Neutrino Oscillations

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Neutrino Oscillations and the PMNS Matrix
What are we made of?

nucleus
protons
neutrons
electrons
What are we made of?

Proton

2 quarks up
1 quark down

Neutron

1 quark up
2 quarks down
Beta Decay

Neutrons are not stable

“neutron→ proton + electron”


Fig. 5. Energy distribution curve of the beta-rays.
Liebe Radioactive Damen und Herren,

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut der Eidg. Technischen Hochschule Zürich

Liebe Radioaktive Damen und Herren,

Dear Radioactive Ladies and Gentlemen,

...I have hit upon a desperate remedy...

...so far I do not dare to publish anything about this idea, and trustfully turn first to you, dear radioactive people, with the question of how likely it is to find experimental evidence for such [a particle]...

--Signed, W. Pauli
Dear Radioactive Ladies and Gentlemen,

- Enrico Fermi (re-)named the new particle ‘neutrino’.
- Discovered in $\beta$ decays from reactors in 1956:

$$\bar{\nu}_e + p \rightarrow n + e^+$$
Hydrogen Bubble chamber, Argonne, 1970

- muon
- proton
- pion
- (muon)-neutrino
Charged and neutral leptons come in pairs.

"Flavour" of outgoing charged lepton defines neutrino flavour.

Neutrinos have three flavours (e, μ, τ).
The “Standard Model”

3 families

• Neutrinos are electrically neutral
• Four forces:
  - Strong force (gluon g)
  - Weak force (W and Z boson)
  - Electromagnetic force (photon $\gamma$)
  - Gravity (graviton)
• Neutrinos only feel the weak force (and gravity)
• Even though neutrinos appear “ghost” like and ephemeral, they play an important role in the Universe.
Neutrinos are the most abundant matter particle: for every proton/neutron/electron the Universe contains a billion neutrinos from the Big Bang.
10^{38} neutrinos per second are produced by fusion processes in the Sun (with a flux of \sim 10^{11}/\text{cm}^2/\text{sec at the Earth})
In a box of 1 cubic meter, somewhere in the Universe, we have on average

• 10 protons
• 300 million neutrinos

left over from the Big Bang.

Neutrinos are the most abundant *matter* particles in the Universe
Neutrino Sources (nuclear processes)

- Nuclear processes are typically the source of electron-neutrinos.
- Typical energies 1-10 MeV
- Discovery of electron-neutrino by Cowan and Reines at the Savannah River power plant in 1956.

\[
p + e^- \rightarrow n + \nu_e \\
n \rightarrow p + e^- + \bar{\nu}_e \\
p + p \rightarrow ^2H + e^+ + \nu_e \\
\]

\[
\begin{align*}
\frac{238}{92}U & \rightarrow \frac{206}{82}Pb + 8\alpha + 6e^- + 6\bar{\nu}_e \\
\frac{235}{92}U & \rightarrow \frac{207}{82}Pb + 7\alpha + 4e^- + 4\bar{\nu}_e \\
\frac{232}{90}Th & \rightarrow \frac{208}{82}Pb + 6\alpha + 4e^- + 4\bar{\nu}_e
\end{align*}
\]
Fusion produces electron-neutrinos in the Sun


pp

\[ p^+ + p^+ \rightarrow ^2\text{H} + e^+ + \nu_e \] 99.77%

\[ ^2\text{H} + p^+ \rightarrow ^3\text{He} + \gamma \] 84.92%

\[ ^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma \] 15.08%

\[ ^7\text{Be} + e^- \rightarrow ^7\text{Li} + \nu_e \] 99.9%

\[ ^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2p^+ \] ppl

\[ ^7\text{Li} + p^+ \rightarrow ^4\text{He} + ^4\text{He} \] ppII

\[ ^7\text{Be} + p^+ \rightarrow ^8\text{B} + \gamma \] 0.1%

\[ ^8\text{B} \rightarrow ^8\text{Be}^* + e^+ + \nu_e \] ppIII

\[ ^8\text{Be}^* \rightarrow ^4\text{He} + ^4\text{He} \]

Source: Manchester
• Dropping an object turns gravitational 'potential energy' into 'kinetic energy' when an object falls.

• As the star falls inward the gravitational energy has to go somewhere:

\[ p + e^- \rightarrow n + \nu_e \]

• Neutrinos only interact weakly, so easiest for them to escape.

• About 99% of the huge binding energy of the neutron star is shed within about 10 seconds in the form of neutrinos.
Neutrino Sources (atmospheric/accelerator)

- Pion/kaon production and decay are main source of accelerator and atmospheric neutrinos.

\[ p + N \rightarrow \pi^\pm + X \]
\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]
\[ \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \]

- Typical energies \( \approx \) GeV, ratio of \( \nu_\mu : \nu_e = 2:1 \).

