Positivity bounds in SMEFT and the inverse problem

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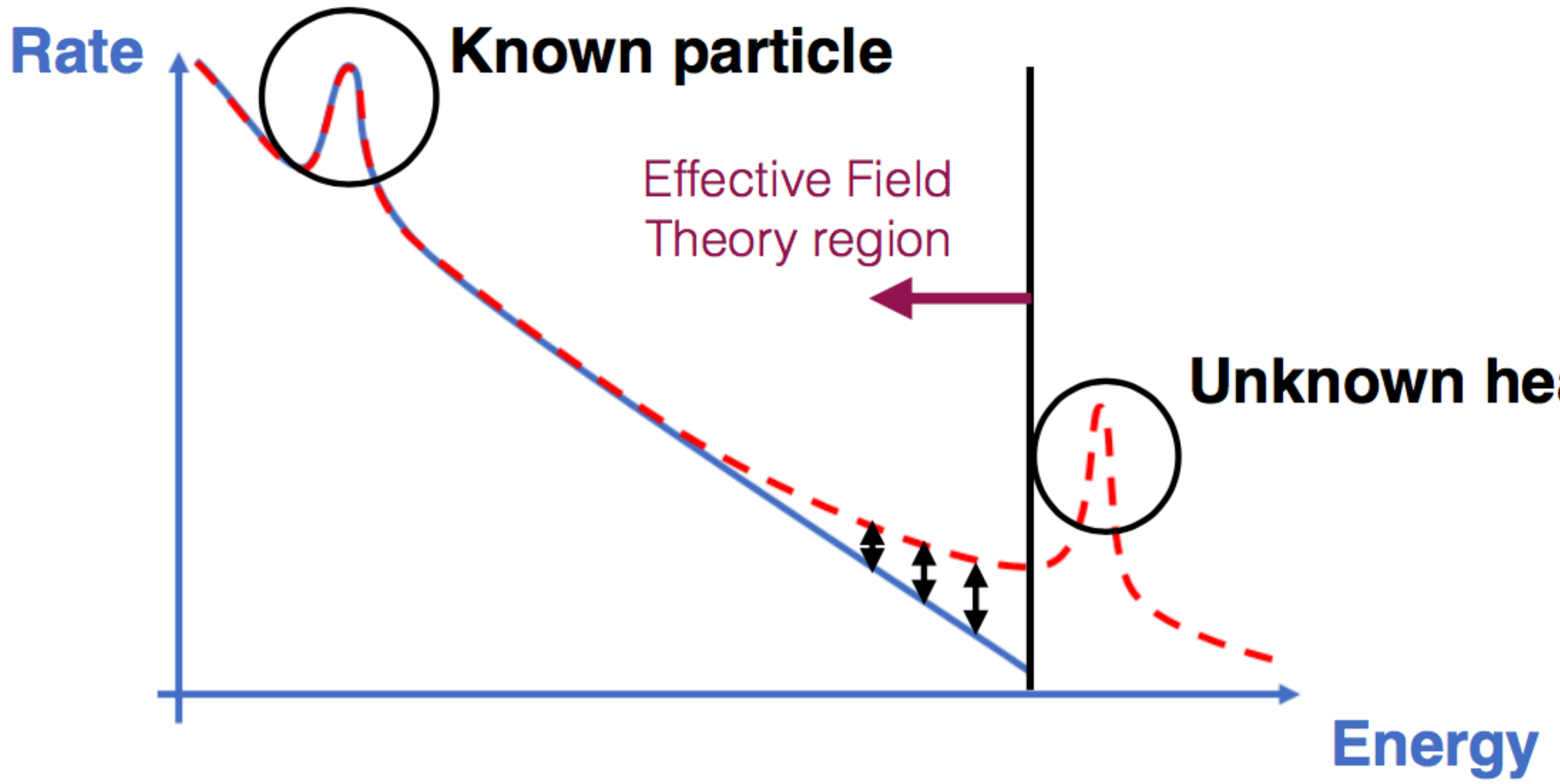
Based on 2005.03047 with S.-Y. Zhou, 2009.02212 with B. Fuks, Y. Liu and S.-Y. Zhou, 2101.01191 with X. Li, H. Xu, C. Yang, S.-Y. Zhou, and ongoing works.

New particles? New interactions?



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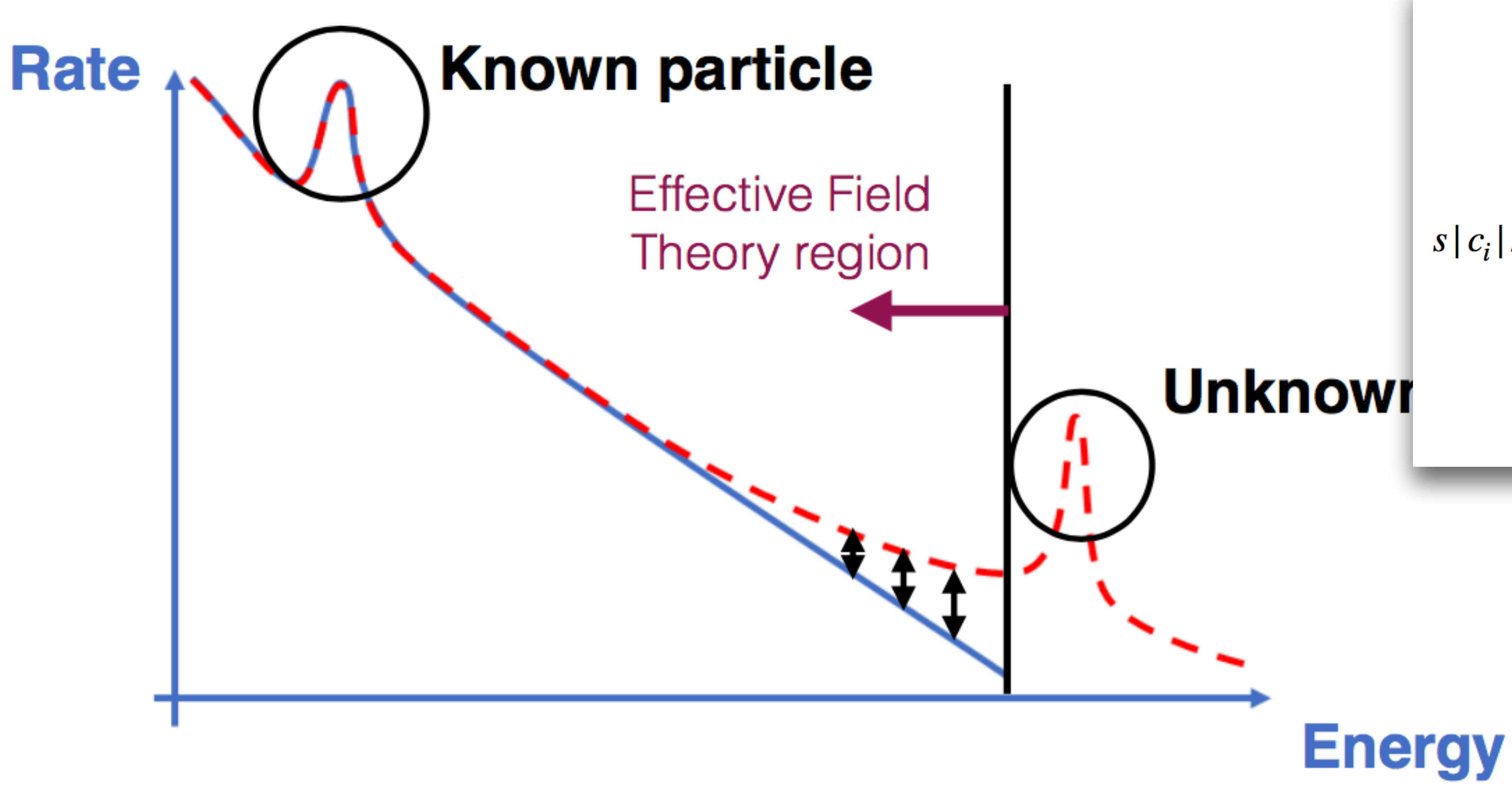
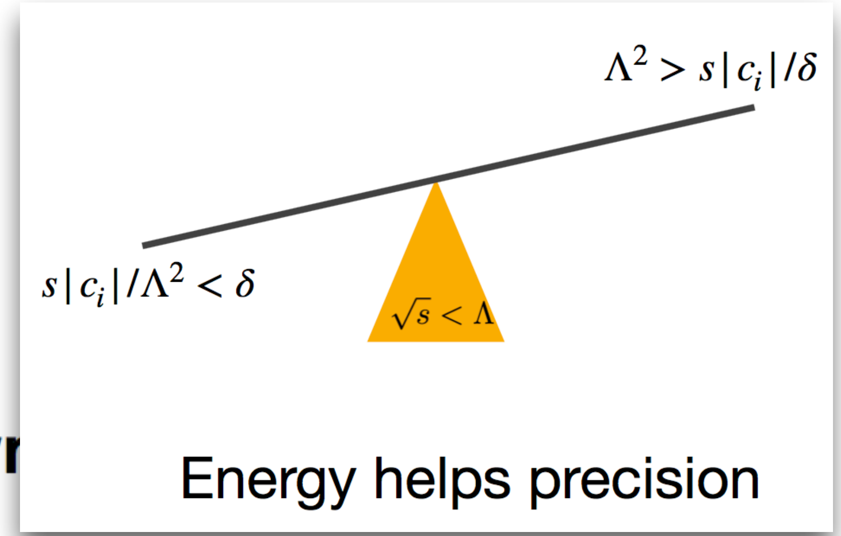




Precise measurements at low energy => probe BSM beyond the collider reach.

$$\mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{C_i^{(6)} O_i^{(6)}}{\Lambda^2} + \sum_i \frac{C_i^{(8)} O_i^{(8)}}{\Lambda^4} + \cdots$$

Dim-6 (84)

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2 \varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger \varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger \varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r) \widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger \varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r) \tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r) \widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger \varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r) \varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r) \tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger \tau^I \varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r) \varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p \gamma_\mu l_r)(\bar l_s \gamma^\mu l_t)$	Q_{ee}	$(\bar e_p \gamma_\mu e_r)(\bar e_s \gamma^\mu e_t)$	Q_{le}	$(\bar l_p \gamma_\mu l_r)(\bar e_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar q_s \gamma^\mu q_t)$	Q_{uu}	$(\bar u_p \gamma_\mu u_r)(\bar u_s \gamma^\mu u_t)$	Q_{lu}	$(\bar l_p \gamma_\mu l_r)(\bar u_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p \gamma_\mu \tau^I q_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p \gamma_\mu d_r)(\bar d_s \gamma^\mu d_t)$	Q_{ld}	$(\bar l_p \gamma_\mu l_r)(\bar d_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p \gamma_\mu l_r)(\bar q_s \gamma^\mu q_t)$	Q_{eu}	$(\bar e_p \gamma_\mu e_r)(\bar u_s \gamma^\mu u_t)$	Q_{qe}	$(\bar q_p \gamma_\mu q_r)(\bar e_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p \gamma_\mu \tau^I l_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p \gamma_\mu e_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar u_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p \gamma_\mu u_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar u_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p \gamma_\mu T^A u_r)(\bar d_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar d_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar d_s \gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$		

[B. Grzadkowski et al., 1008.4884]

Dim-8 classes (993)

#	Class	$N_{\rm type}$	$N_{\rm term}$	$N_{\rm op}$ [10]
1	X^4	7	43	43
2	H^8	1	1	1
3	H^6D^2	1	2	2
4	H^4D^4	1	3	3
5	X^3H^2	3	6	6
6	X^2H^4	5	10	10
7	$X^2H^2D^2$	4	18	18
8	XH^4D^2	2	6	6
9	ψ^2X^2H	16	96	$96n_g^2$
10	ψ^2XH^3	8	22	$22n_g^2$
11	$\psi^2H^2D^3$	6	16	$16n_g^2$
12	ψ^2H^5	3	6	$6n_g^2$
13	ψ^2H^4D	6	13	$13n_g^2$
14	ψ^2X^2D	21	57	$57n_g^2$
15	ψ^2XH^2D	16	92	$92n_g^2$
16	ψ^2XHD^2	8	48	$48n_g^2$
17	$\psi^2H^3D^2$	3	36	$36n_g^2$
$18(B)$	ψ^4H^2	19	75	$n_g^2(67n_g^2+n_g+7)$
$18(\not B)$		$4+3$	$12+8$	$\frac{1}{3}n_g^2(43n_g^2-9n_g+2)$
$19(B)$	ψ^4X	$40+5$	$156+12$	$4n_g^2(40n_g^2-1)$
$19(\not B)$		4	$44+12$	$2n_g^3(21n_g+1)$
$20(B)$	ψ^4HD	16	$134+2$	$n_g^3(135n_g-1)$
$20(\not B)$		7	32	$n_g^3(29n_g+3)$
$21(B)$	ψ^4D^2	18	55	$\frac{11}{2}n_g^2(9n_g^2+1)$
$21(\not B)$		4	$10+2$	$n_g^3(11n_g-1)$
	B	$204+5$	$895+14$	$895(36971),\ n_g=1(3)$
	$\not B$	$19+3$	$98+22$	$98(7836),\ n_g=1(3)$
	Total	$223+8$	$993+36$	$993(44807),\ n_g=1(3)$

[H.-L. Li et al., 2020] [C. Murphy, 2020]

See also:

Dim-7: **[L. Lehman, 14] [Liao & Ma, 16]**

Dim-9: **[H.-L. Li et al., 20] [Liao & Ma, 20]**

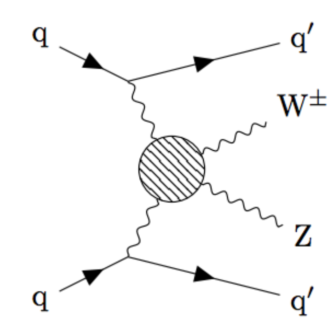
Counting: [Henning, Lu, Melia, Murayama, 1512.03433]

Dim-8

- 4-boson (W,Z,γH) (anomalous quartic-gauge-boson couplings)

$$
\begin{aligned}
O_{S,0} &= [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\mu\Phi)^\dagger D^\nu\Phi] \\
O_{S,1} &= [(D_\mu\Phi)^\dagger D^\mu\Phi] \times [(D_\nu\Phi)^\dagger D^\nu\Phi] \\
O_{T,0} &= \mathrm{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] \times \mathrm{Tr}\left[W_{\alpha\beta}W^{\alpha\beta}\right] \\
O_{T,1} &= \mathrm{Tr}\left[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}\right] \\
O_{M,0} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right] \times [(D_\beta\Phi)^\dagger D^\beta\Phi] \\
O_{M,1} &= \mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}\right] \times [(D_\beta\Phi)^\dagger D^\mu\Phi]
\end{aligned}
$$

	14 TeV		27 TeV	
	$WZjj$	$W^\pm W^\pm jj$	$WZjj$	$W^\pm W^\pm jj$
f_{S_0}/Λ^4	[-8,8]	[-6,6]	[-1.5,1.5]	[-1.5,1.5]
f_{S_1}/Λ^4	[-18,18]	[-16,16]	[-3,3]	[-2.5,2.5]
f_{T_0}/Λ^4	[-0.76,0.76]	[-0.6,0.6]	[-0.04,0.04]	[-0.027,0.027]
f_{T_1}/Λ^4	[-0.50,0.50]	[-0.4,0.4]	[-0.03,0.03]	[-0.016,0.016]
f_{M_0}/Λ^4	[-3.8,3.8]	[-4.0,4.0]	[-0.5,0.5]	[-0.28,0.28]
f_{M_1}/Λ^4	[-5.0,5.0]	[-12,12]	[-0.8,0.8]	[-0.90,0.90]



CA Lee, HL/HE-LHC Jamboree, 1 March 2019 18

- Light-by-light, gg>diphoton [J. Ellis et al. 1703.08450] [Ellis and Ge 1802.02416]

- nTGC, ffZZ/ffZγ/ff$\gamma\gamma$ [C. Degrande 1308.6323] [Bellazzini and Riva 1806.09640] [Ellis, He, Xiao 2008.04298] [J. Ellis et al. 1902.06631] [Gu, Wang, CZ 2011.03055]

- EWPD [Corbett, Helset, Martin, Trott 2102.02819]

- Higgs+Z [Hays, Martin, Sanz, Setford 1808.00442]

- qqqq, llll, qqll, … [Bellazzini, Riva, Serra, Sgarlata 1706.03070] [Fuks, Liu, CZ, Zhou 2009.02212] [Alioli, Boughezal, Mereghetti, Petriello 2003.11615]

$$\mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{C_i^{(6)} O_i^{(6)}}{\Lambda^2} + \sum_i \frac{C_i^{(8)} O_i^{(8)}}{\Lambda^4} + \cdots$$

Dim-6 (84)

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger \varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger \varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r) \widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger \varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r) \tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r) \widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger \varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r) \varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r) \tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger \tau^I \varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r) \varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p \gamma_\mu l_r)(\bar l_s \gamma^\mu l_t)$	Q_{ee}	$(\bar e_p \gamma_\mu e_r)(\bar e_s \gamma^\mu e_t)$	Q_{le}	$(\bar l_p \gamma_\mu l_r)(\bar e_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar q_s \gamma^\mu q_t)$	Q_{uu}	$(\bar u_p \gamma_\mu u_r)(\bar u_s \gamma^\mu u_t)$	Q_{lu}	$(\bar l_p \gamma_\mu l_r)(\bar u_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p \gamma_\mu \tau^I q_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p \gamma_\mu d_r)(\bar d_s \gamma^\mu d_t)$	Q_{ld}	$(\bar l_p \gamma_\mu l_r)(\bar d_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p \gamma_\mu l_r)(\bar q_s \gamma^\mu q_t)$	Q_{eu}	$(\bar e_p \gamma_\mu e_r)(\bar u_s \gamma^\mu u_t)$	Q_{qe}	$(\bar q_p \gamma_\mu q_r)(\bar e_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p \gamma_\mu \tau^I l_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p \gamma_\mu e_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar u_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p \gamma_\mu u_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar u_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p \gamma_\mu T^A u_r)(\bar d_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar d_s \gamma^\mu d_t)$
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$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
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$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$		

[B. Grzadkowski et al., 1008.4884]

Dim-8 classes (993)

#	Class	$N_{\rm type}$	$N_{\rm term}$	$N_{\rm op}$ [10]
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21(B)	ψ^4D^2	18	55	$\frac{11}{2}n_g^2(9n_g^2+1)$
21($\not B$)		4	$10+2$	$n_g^3(11n_g-1)$
	B	$204+5$	$895+14$	$895(36971)$, $n_g=1(3)$
	$\not B$	$19+3$	$98+22$	$98(7836)$, $n_g=1(3)$
	Total	$223+8$	$993+36$	$993(44807)$, $n_g=1(3)$

[H.-L. Li et al., 2020] [C. Murphy, 2020]

See also:

Dim-7: **[L. Lehman, 14] [Liao & Ma, 16]**

Dim-9: **[H.-L. Li et al., 20] [Liao & Ma, 20]**

Counting: [Henning, Lu, Melia, Murayama, 1512.03433]

$$\mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{C_i^{(6)} O_i^{(6)}}{\Lambda^2} + \sum_i \frac{C_i^{(8)} O_i^{(8)}}{\Lambda^4} + \cdots$$

Dim-6 (84)

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu\tau^I q_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu\tau^I l_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$		

[B. Grzadkowski et al., 1008.4884]

Dim-8 classes (993)

#	Class	$N_{\rm type}$	$N_{\rm term}$	$N_{\rm op}$ [10]
1	X^4	7	43	43
2	H^8	1	1	1
3	H^6D^2	1	2	2
4	H^4D^4	1	3	3
5	X^3H^2	3	6	6
6	X^2H^4	5	10	10
7	$X^2H^2D^2$	4	18	18
8	XH…			6
9	ψ^2X…			$96n_g^2$
10	ψ^2X…			$22n_g^2$
11	$\psi^2H^2D^3$	6	16	$16n_g^2$
12	ψ^2H^5	3	6	$6n_g^2$
13	ψ^2H^4D	6	13	$13n_g^2$
14	ψ^2X^2D	21	57	$57n_g^2$
15	ψ^2XH^2D	16	92	$92n_g^2$
16	ψ^2XHD^2	8	48	$48n_g^2$
17	$\psi^2H^3D^2$	3	36	$36n_g^2$
18(B)	ψ^4H^2	19	75	$n_g^2(67n_g^2+n_g+7)$
18($\not B$)		$4+3$	$12+8$	$\frac{1}{3}n_g^2(43n_g^2-9n_g+2)$
19(B)	ψ^4X	$40+5$	$156+12$	$4n_g^2(40n_g^2-1)$
19($\not B$)		4	$44+12$	$2n_g^3(21n_g+1)$
20(B)	ψ^4HD	16	$134+2$	$n_g^3(135n_g-1)$
20($\not B$)		7	32	$n_g^3(29n_g+3)$
21(B)	ψ^4D^2	18	55	$\frac{11}{2}n_g^2(9n_g^2+1)$
21($\not B$)		4	$10+2$	$n_g^3(11n_g-1)$
	B	$204+5$	$895+14$	$895(36971)$, $n_g=1(3)$
	$\not B$	$19+3$	$98+22$	$98(7836)$, $n_g=1(3)$
	Total	$223+8$	$993+36$	$993(44807)$, $n_g=1(3)$

250 OPs constrained by positivity ($\propto E^4$)

[H.-L. Li et al., 2020] [C. Murphy, 2020]

See also:

Dim-7: **[L. Lehman, 14] [Liao & Ma, 16]**

Dim-9: **[H.-L. Li et al., 20] [Liao & Ma, 20]**

Counting: [Henning, Lu, Melia, Murayama, 1512.03433]

Positivity bounds

- Not all SMEFTs have a UV completion.

- Bounds from axiomatic principles of QFT (causality, unitarity, etc.), on the signs of (combinations of) Wilson coefficients.

- 2-to-2 amplitude (1 particle) $A_{2\to 2}(s, t=0) = c_0 + c_2 s^2 + c_4 s^4 + \cdots$

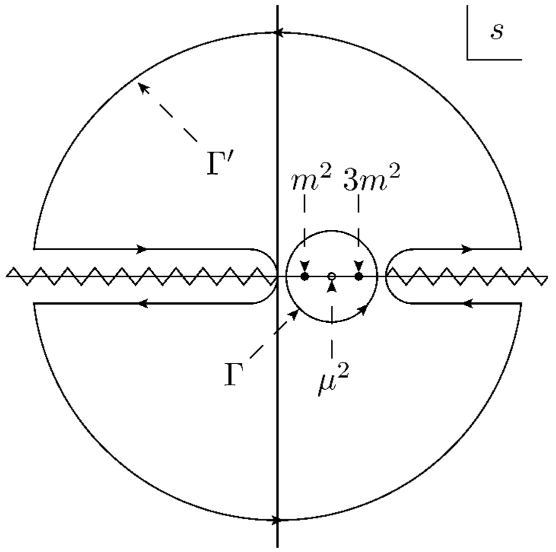
$$\partial_s^2 A_{\mathrm{EFT}}(s,0) > 0$$

[Adams++, 2006]

- $c_2 > 0$; Often in SMEFT: $C^{(8)} > 0$. [A. Adams et al., JHEP 06]
  - General bosonic operators [G. Remmen, N. Rodd 1908.09845]
  - aQGC [Bi, CZ, Zhou 1902.08977] [Yamashita, CZ, Zhou 2009.04490]
  - Fermion/flavor operators [G. Remmen, N. Rodd 2004.02885], [Bonnefoy, Gendy, Grojean 2011.12855]
  - See also [Bellazzini, Riva, Serra, Sgarlata 1706.03070] [Bellazzini and Riva 1806.09640] [Fuks, Liu, CZ, Zhou 2009.02212] [Gu, Wang, CZ 2011.03055] [T. Trott 2011.10058]…

Positivity bounds

$$A_{2\to 2}(s,t=0) = c_0 + c_2 s^2 + c_4 s^4 + \cdots$$



✦ Analyticity: $f = \frac{1}{2\pi i}\oint_\Gamma \mathrm{d}s \frac{A(s,0)}{(s-\mu^2)^3}$

✦ Unitarity + Locality: $A(s,0) < \mathcal{O}(s\ln^2 s)$

$$f = \frac{1}{2\pi i}\oint_\Gamma \mathrm{d}s \frac{A(s,0)}{(s-\mu^2)^3} = \frac{1}{2\pi i}\left(\int_{-\infty}^{0} + \int_{4m^2}^{\infty}\right)\mathrm{d}s \frac{\mathrm{Disc}A(s,0)}{(s-\mu^2)^3}$$

IR

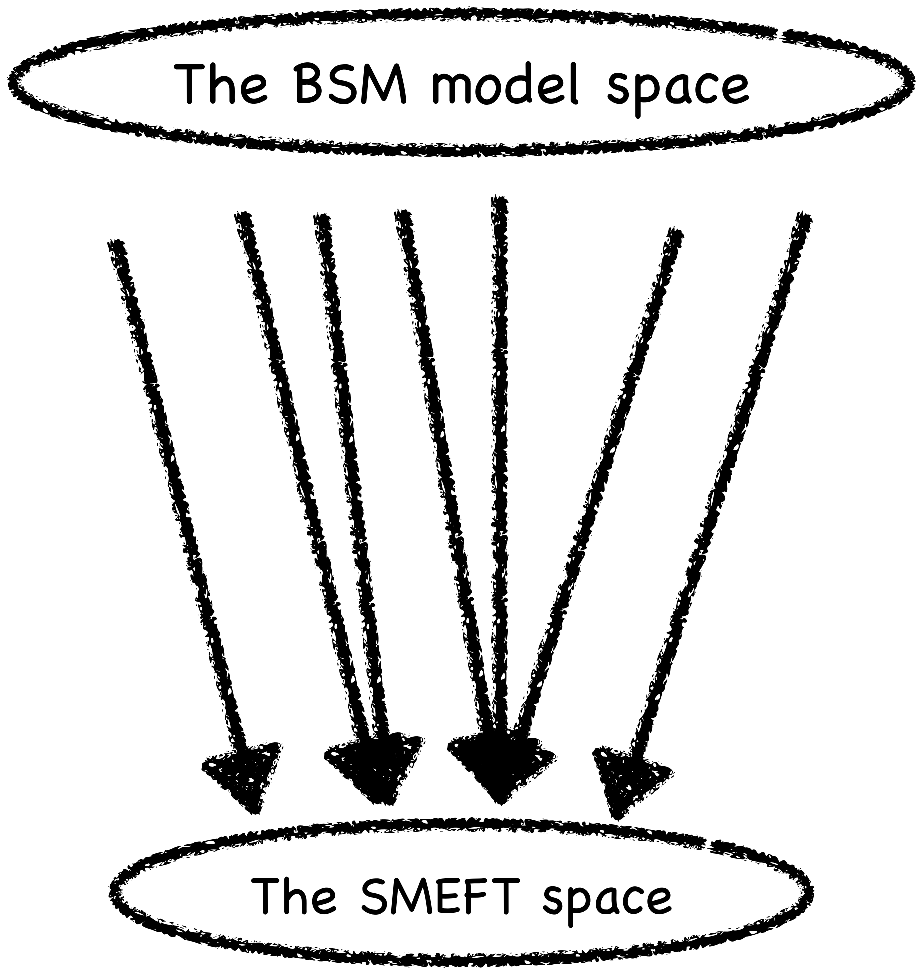
Calculable in EFT

UV

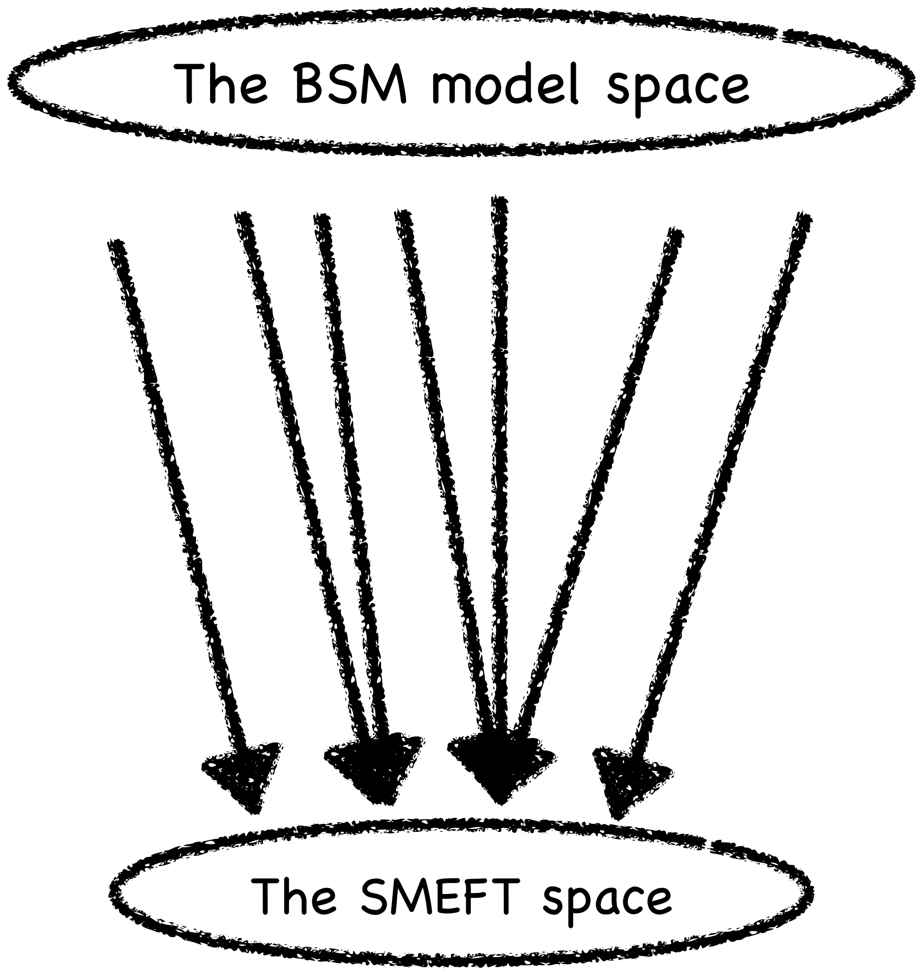
Disc > 0 by optical theorem + (s-u) crossing

$$A_{2\to 2}(s,t=0) = c_0 + c_2 s^2 + c_4 s^4 + \cdots$$

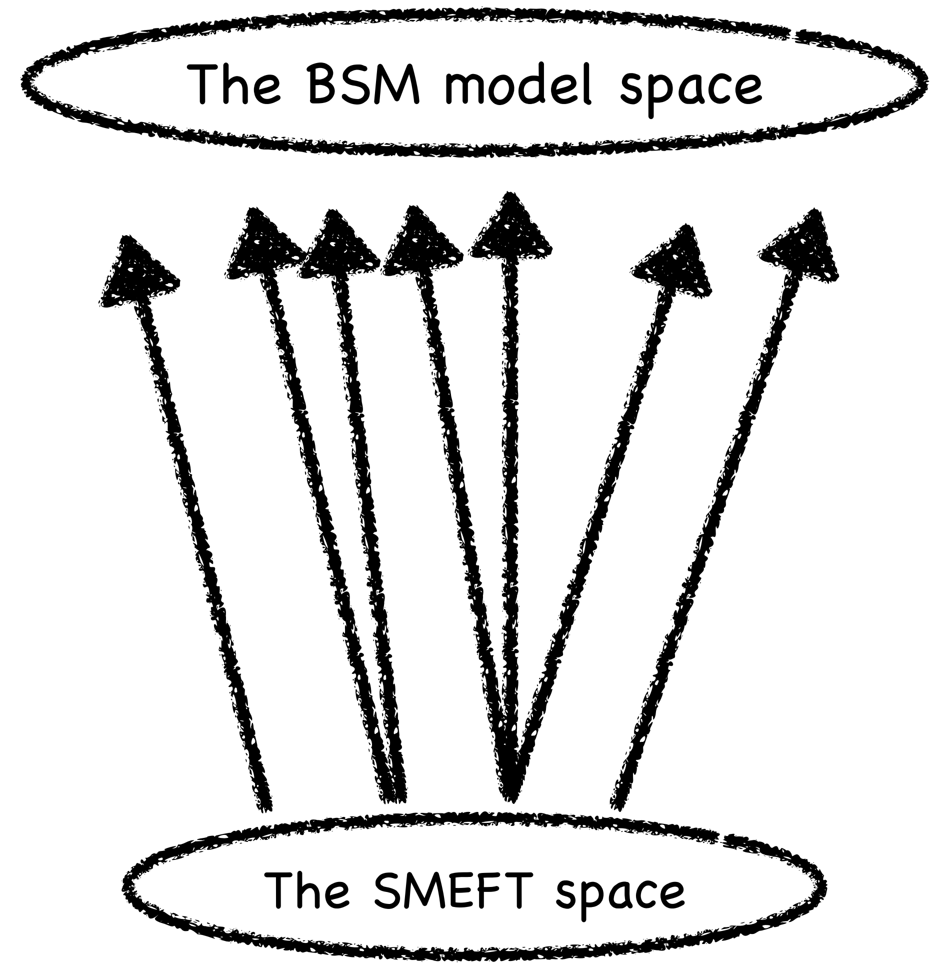
All heavy BSM models can be matched to some EFT



All heavy BSM models can be matched to some EFT



Inverse problem: how do we find the UV completions, given the measured SMEFT?



