



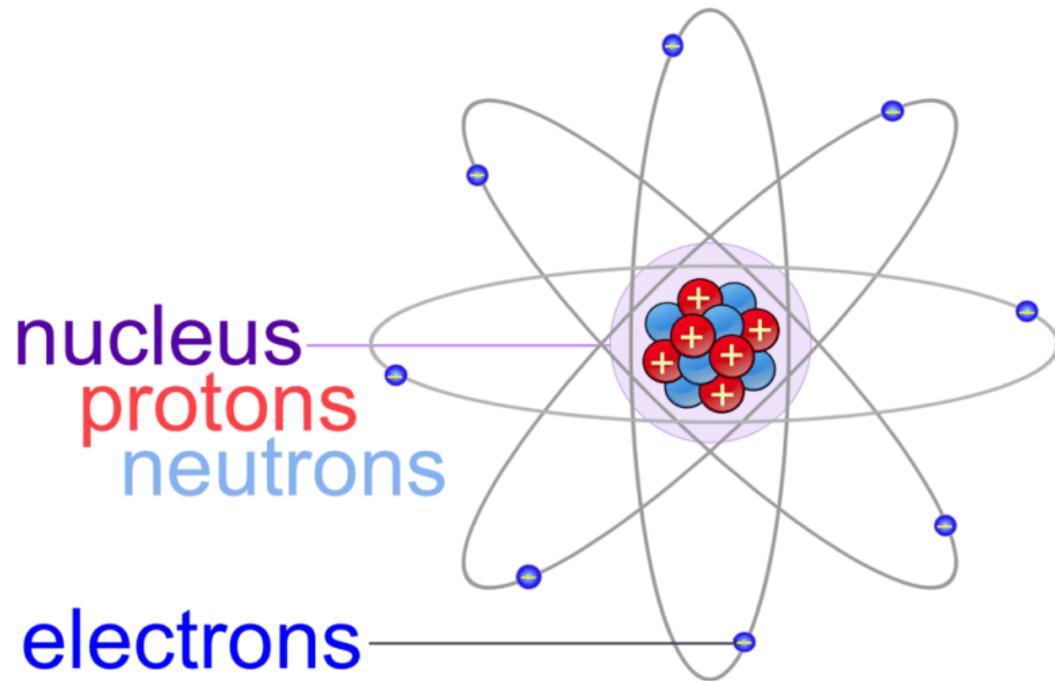
# Neutrino Oscillations

Stefan Söldner-Rembold  
University of Manchester

Academic Training Lecture Programme  
May 2021

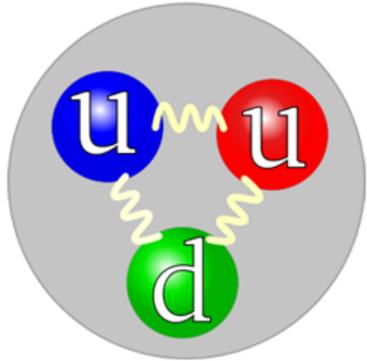
## Neutrino Oscillations and the PMNS Matrix

# What are we made of?



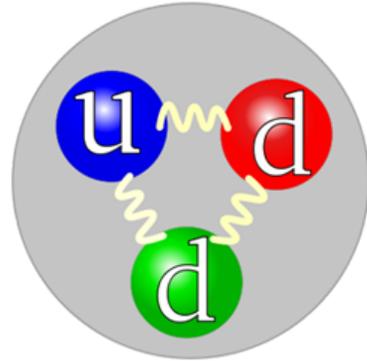
# What are we made of?

Proton



2 quarks up  
1 quark down

Neutron



1 quark up  
2 quarks down



# Beta Decay

Neutrons are not stable

“neutron  $\rightarrow$  proton + electron”

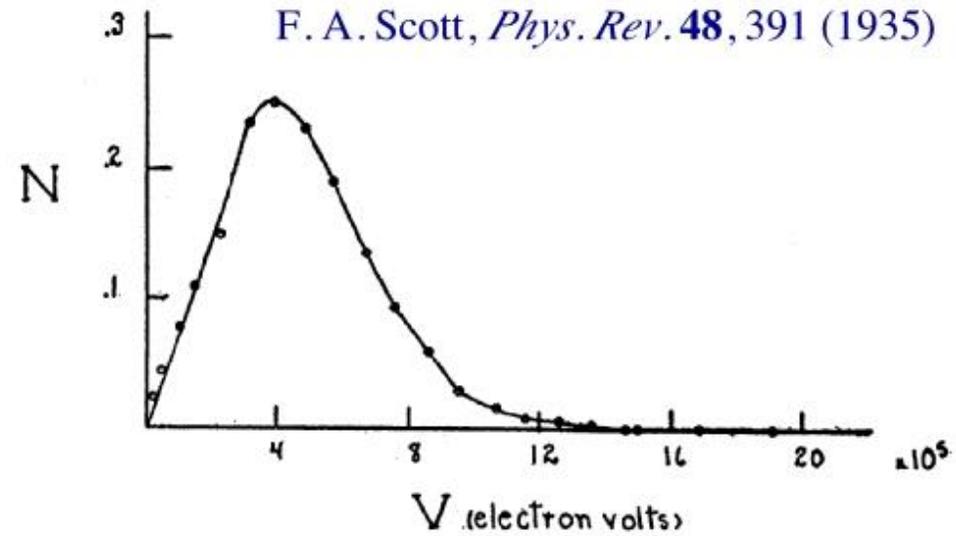
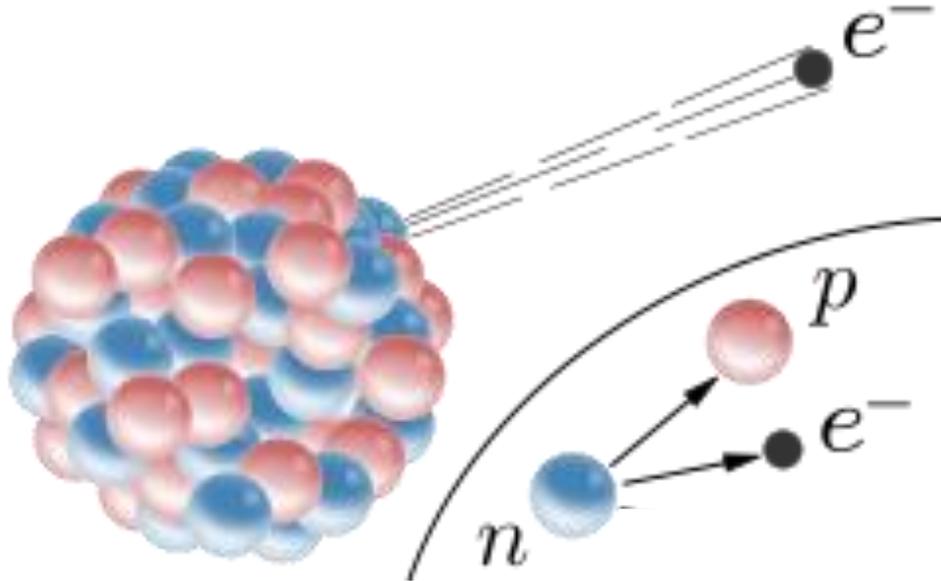


FIG. 5. Energy distribution curve of the beta-rays.

# Liebe Radioactive Damen und Herren,



original - Photocopy of PLC 0393  
Abschrift/15.12.56 **FN**

Offener Brief an die Gruppe der Radioaktiven bei der  
Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut  
der Eidg. Technischen Hochschule  
Zürich

Zürich, 4. Dez. 1930  
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst  
anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich  
angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie  
des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg  
verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz  
zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale  
Teilchen, die ich Neutronen nennen will, in den Kernen existieren,  
welche den Spin  $1/2$  haben und das Ausschliessungsprinzip befolgen und  
sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie  
nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen  
müsste von derselben Grössenordnung wie die Elektronenmasse sein und  
jedenfalls nicht grösser als  $0,01$  Protonenmasse.- Das kontinuierliche  
beta-Spektrum wäre dann verständlich unter der Annahme, dass beim  
beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert  
wird, derart, dass die Summe der Energien von Neutron und Elektron  
konstant ist.

# Dear Radioactive Ladies and Gentlemen,

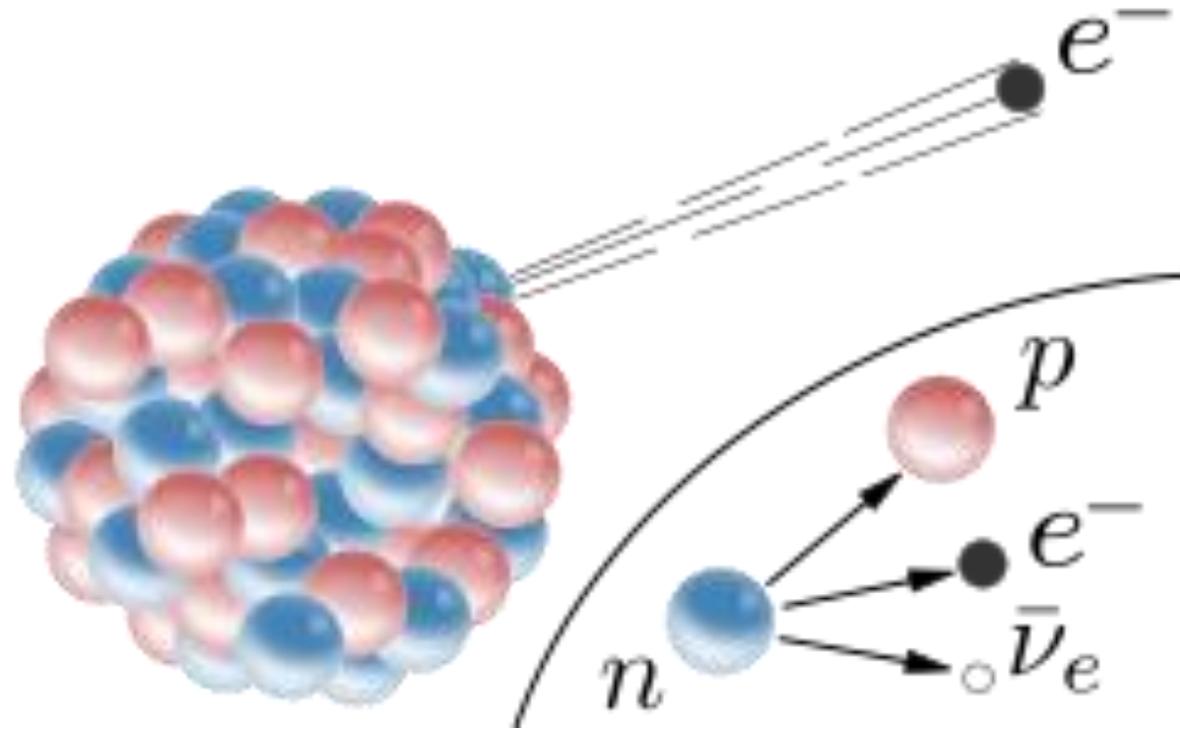


**...I have hit upon a desperate remedy...**

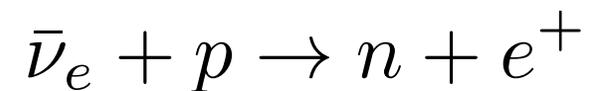
**...so far I do not dare to publish anything about this idea, and trustfully turn first to you, dear radioactive people, with the question of how likely it is to find experimental evidence for such [a particle]...**

**--Signed, W. Pauli**

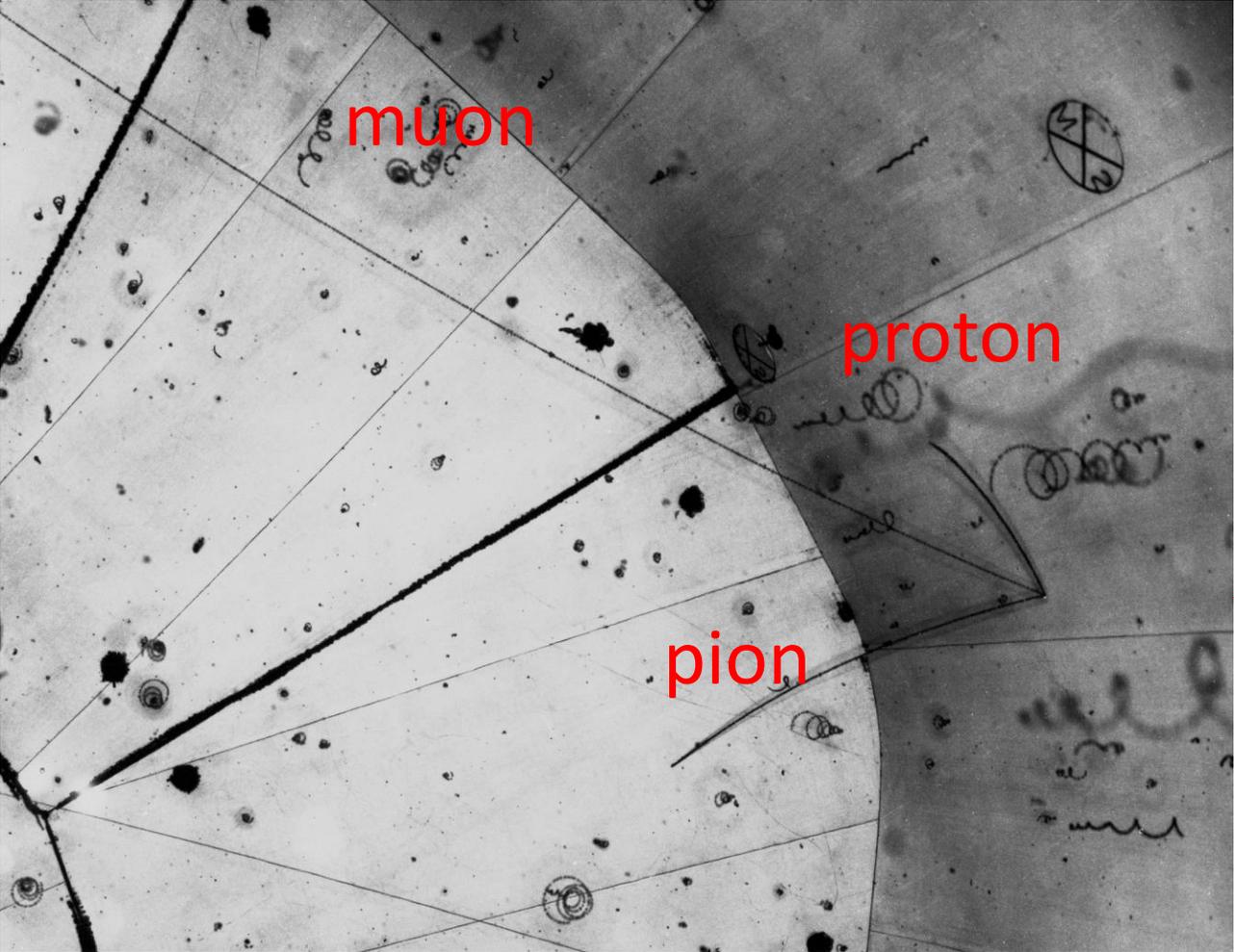
# Dear Radioactive Ladies and Gentlemen,