- Discovery of muon-neutrino by Ledermann, Schwartz, Steinberger at Brookhaven in 1962.
Neutrino sources, flux, and cross section

C. Spiering, arXiv:1207.4952
The missing solar neutrinos

The Ray Davis experiment, Homestake Mine, South Dakota

Filled with 390,000 litres of cleaning fluid (C₂Cl₄)

"Inverse β Decay"

\[ \nu_e + ^{37}\text{Cl} \rightarrow ^{37}\text{Ar} + e^- \]

\[ E_\nu > 0.8 \text{ MeV} \]

~1/3
Filter out argon and search for $^{37}$Ar decay

Detecting ~5 atoms of $^{37}$Ar per day in 390,000 litres of $C_2Cl_4$

Super-Kamiokande

\[ \cos \theta = 1 \]
\[ \cos \theta = -1 \]
Super-Kamiokande

2-flavor analysis update (SK1-4)

\[ \nu_{\mu} \text{-like} \]

\[ \nu_e \text{-like} \]

Expectation
Data
Best fit

\[ \cos \theta = 1 \]

\[ \cos \theta = -1 \]
What happens to the neutrinos?

• Solar neutrinos
  - Only about 1/3 of expected neutrino flux observed (electron neutrinos)
  - Depends on uncertainties of modelling of the Sun, detector effects?

• Atmospheric neutrinos
  - Muon neutrino disappearance increases with distance traveled
  - Direct evidence for neutrino disappearance

• What happens to the neutrinos?
  - Perhaps the neutrinos are decaying?
  - Need a mechanism for flavour change and a complete set of measurement for all flavours
Bruno Pontecorvo

B. PONTECORVO

Joint Institute for Nuclear Research
Submitted to JETP editor October 19, 1957
(January, 1958)

RECENTLY the question was discussed\(^1\) whether there exist other “mixed” neutral particles beside the \(K^0\) mesons,\(^2\) i.e., particles that differ from the corresponding antiparticles, with the transitions between particle and antiparticle states not being strictly forbidden. It was noted that the neutrino might be such a mixed particle, and consequently there exists the possibility of real neutrino \(\rightarrow\) antineutrino transitions in vacuum, provided that lepton (neutrino) charge\(^3\) is not conserved. In the present note we make a more detailed study of this possibility, in which interest has been renewed owing to recent experiments dealing with inverse beta processes.

- Concept of flavour not known at the time
- Pontecorvo hypothesized that the neutrinos oscillated between particle and anti-particle states.
Neutrinos – flavour and mass

- Neutrinos are special:
  - their masses are much smaller than all other particle masses
  - but they are not zero (as we believed for a long time)

- Neutrino masses are not (directly) created by the Higgs boson – something different going on.

- Their small masses make them truly quantum mechanical objects.
Neutrinos are special:
- their masses are much smaller than all other particle masses
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Their small masses make them truly quantum mechanical objects.
• The neutrino “particles” with *masses* are not the same as the “flavour” neutrinos.

• Each neutrino particle (labelled 1,2,3) is made up of a different combination of flavour.

• Property depends on type of measurement.
Neutrinos are truly quantum

- An electron-neutrino emitted by the Sun is a combination of three different masses.
- The corresponding waves travel with different speeds.
- The waves oscillate at different frequencies and therefore interfere.
- The flavours of the neutrino thus change back and forth as the waves interfere constructively and destructively.

Credit: D. Schmidt, P. Vahle
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Credit: D. Schmidt, P. Vahle
“If you think you understand quantum mechanics, you don't understand quantum mechanics.”

Attributed to Richard Feynman
Two-flavour oscillations

Flavour states

\[
\begin{pmatrix}
    \nu_{\alpha} \\
    \nu_{\beta}
\end{pmatrix} =
\begin{pmatrix}
    \cos \theta & \sin \theta \\
    -\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
    \nu_1 \\
    \nu_2
\end{pmatrix}
\]

Mass states

\[
|\nu(t = 0)\rangle = |\nu_{\alpha}\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle
\]
Two-flavour oscillations

\[ |\nu(t)\rangle = e^{i(E_1 t - pL)} \cos(\theta) |\nu_1\rangle + e^{i(E_2 t - pL)} \sin(\theta) |\nu_2\rangle \]

\[ \langle \nu_\beta | \nu(t) \rangle = \sin(\theta) \cos(\theta) (e^{i(E_2 t - pL)} - e^{i(E_1 t - pL)}) \]
Two-flavour oscillations

\[ |\nu(t)\rangle = e^{i(E_1 t - pL)} \cos(\theta) |\nu_1\rangle + e^{i(E_2 t - pL)} \sin(\theta) |\nu_2\rangle \]

plane wave

\[ \langle \nu_\beta |\nu(t)\rangle = \sin(\theta) \cos(\theta) (e^{i(E_2 t - pL)} - e^{i(E_1 t - pL)}) \]

\[ E \approx p + \frac{m_i^2}{2E} \quad \text{and} \quad t = \frac{L}{c} \]

