Quark and gluon contents (partons) of a lepton at high energies

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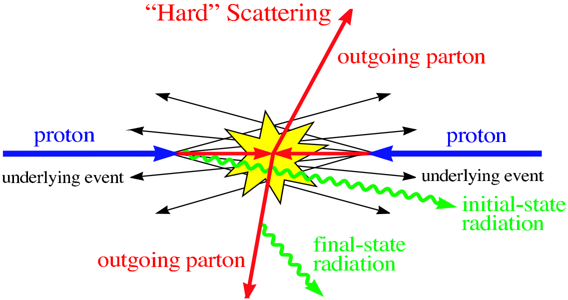
In collaboration with **Tao Han** and **Yang Ma**
2007.14300, 2103.09844

What is a "parton"?

Recall the hadron colliders: $pp(\bar{p})$ collision at Tevatron or LHC



Factorization formalism: PDFs $\otimes$ partonic cross sections

$$\boldsymbol{\sigma}(AB \to X) = \sum_{a,b} \int \mathrm{d}x_a \mathrm{d}x_b f_{a/A}(x_a, Q) f_{b/B}(x_b, Q) \hat{\boldsymbol{\sigma}}(ab \to X) + \cdots$$

- a, b are the "partons" from the beam particles A and B.
- $f_{a/A}$ ($f_{b/B}$) are PDFs, defined as the probabilities of finding partons a (b) from the beam particles A (B) with the momentum fractions x_a (x_b).

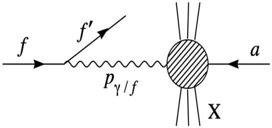
The simplest parton: photon inside of a lepton

"Equivalent photon approximation (EPA)" [Fermi, Z. Phys. 29, 315 (1924), von Weizsacker, Z. Phys. 88, 612 (1934)]

Treat photon as a parton constituent in the lepton [E. J. Williams, Phys. Rev. 45, 729 (1934)]

$$\sigma(\ell^- + a \to \ell^- + X) = \int \mathrm{d}x\, f_{\gamma/\ell}\hat{\sigma}(\gamma a \to X)$$

$$f_{\gamma/\ell,\mathrm{EPA}}(x_\gamma, Q^2) = \frac{\alpha}{2\pi}\frac{1+(1-x_\gamma)^2}{x_\gamma}\ln\frac{Q^2}{m_\ell^2}$$



Improvements:

[Frixione, Mangano, Nason, Ridolfi, 2103.09844]

[Budnev, Ginzburg, Meledin, Serbo, Phys. Rept.(1975)]

Applications at lepton colliders

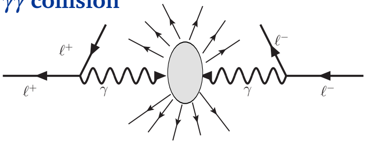
- Production cross sections

$$\sigma(\ell^+\ell^- \to F + X) = \int_{\tau_0}^{1} d\tau \sum_{ij} \frac{d\mathscr{L}_{ij}}{d\tau}\ \hat{\sigma}(ij \to F),\ \tau = \hat{s}/s$$

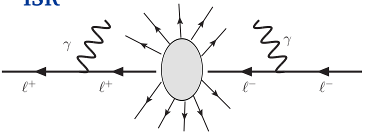
- Partonic luminosities

$$\frac{d\mathscr{L}_{ij}}{d\tau} = \frac{1}{1+\delta_{ij}}\int_{\tau}^{1}\frac{d\xi}{\xi}\left[f_i(\xi, Q^2)f_j\left(\frac{\tau}{\xi}, Q^2\right) + (i \leftrightarrow j)\right]$$

$\gamma\gamma$ collision



ISR



Applications of EPA at high-energy lepton colliders

What are the dominant processes at a high-energy lepton collider?

