60	198.8	1.1σ
δC_9^μ	-1.03 ± 0.13	156.0	6.6σ
δC_{10}	0.34 ± 0.23	197.7	1.5σ
δC_{10}^e	-0.50 ± 0.50	199.0	1.0σ
δC_{10}^μ	0.41 ± 0.23	196.5	1.9σ
δC_{LL}^e	0.33 ± 0.29	198.9	1.1σ
δC_{LL}^μ	-0.75 ± 0.13	167.9	5.7σ

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δC_{LL}^μ	-0.38 ± 0.09	7.1	4.6σ



Clean observables

All observables except $R_{K^{(*)}}, B_{s,d} \rightarrow \mu^+ \mu^-$ ($\chi^2_{\text{SM}} = 200.1$)			
	b.f. value	χ^2_{min}	Pull _{SM}
δC_9	-1.01 ± 0.13	158.2	6.5σ
δC_9^e	0.70 ± 0.60	198.8	1.1σ
δC_9^μ	-1.03 ± 0.13	156.0	6.6σ
δC_{10}	0.34 ± 0.23	197.7	1.5σ
δC_{10}^e	-0.50 ± 0.50	199.0	1.0σ
δC_{10}^μ	0.41 ± 0.23	196.5	1.9σ
δC_{LL}^e	0.33 ± 0.29	198.9	1.1σ
δC_{LL}^μ	-0.75 ± 0.13	167.9	5.7σ



Dependent on the assumptions on the non-factorisable power corrections

$\delta C_{\text{LL}}^\ell$ basis corresponds to $\delta C_9^\ell = -\delta C_{10}^\ell$.



Single operator fits

Comparison of one-operator NP fits:

Only $R_{K^{(*)}}$, $B_{s,d} \rightarrow \mu^+ \mu^-$ ($\chi^2_{\text{SM}} = 28.19$)			
	b.f. value	χ^2_{min}	Pull _{SM}
δC_9	-1.00 ± 6.00	28.1	0.2σ
δC_9^e	0.80 ± 0.21	11.2	4.1σ
δC_9^μ	-0.77 ± 0.21	11.9	4.0σ
δC_{10}	0.43 ± 0.24	24.6	1.9σ
δC_{10}^e	-0.78 ± 0.20	9.5	4.3σ
δC_{10}^μ	0.64 ± 0.15	7.3	4.6σ
δC_{LL}^e	0.41 ± 0.11	10.3	4.2σ
δC_{LL}^μ	-0.38 ± 0.09	7.1	4.6σ



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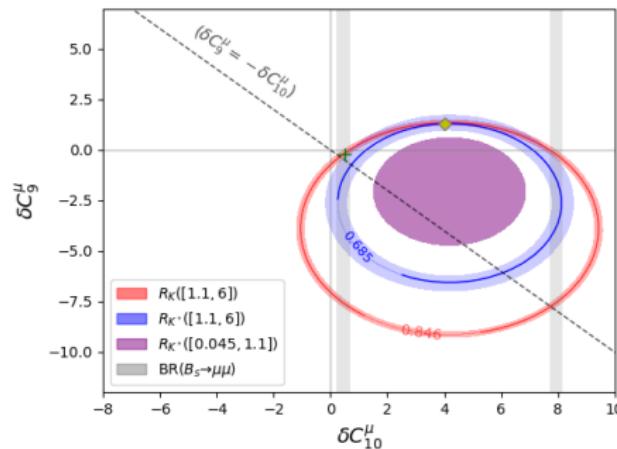
$\delta C_{\text{LL}}^\ell$ basis corresponds to $\delta C_9^\ell = -\delta C_{10}^\ell$.

- Compatible NP scenarios between different sets
- Hierarchy of the preferred NP scenarios have remained the same with updated data (C_9^μ followed by C_{LL}^μ)
- Significance increased by more than 2σ in the preferred scenarios compared to 2019



$R_{K^{(*)}}$ and $B_{s,d} \rightarrow \mu^+ \mu^-$

Clean observables within 1σ of experimental central value:



Colored regions: 1σ range (th + exp uncertainties added in quadrature) with the experimental central value.

