



Neutrino Oscillations

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Academic Training Lecture Programme
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Reactor measurements of θ_{13} CP violation, and current long-baseline experiments

PMNS Matrix

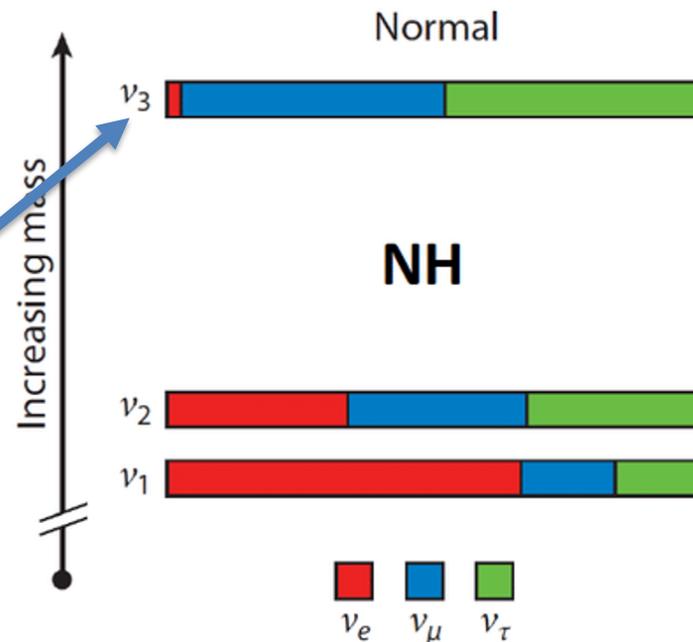
$$U_{\text{PMNS}} = \begin{matrix} & \text{solar} & & \text{"reactor"} & & \text{atmospheric} \\ \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} & \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} & \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \end{matrix}$$

$$\theta_{12} = 33.44^{\circ+0.78^{\circ}}_{-0.75^{\circ}}$$

$$\theta_{23} = 49.0^{\circ+1.1^{\circ}}_{-1.4^{\circ}}$$

$$c_{ij} = \cos \theta_{ij}; s_{ij} = \sin \theta_{ij}$$

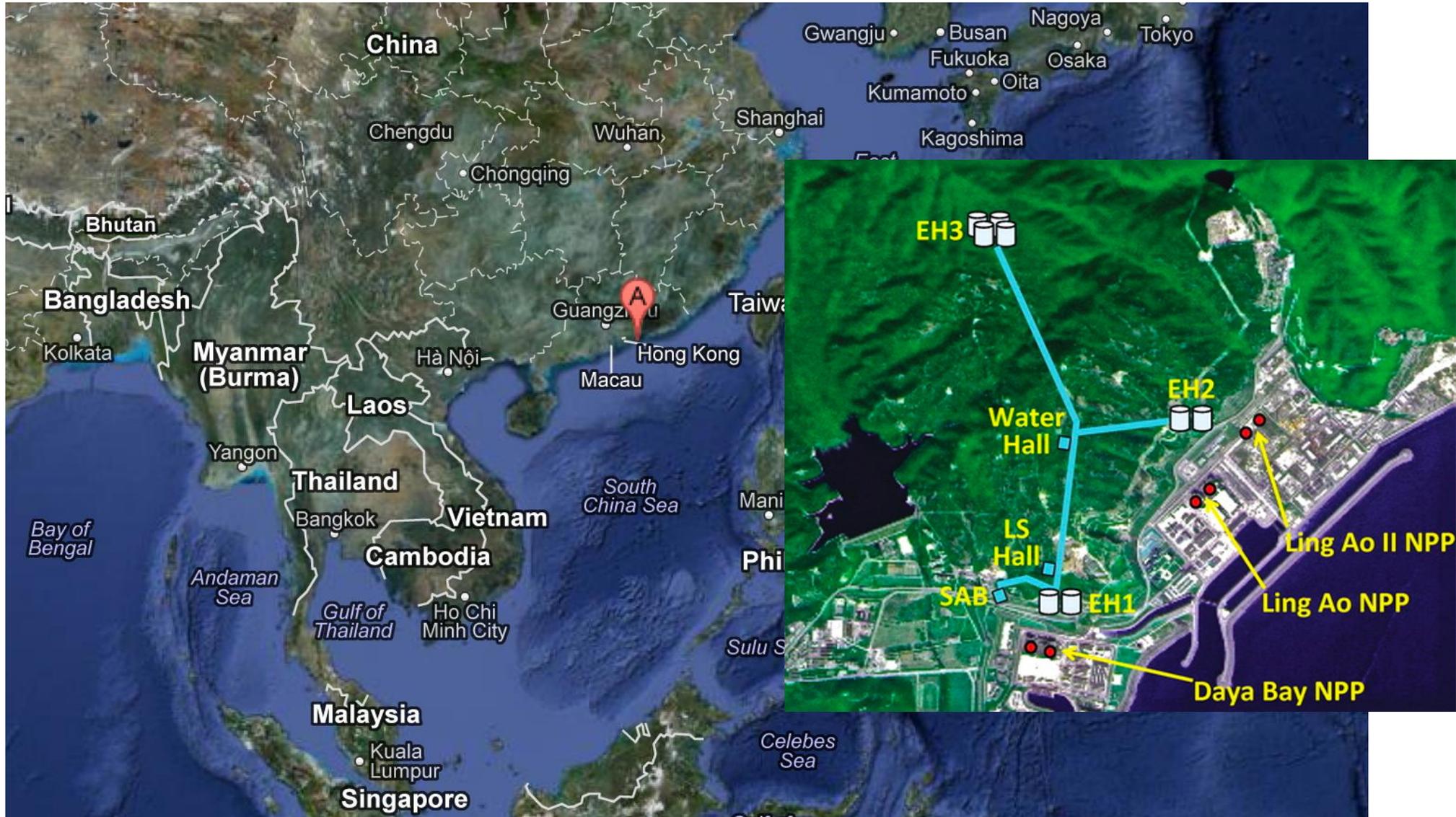
- θ_{12} and θ_{23} are large ("maximal" mixing)
- Angle θ_{13} is small and mixes ν_e with ν_3
- CPV term (δ) $\propto \theta_{13}$
- Look for ν_e mixing driven by Δm_{32}^2



$$\Delta m_{32}^2 = 2.4 \times 10^{-3} \text{ eV}^2$$

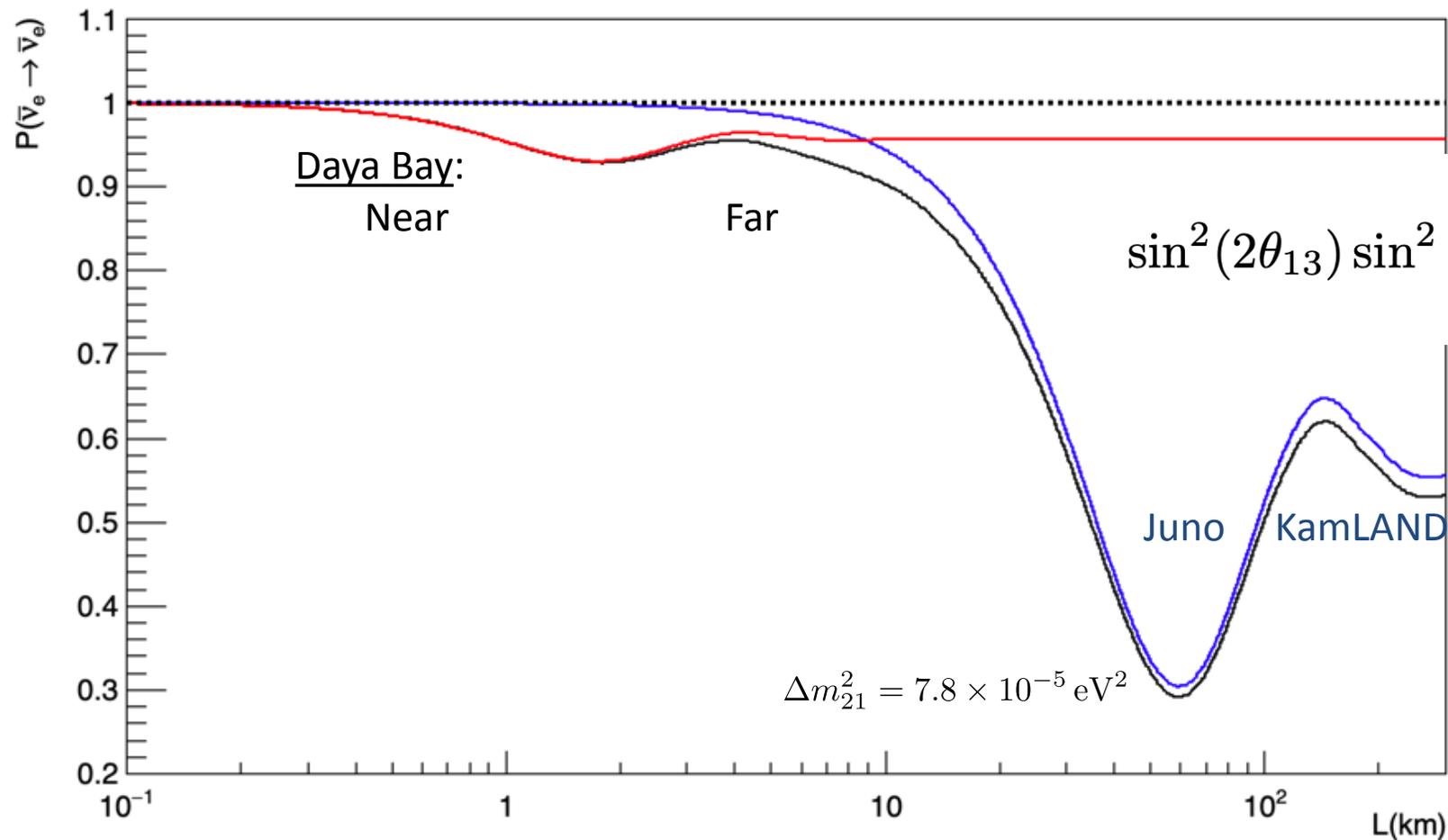
$$\Delta m_{21}^2 = 7.8 \times 10^{-5} \text{ eV}^2$$

Daya Bay reactor



“Reactor” Oscillations

“Survival probability” for anti- ν_e from the reactor ($E \approx 3$ MeV)

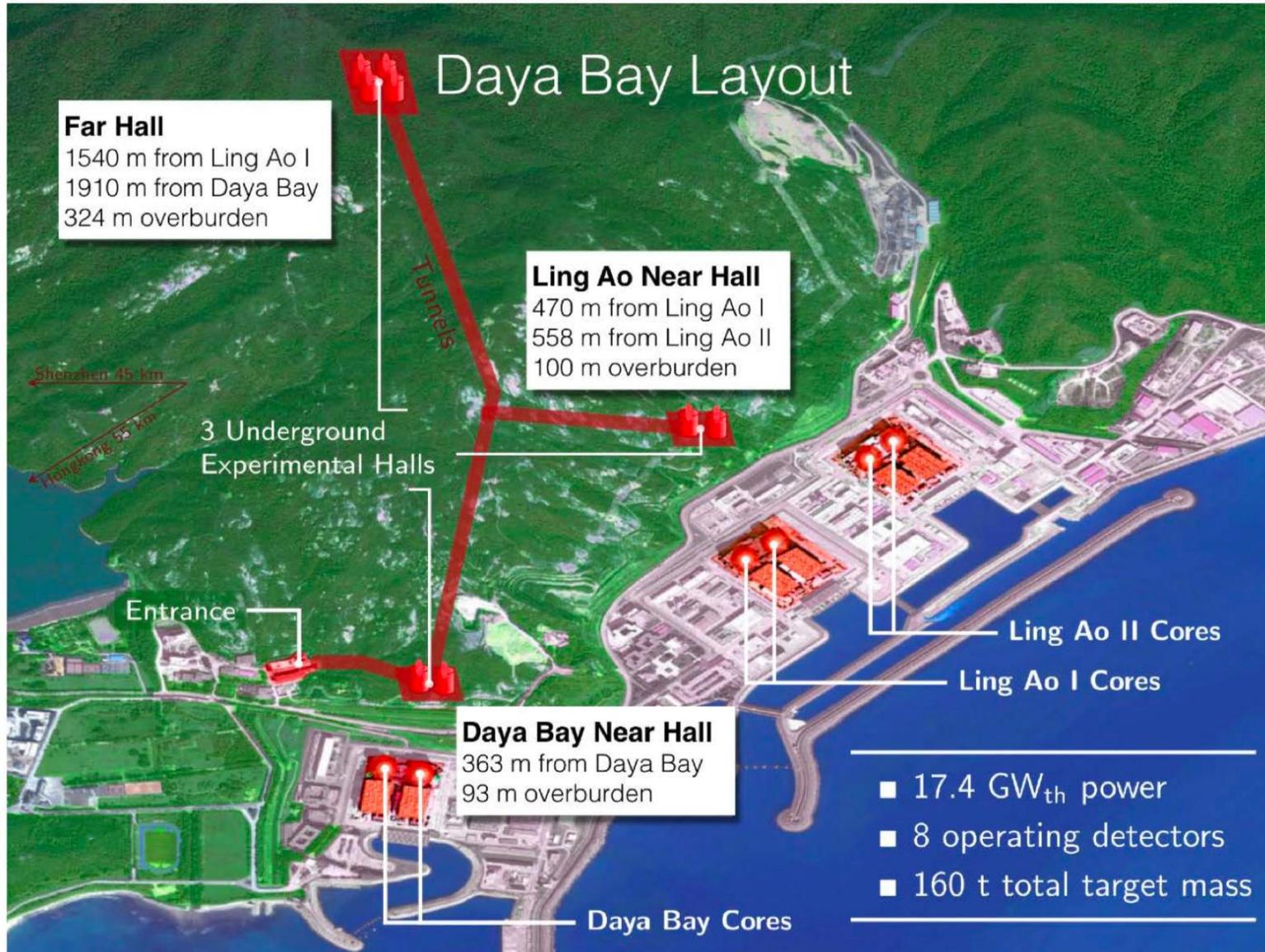


$$\Delta m_{32}^2 = 2.4 \times 10^{-3} \text{ eV}^2$$

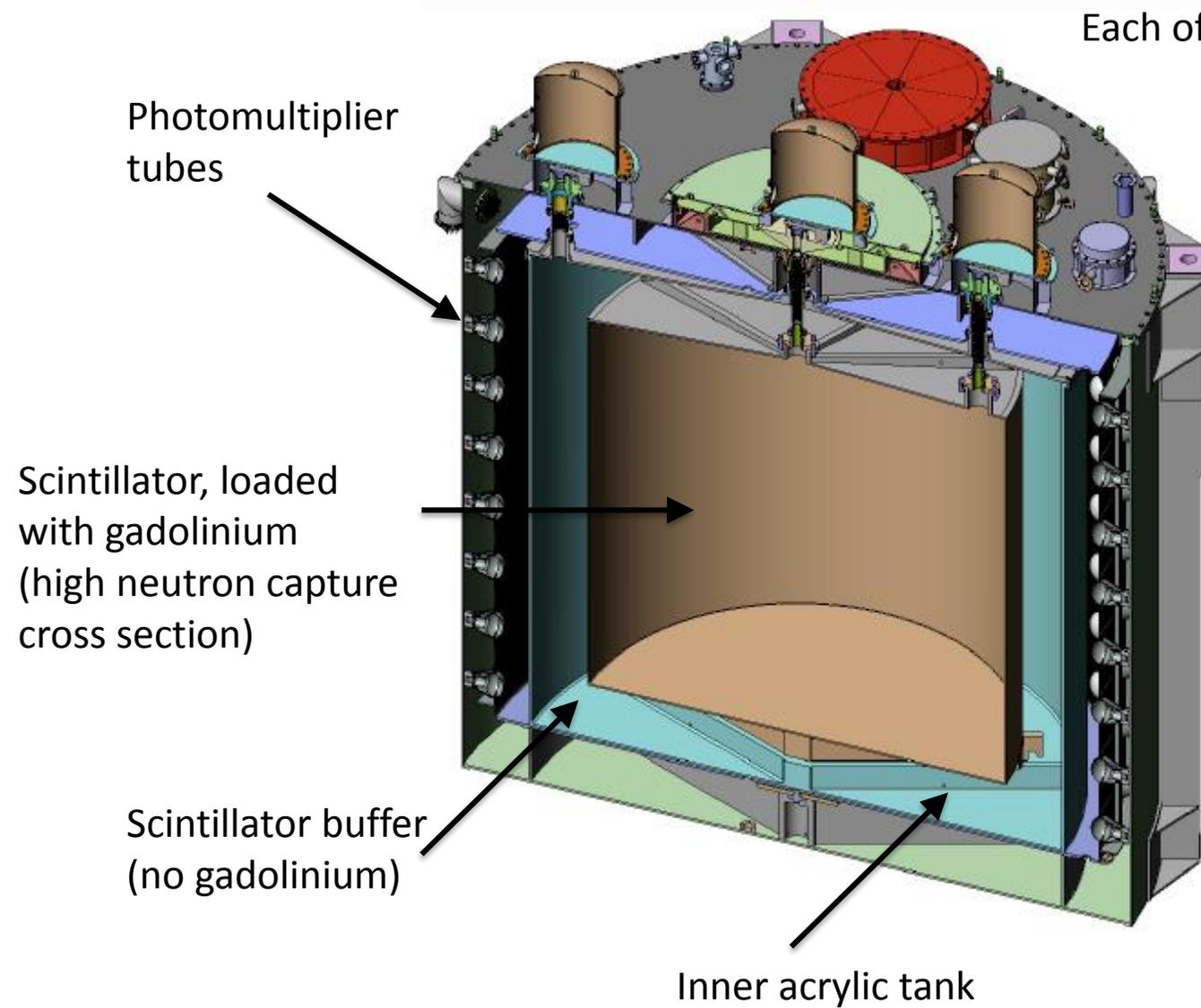
$$\sin^2(2\theta_{13}) \sin^2 \left(1.27 \frac{\Delta m_{32}^2 [\text{eV}^2] L [\text{km}]}{E [\text{GeV}]} \right)$$

J. Ling, Neutrino 2020

Daya Bay Layout

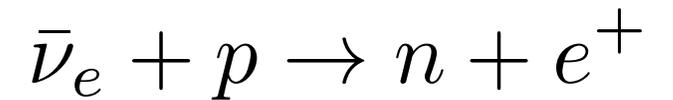
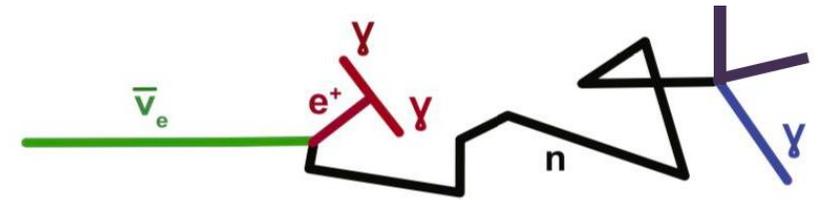


Daya Bay detectors



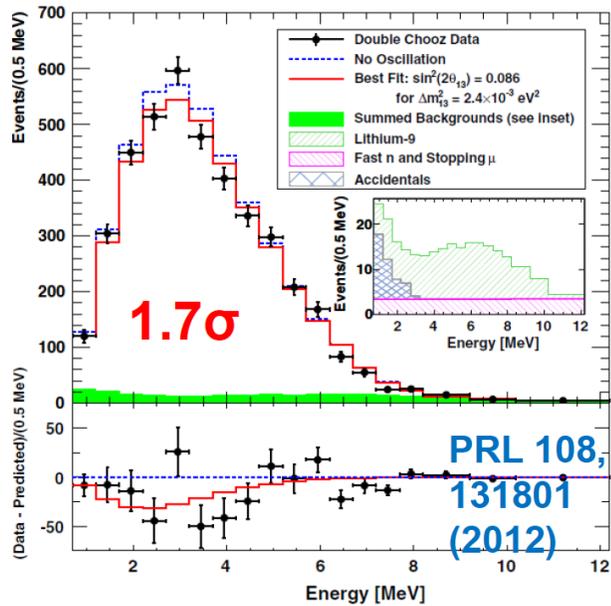
Each of the 8 detector is 20 tons.