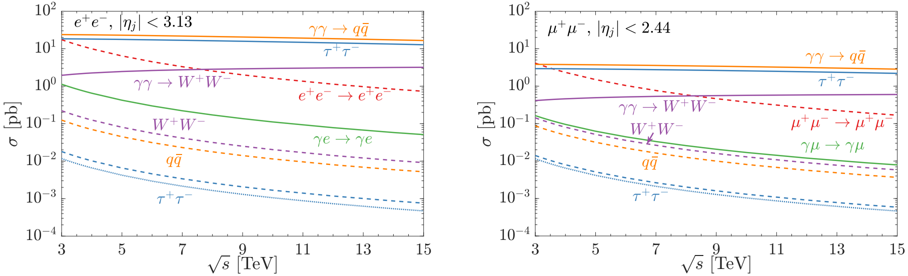
- Annihilation: $\ell^+\ell^- \to \ell^+\ell^-$, $\boldsymbol{\tau^+\tau^-}$, $q\bar{q}$, W^+W^-, and Compton: $\boldsymbol{\gamma\ell \to \gamma\ell}$
- $\boldsymbol{\gamma\gamma}$ fusion: $\boldsymbol{\gamma\gamma \to \tau^+\tau^-}$, $q\bar{q}$, W^+W^- [Backups for other high-energy processes]

Some typical cuts:

- Detector angle: $\theta_{\rm cut} = 5^\circ\,(10^\circ) \Longleftrightarrow |\boldsymbol{\eta}| < 3.13(2.44)$
- Threshold: $m_{ij} > 20$ GeV
- A p_T cut to avoid the nonperturbative hadronic production [details come later]

[Drees and Godbole, PRL 67, 1189; Chen, Barklow, and Peskin, hep-ph/9305247; T. Barklow, et al., LCD-2011-020]

$$p_T > \left(4 + \sqrt{s}/3\,\mathrm{TeV}\right)\,\mathrm{GeV}$$

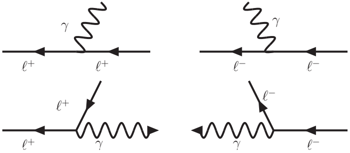


Go beyond the EPA at high-energy lepton colliders

We have been doing:

- $\ell^+\ell^-$ annihilation

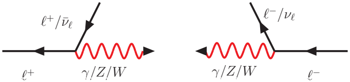
- EPA and ISR



- "Effective W Approx." (EWA)

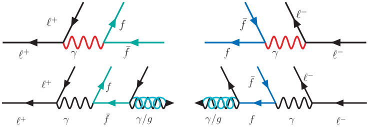
[G. Kane, W. Repko, and W. Rolnick, PLB 148 (1984) 367]

[S. Dawson, NPB 249 (1985) 42]

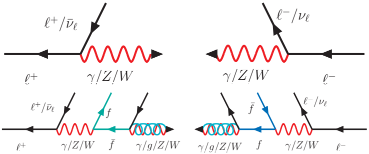


We complete:

- Above $\mu_{\rm QCD}$: QED$\otimes$QCD
  q/g emerge [T. Han, Y. Ma, KX, 2103.09844]



- Above $\mu_{\rm EW} = M_Z$: EW$\otimes$QCD
  EW partons emerge [T. Han, Y. Ma, KX, 2007.14300]



In the end, every content is a parton, i.e. the full SM PDFs.

The PDF evolution: DGLAP

- The DGLAP equations

$$\frac{\mathrm{d}f_i}{\mathrm{d}\log Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P_{ij}^I \otimes f_j$$

- The initial conditions

$$f_{\ell/\ell}(x, m_\ell^2) = \delta(1-x)$$

- Three regions and two matchings
  - $m_\ell < Q < \mu_{\rm QCD}$: QED
  - $Q = \mu_{\rm QCD} \lesssim 1$ GeV: $f_q \propto P_{q\gamma} \otimes f_\gamma, f_g = 0$
  - $\mu_{\rm QCD} < Q < \mu_{\rm EW}$: QED$\otimes$QCD
  - $Q = \mu_{\rm EW} = M_Z$: $f_\nu = f_t = f_W = f_Z = f_{\gamma Z} = 0$
  - $\mu_{\rm EW} < Q$: EW$\otimes$QCD.