Red (blue) solid line: central value of the experimental measurements of R_K and R_{K^*}

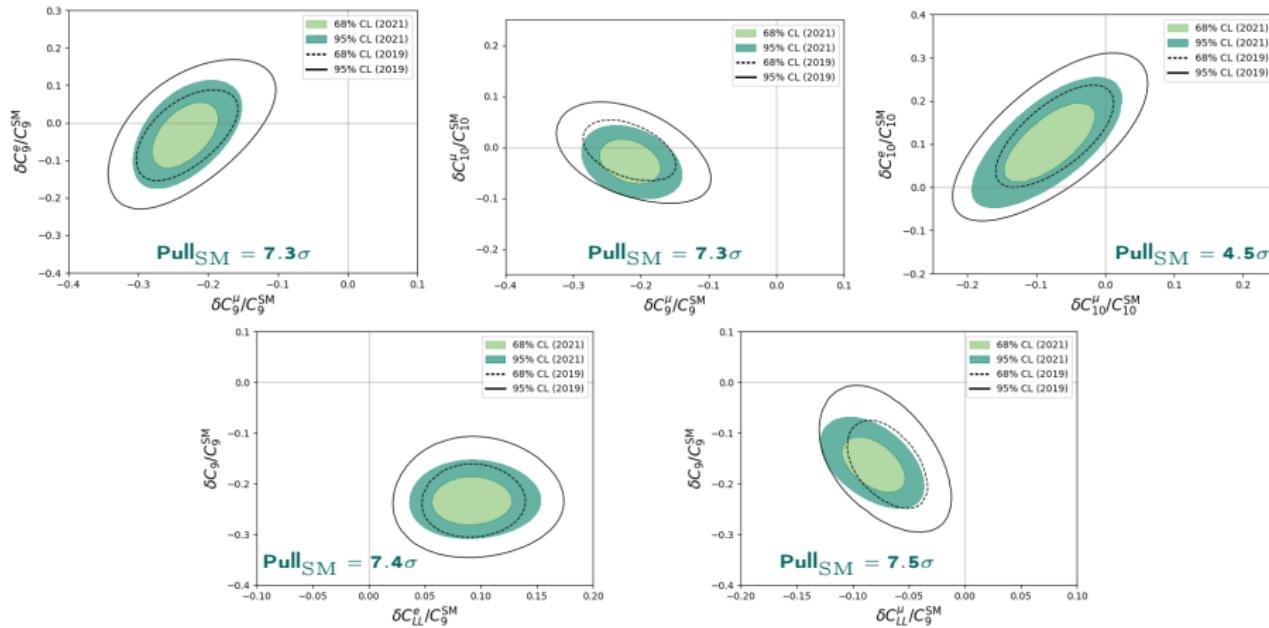
Yellow diamond: best fit point of the fit to only $R_{K^{(*)}}$

Green cross: best fit value when fitting to $R_{K^{(*)}}$ and $\text{BR}(B_{s,d} \rightarrow \mu^+ \mu^-)$.



Two operator fits

Two operator fits to NP to all observables (with the assumption of 10% power corrections):



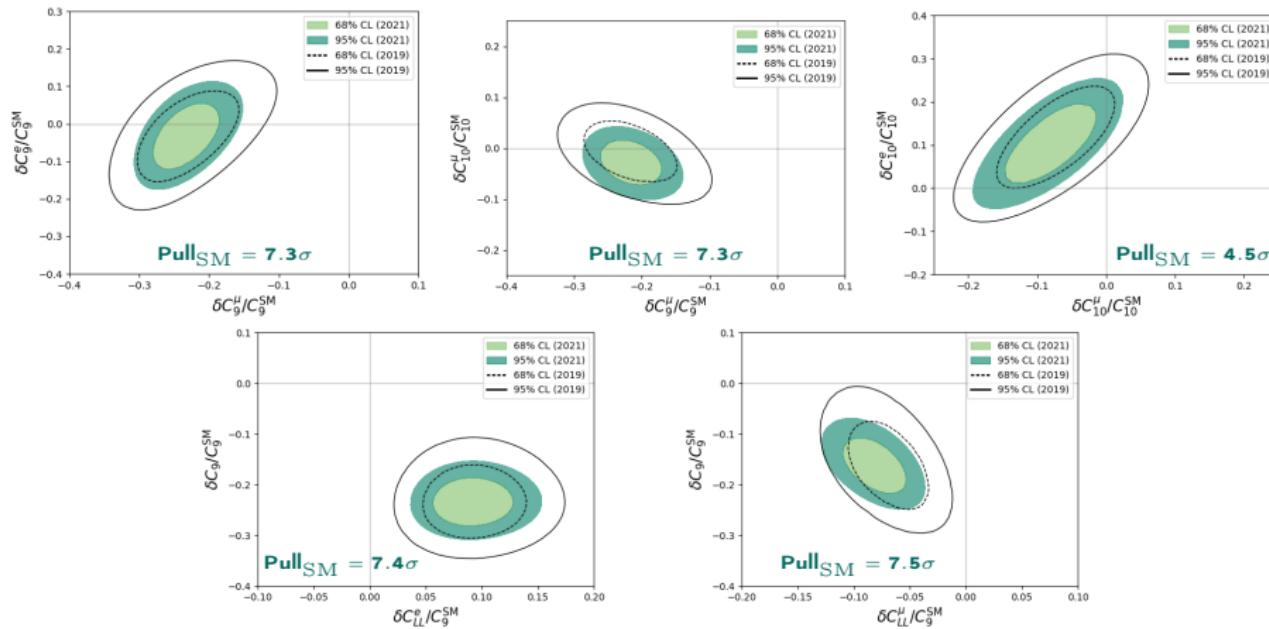
Similar fits by other groups: Geng et al. arXiv:2103.12738, Altmannshofer et al. 2103.13370, Alguero et al. arXiv:2104.08921,