ultra-relativistic
Two-flavour oscillations

\[ |\nu(t)\rangle = e^{i(E_1 t - pL)} \cos(\theta) |\nu_1\rangle + e^{i(E_2 t - pL)} \sin(\theta) |\nu_2\rangle \]

plane wave

\[ \langle \nu_\beta | \nu(t) \rangle = \sin(\theta) \cos(\theta) (e^{i(E_2 t - pL)} - e^{i(E_1 t - pL)}) \]

ultra-relativistic

\[ E \approx p + \frac{m_i^2}{2E} \text{ and } t = \frac{L}{c} \]

\[ \langle \nu_\beta | \nu(t) \rangle = \sin(\theta) \cos(\theta) (e^{i \frac{m_2^2 L}{2E}} - e^{i \frac{m_1^2 L}{2E}}) = \sin(\theta) \cos(\theta) e^{i \frac{\Delta m_i^2 L}{2E}} \]

\[ P(\nu_\alpha \rightarrow \nu_\beta) = \langle \nu_\beta | \nu(t) \rangle^2 = \sin^2(2\theta) \sin^2 \left( \frac{\Delta m_i^2 L}{2E} \right) \]
Two-flavour oscillations

\[ P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta) \sin^2 \left( 1.27 \frac{\Delta m^2_{21}[eV^2]}{E[GeV]} L[km] \right) \]

Rate driven by mass splitting $\Delta m^2$

$\nu_\alpha$ component

$\nu_\beta$ component

Amplitude driven by mixing angle $\sin^2(2\theta)$
• Davis experiment only showed that some of the electron-neutrinos went missing.

• Need a detector that can measure different neutrino flavours to confirm the 3-flavour oscillation model.

• SNO detector – filled with heavy water - is sensitive to Cherenkov light from scattered electrons and from photons produced when neutrons are captured.
Neutrino interactions in SNO

Charged current interaction:
Sensitive only to $\nu_e$

Elastic scattering:
Sensitive to charged and neutral current. $\nu_e$ dominate by a factor of 6
Neutrino interactions in SNO

**Charged Current interaction (CC):**
Sensitive only to $\nu_e$

**Elastic Scattering (ES):**
Sensitive to charged and neutral current. $\nu_e$ dominate by a factor of 6

Sensitive to neutrino direction
In its second phase, SNO was filled with two tons of NaCl.

$^{35}$Cl has a high neutron absorption cross, releases $\gamma$ of up to 8.6 MeV.

Neutral current interaction equally sensitive to all three neutrino flavours.

The salt phase
SNO demonstrates flavour change

What SNO told us

Individual phases

• Measurement of 8B flux without Solar model constraints.
  – Confirm SSM.
• Neutrino flavour change.
  – Non-electron Solar neutrinos observed.
  – Solve the Solar Neutrino Problem
• Measurement of $\theta_{12}$ independent of SSM possible.

Final results from SNO slide 6 of 21
PMNS Matrix

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= U_{\text{PMNS}}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

\[c_{ij} = \cos \theta_{ij}; s_{ij} = \sin \theta_{ij}\]

\[U_{\text{PMNS}} = \begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}\begin{pmatrix}
c_{13} & 0 & s_{13} e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13} e^{i\delta} & 0 & c_{13}
\end{pmatrix}\begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}\]

\[\theta_{13}: \text{mixes } \nu_e \text{ with } \nu_3\]
\[\delta: \text{complex phase}\]

- \(\theta_{12}: \text{“solar mixing angle”}\)
  - mixes \(\nu_e\) with \(\nu_1\) and \(\nu_2\)

- \(\theta_{23}: \text{“atmospheric mixing angle”}\)
  - mixes \(\nu_\mu\) with \(\nu_\tau\)
The MSW Effect

- When neutrinos travel over long distances through dense matter (Sun, Earth), their propagation is modified through coherent forward scattering off particles (electrons).
- This effect modifies the flavour oscillation probability (Mikhaev, Smirnov, Wolfenstein).
- MSW effect can be enhanced through a resonance condition.
- Neutrino and anti-neutrino oscillation probabilities affected differently (not related to CPV).
- MSW effect depends on the sign of $\Delta m^2$.
Solar mixing fixes sign of $\Delta m_{21}$
Where are the tau neutrinos?

Oscillation probabilities for an initial muon neutrino

Baseline/Neutrino energy

Probability

L/E (km/GeV)
CNGS $\nu_\mu$ beam from CERN to Gran Sasso

L/E $\approx 732$ km/$17$ GeV $= 43$
OPERA experiment at Gran Sasso

Target Tracker:

75,000 emulsion/lead modules or "bricks"
First observation of $\nu_\tau \rightarrow \nu_\tau$ appearance

Consistent with $\nu_\tau \rightarrow T^- \rightarrow \pi^- + \pi^0$

6σ significance