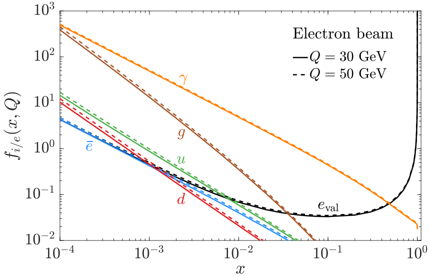
$$\begin{pmatrix} f_B \\ f_{W^3} \\ f_{BW^3} \end{pmatrix} = \begin{pmatrix} c_W^2 & s_W^2 & -2c_W s_W \\ s_W^2 & c_W^2 & 2c_W s_W \\ c_W s_W & -c_W s_W & c_W^2 - s_W^2 \end{pmatrix} \begin{pmatrix} f_\gamma \\ f_Z \\ f_{\gamma Z} \end{pmatrix}$$

- We work in the (B, W) basis. The technical details can be referred to the backup slides.

The QED$\otimes$QCD PDFs for lepton colliders

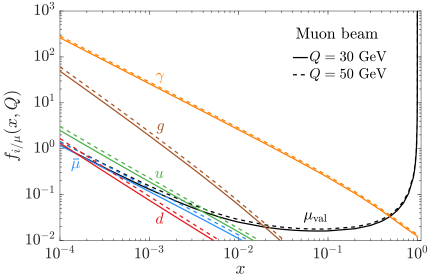
- **Electron beam**:
  $f_{e_{\rm val}}, f_\gamma, f_{\ell_{\rm sea}}, f_q, f_g$
- Scale uncertainty: 10% for $f_{g/e}$
- The averaged momentum fractions
  $\langle x_i \rangle = \int x f_i(x) \mathrm{d}x$

$Q(e^\pm)$	$e_{\rm val}$	γ	ℓsea	q	g
30 GeV	96.6	3.20	0.069	0.080	0.023
50 GeV	96.5	3.34	0.077	0.087	0.026
M_Z	96.3	3.51	0.085	0.097	0.028



- **Muon beam**: $f_{\mu_{\rm val}}, f_\gamma, f_{\ell_{\rm sea}}, f_q, f_g$
- Scale uncertainty: 20% for $f_{g/\mu}$
- The averaged momentum fractions
  $\langle x_i \rangle = \int x f_i(x) \mathrm{d}x$

$Q(\mu^\pm)$	$\mu_{\rm val}$	γ	ℓsea	q	g
30 GeV	98.2	1.72	0.019	0.024	0.0043
50 GeV	98.0	1.87	0.023	0.029	0.0051
M_Z	97.9	2.06	0.028	0.035	0.0062

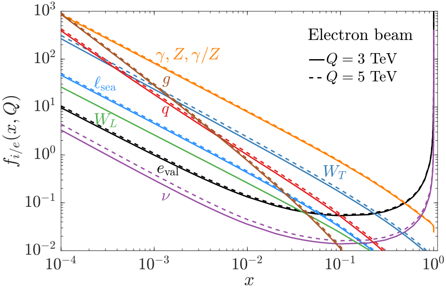


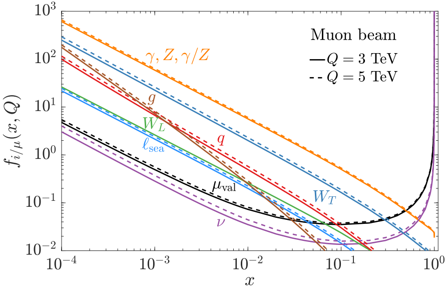
EWPDFs of a lepton

- The sea leptonic and quark PDFs

$$\nu = \sum_i (\nu_i + \bar{\nu}_i), \ \ell\text{sea} = \bar{\ell}_{\text{val}} + \sum_{i \neq \ell_{\text{val}}} (\ell_i + \bar{\ell}_i), \ q = \sum_{i=d}^{t} (q_i + \bar{q}_i)$$