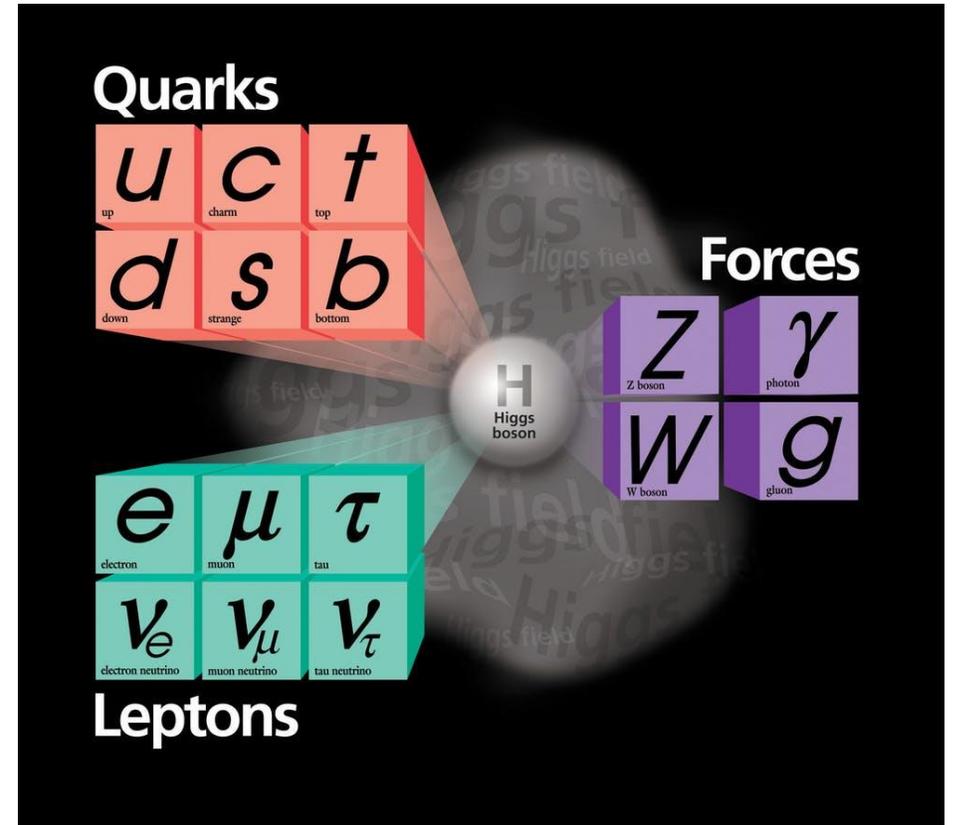
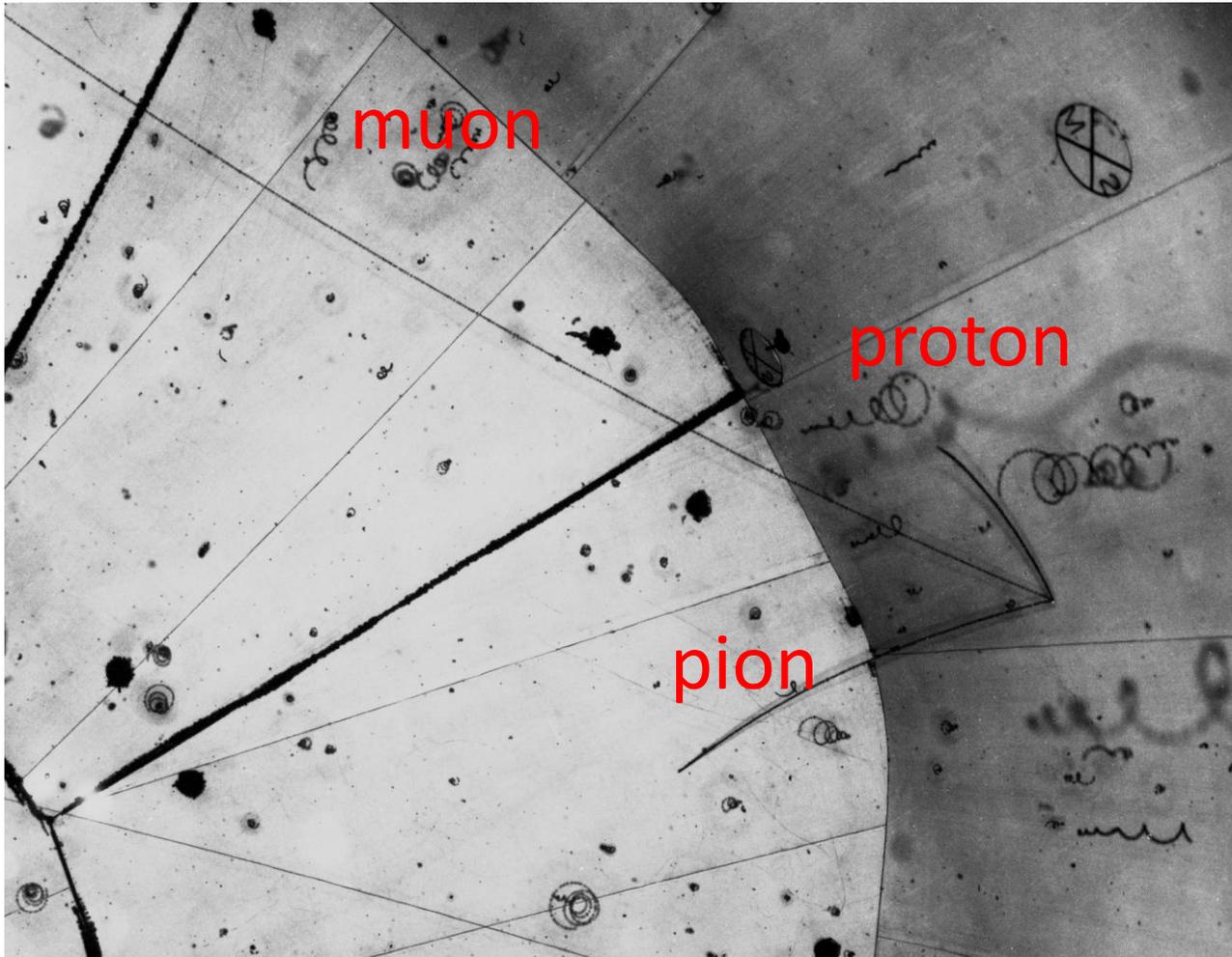
- Enrico Fermi (re-)named the new particle 'neutrino'.
- Discovered in  $\beta$  decays from reactors in 1956:



Hydrogen Bubble chamber, Argonne, 1970



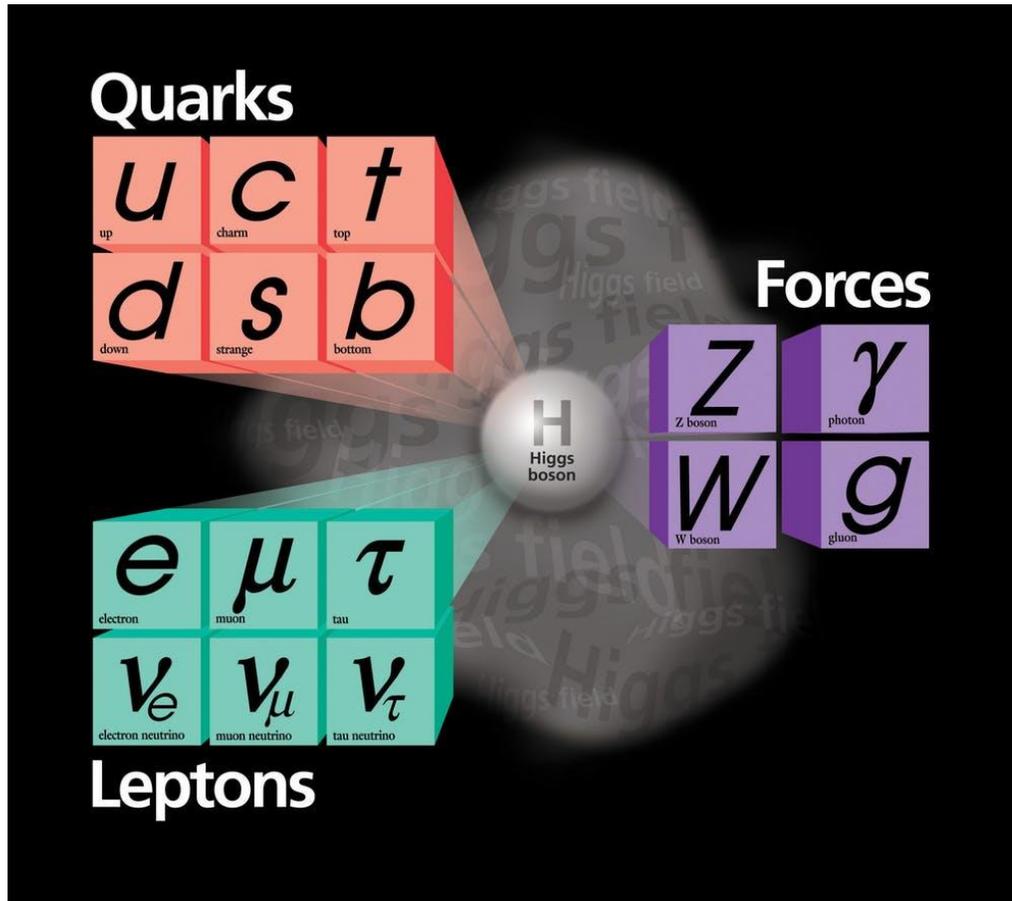
← (muon)-neutrino



- Charged and neutral leptons come in pairs
- "Flavour" of outgoing charged lepton defines neutrino flavour
- Neutrinos have three flavours ( $e, \mu, \tau$ )

# The “Standard Model”

3 families



- Neutrinos are electrically neutral
- Four forces:
  - **Strong force (gluon g)**
  - **Weak force (W and Z boson)**
  - **Electromagnetic force (photon γ)**
  - **Gravity (graviton)**
- Neutrinos only feel the weak force (and gravity)
- Even though neutrinos appear “ghost” like and ephemeral, they play an important role in the Universe.



**Neutrinos are the most abundant matter particle:  
for every proton/neutron/electron the Universe  
contains a billion neutrinos from the Big Bang**



**$10^{38}$  neutrinos per second are produced  
by fusion processes in the Sun  
(with a flux of  $\sim 10^{11}/\text{cm}^2/\text{sec}$  at the Earth)**

# Neutrinos in a box

In a box of 1 cubic meter, somewhere in the Universe, we have on average

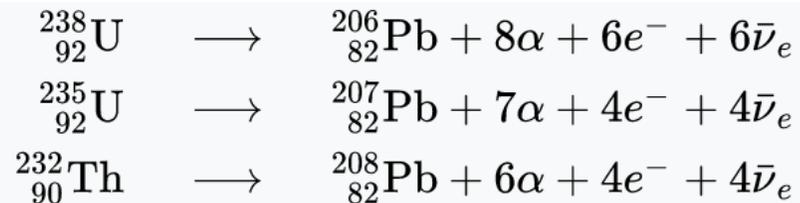
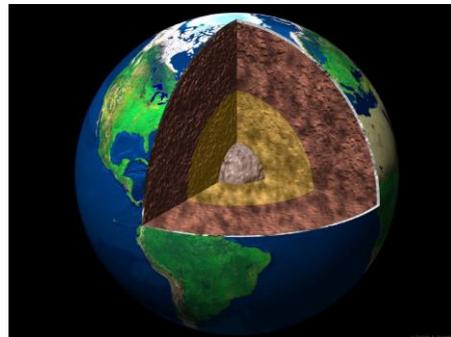
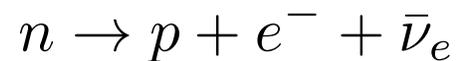
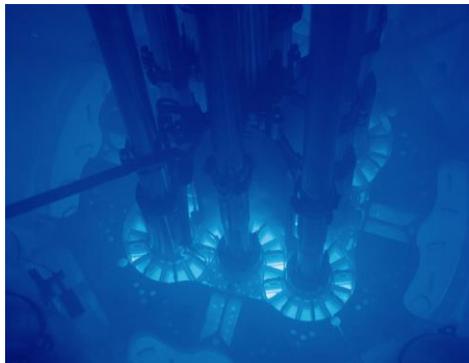
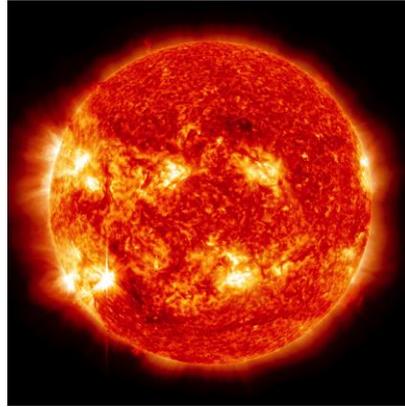
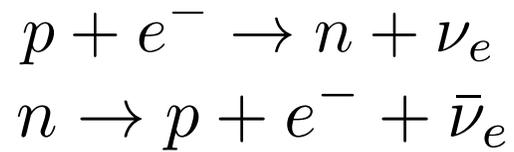
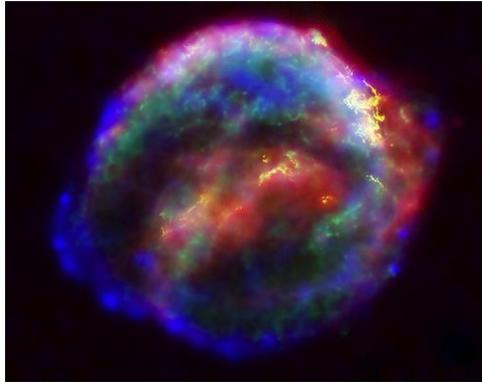
- 10 protons
- 300 million neutrinos

left over from the Big Bang.

Neutrinos are the most abundant *matter particles* in the Universe

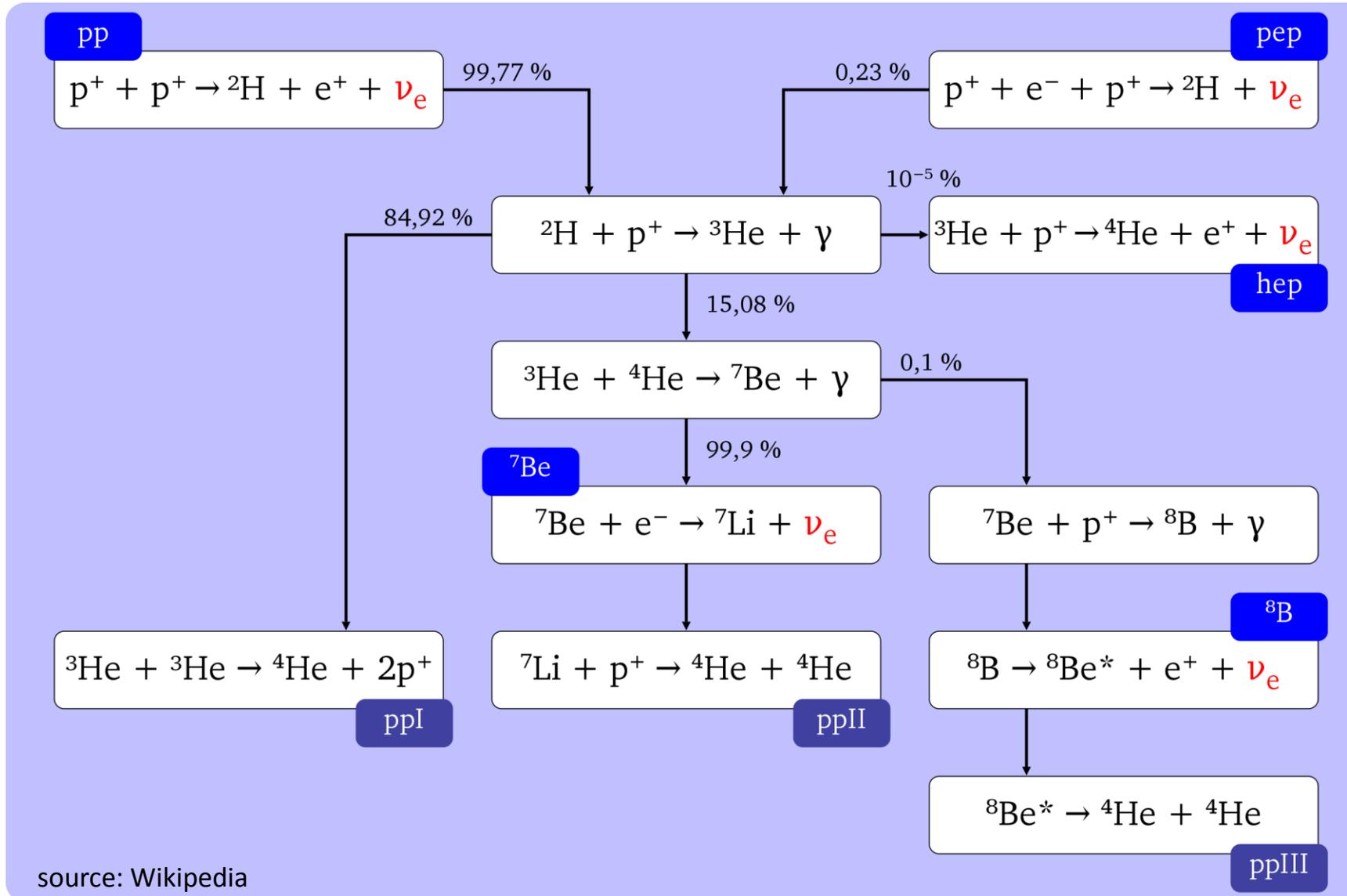


# Neutrino Sources (nuclear processes)

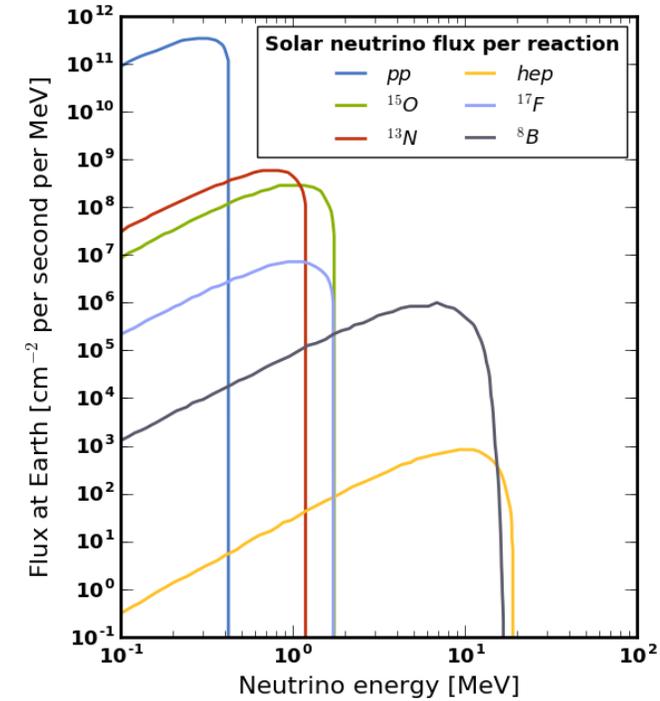


- Nuclear processes are typically the source of electron-neutrinos.
- Typical energies 1-10 MeV
- Discovery of electron-neutrino by Cowan and Reines at the Savannah River power plant in 1956.

