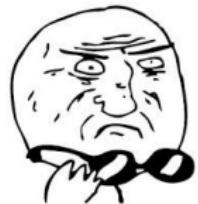


Tau $g-2$ and other probes of lepton flavour

Anomalies and Precision in the Belle II Era

Matteo Fael | Sept. 7, 2021

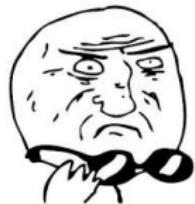
INSTITUTE FOR THEORETICAL PARTICLE PHYSICS - KIT KARLSRUHE



Electron

$$a_e = 115\,965\,218\,091\,(26) \cdot 10^{-12}$$

0.23 parts per billion! Hanneke et al, PRL100 (2008) 120801



MOTHER OF GOD

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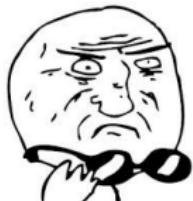
Muon

$$a_\mu = 116\,592\,061\,(41) \cdot 10^{-11}$$

350 parts per billion! FNAL+E821: Phys.Rev.Lett. 126 (2021) 141801



NOT BAD



MOTHER OF GOD

Electron

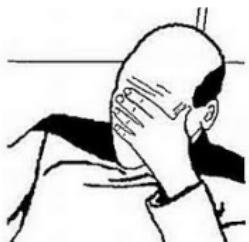
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Muon

$$a_\mu = 116\,592\,061\,(41) \cdot 10^{-11}$$

350 parts per billion! FNAL+E821: Phys.Rev.Lett. 126 (2021) 141801



Tau

$$-0.052 \leq a_\tau \leq 0.013$$

DELPHI - EPJC 35 (2004) 159

Precise SM prediction: $a_\tau^{\text{SM}} = 117\,721\,(5)\,10^{-8}$

S. Eidelman, M. Passera, Mod.Phys.Lett. A22 (2007) 159

The τ dipole moments: SM prediction

The Standard Model prediction for the tau g-2

$$\begin{aligned} a_{\tau}^{\text{SM}} = & +117\,324(2) \times 10^{-8} \quad \text{QED} \\ & +47.4(5) \times 10^{-8} \quad \text{EW} \\ & +337.5(3.7) \times 10^{-8} \quad \text{HLO} \\ & +7.6(2) \times 10^{-8} \quad \text{HHO (vac)} \\ & +5(3) \times 10^{-8} \quad \text{HHO (lbl)} \\ = & \quad \textcolor{red}{117\,721(5)} \times 10^{-8} \end{aligned}$$

Eidelman & Passera, Mod.Phys.Lett. A22 (2007) 159

- $(m_{\tau}/m_{\mu})^2 \sim 283$: good to look for NP! Giudice, Paradisi, Passera JHEP 1211 (2012) 113

	Electron	Muon	Tau
$a^{\text{EW}}/a^{\text{had}}$	1/56	1/45	1/7
$a^{\text{EW}}/\delta a^{\text{had}}$	1.6	4	10

- Lepton EDMs vanish up to 3 loop
Pospelov & Khriplovich, SJNP 53 (1991) 638
- $d_i^{\text{SM}} \sim 10^{-38} - 10^{-35} \text{ e}\cdot\text{cm}$

Tau $g-2$ and EDM: Experimental Bounds

Challenges:

- $\tau_\tau = 2.903(5) \times 10^{-13} \text{ s}$

however $g-2$ measured for Σ^+ baryon
Chen et al. Phys.Rev.Lett. 69 (1992) 3286

■ How to produce polarized τ ?

$$a_\mu : \pi^+ \rightarrow \mu^+ \nu_\mu$$

$$a_\tau : B^+ \rightarrow \tau^+ \nu_\tau, D_s^+ \rightarrow \tau^+ \nu_\tau$$

the PDG value

$$a_\tau = -0.018(17)$$

in $e^+ e^- \rightarrow e^+ e^- \tau^+ \tau^-$ at LEP 2

DELPHI - EPJC35 (2004) 159

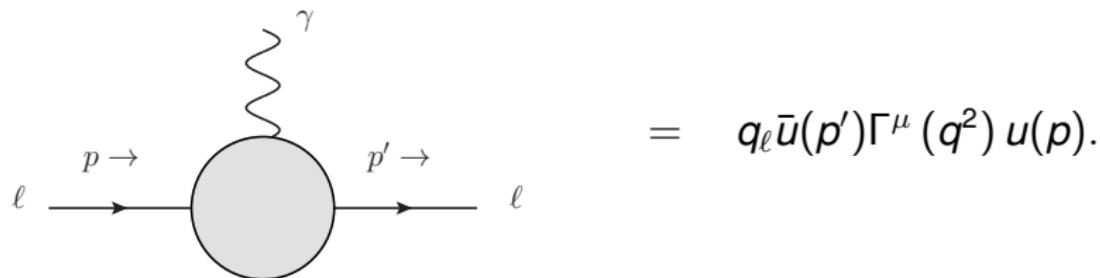
■ Indirect bound from LEP1, SLC, and LEP2 data:

$$-0.007 \leq a_\tau^{\text{NP}} \leq 0.005 \quad \text{González-Sprinberg et al., NPB 582 (2000) 2}$$