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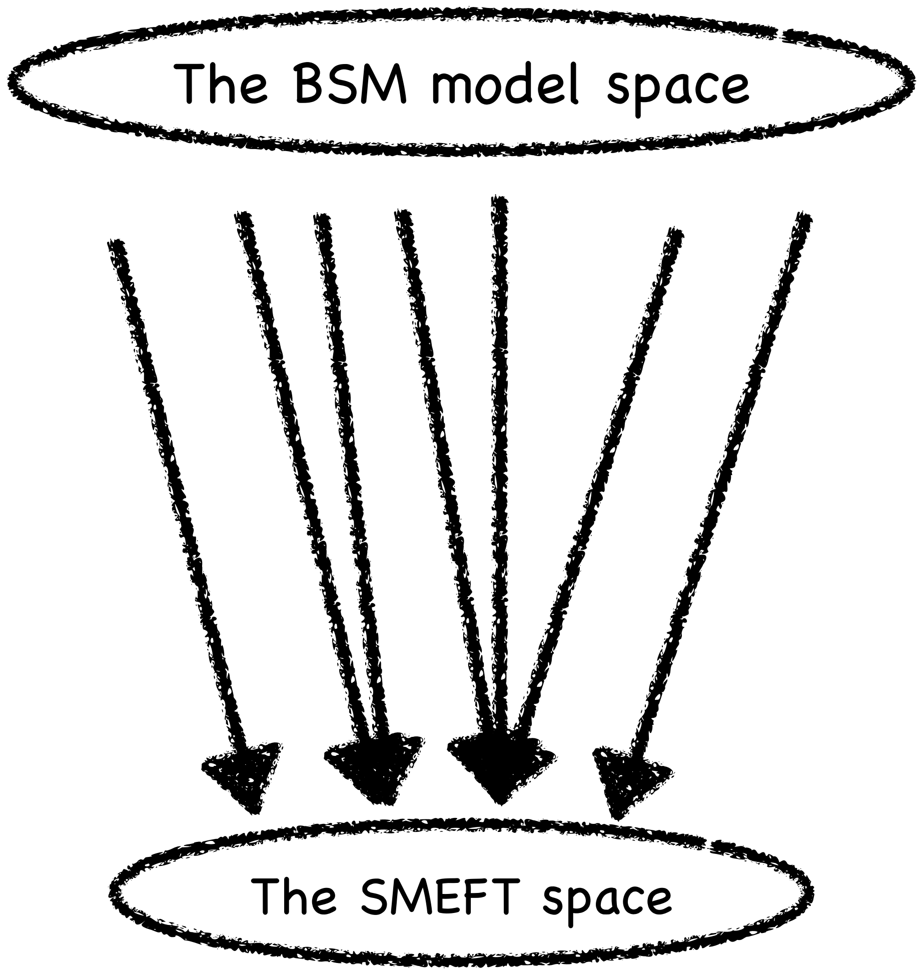

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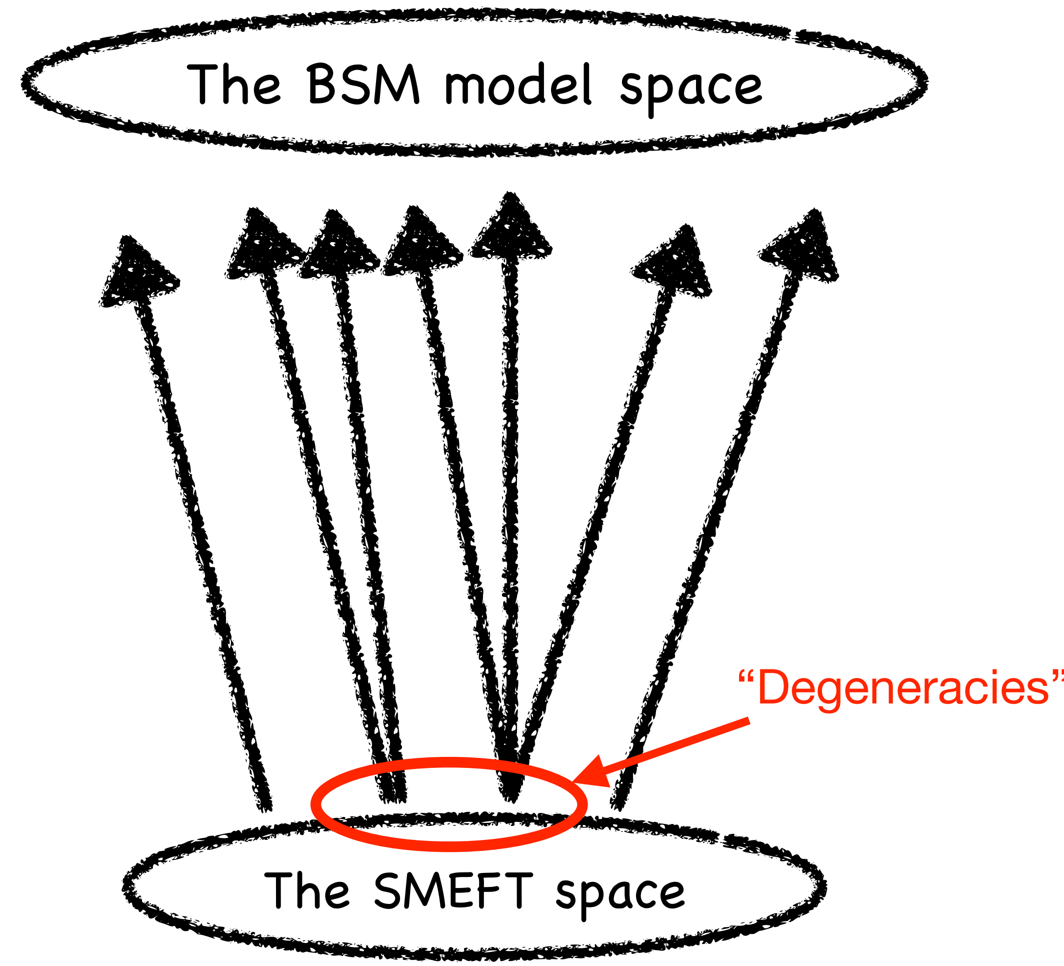

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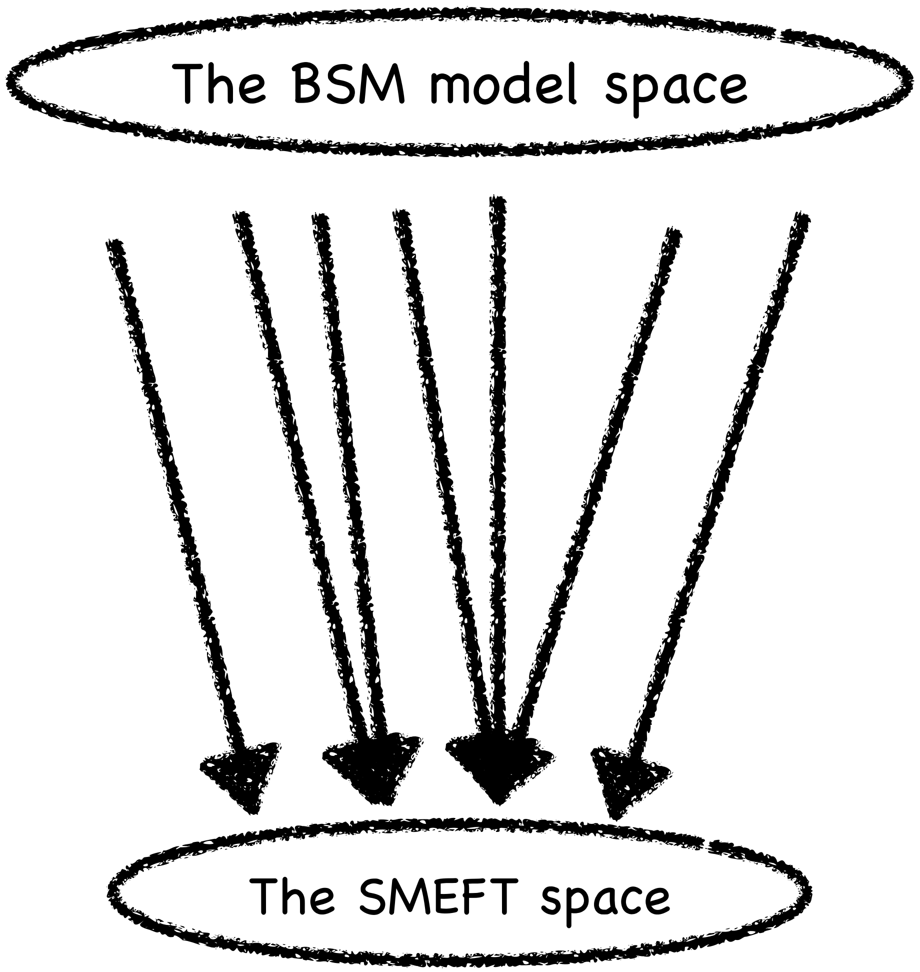

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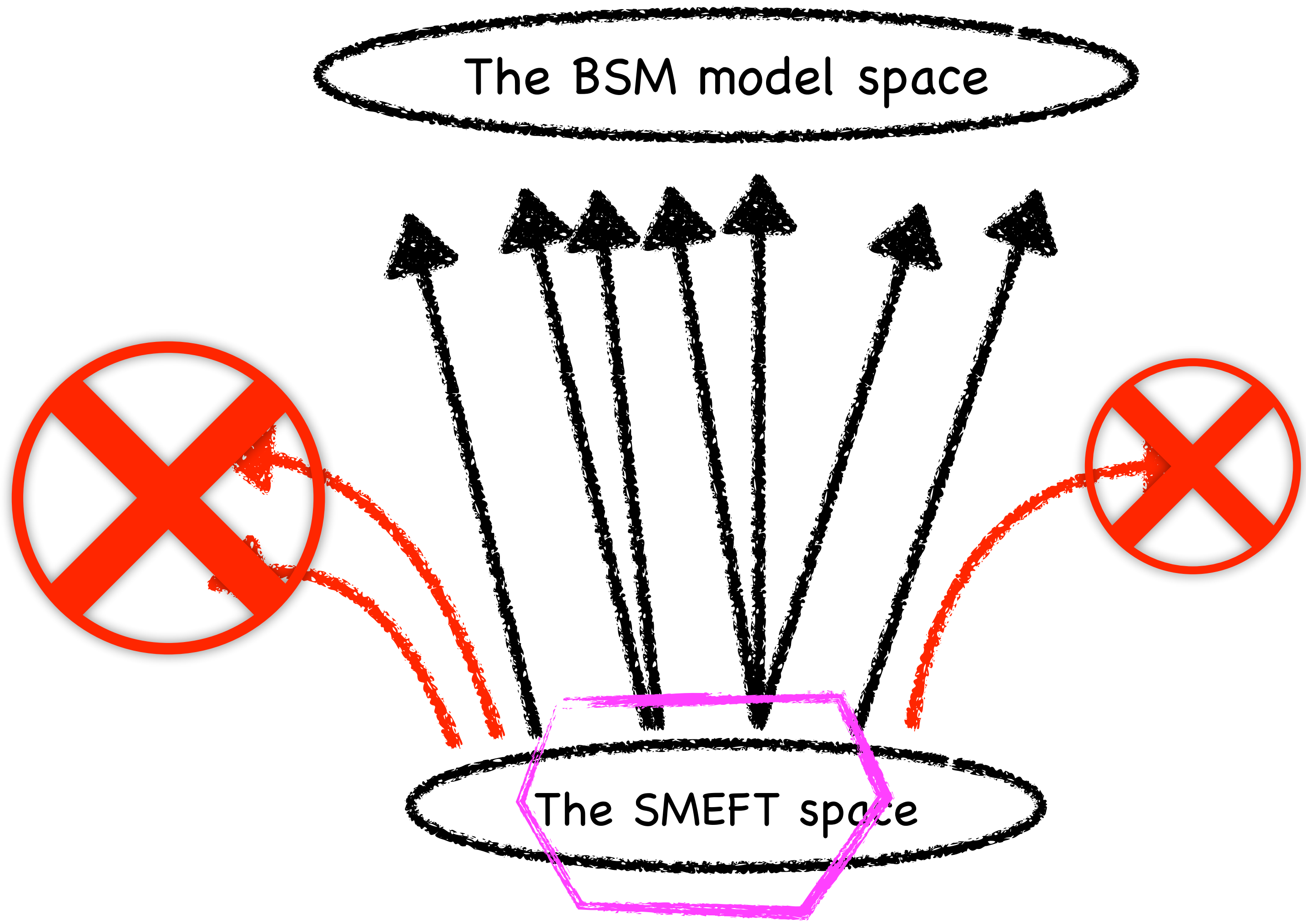

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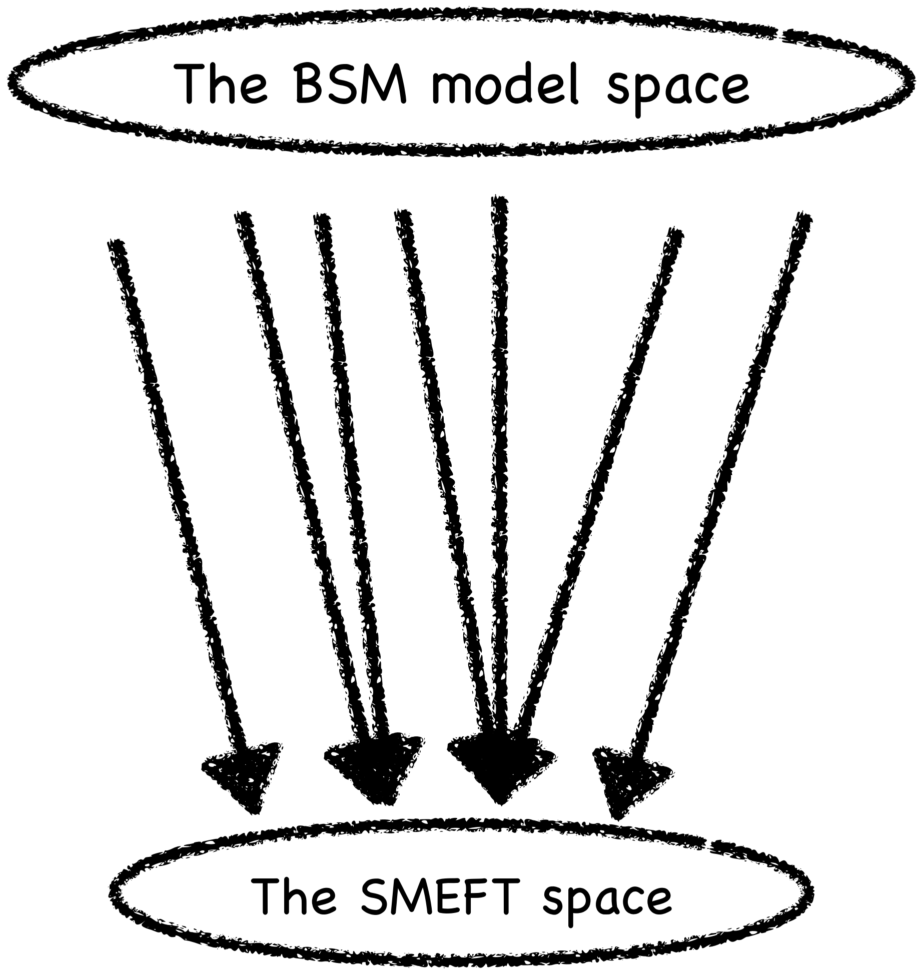

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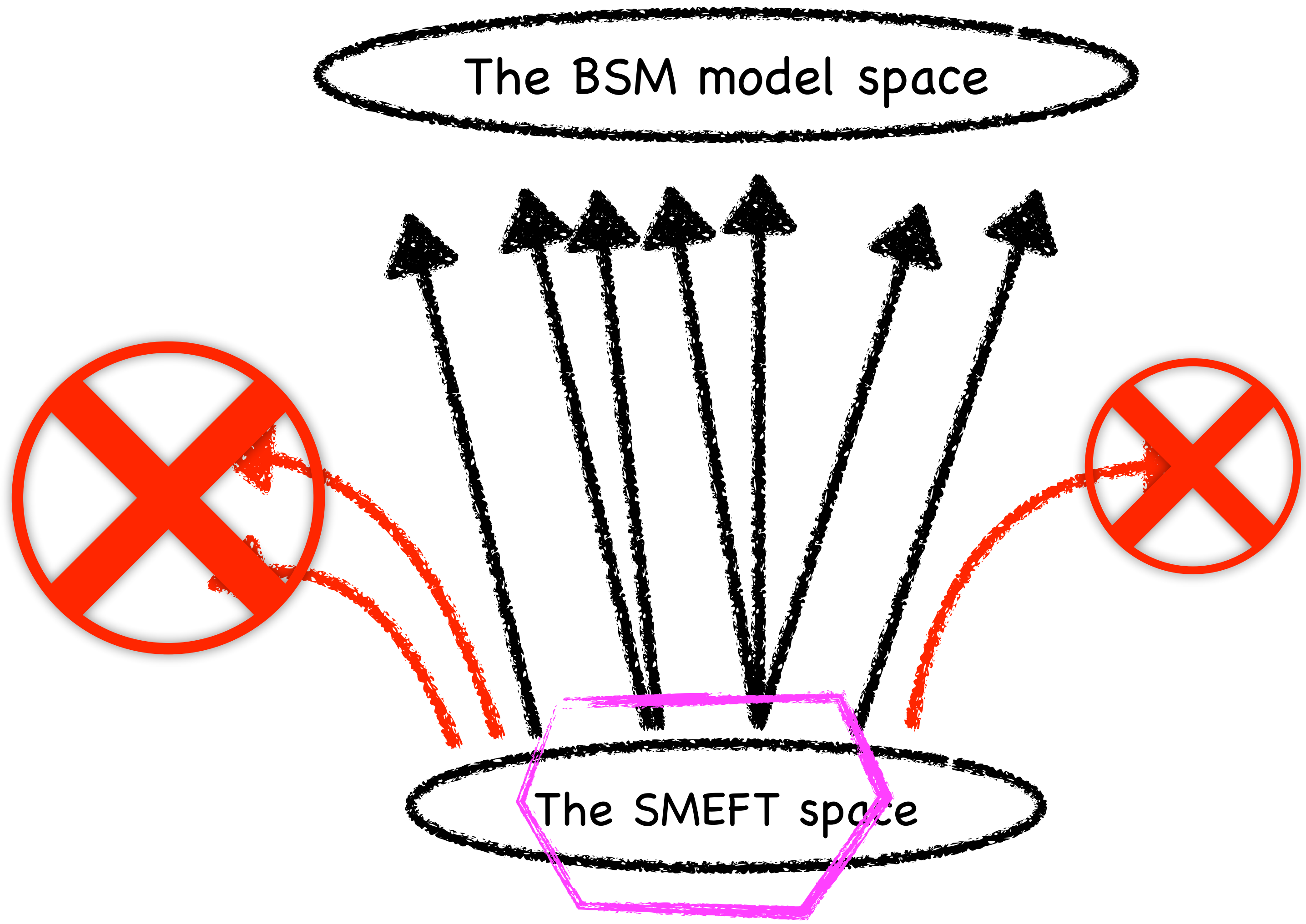
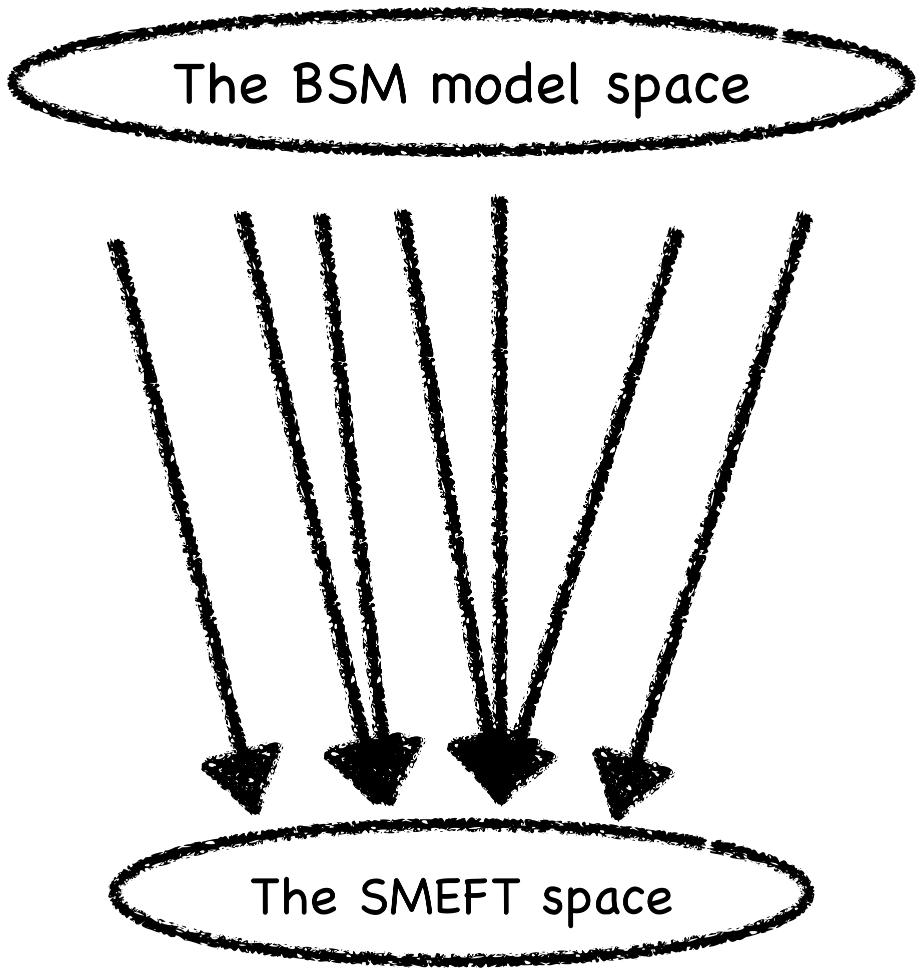


- Where exactly is the boundary between UV-completable SMEFTs and the others?

All heavy BSM models can be matched to some EFT



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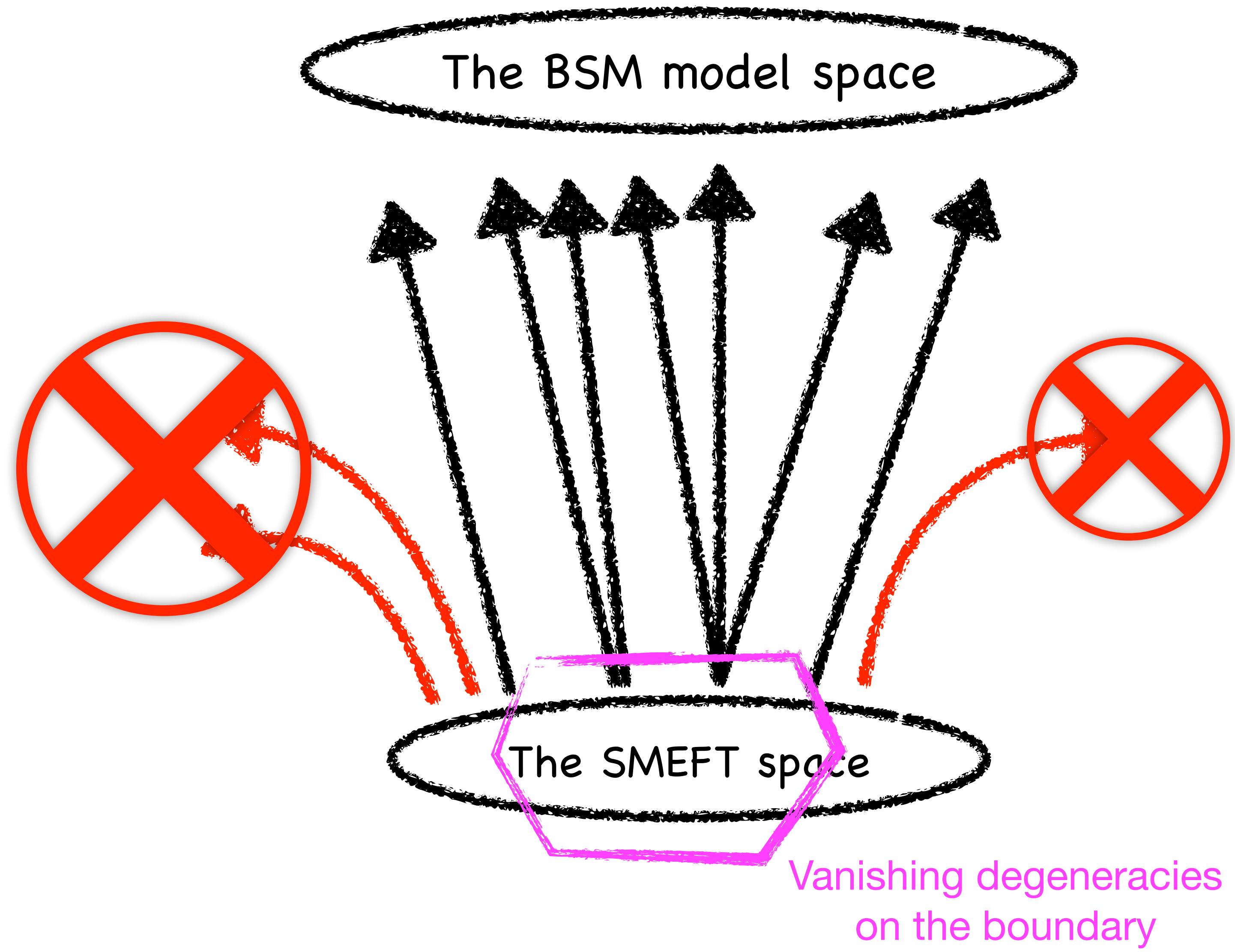

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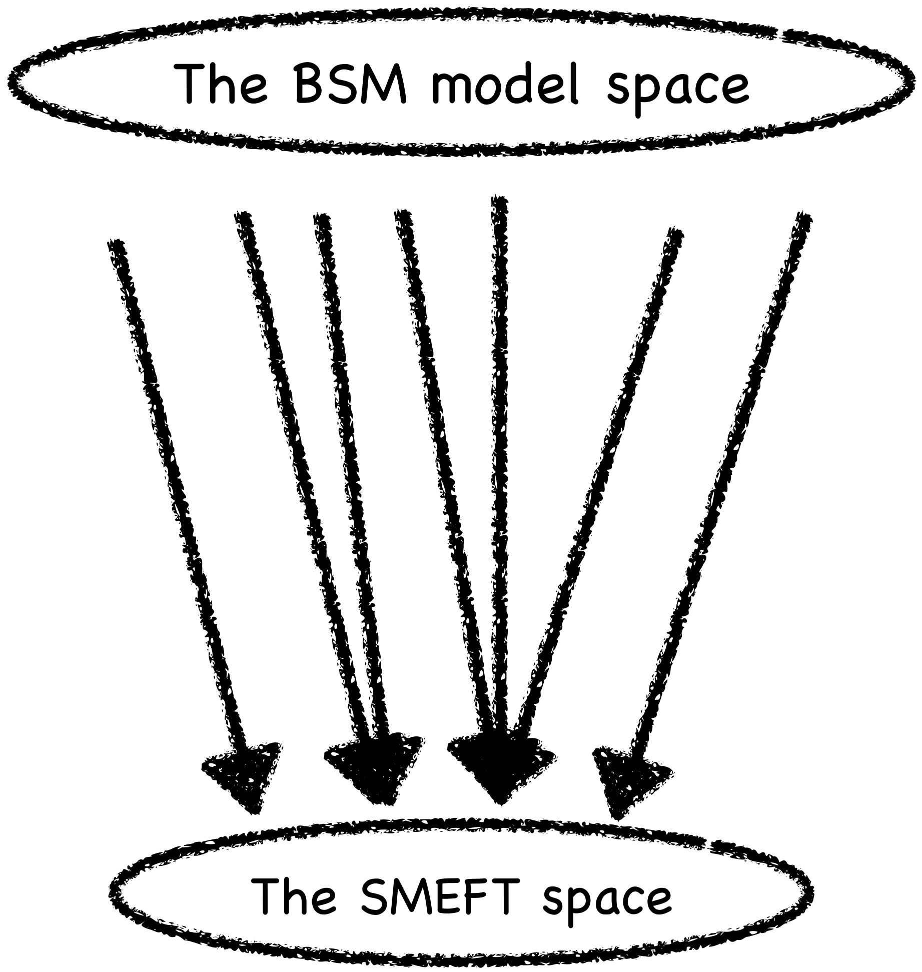

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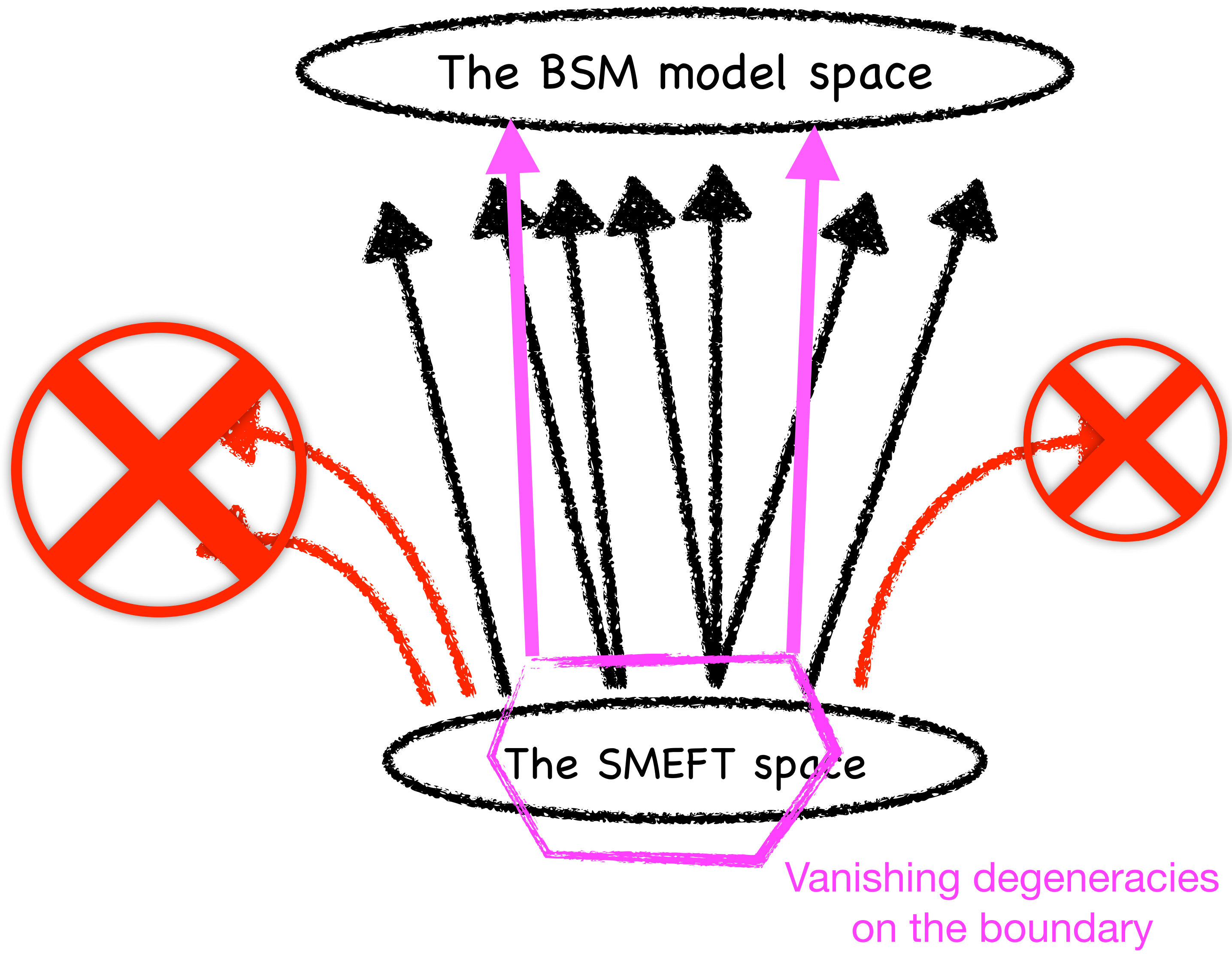
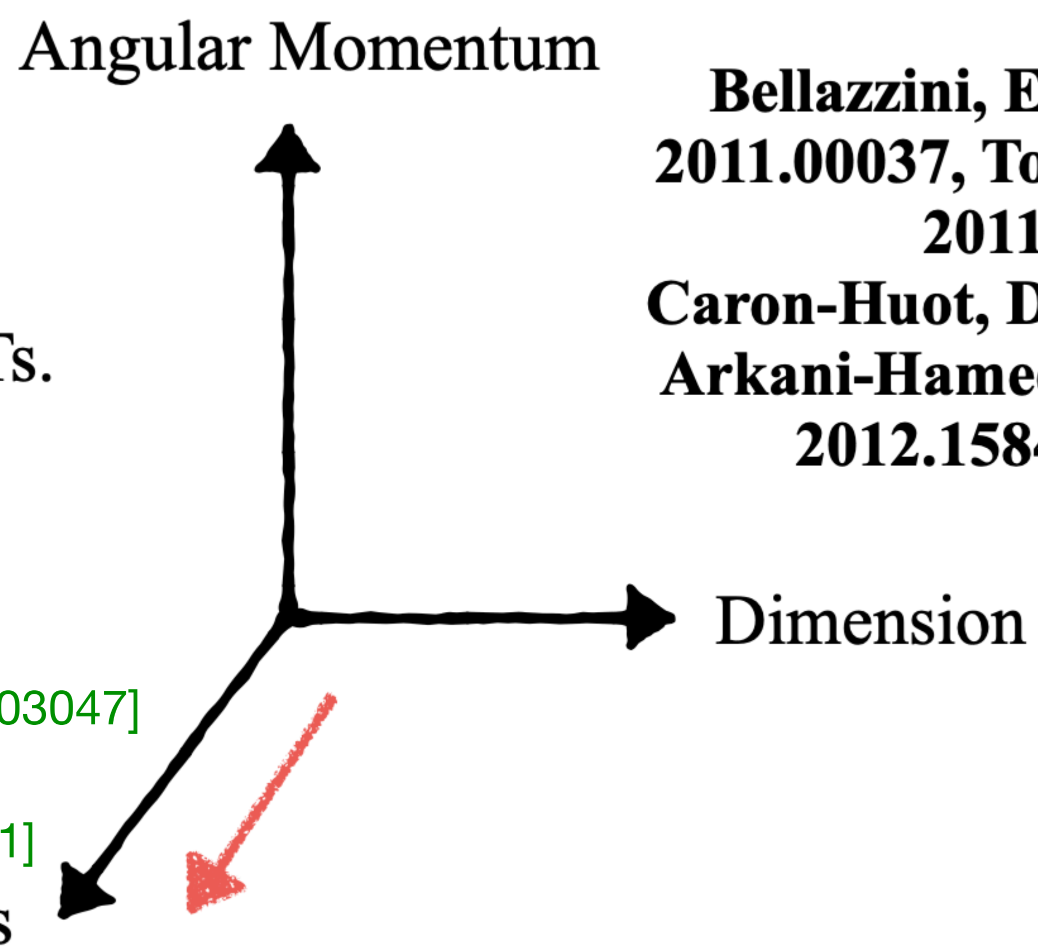


- Where exactly is the boundary between UV-completable SMEFTs and the others?
- How to determine the UV models (in specific regions of the coefficient space)?

Space of interactions in EFTs.



Bellazzini, Elias Miro, et.al. 2011.00037, Tolley, Wang, Zhou 2011.02400, Caron-Huot, Duong 2011.02957, Arkani-Hamed, Huang, Huang 2012.15849 and more.

[CZ, S.-Y. Zhou, 2005.03047]
[T. Trott 2011.10058]
[X. Li et al., 2101.01191]

E.g. gluon = 16 dofs, 7 ops, 48 bounds

Slide by T. Trott at HEFT 2021

Outline

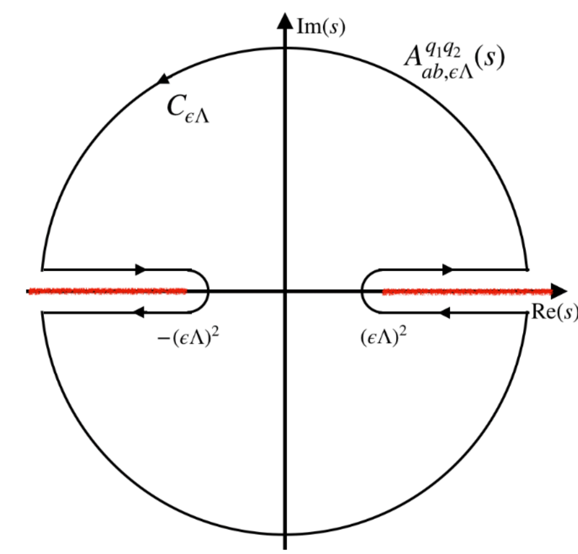
- Exact boundary of UV-completable SMEFTs, positivity cone
  - Two approaches to the positivity region:
    1) find the generators, [CZ, S.-Y. Zhou, 2005.03047]
  - 2) and directly search for the bounds. [X. Li et al., 2101.01191]
- Phenomenological aspects
  - The “inverse problem”
    (i.e. how to find UV completions from EFT measurements)
  - Example: e+e- → e+e- [Fuks, Liu, CZ, Zhou 2009.02212]

Positivity bound from generators

[CZ, S.-Y. Zhou, 2005.03047]

Dispersion keeping track of particle modes

Define $$\mathcal{M}^{ijkl} \equiv \frac{\mathrm{d}^2}{\mathrm{d}s^2} A_{ij\to kl}(s)\Big|_{s\to 0}$$



Dispersion relation with particle indices

$$\mathcal{M}^{ijkl} = \frac{1}{2\pi i}\int_{(\epsilon\Lambda)^2}^{\infty} \mathrm{d}s \frac{\mathrm{Disc}A_{ij\to kl}(s,0) + \mathrm{Disc}A_{i\bar{l}\to k\bar{j}}(s,0)}{(s-2m^2)^3}$$

$\epsilon\Lambda$ is some scale comparable but below cutoff so the EFT is still valid; see “improved positivity” of [C. de Rham et al., 1710.09611]; and the “arc”s in [B. Bellazzini et al., 2011.00037]

Generalized optical theorem (neglecting masses):

$$\mathrm{Disc}A_{ij\to kl}(s) = A_{ij\to kl}(s) - A_{kl\to ij}(s)^* = i\sum_X M_{ij\to X}(s) M_{kl\to X}(s)^*$$

$$\mathcal{M}^{ijkl} = \frac{1}{2\pi}\int_{(\epsilon\Lambda)^2}^{\infty} \frac{ds}{s^3} \sum_X \left[\mathbf{M}_{ij\to X}\mathbf{M}^*_{kl\to X} + (j \leftrightarrow l)\right]$$ (not positive yet)

$$\mathcal{M}^{ijkl} = \frac{1}{2\pi}\int_{(\epsilon\Lambda)^2}^{\infty}\frac{ds}{s^3}\sum_X \left[\mathbf{M}_{ij\to X}\mathbf{M}^*_{kl\to X} + (j \leftrightarrow l)\right]$$

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M^{ijkl} can be mapped to coefficients
e.g. 2-scalar theory, tree level

$$O_{ijkl} = (\partial_\mu\phi_i\partial^\mu\phi_j)(\partial_\mu\phi_k\partial^\mu\phi_l)$$
$$O_1 = O_{1111},\quad O_2 = O_{1122},\quad O_3 = O_{2222},$$
$$O_4 = O_{1212},\quad O_5 = O_{1112},\quad O_6 = O_{1222}\,.$$
$$\bar{C}_2 \equiv C_2 + \tfrac{1}{2}C_4$$

$\mathcal{M}^{ijkl} =$

ij \ kl	$\phi_1\phi_1$	$\phi_2\phi_2$	$\phi_1\phi_2$	$\phi_2\phi_1$
$\phi_1\phi_1$	$4C_1$	$\bar{C}_2$	C_5	C_5
$\phi_2\phi_2$	$\bar{C}_2$	$4C_3$	C_6	C_6
$\phi_2\phi_1$	C_5	C_6	C_4	$\bar{C}_2$
$\phi_1\phi_2$	C_5	C_6	$\bar{C}_2$	C_4

Higher orders always possible

$$\mathcal{M}^{ijkl} = \frac{1}{2\pi}\int_{(\epsilon\Lambda)^2}^{\infty} \frac{ds}{s^3} \sum_X \left[\mathbf{M}_{ij\to X}\mathbf{M}^*_{kl\to X} + (j \leftrightarrow l)\right]$$

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ij \ kl	$\phi_1\phi_1$	$\phi_2\phi_2$	$\phi_1\phi_2$	$\phi_2\phi_1$
$\phi_1\phi_1$	$4C_1$	$\bar{C}_2$	C_5	C_5
$\phi_2\phi_2$	$\bar{C}_2$	$4C_3$	C_6	C_6
$\phi_2\phi_1$	C_5	C_6	C_4	$\bar{C}_2$
$\phi_1\phi_2$	C_5	C_6	$\bar{C}_2$	C_4

Higher orders always possible

$\mathbf{M}_{ij\to X}$ describes <u>unknown</u> UV physics.
Restricted by only symmetries.