# Fusion produces electron-neutrinos in the Sun



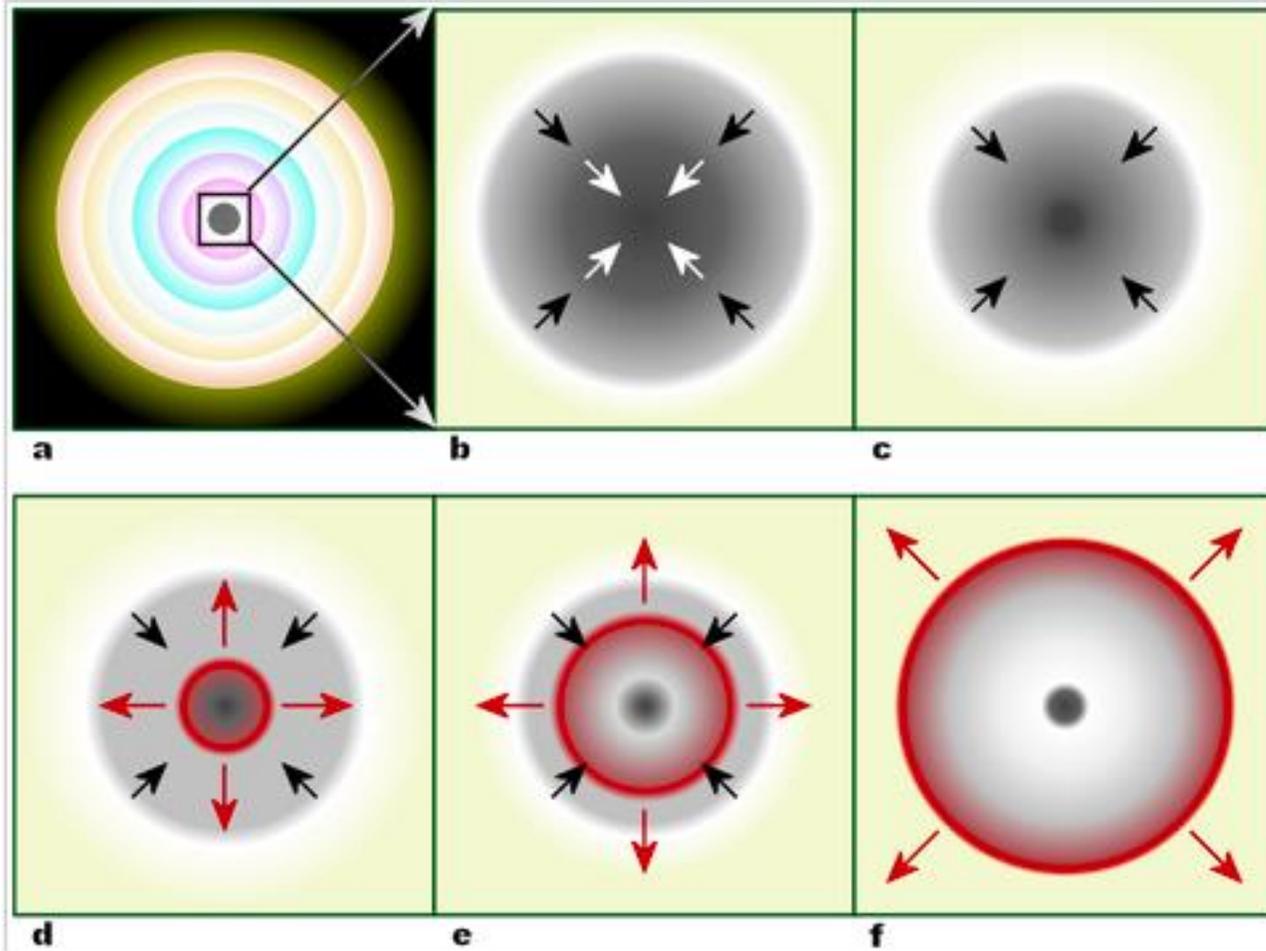
source: Wikipedia



Standard Solar Model

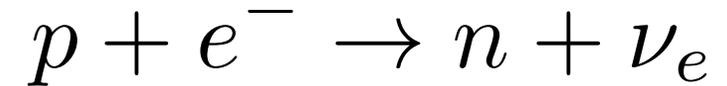
# Type II Supernova

Gravitational collapse of a massive star at the end of its life



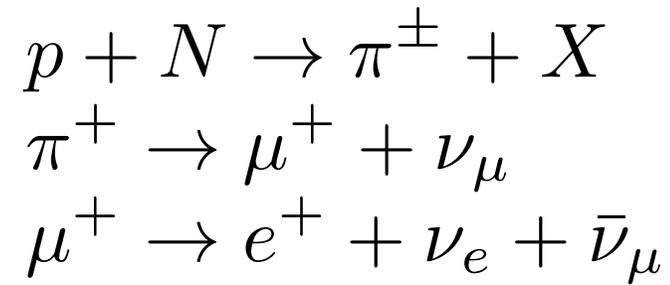
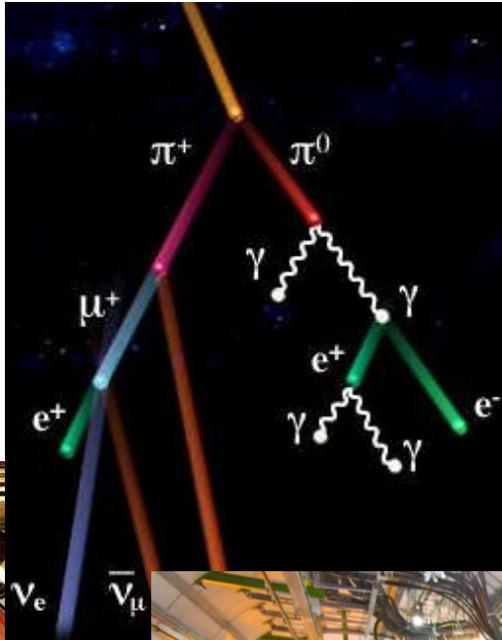
Compact remnant: neutron star or black hole

- Dropping an object turns gravitational **'potential energy'** into **'kinetic energy'** when an object falls.
- As the star falls inward the gravitational energy has to go somewhere:

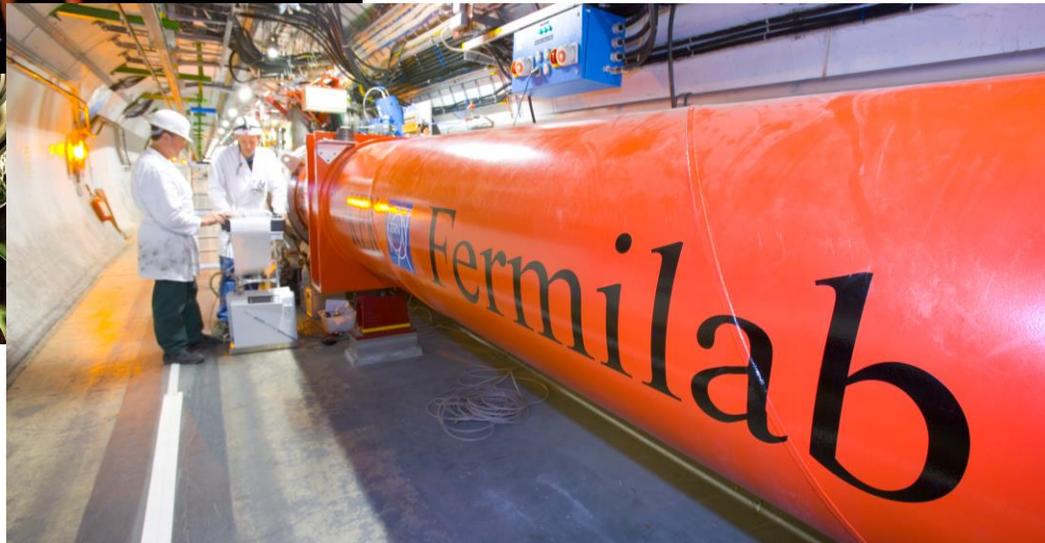
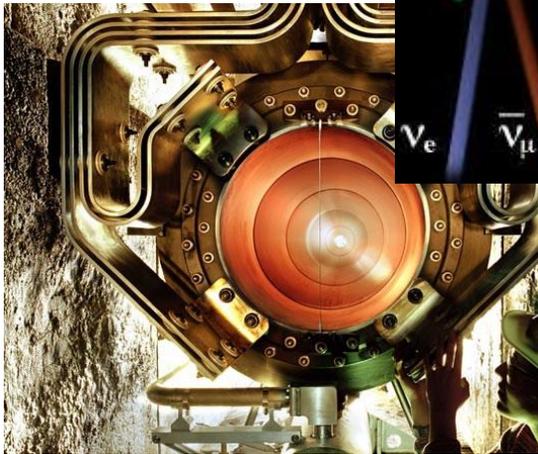


- Neutrinos only interact weakly, so easiest for them to escape.
- **About 99% of the huge binding energy of the neutron star is shed within about 10 seconds in the form of neutrinos.**

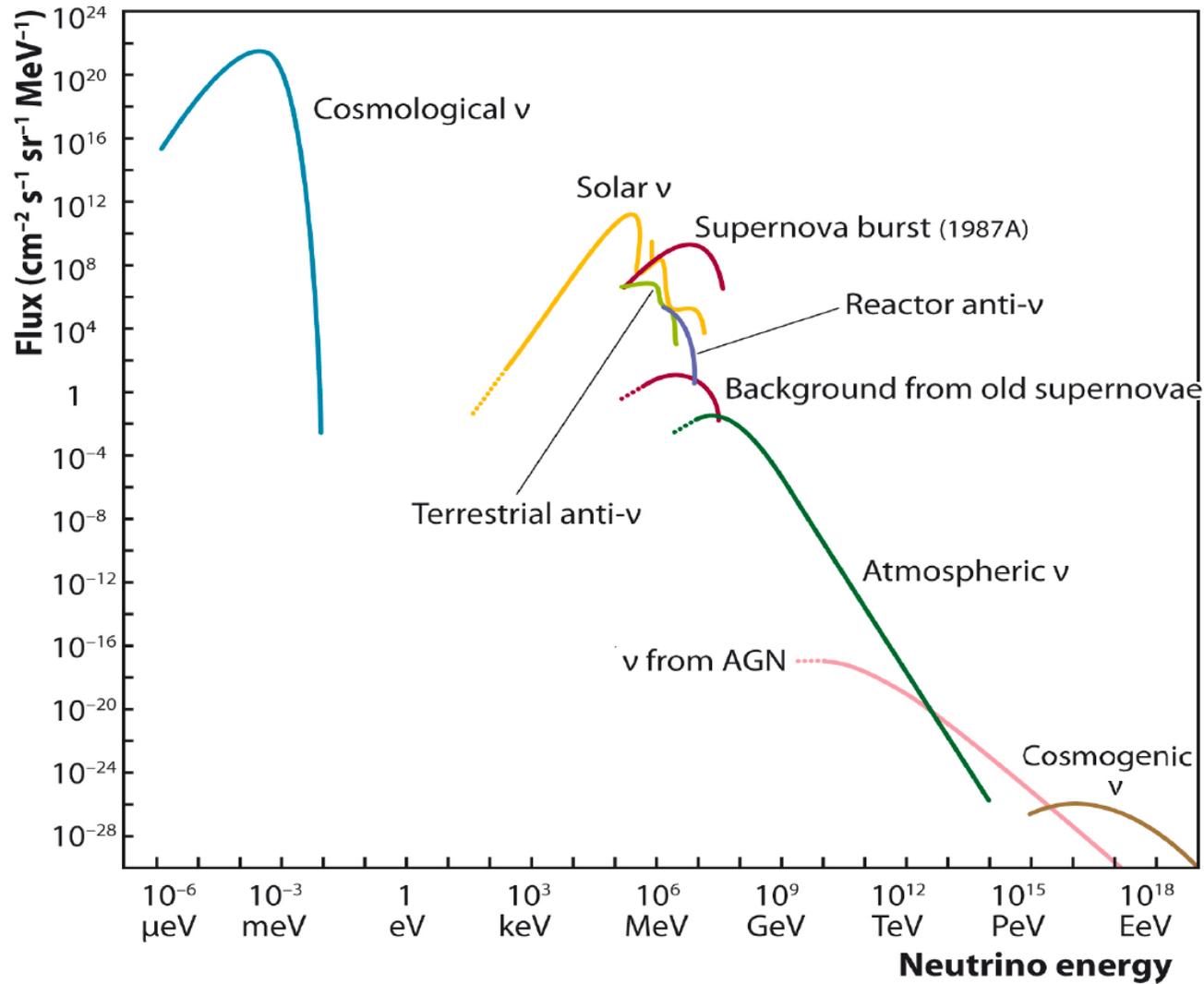
# Neutrino Sources (atmospheric/accelerator)



- Pion/kaon production and decay are main source of accelerator and atmospheric neutrinos
- Typical energies  $\approx$  GeV, ratio of  $\nu_\mu:\nu_e = 2:1$ .
- Discovery of muon-neutrino by Ledermann, Schwartz, Steinberger at Brookhaven in 1962.



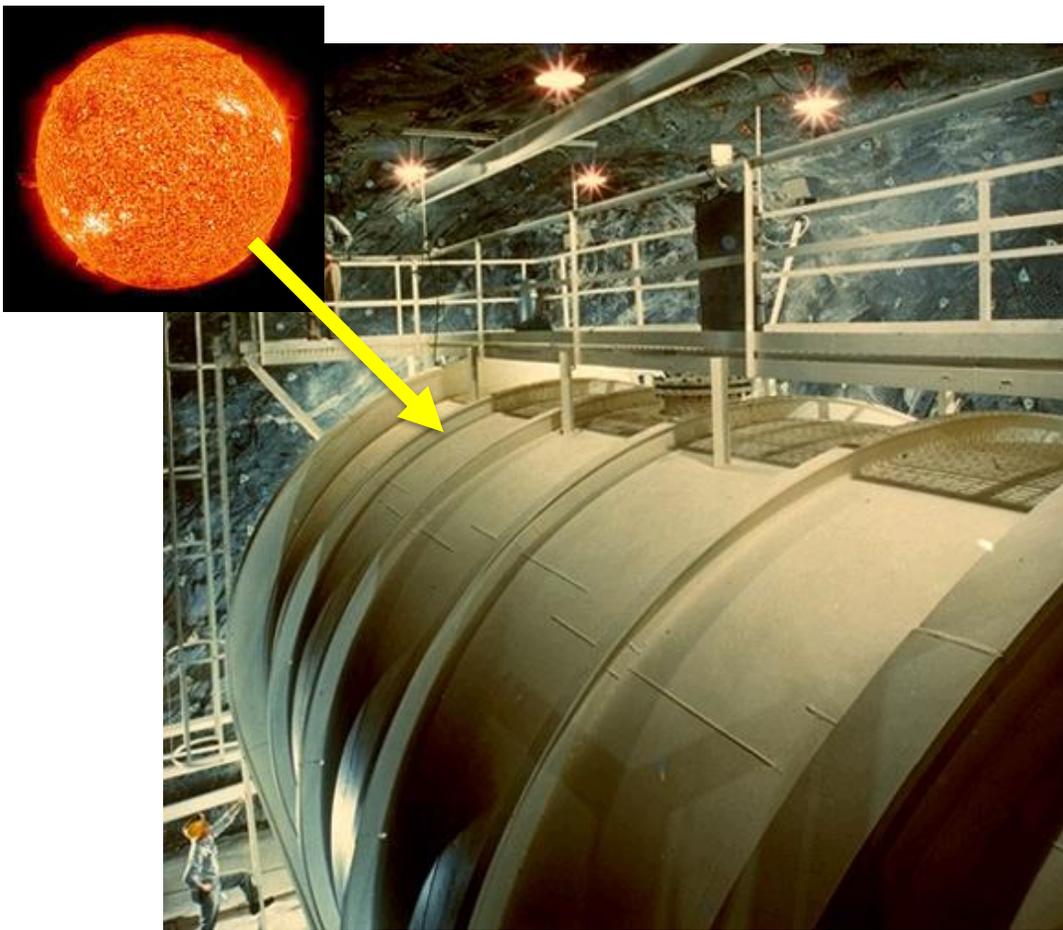
# Neutrino sources, flux, and cross section



C. Spiering, arXiv:1207.4952

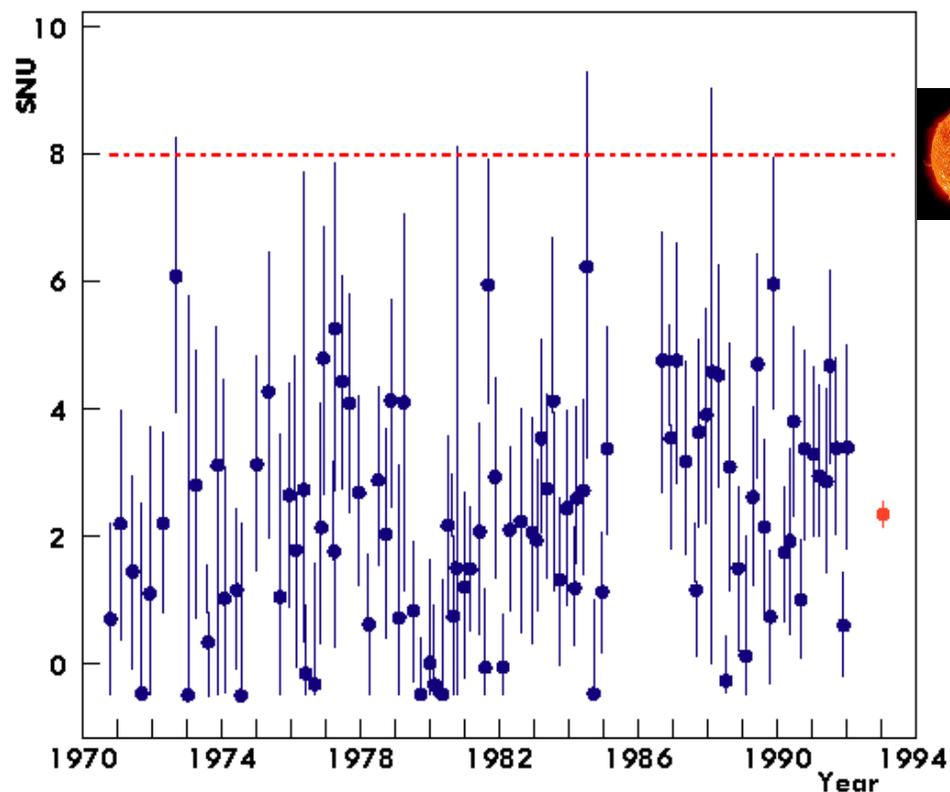
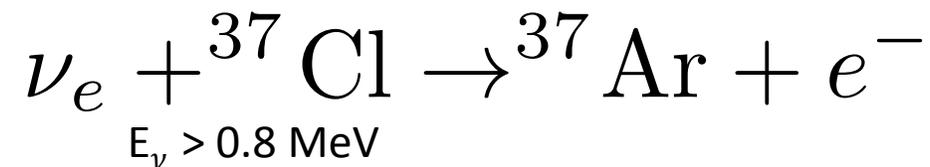
# The missing solar neutrinos

Ray Davis experiment, Homestake Mine, South Dakota



Filled with 390,000 litres of cleaning fluid ( $C_2Cl_4$ )

“Inverse  $\beta$  Decay”



$\sim 1/3$

# Homestake experiment (1970-1994)

- Filter out argon and search for  $^{37}\text{Ar}$  decay

- Detecting ~5 atoms of  $^{37}\text{Ar}$  per day in 390,000 litres of  $\text{C}_2\text{Cl}_4$

VOLUME 20, NUMBER 21

PHYSICAL REVIEW LETTERS

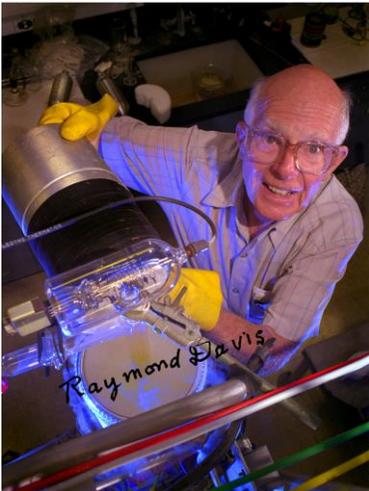
20 MAY 1968

from the bottom of the tank and returned to the tank through a series of 40 eductors arranged along two horizontal header pipes inside the tank. The eductors aspirate the helium from the gas space (2000 liters) above the liquid, and mix it as small bubbles with the liquid in the tank. The pump and eductor system passes helium through the liquid at a total rate of 9000 liters per minute maintaining an effective equilibrium between the argon dissolved in the liquid and the argon in the gas phase.