Inverse β decay



Original θ_{13} measurements (Far/Near)

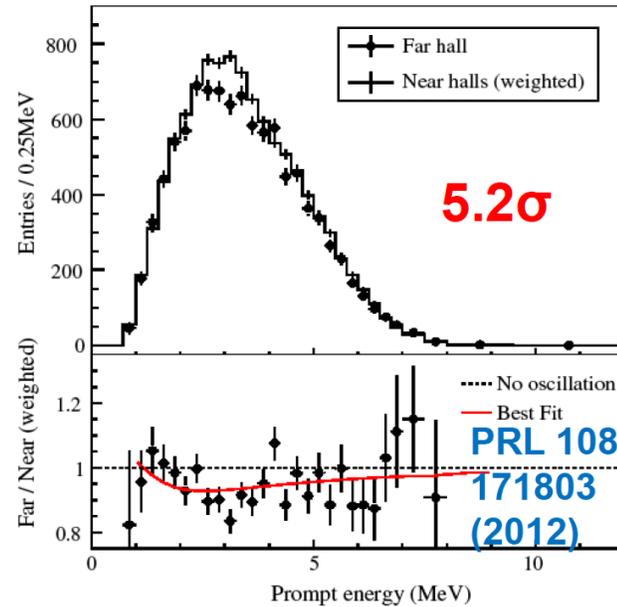
Double Chooz
with only a far detector
(Nov. 2011)



Rate+shape

$$\sin^2 2\theta_{13} = 0.086 \pm 0.041(\text{stat}) \pm 0.030(\text{syst})$$

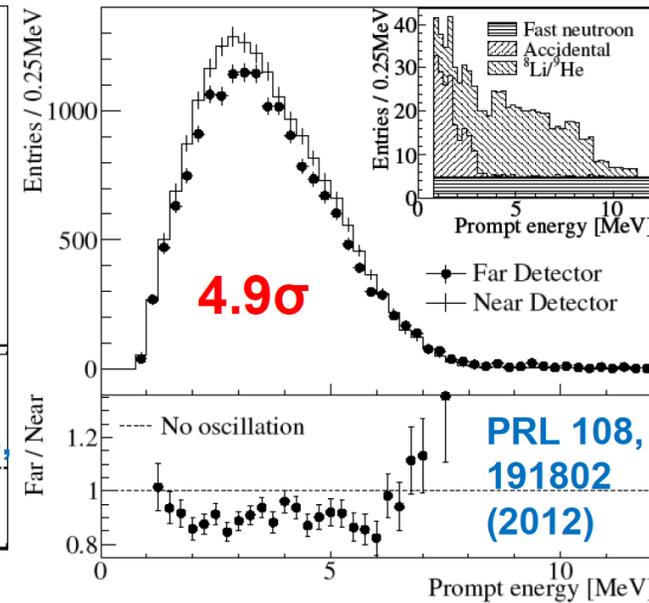
Daya Bay
(March 2012)



Rate only

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat.}) \pm 0.005(\text{syst.})$$

RENO
(April 2012)

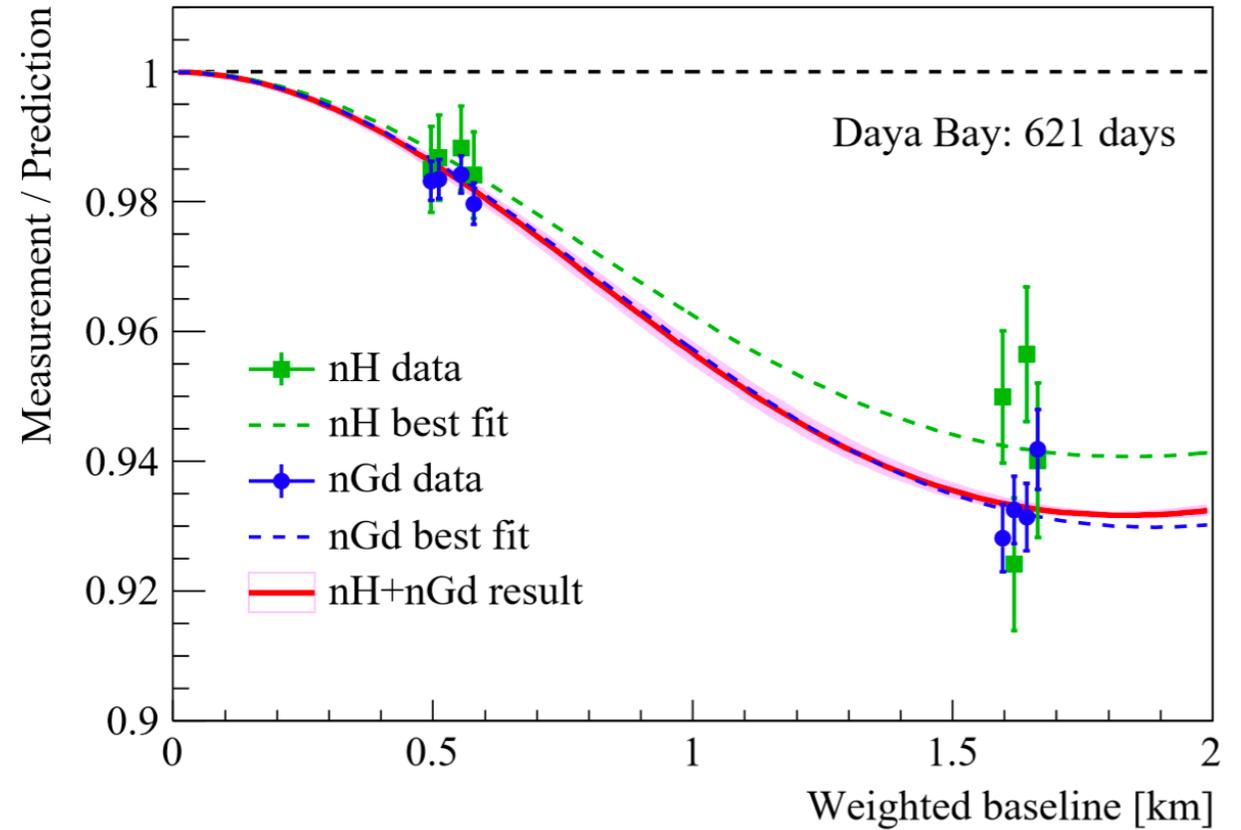
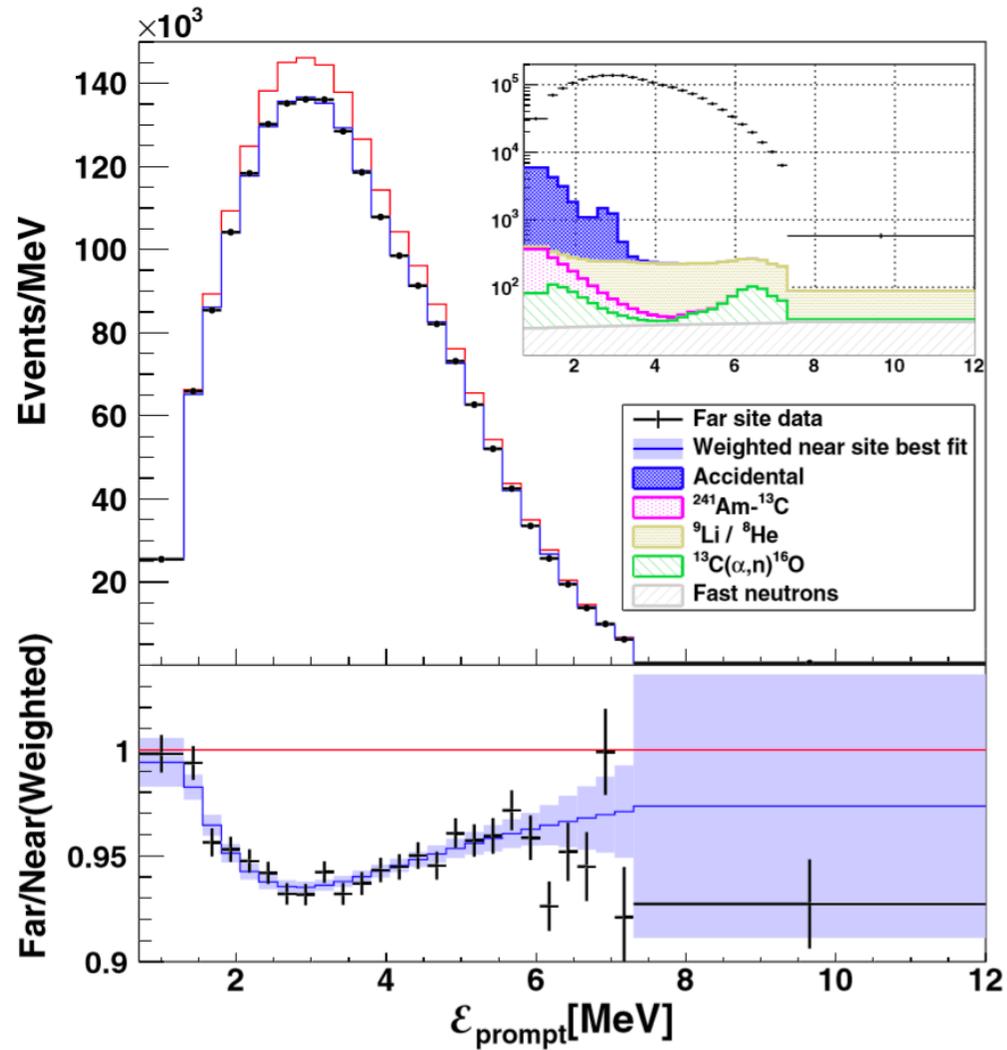


Rate only

$$\sin^2 2\theta_{13} = 0.103 \pm 0.013(\text{stat.}) \pm 0.011(\text{syst.})$$

M.He, NNN

θ_{13} measurement (Daya Bay)



$$\sin^2 2\theta_{13} = 0.0856 \pm 0.0029$$

PMNS Matrix

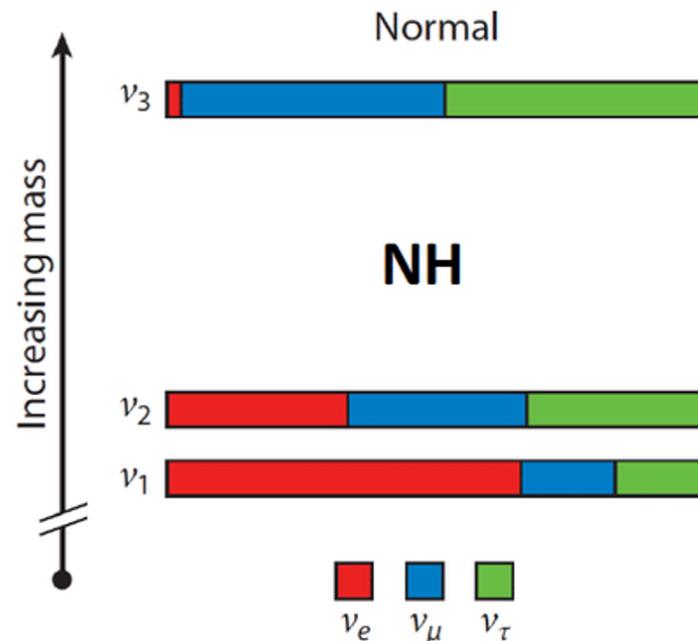
$$U_{\text{PMNS}} = \begin{pmatrix} \text{solar} & & \\ c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \text{reactor} & & \\ c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} \text{atmospheric} & & \\ 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

$c_{ij} = \cos \theta_{ij}; s_{ij} = \sin \theta_{ij}$

$$\theta_{12} = 33.44^{\circ +0.78^{\circ}}_{-0.75^{\circ}}$$

$$\theta_{23} = 49.0^{\circ +1.1^{\circ}}_{-1.4^{\circ}}$$

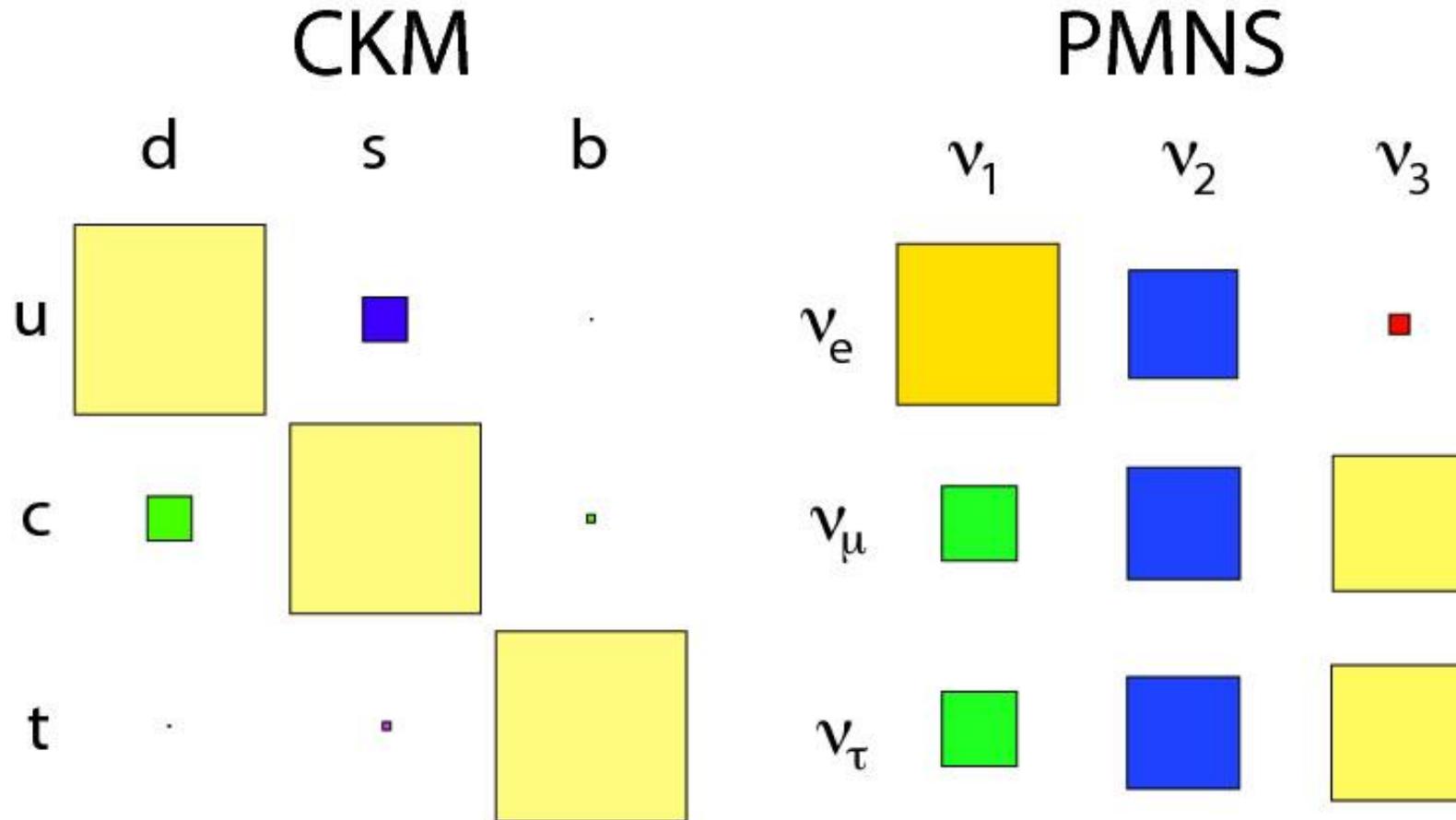
- Small angle θ_{13} mixes ν_e with ν_3
 - Look for ν_e mixing driven by Δm_{32}^2
 - Reactor: anti- ν_e disappearance
 - Accelerator: ν_e appearance in ν_μ beam
- sensitive to θ_{13} and δ



$$\Delta m_{32}^2 = 2.4 \times 10^{-3} \text{ eV}^2$$

$$\Delta m_{21}^2 = 7.8 \times 10^{-5} \text{ eV}^2$$

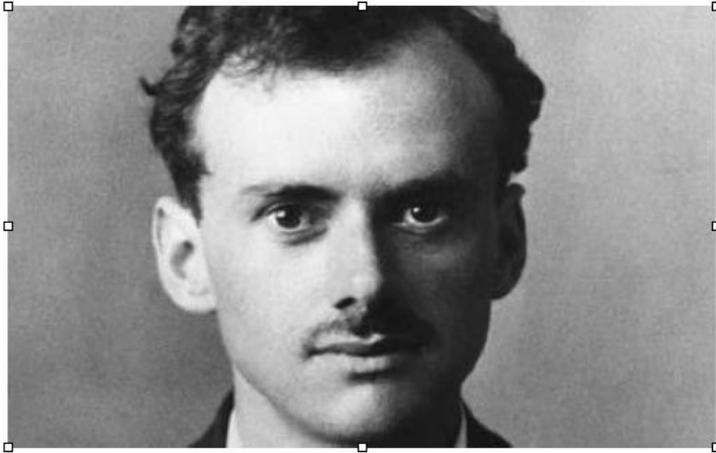
CKM vs PMNS



The CKM matrix is almost diagonal, while the PMNS matrix is almost uniform.

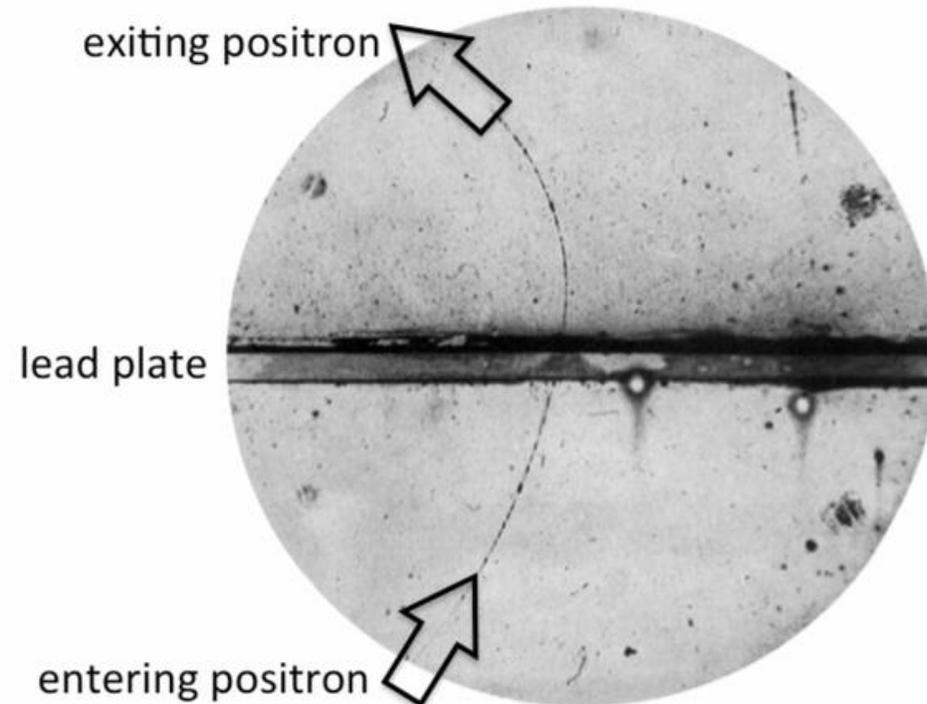
Matter and Anti-matter

Paul Dirac



$$\left(\beta mc^2 + \sum_{k=1}^3 \alpha_k p_k c \right) \psi(\mathbf{x}, t) = i\hbar \frac{\partial \psi(\mathbf{x}, t)}{\partial t}$$

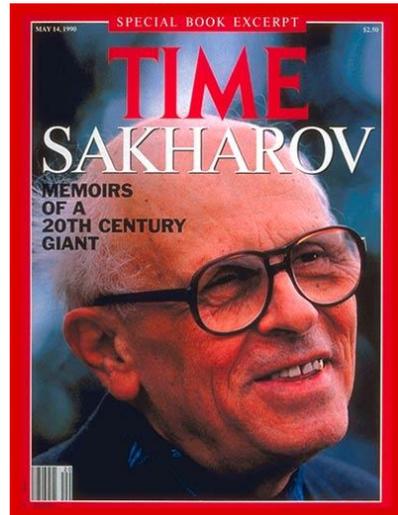
Dirac equation predicts anti-particle states (1928)



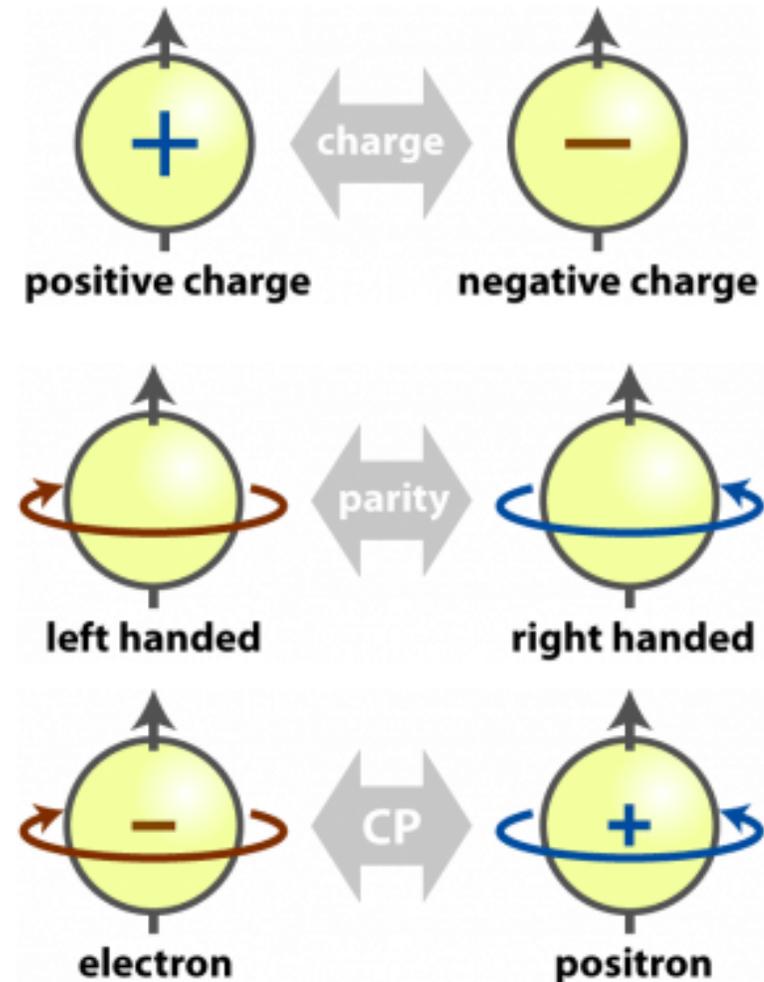
Positron discovered by C.D. Anderson in 1932

Matter-antimatter asymmetry (“CP violation”)

- A tiny ($\approx 10^{-10}$) asymmetry between particle and anti-particles led to our matter dominated universe
- One of the conditions for this asymmetry is violation of *CP symmetry*
- The observation of *CP violation* involving neutrinos could provide support for a theory called *Leptogenesis*



1. Baryon number violation
2. CP violation
3. Departure from thermal equilibrium



CP violation

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

$c_{ij} = \cos \theta_{ij}; s_{ij} = \sin \theta_{ij}$

- A 2x2 "rotation" matrix is real, whereas a 3x3 rotation matrix is imaginary (phase δ).
- CP violation (the difference between a process and its CP conjugate) is only possible when the matrix is imaginary (3 generations!).