- The only necessary information is

$$\mathcal{M}^{ijkl} = \sum_X \lambda_X \left(m_X^{ij} m_X^{*kl} + m_X^{i\bar{l}} m_X^{*k\bar{j}}\right),\ \lambda_X \geq 0$$

i.e. a positively weighted sum of (mm+mm)

Bounds from elastic scattering of two factorized superposed states

$$\mathcal{M}^{ijkl} = \sum_X \lambda_X \left(m_X^{ij} m_X^{*kl} + m_X^{i\bar{l}} m_X^{*k\bar{j}} \right), \ \lambda_X \geq 0$$

As the simplest generalization of 1-species EFT (c2>0), consider two superposed states: $|u\rangle = u^i |i\rangle, |v\rangle = v^j |j\rangle,$

- ✦ The elastic amplitude is $\mathbf{M}_{uv\to uv} \ \Rightarrow \ u^i v^j u^{*k} v^{*l} \mathcal{M}^{ijkl}$
- ✦ Expected to be positive. In fact,

$$u^i v^j u^{*k} v^{*l} \mathcal{M}^{ijkl} = \sum_X \lambda_X u^i v^j u^{*k} v^{*l} \left(m_X^{ij} m_X^{*kl} + m_X^{il} m_X^{*kj} \right)$$

$$= \sum_X \lambda_X \left(\left| u^i m^{ij} v^j \right|^2 + \left| u^i m^{ij} v^{*j} \right|^2 \right) \geq 0$$

- ✦ Many applications in SMEFT.
- ✦ Incomplete. [CZ, S.-Y. Zhou, 2005.03047]

Bounds from a generation point of view

$$\mathcal{M}^{ijkl} = \sum_X \lambda_X \left(m_X^{ij} m_X^{*kl} + m_X^{i\bar{l}} m_X^{*k\bar{j}} \right), \ \lambda_X \geq 0$$

Define the “directional” information of $m_X^{ij} m_X^{*kl} + m_X^{i\bar{l}} m_X^{*k\bar{j}}$ as the “**generator**”

$$\mathcal{G}_X^{ijkl} \equiv m_X^{ij} m_X^{*kl} + m_X^{i\bar{l}} m_X^{*k\bar{j}}$$

Can map to a set of coefficients (C1,C2,…)

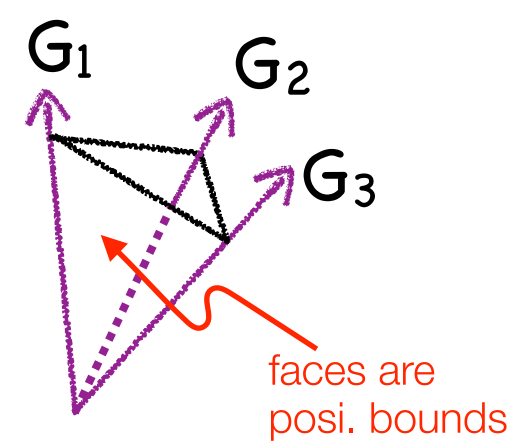
$\mathcal{M}^{ijkl} =$

	$\phi_1\phi_1$	$\phi_2\phi_2$	$\phi_1\phi_2$	$\phi_2\phi_1$
$\phi_1\phi_1$	$4C_1$	$\bar{C}_2$	C_5	C_5
$\phi_2\phi_2$	$\bar{C}_2$	$4C_3$	C_6	C_6
$\phi_2\phi_1$	C_5	C_6	C_4	$\bar{C}_2$
$\phi_1\phi_2$	C_5	C_6	$\bar{C}_2$	C_4

$\mathcal{M}$ lives in the convex cone

$$\mathbf{C} \equiv \left\{ \mathcal{M}^{ijkl} \right\} = \text{cone}\left(\left\{ \mathcal{G}^{ijkl} \right\} \right)$$

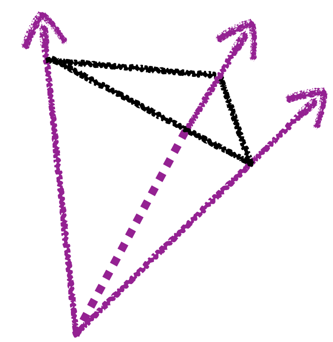
i.e. all positively weighted sum



UV-completable SMEFTs are the conical hull of all generators.

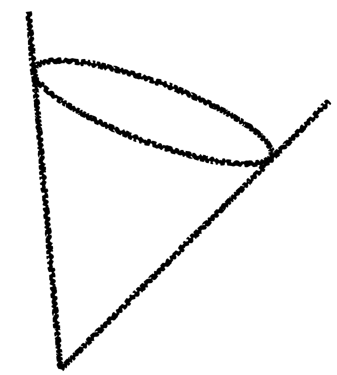
Bounds from a generation point of view

- One species charged under some continuous symmetry (SO/SU groups etc.), **m** $(\mathcal{M}_{ij\to X})$ -> Clebsch Gordan coefficients, **G** -> projective operators
[Bellazzini, Martucci, Torre, 1405.2960]



  - Generators are the edge vectors. Positivity cone is a polytope.

- More generally, can have infinite number of generators, curved boundary.
[CZ, S.-Y. Zhou, 2005.03047]

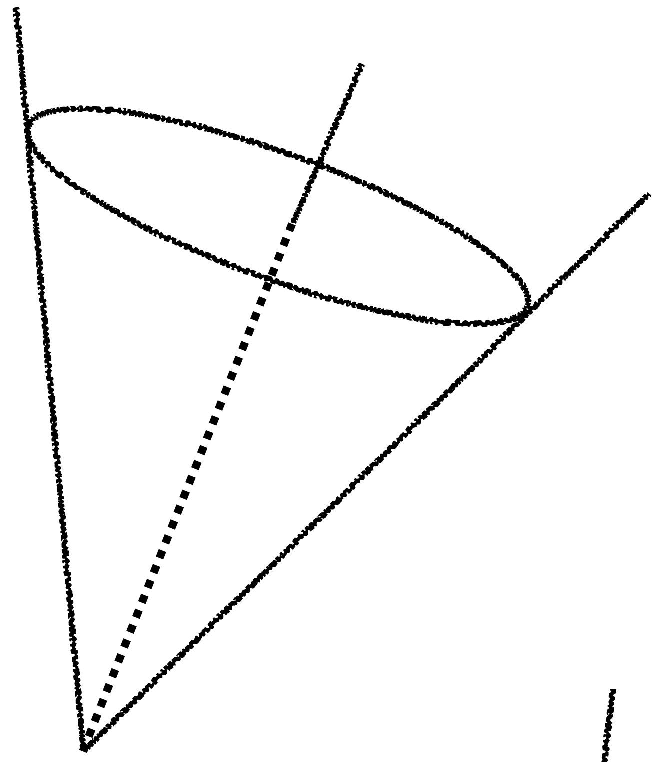


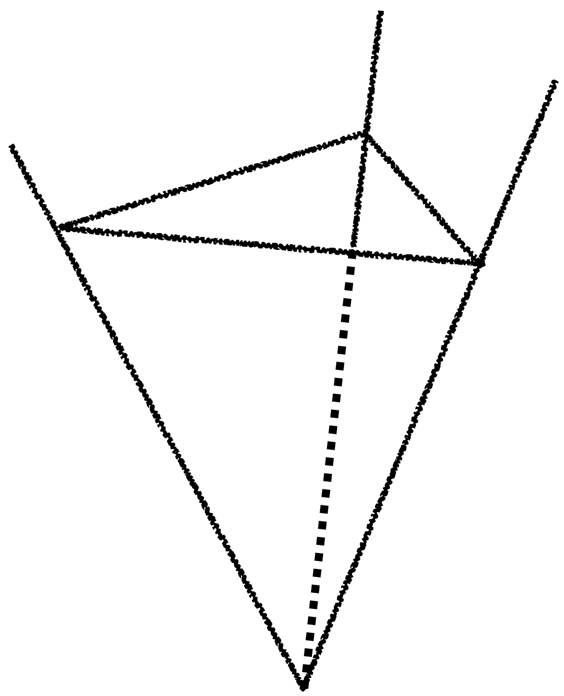
  - Edge vectors -> extremal rays (ERs).
  - ERs are “one-particle SM extensions”
  - Extremality plays a role in the “inverse problem”

Extremal rays

- **Extremal Ray (ER)**: an extremal ray of cone C cannot be split into two other vectors in C, which are linearly independent.

- In polyhedral cones, ERs are the edge vectors.





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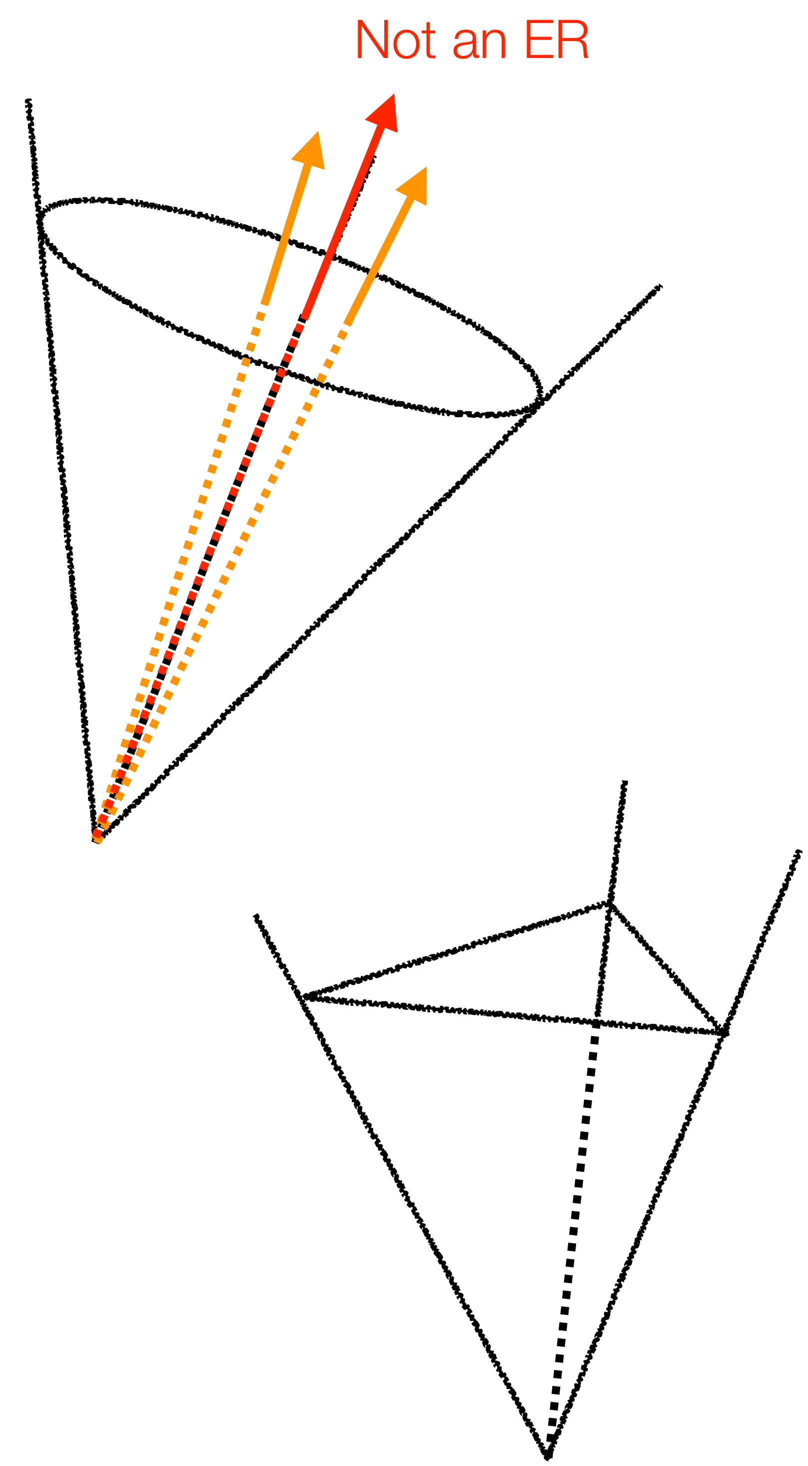

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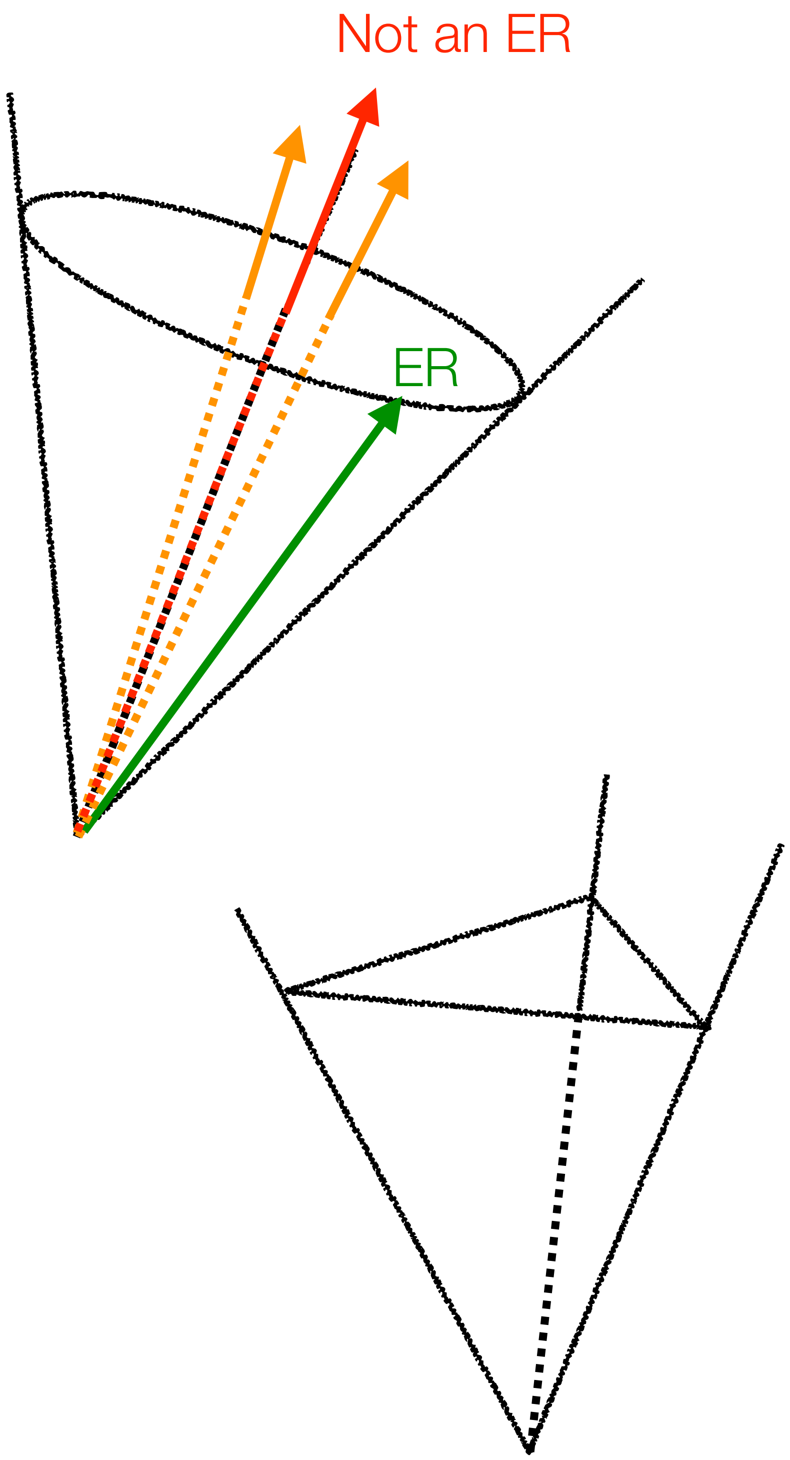

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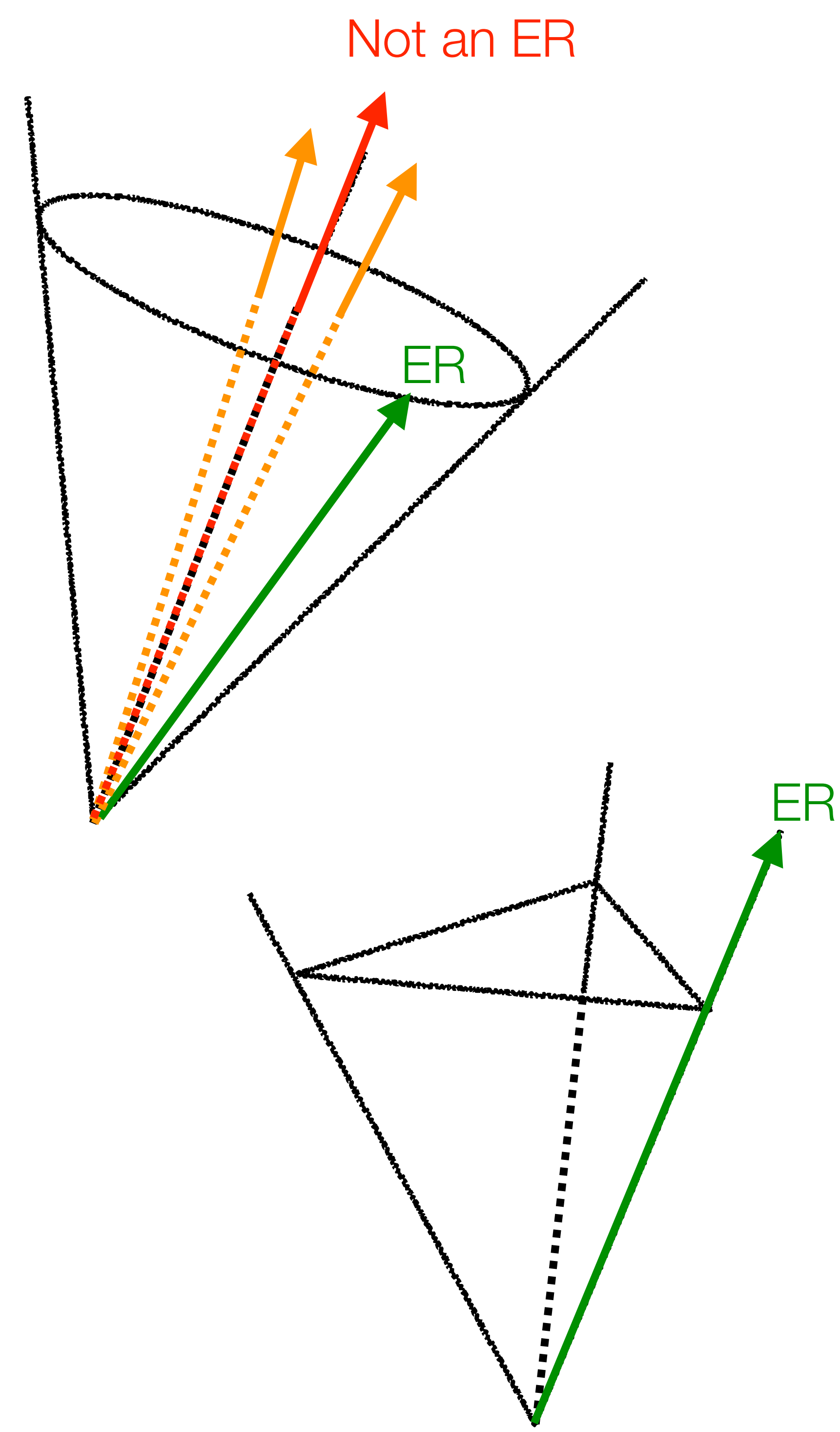

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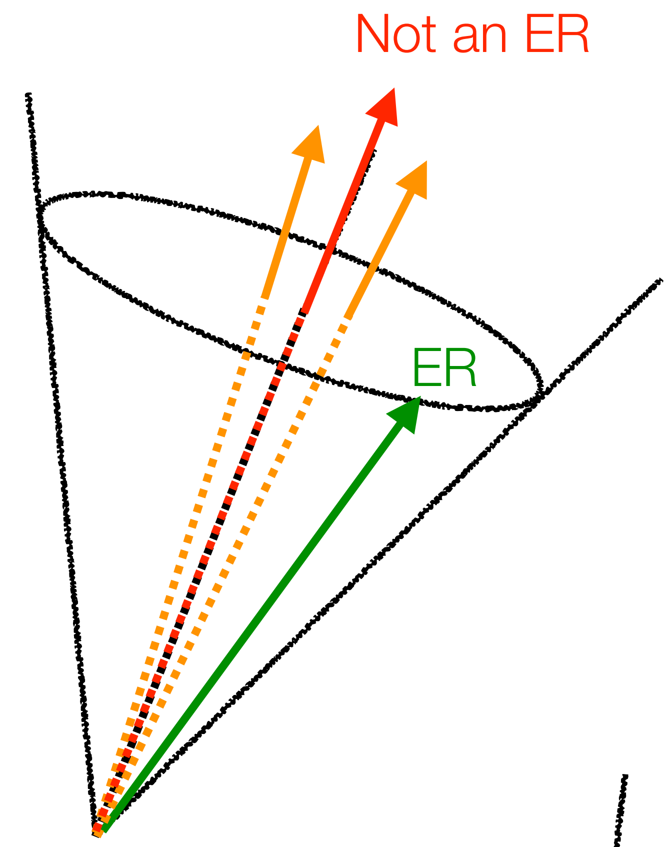
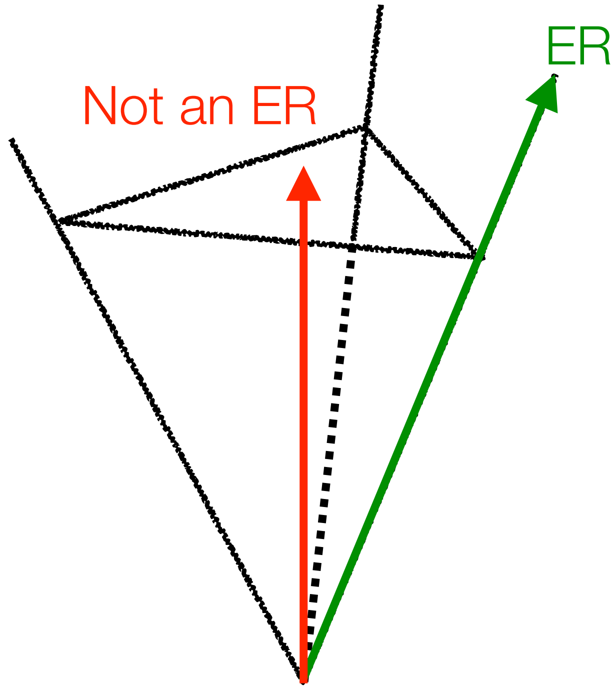
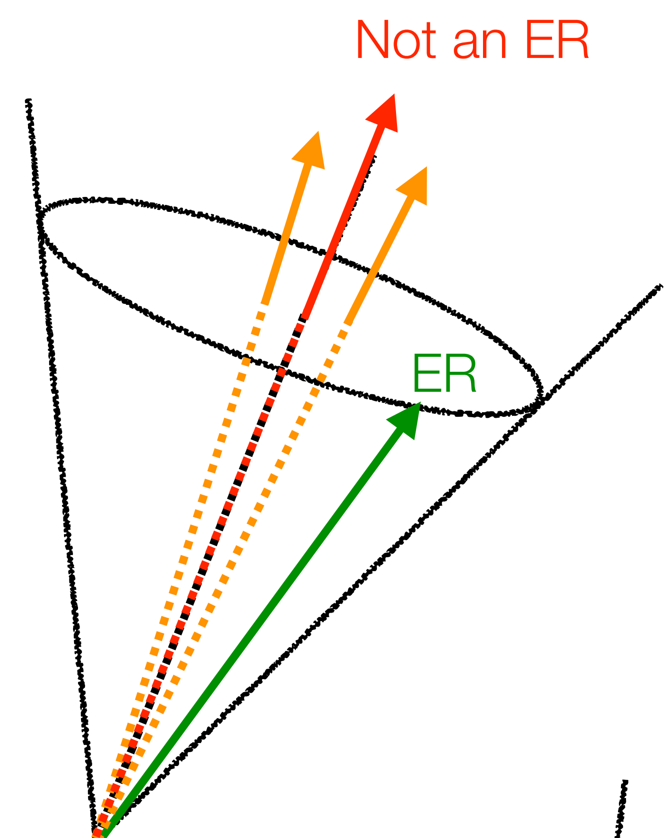
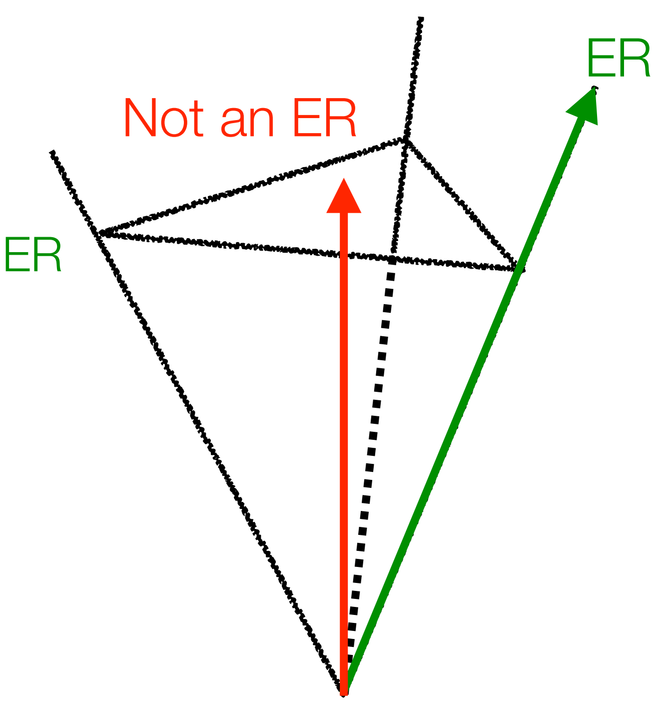




Extremal rays

- **Extremal Ray (ER)**: an extremal ray of cone C cannot be split into two other vectors in C, which are linearly independent.
- In polyhedral cones, ERs are the edge vectors.
- Being **not splittable**, the corresponding UV completion cannot have more than one (type of) particles.
  - If data tells us we are on an ER, the UV particle content is **uniquely determined**. More UV particles -> sum of more than one rays, non-ER
  - Other structures (vertices, faces, etc.) also have similar implication for the UV.
  - Not true at dim-6. No cone -> no ER exists.





SM Higgs

Operators

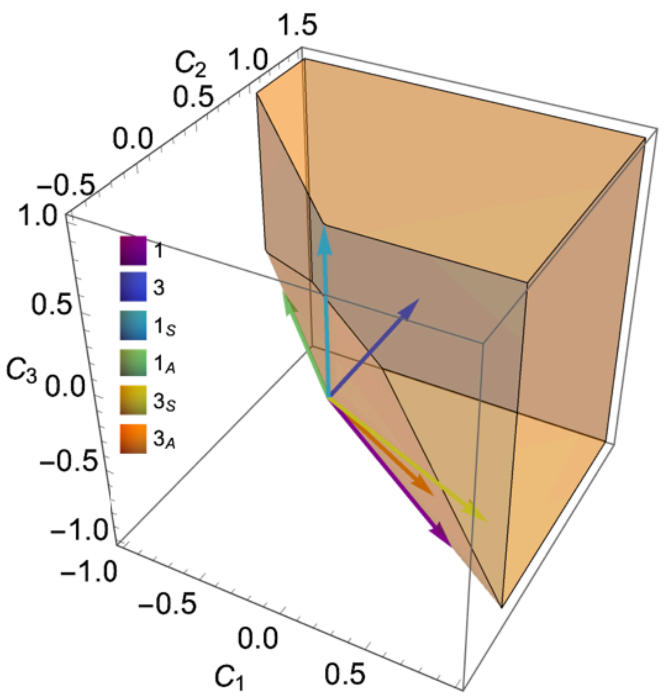
$$Q_{H^4}^{(1)} = \left(D_\mu H^\dagger D_\nu H\right)\left(D^\nu H^\dagger D^\mu H\right)$$
$$Q_{H^4}^{(2)} = \left(D_\mu H^\dagger D_\nu H\right)\left(D^\mu H^\dagger D^\nu H\right)$$
$$Q_{H^4}^{(3)} = \left(D^\mu H^\dagger D_\mu H\right)\left(D^\nu H^\dagger D_\nu H\right)$$

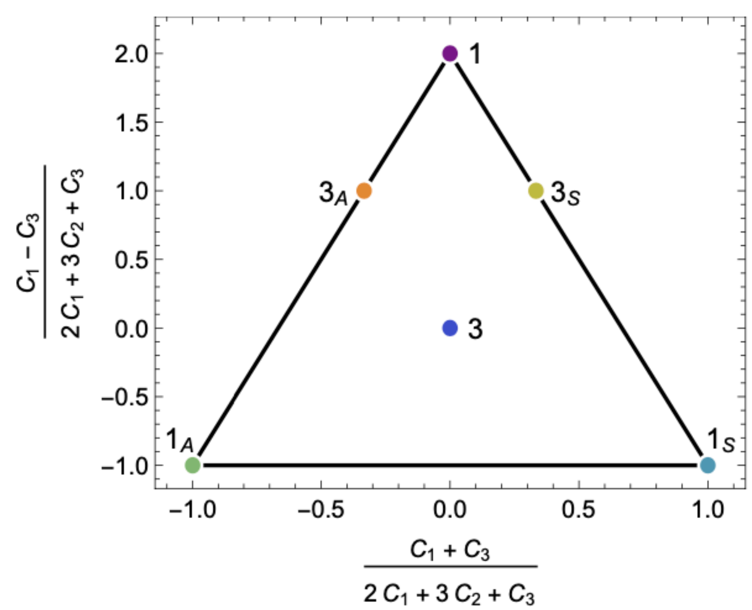
Generators

Either construct from symmetry

Particle	Spin	Charge/irrep	Interaction	ER	$\vec{c}$
$\mathcal{B}_1$	1	1_1	$g\mathcal{B}_1^{\mu\dagger}(H^T\epsilon \overleftrightarrow{D}_\mu H) + h.c.$	✓	$8(1,0,-1)$
Ξ_1	0	3_1	$gM\Xi_1^{I\dagger}(H^T\epsilon\tau^I H) + h.c.$	✗	$8(0,1,0)$
$\mathcal{S}$	0	$1_0(S)$	$gM\mathcal{S}(H^\dagger H)$	✓	$2(0,0,1)$
$\mathcal{B}$	1	$1_0(A)$	$g\mathcal{B}^\mu(H^\dagger \overleftrightarrow{D}_\mu H)$	✓	$2(-1,1,0)$
Ξ_0	0	$3_0(S)$	$gM\Xi_0^I(H^\dagger\tau^I H)$	✗	$2(2,0,-1)$
$\mathcal{W}$	1	$3_0(A)$	$g\mathcal{W}^{\mu I}(H^\dagger\tau^I \overleftrightarrow{D}_\mu H)$	✗	$2(1,1,-2)$

or enumerate all UV particle types





A cross section of the triangular cone

$$C_2 \geq 0, \quad C_1 + C_2 \geq 0, \quad C_1 + C_2 + C_3 \geq 0.$$

First derived with "elastic scattering" by [G. Remmen, N. Rodd 1908.09845]

Figure 5. The positivity cone for 4-Higgs operators, with the corresponding generators. Colors represent different irreps. They are only labeled with SU(2) irrep (1,3) and the exchange symmetry (S,A). The cone is shown in the left plot, while the right plot shows a slice of the cone. The latter can be thought of as intersecting the cone with a hyperplane $2C_1 + 3C_2 + C_3 = 1$.