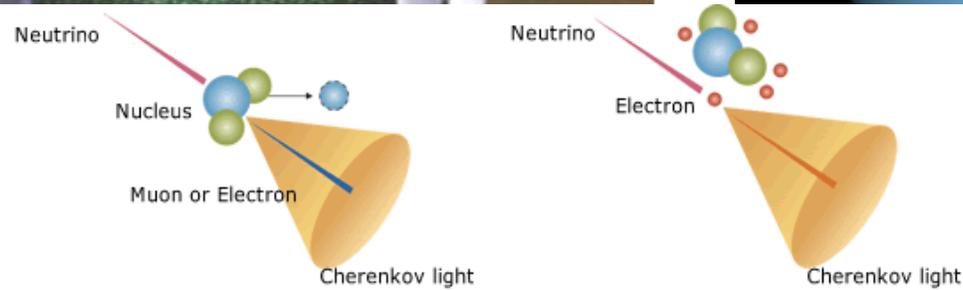
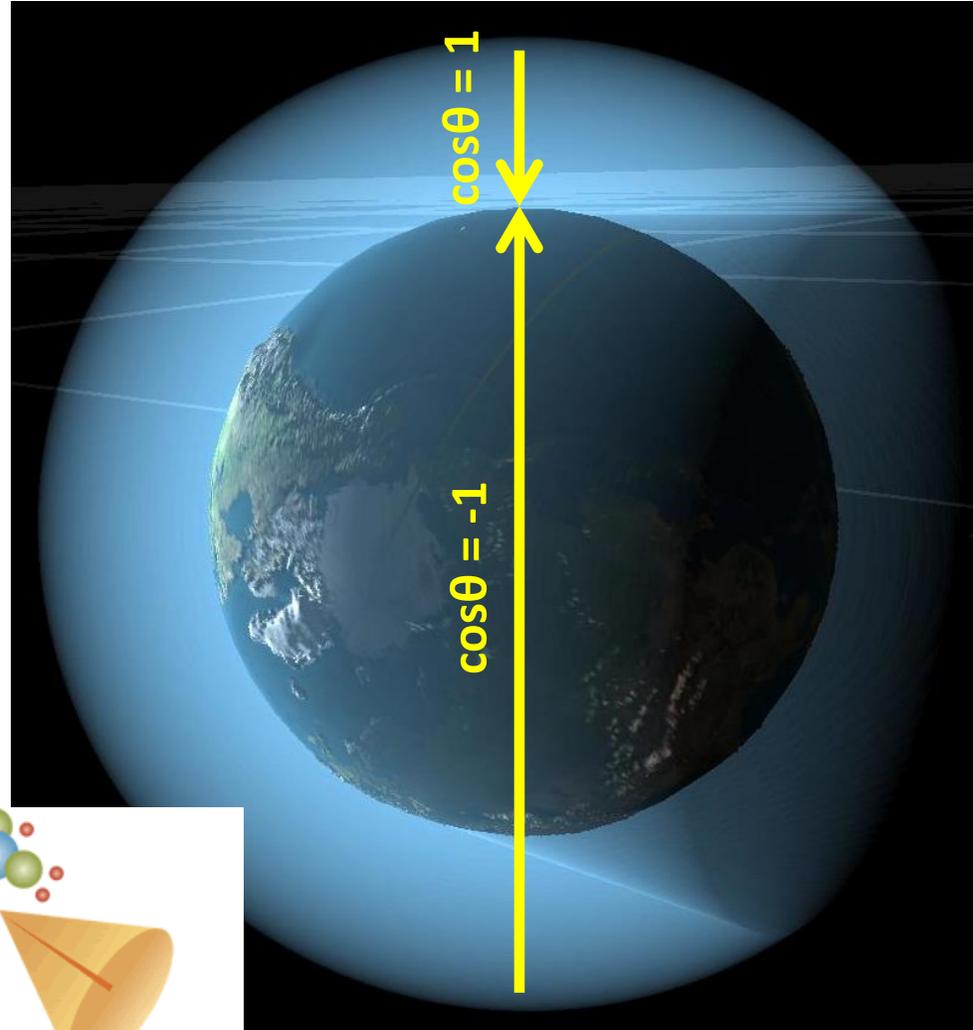
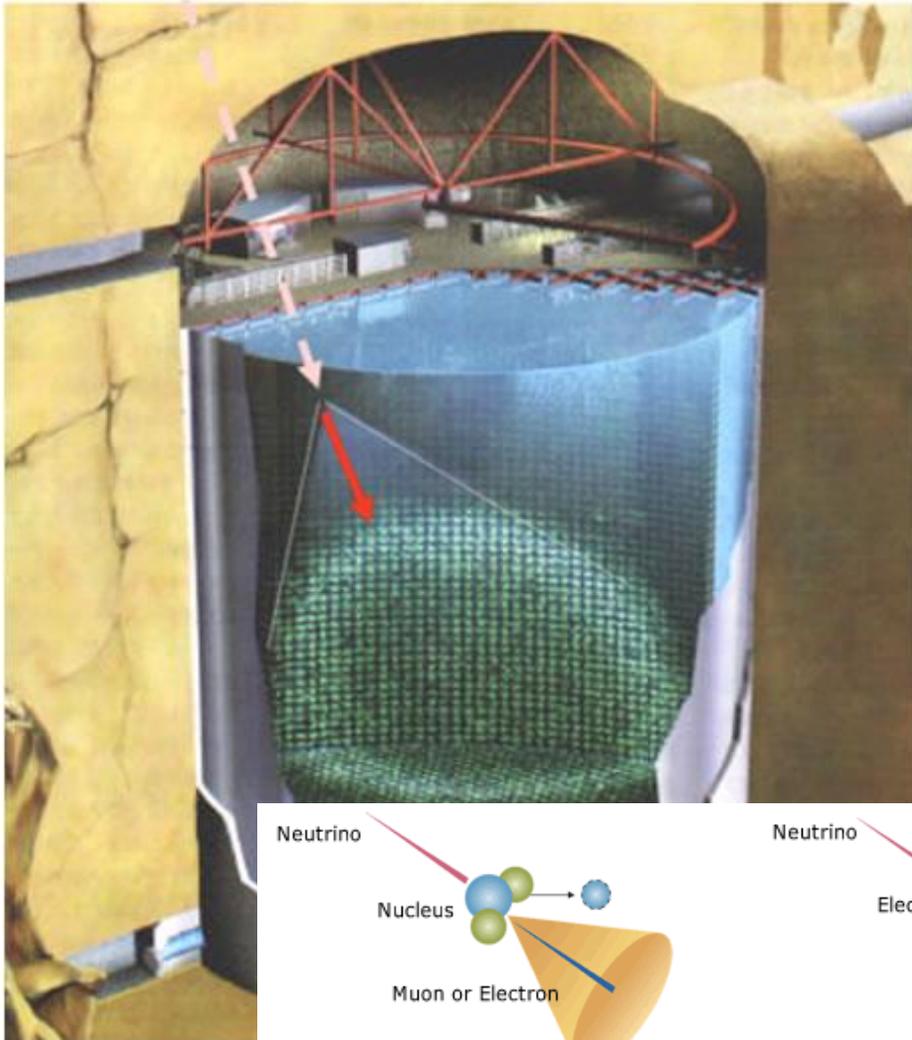
Argon is extracted by circulating the helium from the tank through an argon extraction system. Gas flow is again achieved by a pair of eductors in the tank system, and they maintain a flow rate of 310 liters per minute through the argon extraction system. The tetrachloroethylene vapor is removed by a condenser at  $-40^\circ\text{C}$  followed by a bed of molecular sieve adsorber at room temperature. The helium then passes through a charcoal bed at  $77^\circ\text{K}$  to adsorb the argon, and is finally returned to the tank. This is

in the mine as indicated in the diagram.

The argon sample adsorbed on the charcoal trap is removed by warming the charcoal while a current of helium is passed through it. The argon and other rare gases from the effluent gas stream are collected on a small liquid-nitrogen-cooled charcoal trap (1 cm diam by 10 cm long). Finally, the gases from this trap are desorbed and heated over titanium metal at  $1000^\circ\text{C}$  to remove all traces of chemically reactive gases. The resulting rare gas contains krypton and xenon in addition to argon. These higher rare gases were dissolved from the atmosphere during exposure of the liquid during the various manufacturing, storage, and transfer operations. Krypton and xenon are much more soluble in tetrachloroethylene than argon, and, therefore, they are more slowly removed from the liquid by sweeping with helium. Since the volume of krypton and xenon in an experimental run is comparable with or exceeds the volume of argon, it is necessary to remove these higher rare gases

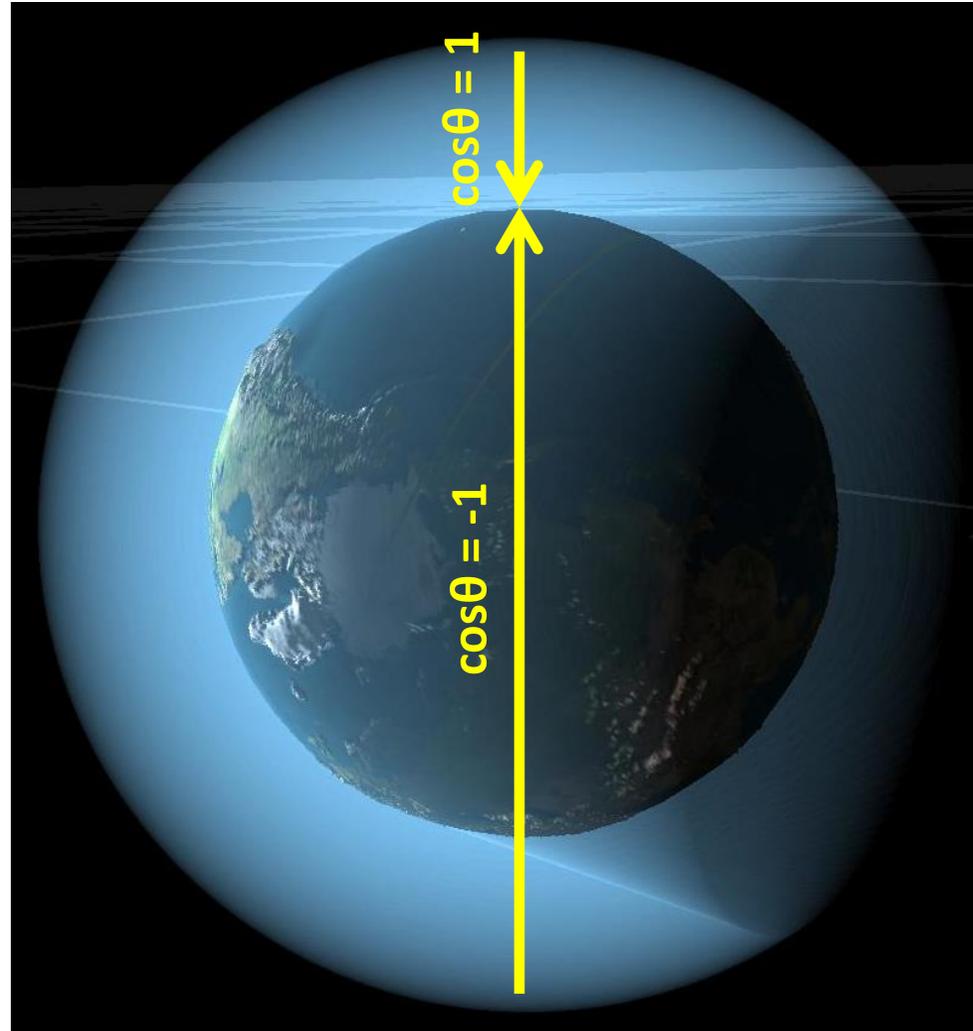
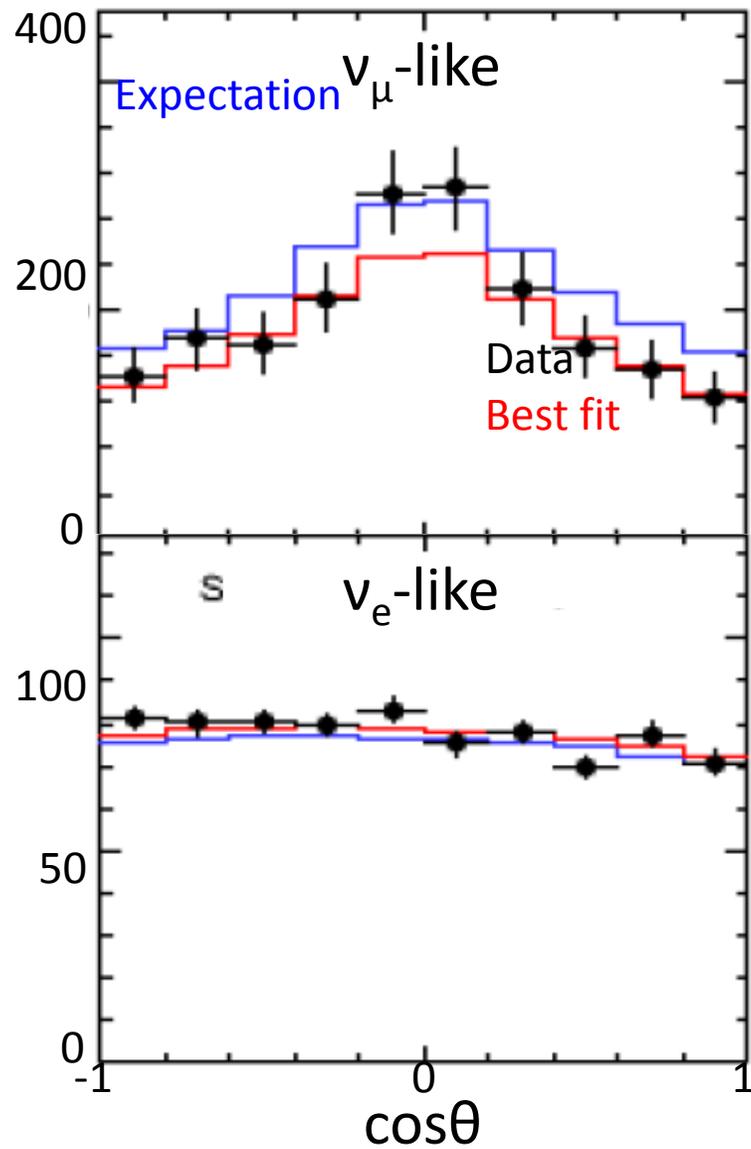


# Super-Kamiokande



The generated charged particle emits the Cherenkov light.