Ciuchini et al. arXiv:2011.01212, Datta et al. 1903.10086, Kowalska et al., arXiv:1903.10932, ...



Two operator fits

Two operator fits to NP to all observables (with the assumption of 10% power corrections):



Colored bands (black contours): 68 and 95% CL regions considering 2021 (2019) data

Similar fits by other groups: Geng et al. arXiv:2103.12738, Altmannshofer et al. 2103.13370, Alguero et al. arXiv:2104.08921,

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More complete analyses

In a New Physics model:

- new vector bosons: C_7, C_9, C_{10}
- new fermions: C_7, C_8, C_9, C_{10}
- extended Higgs sector/new scalars: C_S, C_P

e.g. in the MSSM, 2HDM, ...: $C_7, C_8, C_9, C_{10}, C_S, C_P$

Considering only one or two Wilson coefficients may not give the full picture!

A generic set of Wilson coefficients:

complex $C_7, C_8, C_9^\ell, C_{10}^\ell, C_S^\ell, C_P^\ell$ + primed coefficients

The available observables are mainly insensitive to the imaginary parts, one can limit the set to

real $C_7, C_8, C_9^\ell, C_{10}^\ell, C_S^\ell, C_P^\ell$ + primed coefficients

corresponding to 20 degrees of freedom.

Considering the most general NP description, look-elsewhere effect is avoided!



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Full fit - results

Set: real $C_7, C_8, C_9^\ell, C_{10}^\ell, C_S^\ell, C_P^\ell$ + primed coefficients, 20 degrees of freedom

All observables with $\chi^2_{\text{SM}} = 225.8$ $\chi^2_{\min} = 151.6$; $\text{Pull}_{\text{SM}} = 5.5(5.6)\sigma$			
δC_7		δC_8	
0.05 ± 0.03		-0.70 ± 0.40	
$\delta C'_7$		$\delta C'_8$	
-0.01 ± 0.02		0.00 ± 0.80	
δC_9^μ	δC_9^e	δC_{10}^μ	δC_{10}^e
-1.16 ± 0.17	-6.70 ± 1.20	0.20 ± 0.21	degenerate w/ C'_{10}^e
$\delta C_9'^\mu$	$\delta C_9'^e$	$\delta C_{10}'^\mu$	$\delta C_{10}'^e$
0.09 ± 0.34	1.90 ± 1.50	-0.12 ± 0.20	degenerate w/ C_{10}^e
$C_{Q_1}^\mu$	$C_{Q_1}^e$	$C_{Q_2}^\mu$	$C_{Q_2}^e$
0.04 ± 0.10 [-0.08 ± 0.11]	-1.50 ± 1.50 [-0.20 ± 1.60]	-0.09 ± 0.10 [-0.11 ± 0.10]	-4.10 ± 1.5 [4.50 ± 1.5]
$C_{Q_1}'^\mu$	$C_{Q_1}'^e$	$C_{Q_2}'^\mu$	$C_{Q_2}'^e$
0.15 ± 0.10 [0.02 ± 0.12]	-1.70 ± 1.20 [-0.30 ± 1.10]	-0.14 ± 0.11 [-0.16 ± 0.10]	-4.20 ± 1.2 [4.40 ± 1.2]

- No real improvement in the fits when going beyond the C_9^μ case
- Many parameters are weakly constrained at the moment
- Effective d.o.f is (19) leading to 5.6σ significance



Wilks' test

Pull_{SM} of 1, 2, 6, 10 and 20 dimensional fit:

Set of WC	param.	χ^2_{\min}	Pull_{SM}	Improvement
SM	0	225.8	-	-
C_9^μ	1	168.6	7.6σ	7.6σ
C_9^μ, C_{10}^μ	2	167.5	7.3σ	1.0σ
$C_7, C_8, C_9^{(e,\mu)}, C_{10}^{(e,\mu)}$	6	158.0	7.1σ	2.0σ
All non-primed WC	10	157.2	6.5σ	0.1σ
All WC (incl. primed)	20 (19)	151.6	$5.5 (5.6)\sigma$	$0.2 (0.3)\sigma$

The "All non-primed WC" includes in addition to the previous row, the scalar and pseudoscalar Wilson coefficients.