■ Belle bounds on τ EDM from $e^+ e^- \rightarrow \tau^+ \tau^-$: Inami et al., Phys.Lett.B 551 (2003) 16

$$2.2 \leq \text{Re}(d_\tau) \leq 4.5 (10^{-17} e \cdot \text{cm})$$
$$2.5 \leq \text{Im}(d_\tau) \leq 0.8 (10^{-17} e \cdot \text{cm})$$

What can be measured exactly?



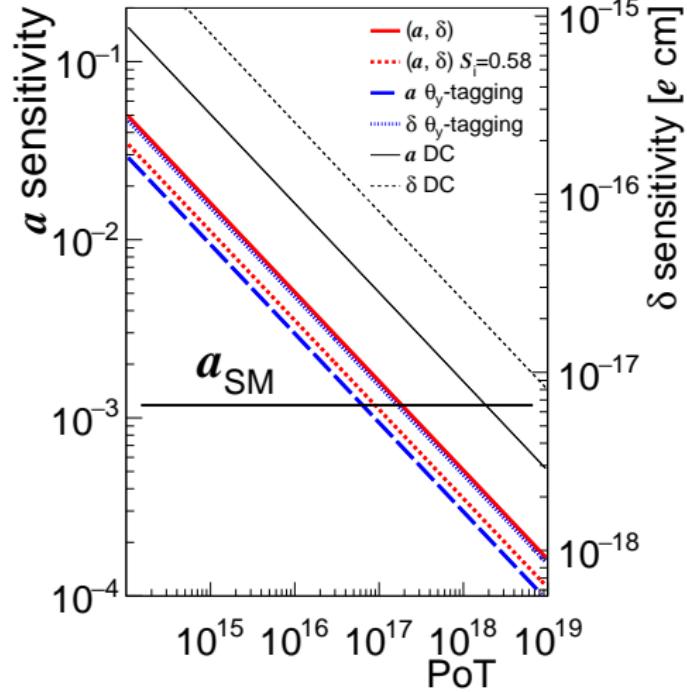
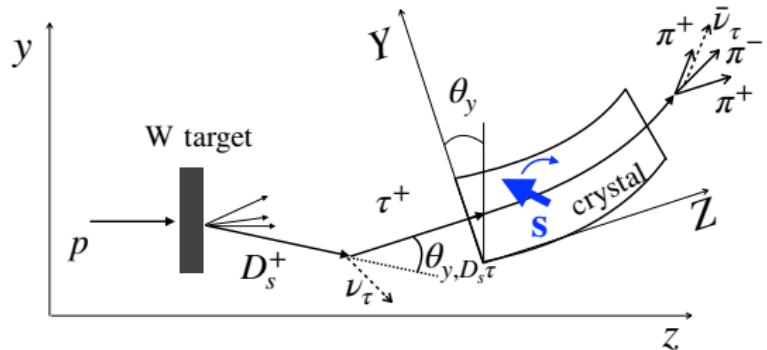
Vertex Function:

$$\begin{aligned}\Gamma^\mu(q^2) &= \gamma^\mu F_1(q^2) + F_A(q^2) (\gamma^\mu q^2 - 2m_\ell q^\mu) \gamma_5 \\ &\quad + \frac{\sigma^{\mu\nu}}{2m_\ell} q_\nu [iF_2(q^2) + F_3(q^2) \gamma_5]\end{aligned}$$