There is even neutrino due to the EW sector, **every constitute is a parton!**





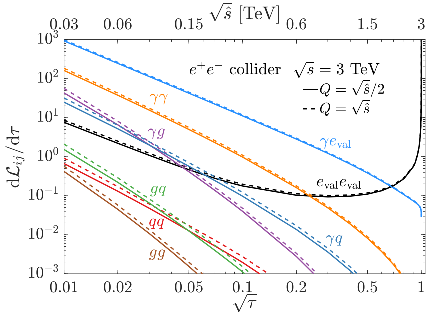
- **All SM particles are partons** [T. Han, Y. Ma, KX, 2007.14300]
- $W_L(Z_L)$ does not evolve: **Bjorken-scaling restoration**: $f_{W_L}(x) = \frac{\alpha_2}{4\pi}\frac{1-x}{x}$.
- The EW correction can be large: $\sim 50\%$ (100%) for $f_{d/e}$ $(f_{d/\mu})$ due to the relatively **large SU(2) gauge coupling**. [T. Han, Y. Ma, KX, 2103.09844]
- Scale uncertainty: $\sim 15\%$ (20%) between $Q = 3$ TeV and $Q = 5$ TeV

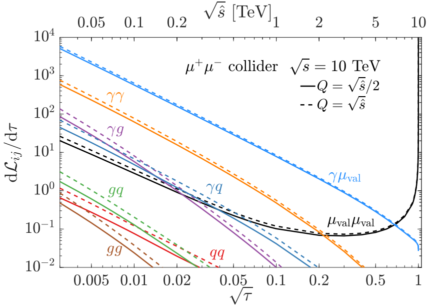
Parton luminosities at high-energy lepton colliders

Consider a 3 TeV e^+e^- machine and a 10 TeV $\mu^+\mu^-$ machine

- Partonic luminosities for

$$\ell^+\ell^-, \gamma\ell, \gamma\gamma, qq, \gamma q, \gamma g, gq, \text{ and } gg$$





- The partonic luminosity of $\gamma g + \gamma q$ is $\sim 50\%$ (20%) of the $\gamma\gamma$ one
- The partonic luminosities of qq, gq, and gg are $\sim 2\%$ (0.5%) of the $\gamma\gamma$ one
- Given the stronger QCD coupling, **sizable QCD cross sections are expected**.
- Scale uncertainty is $\sim 20\%$ (50%) for photon (gluon) initiated processes.

$\gamma\gamma \to$hadrons at CLIC

- Large photon induced non-perturbative hadronic production

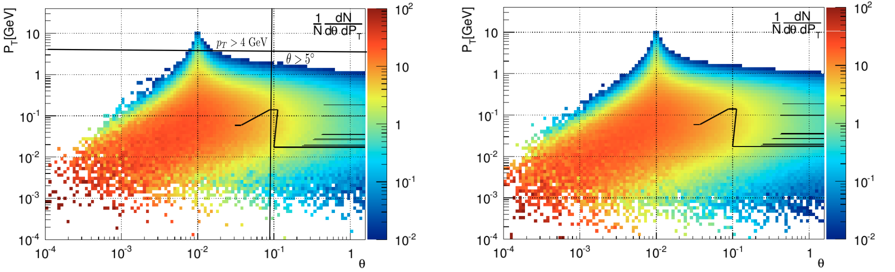
  [Drees and Godbole, PRL 67 (1991) 1189, hep-ph/9203219]

  [Chen, Barklow, and Peskin, hep-ph/9305247; Godbole et al., Nuovo Cim. C 034S1 (2011)]

  - $\sigma_{\gamma\gamma\to \text{ hadrons}}$ may reach micro-barns level at TeV c.m. energies
  - $\sigma_{\ell\ell\to \text{ hadrons}}$ may reach nano-barns, after folding in the $\gamma\gamma$ luminosity

- The events populate at low p_T regime

  So we can separate from this non-perturbative range via a p_T cut.



(a) Pythia sample

(b) SLAC sample

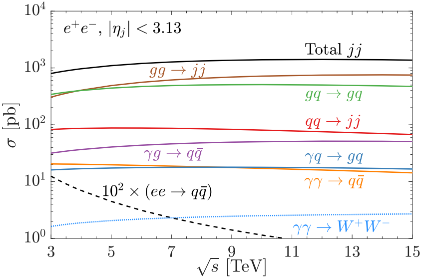
[T. Barklow, D. Dannheim, M. O. Sahin, and D. Schulte, LCD-2011-020]

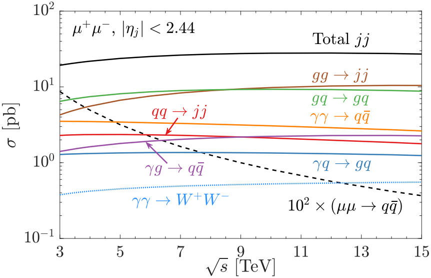
Di-jet production at possible lepton colliders