CP violation

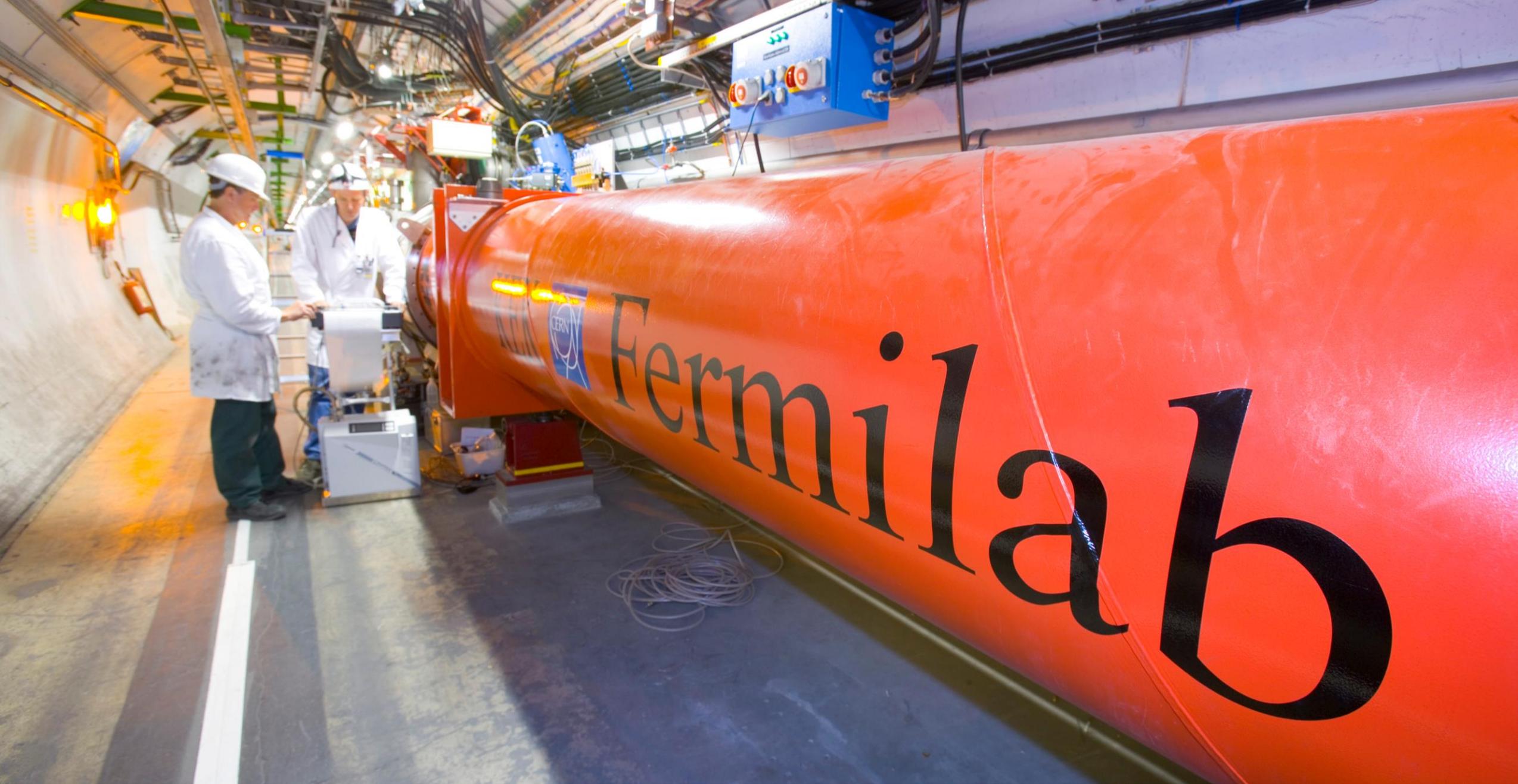
- The same is true for the CKM matrix, where CP violation has been observed for quark processes.
- CP violation in the quark sector is too small to describe the matter dominance in the Universe.
- Discovery of CP violation with neutrinos would lend support to the Leptogenesis model – Leptogenesis would happen at large scales, e.g. through a heavy right-handed neutrino N_R (see-saw mechanism).



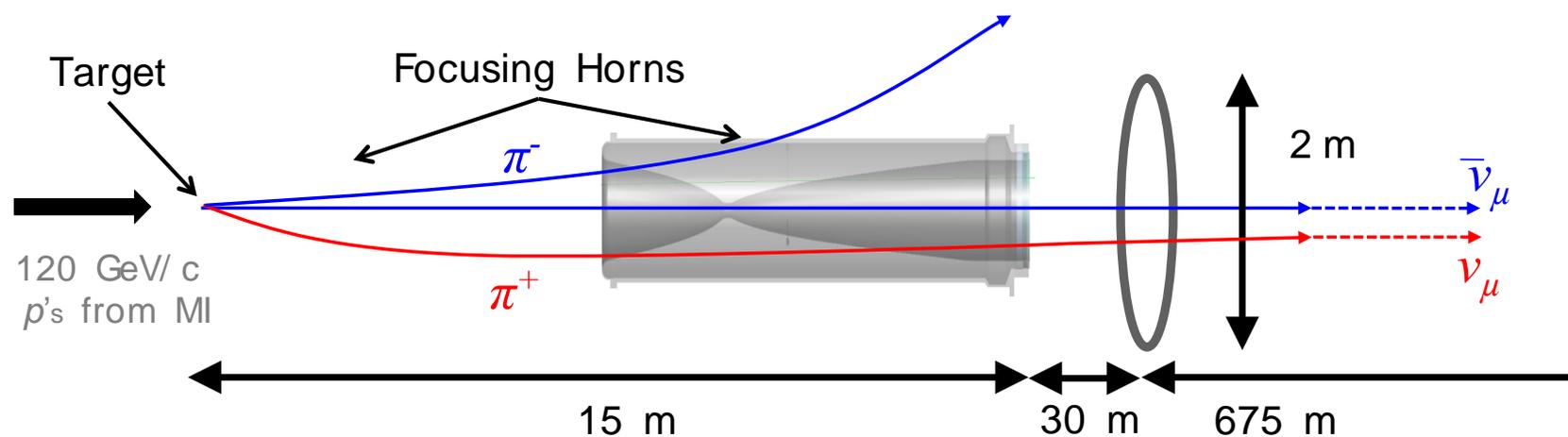
CKM vs PMNS – an aside

$$U_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix}$$

- Neutrinos differ from quarks (Dirac particles), as they can be their own anti-particles (Majorana particles).
- This gives rise to additional complex phases in the PMNS matrix.
- These complex phases appear on the diagonal of the matrix so they have no impact on neutrino oscillations.



Making a neutrino beam

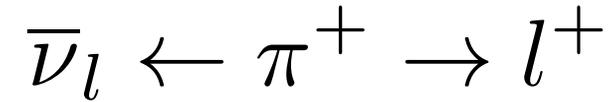


- As neutrinos are neutral, they cannot be focused, and a magnetic horn is thus used to focus the pions.
- Invented by Simon van der Meer at CERN



Making a neutrino beam

Pion decay at rest:



$$E_\nu^* = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} = 29.8 \text{ MeV}$$

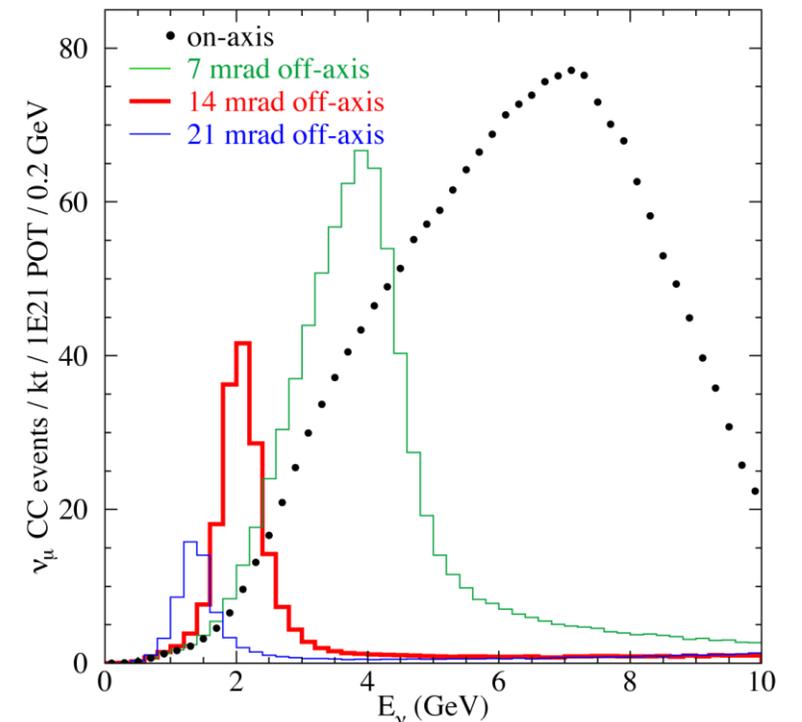
$$E_\nu = \frac{m_\pi E_\nu^*}{E_\pi - p_\pi \cos \theta} = \frac{E_\nu^*}{\gamma_\pi (1 - \beta_\pi \cos \theta)}$$

Boost into lab system:

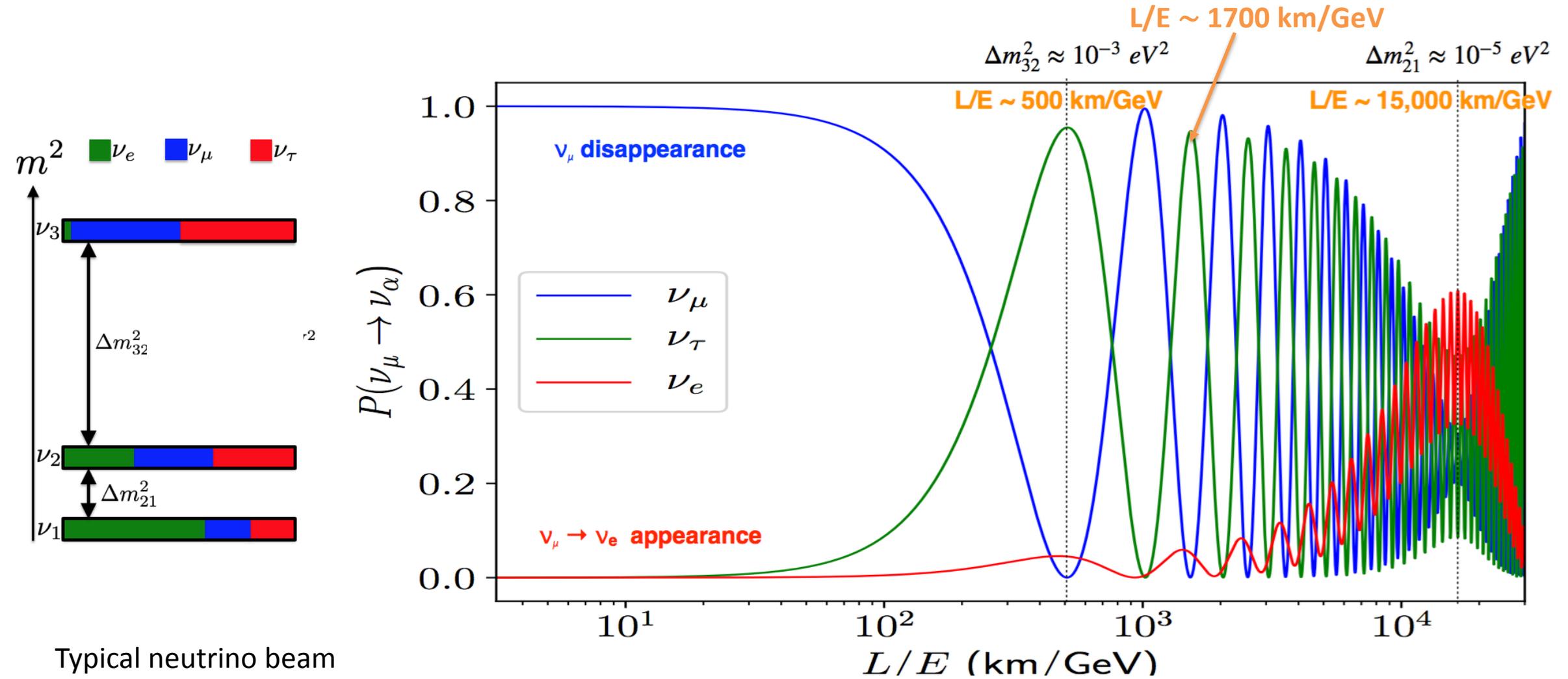
$$E_\pi = 9 \text{ GeV at } \theta = 0 :$$

$$\gamma_\pi = 64.5 \implies E_\nu = 3.8 \text{ GeV}$$

Medium Energy Tune



Finding the oscillation maximum



Typical neutrino beam energy is around 2.5 GeV

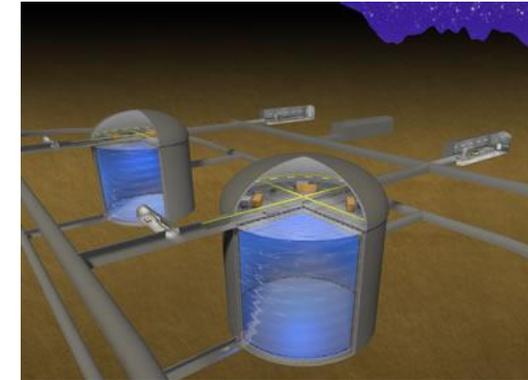
Baseline/Neutrino energy

Optimizing L/E for neutrino oscillations

$L \approx 300 \text{ km}$

$$\frac{\Delta m_{31}^2 L}{4E} \sim \frac{\pi}{2}$$

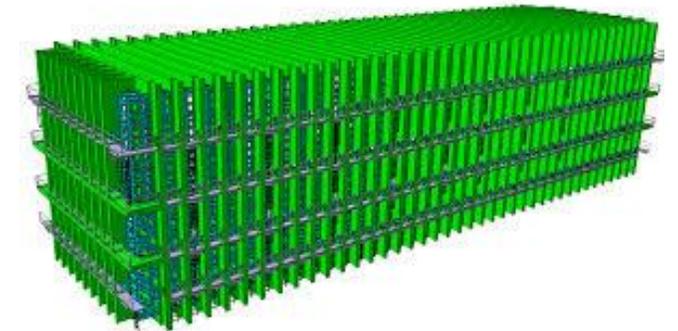
- $L/E = 300 \text{ km} / 0.6 \text{ GeV} = 500 \text{ km/GeV}$
- no matter effects; first oscillation maximum.
- use narrow width neutrino beam (off axis) with $E < 1 \text{ GeV}$



Water Cherenkov (T2K, HK)

$L = 1300 \text{ km}$

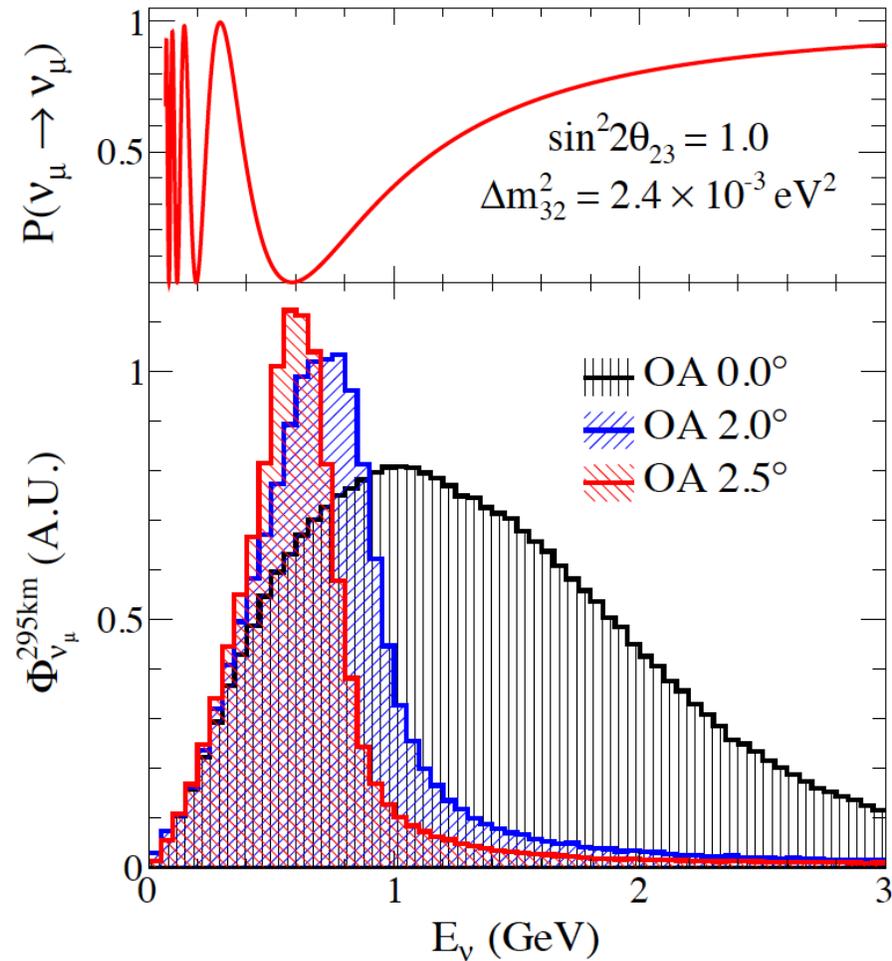
- $L/E = 1300 \text{ km} / 2.5 \text{ GeV} = 500 \text{ km/GeV}$ (1st max),
- $L/E = 1300 \text{ km} / 0.8 \text{ GeV} = 1700 \text{ km/GeV}$ (2nd max)
- matter effects; first and second oscillation maximum.
- use broad-band neutrino beam (on axis).