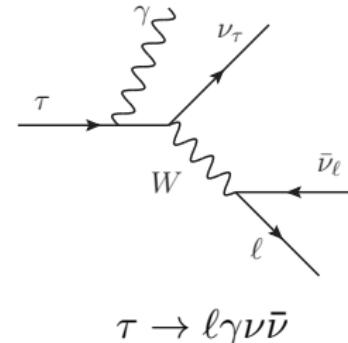
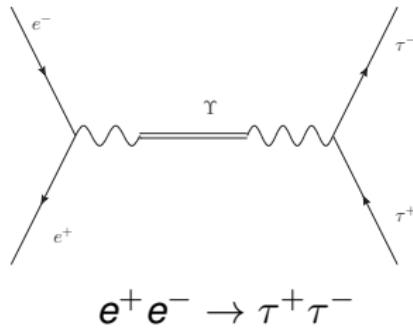
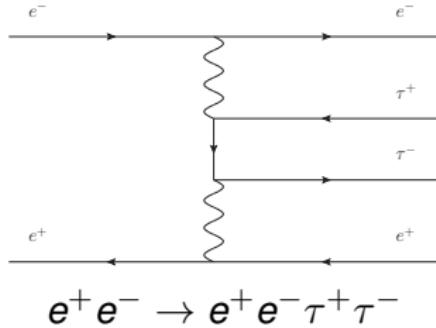
- $F_1(q^2 = 0) = 1$
- $F_2(q^2 = 0) = a_\ell$
- $F_3(q^2 = 0) = d_\ell \frac{2m_\ell}{e}$

Direct Probes at LHC

Fu et al, Phys.Rev.Lett. 123 (2019) 1, 011801
Fomin, Korchin, Stocchi, Barsuk, Robbe, JHEP 1903 (2019) 156



Indirect probes of τ dipole moments



- Not directly sensitive to the form factors at $q^2 = 0$.
- Processes are sensitive to $F(q^2 > 0)$.
- $F(q^2 > 0)$ are non physical, i.e. contains IR divergences.
- Higher QED corrections must be considered.

Effective Lagrangian Approach

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \left[e \frac{\tilde{a}}{4m_\tau} \bar{\tau} \sigma^{\mu\nu} \tau - i \frac{\tilde{d}}{2} \bar{\tau} \sigma^{\mu\nu} \gamma^5 \tau \right] F_{\mu\nu}$$

Eidelman, Epifanov, MF, Mercolli, Passera, JHEP 03 (2016) 140

see also Bernabeu et al, NPB 790 (2008) 160; Gonzalez-Sprinberg, Santamaria, Vidal, NPB 582 (2000) 3; Escribano, Masso, Phys.Lett. B 395 (1997) 369.

- \tilde{a}_τ and \tilde{d}_τ : **BSM contributions** to τ dipole moments from heavy NP.

$$F_2(0) = a_\tau^{\text{SM}} + \tilde{a}_\tau \quad \frac{e}{2m_\tau} F_3(0) = d_\tau^{\text{SM}} + \tilde{d}_\tau$$

- Indirect bounds at e^+e^- collider: $\sigma_{\text{SM}} + \tilde{a} \sigma_a + \tilde{d} \sigma_d$

Complex formalism:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{c_\tau}{\Lambda} \bar{\tau} \sigma^{\mu\nu} P_L \tau F_{\mu\nu} + \text{h.c.}$$

Real and imaginary part of $c\tau$:

$$c_\tau = \tilde{a}_\tau \frac{e}{2m_\tau} - i \tilde{d}_\tau$$

WARNING

- Belle measurement of $\text{Re}(d_\tau)$, $\text{Im}(d_\tau)$ assumes non-Hermitian Lagrangian!
Inami, Phys.Lett.B 551 (2003) 16; Bernreuther et al., Phys.Rev.D 48 (1993) 78, Nucl.Phys.B 388 (1992) 53
- $\text{Im } \tilde{d}_\tau$ violates CPT.
- EFT for CPT/Lorentz violation: SM Extension
Colladay, Kostelecky, PRD 55 (11) 6760

Dipole moments and SM EFT

Tree-level contribution to τ dipole moments (dimension-six):

$$Q_{IW}^{33} = (\bar{\ell}_L \sigma^{\mu\nu} \tau_R) \sigma^I \phi W_{\mu\nu}^I \quad \mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} [C_{IW}^{33} Q_{IW}^{33} + C_{IB}^{33} Q_{IB}^{33} + \text{h.c.}]$$
$$Q_{IB}^{33} = (\bar{\ell}_L \sigma^{\mu\nu} \tau_R) \phi B_{\mu\nu}$$

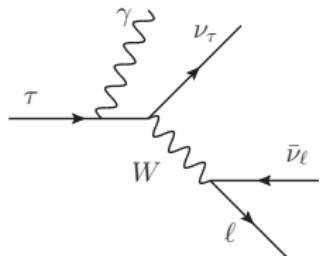
Grzadkowski et al, JHEP10(2010) 085; Crivellin, Najjari, Rosiek, JHEP 04 (2014) 167; Eidelman, Epifanov, MF, Mercolli, Passera, JHEP 03 (2016) 140.