- High-p_T range $[p_T > (4 + \sqrt{s}/3\,\mathrm{TeV})\ \mathrm{GeV}]$: perturbatively computable

$$\gamma\gamma \to q\bar{q},\ \gamma g \to q\bar{q},\ \gamma q \to gq,$$
$$qq \to qq\ (gg),\ gq \to gq \text{ and } gg \to gg\ (q\bar{q}).$$

- Large $\alpha_s \ln(Q^2)$ brings a $6\% \sim 15\%$ $(30\% \sim 40\%)$ enhancement if $Q = \sqrt{\hat{s}}/2 \to Q = \sqrt{\hat{s}}$





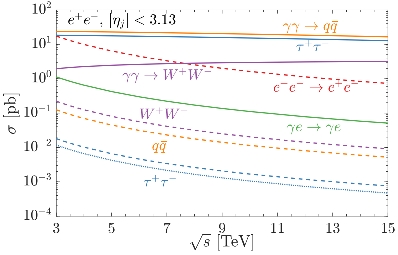
- Including the QCD contribution leads to much larger total cross section.
- gg initiated cross sections are large for its large multiplicity;
- gq initiated cross sections are large for its large luminosity.
- $\gamma\gamma$ initiated cross sections here are smaller than the EPA results.

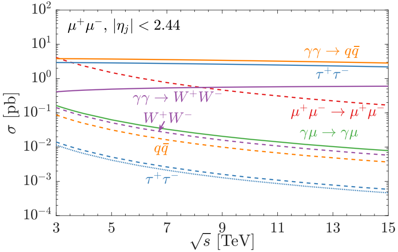
Refresh the picture of high-energy lepton colliders

What is the dominant process at a high-energy muon collider?

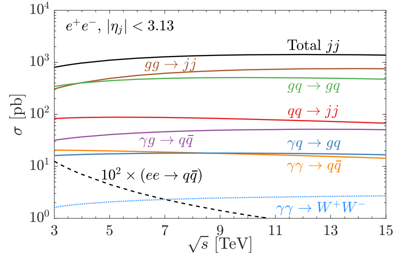
- **Quark/gluon initiated jet production dominates**

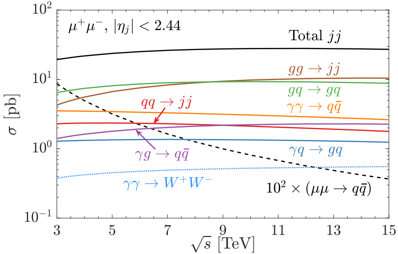
EPA:





q/g PDFs:

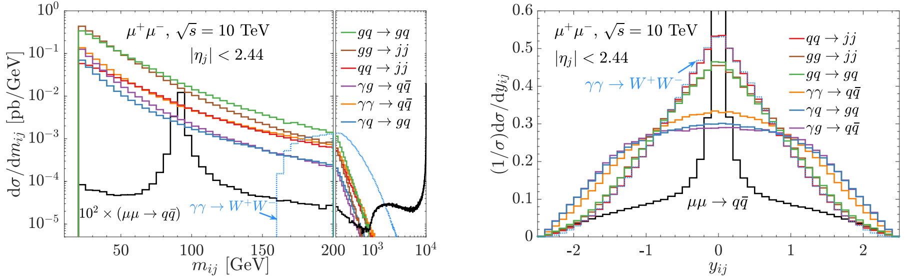




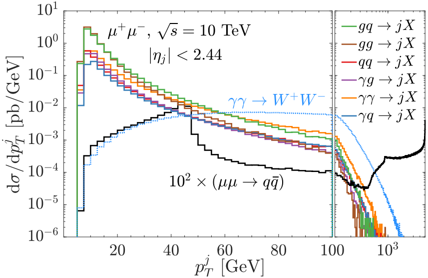
Di-jet distributions at a muon collider

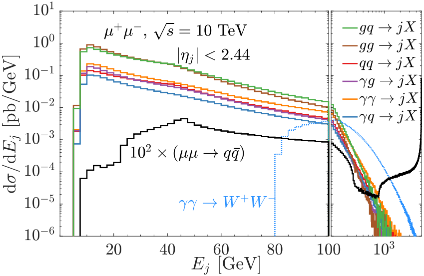
Rather a conservative set up: $\theta = 10^\circ$