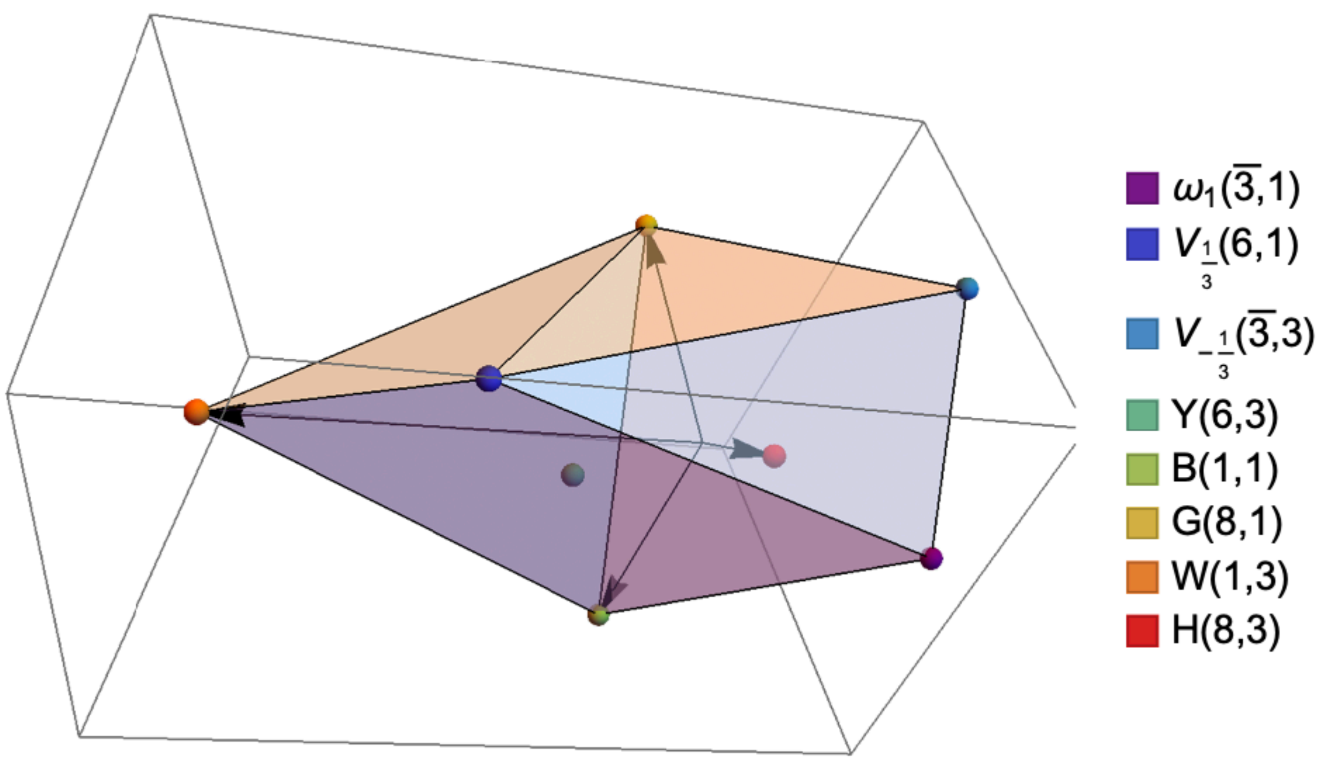
Quark doublets [SU(3) x SU(2)]

Operators

$$O_1 = \partial_\mu \left(\bar{q}\gamma_\nu q\right) \partial^\mu \left(\bar{q}\gamma^\nu q\right),$$
$$O_2 = \partial_\mu \left(\bar{q}\gamma_\nu \tau^I q\right) \partial^\mu \left(\bar{q}\gamma^\nu \tau^I q\right),$$
$$O_3 = \partial_\mu \left(\bar{q}\gamma_\nu T^A q\right) \partial^\mu \left(\bar{q}\gamma^\nu T^A q\right)$$
$$O_4 = \partial_\mu \left(\bar{q}\gamma_\nu \tau^I T^A q\right) \partial^\mu \left(\bar{q}\gamma^\nu \tau^I T^A q\right)$$

Generators

State	Spin	Charge/irrep	Interaction	ER	$\vec{c}$
ω_1	0	$(3,1)_{-\frac{1}{3}}$	$\omega_1^a \epsilon_{abc}\bar{q}^{cb}\epsilon q^c$	✓	$\frac{1}{3}(-1,1,3,-3)$
$\mathcal{V}_{-\frac{1}{3}}$	1	$(3,3)_{-\frac{1}{3}}$	$\mathcal{V}^{aI}_{-\frac{1}{3}}\epsilon_{abc}\bar{q}^{cb}\epsilon\tau^I i\overleftrightarrow{D}_\mu q^c$	✓	$\frac{1}{3}(3,1,-9,-3)$
$\mathcal{V}_{\frac{1}{3}}$	1	$(6,1)_{\frac{1}{3}}$	$\mathcal{V}^{\dagger ab\mu}_{\frac{1}{3}}\bar{q}^{c(a}\epsilon i\overleftrightarrow{D}_\mu q^{b)}$	✓	$\frac{1}{6}(2,-2,3,-3)$
Υ	0	$(6,3)_{\frac{1}{3}}$	$\Upsilon^{\dagger Iab}\bar{q}^{c(a}\epsilon\tau^I q^{b)}$	✗	$\frac{1}{6}(-6,-2,-9,-3)$
$\mathcal{B}$	1	$(1,1)_0$	$\mathcal{B}^\mu\bar{q}\gamma_\mu q$	✓	$\frac{1}{2}(-1,0,0,0)$
$\mathcal{W}$	1	$(1,3)_0$	$\mathcal{W}^{I\mu}\bar{q}\gamma_\mu\tau^I q$	✓	$\frac{1}{2}(0,-1,0,0)$
$\mathcal{G}$	1	$(8,1)_0$	$\mathcal{G}^{A\mu}\bar{q}\gamma_\mu T^A q$	✓	$\frac{1}{2}(0,0,-1,0)$
$\mathcal{H}$	1	$(8,3)_0$	$\mathcal{H}^{AI\mu}\bar{q}\gamma_\mu T^A\tau^I q$	✗	$\frac{1}{2}(0,0,0,-1)$



"Elastic bounds" incomplete [Remmen, Rodd 2004.02885]

First completed by [T. Trott 2011.10058]

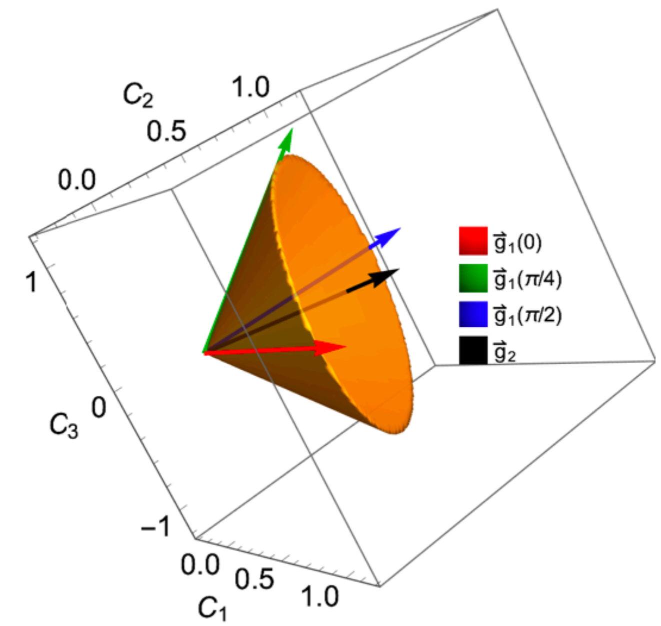
$$\begin{pmatrix} 3 & 3 & 1 & 1 \\ 0 & 3 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 12 & 0 & 1 & 9 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} \le 0$$

$u^i v^j \Rightarrow$

$$\frac{1}{2}\left(|\mathbf{2};1\rangle|\mathbf{2};2\rangle - |\mathbf{2};2\rangle|\mathbf{2};1\rangle\right)\left(|\mathbf{3};1\rangle|\mathbf{3};2\rangle - |\mathbf{3};2\rangle|\mathbf{3};1\rangle\right)$$
$$+\frac{3}{2}\left(|\mathbf{2};1\rangle|\mathbf{2};2\rangle + |\mathbf{2};2\rangle|\mathbf{2};1\rangle\right)\left(|\mathbf{3};1\rangle|\mathbf{3};2\rangle + |\mathbf{3};2\rangle|\mathbf{3};1\rangle\right)$$

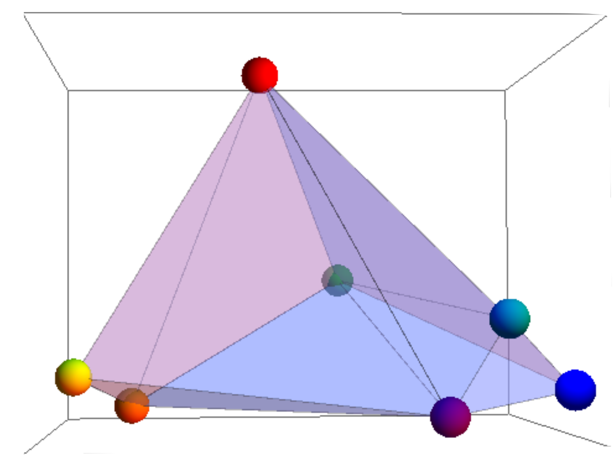
Photon (parity violating)

$$O_1 = (B_{\mu\nu}B^{\mu\nu})\,(B_{\rho\sigma}B^{\rho\sigma})$$
$$O_2 = \left(B_{\mu\nu}\tilde{B}^{\mu\nu}\right)\left(B_{\rho\sigma}\tilde{B}^{\rho\sigma}\right)$$
$$O_3 = \left(B_{\mu\nu}B^{\mu\nu}\right)\left(B_{\rho\sigma}\tilde{B}^{\rho\sigma}\right)$$



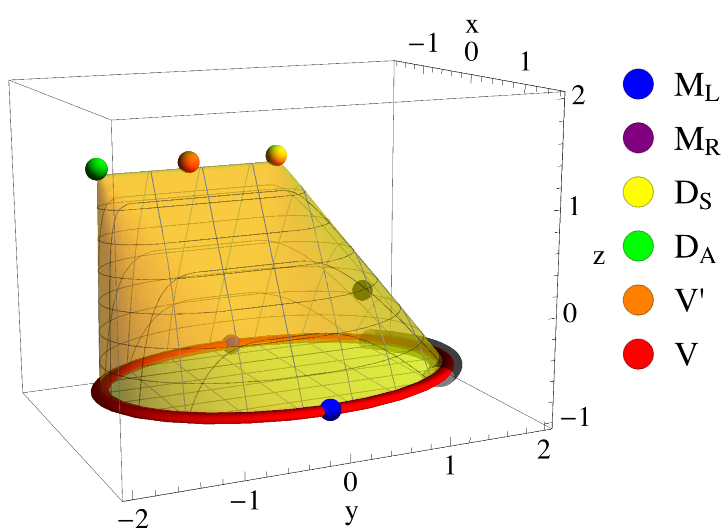
W-boson (with parity)

$$O_{T,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}]$$
$$O_{T,2} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}]$$
$$O_{T,1} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]$$
$$O_{T,10} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\tilde{W}^{\alpha\beta}]$$



Lepton (L+R)

$$O_1 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{e}\gamma_\mu e)\ ,$$
$$O_2 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{l}\gamma_\mu l)\ ,$$
$$O_3 = D^\alpha(\bar{l}e)\ D_\alpha(\bar{e}l),$$
$$O_4 = \partial^\alpha(\bar{l}\gamma^\mu l)\ \partial_\alpha(\bar{l}\gamma_\mu l)\ ,$$
$$O_5 = D^\alpha(\bar{l}\gamma^\mu\tau^I l)\ D_\alpha(\bar{l}\gamma_\mu\tau^I l)$$



Bounds on all self-quartic operators (i.e. with 4 identical fields) are known

Why this is better than "elastic scattering"

- Elastic scattering of two superposed states: $\mathbf{M}_{uv \to uv} \;\; \Rightarrow \;\; u^i v^j u^{*k} v^{*l} \mathcal{M}^{ijkl}$

  - Initial = final = $|uv\rangle = u^i v^j \, |i\rangle \otimes |j\rangle$

R(s): $u^i v^j u^k v^l m^{ij} m^{kl} \geq 0,$

L(u): $u^i v^l u^k v^j m^{ij} m^{kl} \geq 0$

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✦ The four quark bound $\begin{pmatrix} 3 & 3 & 1 & 1 \\ 0 & 3 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 12 & 0 & 1 & 9 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} \leq 0$ is an elastic scattering of

$$i=f= \frac{1}{2}\left(|\mathbf{2};1\rangle|\mathbf{2};2\rangle - |\mathbf{2};2\rangle|\mathbf{2};1\rangle\right)\left(|\mathbf{3};1\rangle|\mathbf{3};2\rangle - |\mathbf{3};2\rangle|\mathbf{3};1\rangle\right) + \frac{3}{2}\left(|\mathbf{2};1\rangle|\mathbf{2};2\rangle + |\mathbf{2};2\rangle|\mathbf{2};1\rangle\right)\left(|\mathbf{3};1\rangle|\mathbf{3};2\rangle + |\mathbf{3};2\rangle|\mathbf{3};1\rangle\right)$$

(T. Trott, KITP seminar)

which does not have a uv -> uv interpretation.

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  - In general, if initial = final = $U^{ij} |i\rangle \otimes |j\rangle$

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  - Still, may work for specific U matrices.

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  - Still, may work for <u>specific U matrices</u>.
- How to find them? With symmetries, just enumerate the generators. Otherwise…

Positivity bound from spectrahedrons

[X. Li et al., 2101.01191]

C is dual to a "spectrahedral cone"

$$\mathbf{C}^{n^4} = \mathrm{cone}\left(\left\{m^{ij}m^{kl} + m^{il}m^{kj}\right\}\right)$$

$\mathbf{C}$ is dual to a "spectrahedral cone"

$$\mathbf{C}^{n^4} = \text{cone}\left(\left\{m^{ij}m^{kl} + m^{il}m^{kj}\right\}\right)$$

- Define the "crossing symmetric" subspace of r4 tensors,

$$\mathbf{C}^{n^4} \subset \vec{\mathbf{S}}^{n^4} \equiv \left\{\mathcal{T} \mid \mathcal{T}^{ijkl} = \mathcal{T}^{ilkj} = \mathcal{T}^{kjil} = \mathcal{T}^{jilk}\right\}$$

C is dual to a "spectrahedral cone"

$$\mathbf{C}^{n^4} = \text{cone}\left(\left\{m^{ij}m^{kl} + m^{il}m^{kj}\right\}\right)$$

- Define the "crossing symmetric" subspace of r4 tensors,

$$\mathbf{C}^{n^4} \subset \vec{\mathbf{S}}^{n^4} \equiv \left\{\mathcal{T} \mid \mathcal{T}^{ijkl} = \mathcal{T}^{ilkj} = \mathcal{T}^{kjil} = \mathcal{T}^{jilk}\right\}$$

- Within **S**, **C** is dual to a "spectrahedron" (dual is the set of valid bounds, Q.M>0)

$$\boxed{\mathbf{Q}^{n^4} = \mathbf{S}_{+}^{n^2 \times n^2} \cap \vec{\mathbf{S}}^{n^4}} \qquad \mathcal{Q}^{ijkl}\sum_{\alpha}\left(m_{\alpha}^{ij}m_{\alpha}^{kl} + m_{\alpha}^{il}m_{\alpha}^{kj}\right) = 2\sum_{\alpha} m_{\alpha}^{ij}\mathcal{Q}^{ijkl}m_{\alpha}^{kl} \geq 0$$

PSD matrix cone

Crossing sym. subspace

C is dual to a "spectrahedral cone"

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PSD matrix cone

Crossing sym. subspace

- ✦ Completeness: **C**** = **C**, $\mathbf{C}^{n^4} = \left\{\mathcal{M} \in \vec{\mathbf{S}}^{n^4} \mid \mathcal{Q}\cdot\mathcal{M} \geq 0 \; \forall \mathcal{Q} \in \mathbf{Q}^{n^4}\right\}$

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PSD matrix cone — Crossing sym. subspace

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- Independence: if $\mathcal{Q}_1 = \mathcal{Q}_2 + \mathcal{Q}_3$, remove Q_1, i.e. only keep the ERs.

C is dual to a "spectrahedral cone"

$$\mathbf{C}^{n^4} = \text{cone}\left(\left\{m^{ij}m^{kl} + m^{il}m^{kj}\right\}\right)$$

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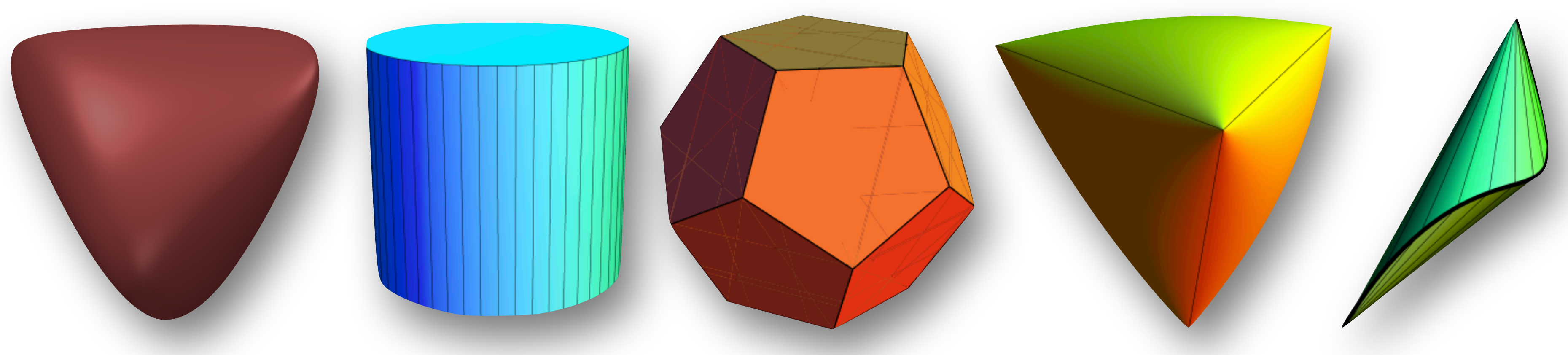
PSD matrix cone　　Crossing sym. subspace

- Completeness: **C**** = **C**, $\mathbf{C}^{n^4} = \left\{\mathcal{M} \in \vec{\mathbf{S}}^{n^4} \mid \mathcal{Q}\cdot\mathcal{M} \geq 0 \ \forall\mathcal{Q} \in \mathbf{Q}^{n^4}\right\}$
- Independence: if $\mathcal{Q}_1 = \mathcal{Q}_2 + \mathcal{Q}_3$, remove Q_1, i.e. only keep the ERs.
- Finding positivity bounds = finding the ERs of some "spectrahedron".

$$\mathbf{C}^{n^4} = \left\{\mathcal{M} \in \vec{\mathbf{S}}^{n^4} \mid \mathcal{Q}\cdot\mathcal{M} \geq 0, \ \forall\mathcal{Q} \in \boxed{\text{ext}\left(\mathbf{Q}^{n^4}\right)}\right\}$$

Spectrahedron is…

- Wiki: the set of n × n positive semidefinite matrices forms a convex cone, and a spectrahedron is a shape that can be formed by intersecting this cone with a linear affine subspace.
- Let $Q_i, \ i = 0, 1, \ldots, m$ be the basis matrices of the subspace, $Q(x) \equiv x_i Q_i$
- The spectrahedron $G = \{x \mid Q(x) \succeq 0\}$
- How do they look like? From google:



Example: 2-scalar EFT

Scalar EFT with ϕ_1, ϕ_2 , with Z_2 symmetry $\phi_1 \to -\phi_1$

$$O_{1111} = (\partial^\mu \phi_1 \partial_\mu \phi_1)^2$$
$$O_{2222} = (\partial^\mu \phi_2 \partial_\mu \phi_2)^2$$
$$O_{1212} = (\partial^\mu \phi_1 \partial_\mu \phi_2)^2$$
$$O_{1122} = (\partial^\mu \phi_1 \partial_\mu \phi_1)(\partial^\nu \phi_2 \partial_\nu \phi_2)$$

$$\mathcal{M}^{ijkl} = \begin{array}{c} \\ ij=11 \\ 22 \\ 12 \\ 21 \end{array} \begin{array}{c} \begin{array}{cccc} kl=11 & 22 & 12 & 21 \end{array} \\ \begin{bmatrix} 4C_{1111} & C'_{1122} & & \\ C'_{1122} & 4C_{2222} & & \\ & & C_{1212} & C'_{1122} \\ & & C'_{1122} & C_{1212} \end{bmatrix} \end{array}$$

$$\mathbf{Q}^{2^4} \ni \mathcal{Q} = \begin{array}{c} \\ ij=11 \\ 22 \\ 12 \\ 21 \end{array} \begin{array}{c} \begin{array}{cccc} kl=11 & 22 & 12 & 21 \end{array} \\ \begin{pmatrix} a & b & & \\ b & c & & \\ & & d & b \\ & & b & d \end{pmatrix} \end{array}$$

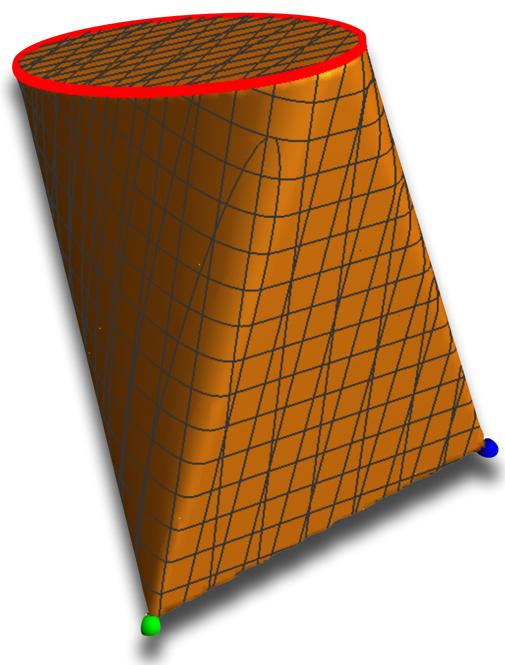
$$\mathcal{Q} \succeq 0 \;\Rightarrow\; a \geq 0,\; c \geq 0,\; ac \geq b^2,\; d \geq |b|$$

ERs

$$Q_{\text{ex1}}(r) = \begin{bmatrix} 1 & r & 0 & 0 \\ r & r^2 & 0 & 0 \\ 0 & 0 & |r| & r \\ 0 & 0 & r & |r| \end{bmatrix}, \; Q^{12}_{\text{ex2}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

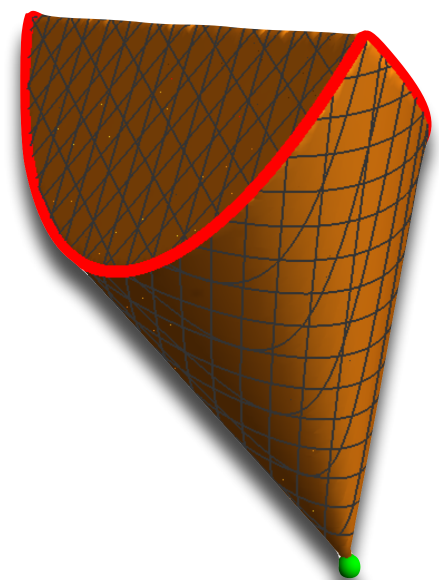
Amplitude space

$\mathbf{C}^{n^4}$



Dual space (spectrahedron)

$\mathbf{Q}^{n^4}$

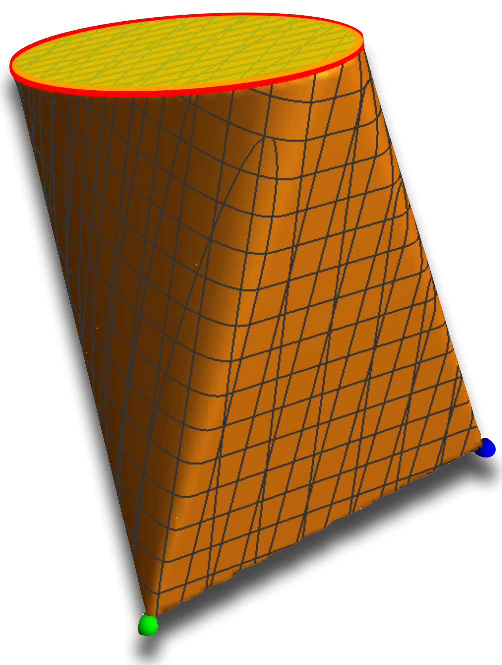


ERs = posi. bounds

$$C_{1111} \geq 0,\ C_{2222} \geq 0,\ C_{1212} \geq 0$$
$$4\sqrt{C_{1111}C_{2222}} \geq \pm(2C_{1122} + C_{1212}) - C_{1212}$$

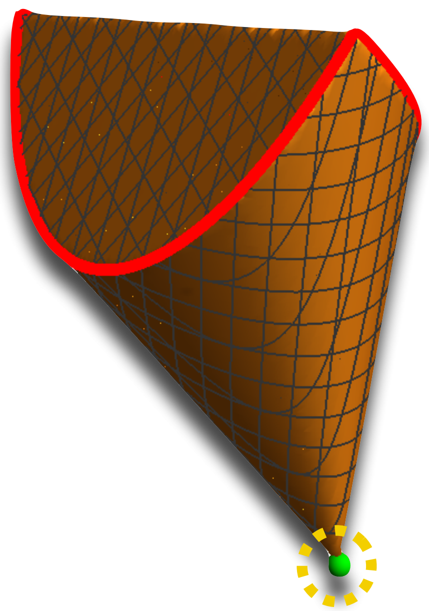
Amplitude space

$\mathbf{C}^{n^4}$



Dual space (spectrahedron)

$\mathbf{Q}^{n^4}$



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Amplitude space

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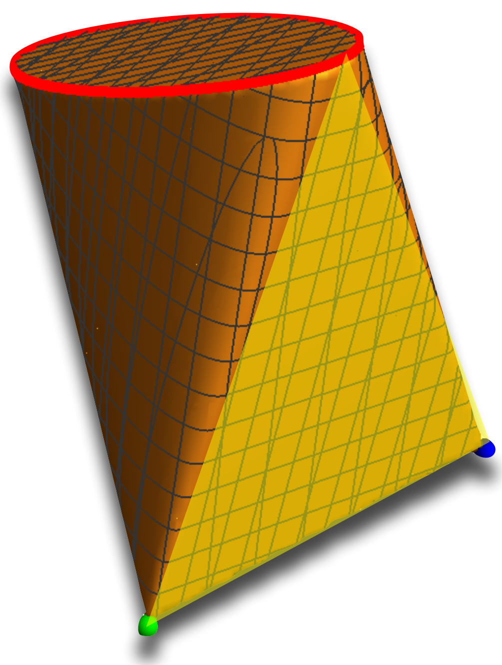
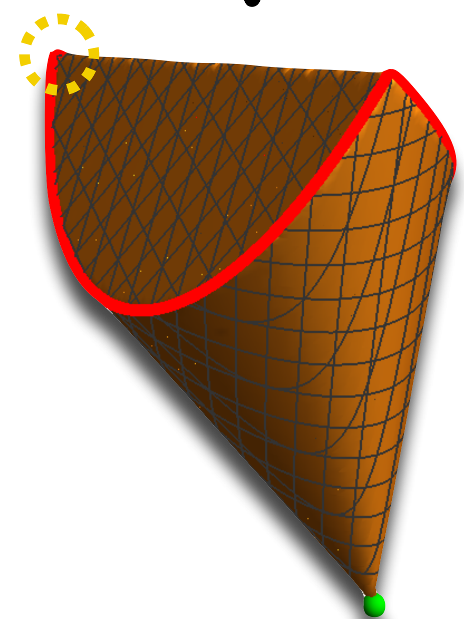


Dual space (spectrahedron)

$\mathbf{Q}^{n^4}$



ERs = posi. bounds

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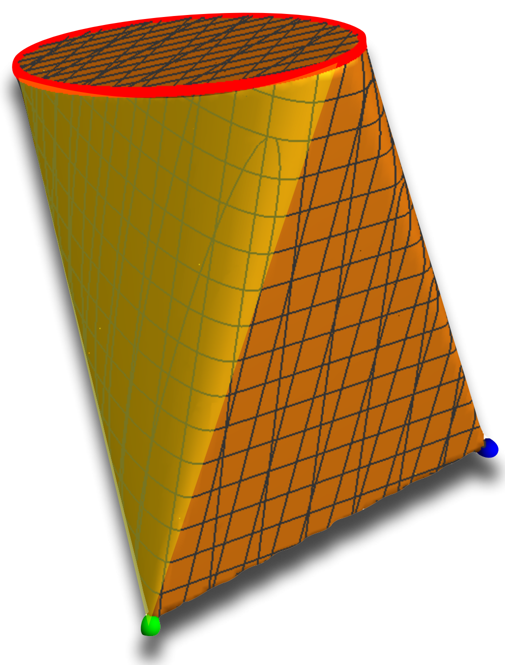
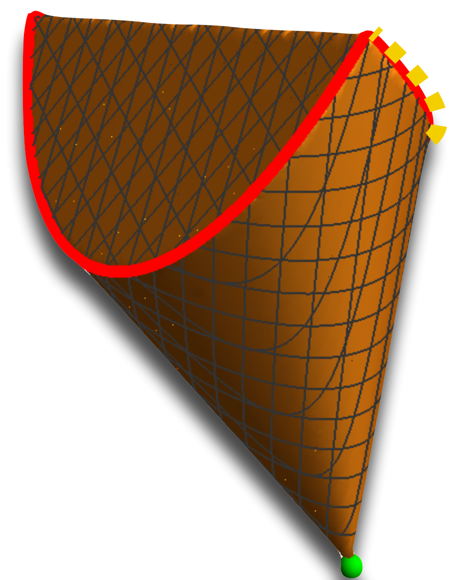
Amplitude space

$\mathbf{C}^{n^4}$



Dual space (spectrahedron)

$\mathbf{Q}^{n^4}$



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Amplitude space

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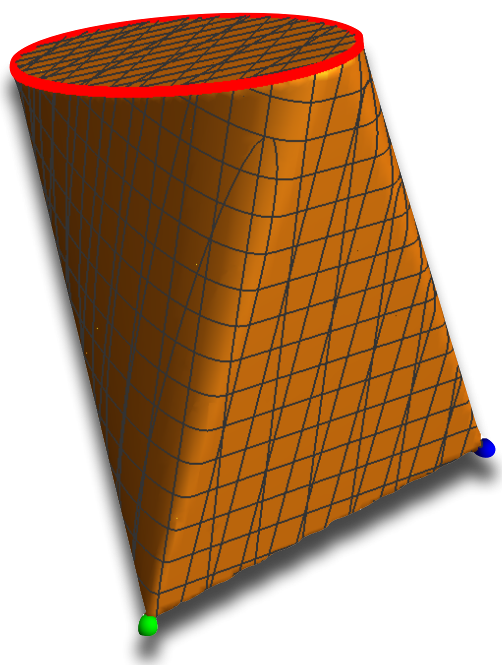

Dual space (spectrahedron)

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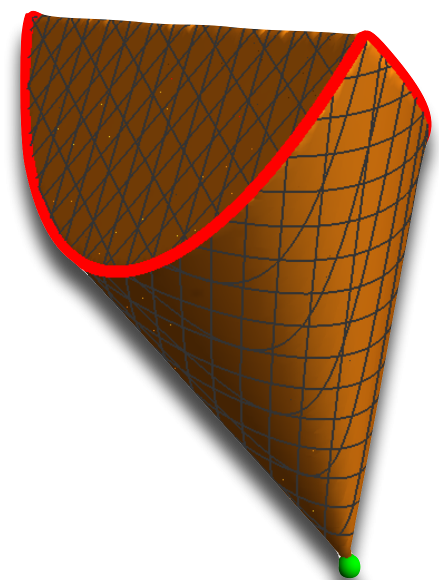

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Example: 2-scalar EFT

Without Z_2 symmetry

$$O_{1111} = (\partial^\mu \phi_1 \partial_\mu \phi_1)^2 \qquad O_{1112} = (\partial^\mu \phi_1 \partial_\mu \phi_1)(\partial^\mu \phi_1 \partial_\mu \phi_2)$$
$$O_{2222} = (\partial^\mu \phi_2 \partial_\mu \phi_2)^2 \qquad O_{1222} = (\partial^\mu \phi_1 \partial_\mu \phi_2)(\partial^\mu \phi_2 \partial_\mu \phi_2)$$
$$O_{1212} = (\partial^\mu \phi_1 \partial_\mu \phi_2)^2$$
$$O_{1122} = (\partial^\mu \phi_1 \partial_\mu \phi_1)(\partial^\nu \phi_2 \partial_\nu \phi_2)$$

	kl=11	22	12	21
ij=11	$4C_{1111}$	C'_{1122}	C_{1112}	C_{1112}
22	C'_{1122}	$4C_{2222}$	C_{1222}	C_{1222}
12	C_{1112}	C_{1222}	C_{1212}	C'_{1122}
21	C_{1112}	C_{1222}	C'_{1122}	C_{1212}

$$\mathcal{M}^{ijkl} = \begin{bmatrix} 4C_{1111} & C'_{1122} & C_{1112} & C_{1112} \\ C'_{1122} & 4C_{2222} & C_{1222} & C_{1222} \\ C_{1112} & C_{1222} & C_{1212} & C'_{1122} \\ C_{1112} & C_{1222} & C'_{1122} & C_{1212} \end{bmatrix}$$

$$C'_{1122} \equiv C_{1122} + \frac{1}{2} C_{1212}$$

ERs: $$\begin{bmatrix} a^2 & ab & ac & ac \\ ab & b^2 & bc & bc \\ ac & bc & 2c^2 - ab & ab \\ ac & bc & ab & 2c^2 - ab \end{bmatrix} \qquad c^2 \geq ab$$

Bounds

$C_{1111} \geq 0 \quad \text{and} \quad 4C_{1111}C_{1212} - C_{1112}^2 \geq 0$

and $\Big\{ -C_{2222}C_{1112}^2 + C_{1122}C_{1222}C_{1112} - C_{1111}C_{1222}^2 - C_{1122}^2 C_{1212} + 4C_{1111}C_{1212}C_{2222} \geq 0$

or $\Big[\Delta \equiv C_{1122}^2 + 2C_{1212}C_{1122} + C_{1212}^2 - 3C_{1112}C_{1222} + 12C_{1111}C_{2222} \geq 0$

and $\sqrt{\Delta} \leq C_{1212} - 2C_{1122}$

and $\sqrt{\Delta} \geq \frac{3C_{1112}^2}{4C_{1111}} - 2(C_{1122} + C_{1212})$

and $2\Delta^{3/2} \geq 2C_{1122}^3 + 6C_{1212}C_{1122}^2 + 6C_{1212}^2 C_{1122} - 9(C_{1112}C_{1222} + 8C_{1111}C_{2222})C_{1122}$
$+ 2C_{1212}^3 - 9C_{1212}(C_{1112}C_{1222} + 8C_{1111}C_{2222}) + 27(C_{2222}C_{1112}^2 + C_{1111}C_{1222}^2)\Big]\Big\}$

$\mathbf{M}_{uv \to uv}$ interpretation:

$$u = (a, c - \sqrt{c^2 - ab})$$
$$v = (a, c + \sqrt{c^2 - ab})$$

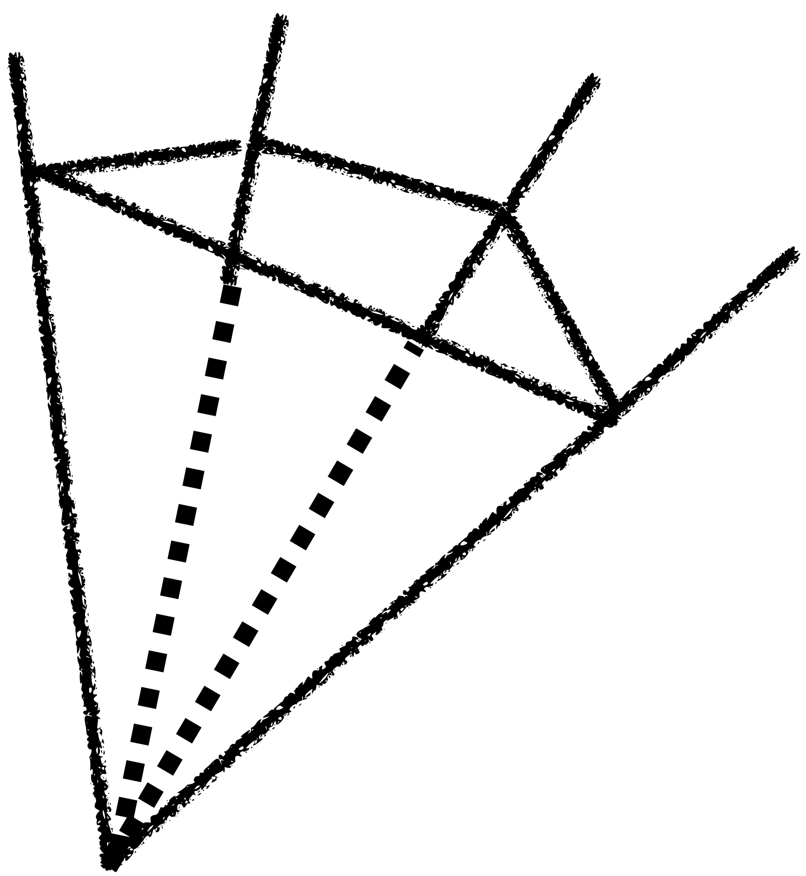
The ERs are:

$$u^i v^j u^k v^l + v^i u^j v^k u^l$$

Not possible for n>2

Numerical approach 1: random sampling

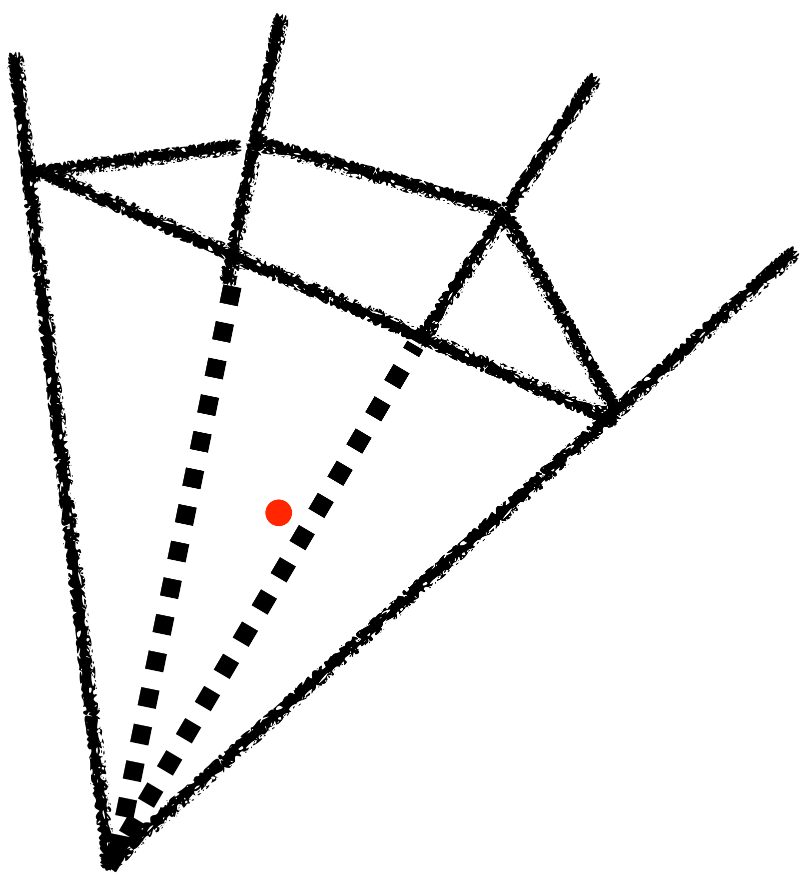
i.e. a “Monte Carlo generator” of positivity bounds