Four independent parameters:

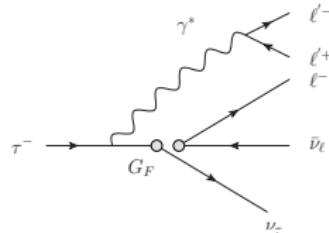
$$\tilde{a}_\tau = \frac{2m_\tau}{e} \frac{\sqrt{2}\nu}{\Lambda^2} \operatorname{Re} [\cos \theta_W C_{IB}^{33} - \sin \theta_W C_{IW}^{33}] \quad \tilde{d}_\tau = \frac{\sqrt{2}\nu}{\Lambda^2} \operatorname{Im} [\cos \theta_W C_{IB}^{33} - \sin \theta_W C_{IW}^{33}]$$
$$\tilde{a}_\tau^W = \frac{2m_\tau}{e} \frac{\sqrt{2}\nu}{\Lambda^2} \operatorname{Re} [\sin \theta_W C_{IB}^{33} + \cos \theta_W C_{IW}^{33}] \quad \tilde{d}_\tau^W = -\frac{\sqrt{2}\nu}{\Lambda^2} \operatorname{Im} [\sin \theta_W C_{IB}^{33} + \cos \theta_W C_{IW}^{33}]$$

Leptonic Tau Decays

- Michel decay $\tau \rightarrow \ell \nu \bar{\nu}$
- Radiative decay $\tau \rightarrow \ell \gamma \nu \bar{\nu}$



- Rare decay $\tau \rightarrow \ell(\ell'^+ \ell'^-) \nu \bar{\nu}$



- Precise SM predictions diff. rate

- $\tau \rightarrow \ell \gamma \nu \bar{\nu}$ up to NLO
MF, Mercolli, Passera, JHEP 07 (2015) 153;
Pruna, Signer, Ulrich, Phys.Lett.B 772 (2017) 452
- $\tau \rightarrow \ell(\ell'^+ \ell'^-) \nu \bar{\nu}$ up to NLO
Greub, MF, JHEP 1701 (2017) 084,
Pruna, Signer, Ulrich, Phys.Lett. B765 (2017) 280;
MF, SciPost Phys. Proc. 1 (2019) 009,

- Optimal probe of NP effects

- Michel Parameters
Shimizu (Belle coll.), PTEP 2018 (2018) 023C01;
Arbuzov, Kopylova, JHEP 09 (2016) 109;
Flores-Talpa, Lopez Castro, Roig, JHEP 04 (2016) 185
- SM EFT
- Dipole moments
MF, Eidelman, Epifanov, Mercolli, Passera, JHEP 03 (2016) 140

	$\tau \rightarrow e\bar{\nu}\nu\gamma$	$\tau \rightarrow \mu\bar{\nu}\nu\gamma$
\mathcal{B}_{LO}	1.834×10^{-2}	3.663×10^{-3}
$\mathcal{B}_{\text{NLO}}^{\text{Inc}}$	$1.728(10)_{\text{th}}(3)_{\tau} \times 10^{-2}$	$3.605(2)_{\text{th}}(6)_{\tau} \times 10^{-3}$
$\mathcal{B}_{\text{NLO}}^{\text{Exc}}$	$1.645(19)_{\text{th}}(3)_{\tau} \times 10^{-2}$	$3.572(3)_{\text{th}}(6)_{\tau} \times 10^{-3}$
K (Inc)	0.94	0.98
K (Exc)	0.90	0.97
Babar [†]	$(1.847 \pm 0.015 \pm 0.052) \times 10^{-2}$	$(3.69 \pm 0.03 \pm 0.10) \times 10^{-3}$
Belle [*]	$(1.79 \pm 0.02 \pm 0.10) \times 10^{-2}$	$(3.63 \pm 0.02 \pm 0.15) \times 10^{-3}$

[†][BABAR - PRD 91 \(2015\) 051103](#)

^{*}[N. Shimizu - Belle - PTEP 2018 \(2018\) 023C01](#)

- $E_{\gamma} \geq 10 \text{ MeV}$
- Exclusive BR: $n = 1$ photon
- Inclusive BR: $n \geq 1$ photons

See also: [L. Mercolli, MF, M. Passera JHEP 1507 \(2015\) 153](#)