# Super-Kamiokande



# What happens to the neutrinos?

- Solar neutrinos
  - Only about 1/3 of expected neutrino flux observed (electron neutrinos)
  - Depends on uncertainties of modelling of the Sun, detector effects?
- Atmospheric neutrinos
  - Muon neutrino disappearance increases with distance traveled
  - Direct evidence for neutrino disappearance
- What happens to the neutrinos?
  - Perhaps the neutrinos are decaying?
  - Need a mechanism for flavour change and a complete set of measurement for all flavours

# Bruno Pontecorvo

## B. PONTECORVO

Joint Institute for Nuclear Research

Submitted to JETP editor October 19, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 247-249  
(January, 1958)

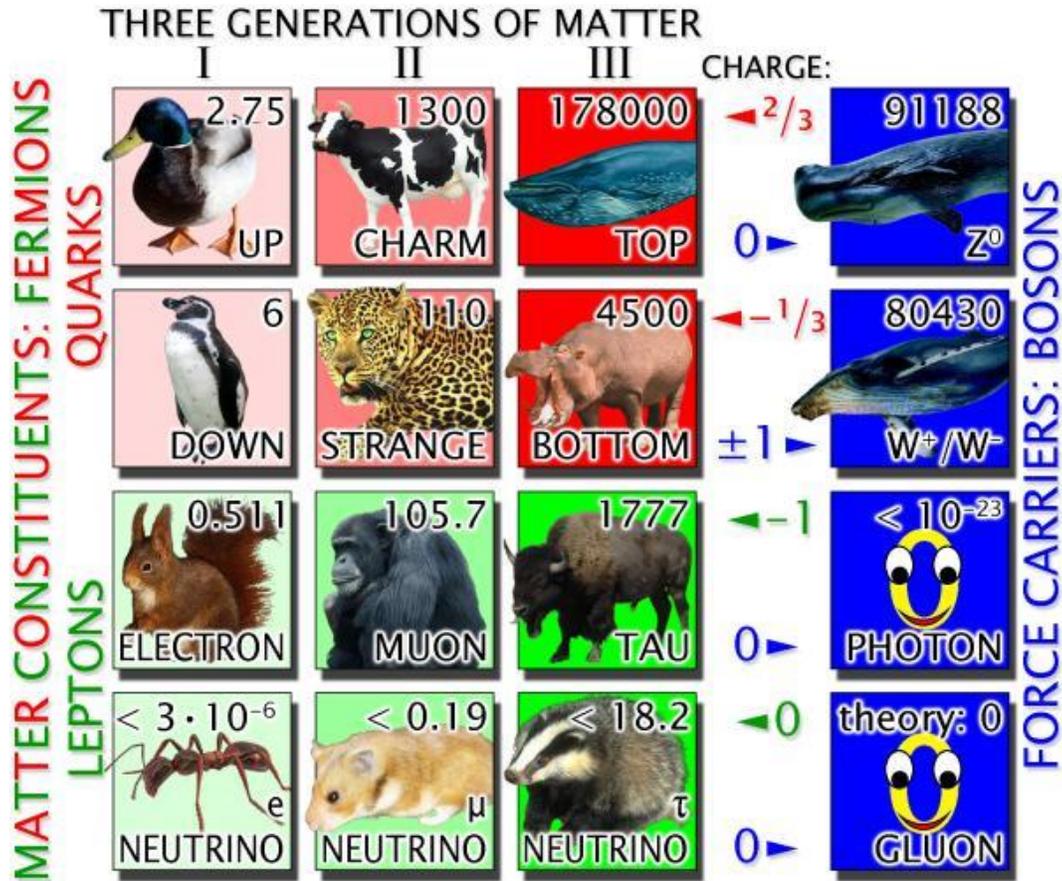
RECENTLY the question was discussed<sup>1</sup> whether there exist other “mixed” neutral particles beside the  $K^0$  mesons,<sup>2</sup> i.e., particles that differ from the corresponding antiparticles, with the transitions between particle and antiparticle states not being strictly forbidden. It was noted that the neutrino might be such a mixed particle, and consequently there exists the possibility of real neutrino  $\rightleftharpoons$  antineutrino transitions in vacuum, provided that lepton (neutrino) charge<sup>3</sup> is not conserved. In the present note we make a more detailed study of this possibility, in which interest has been renewed owing to recent experiments dealing with inverse beta processes.



Бруно Понтекорво

- Concept of flavour not known at the time
- Pontecorvo hypothesized that the neutrinos oscillated between particle and anti-particle states.

# Neutrinos – flavour and mass

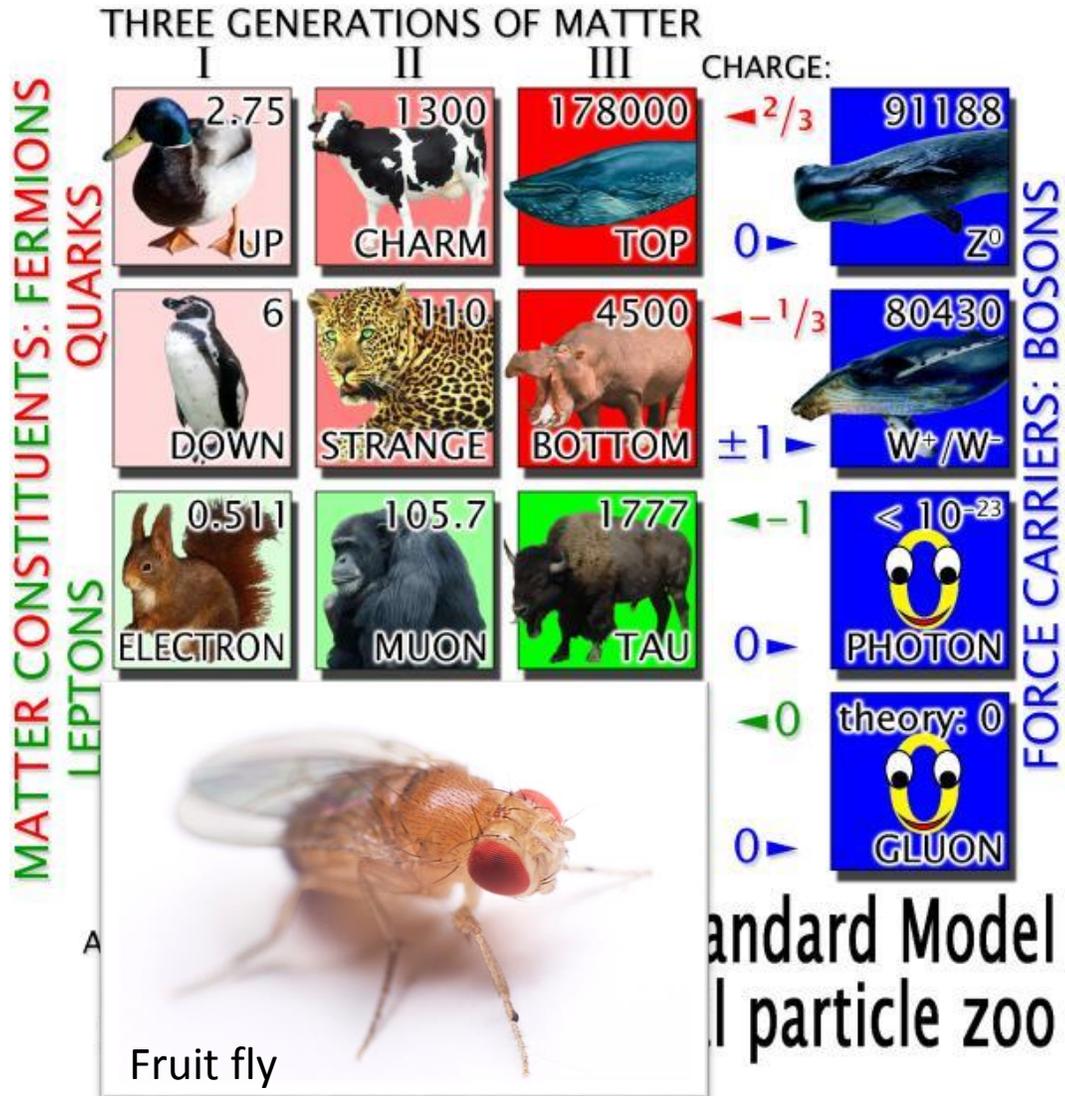


ALL MASSES IN MEV;  
ANIMAL MASSES  
SCALE WITH  
PARTICLE MASSES  
Fruit fly

## The Standard Model fundamental particle zoo

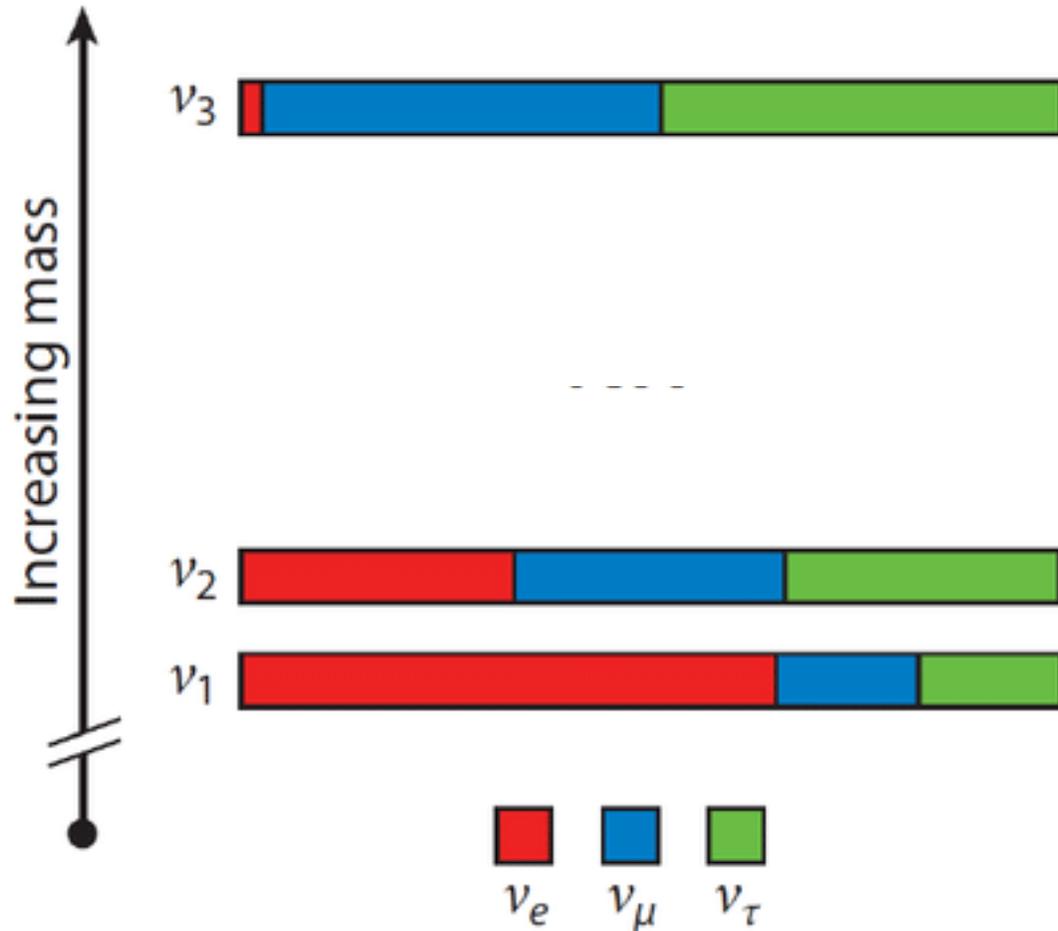
- Neutrinos are special:
  - their masses are much smaller than all other particle masses
  - but they are not zero (as we believed for a long time)
- Neutrino masses are not (directly) created by the Higgs boson – something different going on.
- Their small masses make them truly quantum mechanical objects.

# Neutrinos and mass



- Neutrinos are special:
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- Their small masses make them truly quantum mechanical objects.

# Mass and Flavour



- The neutrino “particles” with *masses* are not the same as the “*flavour*” neutrinos.
- Each neutrino particle (labelled 1,2,3) is made up of a different combination of flavour.
- Property depends on type of measurement.

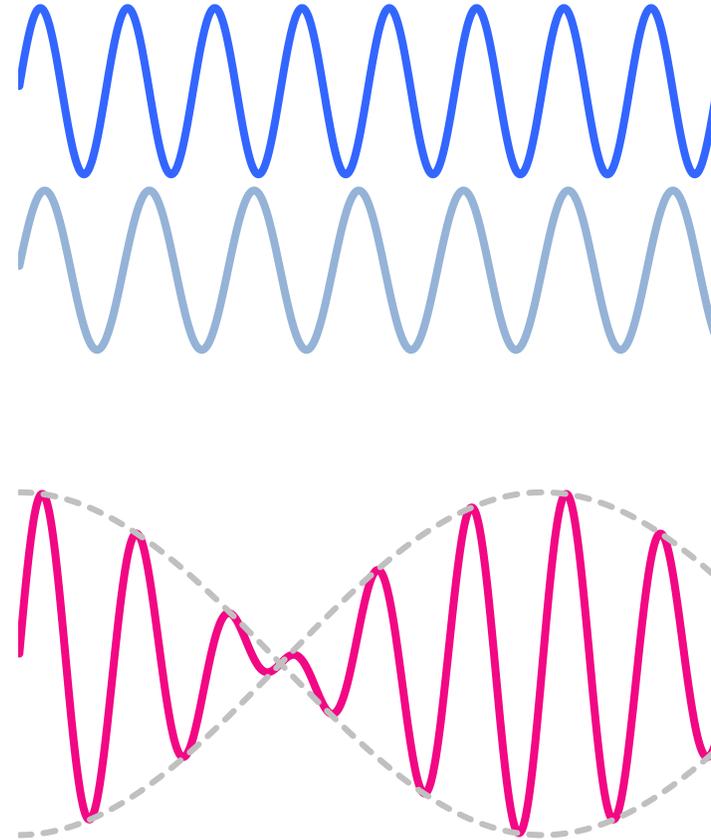
# Neutrinos are truly quantum

- An electron-neutrino emitted by the Sun is a combination of three different masses.
- The corresponding waves travel with different speeds.
- The waves oscillate at different frequencies and therefore interfere.
- The flavours of the neutrino thus change back and forth as the waves interfere constructively and destructively.

Credit: D. Schmidt, P. Vahle

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Credit: D. Schmidt, P. Vahle

“If you think you understand quantum mechanics, you don't understand quantum mechanics.”

Attributed to Richard Feynman



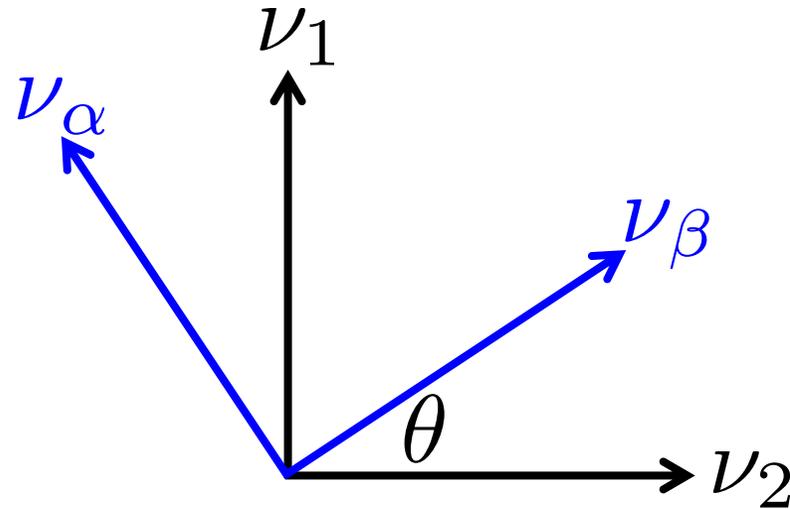
# Two-flavour oscillations

Flavour states

“Rotation Matrix”

Mass states

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$



$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle$$

# Two-flavour oscillations

$$|\nu(t)\rangle = e^{i(E_1 t - pL)} \cos(\theta) |\nu_1\rangle + e^{i(E_2 t - pL)} \sin(\theta) |\nu_2\rangle$$

plane wave

$$\langle \nu_\beta | \nu(t) \rangle = \sin(\theta) \cos(\theta) (e^{i(E_2 t - pL)} - e^{i(E_1 t - pL)})$$

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$$E \approx p + \frac{m_i^2}{2E} \quad \text{and} \quad t = \frac{L}{c} \quad \text{ultra-relativistic}$$

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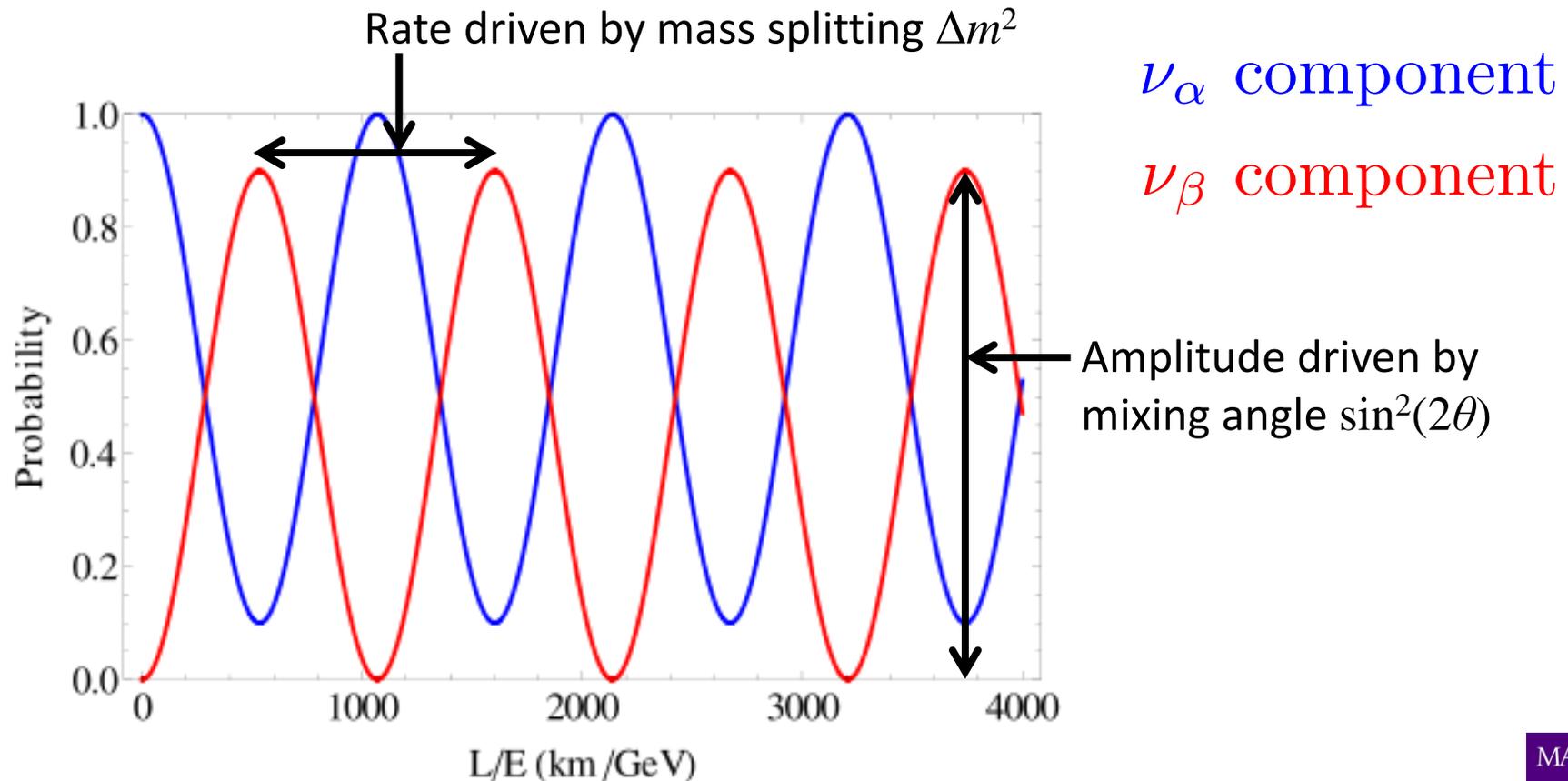
$$E \approx p + \frac{m_i^2}{2E} \quad \text{and} \quad t = \frac{L}{c} \quad \text{ultra-relativistic}$$