Numerical approach 1: random sampling

i.e. a "Monte Carlo generator" of positivity bounds

- Start with a random point



Numerical approach 1: random sampling

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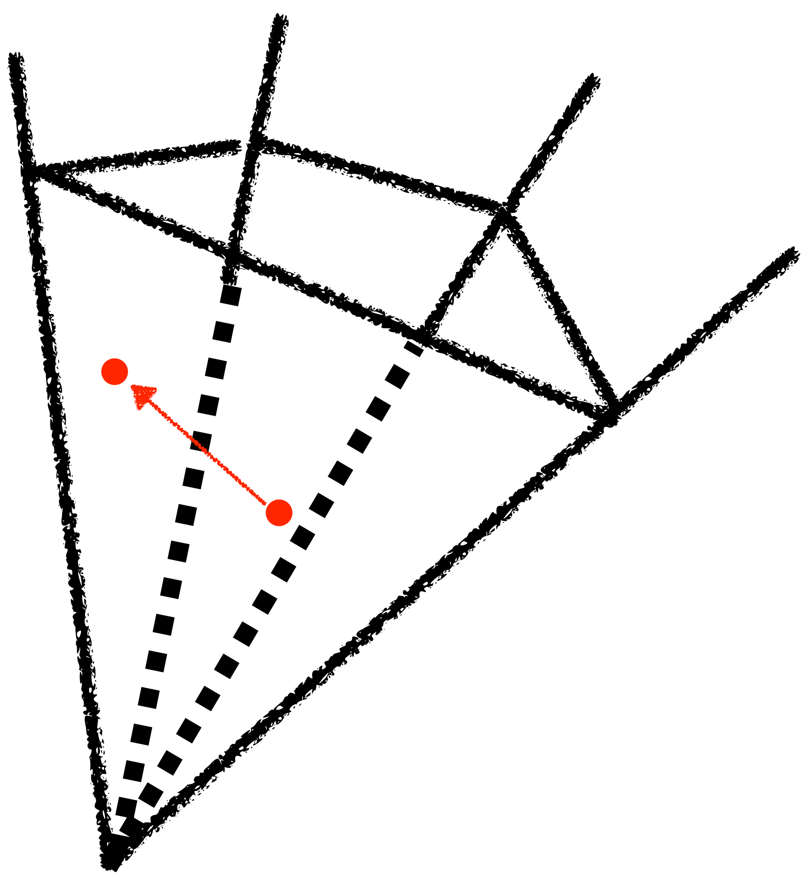

- Start with a random point
- Take a random direction, keep going until hitting the boundary (this is a SDP)

Numerical approach 1: random sampling

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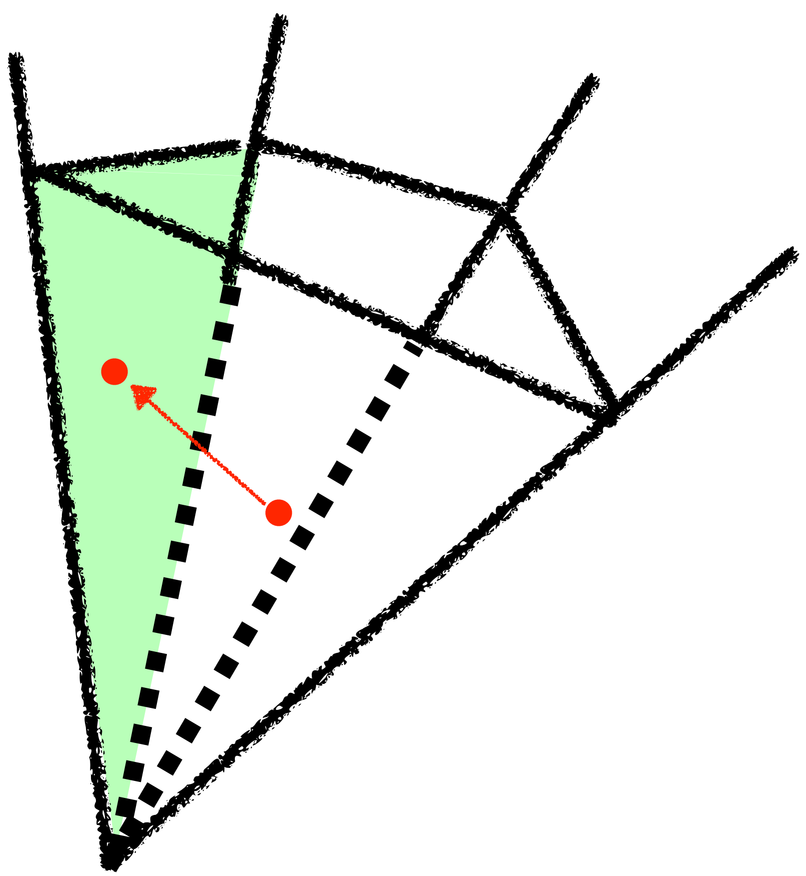
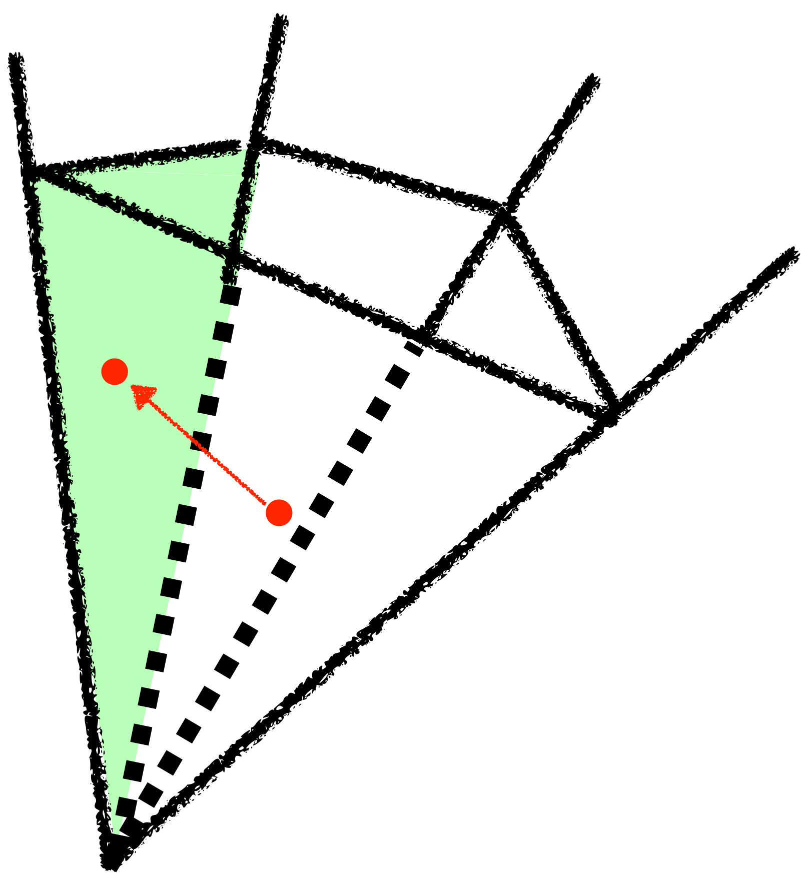


- Start with a random point
- Take a random direction, keep going until hitting the boundary (this is a SDP)
- Compute the face F(x) that contains the hitting point x. (Calculating F(x) follows [Ramana & Goldman ’95])

Numerical approach 1: random sampling

i.e. a "Monte Carlo generator" of positivity bounds



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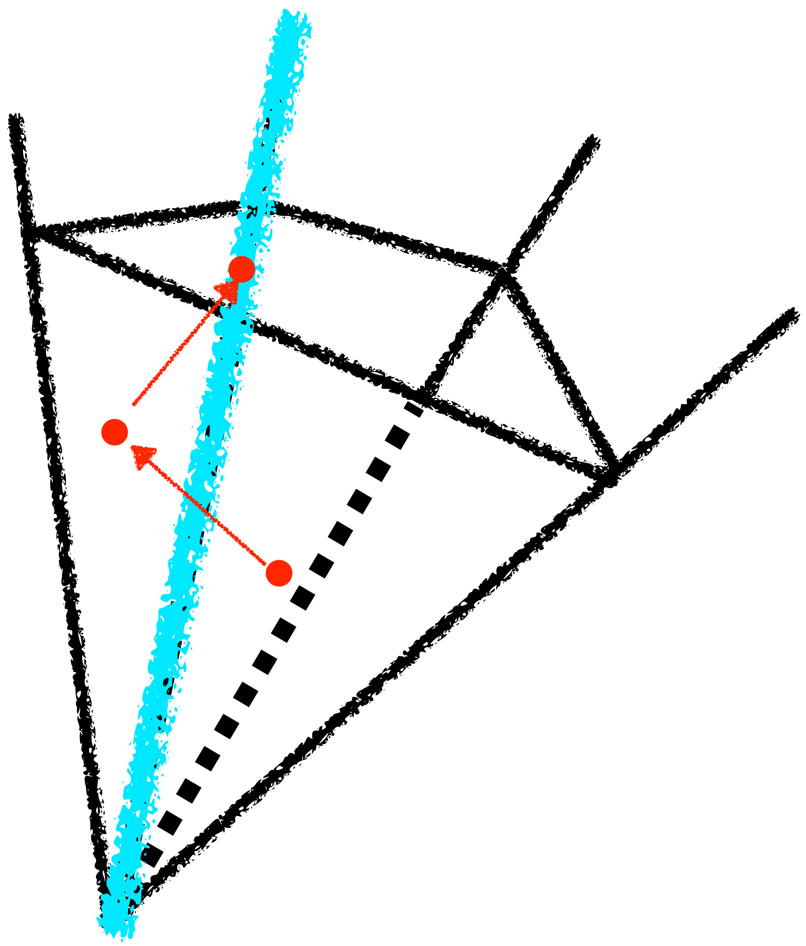
- A face of a spectrahedron is another spectrahedron (of lower dimension)

Numerical approach 1: random sampling

i.e. a “Monte Carlo generator” of positivity bounds



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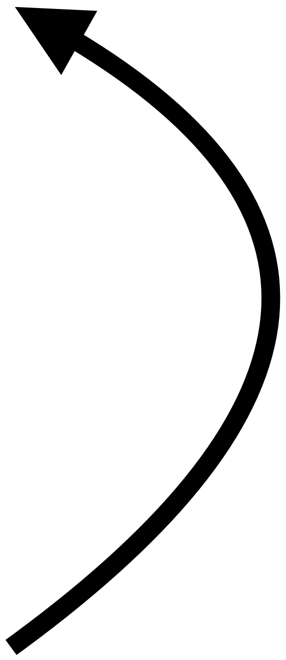
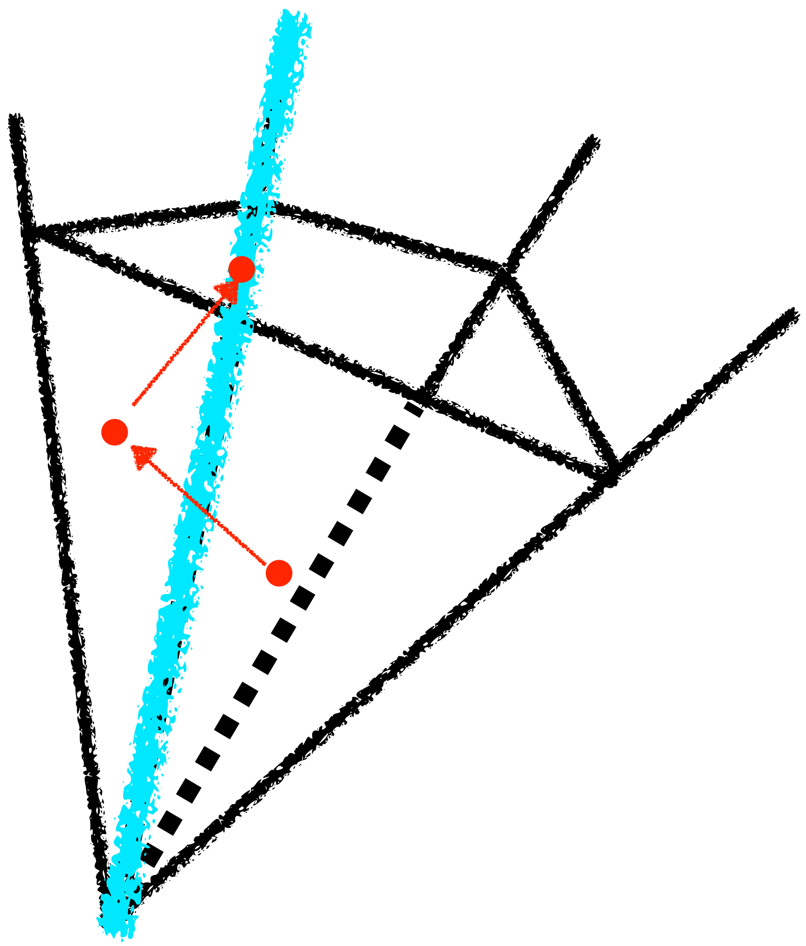


Numerical approach 1: random sampling

i.e. a “Monte Carlo generator” of positivity bounds



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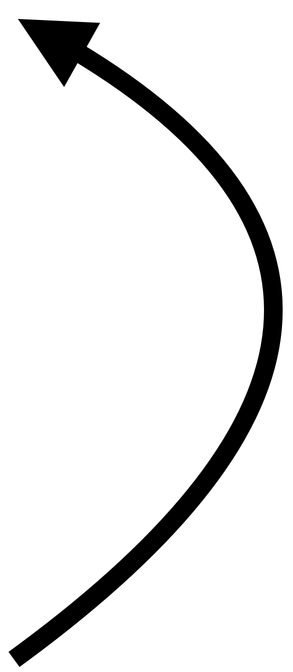
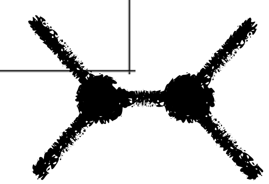
- A face of a spectrahedron is another spectrahedron (of lower dimension)
- Iterate
- Dimension = 1: an ER is found



Gluon operators

$Q_{G^4}^{(1)} = \left(G^A_{\mu\nu} G^{A\mu\nu}\right)\left(G^B_{\rho\sigma} G^{B\rho\sigma}\right)$
$Q_{G^4}^{(2)} = \left(G^A_{\mu\nu} \tilde{G}^{A\mu\nu}\right)\left(G^B_{\rho\sigma} \tilde{G}^{B\rho\sigma}\right)$
$Q_{G^4}^{(3)} = \left(G^A_{\mu\nu} G^{B\mu\nu}\right)\left(G^A_{\rho\sigma} G^{B\rho\sigma}\right)$
$Q_{G^4}^{(4)} = \left(G^A_{\mu\nu} \tilde{G}^{B\mu\nu}\right)\left(G^A_{\rho\sigma} \tilde{G}^{B\rho\sigma}\right)$
$Q_{G^4}^{(7)} = d^{ABE} d^{CDE}\left(G^A_{\mu\nu} G^{B\mu\nu}\right)\left(G^C_{\rho\sigma} G^{D\rho\sigma}\right)$
$Q_{G^4}^{(8)} = d^{ABE} d^{CDE}\left(G^A_{\mu\nu} \tilde{G}^{B\mu\nu}\right)\left(G^C_{\rho\sigma} \tilde{G}^{D\rho\sigma}\right)$
$O_G = f^{ABC} G^{A\nu}_{\mu} G^{B\rho}_{\nu} G^{C\mu}_{\rho}$



$$\vec{x} \cdot \vec{c} \geq 0,$$

$$\vec{c} = [C_{G^4}^{(1)}, C_{G^4}^{(2)}, C_{G^4}^{(3)}, C_{G^4}^{(4)}, C_{G^4}^{(7)}, C_{G^4}^{(8)}, c_G^2]$$

$\vec{x}$:

$[0,0,0,1,0,0,0]$	$[0,0,6,3,7,2,0]$	$[24,0,12,21,15,14,0]$	$[0,0,96,24,64,40,-81]$
$[0,0,1,1,1,0,0]$	$[8,6,1,6,0,2,0]$	$[24,32,24,4,8,0,-27]$	$[40,32,80,4,0,0,-189]$
$[2,0,1,0,0,0,0]$	$[0,6,3,12,5,0,0]$	$[48,36,21,27,25,0,0]$	$[0,0,24,120,40,104,-81]$
$[0,2,0,1,0,0,0]$	$[8,6,1,12,0,0,0]$	$[32,40,4,80,0,0,-27]$	$[0,0,120,24,104,40,-81]$
$[0,0,3,0,2,0,0]$	$[0,6,6,9,10,4,0]$	$[0,48,0,48,0,40,-81]$	$[96,0,144,24,64,40,-81]$
$[0,0,0,3,0,2,0]$	$[0,12,0,14,0,0,-9]$	$[24,0,36,24,16,40,-81]$	$[48,0,96,24,0,40,-243]$
$[1,1,2,2,0,0,0]$	$[0,0,8,8,0,8,-27]$	$[0,0,48,24,32,40,-81]$	$[0,192,168,96,112,120,-405]$
$[6,0,3,0,2,0,0]$	$[12,0,14,0,0,0,-27]$	$[0,0,24,48,16,56,-81]$	$[168,480,168,156,56,160,-729]$
$[4,2,2,1,2,0,0]$	$[6,8,12,1,0,0,-27]$	$[88,32,56,4,40,0,-27]$	$[264,384,156,168,16,200,-729]$
$[0,0,4,0,0,0,-9]$	$[8,16,4,8,0,8,-27]$	$[96,42,27,84,25,0,0]$	$[288,384,216,168,0,200,-891]$
$[6,0,6,0,5,0,0]$	$[0,24,0,12,0,8,-27]$	$[96,66,42,39,50,4,0]$	$[480,384,480,168,160,200,-729]$
$[0,0,3,6,5,4,0]$	$[8,22,1,14,0,10,-27]$	$[120,42,39,42,40,14,0]$	$[336,768,672,216,0,200,-2187]$

48 total

Numerical approach 2: SDP

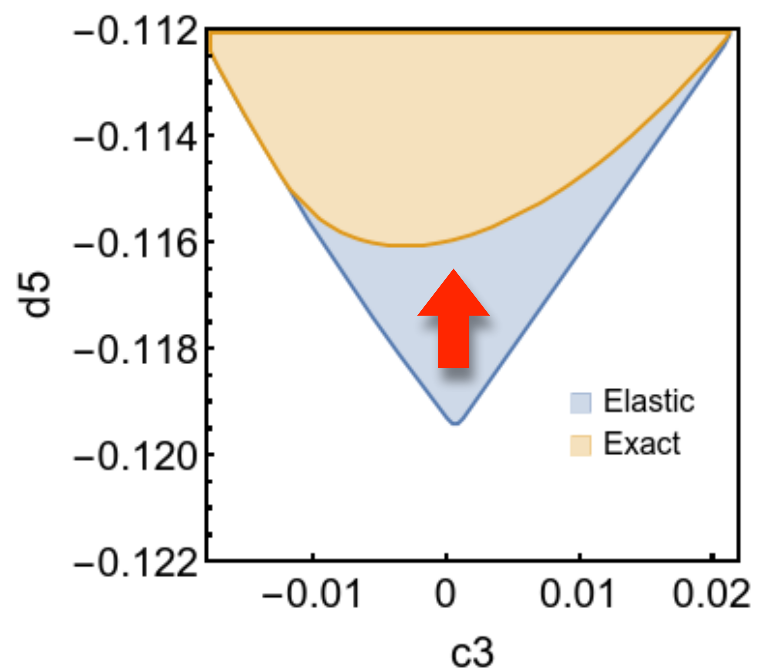
- Given a set of measured coefficients, or *M*, how to check if its in the positivity cone?

$$\mathbf{C}^{n^4} = \left\{ \mathcal{M} \in \vec{\mathbf{S}}^{n^4} \mid \mathcal{Q} \cdot \mathcal{M} \geq 0 \ \forall \mathcal{Q} \in \mathbf{Q}^{n^4} \right\}$$

- This is a semi-definite programming

$$\begin{aligned} &\min \quad \mathcal{Q} \cdot \mathcal{M} \\ &\text{subject to} \quad \mathcal{Q} \in \mathbf{Q}^{n^4} \end{aligned}$$

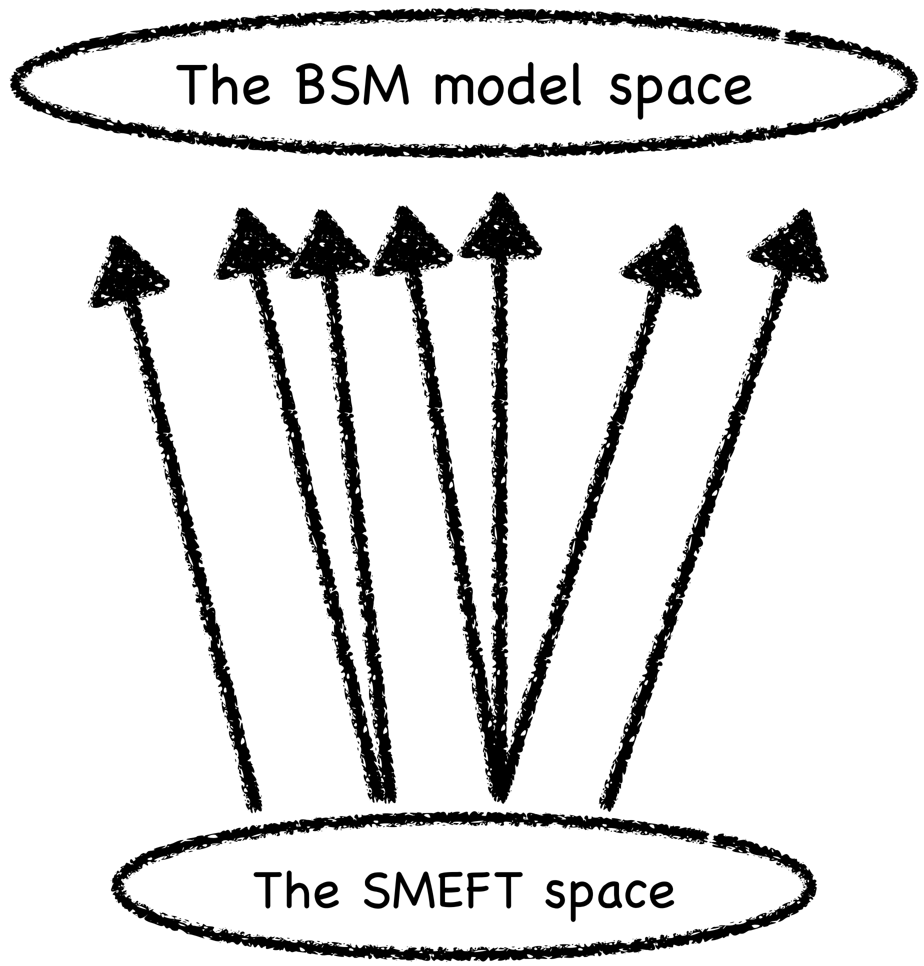
- If there is a (positive) minimum, positivity is satisfied.
  - Solvable in polynomial complexity (in contrast to superposed elastic scattering, which is NP hard)
  - Applications: aQGC, flavor operators, massive gravity (improved bounds on dRGT from [Cheung, Remmen,1601.04068]), etc.



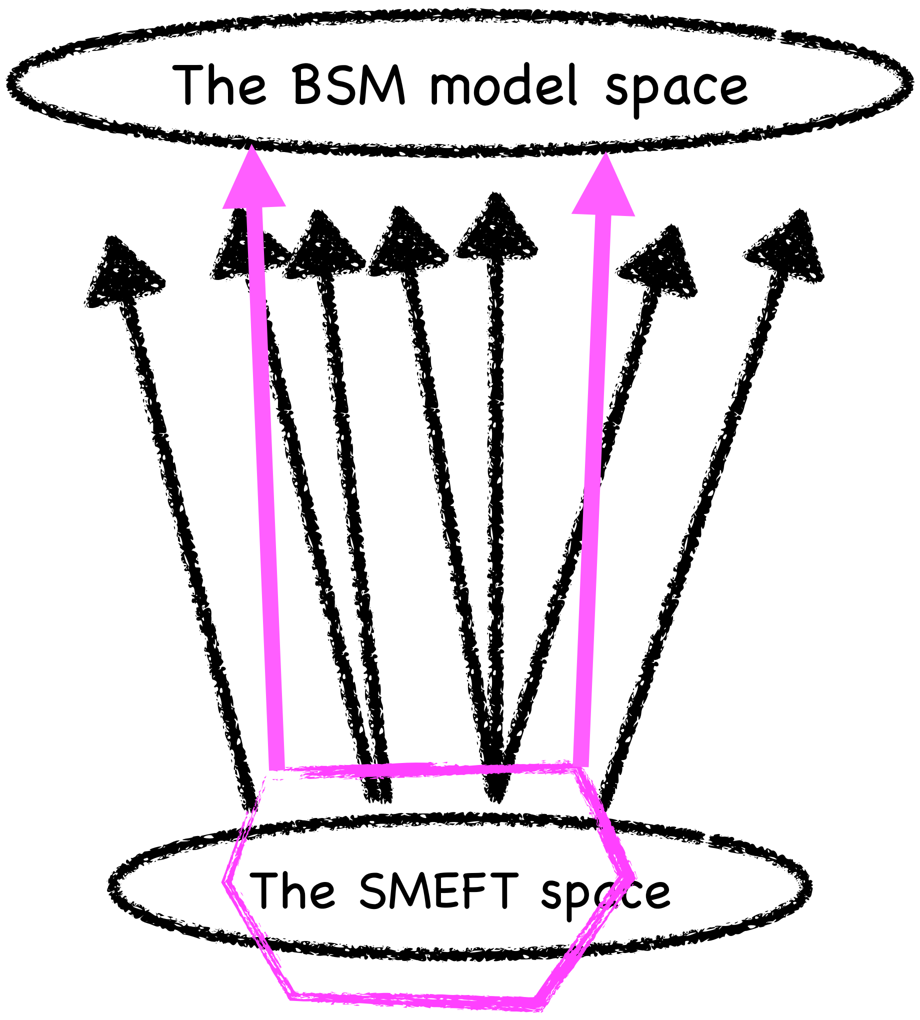
The inverse problem

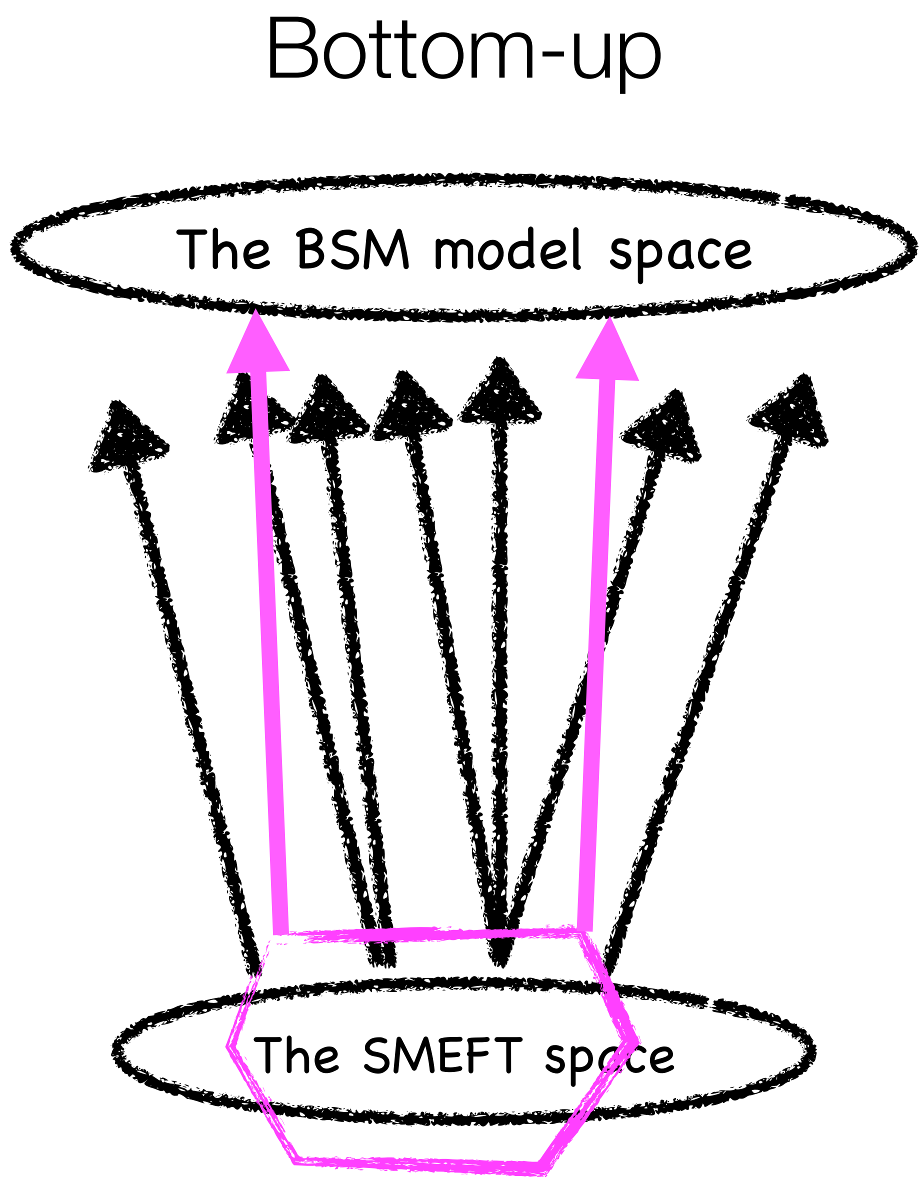
W.I.P

Bottom-up



Bottom-up





$$\mathcal{M}^{ijkl}\left((\epsilon\Lambda)^2\right) = \frac{1}{2\pi}\int_{(\epsilon\Lambda)^2}^{\infty}\frac{ds}{s^3}\left[\mathbf{M}_{ij\to X}(s)\mathbf{M}^*_{kl\to X}(s) + (j \leftrightarrow l)\right]$$

Assume $\mathcal{M}^{ijkl}\left((\epsilon\Lambda)^2\right) \in \mathbf{C}$ is extremal

M not splittable implies:

$$\mathbf{M}_{ij\to X}(s)\mathbf{M}^*_{kl\to X}(s) + (j \leftrightarrow l) \propto \mathcal{M}^{ijkl}\left((\epsilon\Lambda)^2\right) \quad \text{for all } s$$

I.e. fix UV interactions at all scales, but not the spectrum (mass, width, shapes,…)

A toy example

- Consider a two scalar EFT, with two discrete symmetries:
  - $\phi_1 \to -\phi_1$
  - $\phi_1 \leftrightarrow \phi_2$

- 3 dim-8 operators (E[4]):

$$O_1^8 = \partial_\mu\phi_1\partial^\mu\phi_1\partial_\nu\phi_1\partial^\nu\phi_1 + \partial_\mu\phi_2\partial^\mu\phi_2\partial_\nu\phi_2\partial^\nu\phi_2$$
$$O_2^8 = \partial_\mu\phi_1\partial^\mu\phi_1\partial_\nu\phi_2\partial^\nu\phi_2$$
$$O_3^8 = \partial_\mu\phi_1\partial^\mu\phi_2\partial_\nu\phi_1\partial^\nu\phi_2$$

- Generators -> tree-level UV completions 

particle	spin	parities $(\phi_1 \to -\phi_1, \phi_1 \leftrightarrow \phi_2)$	Interaction	ER	$\vec{c} = (C_1, C_2, C_3)$
S_1	0	$++$	$g_1 M_1(\phi_1^2 + \phi_2^2)S_1$	✓	$2 \times (1, 2, 0)$
S_2	0	$+-$	$g_2 M_2(\phi_1^2 - \phi_2^2)S_2$	✓	$2 \times (1, -2, 0)$
S_3	0	$-+$	$g_3 M_3 \phi_1\phi_2 S_3$	✓	$2 \times (0, 0, 1)$
V_4	1	$--$	$g_4(\phi_1 \overleftrightarrow{D}_\mu \phi_2)V_4^\mu$	✓	$2 \times (0, -1, 1)$

$\times \frac{g^2}{M^4}$

“One-particle extension” $\xrightarrow{\text{Top down}}$ extremal ray

particle	spin	parities $(\phi_1 \to -\phi_1, \phi_1 \leftrightarrow \phi_2)$	Interaction	ER	$\vec{c} = (C_1, C_2, C_3)$
S_1	0	$++$	$g_1 M_1(\phi_1^2 + \phi_2^2) S_1$	✓	$2 \times (1, 2, 0)$
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S_3	0	$-+$	$g_3 M_3 \phi_1 \phi_2 S_3$	✓	$2 \times (0, 0, 1)$
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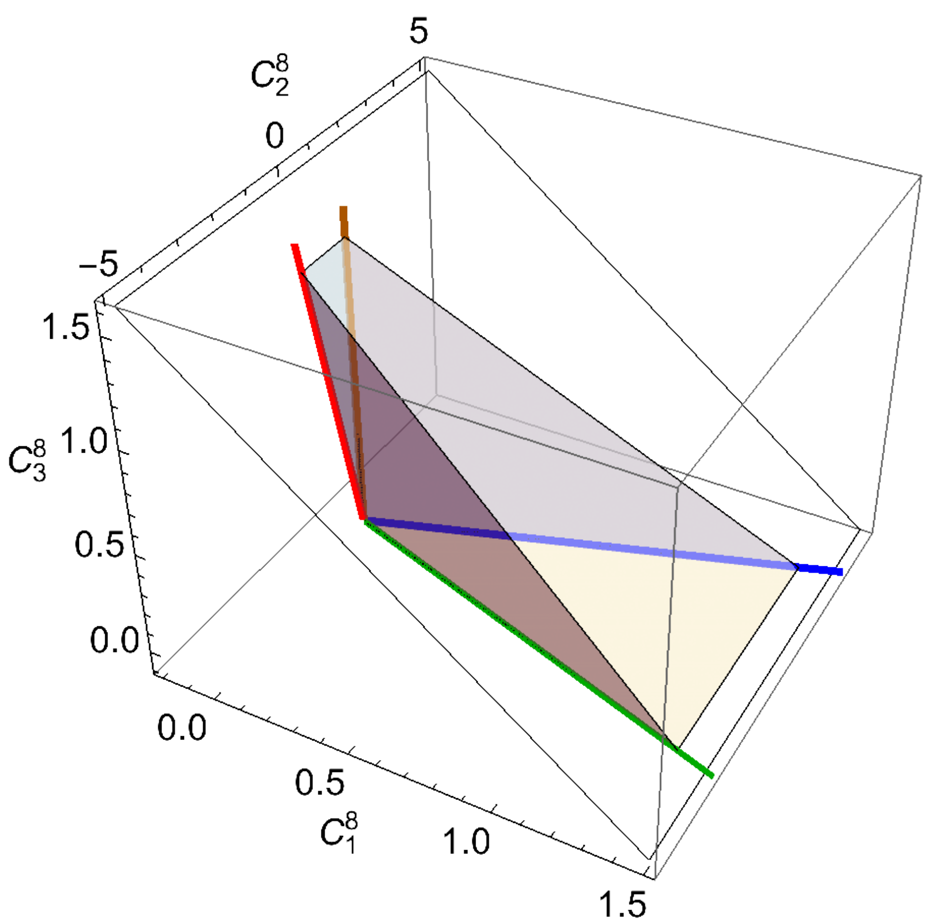
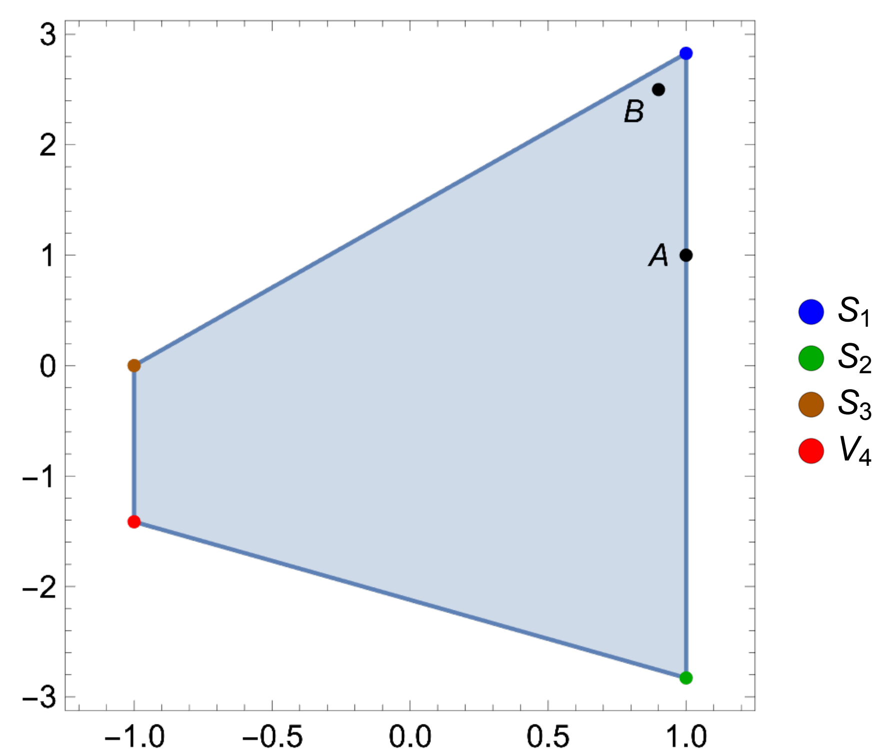
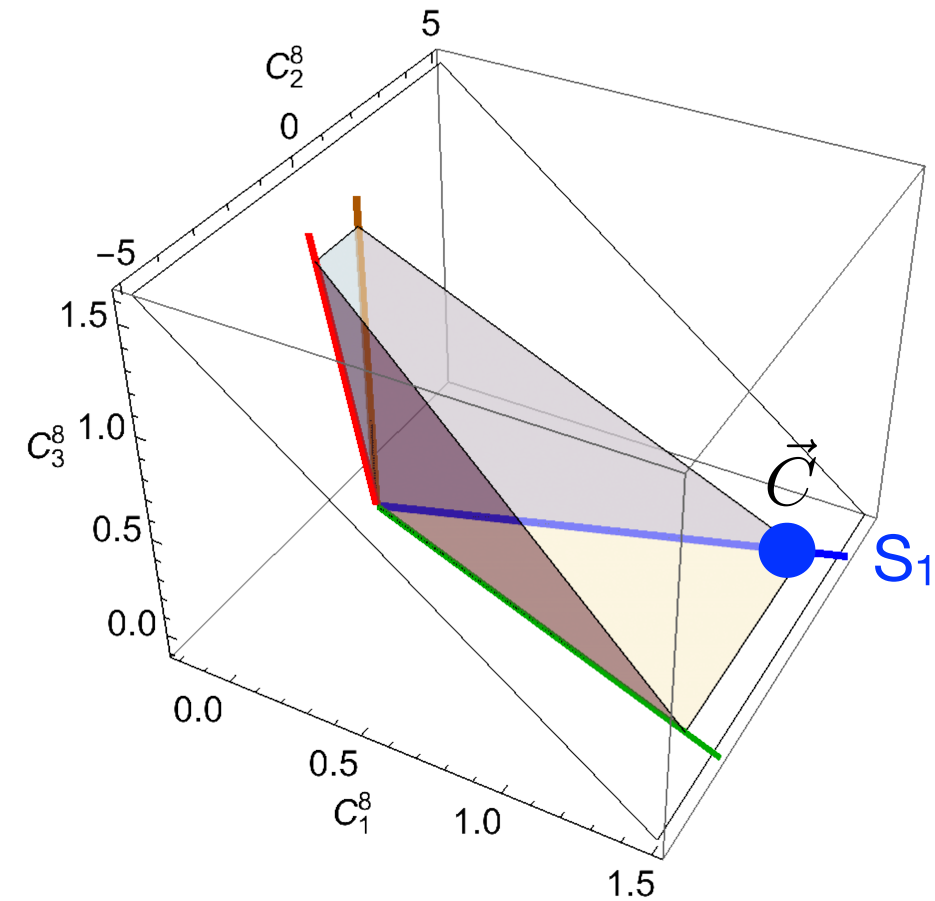