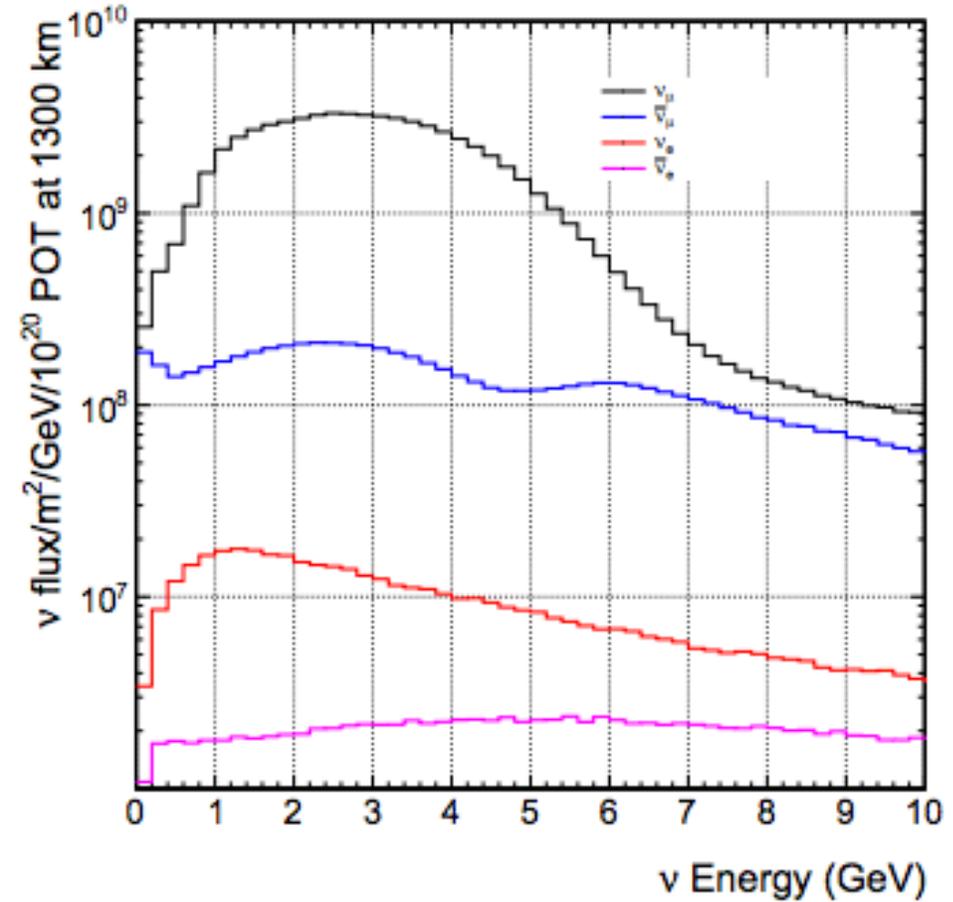
Liquid argon (DUNE)

Off-axis vs on-axis beams

T2K at 2.5 degrees

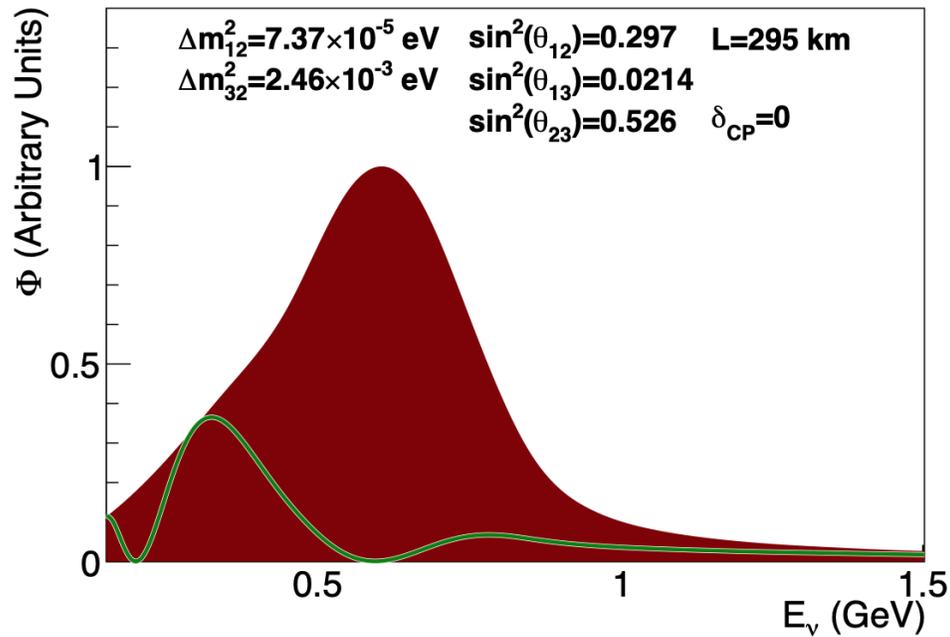


DUNE on-axis beam

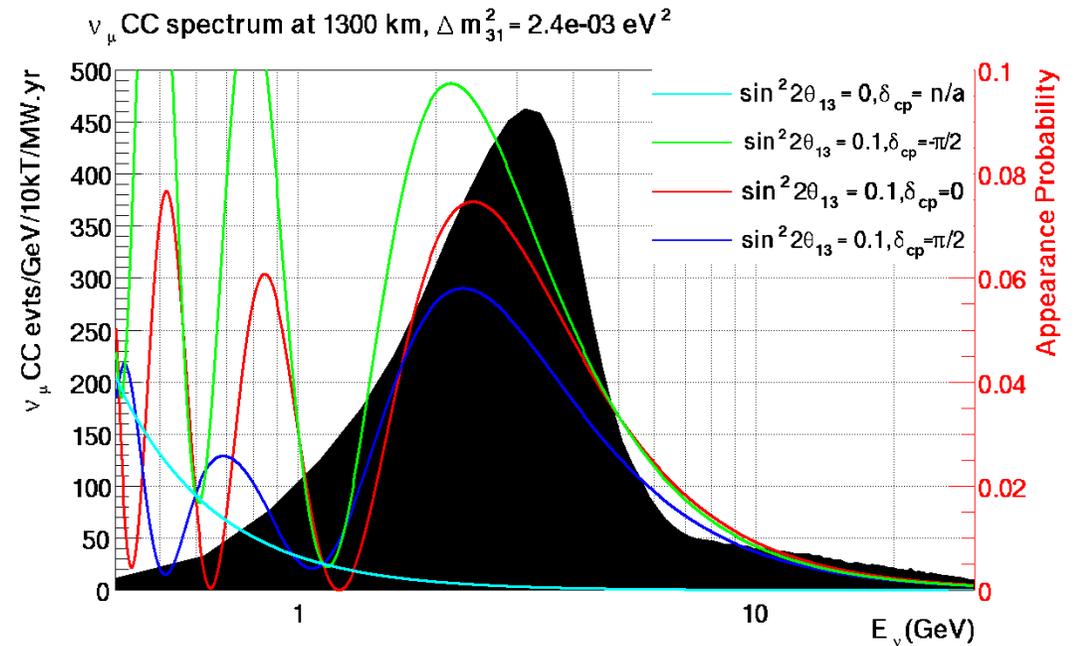


Off-axis vs on-axis beams

T2K at 2.5 degrees



DUNE on-axis beam



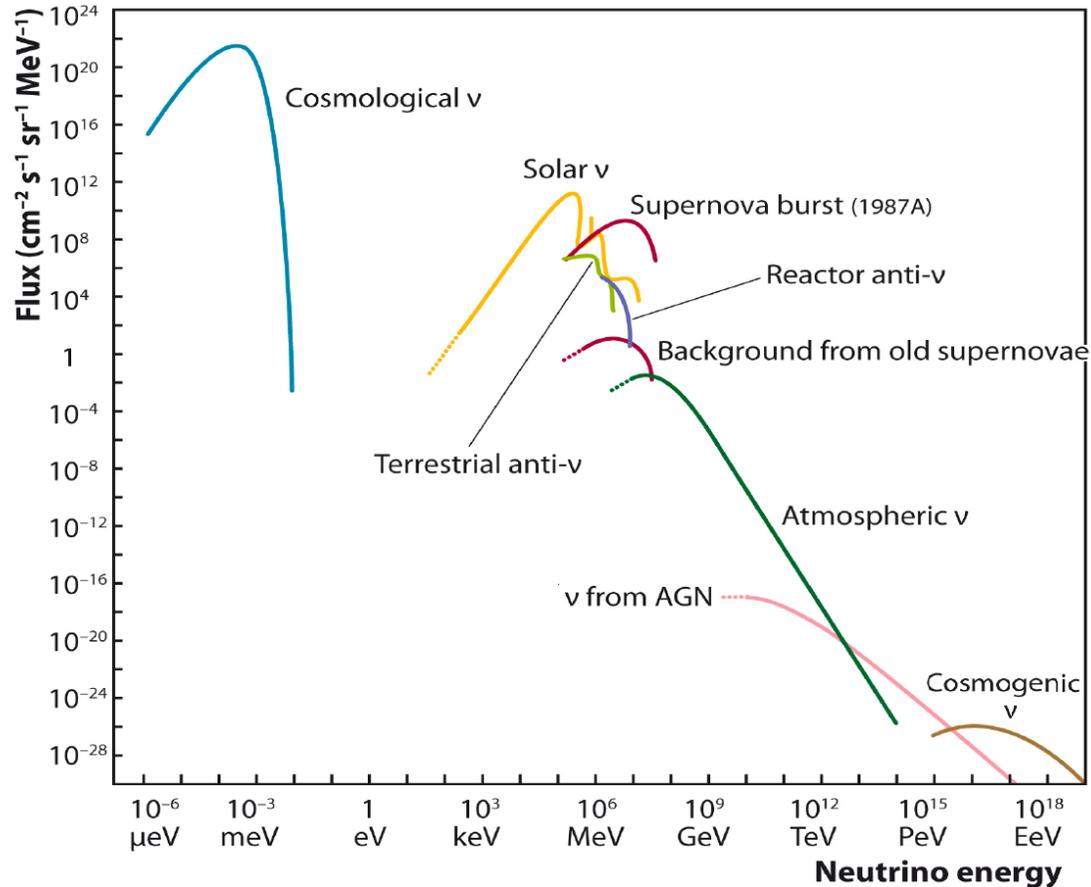
How to measure LBL neutrino oscillations

- Measure flavour change as a function of energy over a long distance.
- Starting with a muon-neutrino beam, we observe muon-neutrino disappearance and electron-neutrino appearance.
- We measure event rates and not the flux directly.
- Measurement is a convolution of the oscillation probabilities P , the neutrino flux Φ , the cross sections σ , and the detector response T .

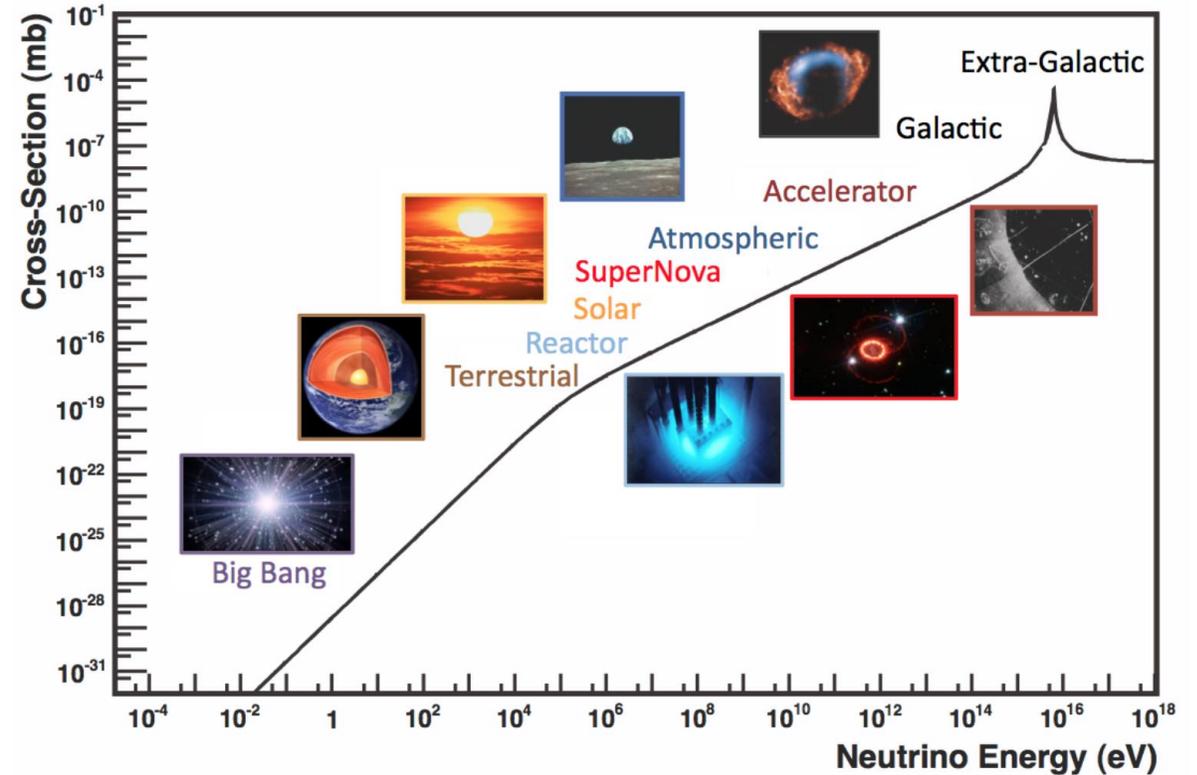
$$\frac{N_{\mu}^{\text{FD}}}{N_{\mu}^{\text{ND}}}(E_{\text{rec}}) = \frac{\int \Phi_{\mu}^{\text{FD}}(E) \cdot P_{\mu\tau}(E) \cdot \sigma_{\mu}^{\text{Ar}} \cdot T_{\mu}^{\text{FD}}(E, E_{\text{rec}}) dE}{\int \Phi_{\mu}^{\text{ND}}(E) \cdot \sigma_{\mu}^{\text{X}} \cdot T_{\mu}^{\text{ND}}(E, E_{\text{rec}}) dE}$$

Neutrino sources, flux, and cross sections

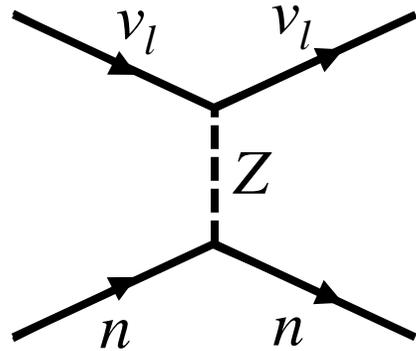
C. Spiering, arXiv:1207.4952



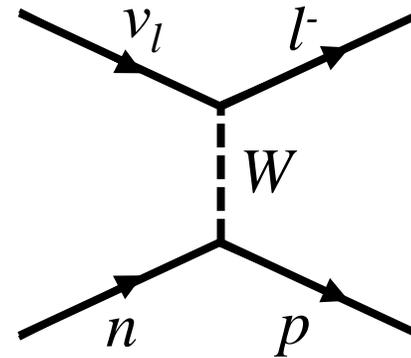
J. Formaggio, G.P. Zeller, arXiv:1305.7513



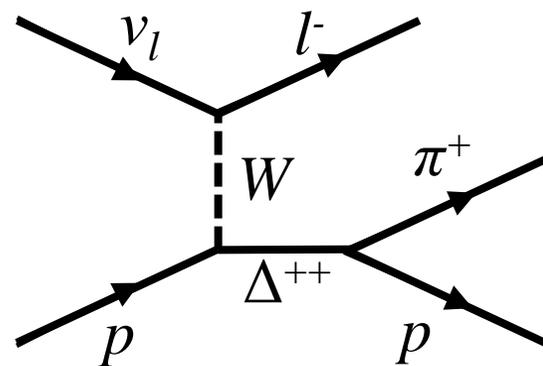
Neutrino-nucleon interaction



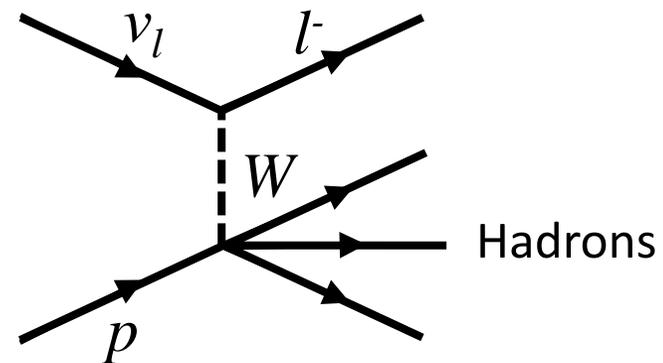
Elastic scattering



Quasi-elastic scattering
(lowest energies)



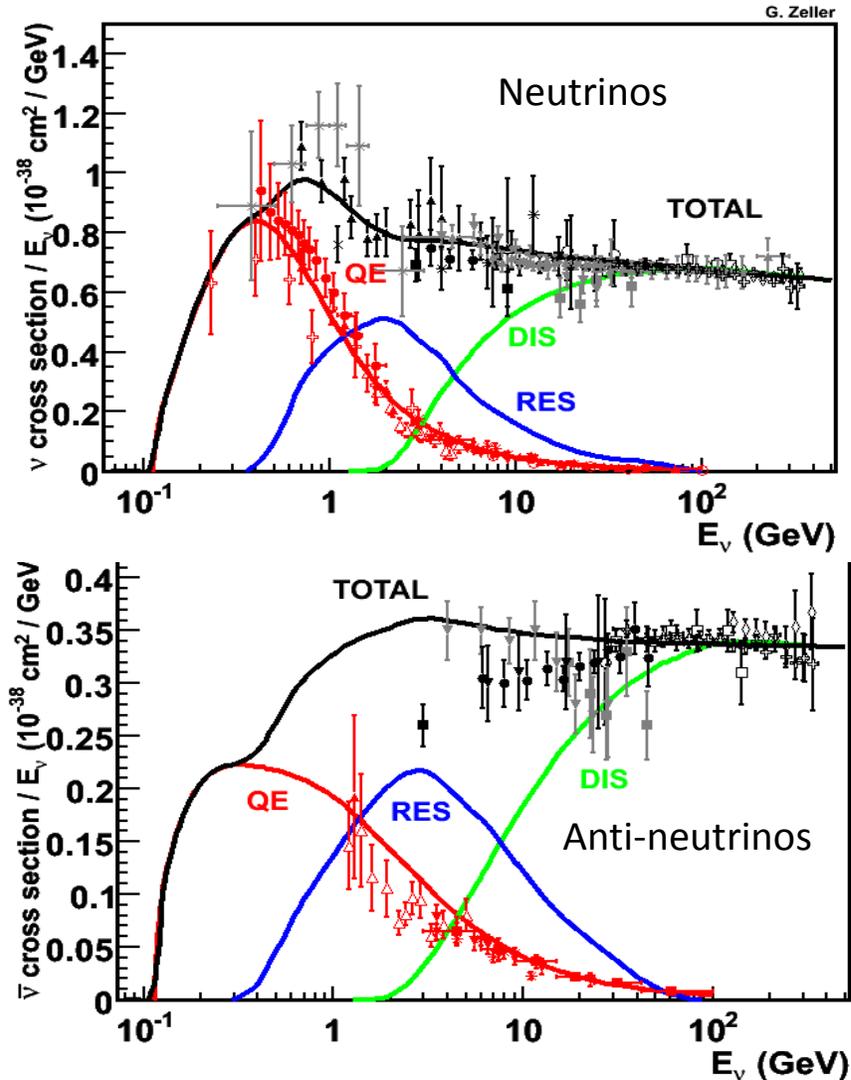
Resonance
(Energies ~ 1 GeV)



Deep inelastic scattering
Highest energies (>1 GeV)

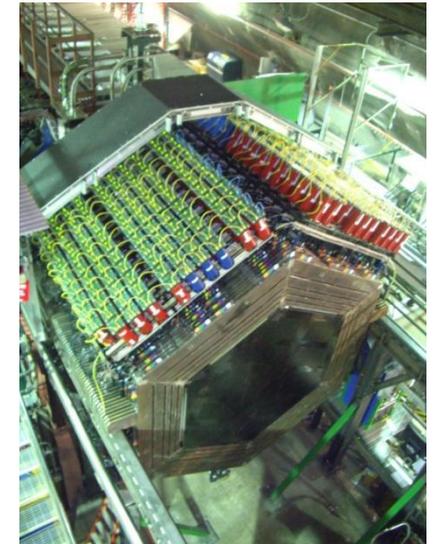
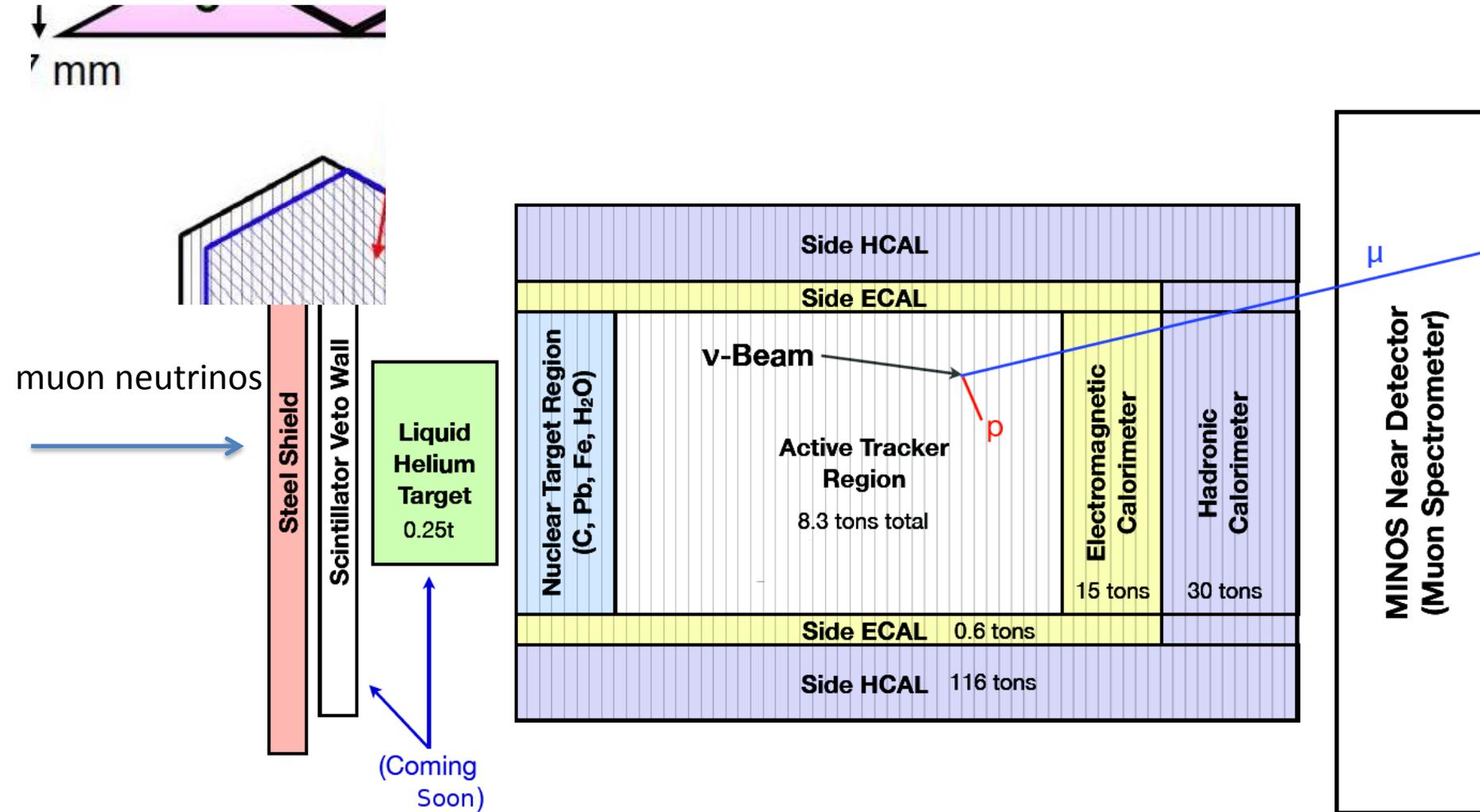
Neutrino-nucleon interactions and cross sections