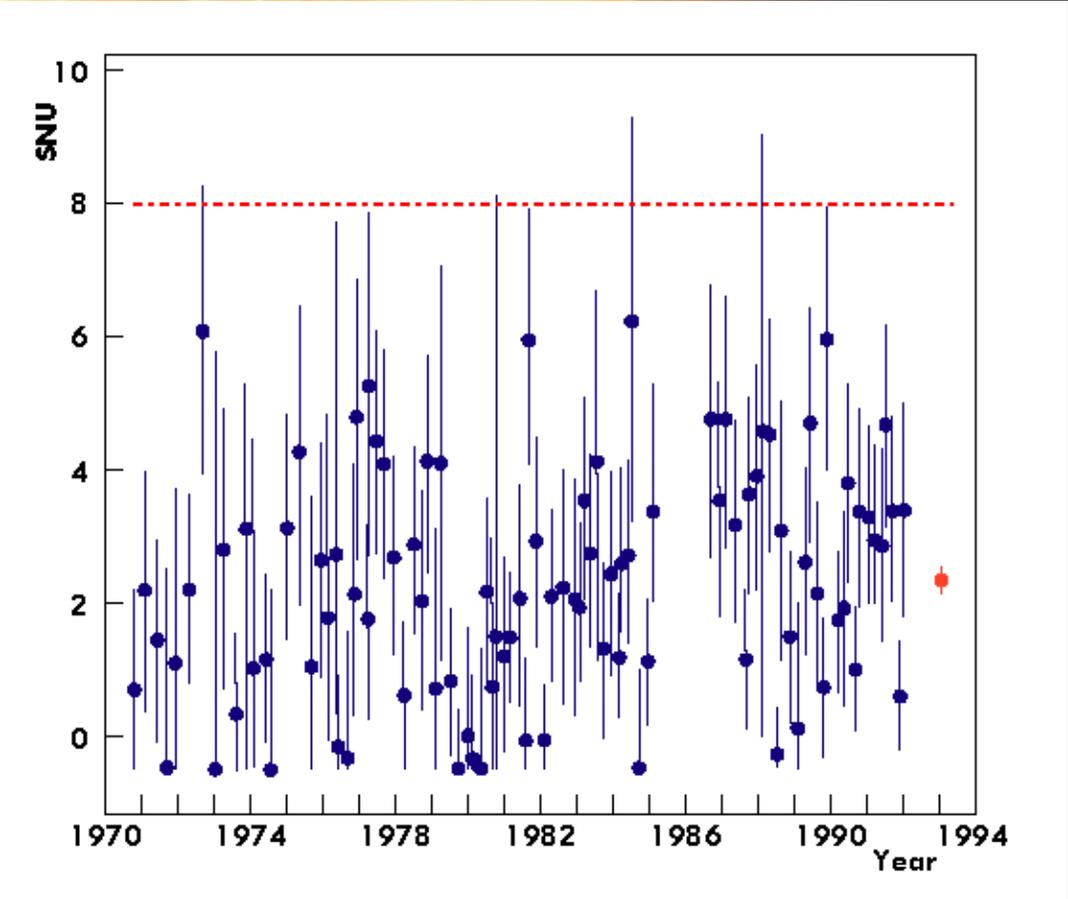
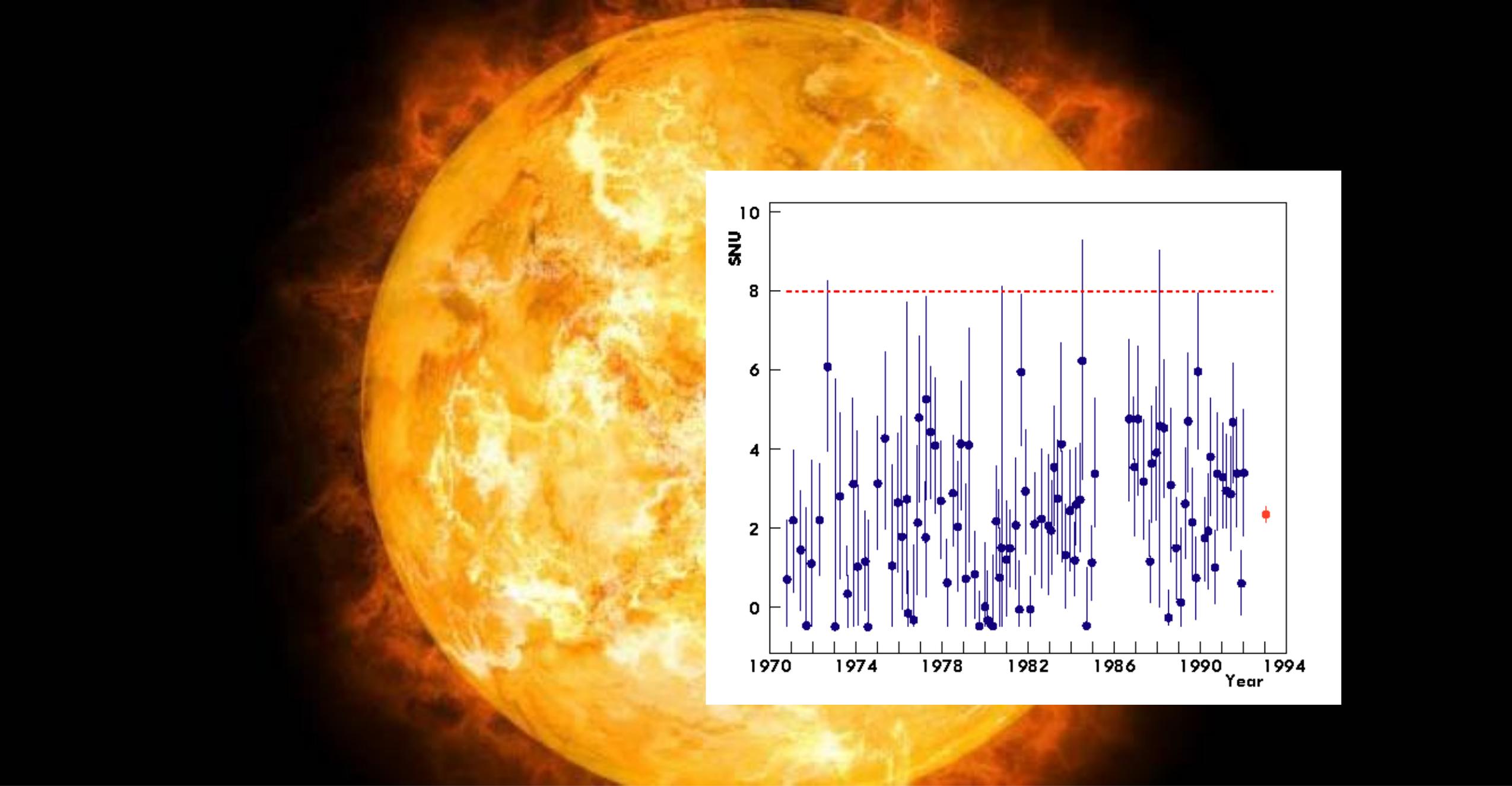
$$\langle \nu_\beta | \nu(t) \rangle = \sin(\theta) \cos(\theta) (e^{i \frac{m_2^2 L}{2E}} - e^{i \frac{m_1^2 L}{2E}}) = \sin(\theta) \cos(\theta) e^{i \frac{\Delta m_i^2 L}{2E}}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \langle \nu_\beta | \nu(t) \rangle^2 = \sin^2(2\theta) \sin^2\left(\frac{\Delta m_i^2 L}{2E}\right)$$

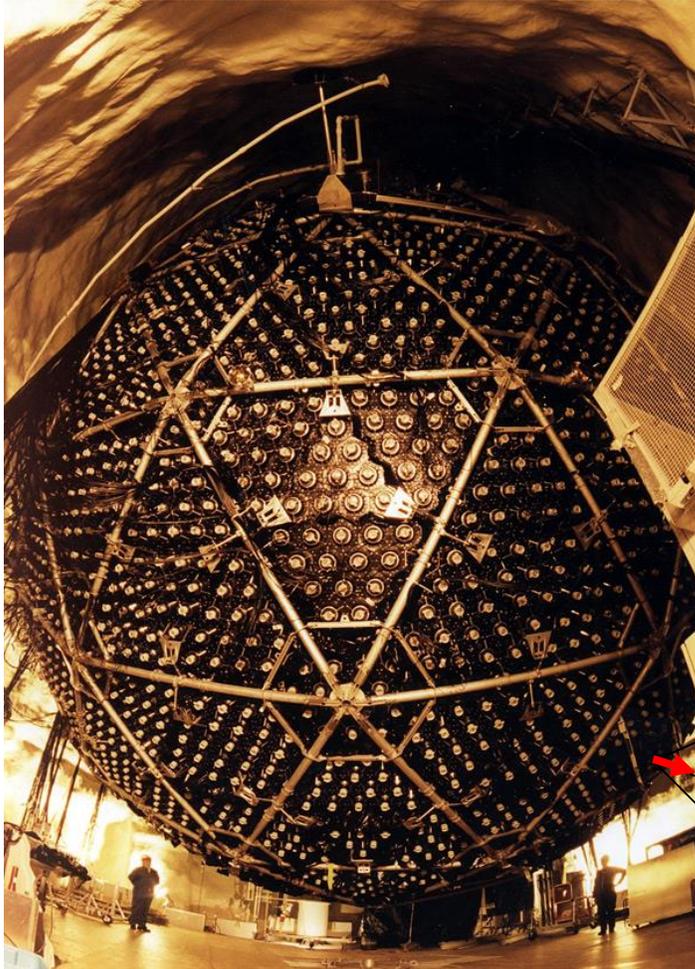
# Two-flavour oscillations

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2(2\theta) \sin^2 \left( 1.27 \frac{\Delta m_{21}^2 [\text{eV}^2] L [\text{km}]}{E [\text{GeV}]} \right)$$

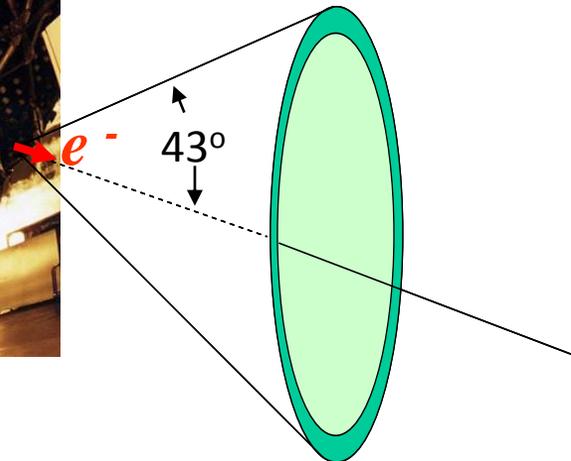




# SNO Detector

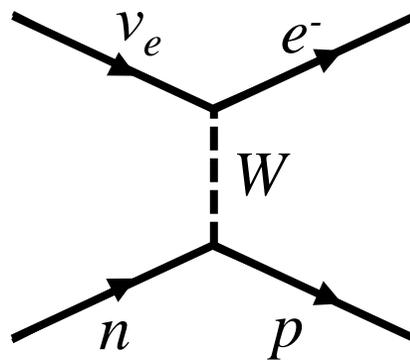
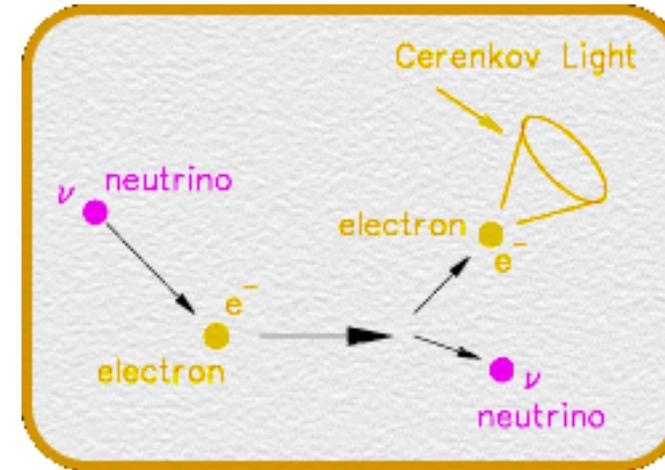
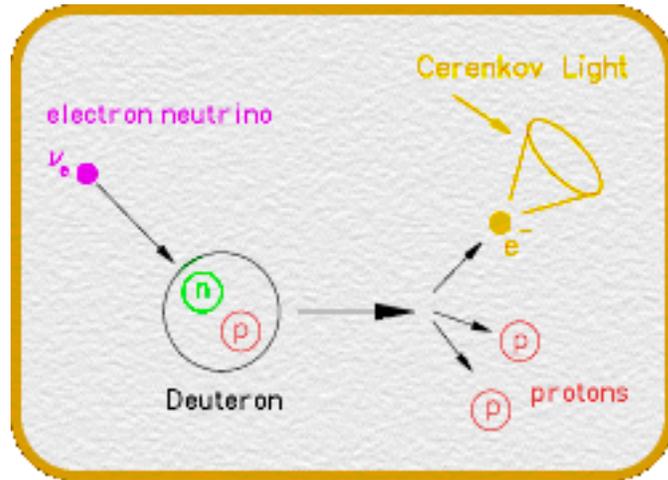


Cherenkov cone

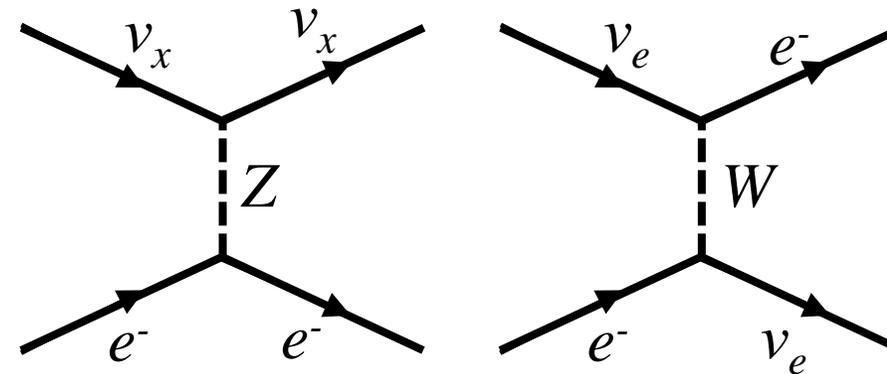


- Davis experiment only showed that some of the electron-neutrinos went missing.
- Need a detector that can measure different neutrino flavours to confirm the 3-flavour oscillation model.
- SNO detector – filled with heavy water - is sensitive to Cherenkov light from scattered electrons and from photons produced when neutrons are captured.