“One-particle extension” ⟵ extremal ray

Bottom up

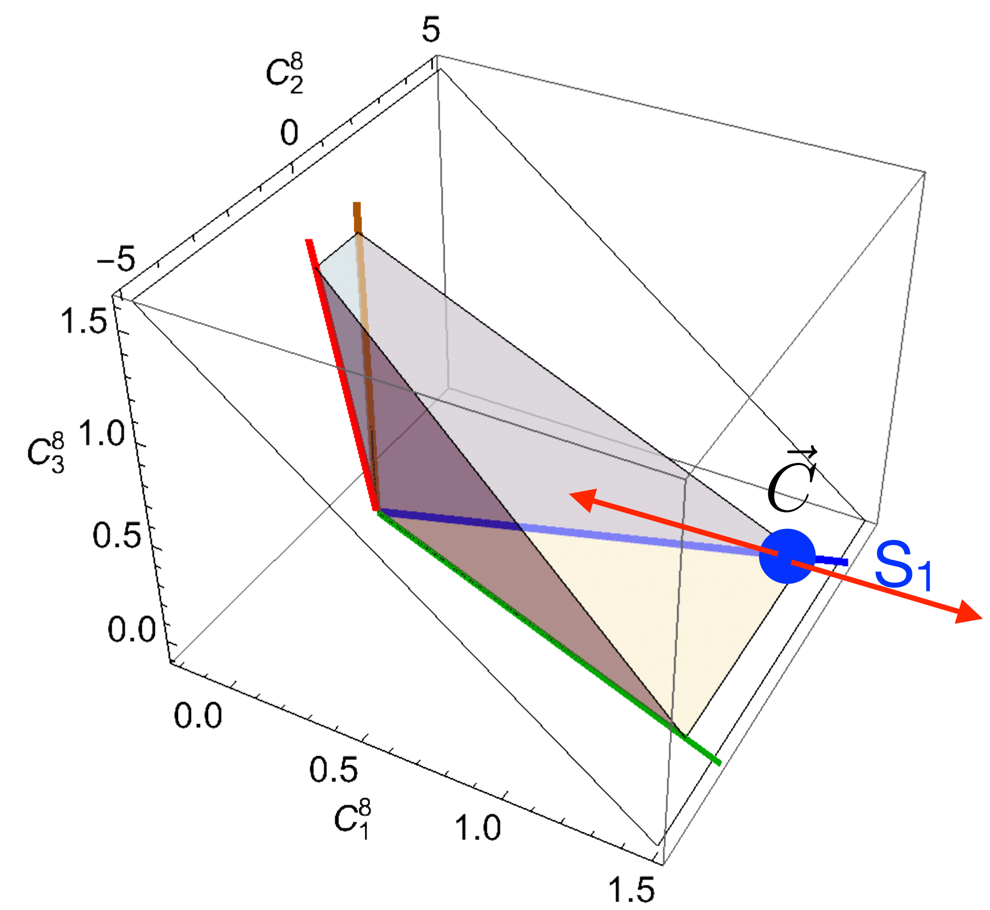
- Suppose the data tells us the coefficients are on the S1 generator (which is extremal).
    - ➡ S1 is the only possible UV particle(s).



“One-particle extension” ⟵ extremal ray

Bottom up

- Suppose the data tells us the coefficients are on the S1 generator (which is extremal).
    - ➡ S1 is the only possible UV particle(s).
- Extremality: the S1 generator cannot be split. This suggests that the UV completion cannot have more than one (type of) particles.



Bottom up

"One-particle extension" ← extremal ray

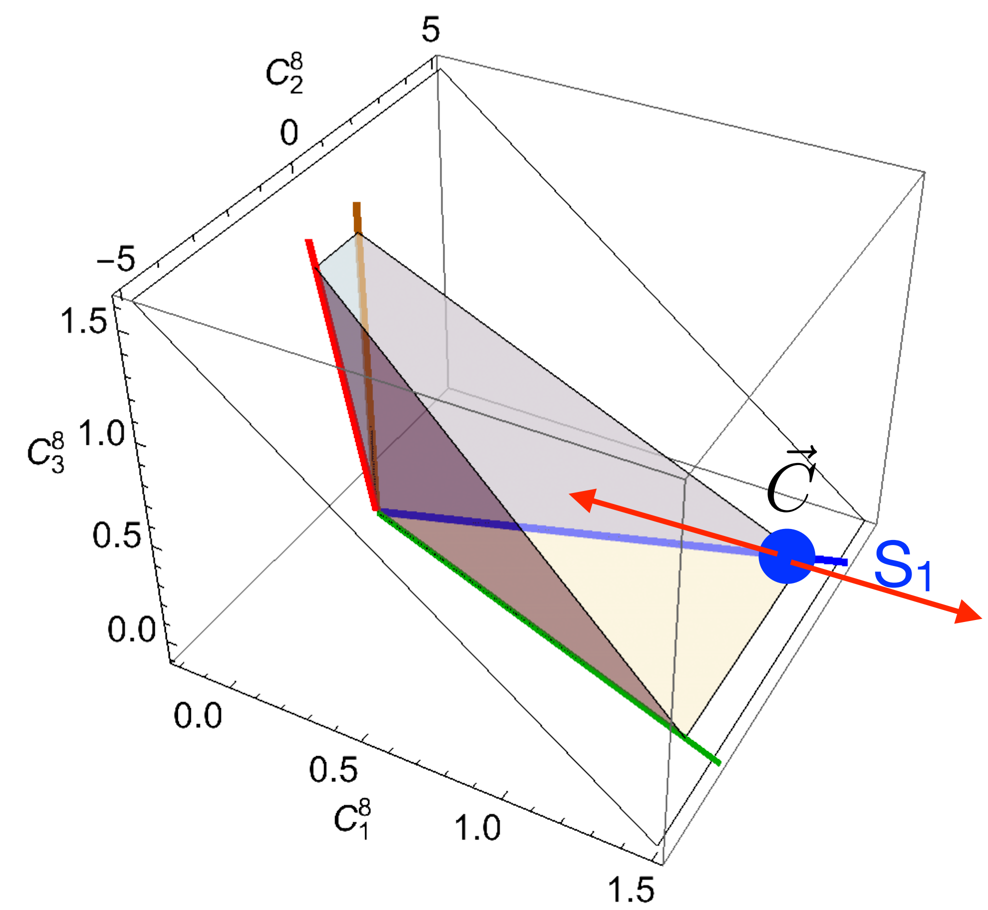
- Suppose the data tells us the coefficients are on the S1 generator (which is extremal).
  - ➡ S1 is the only possible UV particle(s).
- Extremality: the S1 generator cannot be split. This suggests that the UV completion cannot have more than one (type of) particles.
- Could also check the bounds:

$$C_1^8 = 2\left(\frac{g_1^2}{M_1^4} + \frac{g_2^2}{M_2^4}\right) \geq 0$$

$$2C_1^8 + C_2^8 + C_3^8 = 2\left(4\frac{g_1^2}{M_1^4} + \frac{g_3^2}{M_3^4}\right) \geq 0$$

$$2C_1^8 - C_2^8 = 2\left(4\frac{g_2^2}{M_2^4} + \frac{g_4^2}{M_4^4}\right) \geq 0$$

$$C_3^8 = 2\left(\frac{g_3^2}{M_3^4} + \frac{g_4^2}{M_4^4}\right) \geq 0$$



"One-particle extension" ← extremal ray

Bottom up

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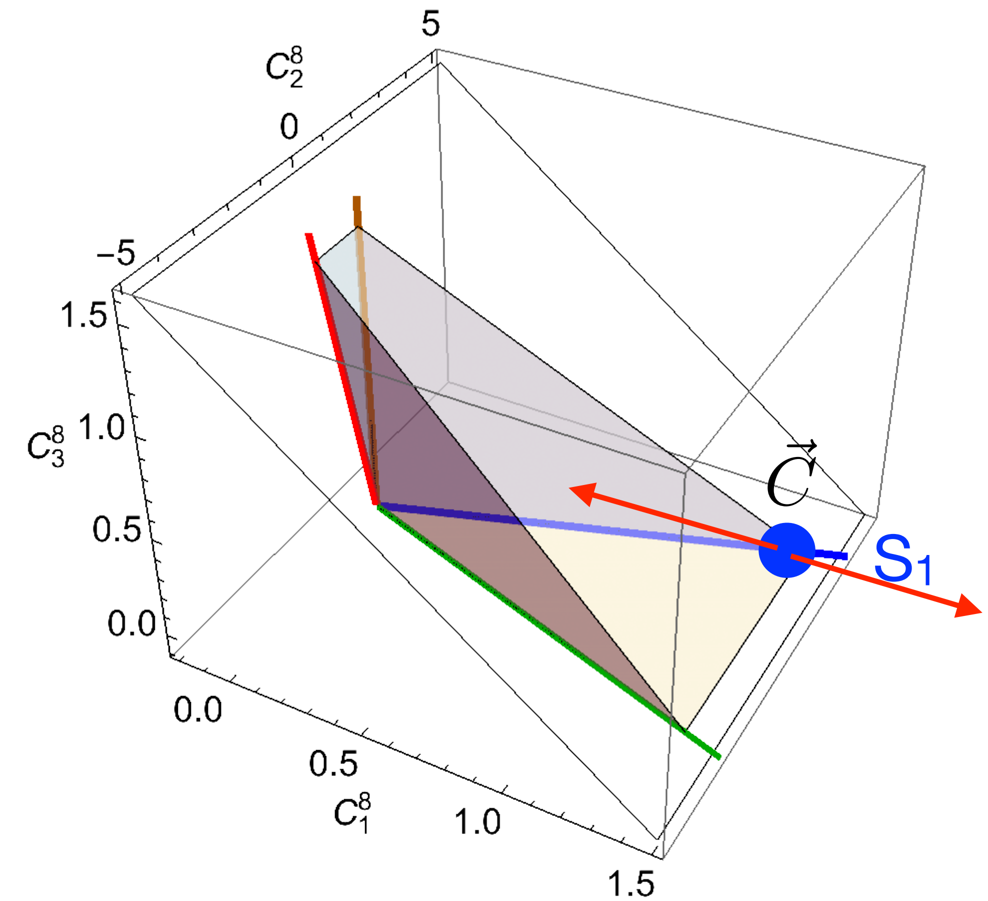

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Excludes S2, S3, V4

“One-particle extension” ⬅ extremal ray

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  - ➡ S1 is the only possible UV particle(s).
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- Could also check the bounds:

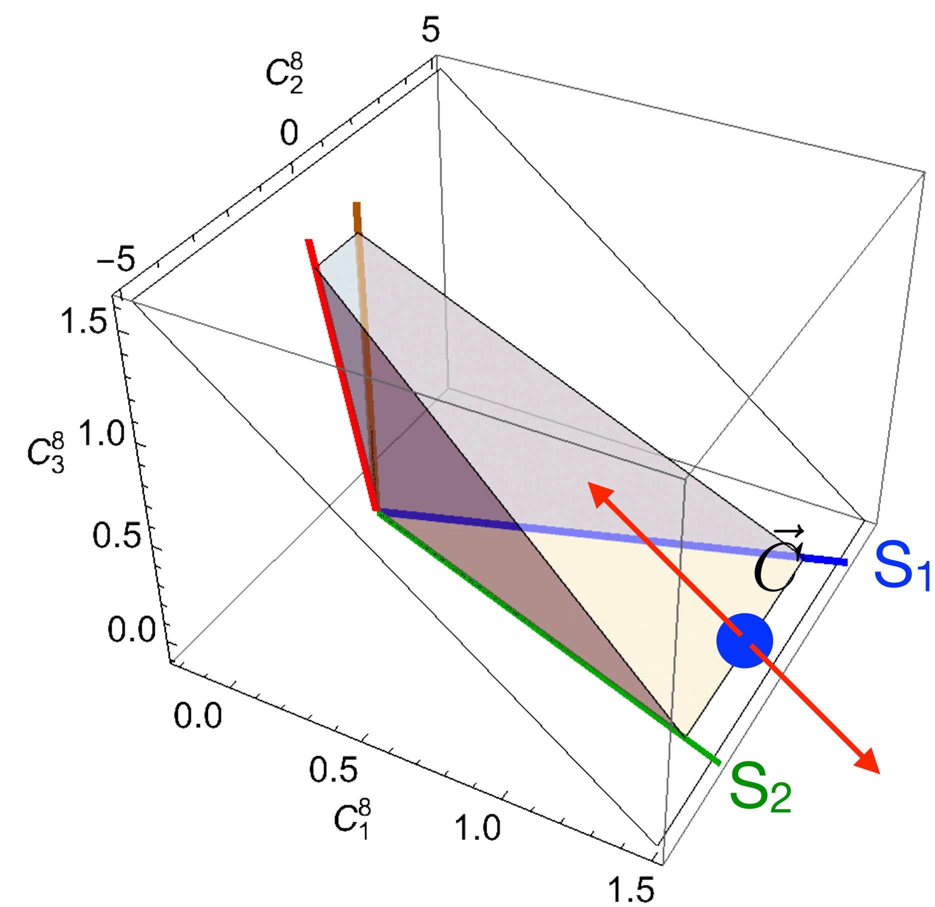
$$C_1^8 = 2\left(\frac{g_1^2}{M_1^4} + \frac{g_2^2}{M_2^4}\right) \geq 0$$

$$2C_1^8 + C_2^8 + C_3^8 = 2\left(4\frac{g_1^2}{M_1^4} + \frac{g_3^2}{M_3^4}\right) \geq 0$$

$$2C_1^8 - C_2^8 = 2\left(4\frac{g_2^2}{M_2^4} + \frac{g_4^2}{M_4^4}\right) \geq 0$$

$$C_3^8 = 2\left(\frac{g_3^2}{M_3^4} + \frac{g_4^2}{M_4^4}\right) \geq 0$$

Excludes S3, V4



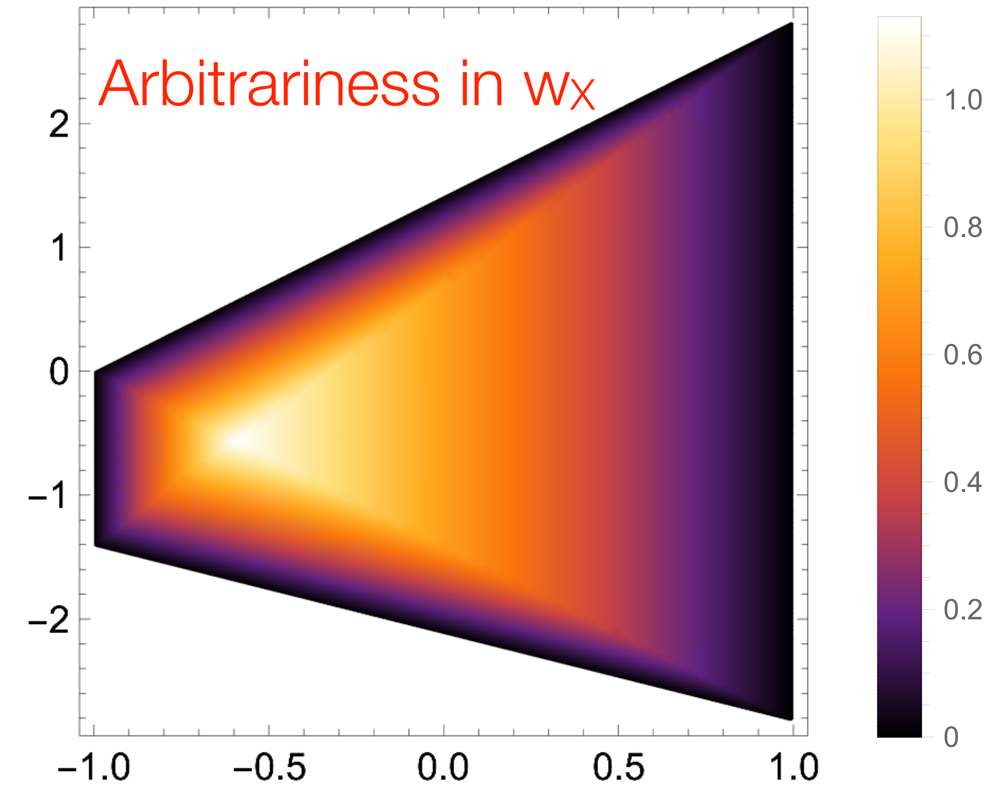
UV particle content fixed for SMEFTs on the ER & boundary

Degeneracies in the space

$$\vec{C} = \sum_{X=S_{1,2,3},V} w_X \vec{c}_X, \quad w_X \equiv \frac{g_X^2}{M_X^4} \geq 0$$

particle	$\vec{c}$
S_1	$2 \times (1, 2, 0)$
S_2	$2 \times (1, -2, 0)$
S_3	$2 \times (0, 0, 1)$
V_4	$2 \times (0, -1, 1)$

- Inverse: what extend can we determine the weights w, given the measured coefficients C?
- Vertex, edge, facet: unique solution
  - Vertex (C=0): no BSM states
  - Edge: one (type of) UV state
  - Facet: two (types of) UV states
- Inside: uncertainty is finite. Can set exclusion limit on each BSM state.

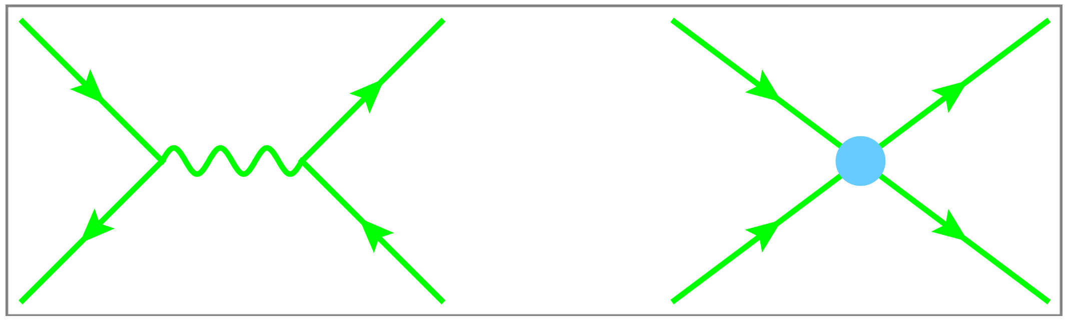


(No model assumption. Only rely on positivity: if w1 is too large, fixing C, the other states will contribute outside the positivity cone)

e+e- scattering at ILC

2009.02212 with B. Fuks, Y. Liu and S.-Y. Zhou

ee scattering at future lepton collider



$$O_1 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{e}\gamma_\mu e)\ ,$$
$$O_2 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{l}\gamma_\mu l)\ ,$$
$$O_3 = D^\alpha(\bar{l}e)\ D_\alpha(\bar{e}l),$$
$$O_4 = \partial^\alpha(\bar{l}\gamma^\mu l)\ \partial_\alpha(\bar{l}\gamma_\mu l)\ ,$$
~~$O_5 = D^\alpha(\bar{l}\gamma^\mu\tau^I l)\ D_\alpha(\bar{l}\gamma_\mu\tau^I l)$~~

In ee → ee, C_5 does not give an independent contribution:

$$\vec{C}^{(8)} = (C_1, C_2, C_3, C_4)$$

UV states and interactions

Scalar			Vector	
$D \equiv \mathbf{2}_{1/2}$	$M_L \equiv \mathbf{1}_1$	$M_R \equiv \mathbf{1}_2$	$V \equiv \mathbf{1}_0$	$V' \equiv \mathbf{2}_{-3/2}$

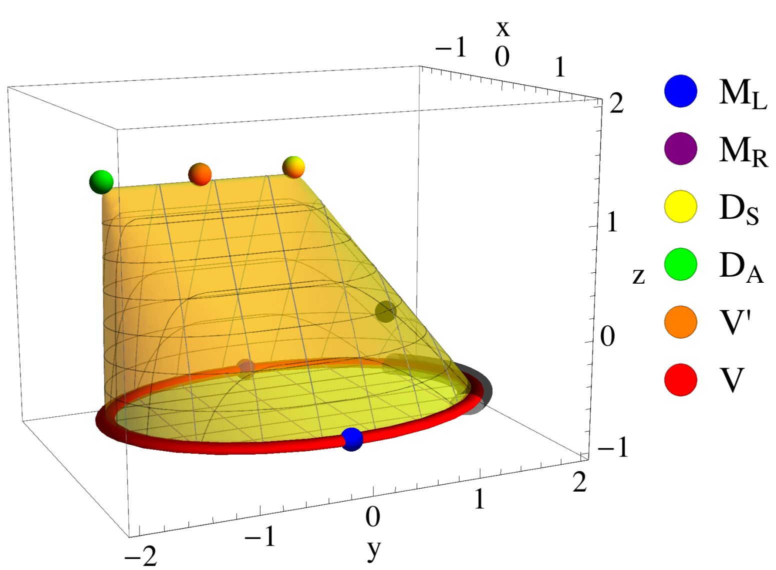
$$\mathcal{L}_{\rm int} = g_{Di}\bar{L}eD_i + g_{M_L i}\bar{L}^c\epsilon L M_{Li} + g_{M_R i}\bar{e}^c e M_{Ri}$$
$$+\, g_{Vi}\left(\bar{L}\gamma^\mu L + \kappa_i \bar{e}\gamma^\mu e\right)V_{i\mu} + g_{V'i}(\bar{e}^c\gamma^\mu L)V_i'^\dagger$$
$$+\,\text{h.c.},$$

Generators:

$$\vec{c}_D^{(8)} = (0,0,1,0),$$
$$\vec{c}_{M_L}^{(8)} = (0,0,0,-1),$$
$$\vec{c}_{M_R}^{(8)} = (-1,0,0,0),$$
$$\vec{c}_{V'}^{(8)} = (0,0,-1,2),$$
$$\vec{c}_{V(\kappa)}^{(8)} = (-\kappa^2/2, -\kappa, 0, -1/2).$$

Bounds

$$C_5 \le 0, \quad C_1 \le 0,$$
$$C_3 \ge 0, \quad C_4 + C_5 \le 0,$$
$$2\sqrt{C_1(C_4+C_5)} \ge C_2,$$
$$2\sqrt{C_1(C_4+C_5)} \ge -(C_2+C_3).$$



ee scattering at future lepton collider

Scenario	Beam polarization $P(e^-, e^+)$	Runs (luminosity @ energy), [ab^{-1}] @ [GeV] 1	2	3	4
CEPC	None	2.6@161	5.6@240		
FCC-ee	None	10@161	5@240	0.2@350	1.5@365
ILC-500	(−80%, 30%)	0.9@250	0.135@350	1.6@500	
	(80%, −30%)	0.9@250	0.045@350	1.6@500	
ILC-1000	(−80%, 30%)	0.9@250	0.135@350	1.6@500	1.25@1000
	(80%, −30%)	0.9@250	0.045@350	1.6@500	1.25@1000
CLIC	(−80%, 0%)	0.5@380	2@1500	4@3000	
	(80%, 0%)	0.5@380	0.5@1500	1@3000	

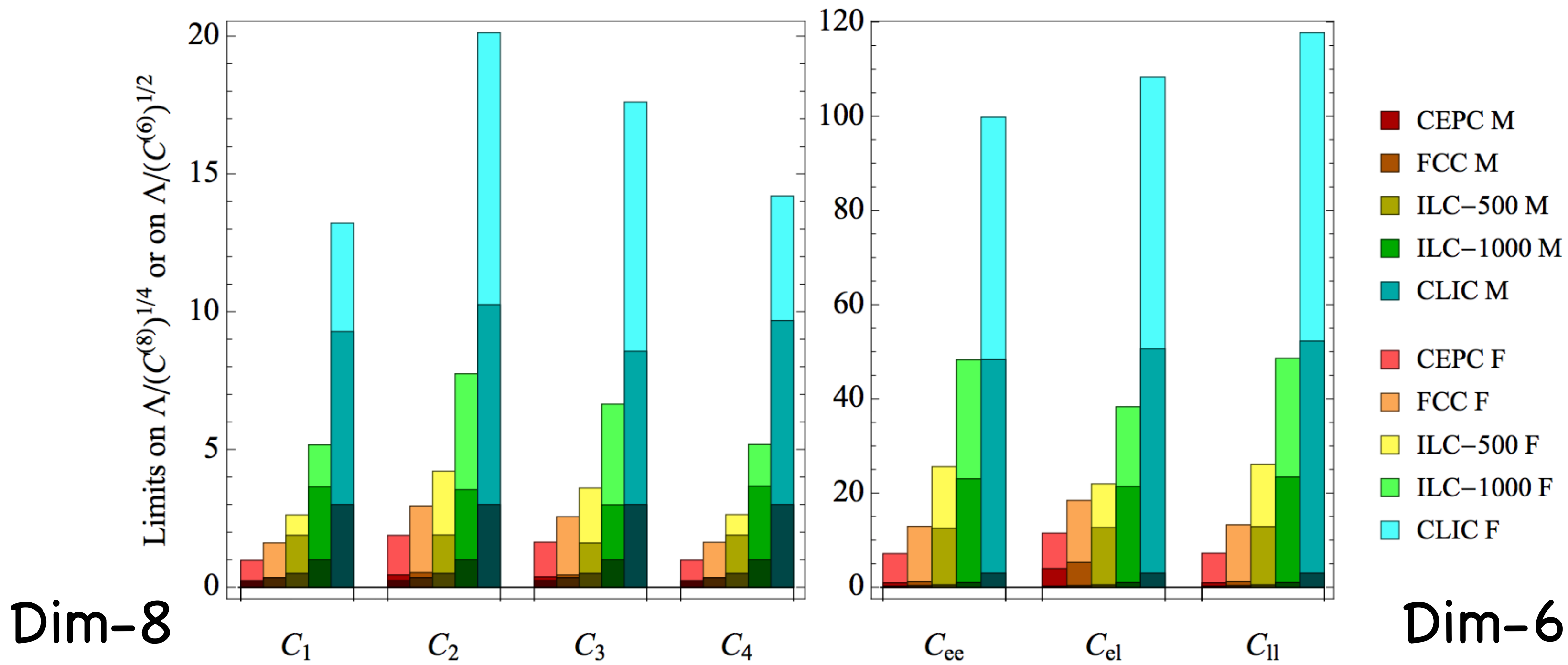


FIG. 3. Limits on the new physics characterization scale Λ_c (in TeV) for the various considered future lepton colliders. 'M' denotes marginalized limits (all other coefficients being floating) while 'F' denotes individual limits (all other coefficients being vanishing). In addition, we represent by the darkest color the largest center-of-mass energy of each collider project.

UV states

- Assume D-type scalar extension, $g_D = 0.8$, $M_D = 2$ TeV

Scalar			Vector	
$D \equiv \mathbf{2}_{1/2}$	$M_L \equiv \mathbf{1}_1$	$M_R \equiv \mathbf{1}_2$	$V \equiv \mathbf{1}_0$	$V' \equiv \mathbf{2}_{-3/2}$

$$\mathcal{L}_{\rm int} = g_{Di}\bar{L}eD_i + g_{M_L i}\bar{L}^c\epsilon L M_{Li} + g_{M_R i}\bar{e}^c e M_{Ri} + g_{Vi}\left(\bar{L}\gamma^\mu L + \kappa_i \bar{e}\gamma^\mu e\right)V_{i\mu} + g_{V'i}(\bar{e}^c\gamma^\mu L)V_i'^\dagger + \text{h.c.},$$

- At ILC (with 1 TeV run), global fit ->

$$C_{ee} = 0 \pm 0.0024, \quad C_{el} = -0.08 \pm 0.0035,$$
$$C_{ll} = 0 \pm 0.0023,$$
$$C_1 = 0 \pm 0.0074, \quad C_2 = 0 \pm 0.0077,$$
$$C_3 = 0.04 \pm 0.020, \quad C_4 = 0 \pm 0.0071.$$

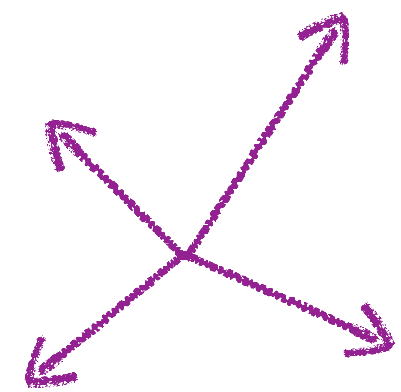
- What to conclude at dim-6?
  - If assume the SM is only supplemented by D-type scalar,

$$M_D/g_D \in [2.45, 2.56]\ \text{TeV}.$$

  - If assume the SM is extend by D and V',

$$\frac{g_D^2}{2M_D^2} - \frac{g_{V'}^2}{M_{V'}^2} = 0.08 \pm 0.0035\ \text{TeV}^{-2}.$$

  - If assuming more complicated models, not much to be concluded about the existence of UV states.