Muon neutrino data



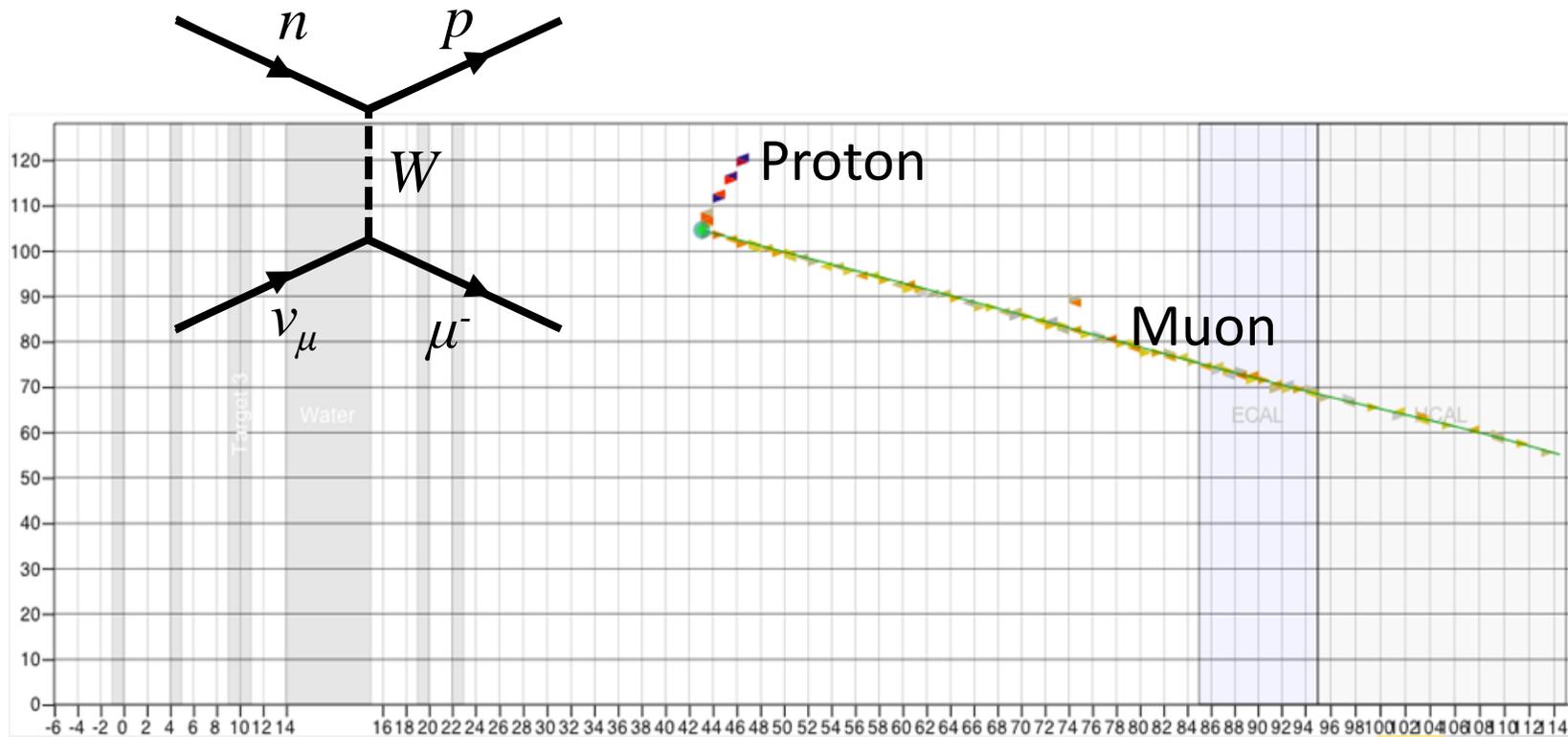
- At lower energies, quasi-elastic dominates.
- At higher energies, resonance production and deep-inelastic scattering
- An important systematic limitation for long-baseline neutrino experiments
- Interaction modelling will affect energy reconstruction.

Measuring neutrino cross sections with MINER ν A



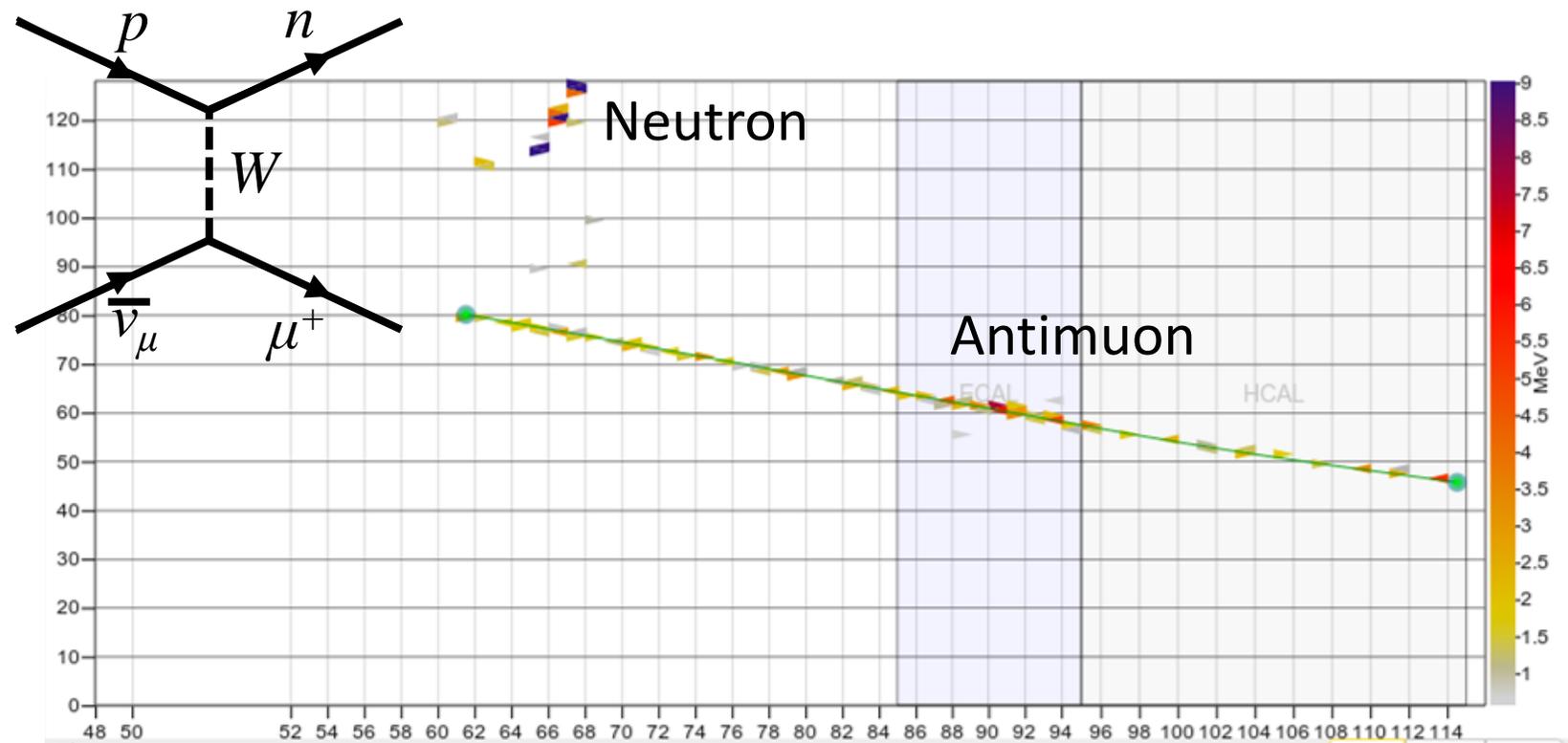
MINER ν A ν_{μ} quasi-elastic interaction

Quasielastic scattering

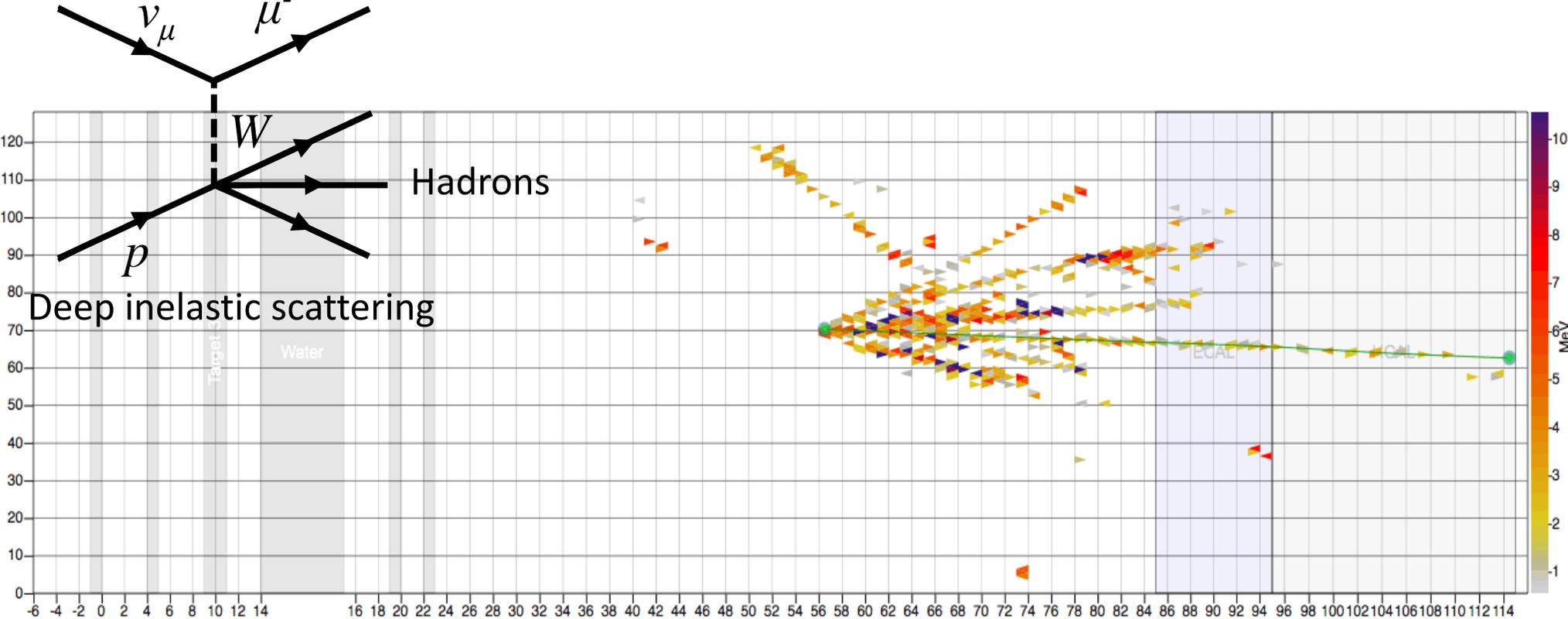


MINER ν A ν_{μ} quasi-elastic interaction

Quasielastic scattering



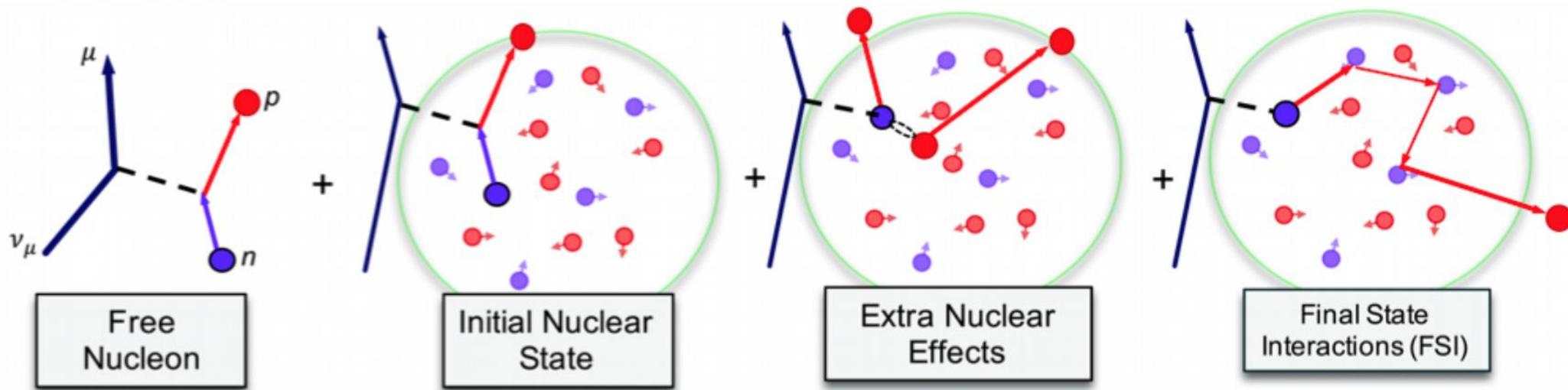
MINER ν A deep inelastic scattering event



Another Complication

Need to understand nuclear effects – which are messy!

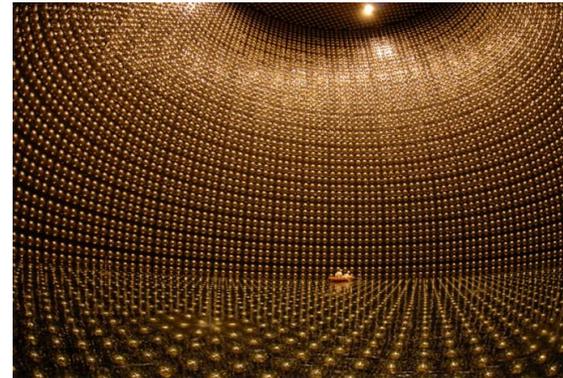
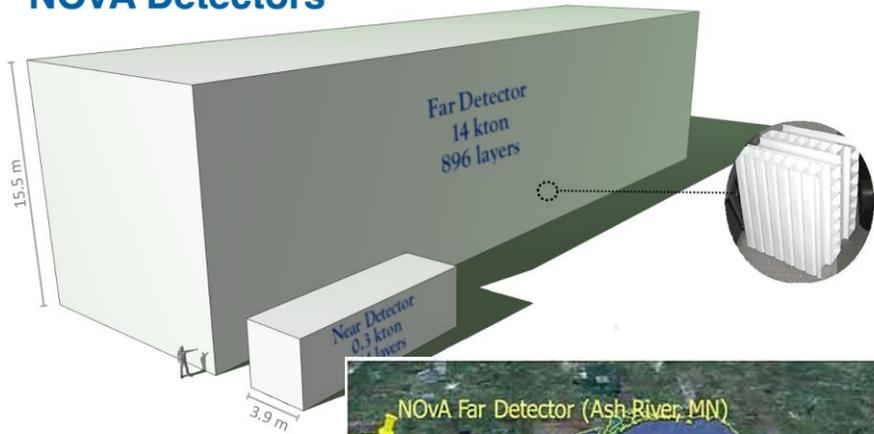
Nucleus



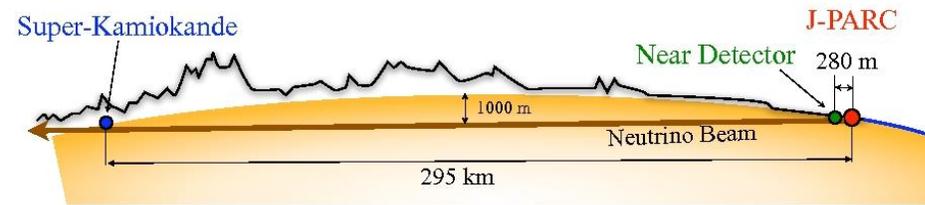
Some effects can be mitigated by use of same nuclear targets

Operating Long-baseline experiments

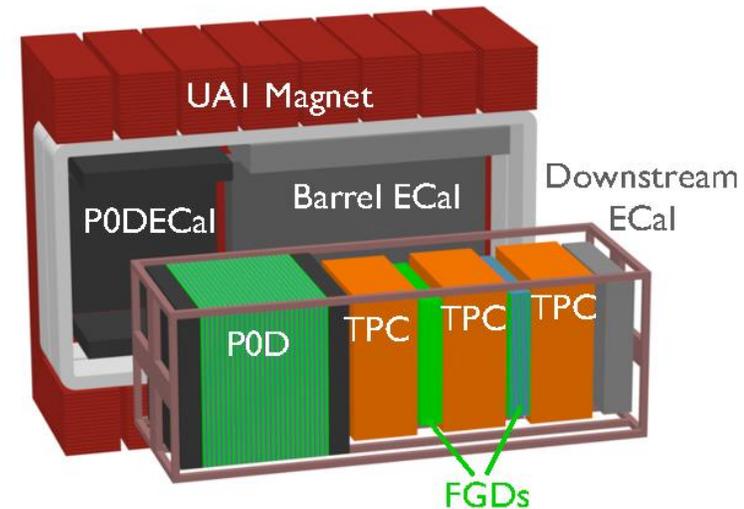
NOvA Detectors



T2K baseline:
295 km



NOvA baseline:
810 km



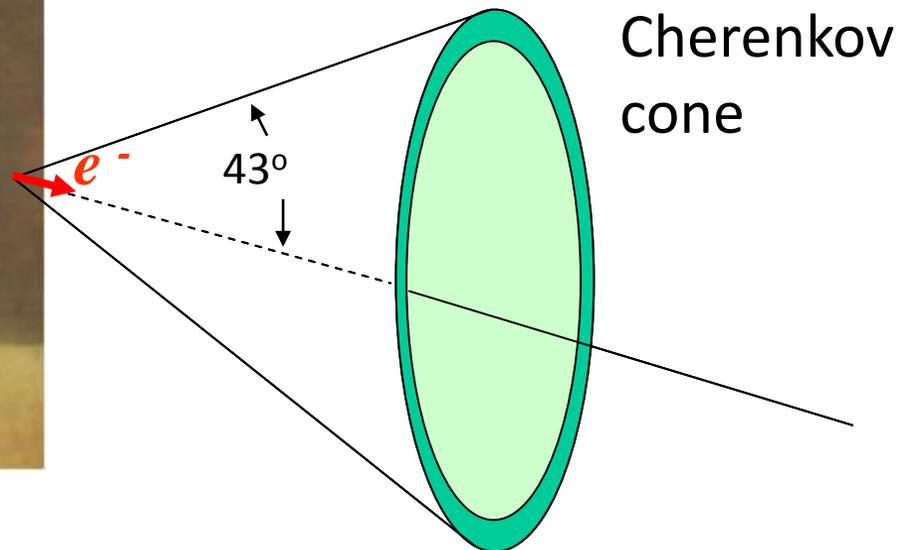
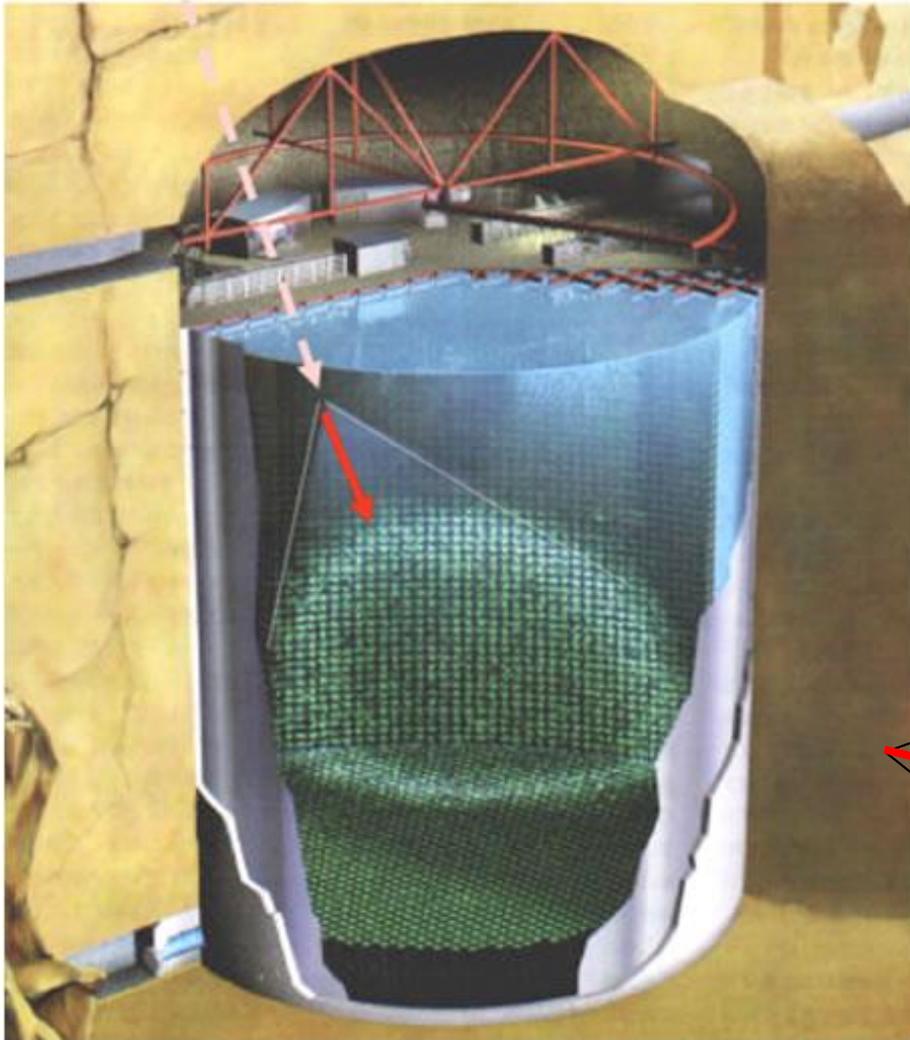
T2K Experiment