The last row also includes the chirality-flipped counterparts of the Wilson coefficients.

In the last column the significance of improvement of the fit compared to the scenario of the previous row is given.

The number in parentheses corresponds to the effective degrees of freedom (19).



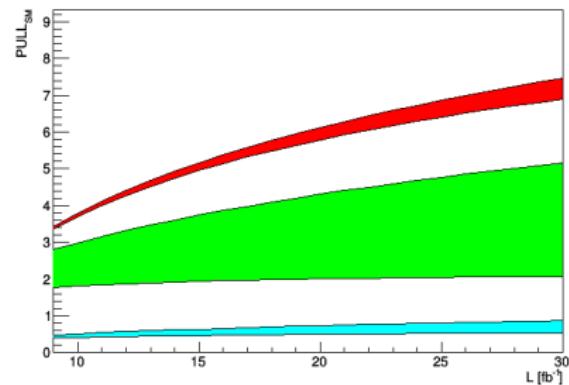
Future prospects for clean observables



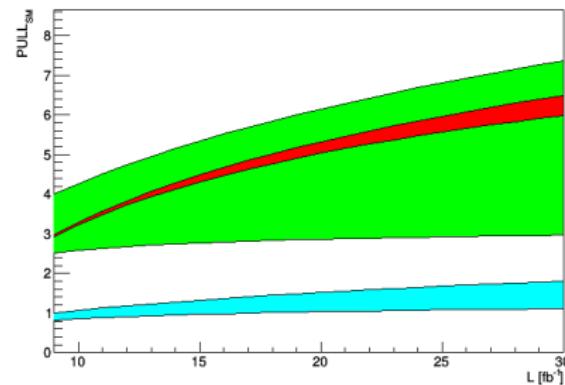
Experimental prospects

Evolution of the tension between the SM and the experimental values

Assuming the best fit values of C_9^μ (left) and C_{10}^μ (right)



red: $R_K^{[1.1, 6]}$ green: $R_{K^*}^{[1.1, 6]}$ blue: $R_{K^*}^{[0.045, 1.1]}$



Upper limit: assuming ultimate systematic uncertainties (1% for ratios & 4% for $B_s \rightarrow \mu^+ \mu^-$)

Lower limit: assuming current systematic uncertainties do not improve



Projections

Predictions of Pull_{SM} for the fit to δC_9^μ , δC_{10}^μ and δC_{LL}^μ

For LHCb upgrade scenarios with 18, 50 and 300 fb^{-1} collected luminosity:

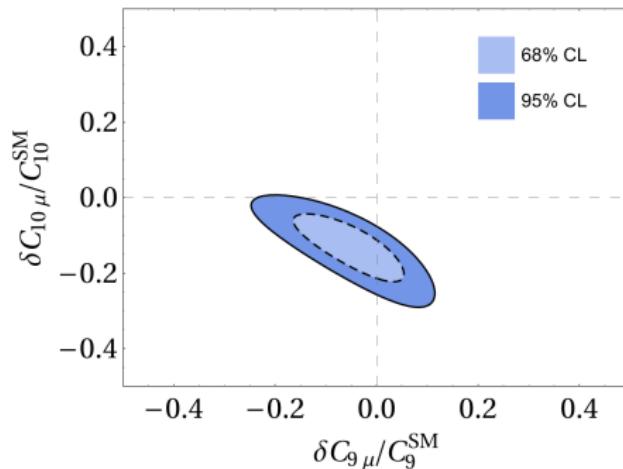
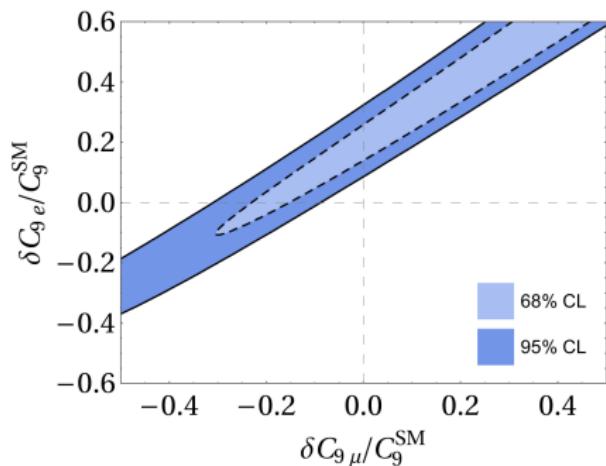
Pull _{SM} with $R_{K(*)}$ and $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ prospects			
LHCb lum.	18 fb^{-1}	50 fb^{-1}	300 fb^{-1}
δC_9^μ	6.5σ	14.7σ	21.9σ
δC_{10}^μ	7.1σ	16.6σ	25.1σ
δC_{LL}^μ	7.5σ	17.7σ	26.6σ

For all three scenarios, NP significance will be larger than 6σ already with 18 fb^{-1} !