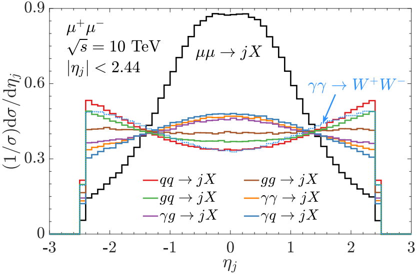
- Some physics:
  Two different mechanisms: $\mu^+\mu^-$ **annihilation** v.s. **fusion processes**
  - Annihilation is more than 2 orders of magnitude smaller than fusion process.
  - Annihilation peaks at $m_{ij} \sim \sqrt{s}$;
  - Fusion processes peak near m_{ij} threshold.
  - Annihilation is very central, spread out due to ISR;
  - Fusion processes spread out, especially for γq and γg initiated ones.



Inclusive jet distributions at a muon collider







- Jet production dominates over WW production until $p_T > 60$ GeV;
- WW production takes over around energy ~ 200 GeV.
- QCD contributions are mostly forward-backward; $\gamma\gamma$, γq, and γg initiated processes are more isotropic.

Summary and prospects

EWPDF is important and necessary:

- At very high energies, the collinear splittings dominate. **All SM particles should be treated as partons that described by proper PDFs.**
  - The large collinear logarithm needs to be resummed via solving the DGLAP equations, so the **QCD partons (quarks and gluons) emerge**.
  - When $Q > M_Z$, the EW splittings are activated: the EW partons appear, and the existing QED$\otimes$QCD PDFs may receive big corrections.

A high-energy muon collider is an EW version HE LHC

- There are many directions to work on: SUSY, DM, Higgs, etc.
- Two classes of processes: $\mu^+\mu^-$ annihilation v.s. VBF

  [T. Han, Y. Ma, KX, 2007.14300]
- **The main mechanisms of hadron production**:
  - Low p_T range: non-perturbative $\gamma\gamma$ initiated hadronic production dominates

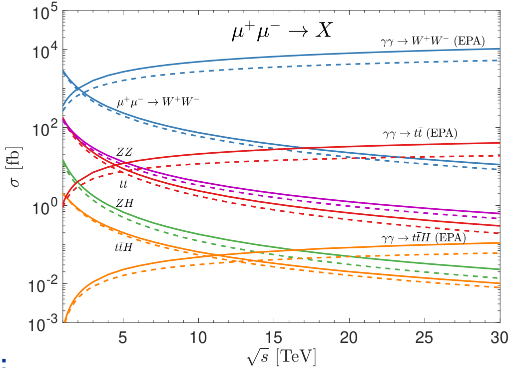
    [Chen, Barklow, and Peskin, hep-ph/9305247; Drees and Godbole, PRL 67, 1189, T. Barklow, et al., LCD-2011-020]
  - High p_T range, perturbative q and g initiated jet production dominates

    [T. Han, Y. Ma, KX, 2103.09844]

EPA for high-energy electroweak processes

What do people expect from a high-energy lepton (muon) collider?

[T. Han, Y. Ma, KX, 2007.14300]



General features:

- The annihilations decrease as $1/s$.
- ISR needs to be considered, which can give over 10% enhancement.
- The fusions increase as $\ln^p(s)$, which take over at high energies.
- The large collinear logarithm $\ln(s/m_\ell^2)$ needs to be resummed, set $Q = \sqrt{\hat{s}}/2$,
- $\gamma\gamma \to W^+W^-$ production has the largest cross section.