	\mathcal{B}_{LO}	$\delta\mathcal{B}_{\text{NLO}}$	$\delta\mathcal{B}_{\text{had}}$	$\delta\mathcal{B}/\mathcal{B}$
$\tau \rightarrow eeee\nu\bar{\nu}$	$4.2488(4) \times 10^{-5}$	$-4.3(1) \times 10^{-8}$	-1.0×10^{-9}	-0.1%
$\tau \rightarrow \mu eee\nu\bar{\nu}$	$1.989(1) \times 10^{-5}$	$4.3(1) \times 10^{-8}$	-6.6×10^{-10}	0.2%
$\tau \rightarrow e\mu\mu\nu\bar{\nu}$	$1.2513(6) \times 10^{-7}$	$2.31(1) \times 10^{-9}$	-3.6×10^{-10}	1.8%
$\tau \rightarrow \mu\mu\mu\nu\bar{\nu}$	$1.1837(1) \times 10^{-7}$	$1.925(2) \times 10^{-9}$	-3.5×10^{-10}	1.6%
$\mu \rightarrow eeee\nu\bar{\nu}$	$3.6054(1) \times 10^{-5}$	$-6.69(5) \times 10^{-8}$	-1.8×10^{-11}	0.2%

MF, SciPost Phys. Proc. 1 (2019) 009

- Belle expect to observe first two modes, upper limits on the last two.

Sasaki (Belle) J.Phys.Conf.Ser. 912 (2017) 012002

Tau Dipole Moments in $\tau \rightarrow \ell \gamma \nu \bar{\nu}$

- Suggested already in the '80

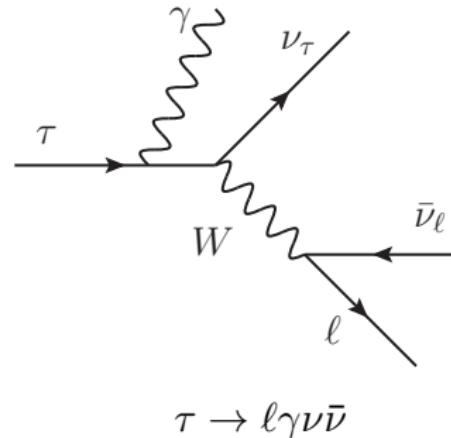
Laursen et al., PRD 29 (1984) 2652

- Exploit the amplitude's **radiation zeros**:

$$\cos \theta_{\ell\gamma} \simeq -1 \quad E_\ell \simeq \frac{m_\tau^2 + m_\ell^2}{2m_\tau}$$

- Achievable sensitivity at Belle

$$|\tilde{a}_\tau| < 2$$



Eideman, Epifanov, MF, Mercolli, Passera, JHEP 03 (2016) 140

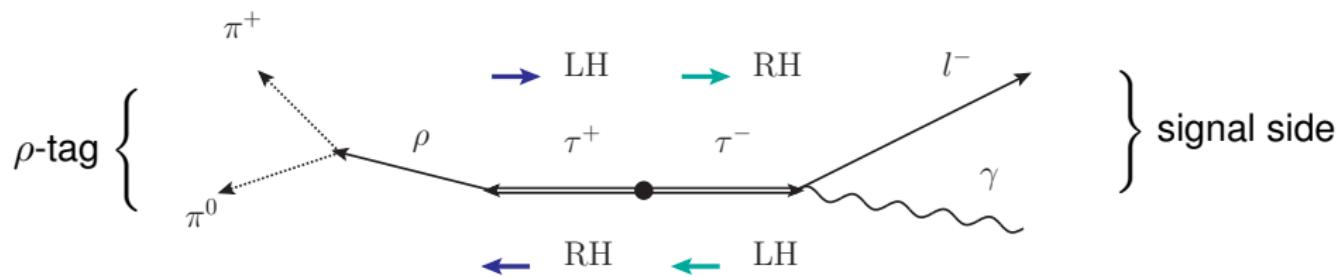
$$\text{BR} = \int d\text{Lips} (d\Gamma^{SM} + \tilde{a}_\tau d\Gamma_a + \tilde{d}_\tau d\Gamma_d) \approx \begin{array}{rcl} 1.6 \times 10^{-2} & + & \tilde{a}_\tau 8.6 \times 10^{-6}, \quad l = e \\ 3.6 \times 10^{-3} & + & \tilde{a}_\tau 8.2 \times 10^{-6}, \quad l = \mu \end{array}$$

Feasibility Study at Belle and Belle II

Eidelman, Epifanov, MF, Mercolli, Passera, JHEP 03 (2016) 140

- Exploit polarised **fully differential rate**, taus spin correlation.
- Goal 10^{-3} precision target: $d\Gamma = d\Gamma_{\text{SM}} + \tilde{a}_\tau d\Gamma_a + \tilde{d}_\tau d\Gamma_\tau$
Include NLO QED + m_τ^2/M_W^2 corrections

MF, Mercolli, Passera, PRD 88 (2013) 093011; JHEP 07 (2015) 153



- 3 Neutrinos: taus' direction constrained on an arc.
- **Strategy:** extract \tilde{a}_τ and \tilde{d}_τ via **unbinned maximum likelihood**.