Projections

With $B_s \rightarrow \mu^+ \mu^-$ and $R_{K^{(*)}}$ only

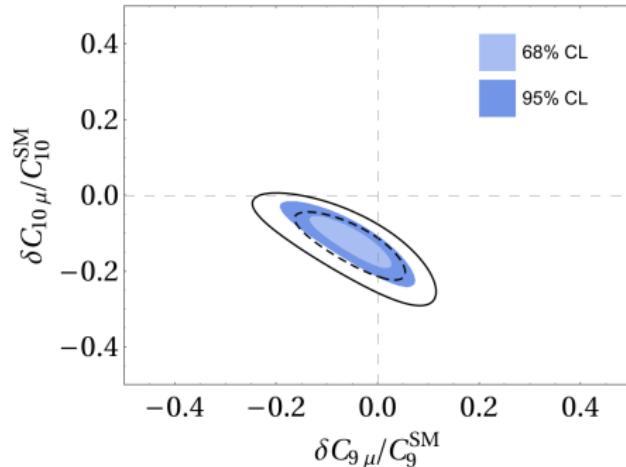
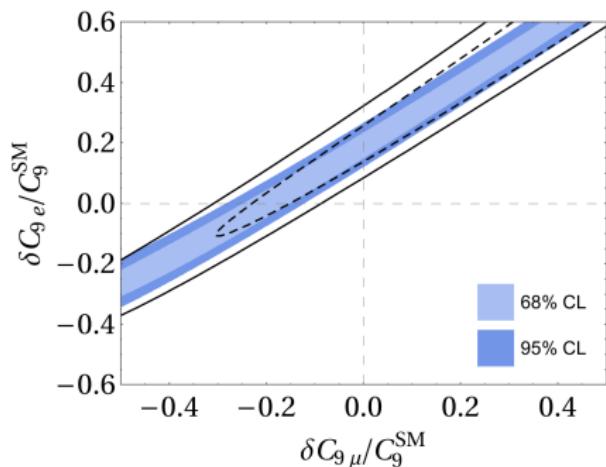


Current data



Projections

With $B_s \rightarrow \mu^+ \mu^-$ and $R_{K^{(*)}}$ only

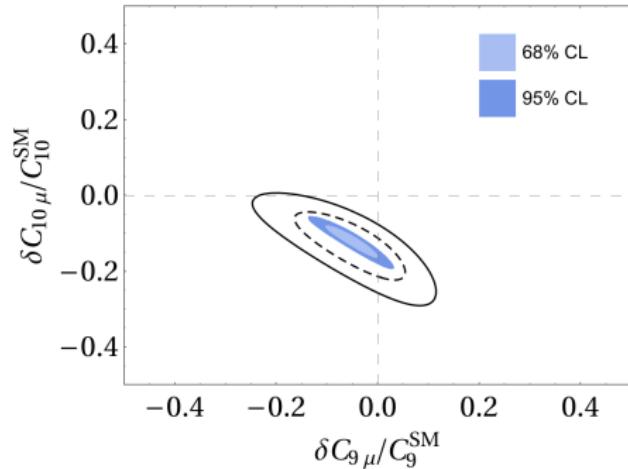
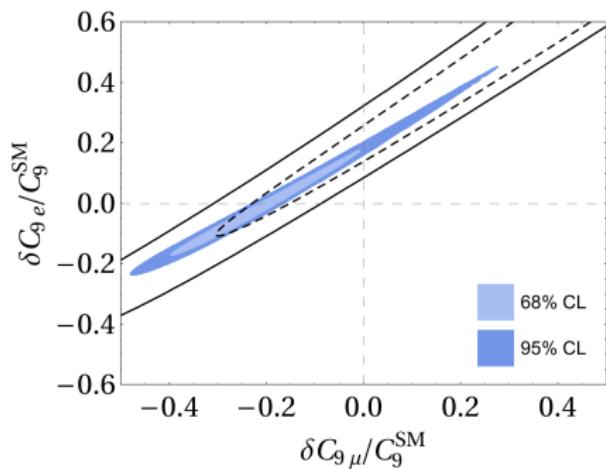