Excluding UV states

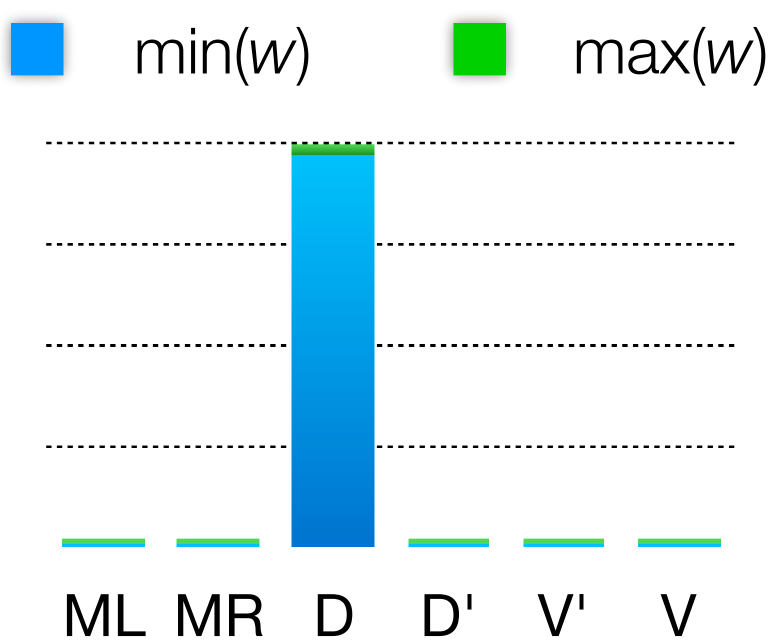
- ✦ What can we conclude at dim-8?
- ✦ Upper bound on all states

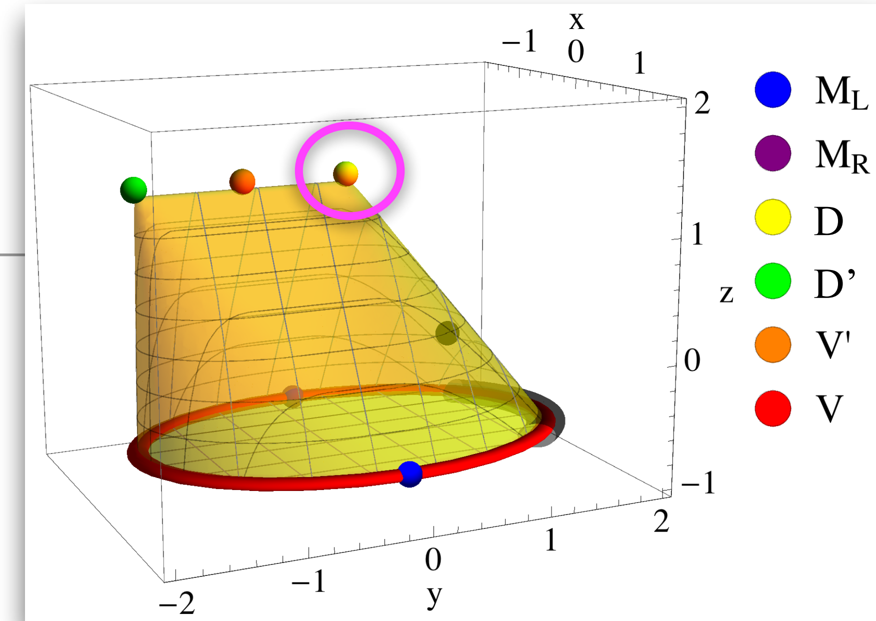
$$\vec{C} = \sum_i w_i \vec{g}_i$$

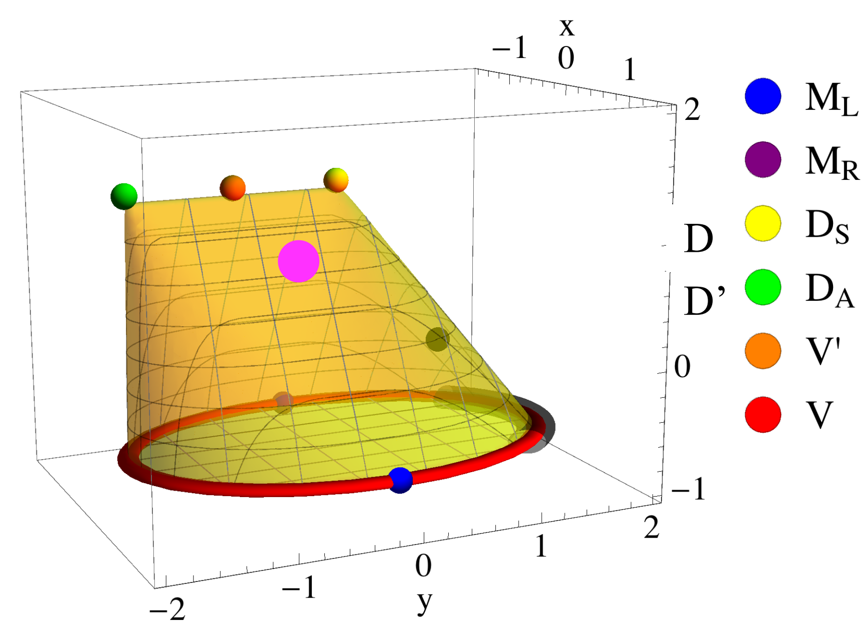
$$w_i = \frac{g_i^2}{M_i^4}$$

$$\vec{C}(\lambda) \equiv \vec{C} - \lambda \vec{g}_k = \sum_{i \neq k} w_i \vec{g}_i + (w_k - \lambda)\vec{g}_k$$

$$\lambda_M = \max_{\vec{C}(\lambda) \in \mathbf{C}} \lambda \qquad \geq w_k$$







Excluding UV states

- What can we conclude at dim-8? $\vec{C} = \sum_i w_i \vec{g}_i$
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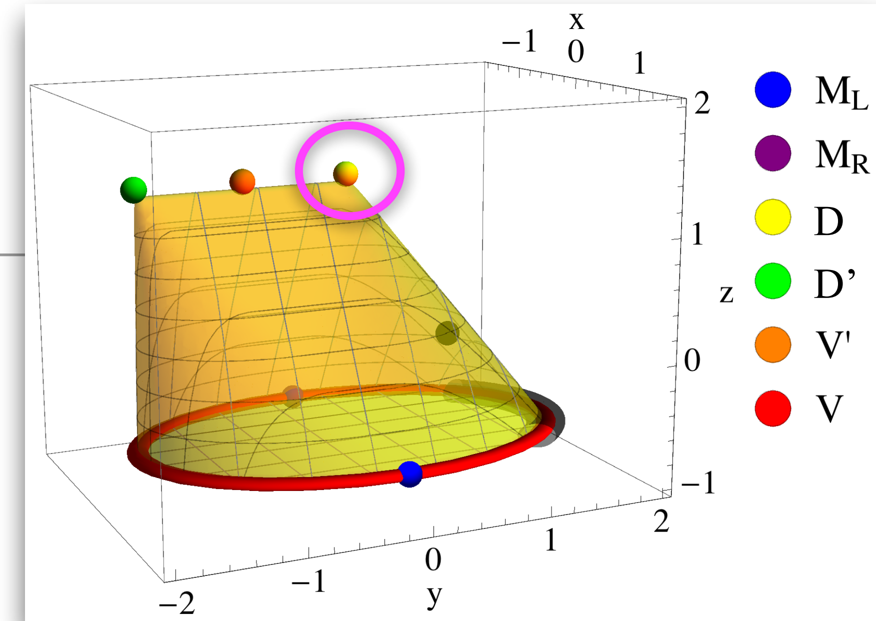

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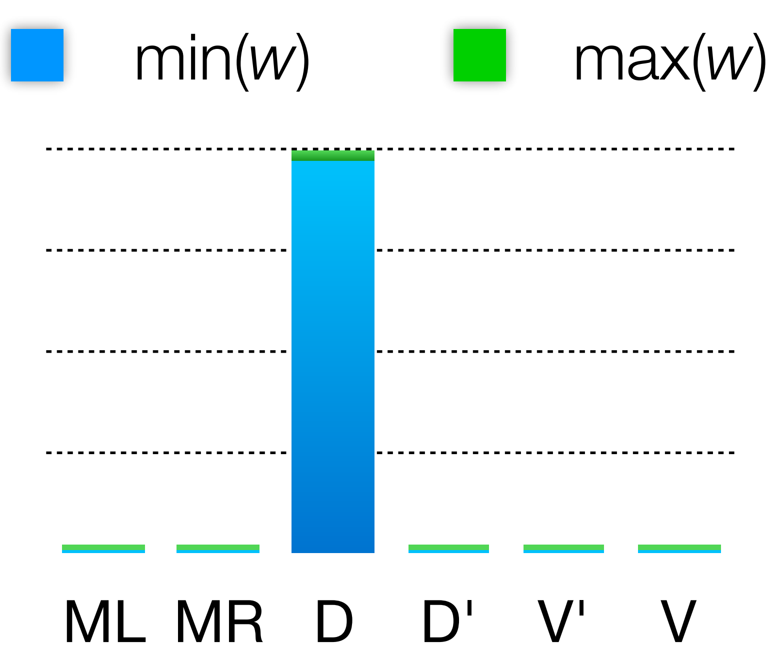

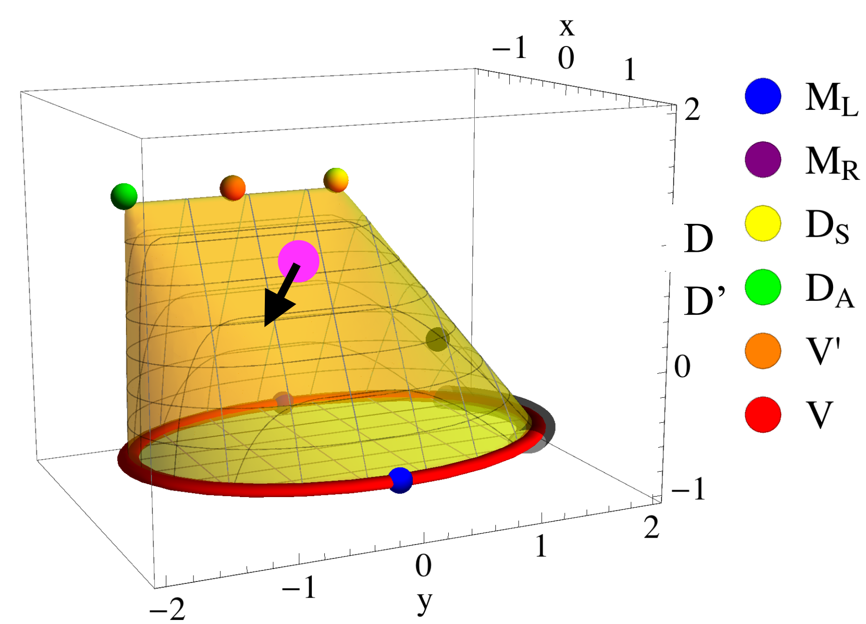

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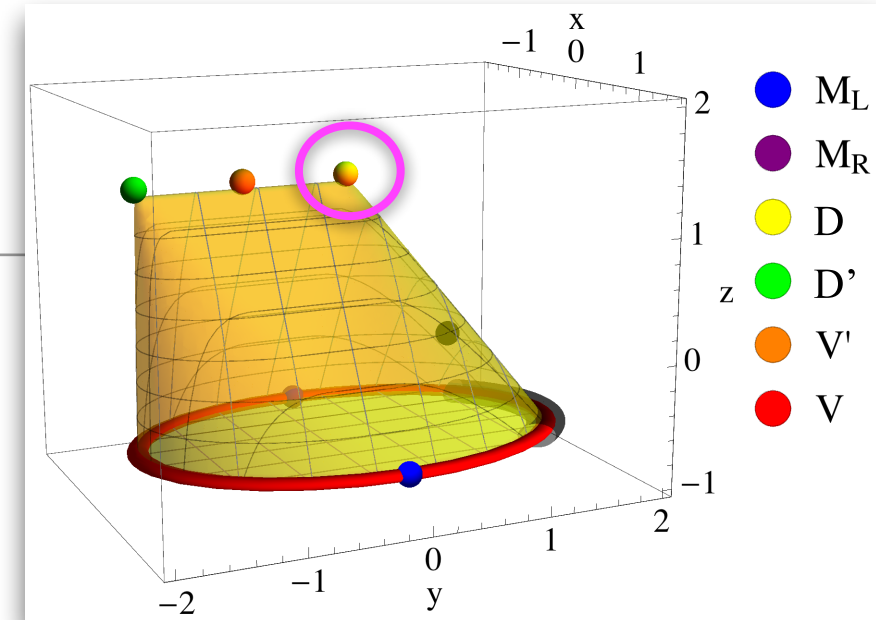

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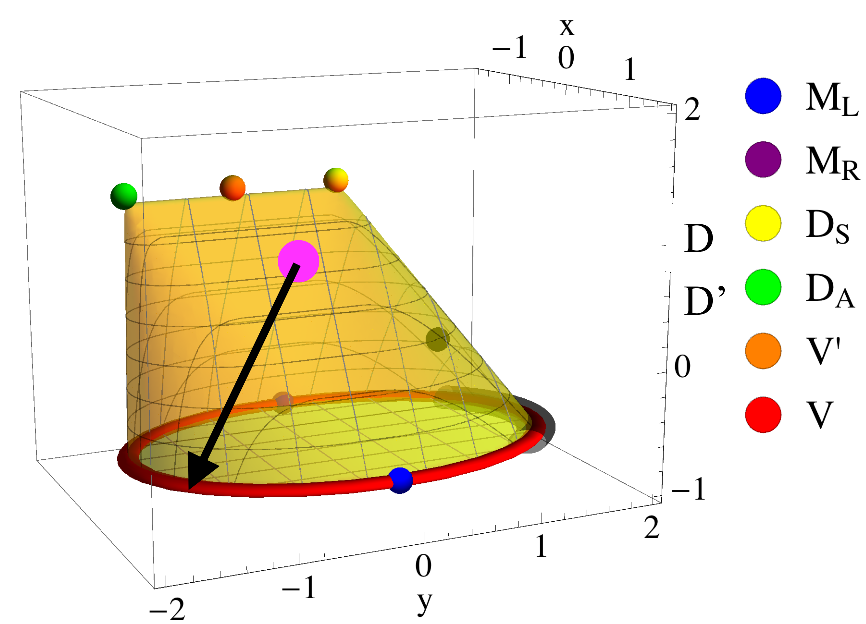
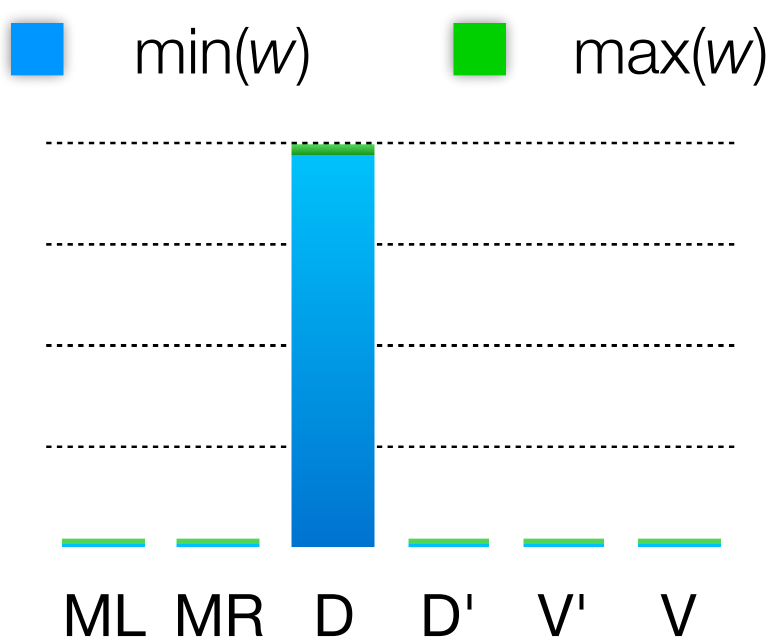
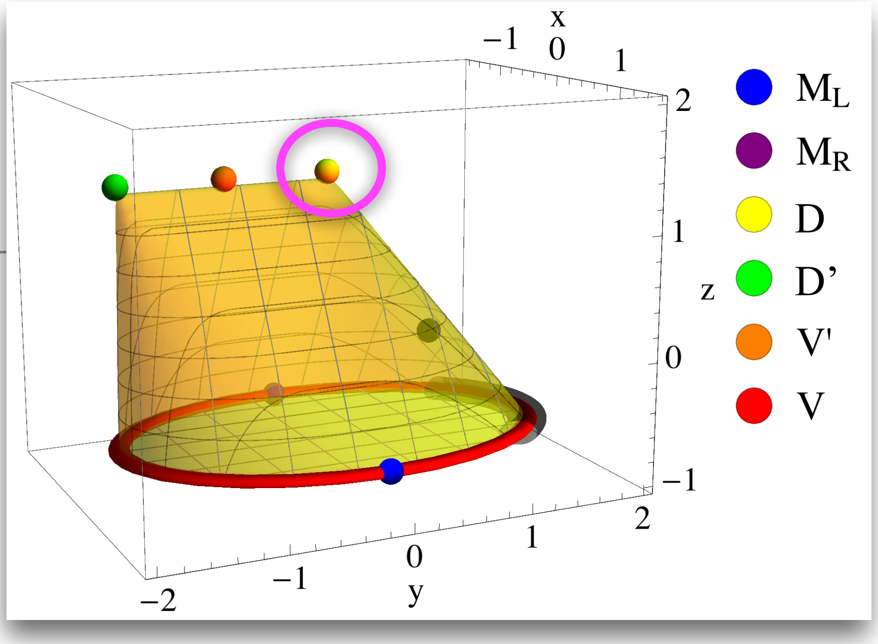




Excluding UV states



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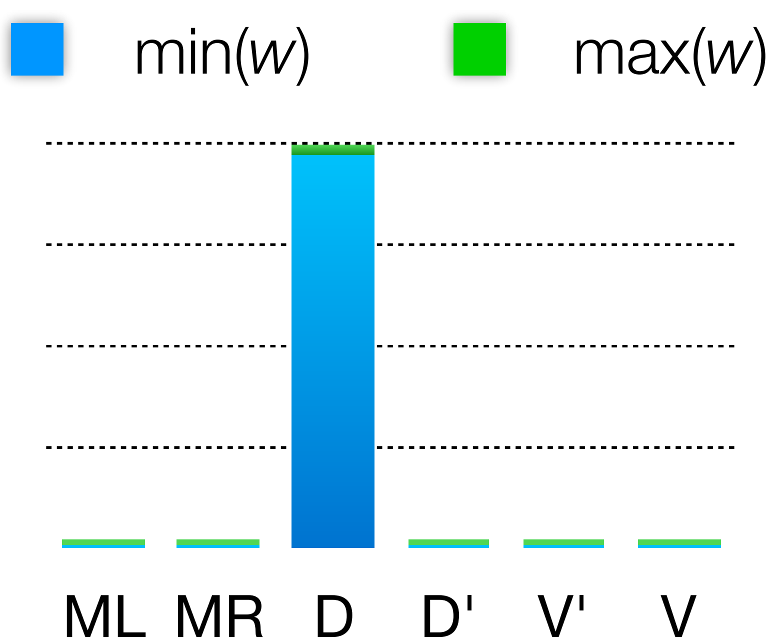
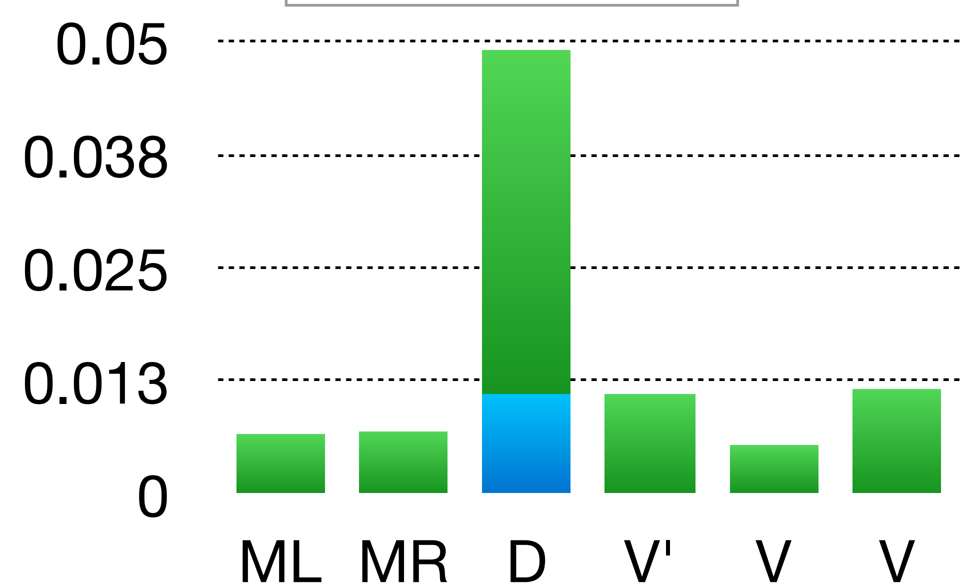
$$\begin{aligned} &\text{maximum} \quad \lambda \\ &\text{subject to} \quad \vec{C} - \lambda \vec{C}_k \in \mathbf{C} \\ &\qquad\quad \text{and} \quad \chi^2\left(\vec{C}, \vec{C}_{\mathrm{EXP}}\right) \leq \chi_c^2 \end{aligned}$$



→

With exp. error:



Excluding UV states

Scalar			Vector	
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$$\mathcal{L}_{\text{int}} = g_{Di}\bar{L}eD_i + g_{M_L i}\bar{L}^c\epsilon L M_{Li} + g_{M_R i}\bar{e}^c e M_{Ri} + g_{Vi}\left(\bar{L}\gamma^\mu L + \kappa_i \bar{e}\gamma^\mu e\right)V_{i\mu} + g_{V'i}(\bar{e}^c\gamma^\mu L)V_i'^\dagger + \text{h.c.},$$

- Dim-8 measurement would universally exclude all alternative hypothesis, independent of any model assumptions.

X	$\vec{c}_X^{\,(8)}$	$\lambda_{\max}$	$M_X/\sqrt{g_X}$
M_L	$(0,0,0,-1)$	0.0067	≥ 3.5 TeV
M_R	$(-1,0,0,0)$	0.0069	≥ 3.5 TeV
V (with $\kappa = 1$)	$(-1/2,-1,0,-1/2)$	0.0055	≥ 3.7 TeV
V (with $\kappa = -1$)	$(-1/2,1,0,-1/2)$	0.0116	≥ 3.0 TeV
V'	$(0,-1,2,0)$	0.0109	≥ 3.1 TeV

$$M_D/\sqrt{g_D} \in [2.1, 3.1]\text{TeV}$$

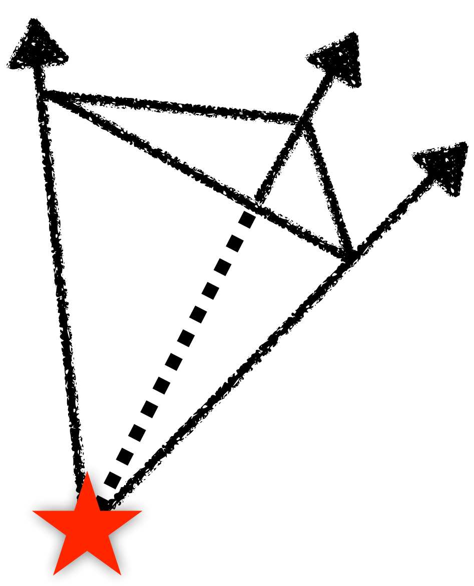
Testing the SM

- If all dim-8 coefficients are consistent with 0, all states can be excluded to above certain scales

  - Not possible at dim-6

$$
\begin{array}{rl}
\text{maximum} & \lambda \\
\text{subject to} & \vec{C} - \lambda \vec{C}_k \in \mathbf{C} \\
\text{and} & \chi^2\left(\vec{C}, \vec{C}_{\mathrm{EXP}}\right) \leq \chi_c^2
\end{array}
$$

X	$\lambda_{\max}$	$M_X/\sqrt{g_X}$
D	0.0076	≥ 3.4 TeV
M_L	0.0053	≥ 3.7 TeV
M_R	0.0054	≥ 3.7 TeV
V'	0.0056	≥ 3.7 TeV
V (with $\kappa = 1$)	0.0041	≥ 4.0 TeV
V (with $\kappa = -1$)	0.0041	≥ 4.0 TeV



Summary

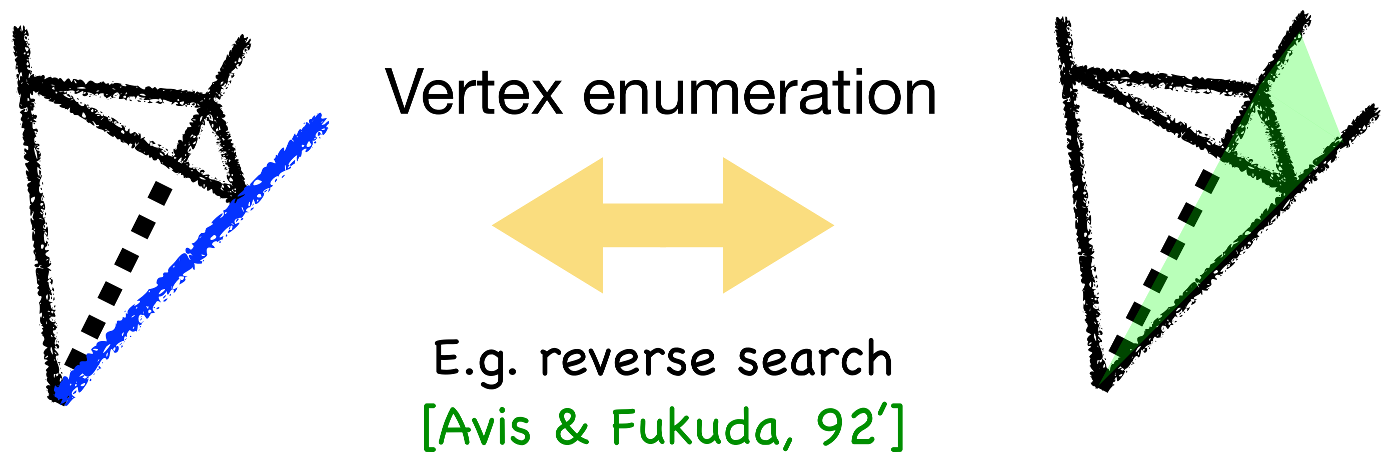
- Two ways to derive positivity bounds for SMEFT at dim-8.
  - Enumerate the generators and take convex hull.
  - Find the ERs in the dual spetrahedron or SDP.
- Positivity seems connected to the inverse problem.
  - Vanishing degeneracy on the boundary allows to determine the UV particles and their quantum numbers.
  - A motivation to move from dim-6 to dim-8 SMEFT: we care about UV model rather than just coefficient measurements. The 250 dim-8 operators (with s^2 dependence) clearly contain important information.

Thank you

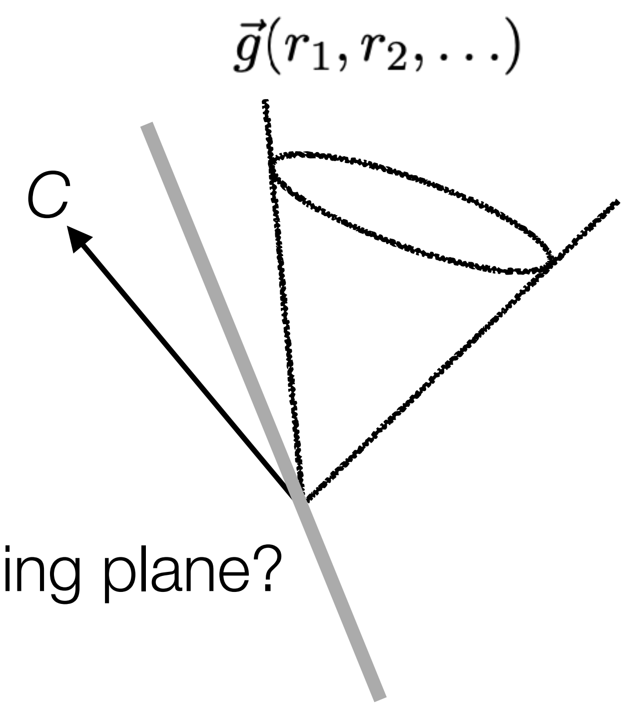
Backups

From extremal rays to bounds

- ✦ Polytope cones: vertex enumeration (for large dim, #ERs >> #dim)



- ✦ None polytope, generators take continuous values. Can convert to programming



Find a separating plane?

$$\vec{n} \cdot \vec{g}(r_1, r_2, \ldots) \geq 0$$
$$\vec{n} \cdot \vec{C} < 0$$

$$\begin{aligned} &\min \quad \vec{n} \cdot \vec{C} \\ &\text{subject to} \quad \vec{n} \cdot \vec{g}(r_1, r_2, \ldots) \geq 0 \end{aligned}$$

E.g. discretization, [Yamashita, CZ, Zhou 2009.04490]

Example: SM Higgs

✦ Operators

[C. Murphy, 2005.00059]

	$4: H^4 D^4$
$Q_{H^4}^{(1)}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$Q_{H^4}^{(2)}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$Q_{H^4}^{(3)}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$

✦ Coefficients

C_1

C_2

C_3

✦ Amplitude in terms complex fields

$\mathcal{M}^{ijkl} =$

	$h_c h_d$	$\bar{h}^c \bar{h}^d$	$h_c \bar{h}^d$	$\bar{h}^c h_d$
$h^a h^b$	$\mathcal{M}(hh \to hh)^{ab}{}_{cd}$			
$\bar{h}_a \bar{h}_b$		$\mathcal{M}(\bar{h}\bar{h} \to \bar{h}\bar{h})_{ab}{}^{cd}$		
$h^a \bar{h}_b$			$\mathcal{M}(h\bar{h} \to h\bar{h})^a{}_{bc}{}^d$	$\mathcal{M}(h\bar{h} \to \bar{h}h)^a{}_b{}^c{}_d$
$\bar{h}_a h^b$			$\mathcal{M}(\bar{h}h \to h\bar{h})_a{}^b{}_c{}^d$	$\mathcal{M}(\bar{h}h \to \bar{h}h)_a{}^{bc}{}_d$

$$\mathcal{M}(hh \to hh)^{ab}{}_{cd} = \frac{1}{2}\left[(C_2 + C_3)\,\delta^a_d \delta^b_c + (C_1 + C_2)\,\delta^a_c \delta^b_d\right]$$

$$\mathcal{M}(\bar{h}\bar{h} \to \bar{h}\bar{h})_{ab}{}^{cd} = \frac{1}{2}\left[(C_2 + C_3)\,\delta^d_a \delta^c_b + (C_1 + C_2)\,\delta^c_a \delta^d_b\right]$$

$$\mathcal{M}(h\bar{h} \to h\bar{h})^a{}_{bc}{}^d = \frac{1}{2}\left[(C_1 + C_2)\,\delta^a_c \delta^d_b + (C_2 + C_3)\,\delta^a_b \delta^d_c\right]$$

$$\mathcal{M}(\bar{h}h \to \bar{h}h)_a{}^{bc}{}_d = \frac{1}{2}\left[(C_1 + C_2)\,\delta^c_a \delta^b_d + (C_2 + C_3)\,\delta^b_a \delta^c_d\right]$$

$$\mathcal{M}(h\bar{h} \to \bar{h}h)^a{}_b{}^c{}_d = \frac{1}{2}(C_1 + C_3)\,(\delta^a_d \delta^c_b + \delta^a_b \delta^c_d)$$

$$\mathcal{M}(\bar{h}h \to h\bar{h})_a{}^b{}_c{}^d = \frac{1}{2}(C_1 + C_3)\left(\delta^d_a \delta^b_c + \delta^b_a \delta^d_c\right)$$

Example: SM Higgs

- Intermediate states couple to $hh, \bar{h}\bar{h}, h\bar{h}, \bar{h}h$: $\mathbf{1}_1, \mathbf{3}_1, \mathbf{1}_{0S}, \mathbf{1}_{0A}, \mathbf{3}_{0S}, \mathbf{3}_{0A}$

- Generators

$$m_1 = \begin{array}{c|cc} & h^b & \bar{h}_b \\ \hline h^a & \epsilon^{ab} & 0 \\ \bar{h}_a & 0 & 0 \end{array}, \quad m_3^I = \begin{array}{c|cc} & h^b & \bar{h}_b \\ \hline h^a & [\epsilon\tau^I]^{ab} & 0 \\ \bar{h}_a & 0 & 0 \end{array} \qquad m_{1[S,A]} = \begin{array}{c|cc} & h^b & \bar{h}_b \\ \hline h^a & 0 & \delta^a_b \\ \bar{h}_a & [\pm]\delta^b_a & 0 \end{array}, \quad m^I_{3[S,A]} = \begin{array}{c|cc} & h^b & \bar{h}_b \\ \hline h^a & 0 & {\tau^I}^a{}_b \\ \bar{h}_a & [\pm]{\tau^I}^b{}_a & 0 \end{array}$$

$\mathcal{G}^{ijkl}_{1,3}$

${P_{1,3}}^{ab}{}_{cd}$			
	${P_{1,3}}^{cd}{}_{ab}$		
		${P_{1,3}}^{ad}{}_{cb}$	
			${P_{1,3}}^{cb}{}_{ad}$

$\mathcal{G}^{ijkl}_{1,3[S,A]}$

${P_{1,3}}^{a}{}_{dc}{}^{b}$			
	${P_{1,3}}^{c}{}_{ba}{}^{d}$		
		${P_{1,3}}^{a}{}_{bc}{}^{d}$	$\pm {P_{1,3}}^{a}{}_{bd}{}^{c} \pm {P_{1,3}}^{a}{}_{db}{}^{c}$
		$\pm {P_{1,3}}^{b}{}_{ac}{}^{d} \pm {P_{1,3}}^{b}{}_{ca}{}^{d}$	${P_{1,3}}^{c}{}_{da}{}^{b}$

$$\vec{g} = (C_1, C_2, C_3)$$

$$\vec{g}_1 = (1, 0, -1) \quad \vec{g}_{1S} = (0, 0, 2) \quad \vec{g}_{3S} = (4, 0, -2)$$

$$\vec{g}_3 = (0, 1, 0) \quad \vec{g}_{1A} = (-2, 2, 0) \quad \vec{g}_{3A} = (2, 2, -4)$$

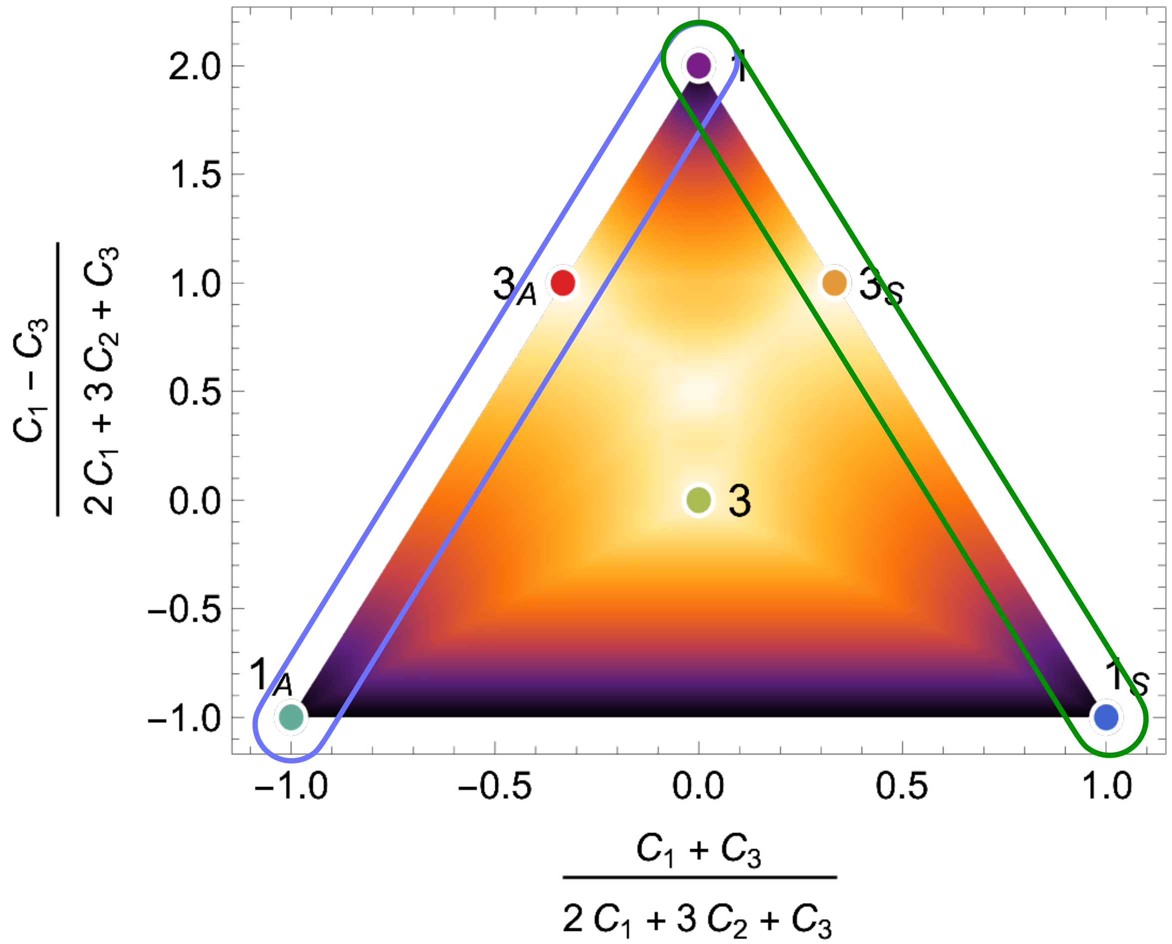
Example: SM Higgs

- 1, 1S, 1A are extremal. Triangular cone.