Feasibility study: results

- $\tau^+\tau^-$ events with KKMC/TAUOLA/PHOTOS
- Events processes by GEANT3 based program
- Analyzed ($\ell^\mp\nu\bar{\nu}\gamma, \pi^\pm\pi^0\nu$) events in the 12-dimensional space.

Two scenarios:

- ρ -tag mode: $\tau \rightarrow \rho\nu_\tau$, BR= 25.5%.
- full-tag mode: six modes, BR= 90%.

Stat. sensitivity to \tilde{a}_τ and \tilde{d}_τ		
	\tilde{a}_τ	$\tilde{d}_\tau(2m_\tau)/e$
Belle (ρ -tag)	0.16	0.15
Belle-II (ρ -tag)	0.023	0.021
Belle (full tag)	0.085	0.080
Belle-II (full tag)	0.012	0.011
DELPHI [†]	0.017	—
Belle*	—	0.0015

Eidelman, Epifanov, MF, Mercolli, Passera, JHEP 03 (2016) 140

†

DELPHI - EPJC35 (2004) 159

*

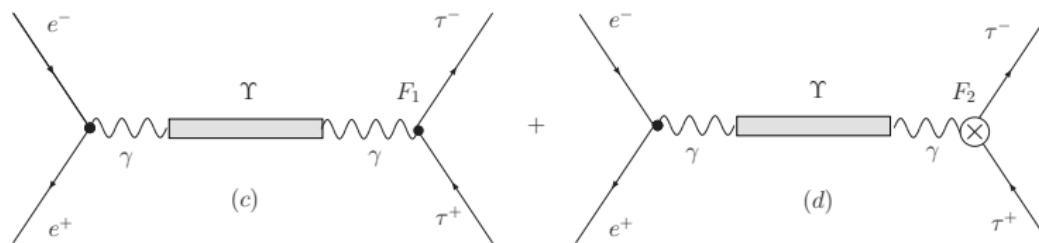
Belle coll. PLB 551 (2003) 16.

Dipole Moments in $e^+e^- \rightarrow \tau^+\tau^-$



- $F_{2V}(q^2 = M_\tau^2)$ physical and measurable at B factories in $e^+e^- \rightarrow \Upsilon \rightarrow \tau^+\tau^-$.

Bernabéu, Gonzalez-Sprinberg, Papavassiliou, Vidal, NPB 790 (2008) 160.

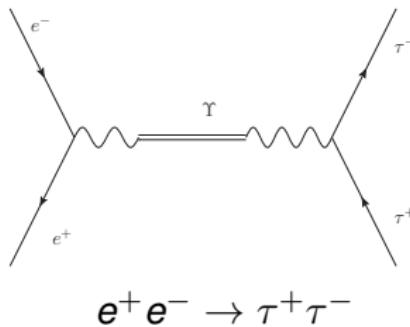


- Expected sensitivity: $|F_2(M_\tau)| < \mathcal{O}(10^{-6})$ with $\mathcal{L} = 15 \text{ ab}^{-1}$.
- Beam energy spread σ_W at Belle (Belle-II):

$$\Gamma_{\Upsilon(1s), \Upsilon(2s), \Upsilon(3s)} \sim \mathcal{O}(10 \text{ keV}) \quad \ll \quad \sigma_W = 5.24 \text{ (5.45) MeV}$$

Dipole Moments in $e^+e^- \rightarrow \tau^+\tau^-$

Chen, Wu, JHEP 10 (2019) 089



- Use effective Lagrangian approach ($\tilde{a}_\tau, \tilde{d}_\tau$).
- **Strategy:** construct optimal observables for $e^+e^- \rightarrow \tau^+\tau^-$ events.
- Focus on hadronic tau decays for the reconstruction
 $\tau \rightarrow \pi\nu, \tau \rightarrow \pi\pi^0\nu, \tau \rightarrow \pi\pi\pi\nu$.
- Full $\tau^+\tau^-$ reconstruction using impact parameters.
- Neutrino momenta determined minimising

$$\chi^2 = \sum_{i=0}^3 \left(\frac{p_{\text{evt},i}^{\text{fit}} - p_{\text{evt},i}}{\sigma_{\text{evt}}} \right)^2 + \left(\frac{m_{\tau 1,2}^{\text{fit}} - m_\tau}{\sigma_{m_\tau}} \right)^2 + \sum_i \chi_{IP,i}^2$$