# Neutrino interactions in SNO

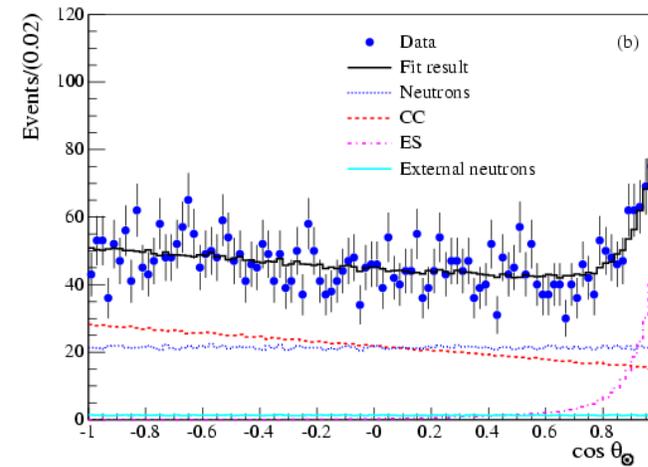
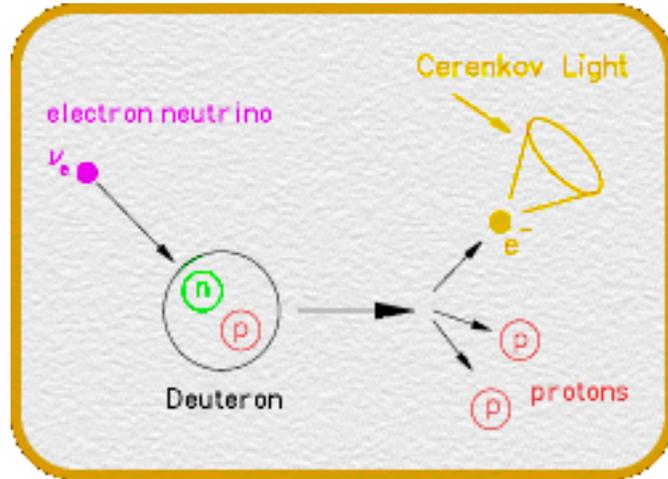


Charged current interaction:  
Sensitive only to  $\nu_e$

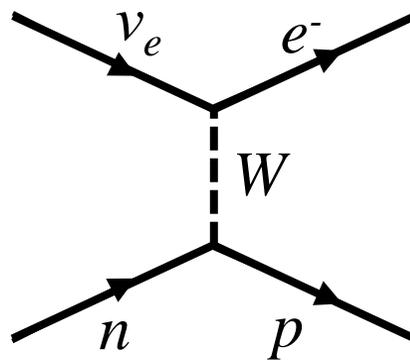


Elastic scattering:  
Sensitive to charged and neutral current.  
 $\nu_e$  dominate by a factor of 6

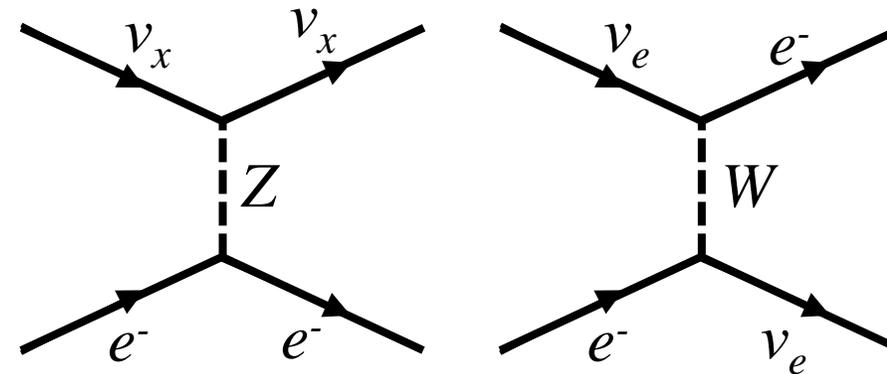
# Neutrino interactions in SNO



Sensitive to neutrino direction



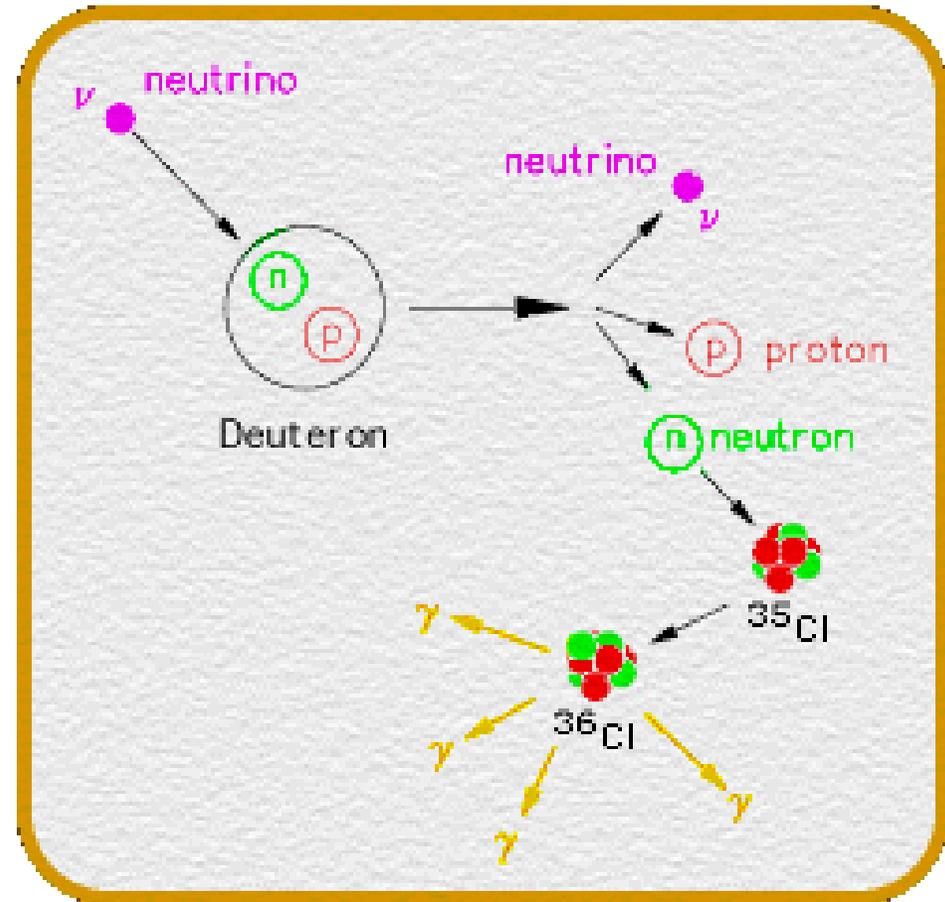
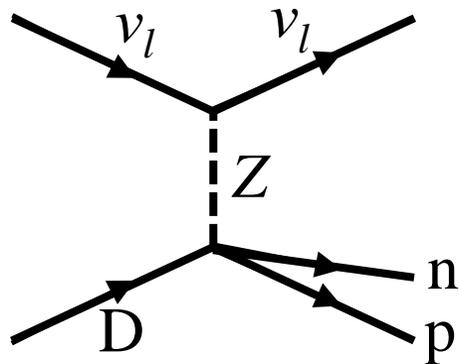
Charged Current interaction (CC):  
Sensitive only to  $\nu_e$



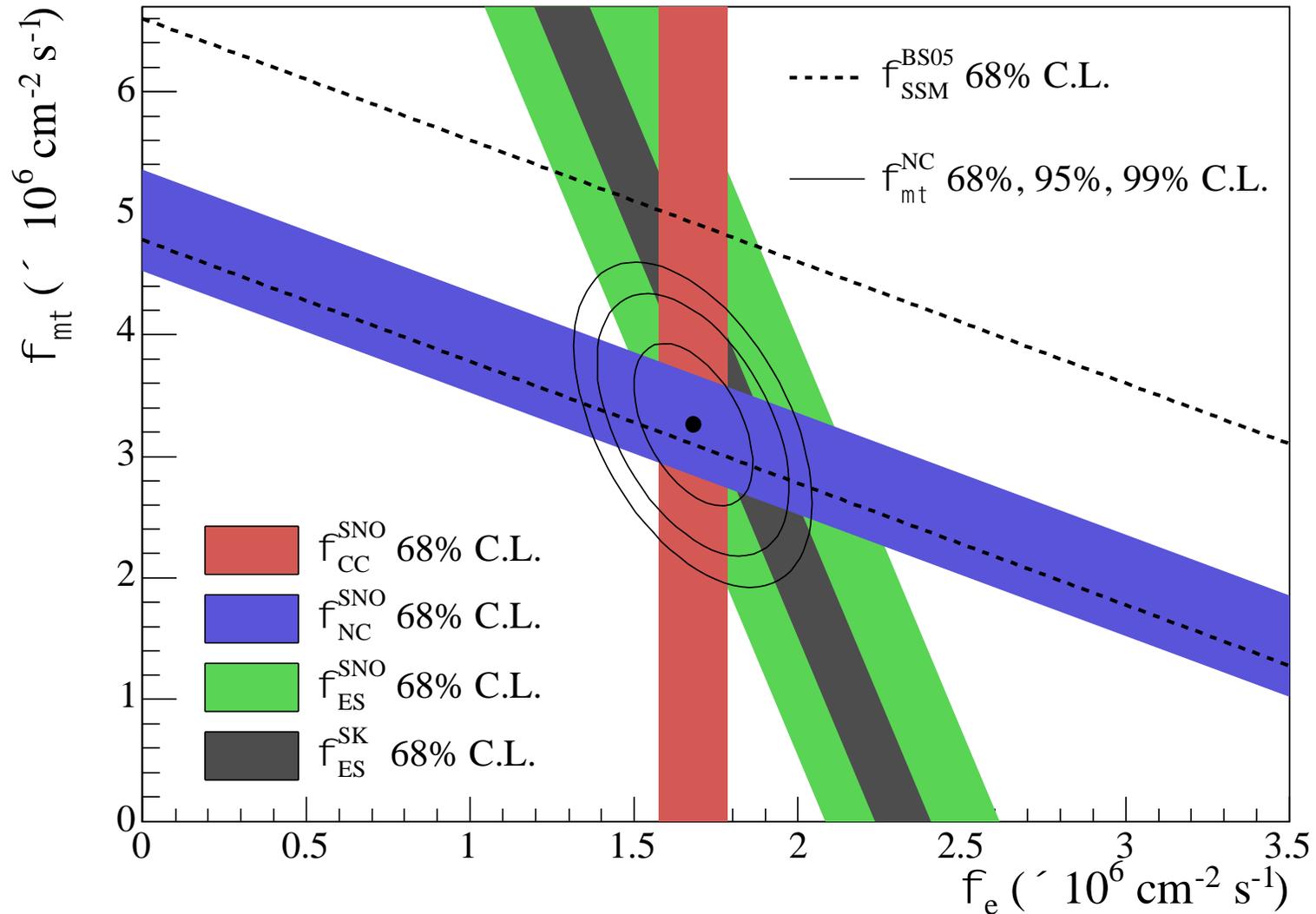
Elastic Scattering (ES):  
Sensitive to charged and neutral current.  
 $\nu_e$  dominate by a factor of 6

# The salt phase

- In its second phase, SNO was filled with two tons of NaCl
- $^{35}\text{Cl}$  has a high neutron absorption cross, releases  $\gamma$  of up to 8.6 MeV
- Neutral current interaction equally sensitive to all three neutrino flavours



# SNO demonstrates flavour change



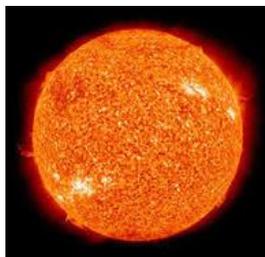
# PMNS Matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

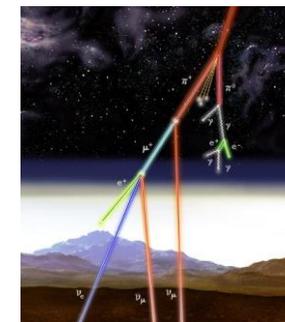
Pontecorvo–Maki–Nakagawa–Sakata

$$c_{ij} = \cos \theta_{ij}; s_{ij} = \sin \theta_{ij}$$

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$



$\theta_{13}$ : mixes  $\nu_e$  with  $\nu_3$   
 $\delta$ : complex phase

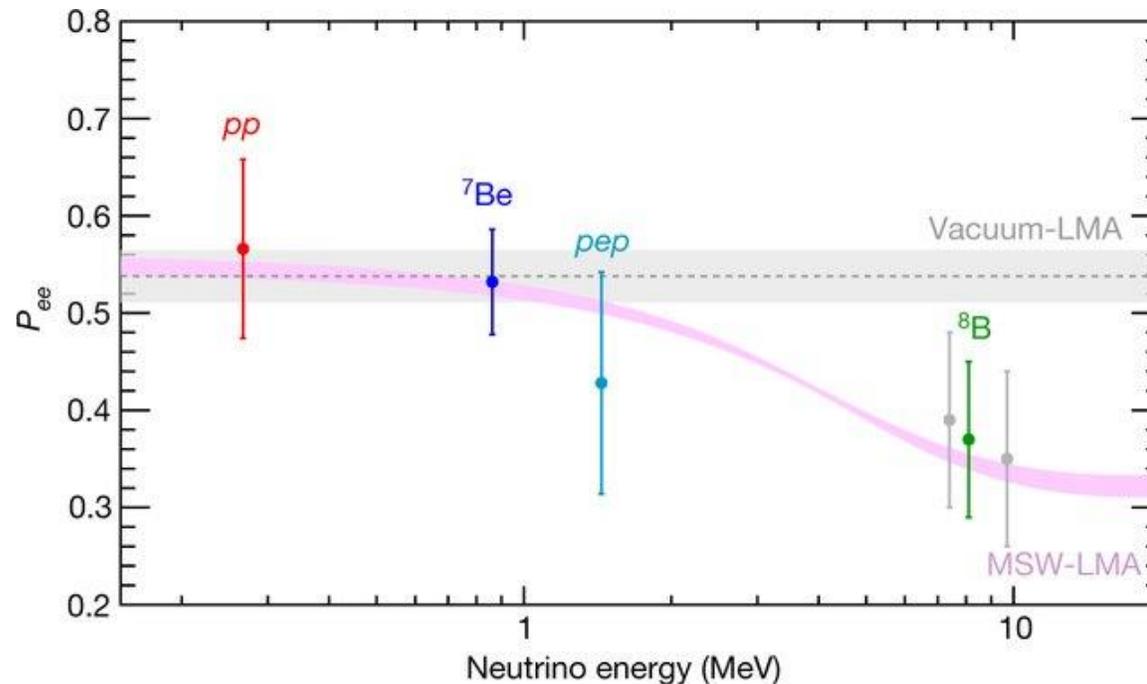


- $\theta_{12}$ : “solar mixing angle”
- mixes  $\nu_e$  with  $\nu_1$  and  $\nu_2$

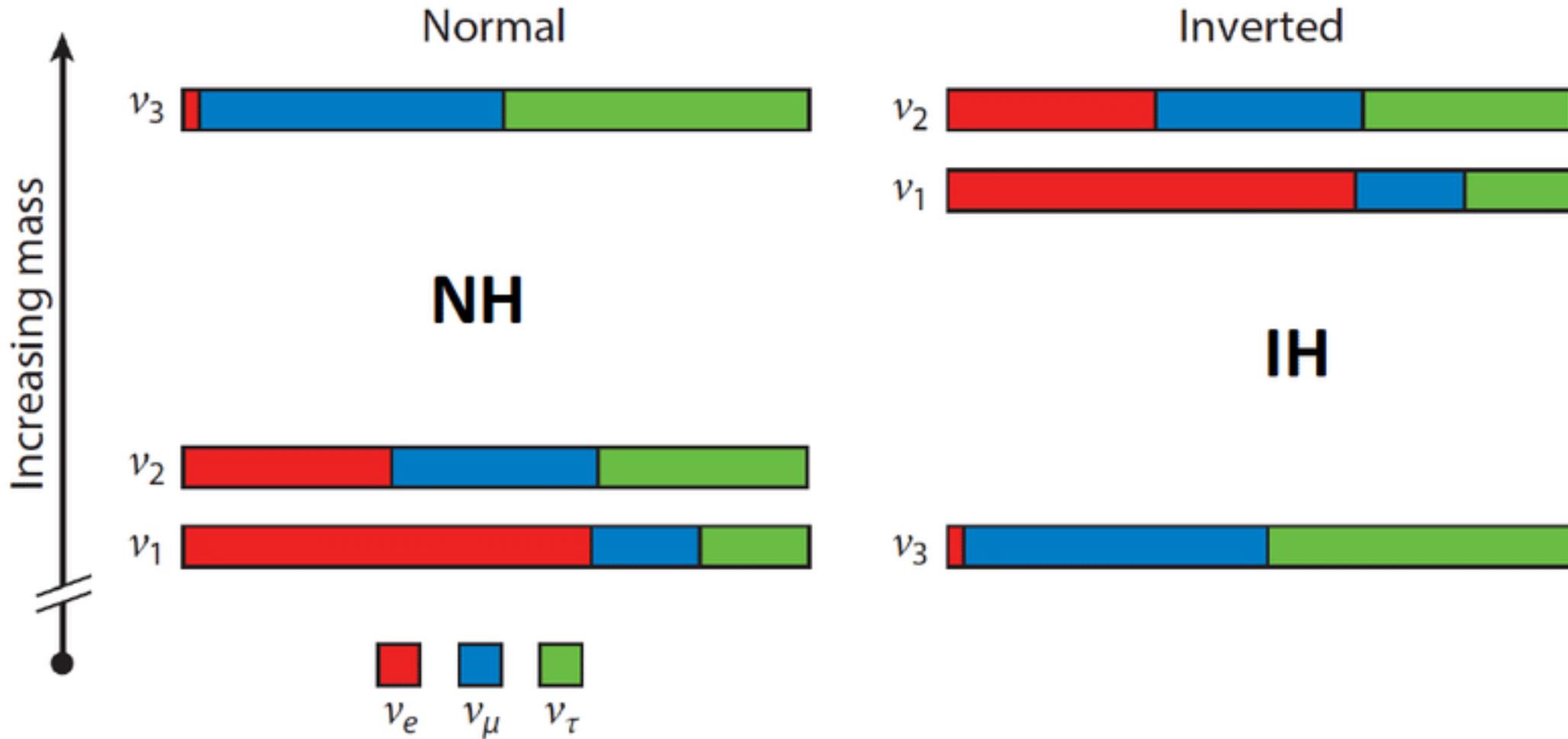
- $\theta_{23}$ : “atmospheric mixing angle”
- mixes  $\nu_\mu$  with  $\nu_\tau$