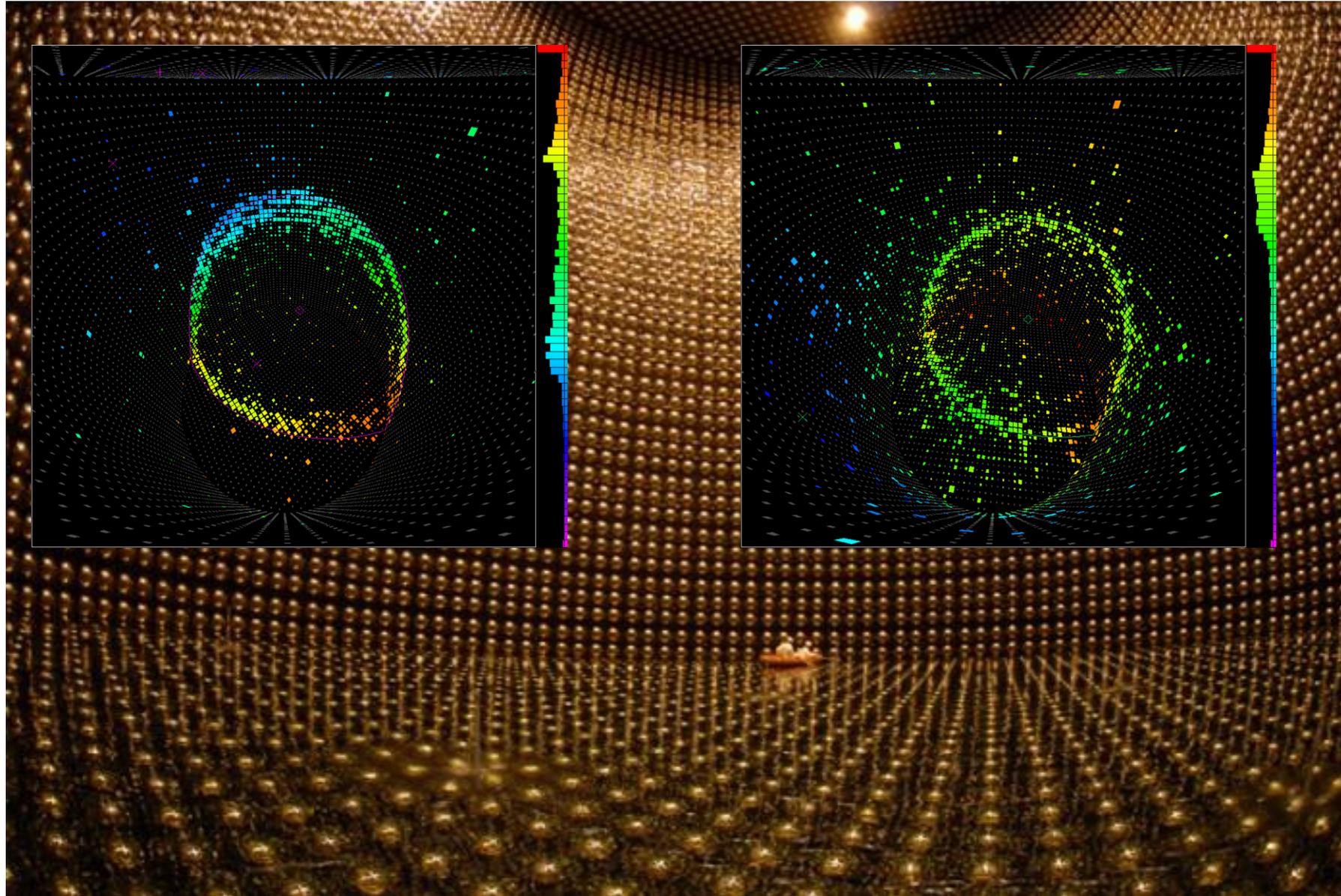
- Muon (anti) neutrino beam generated at J-PARC
- Beam travels 295 km to large SK far detector to be measured after oscillations
- Near detector complex, ND280 constrains beam flux and interaction cross-section before oscillation
- Important to constrain non-oscillation parts of model to avoid bias

Super-Kamiokande

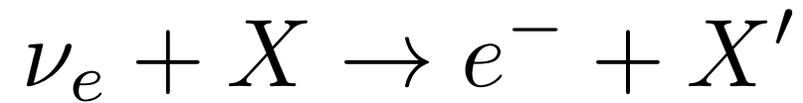
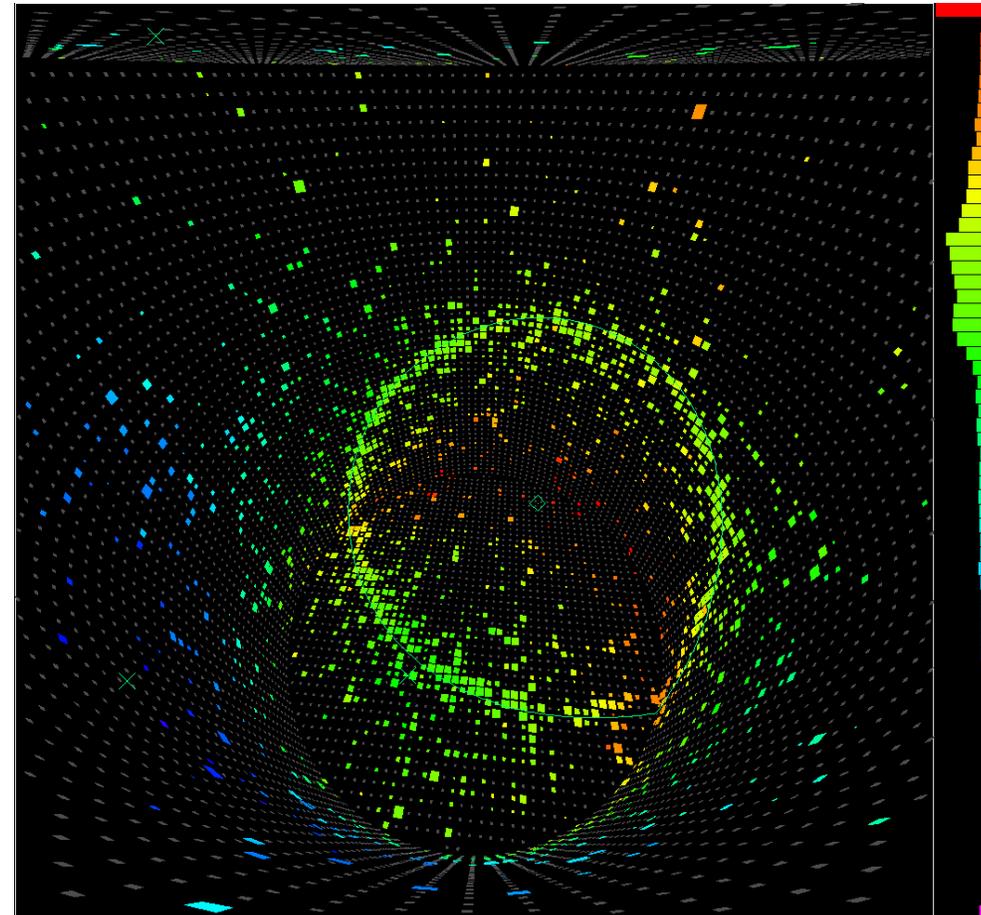
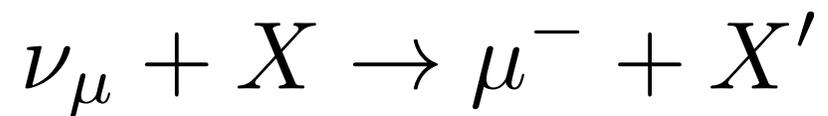
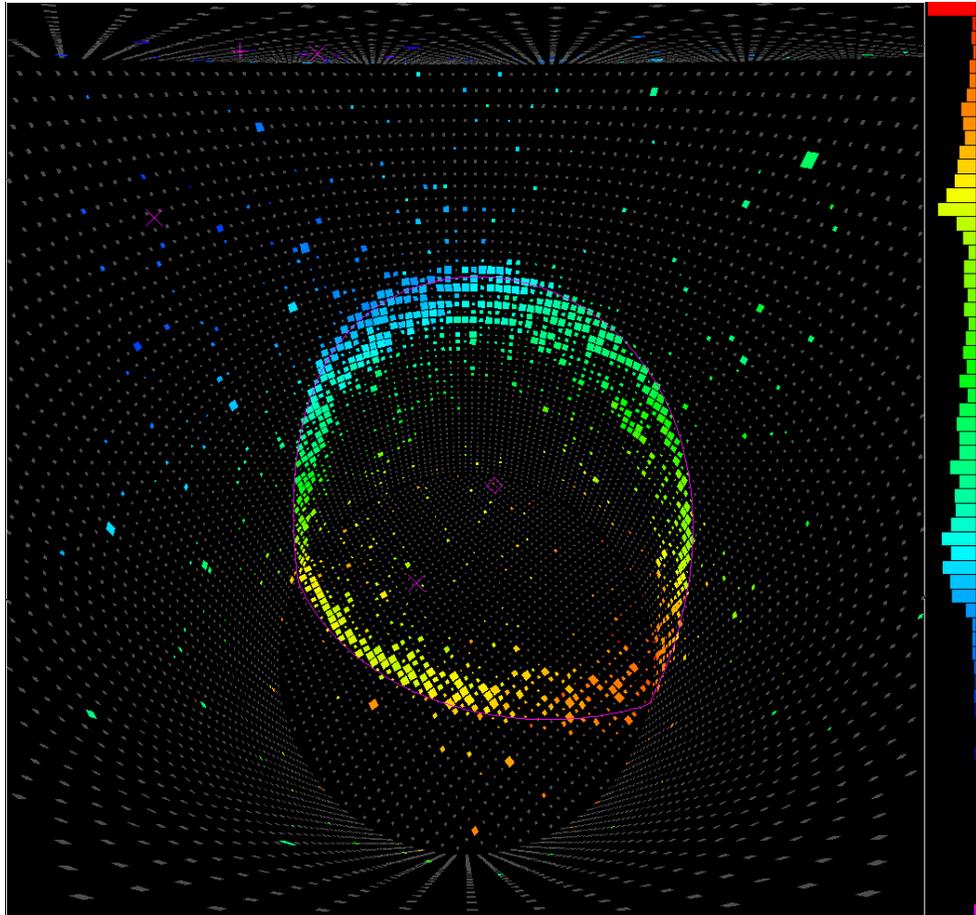
- 50,000 tons of water surrounded by 11,000 PMTs (20 inch).
- 1 km rock overburden
- 39.3m in diameter and 41.4m in height



Super-Kamiokande – electron or muon ring?

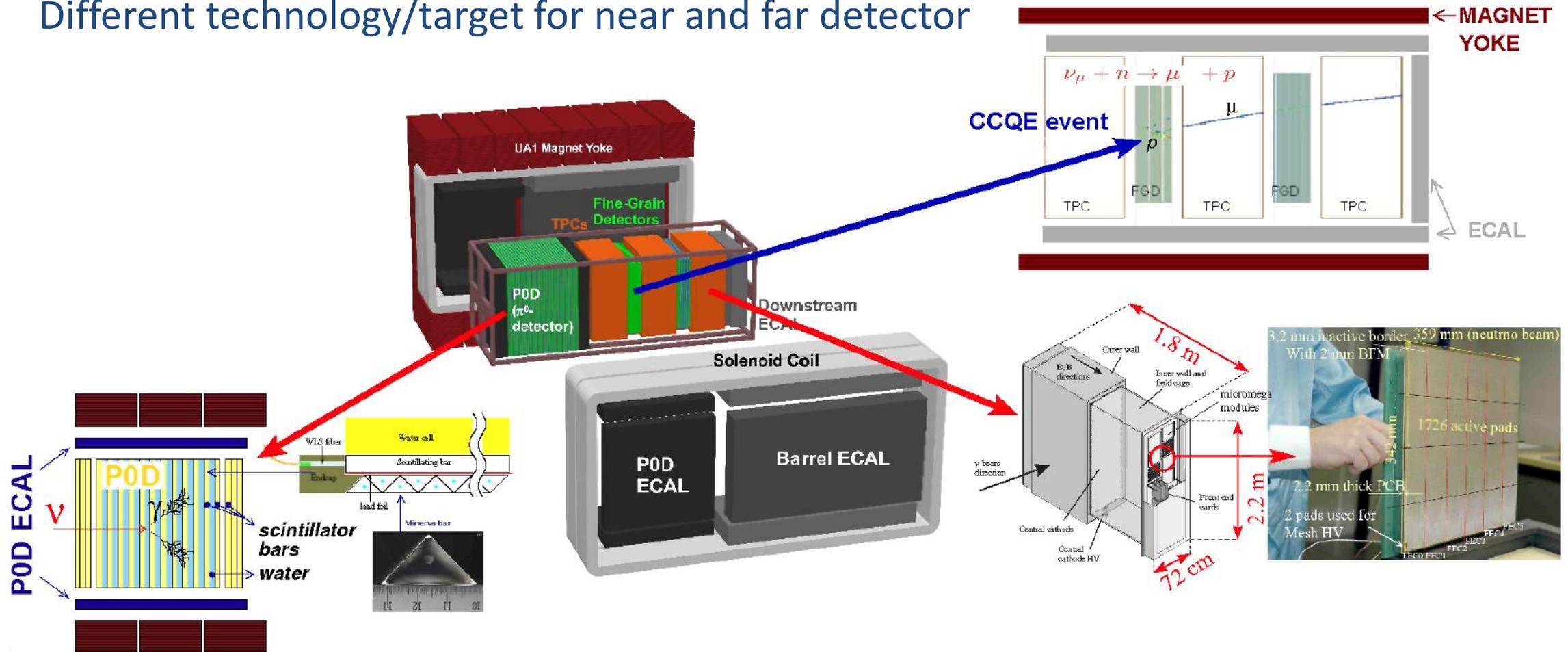


Super-Kamiokande – electron or muon ring?

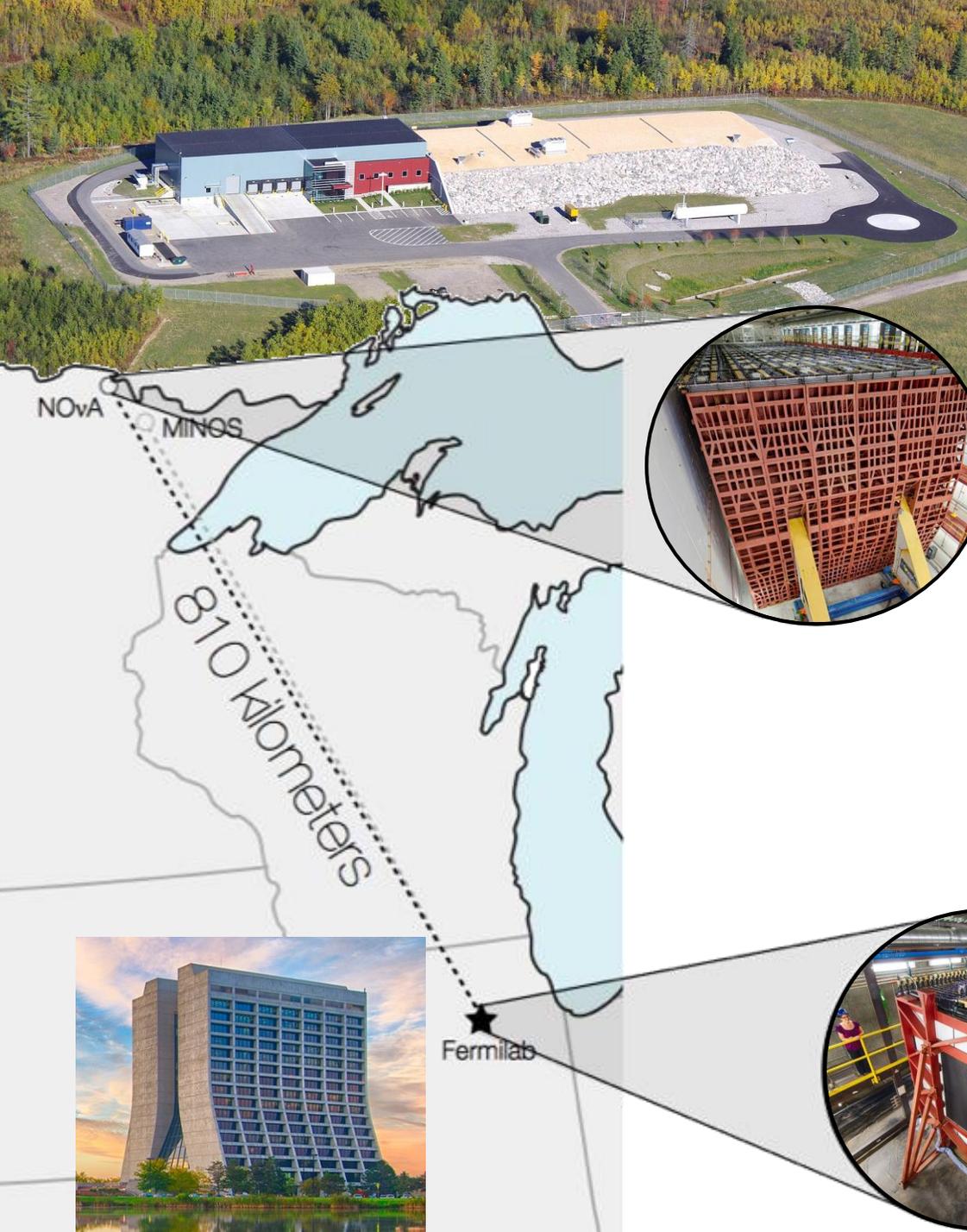


The T2K Near Detector (ND280)

Different technology/target for near and far detector

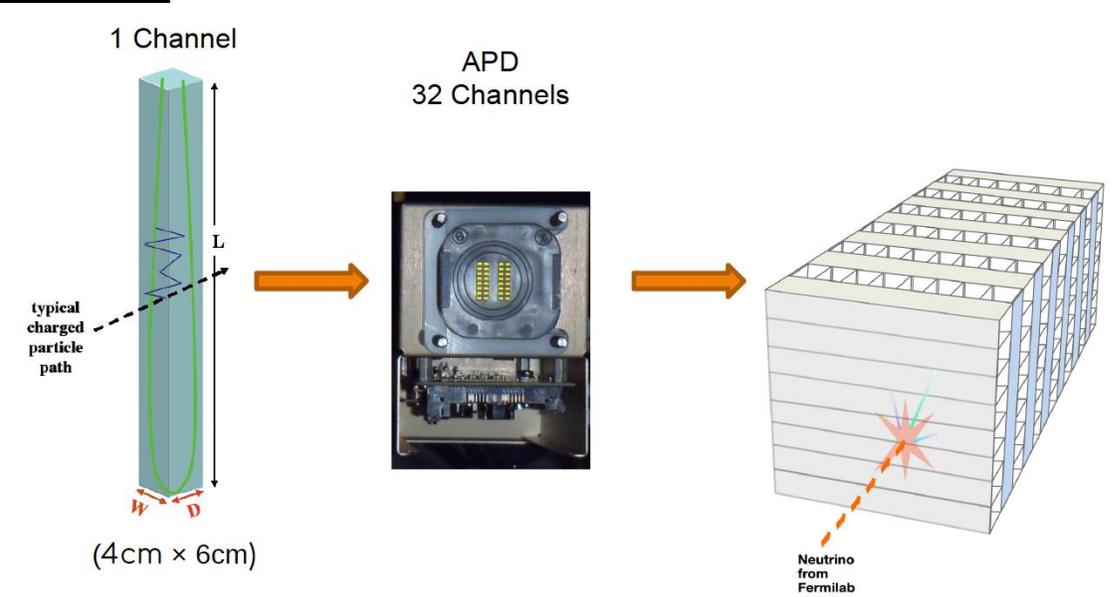
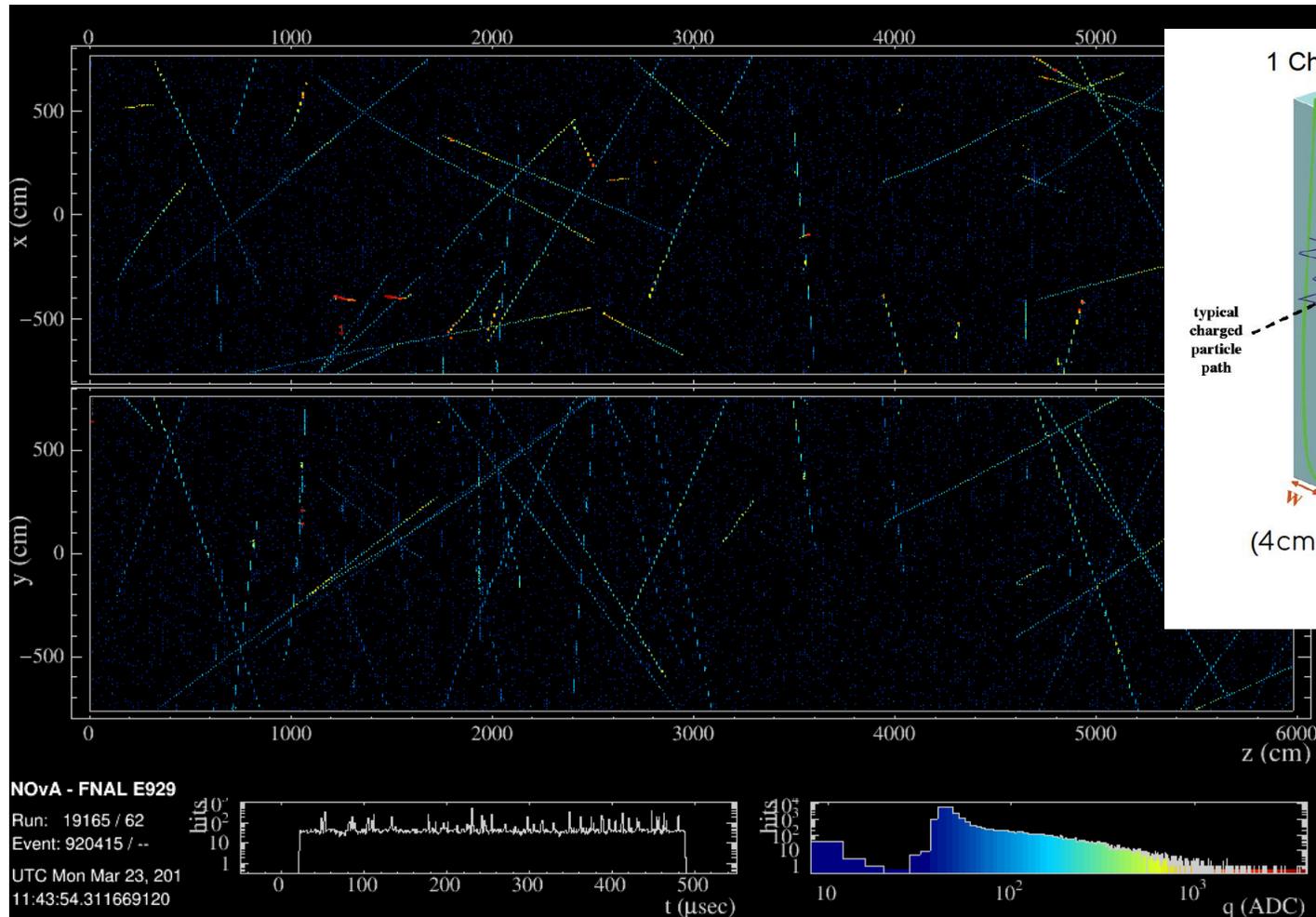