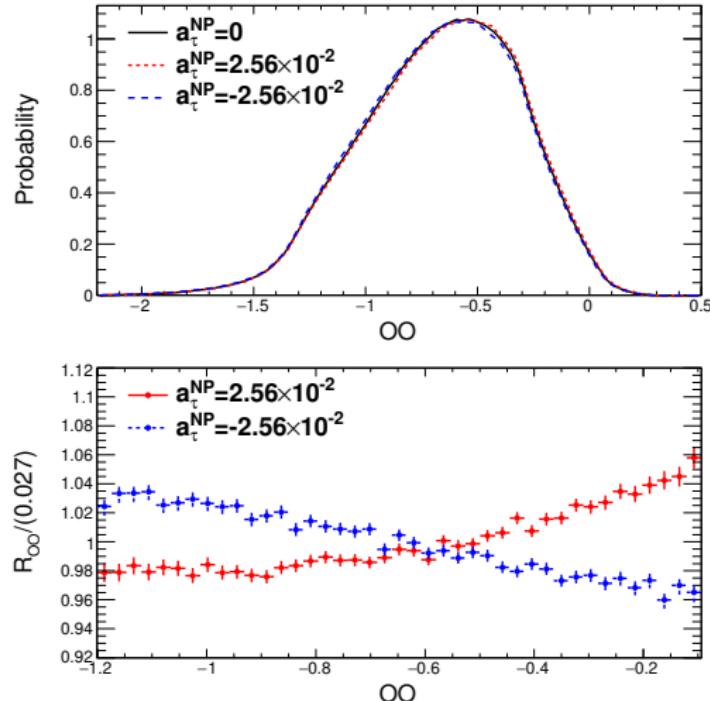
Optimal Observable

Matrix element prediction

$$|\mathcal{M}|^2 = M_{\text{SM}} + \tilde{a}_\tau M_a + \tilde{d}_\tau \frac{2m_\tau}{e} M_d$$

Optimal Observable

$$\begin{aligned} OO_a(p_{\tau^+}, p_{\tau^-}) &= \frac{M_a(p_{\tau^+}, p_{\tau^-})}{M_{\text{SM}}(p_{\tau^+}, p_{\tau^-})} \\ R_{OO} &= \frac{OO_a}{OO_a|_{\tilde{a}=0}} \\ &= 1 + b(OO_a - x_0) \end{aligned}$$



Chen, Wu, JHEP 10 (2019) 089

■ Statistical sensitivity

\mathcal{L}	1 ab ⁻¹	10 ab ⁻¹	50 ab ⁻¹
$ \tilde{d}_\tau $ (e·cm)	1.44×10^{-18}	4.56×10^{-19}	2.04×10^{-19}
$ \tilde{a}_\tau $	1.24×10^{-4}	3.92×10^{-5}	1.75×10^{-5}

Chen, Wu, JHEP 10 (2019) 089

■ Open questions:

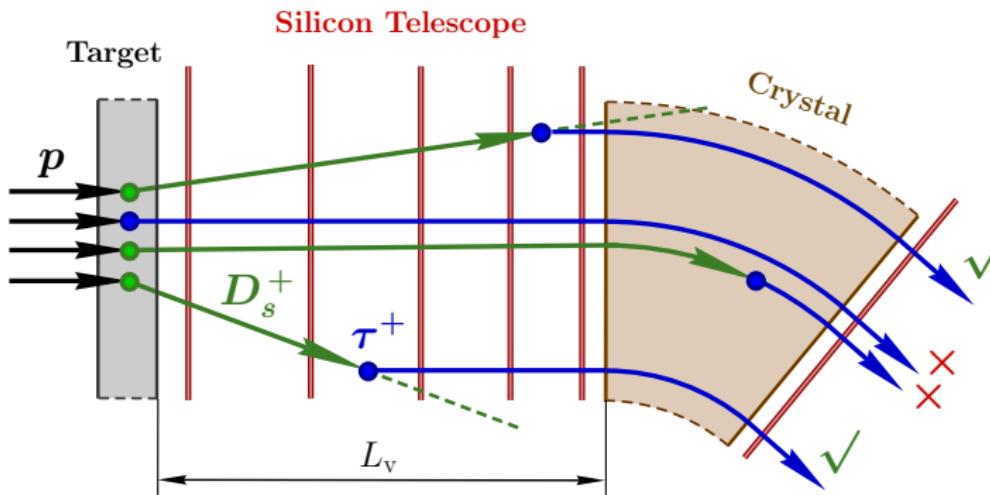
- Accuracy in M_{SM} prediction, e.g. radiative corrections, tau decay modelling.
- Experimental systematic.