Projections for 18 fb⁻¹



Projections

With $B_s \rightarrow \mu^+ \mu^-$ and $R_{K^{(*)}}$ only

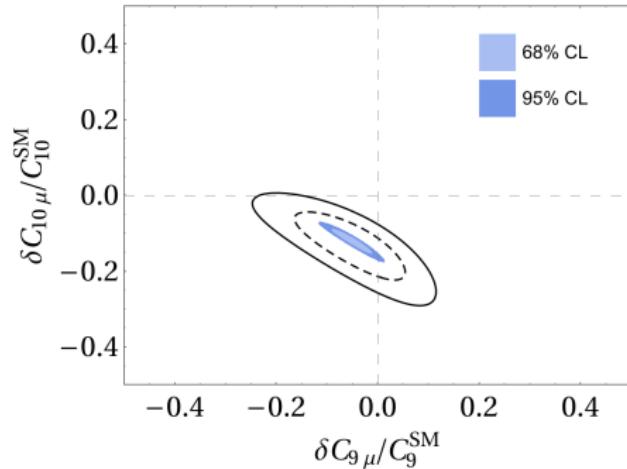
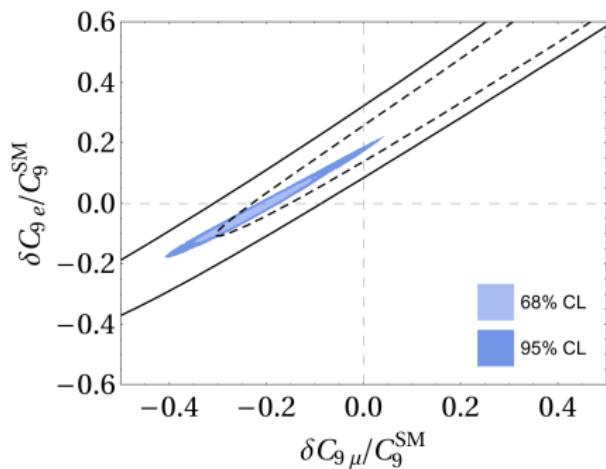


Projections for 50 fb^{-1}



Projections

With $B_s \rightarrow \mu^+ \mu^-$ and $R_{K^{(*)}}$ only



Projections for 300 fb^{-1}



Predictions for other ratios

Predictions for various μ/e ratios

Assuming the central values of the Wilson coefficients remain the same

Obs.	Predictions assuming 50 fb^{-1} luminosity					
	C_9^μ	C_9^e	C_{10}^μ	C_{10}^e	C_{LL}^μ	C_{LL}^e
$R_{F_1}^{[1.1, 6.0]}$	[0.922, 0.932]	[0.941, 0.944]	[0.995, 0.998]	[0.996, 0.997]	[0.961, 0.964]	[1.006, 1.010]
$R_{A_{FB}}^{[1.1, 6.0]}$	[4.791, 5.520]	[-0.416, -0.358]	[0.938, 0.939]	[0.963, 0.970]	[2.822, 3.089]	[0.279, 0.307]
$R_{S_3}^{[1.1, 6.0]}$	[0.922, 0.931]	[0.914, 0.922]	[0.832, 0.852]	[0.858, 0.870]	[0.853, 0.870]	[1.027, 1.032]
$R_{S_5}^{[1.1, 6.0]}$	[0.453, 0.543]	[0.723, 0.742]	[1.014, 1.014]	[1.040, 1.048]	[0.773, 0.801]	[1.298, 1.361]
$R_F^{[15, 19]}$	[0.998, 0.999]	[0.998, 0.998]	[0.998, 0.998]	[0.998, 0.998]	[0.998, 0.998]	[0.998, 0.998]
$R_{A_{FB}}^{[15, 19]}$	[0.929, 0.944]	[0.988, 0.989]	[1.009, 1.010]	[1.036, 1.042]	[0.996, 0.996]	[1.023, 1.028]
$R_{S_3}^{[15, 19]}$	[0.998, 0.998]	[0.998, 0.998]	[0.999, 0.999]	[0.999, 0.999]	[0.999, 0.999]	[0.998, 0.998]
$R_{S_5}^{[15, 19]}$	[0.929, 0.944]	[0.988, 0.989]	[1.009, 1.010]	[1.036, 1.042]	[0.996, 0.996]	[1.023, 1.028]
$R_{K^*}^{[15, 19]}$	[0.825, 0.847]	[0.815, 0.835]	[0.828, 0.846]	[0.799, 0.820]	[0.804, 0.825]	[1.093, 1.107]
$R_K^{[15, 19]}$	[0.823, 0.847]	[0.819, 0.838]	[0.854, 0.870]	[0.825, 0.844]	[0.820, 0.839]	[1.098, 1.113]
$R_\phi^{[1.1, 6.0]}$	[0.862, 0.879]	[0.841, 0.858]	[0.824, 0.843]	[0.795, 0.816]	[0.819, 0.839]	[1.070, 1.080]
$R_\phi^{[15, 19]}$	[0.825, 0.847]	[0.815, 0.835]	[0.826, 0.845]	[0.797, 0.819]	[0.803, 0.824]	[1.093, 1.107]

→ Possible to discriminate between different new physics hypotheses!