NOvA Experiment



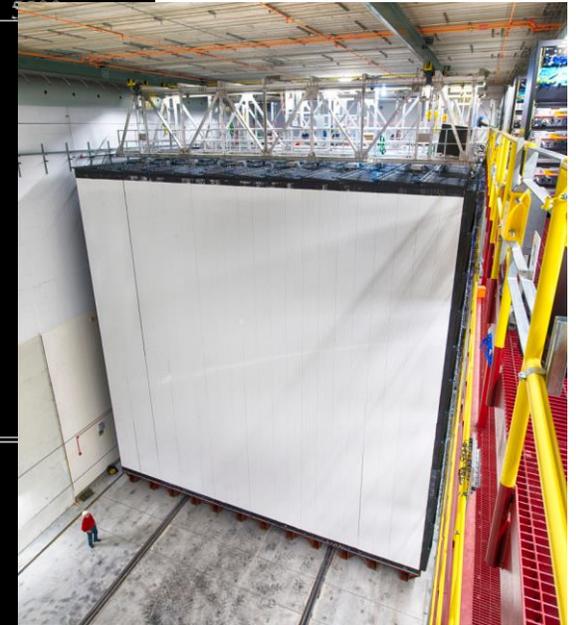
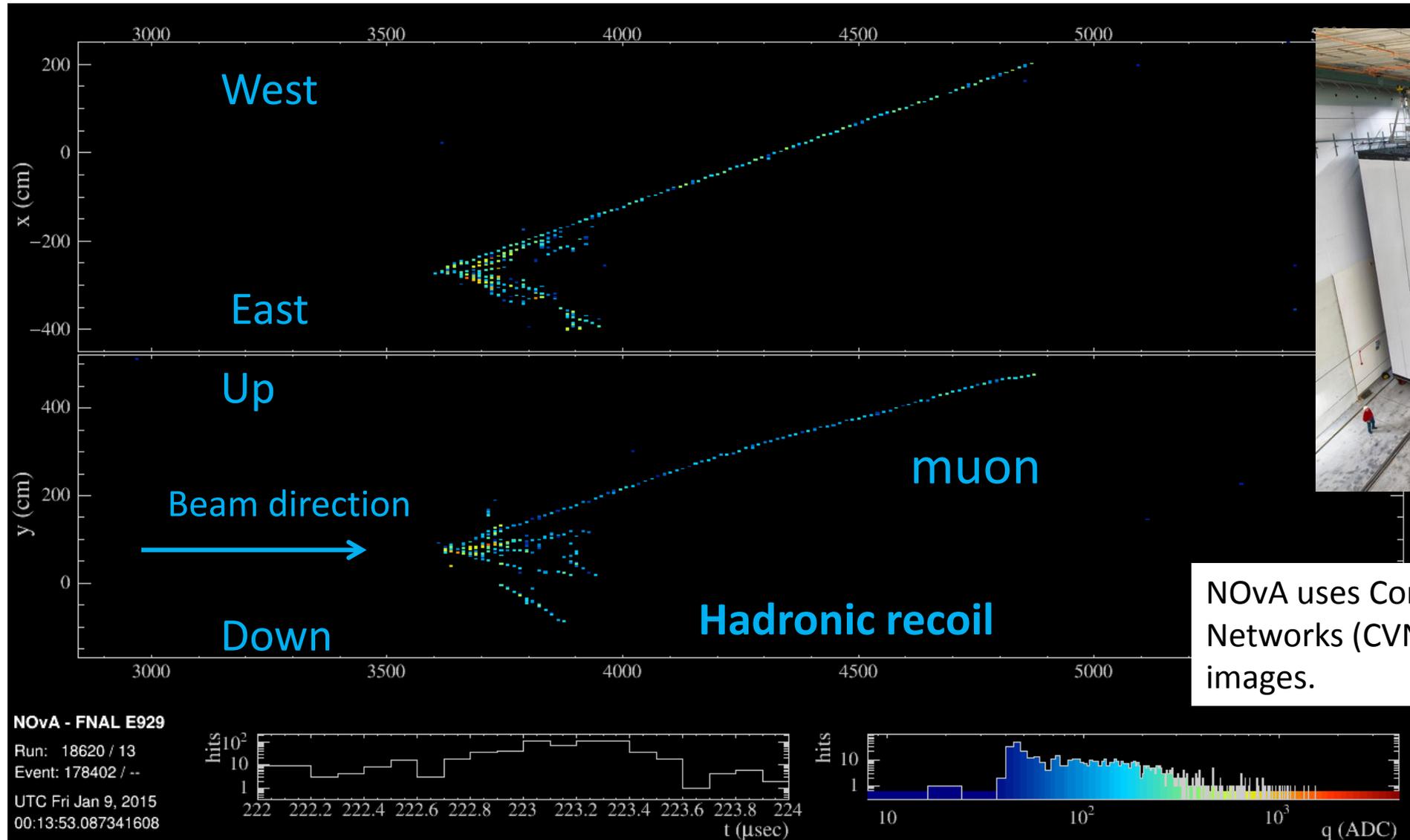
- NuMI beam: ν_{μ} or $\bar{\nu}_{\mu}$
- 2 functionally identical, tracking calorimeter detectors
 - Near: 300 T underground
 - Far: 14 kT on the surface
 - Placed off-axis to produce a narrow-band spectrum
- 810 km baseline
 - Longest baseline of current experiments.

NOvA is on the surface..

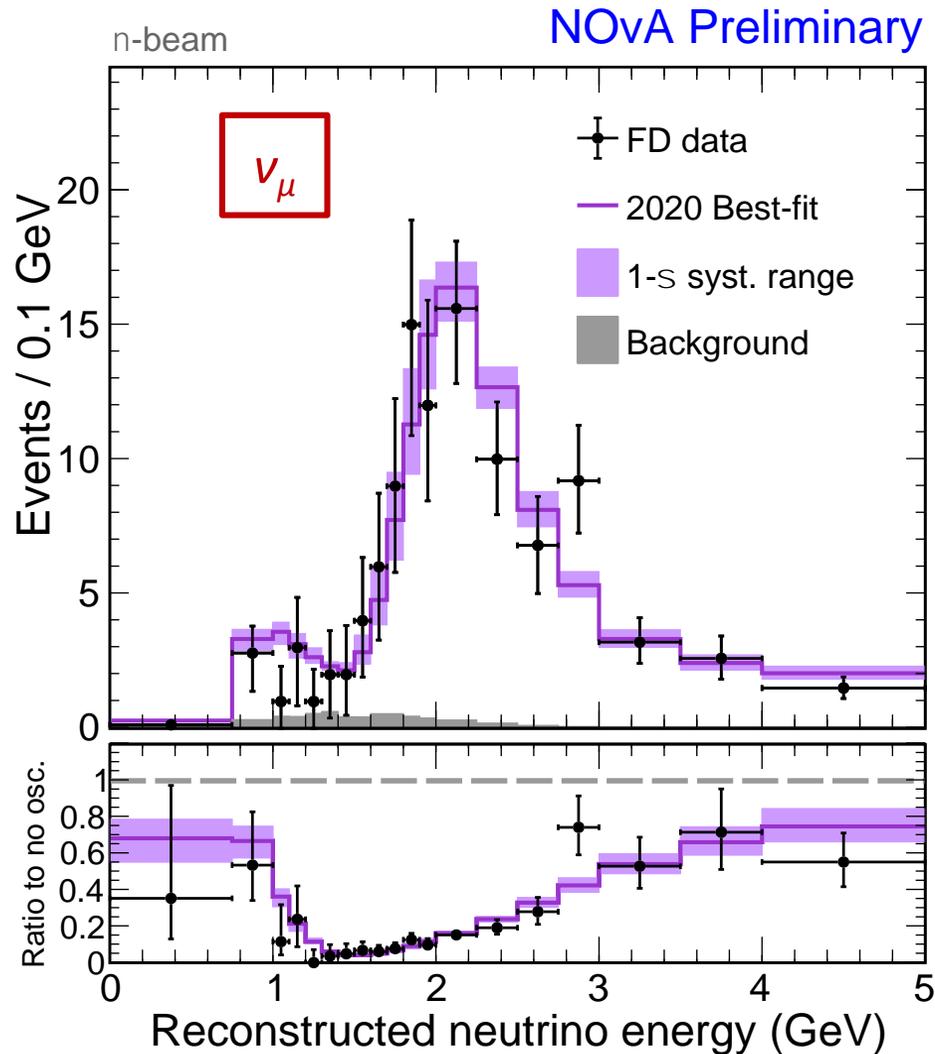


- 14 kt Far Detector
- Equivalent Near Detector
- Liquid scintillator (oil)
- Readout by APDs

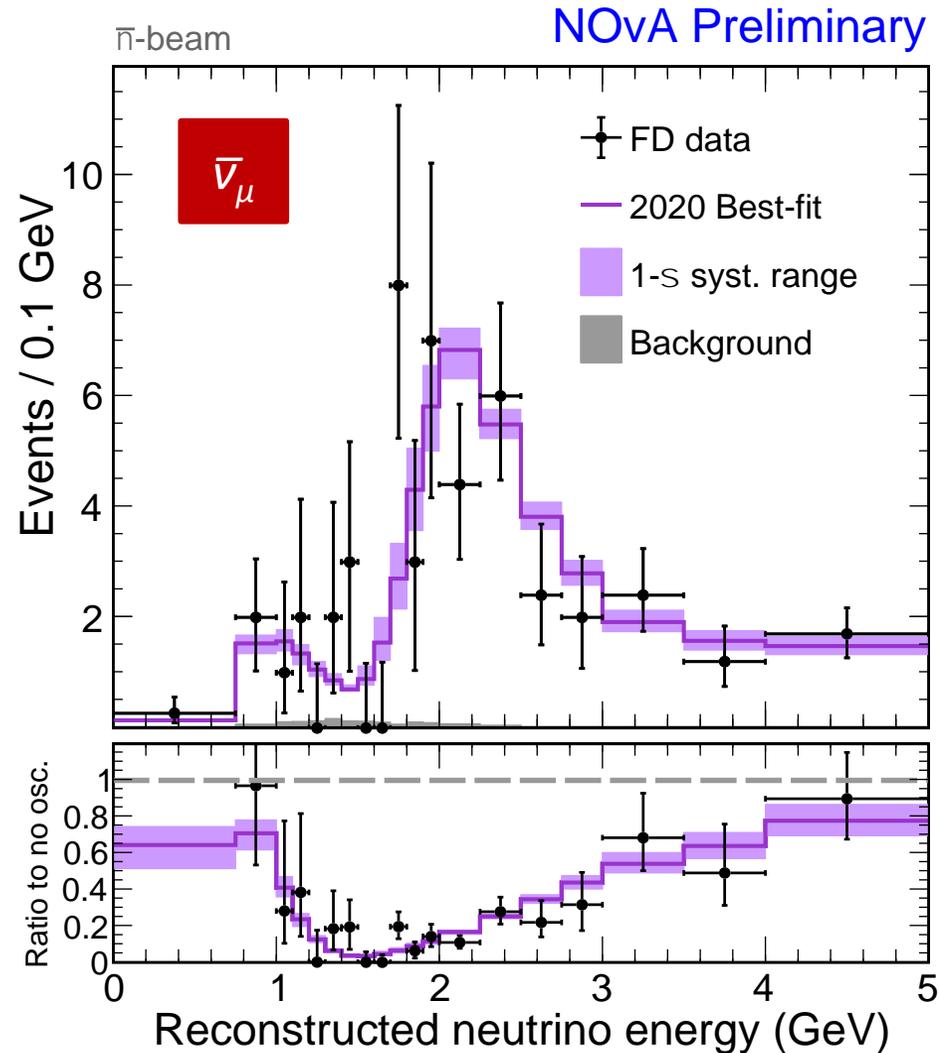
NOvA Detector



ν_μ and $\bar{\nu}_\mu$ disappearance at the NOvA Far Detector



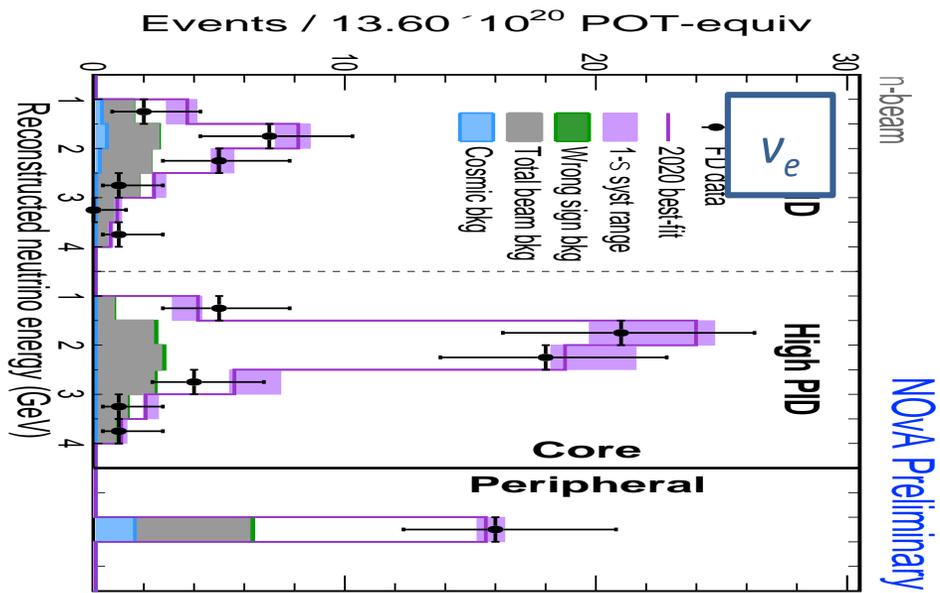
211 events, 8.2 background



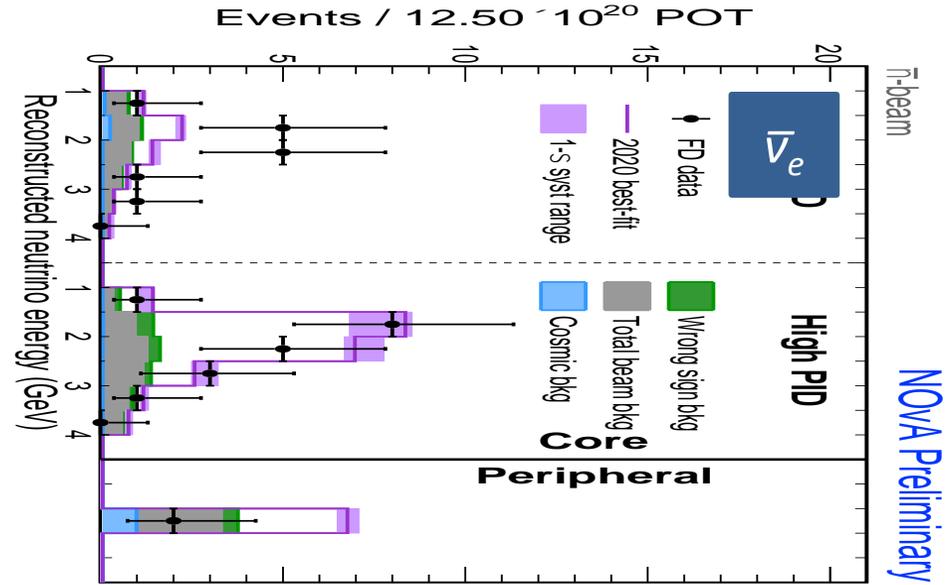
105 events, 2.1 background

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Neutrino 2020

ν_e and $\bar{\nu}_e$ appearance at the NOvA Far Detector



Total Observed	82	Range
Total Prediction	85.8	52-110
Total Bkgd.	26.8	26-28

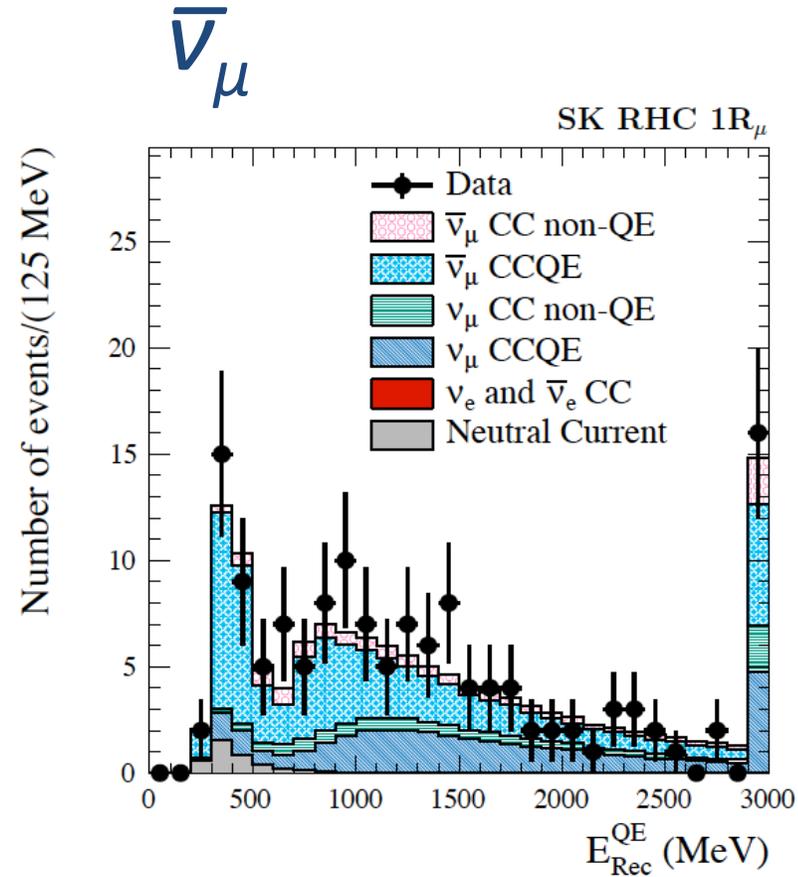
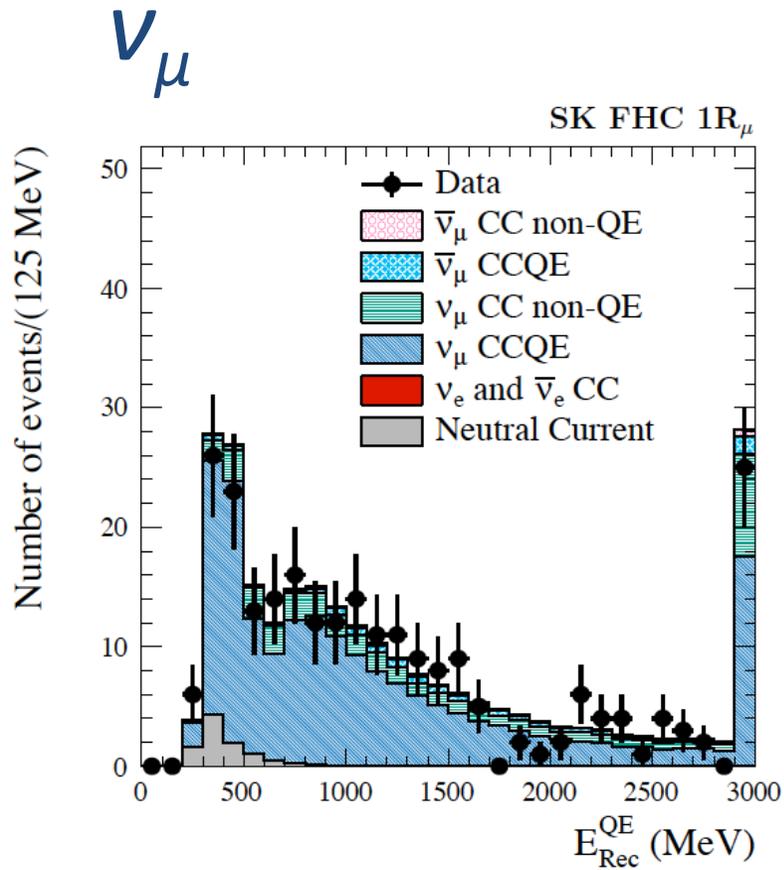