Solving the DGLAP: Singlet and Non-singlet PDFs

The singlets

$$f_L = \sum_{i=e,\mu,\tau}(f_{\ell_i}+f_{\bar{\ell}_i}),\ f_U = \sum_{i=u,c}(f_{u_i}+f_{\bar{u}_i}),\ f_D = \sum_{i=d,s,b}(f_{d_i}+f_{\bar{d}_i})$$

The non-singlets

- The only non-trivial singlet $f_{e,NS} = f_e - f_{\bar{e}}$
- the leptons

$$f_{\ell_i,NS} = f_{\ell_i} - f_{\bar{\ell}_i}(i=2,3),\ f_{\ell,12} = f_{\bar{e}} - f_{\bar{\mu}},\ f_{\ell,13} = f_{\bar{e}} - f_{\bar{\tau}};$$

- the up-type quarks

$$f_{u_i,NS} = f_{u_i} - f_{\bar{u}_i},\ f_{u,12} = f_u - f_c;$$

- and the down-type quarks

$$f_{d_i,NS} = f_{d_i} - f_{\bar{d}_i},\ f_{d,12} = f_d - f_s,\ f_{d,13} = f_d - f_b.$$

Reconstruction:

$$f_e = \frac{f_L + (2N_\ell - 1)f_{e,NS}}{2N_\ell},\ f_{\bar{e}} = f_\mu = f_{\bar{\mu}} = f_\tau = f_{\bar{\tau}} = \frac{f_L - f_{e,NS}}{2N_\ell}.$$

$$f_u = f_{\bar{u}} = f_c = f_{\bar{c}} = \frac{f_U}{2N_u},\ f_d = f_{\bar{d}} = f_s = f_{\bar{s}} = f_b = f_{\bar{b}} = \frac{f_D}{2N_d}.$$

The QED⊗QCD case

- The singlets and gauge bosons ($L = \log Q^2$)

$$\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix} = \begin{pmatrix} P_{\ell\ell} & 0 & 0 & 2N_\ell P_{\ell\gamma} & 0 \\ 0 & P_{uu} & 0 & 2N_u P_{u\gamma} & 2N_u P_{ug} \\ 0 & 0 & P_{dd} & 2N_d P_{d\gamma} & 2N_d P_{dg} \\ P_{\gamma\ell} & P_{\gamma u} & P_{\gamma d} & P_{\gamma\gamma} & 0 \\ 0 & P_{gu} & P_{gd} & 0 & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix}$$

- The non-singlets

$$\frac{\mathrm{d}}{\mathrm{d}L} f_{NS} = P_{ff} \otimes f_{NS}.$$

- The averaged momentum fractions of the PDFs: $f_{\ell_{\mathrm{val}}}, f_\gamma, f_{\ell_{\mathrm{sea}}}, f_q, f_g$

$$\langle x_i \rangle = \int x f_i(x)\mathrm{d}x,\ \sum_i \langle x_i \rangle = 1$$

$$\frac{\langle x_q \rangle}{\langle x_{\ell\mathrm{sea}} \rangle} \lesssim \frac{N_c\left[\sum_i (e_{u_i}^2 + e_{\bar{u}_i}^2) + \sum_i (e_{d_i}^2 + e_{\bar{d}_i}^2)\right]}{e_{\bar{\ell}_{\mathrm{val}}}^2 + \sum_{i\neq \ell\mathrm{val}} (e_{\ell_i}^2 + e_{\bar{\ell}_i}^2)} = \frac{22/3}{5}$$

The DGLAP for the full SM

$$\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix} f_L^{0\pm} \\ f_Q^{0\pm} \\ f_E^{0\pm} \\ f_U^{0\pm} \\ f_D^{0\pm} \\ f_B^{0\pm} \\ f_W^{0\pm} \\ f_g^{0\pm} \end{pmatrix} = \begin{pmatrix} P_{LL}^{0\pm} & 0 & 0 & 0 & 0 & P_{LB}^{0\pm} & P_{LW}^{0\pm} & 0 \\ 0 & P_{QQ}^{0\pm} & 0 & 0 & 0 & P_{QB}^{0\pm} & P_{QW}^{0\pm} & P_{Qg}^{0\pm} \\ 0 & 0 & P_{EE}^{0\pm} & 0 & 0 & P_{EB}^{0\pm} & 0 & 0 \\ 0 & 0 & 0 & P_{UU}^{0\pm} & 0 & P_{UB}^{0\pm} & 0 & P_{Ug}^{0\pm} \\ 0 & 0 & 0 & 0 & P_{DD}^{0\pm} & P_{DB}^{0\pm} & 0 & P_{Dg}^{0\pm} \\ P_{BL}^{0\pm} & P_{BQ}^{0\pm} & P_{BE}^{0\pm} & P_{BU}^{0\pm} & P_{BD}^{0\pm} & P_{BB}^{0\pm} & 0 & 0 \\ P_{WL}^{0\pm} & P_{WQ}^{0\pm} & 0 & 0 & 0 & 0 & P_{WW}^{0\pm} & 0 \\ 0 & P_{gQ}^{0\pm} & 0 & P_{gU}^{0\pm} & P_{gD}^{0\pm} & 0 & 0 & P_{gg}^{0\pm} \end{pmatrix} \otimes \begin{pmatrix} f_L^{0\pm} \\ f_Q^{0\pm} \\ f_E^{0\pm} \\ f_U^{0\pm} \\ f_D^{0\pm} \\ f_B^{0\pm} \\ f_W^{0\pm} \\ f_g^{0\pm} \end{pmatrix}$$