Conclusion

- The latest LHCb measurements still show persistent tensions with the SM predictions in $b \rightarrow s\ell\ell$ transitions
- With the updated data the hierarchy of preferred NP scenarios remains the same with increased significance
- Fit to clean observables and the rest of $b \rightarrow s\ell\ell$ observables point to compatible NP scenarios
- Assuming the central values of the current fits, with already 18 fb^{-1} significances above 6σ can be reached with only clean observables
- The LHCb upgrade will have enough precision to distinguish between NP scenarios



Backup

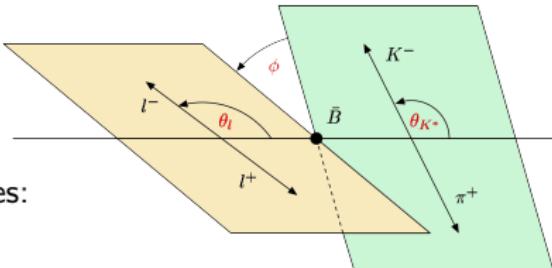
Backup



$b \rightarrow sl^+l^-$ transitions: $B \rightarrow K^*\mu^+\mu^-$

Angular distributions

The full angular distribution of the decay $\bar{B}^0 \rightarrow \bar{K}^{*0}\ell^+\ell^-$ ($\bar{K}^{*0} \rightarrow K^-\pi^+$) is completely described by four independent kinematic variables: q^2 (dilepton invariant mass squared), θ_ℓ , θ_{K^*} , ϕ



Differential decay distribution:

$$\frac{d^4\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_{K^*} d\phi} = \frac{9}{32\pi} J(q^2, \theta_\ell, \theta_{K^*}, \phi)$$

$$J(q^2, \theta_\ell, \theta_{K^*}, \phi) = \sum_i J_i(q^2) f_i(\theta_\ell, \theta_{K^*}, \phi)$$

↗ angular coefficients J_{1-9}

↗ functions of the spin amplitudes A_0 , $A_{||}$, A_{\perp} , A_t , and A_S

Spin amplitudes: functions of Wilson coefficients and form factors

Main operators:

$$\mathcal{O}_9 = \frac{e^2}{(4\pi)^2} (\bar{s}\gamma^\mu b_L)(\bar{\ell}\gamma_\mu \ell), \quad \mathcal{O}_{10} = \frac{e^2}{(4\pi)^2} (\bar{s}\gamma^\mu b_L)(\bar{\ell}\gamma_\mu \gamma_5 \ell)$$

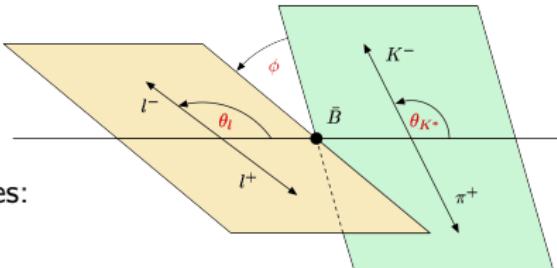
$$\mathcal{O}_5 = \frac{e^2}{16\pi^2} (\bar{s}_L^\alpha b_R^\alpha)(\bar{\ell}\ell), \quad \mathcal{O}_P = \frac{e^2}{16\pi^2} (\bar{s}_L^\alpha b_R^\alpha)(\bar{\ell}\gamma_5 \ell)$$



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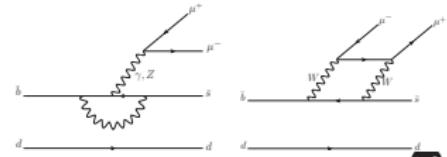
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$B \rightarrow K^* \mu^+ \mu^-$ observables

Optimised observables: form factor uncertainties cancel at leading order

$$\langle P_1 \rangle_{\text{bin}} = \frac{1}{2} \frac{\int_{\text{bin}} dq^2 [J_3 + \bar{J}_3]}{\int_{\text{bin}} dq^2 [J_{2s} + \bar{J}_{2s}]}$$

$$\langle P_2 \rangle_{\text{bin}} = \frac{1}{8} \frac{\int_{\text{bin}} dq^2 [J_{6s} + \bar{J}_{6s}]}{\int_{\text{bin}} dq^2 [J_{2s} + \bar{J}_{2s}]}$$

$$\langle P'_4 \rangle_{\text{bin}} = \frac{1}{N'_{\text{bin}}} \int_{\text{bin}} dq^2 [J_4 + \bar{J}_4]$$

$$\langle P'_5 \rangle_{\text{bin}} = \frac{1}{2N'_{\text{bin}}} \int_{\text{bin}} dq^2 [J_5 + \bar{J}_5]$$

$$\langle P'_6 \rangle_{\text{bin}} = \frac{-1}{2N'_{\text{bin}}} \int_{\text{bin}} dq^2 [J_7 + \bar{J}_7]$$

$$\langle P'_8 \rangle_{\text{bin}} = \frac{-1}{N'_{\text{bin}}} \int_{\text{bin}} dq^2 [J_8 + \bar{J}_8]$$

with

$$N'_{\text{bin}} = \sqrt{- \int_{\text{bin}} dq^2 [J_{2s} + \bar{J}_{2s}] \int_{\text{bin}} dq^2 [J_{2c} + \bar{J}_{2c}]}$$