Total Observed	33	Range
Total Prediction	33.2	25-45
Total Bkgd.	14.0	13-15

>4 σ evidence of $\bar{\nu}_e$ appearance

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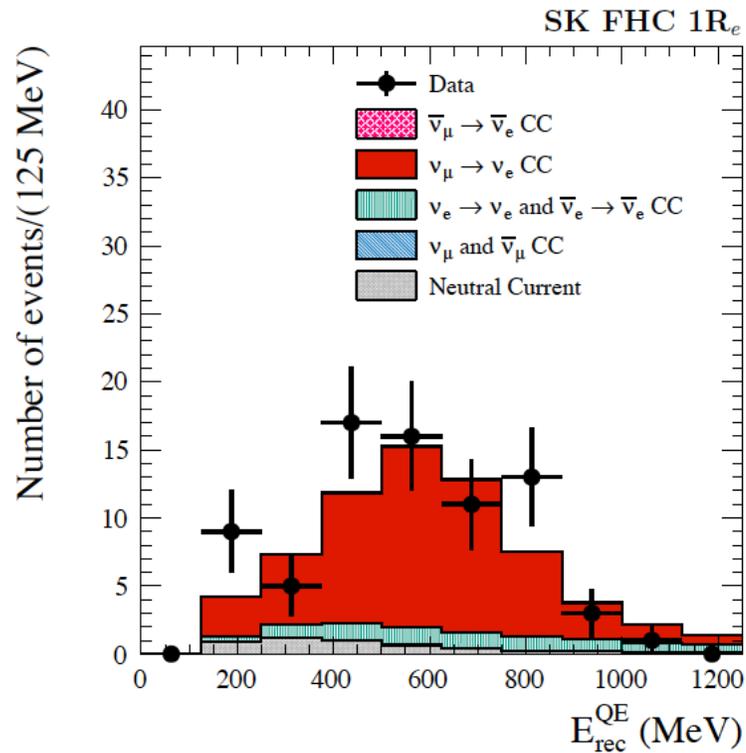
ν_μ and $\bar{\nu}_mu$ disappearance at the T2K Far Detector (SK)



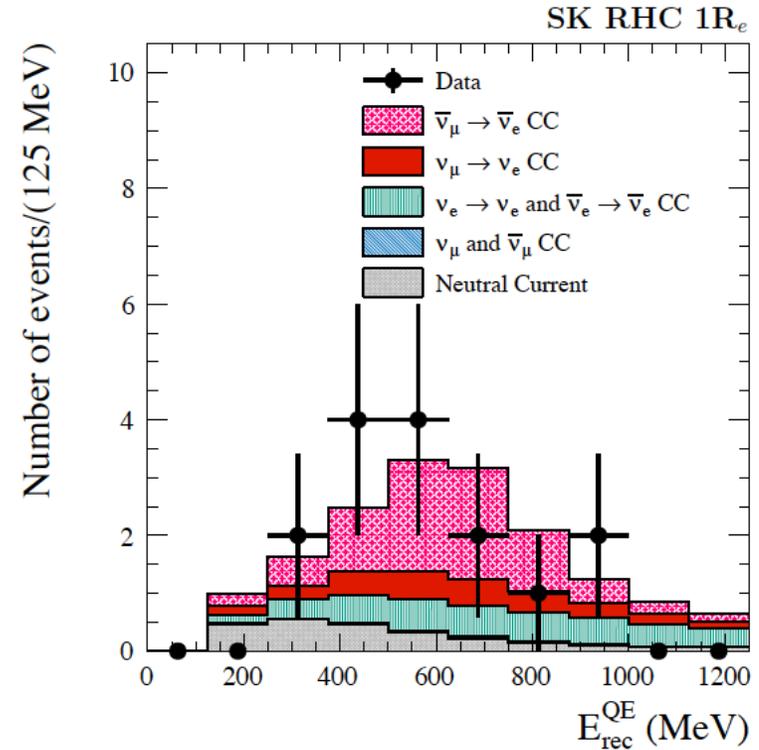
Muon-like rings

ν_e and $\bar{\nu}_e$ appearance at the T2K Far Detector (SK)

ν_e

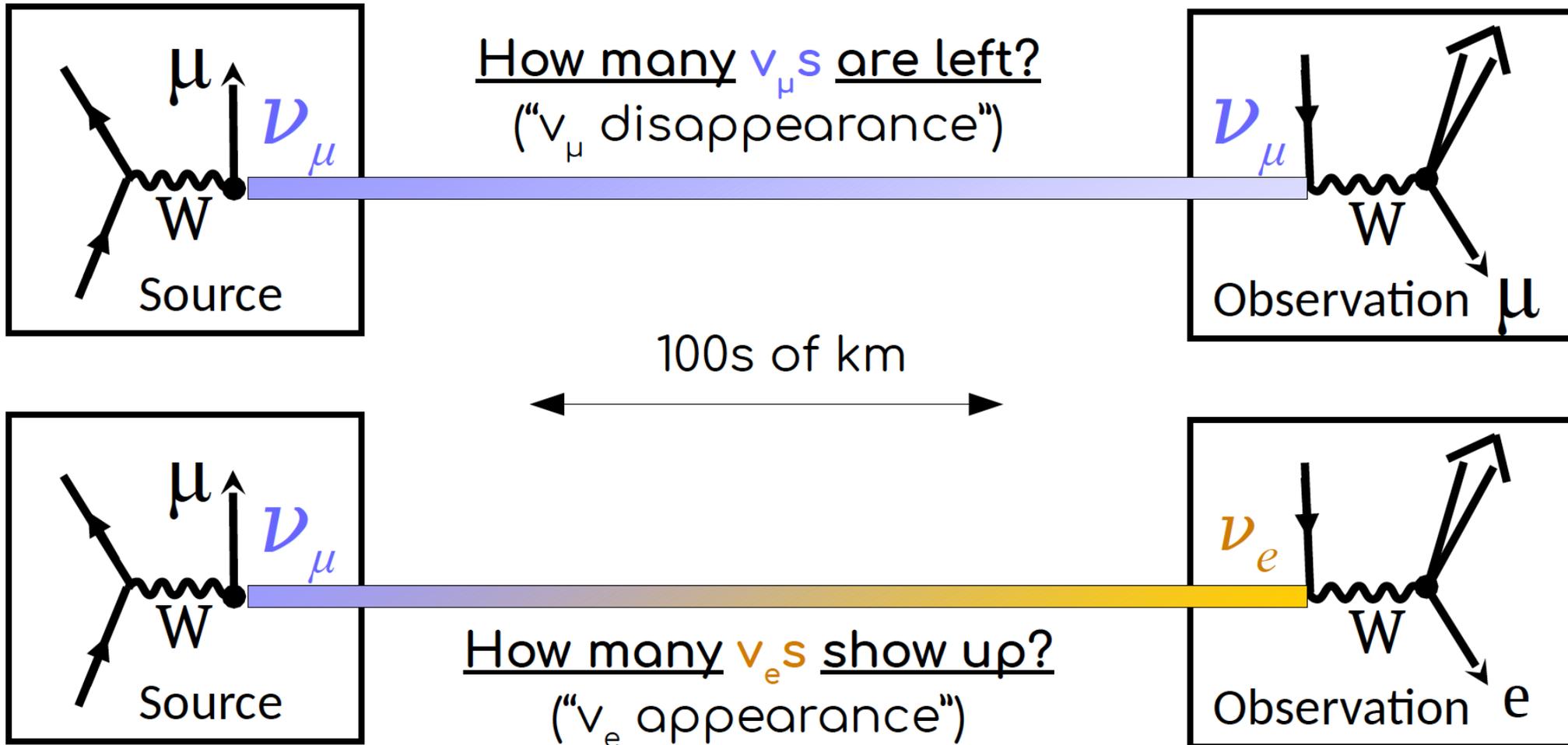


$\bar{\nu}_e$

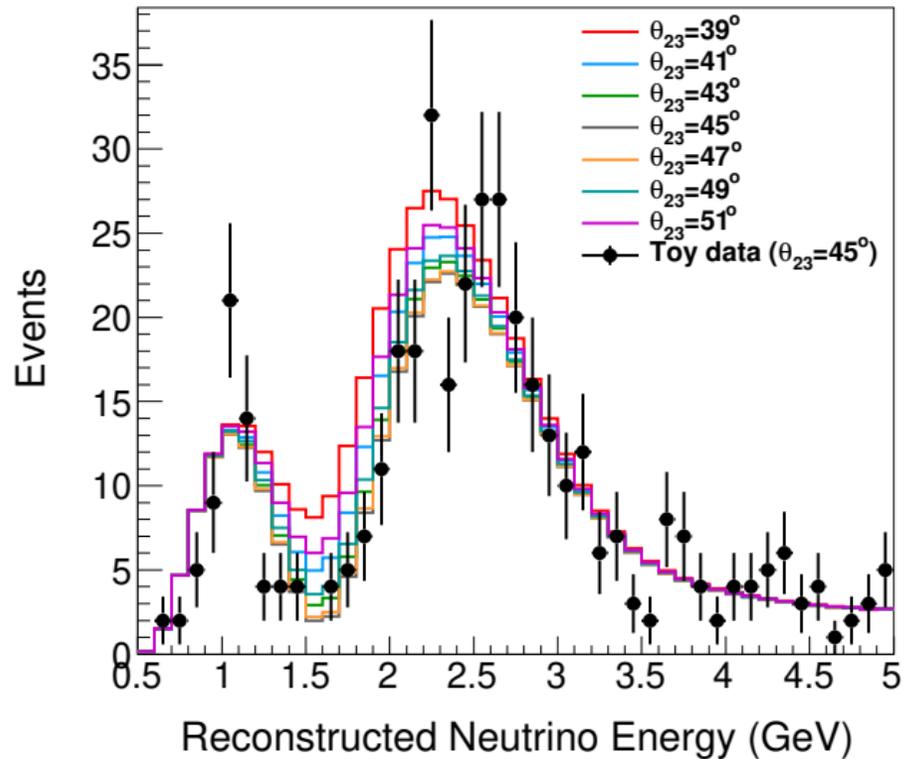


electron-like rings ($0 \pi^0$)

Extracting the Information



Extracting the Information



Simultaneous fits of

- Data samples in Near and Far Detector
- Flux model, incl. beam monitor and hadron production (NA61-SHINE)
- Cross section models
- Detector models for Near and Far Detector
- Error correlation matrix
- Oscillations Parameters

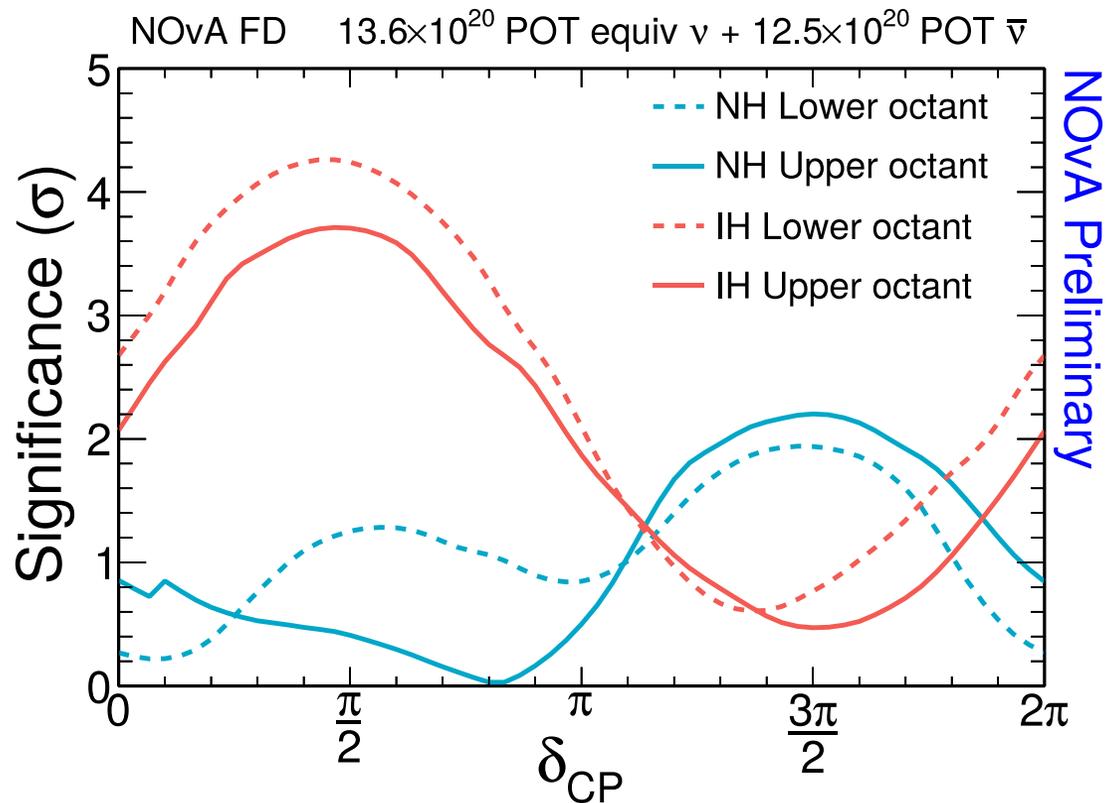
To give you a “flavour”

$$\begin{aligned}
 P(\nu_\mu \rightarrow \nu_e) &= \sin^2 \theta_{23} \sin^2 2\theta_{13} \frac{\sin^2(\Delta_{31} - aL)}{(\Delta_{31} - aL)^2} \Delta_{31}^2 \\
 &+ \sin 2\theta_{23} \sin 2\theta_{13} \sin 2\theta_{12} \frac{\sin(\Delta_{31} - aL)}{(\Delta_{31} - aL)} \Delta_{31} \frac{\sin(aL)}{aL} \Delta_{21} \cos(\Delta_{31} - \delta) \\
 &+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin(aL)}{aL} \Delta_{21}^2
 \end{aligned}$$

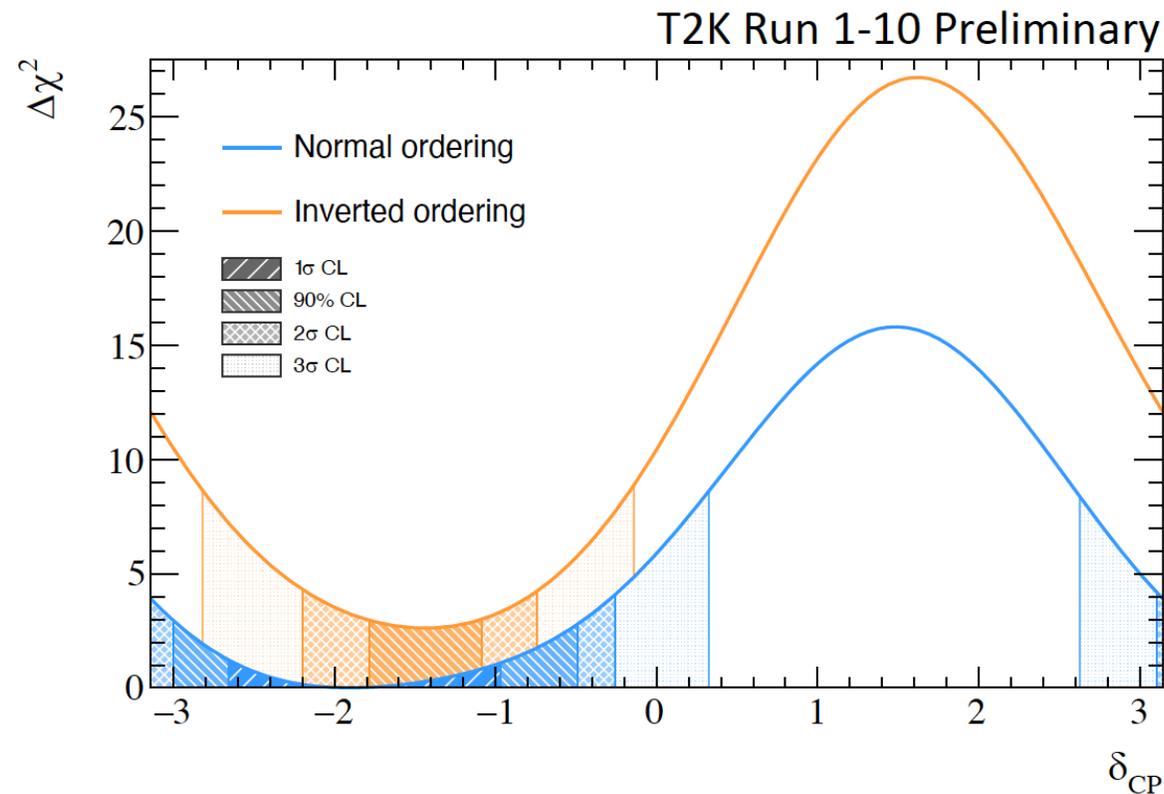
$$\begin{aligned}
 a &= \frac{G_F N_e}{2} \\
 \Delta_{ij} &= \frac{\Delta m_{ij}^2 L}{4E}
 \end{aligned}$$

- Electron-neutrino appearance is sensitive to the CP phase.
- Mass ordering effect depends on electron density N_e .
- Simultaneous determination of mass ordering and CP phase only possible with long-baselines by fitting the energy dependence of the flux modulation (unless we add atmospheric data).

CP Violation Phase



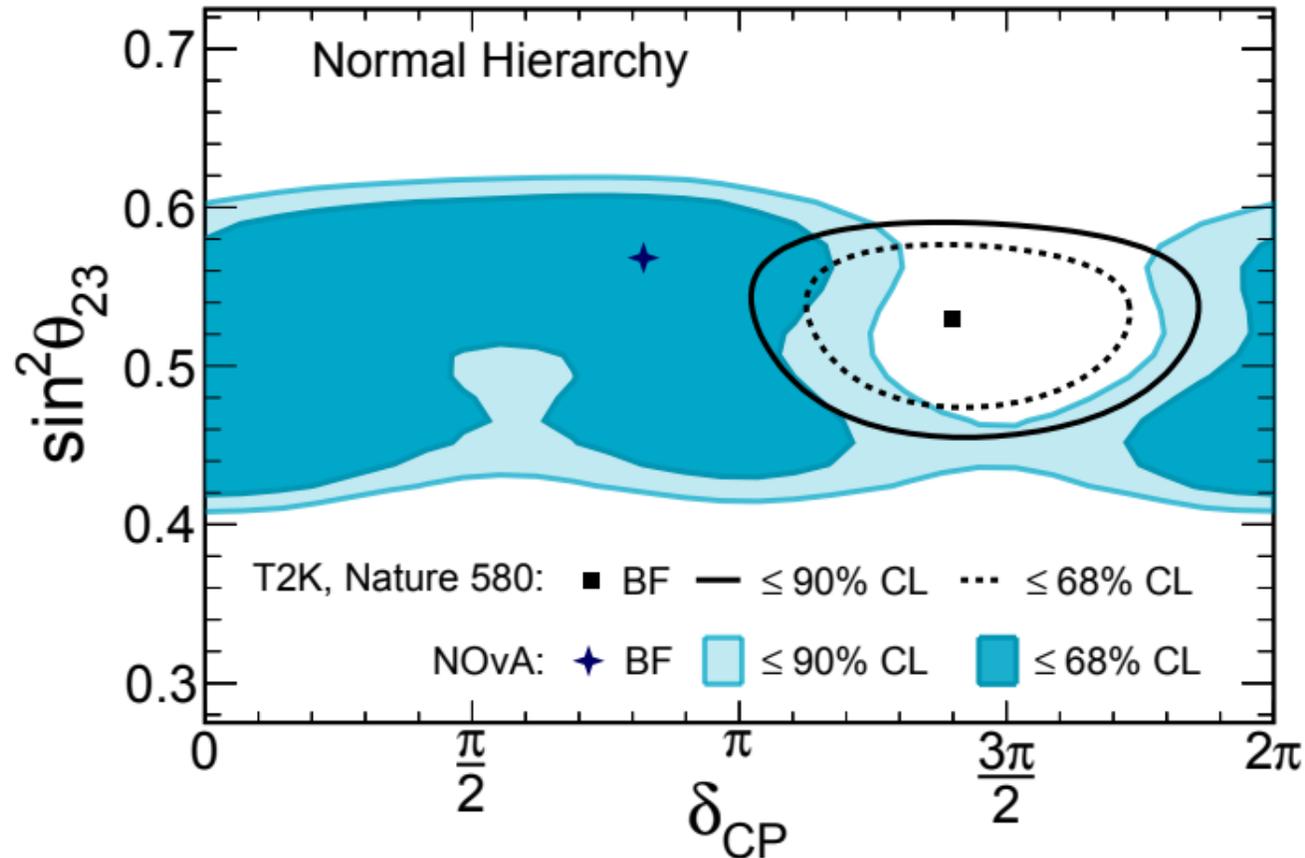
- Sensitivity to combination of CP phase and mass ordering.
- Normal ordering slightly preferred (1σ)
- Exclude IO, $\delta = \pi/2$ at $> 3\sigma$
- Disfavour NO, $\delta = 3\pi/2$ at $\sim 2\sigma$



- CP conversation $(0, \pi)$ excluded at 90% confidence level
- Normal ordering preferred

The Status Quo

NOvA Preliminary



- Current data are inconclusive – expect some improvements with further running
- Need next-generation experiments to discover CPV and resolve mass ordering