$$\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix} f_L^{1\pm} \\ f_Q^{1\pm} \\ f_W^{1\pm} \\ f_{BW}^{1\pm} \end{pmatrix} = \begin{pmatrix} P_{LL}^{1\pm} & 0 & P_{LW}^{1\pm} & P_{LM}^{1\pm} \\ 0 & P_{QQ}^{1\pm} & P_{QW}^{1\pm} & P_{QM}^{1\pm} \\ P_{WL}^{1\pm} & P_{WQ}^{1\pm} & P_{WW}^{1\pm} & 0 \\ P_{ML}^{1\pm} & P_{MQ}^{1\pm} & 0 & P_{MM}^{1\pm} \end{pmatrix} \otimes \begin{pmatrix} f_L^{1\pm} \\ f_Q^{1\pm} \\ f_W^{1\pm} \\ f_{BW}^{1\pm} \end{pmatrix}$$

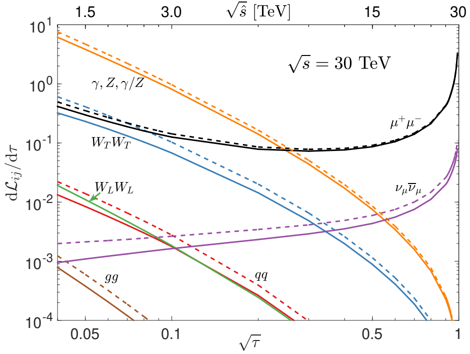
$$\frac{\mathrm{d}}{\mathrm{d}L} f_W^{2\pm} = P_{WW}^{2\pm} \otimes f_{WW}^{2\pm}$$

The splitting functions can be constructed in terms of Refs.

[Chen et al., 1611.00788, Bauer et al., 1703.08562, 1808.08831]

The EW parton luminosities at a high-energy muon collider

- All SM particles are partons when the machine energy is high
- We are able to determine the partons with their different polarizations

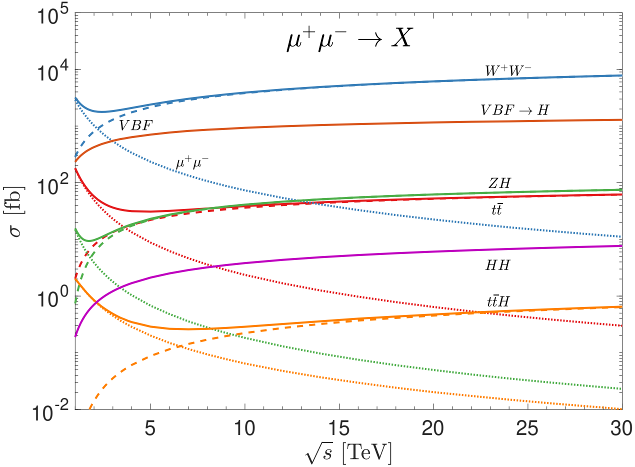


[T. Han, Y. Ma, KX, 2007.14300]

The full picture: Semi-inclusive processes

Just like in hadronic collisions:

$$\mu^+\mu^- \to \text{exclusive particles} + \text{remnants}$$



[T. Han, Y. Ma, KX, 2007.14300]