+ CP violating clean observables and other combinations

U. Egede et al., JHEP 0811 (2008) 032, JHEP 1010 (2010) 056

J. Matias et al., JHEP 1204 (2012) 104

S. Descotes-Genon et al., JHEP 1305 (2013) 137

Or alternatively:

$$S_i = \frac{J_{i(s,c)} + \bar{J}_{i(s,c)}}{\frac{d\Gamma}{dq^2} + \frac{d\bar{\Gamma}}{dq^2}}, \quad P'_{4,5,8} = \frac{S_{4,5,8}}{\sqrt{F_L(1 - F_L)}}$$

W. Altmannshofer et al., JHEP 0901 (2009) 019

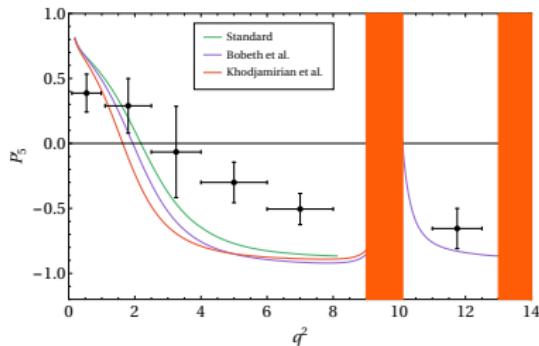


Estimates of hadronic effects

Various methods for hadronic effects

$$\frac{e^2}{q^2} \epsilon_\mu L_V^\mu \left[Y(q^2) \tilde{V}_\lambda + \text{LO in } \mathcal{O}\left(\frac{\Lambda}{m_b}, \frac{\Lambda}{E_{K^*}}\right) + h_\lambda(q^2) \right]$$

	factorisable	non-factorisable	power corrections (soft gluon)	region of calculation	physical region of interest
Standard	✓	✓	✗	$q^2 \lesssim 7 \text{ GeV}^2$	directly
Khodjamirian et al. [1006.4945]	✓	✗	✓	$q^2 < 1 \text{ GeV}^2$	extrapolation by dispersion relation
Bobeth et al. [1707.07305]	✓	✓	✓	$q^2 < 0 \text{ GeV}^2$	extrapolation by analyticity

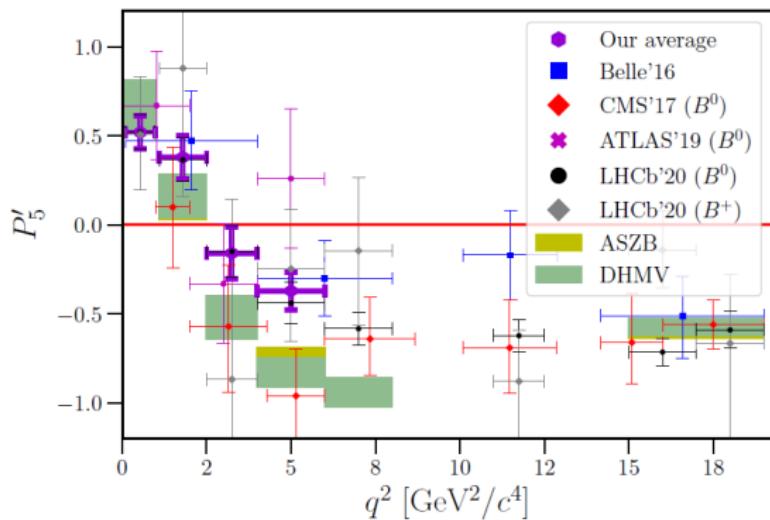


Recent revisit of Khodjamirian et al. calculation ([N. Gubernari, D. van Dyk, J. Virto, JHEP 02 \(2021\) 088](#)):
soft-gluon effect is two orders of magnitude smaller than the previous calculation!



The LHCb anomalies

Current picture



LHCb, JHEP 02 (2016) 104; Belle, PRL 118 (2017); ATLAS, ATLAS-CONF-2017-023; CMS, CMS-PAS-BPH-15-008