Conclusions

- Tau lepton $g-2$ would be a clean test of NP, if only we could measure it!
- Indirect searches at Belle II cannot directly probe $F_2(0)$ and $F_3(0)$.
- NP contributions to the tau dipole moments can be constrained via an EFT approach.
- Measurement strategy is closely related to NP searches in B decays, i.e. we parametrize NP contributions in terms of Wilson coefficients.
- Belle II has the potential to improve upon current bound from LEP.
- Radiative tau decays and $e^+e^- \rightarrow \tau^+\tau^-$ offers two independent avenues to the measurement of tau dipole moments.



$$p p \rightarrow D_s^+ \dots \rightarrow \tau^+ \dots \rightarrow 2\pi^+ \pi^- \dots$$

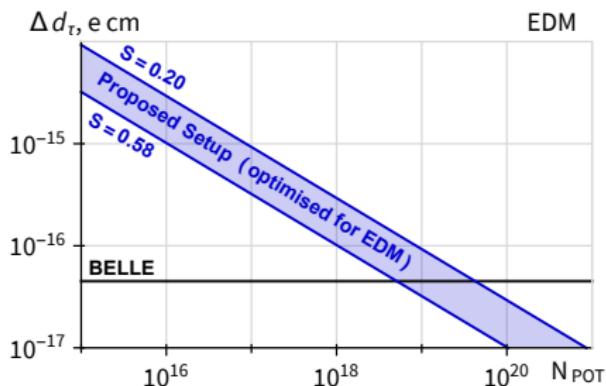
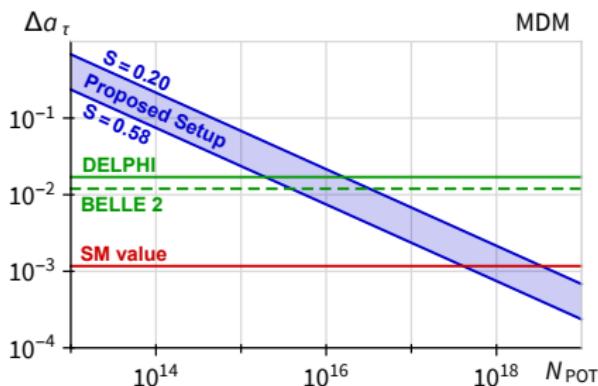


- we need to select events: $D_s^+ \rightarrow \tau^+ \rightarrow 2\pi^+ \pi^-$ out of the background: $X^+ \rightarrow 2\pi^+ \pi^-$
- directions of D_s^+ and τ^+ momenta should be measured very accurately $\Delta\theta < 100 \mu\text{rad}$

SENSITIVITY STUDIES: Precision of anomalous MDM versus number of protons on target.

$$\Delta a_\tau = \sqrt{\frac{1}{N_p \eta_{\text{det}} S^2 \eta_{\text{setup}}}}$$

$$\text{setup} = \left(\frac{d''}{d''} \right)^2 \frac{\partial N^{\text{def}}}{\partial ''} \left(\frac{P^2}{f/a} \right) \left| \frac{4 \alpha}{(1 + \alpha)^2} \right|$$



- Target (W)

$L_{\text{tar}} = 1 \text{ cm}$

- Crystal 1 (Ge)

$L_D = 3 \text{ cm}$,

$R_D = 10 \text{ m}$,

$\theta_p = 100 \mu\text{rad}$

- Crystal 2 (Ge)

$L = 10 \text{ cm}$,

$R = 7 \text{ m}$,

$\theta_V = 80 \mu\text{rad}$

Fomin, Korchin, Stoechi, Barsuk, Bobbe, JHEP 1903 (2019) 156

— Double Crystal Setup at LHC $Ds^+ \rightarrow \tau^+ \bar{\nu}_\tau, \quad \tau^+ \rightarrow \pi^+ \pi^+ \pi^- \bar{\nu}_\tau$

— DELPHI: limit on aMDM from LEP2 experiment $\gamma \gamma \rightarrow \tau^+ \tau^-$
J. Abdallah et al. Eur. Phys. J., C35:159–170, 2004

— BELLE 2: limit on aMDM expected for the BELLE 2 experiment
S. Eidelman, et al. JHEP, 03:140, 2016.

— SM value: aMDM value predicted by a Standard Model
S. Eidelman et al. Mod. Phys. Lett., A22:159–179, 2007.