# The MSW Effect

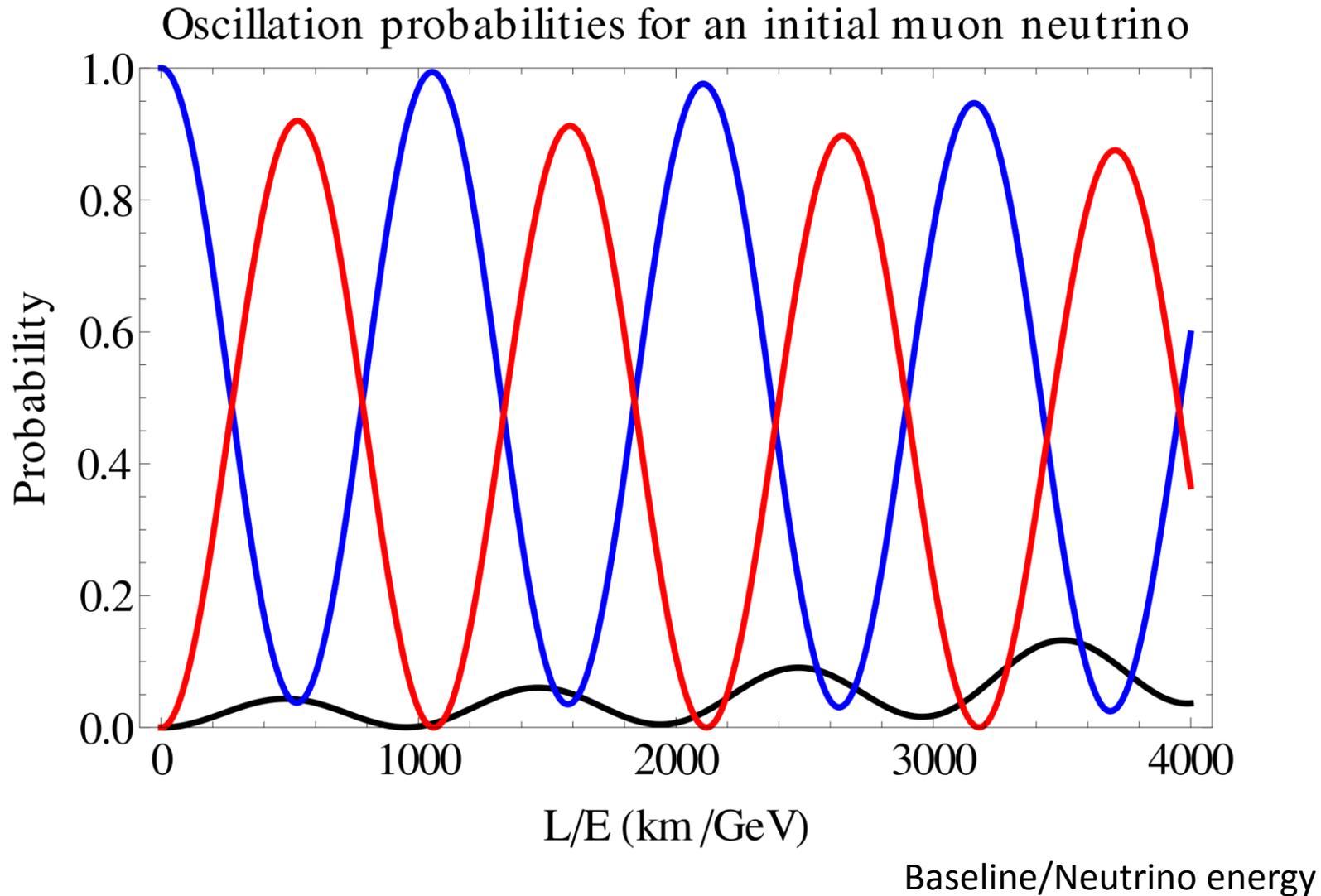
- When neutrinos travel over long distances through dense matter (Sun, Earth), their propagation is modified through coherent forward scattering off particles (electrons)
- This effect modifies the flavour oscillation probability (Mikhaev, Smirnov, Wolfenstein)
- MSW effect can be enhanced through a resonance condition
- Neutrino and anti-neutrino oscillation probabilities affected differently (not related to CPV)
- MSW effect depends on the sign of  $\Delta m^2$



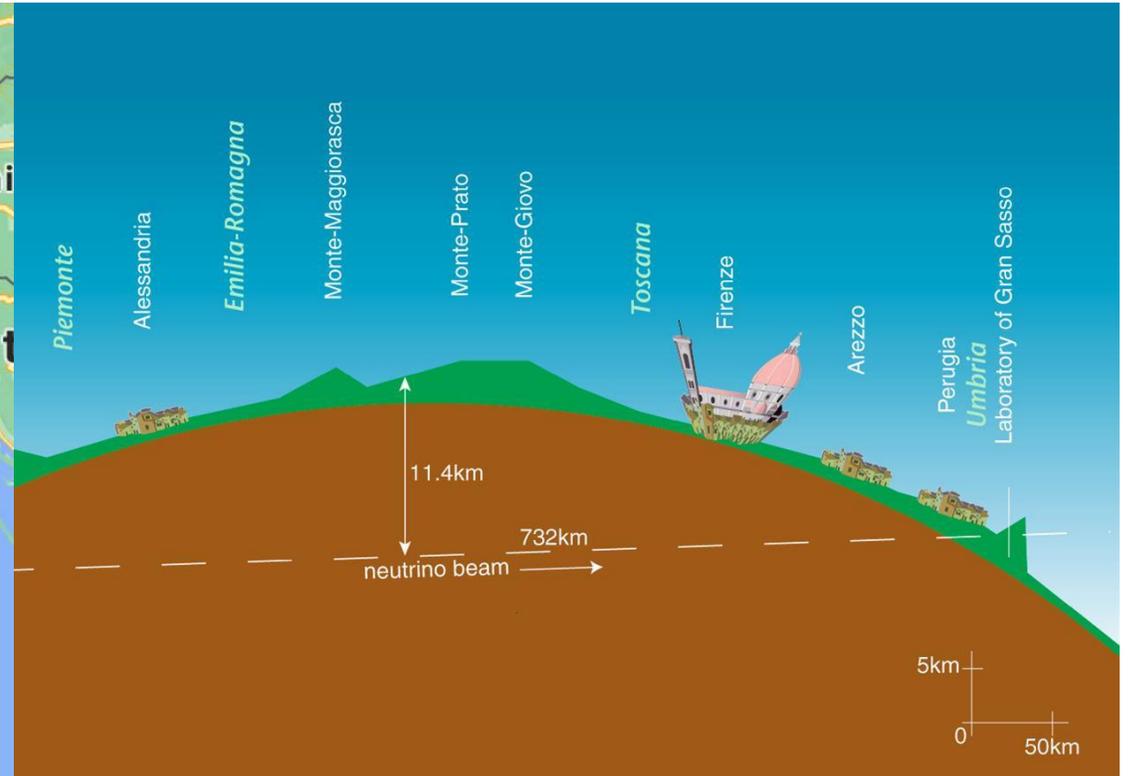
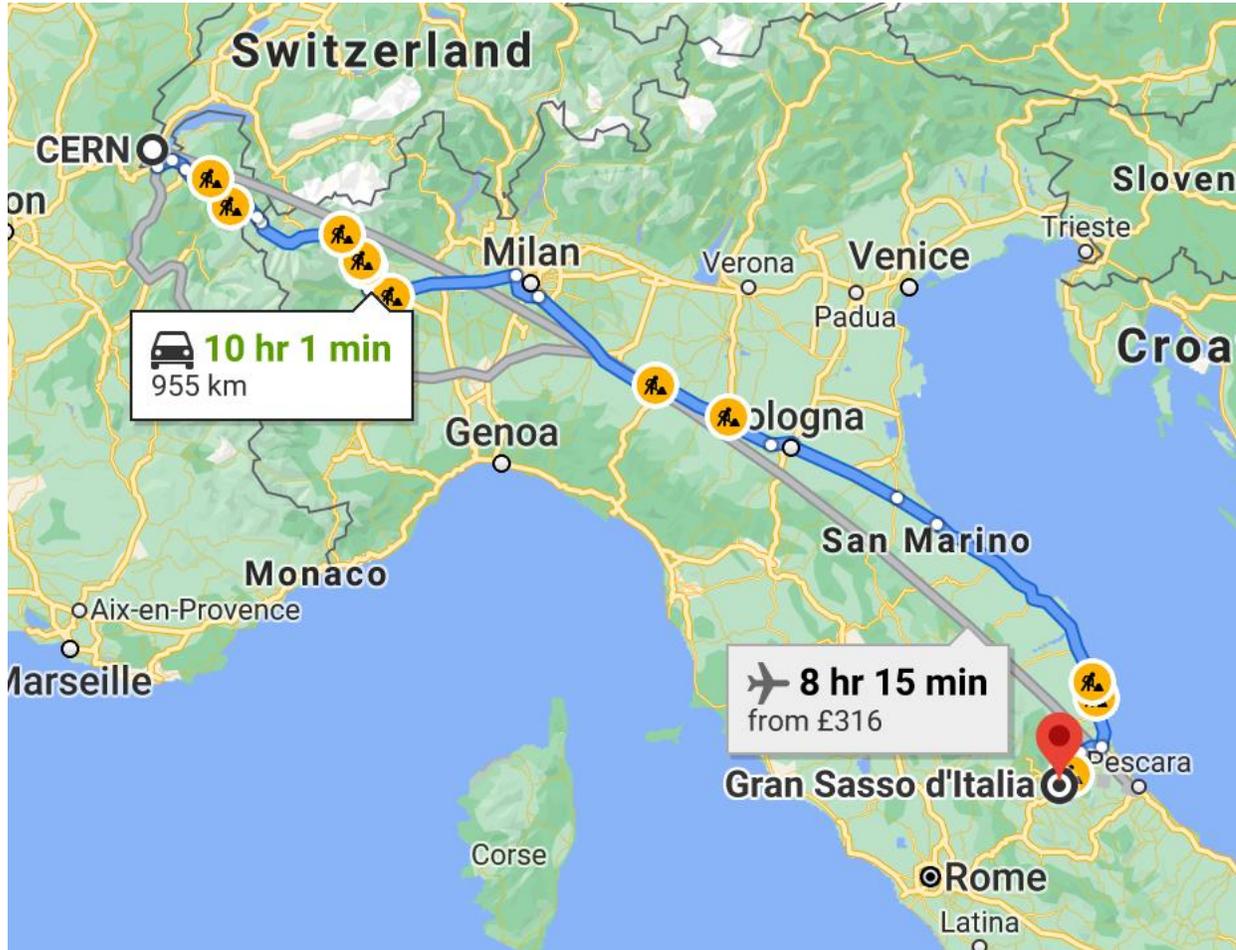
# Solar mixing fixes sign of $\Delta m_{21}$



# Where are the tau neutrinos?

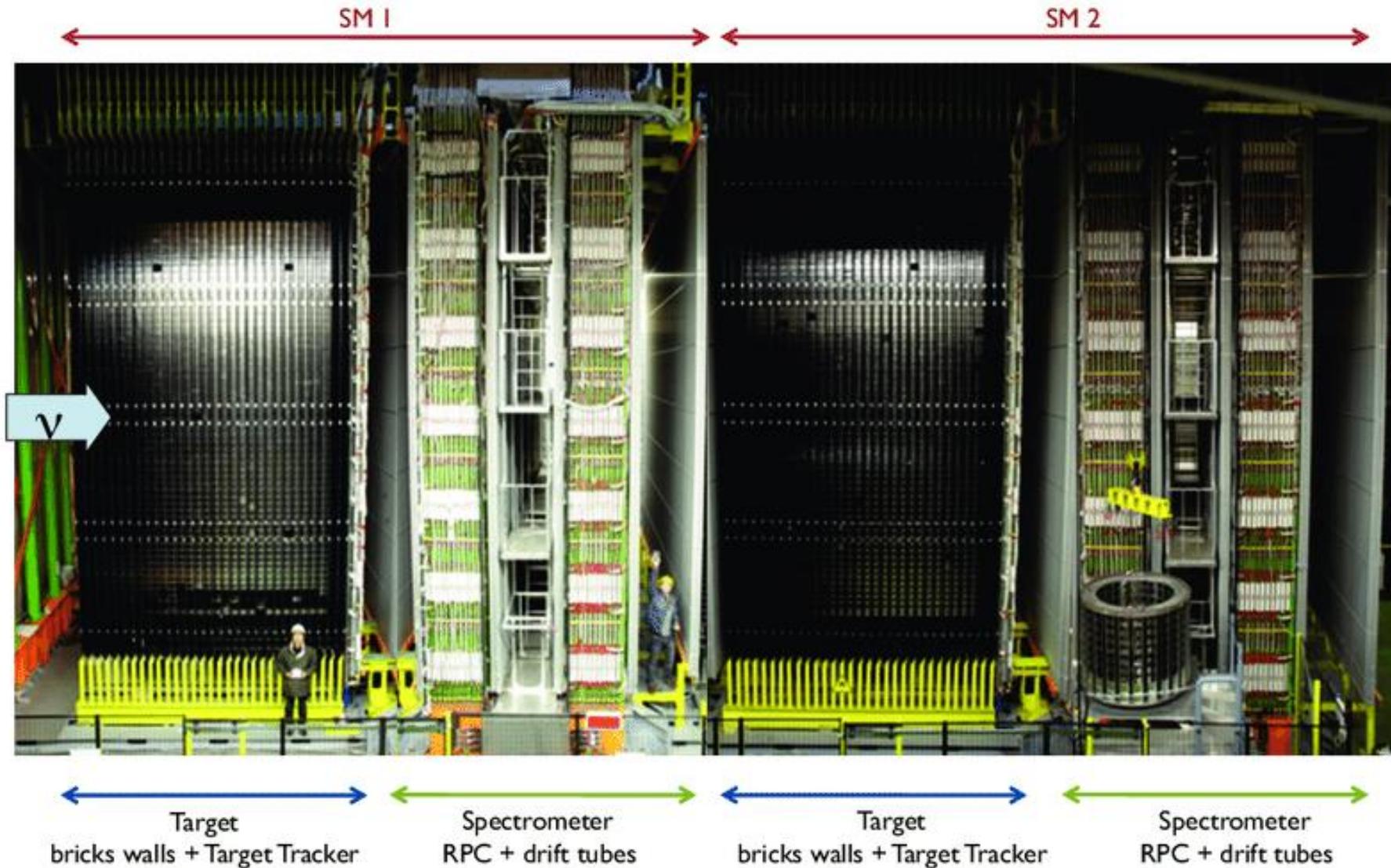


# CNGS $\nu_\mu$ beam from CERN to Gran Sasso



$$L/E \approx 732 \text{ km}/17 \text{ GeV} = 43$$

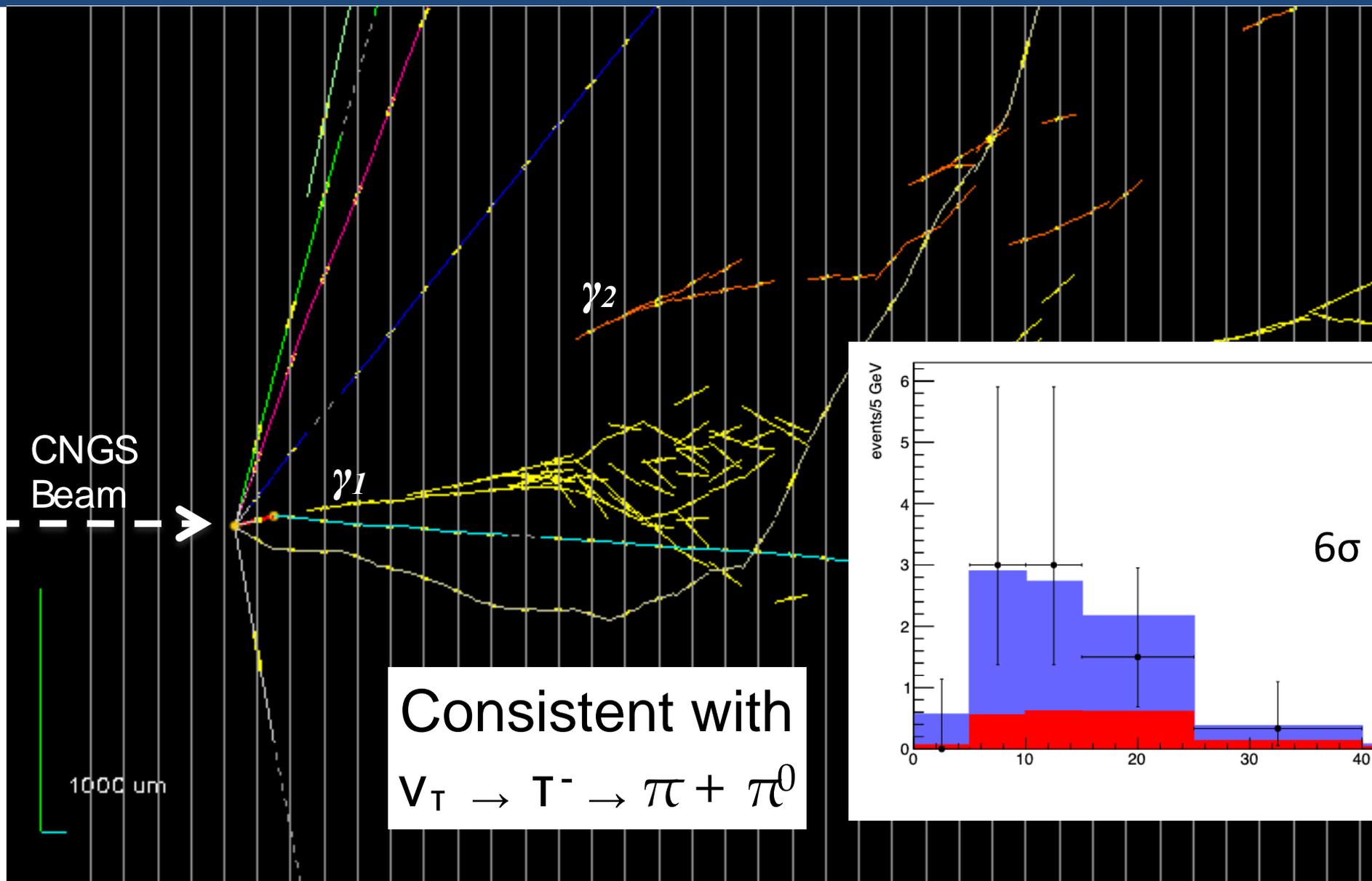
# OPERA experiment at Gran Sasso



Target Tracker:

75,000 emulsion/lead modules or "bricks"

# First observation of $\nu_\tau \rightarrow \nu_\tau$ appearance



Consistent with  
 $\nu_\tau \rightarrow \tau^- \rightarrow \pi + \pi^0$