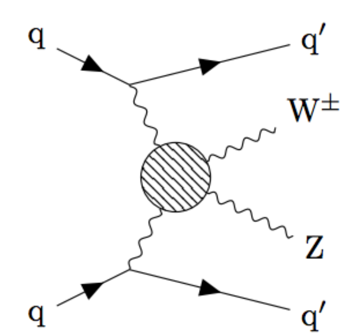
$\vec{g}_1 = (1,0,-1)$ $\quad \vec{g}_{1S} = (0,0,2)$ $\quad \vec{g}_{3S} = (4,0,-2)$

$\vec{g}_3 = (0,1,0)$ $\quad \vec{g}_{1A} = (-2,2,0)$ $\quad \vec{g}_{3A} = (2,2,-4)$

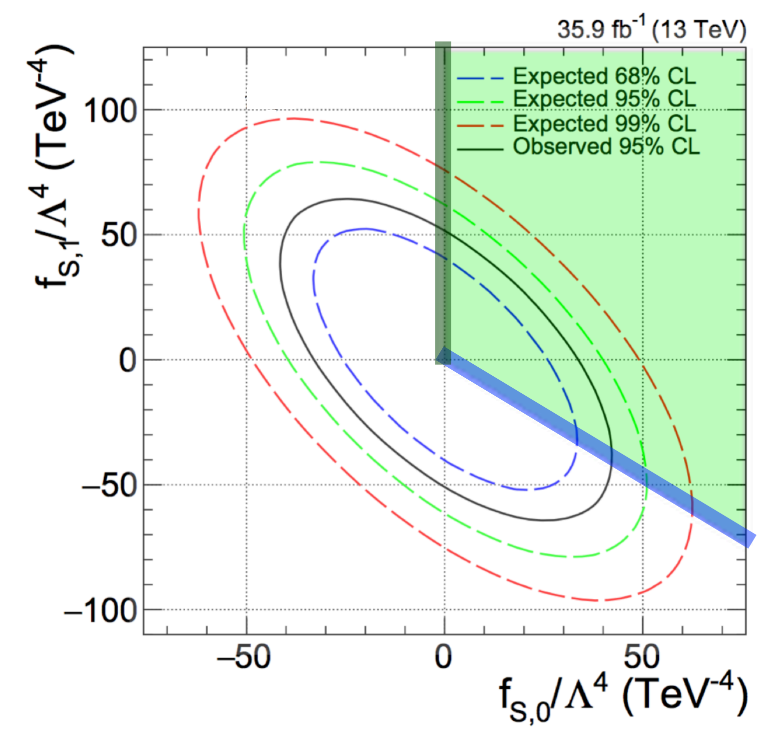
- Take the cross section of triangular cone



- Bounds on "aQGC"



$$\mathcal{O}_{S,0} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\mu\Phi)^\dagger D^\nu\Phi\right]$$
$$\mathcal{O}_{S,1} = \left[(D_\mu\Phi)^\dagger D^\mu\Phi\right] \times \left[(D_\nu\Phi)^\dagger D^\nu\Phi\right]$$
$$\mathcal{O}_{S,2} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\nu\Phi)^\dagger D^\mu\Phi\right]$$



WZjj (CMS-PAS-SMP-18-001)

SM Higgs cone as an example

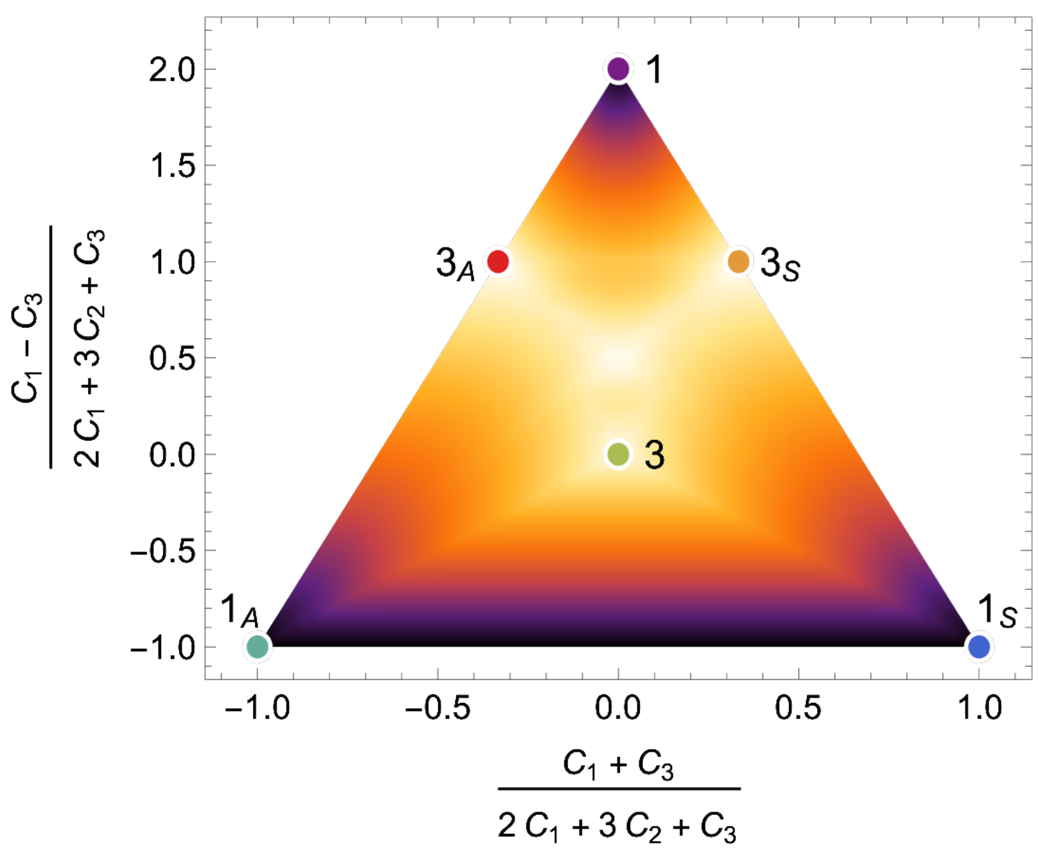
$4: H^4D^4$

$Q_{H^4}^{(1)}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$Q_{H^4}^{(2)}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$Q_{H^4}^{(3)}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$

Particle	Spin	Charge/irrep	Interaction	ER	$\vec{c}$
$\mathcal{B}_1$	1	1_1	$g\mathcal{B}_1^{\mu\dagger}(H^T\epsilon\overleftrightarrow{D}_\mu H) + h.c.$	✓	$8(1,0,-1)$
Ξ_1	0	3_1	$gM\Xi_1^{I\dagger}(H^T\epsilon\tau^I H) + h.c.$	✗	$8(0,1,0)$
$\mathcal{S}$	0	$1_0(S)$	$gM\mathcal{S}(H^\dagger H)$	✓	$2(0,0,1)$
$\mathcal{B}$	1	$1_0(A)$	$g\mathcal{B}^\mu(H^\dagger\overleftrightarrow{D}_\mu H)$	✓	$2(-1,1,0)$
Ξ_0	0	$3_0(S)$	$gM\Xi_0^I(H^\dagger\tau^I H)$	✗	$2(2,0,-1)$
$\mathcal{W}$	1	$3_0(A)$	$g\mathcal{W}^{\mu I}(H^\dagger\tau^I\overleftrightarrow{D}_\mu H)$	✗	$2(1,1,-2)$

$$\vec{C} = \sum_\alpha w_\alpha \vec{g}_\alpha \qquad w_\alpha = g_\alpha^2/M_\alpha^4$$

Uncertainty of $\vec{w} = (w_1, w_2, \cdots, w_6)$
(max distance between two valid w's)



1. Degeneracy vanishes at the ERs, and one of the faces.
2. Origin -> excluding all BSM states. Confirms SM.
3. Degeneracies smaller outside, larger inside.
4. Always finite -> exclude UV particles to certain scales
5. At dim-6, always infinite.

Example: SM W boson, gluons

✦ *W* boson: SO(2) x SU(2)

$$O_{T,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}]$$
$$O_{T,2} = \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}]$$
$$O_{T,1} = \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}]\mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]$$
$$O_{T,10} = \mathrm{Tr}[\hat{W}_{\mu\nu}\tilde{W}^{\mu\nu}]\mathrm{Tr}[\hat{W}_{\alpha\beta}\tilde{W}^{\alpha\beta}]$$
$$O_W = \varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$$

✦ Bounds (elastic only covers first 4)

$$F_{T,2} \geq 0,$$
$$4F_{T,1} + F_{T,2} \geq 36\bar{a}_W^2,$$
$$F_{T,2} + 8F_{T,10} \geq 36\bar{a}_W^2,$$
$$8F_{T,0} + 4F_{T,1} + 3F_{T,2} \geq 0,$$
$$12F_{T,0} + 4F_{T,1} + 5F_{T,2} + 4F_{T,10} \geq 0,$$
$$4F_{T,0} + 4F_{T,1} + 3F_{T,2} + 12F_{T,10} \geq 72\bar{a}_W^2$$

✦ See [Yamashita, Zhou, CZ, 2009.04490] for more W+B cases and applications in aQGC.

✦ Gluon: SO(2) x SU(3)

$Q_{G^4}^{(1)}$	$(G_{\mu\nu}^A G^{A\mu\nu})(G_{\rho\sigma}^B G^{B\rho\sigma})$
$Q_{G^4}^{(2)}$	$(G_{\mu\nu}^A \widetilde{G}^{A\mu\nu})(G_{\rho\sigma}^B \widetilde{G}^{B\rho\sigma})$
$Q_{G^4}^{(3)}$	$(G_{\mu\nu}^A G^{B\mu\nu})(G_{\rho\sigma}^A G^{B\rho\sigma})$
$Q_{G^4}^{(4)}$	$(G_{\mu\nu}^A \widetilde{G}^{B\mu\nu})(G_{\rho\sigma}^A \widetilde{G}^{B\rho\sigma})$
$Q_{G^4}^{(7)}$	$d^{ABE}d^{CDE}(G_{\mu\nu}^A G^{B\mu\nu})(G_{\rho\sigma}^C G^{D\rho\sigma})$
$Q_{G^4}^{(8)}$	$d^{ABE}d^{CDE}(G_{\mu\nu}^A \widetilde{G}^{B\mu\nu})(G_{\rho\sigma}^C \widetilde{G}^{D\rho\sigma})$

[C. Murphy, 2005.00059]

+ D6 3gluon operator

✦ Bounds $\vec{x} \cdot \vec{c} \geq 0$
x’s are:

$[0,0,0,1,0,0,0]$	$[0,0,6,3,7,2,0]$	$[24,0,12,21,15,14,0]$	$[0,0,96,24,64,40,-81]$
$[0,0,1,1,1,0,0]$	$[8,6,1,6,0,2,0]$	$[24,32,24,4,8,0,-27]$	$[40,32,80,4,0,0,-189]$
$[2,0,1,0,0,0,0]$	$[0,6,3,12,5,0,0]$	$[48,36,21,27,25,0,0]$	$[0,0,24,120,40,104,-81]$
$[0,2,0,1,0,0,0]$	$[8,6,1,12,0,0,0]$	$[32,40,4,80,0,0,-27]$	$[0,0,120,24,104,40,-81]$
$[0,0,3,0,2,0,0]$	$[0,6,6,9,10,4,0]$	$[0,48,0,48,0,40,-81]$	$[96,0,144,24,64,40,-81]$
$[0,0,0,3,0,2,0]$	$[0,12,0,14,0,0,-9]$	$[24,0,36,24,16,40,-81]$	$[48,0,96,24,0,40,-243]$
$[1,1,2,2,0,0,0]$	$[0,0,8,8,0,8,-27]$	$[0,0,48,24,32,40,-81]$	$[0,192,168,96,112,120,-405]$
$[6,0,3,0,2,0,0]$	$[12,0,14,0,0,0,-27]$	$[0,0,24,48,16,56,-81]$	$[168,480,168,156,56,160,-729]$
$[4,2,2,1,2,0,0]$	$[6,8,12,1,0,0,-27]$	$[88,32,56,4,40,0,-27]$	$[264,384,156,168,16,200,-729]$
$[0,0,4,0,0,0,-9]$	$[8,16,4,8,0,8,-27]$	$[96,42,27,84,25,0,0]$	$[288,384,216,168,0,200,-891]$
$[6,0,6,0,5,0,0]$	$[0,24,0,12,0,8,-27]$	$[96,66,42,39,50,4,0]$	$[480,384,480,168,160,200,-729]$
$[0,0,3,6,5,4,0]$	$[8,22,1,14,0,10,-27]$	$[120,42,39,42,40,14,0]$	$[336,768,672,216,0,200,-2187]$

7D polyhedral cone with 48 faces

[X. Li et al., 2101.01191]

Dual cone

$$\mathbf{C}^{n^4} = \text{cone}\left(\left\{m^{ij}m^{kl} + m^{il}m^{kj}\right\}\right)$$

- A linear bound is represented by a r4 tensor Q: $\mathcal{Q} \cdot \mathcal{M} \geq 0$
- Collect all valid linear bounds: $\mathbf{C}^{n^4 *} = \{\mathcal{Q} \mid \mathcal{Q} \cdot \mathcal{M} \geq 0, \ \forall \mathcal{M} \in \mathbf{C}^{n^4}\}$
  - This is the dual cone of **C**
  - Exact bound (thanks to hyperplane separation for convex bodies)

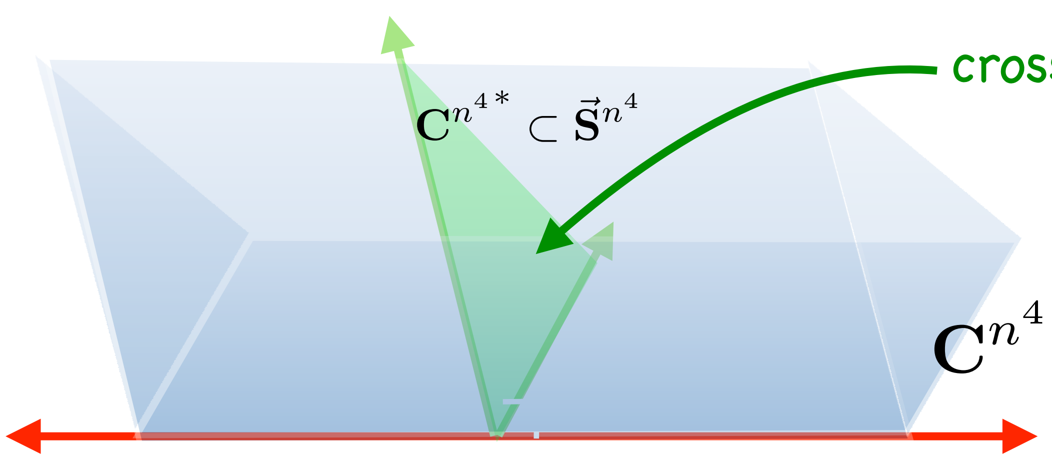
$$\mathbf{C}^{n^4} = \{\mathcal{M} \mid \mathcal{Q} \cdot \mathcal{M} \geq 0, \ \forall \mathcal{Q} \in \mathbf{C}^{n^4 *}\} \qquad \text{or } \mathbf{C}^{**} = \mathbf{C}$$

  - But they are not independent: e.g. if $\mathcal{Q}_1 = \mathcal{Q}_2 + \mathcal{Q}_3$
- If the dual cone is salient, its generators are its ERs. Independent & complete bounds are: $\mathbf{C}^{n^4} = \left\{\mathcal{M} \mid \mathcal{Q} \cdot \mathcal{M} \geq 0, \ \forall \mathcal{Q} \in \text{ext}\left(\mathbf{C}^{n^4 *}\right)\right\}$

- The amplitudes satisfy crossing symmetries by construction
  - i<->k, and j<->l exchanges. These are s<->u crossing when t->0.
  - In additional, (i<->j) and (k<->l) simultaneously. Or equivalently, m matrices are symmetric or anti-symmetric. (implies P-conservation)
  - Define the crossing symmetric subspace

$$\mathbf{C}^{n^4} \subset \vec{\mathbf{S}}^{n^4} \equiv \left\{\mathcal{T} \mid \mathcal{T}^{ijkl} = \mathcal{T}^{ilkj} = \mathcal{T}^{kjil} = \mathcal{T}^{jilk}\right\}$$

  - $\mathbf{C}^{n^4 *}$ contains straight lines perpendicular to this subspace. Not salient.



$$\mathbf{Q}^{n^4} \equiv \mathbf{C}^{n^4 *} \cap \vec{\mathbf{S}}^{n^4}$$

$$\mathbf{C}^{n^4} = \left\{\mathcal{M} \in \vec{\mathbf{S}}^{n^4} \mid \mathcal{Q} \cdot \mathcal{M} \geq 0 \ \forall \mathcal{Q} \in \mathbf{Q}^{n^4}\right\}$$

- Use the fact that physics amplitudes from EFT are crossing symmetric

$$\mathbf{C}^{n^4} \subset \vec{\mathbf{S}}^{n^4} \equiv \{\mathcal{T} \mid \mathcal{T}^{ijkl} = \mathcal{T}^{ilkj} = \mathcal{T}^{kjil} = \mathcal{T}^{jilk}\}$$

  - Define the duality within the symmetric subspace.

$$\mathbf{Q}^{n^4} \equiv \mathbf{C}^{n^4 *} \cap \vec{\mathbf{S}}^{n^4}$$

$$\mathbf{C}^{n^4} = \left\{\mathcal{M} \in \vec{\mathbf{S}}^{n^4} \mid \mathcal{Q} \cdot \mathcal{M} \geq 0 \ \forall \mathcal{Q} \in \mathbf{Q}^{n^4}\right\}$$

- What is $\mathbf{Q}^{n^4}$?

  - With crossing symmetry:

$$\mathcal{Q} \cdot \mathcal{M} \geq 0$$

$$\Rightarrow \mathcal{Q}^{ijkl} \sum_\alpha \left(m_\alpha^{ij} m_\alpha^{kl} + m_\alpha^{il} m_\alpha^{kj}\right) = 2 \sum_\alpha m_\alpha^{ij} \mathcal{Q}^{ijkl} m_\alpha^{kl} \geq 0 \ \forall m \in \mathbb{R}^{n^2}$$

$$\Rightarrow \mathcal{Q}^{(ij),(kl)} \succeq 0 \qquad \Rightarrow \mathcal{Q} \in \mathbf{S}_+^{(n^2 \times n^2)}$$

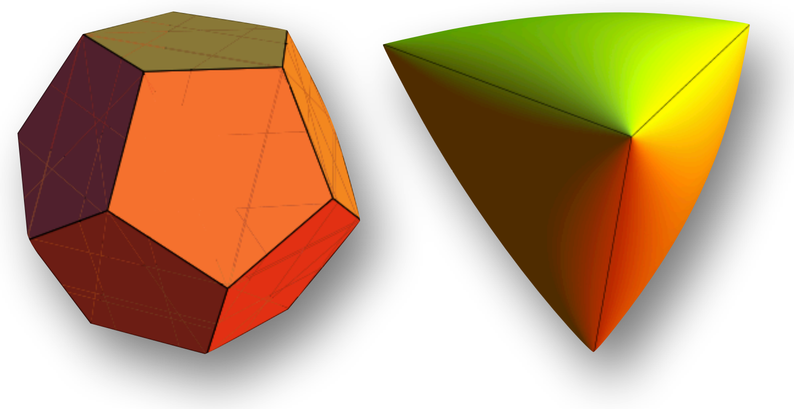
$$\boxed{\mathbf{Q}^{n^4} = \mathbf{S}_+^{n^2 \times n^2} \cap \vec{\mathbf{S}}^{n^4}}$$

  - Finding positivity bounds = finding the ERs of some “spectrahedron”.

$$\mathbf{C}^{n^4} = \left\{\mathcal{M} \in \vec{\mathbf{S}}^{n^4} \mid \mathcal{Q} \cdot \mathcal{M} \geq 0, \ \forall \mathcal{Q} \in \boxed{\mathrm{ext}\left(\mathbf{Q}^{n^4}\right)}\right\}$$

Faces

- Each point x in a spectrahedron is contained in (the relative interior of) a unique face, F(x)



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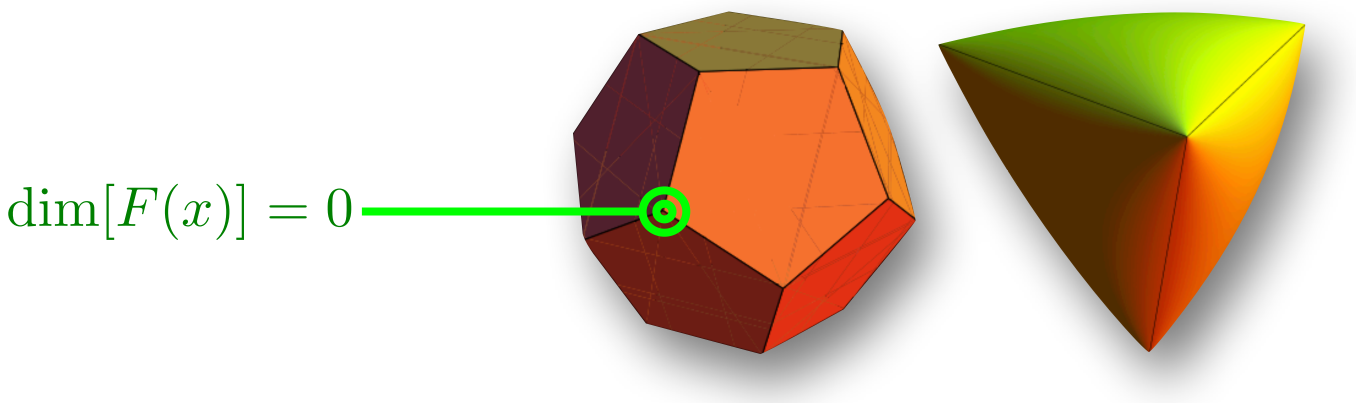

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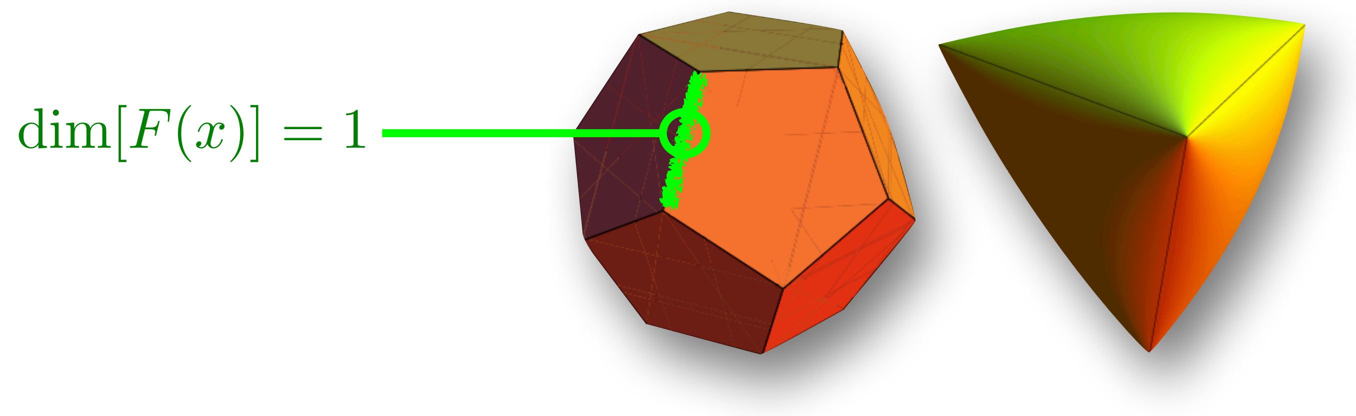

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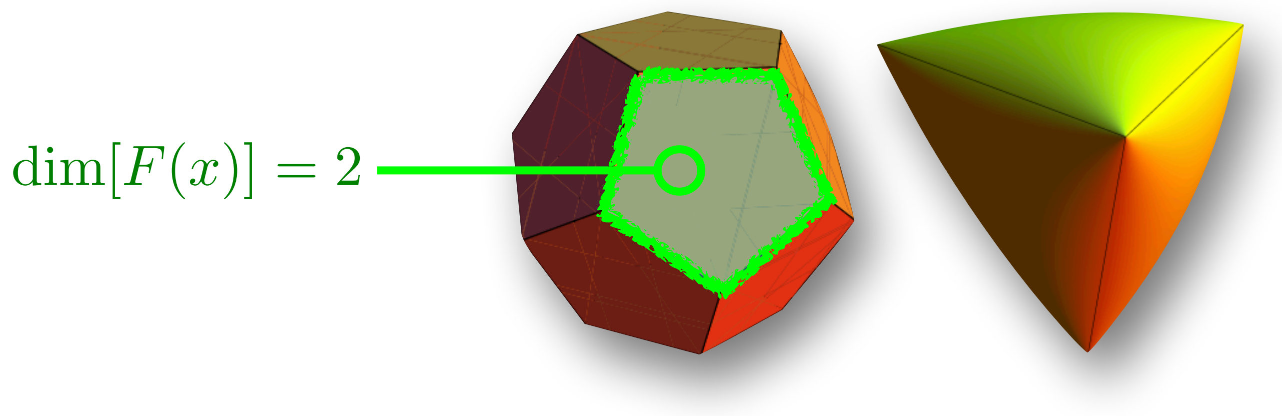

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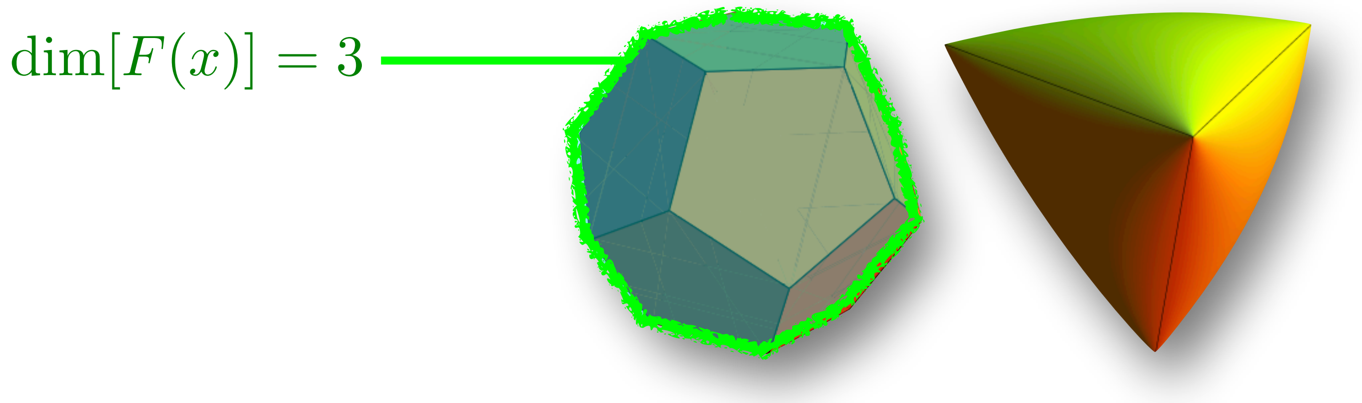

Faces

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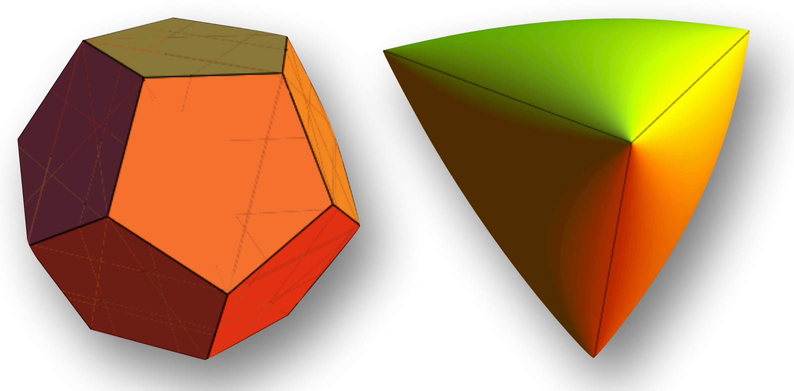

- The null space of Q(x) is constant on F(x) -> numerically identify F(x) for any x [Ramana & Goldman '95]
  - Let $\{u_i\}$ be basis of Null(Q(x)), then Null(B) is the linear span of F(x)

$$B = \begin{bmatrix} \mathcal{Q}_1 u_1 & \cdots & \mathcal{Q}_m u_1 \\ \vdots & \ddots & \vdots \\ \mathcal{Q}_1 u_k & \cdots & \mathcal{Q}_m u_k \end{bmatrix}$$

Application to dRGT

- ✦ dRGT massive gravity (n=5): improves slightly the minimum value of d5

Positive Signs in Massive Gravity

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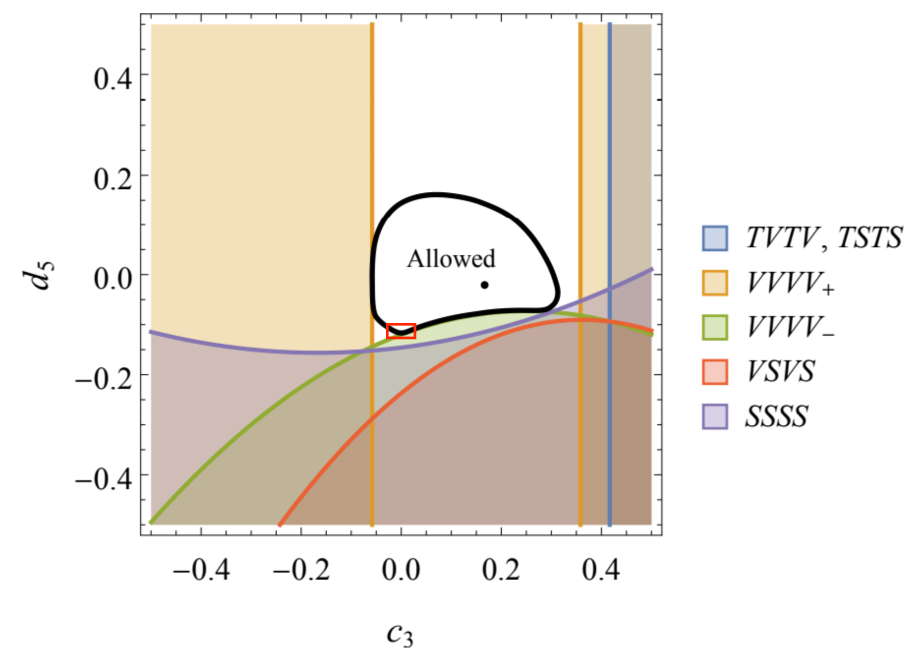
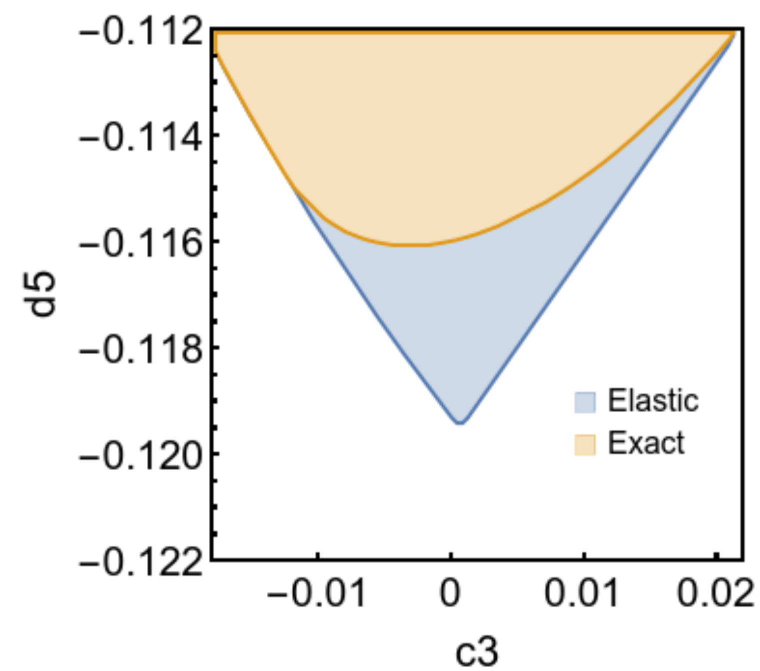
Abstract

We derive new constraints on massive gravity from unitarity and analyticity of scattering amplitudes. Our results apply to a general effective theory defined by Einstein gravity plus the leading soft diffeomorphism-breaking corrections. We calculate scattering amplitudes for all combinations of tensor, vector, and scalar polarizations. The high-energy behavior of these amplitudes prescribes a specific choice of couplings that ameliorates the ultraviolet cutoff, in agreement with existing literature. We then derive consistency conditions from analytic dispersion relations, which dictate positivity of certain combinations of parameters appearing in the forward scattering amplitudes. These constraints exclude all but a small island in the parameter space of ghost-free massive gravity. While the theory of the "Galileon" scalar mode alone is known to be inconsistent with positivity constraints, this is remedied in the full massive gravity theory.

Elastic:



SDP:



- Two sources of degeneracies:
  - EXP: finite resolution in real measurements
  - TH: intrinsic degeneracy at any truncated mass dimension
- Example at dim-6:

$$V_1: \quad g_1 V_1^{\mu}(\bar{e}_R \gamma_\mu e_R)$$
$$S_2: \quad g_2 S_2(\bar{e}_R^c e_R) + h.c.$$

$$O_{ee}^{(6)} = (\bar{e}_R \gamma_\mu e_R)(\bar{e}_R \gamma^\mu e_R)$$
$$\frac{C_{ee}^{(6)}}{\Lambda^2} = -\frac{g_1^2}{2M_1^2} + \frac{g_2^2}{2M_2^2}$$

- Cannot resolve the flat direction $g_1^2/M_1^2 - g_2^2/M_2^2 = \text{const.}$

  This prevents us from determining the coupling/mass ratio of each UV particle type.

- Situation changes at dim-8. $-\frac{C_{ee}^{(8)}}{\Lambda^4} = \frac{g_1^2}{2M_1^4} + \frac{g_2^2}{M_2^4}$

Inverse problem in the PSD matrix cone

- Q: Knowing $\vec{C} = \sum_\alpha w_\alpha \vec{g}_\alpha$, or

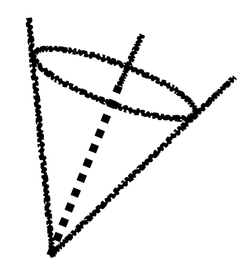
$$\mathcal{M}^{ijkl} = \sum_\alpha w_\alpha \left(m_\alpha^{ij} m_\alpha^{kl} + m_\alpha^{il} m_\alpha^{kj} \right) \qquad w_\alpha = g_\alpha^2 / M_\alpha^4$$

How do we determine w_α ? (Matching: RHS -> LHS; Inverse: LHS -> RHS)

- In general impossible: more g (generators) than # coefficients.

- There are exceptions: e.g. *M* is extremal.

- For illustration, neglect for the moment the L cut



$$\begin{pmatrix} 1 & r \\ r & r^2 \end{pmatrix}$$

$$\mathcal{M}^{ijkl} = \sum_\alpha w_\alpha \left(m_\alpha^{ij} m_\alpha^{kl} + \cancel{m_\alpha^{il} m_\alpha^{kj}} \right) \qquad \mathcal{M}^{ij} = \sum_\alpha w_\alpha m_\alpha^i m_\alpha^j$$

Inverse problem in the PSD matrix cone

$$\mathcal{M}^{ij} = \sum_\alpha w_\alpha m^i_\alpha m^j_\alpha$$

What can we say about w_α ?

- We expect the following four statements about w_α to be physically interesting.
  - Rank-1: only one w can be nonzero. No degeneracy.
    (Uniquely determines the "1-particle extensions")
  - Rank-r, r>1: suppose $b^i_\beta,\ \beta = 1, 2, \cdots, n-r$
    are the basis vector of the dim-(n-r) null space of M, we have
    $$\mathcal{M}^{ij} b^i_\beta b^j_\beta = \sum_\alpha w_\alpha \left(m^i_\alpha b^i_\beta\right)^2 = 0 \qquad \forall\beta$$
    i.e. all generators not on the row space of M are excluded (model independent)
  - Rank-0: all w have to vanish! Confirms the SM. (model independent)
  - Full rank: all w's are bound from above. Exclusion limits on all possible UV states (model independent)

Inverse problem in the PSD matrix cone

$$\mathcal{M}^{ij} = \sum_{\alpha} w_\alpha m_\alpha^i m_\alpha^j$$

What can we say about w_α ?

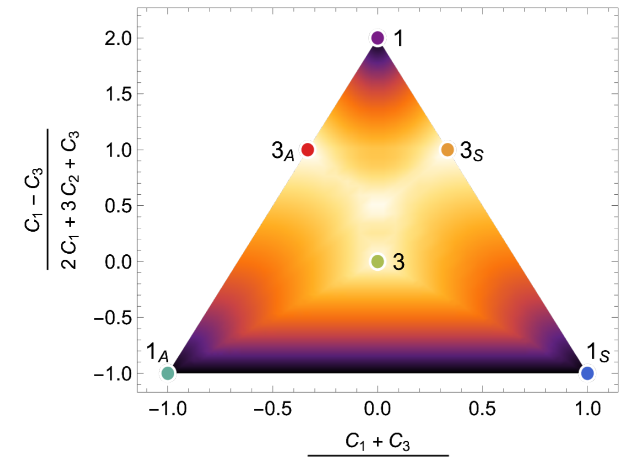
- We expect the following four statements about w_α to be physically interesting.
  - Rank-1 -> ER.
  - Rank-r -> Faces (of dim-r(r+1)/2)
  - Rank-0 -> Vertex.
  - Full rank -> Interior of the cone.

Generalized to all salient cones

$$\mathcal{M}^{ijkl} = \sum_X \lambda_X \left(m_X^{ij} m_X^{*kl} + m_X^{i\bar{l}} m_X^{*k\bar{j}} \right), \ \lambda_X \geq 0$$ is a salient cone

(Positive projection on $\delta^{ik}\delta^{jl}$)

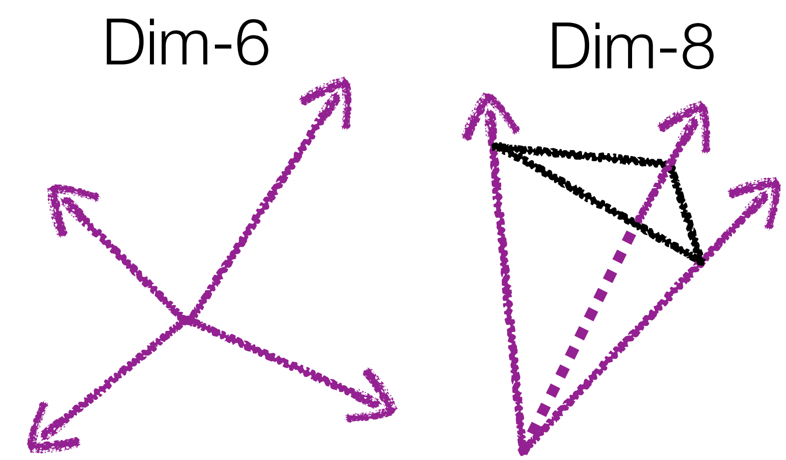
- ✦ We expect the following four statements about w_α to be physically interesting.
  - ✦ Rank-1 -> ER.
    Only one w can be nonzero. No degeneracy.
    (Uniquely determines the "1-particle extensions")



  - ✦ Rank-r, r>1 -> Faces.
    i.e. all generators not on the same face are excluded (model independent)
  - ✦ Rank-0 -> Origin.
    All w have to vanish! Confirms the SM. (model independent)
  - ✦ Full rank -> Interior.
    all w's are bound from above. Exclusion limits on all possible UV states (model independent)

Dim-6 vs dim-8

- Generators at dim-8 form a **salient** cone; at dim-6 this is **not true**.
  - $\mathcal{G}_X^{ijkl} \equiv m_X^{ij} m_X^{*kl} + m_X^{i\bar{l}} m_X^{*k\bar{j}}$ always have positive projects on $\delta^{ik}\delta^{jl}$
  - $\vec{C} = \sum_\alpha w_\alpha \vec{g}_\alpha$ has <u>very different</u> implications



	Dim-6	Dim-8
Unique solutions	No: $0 = \sum_\alpha \bar{w}_\alpha^{(6)} \vec{g}_\alpha^{(6)}$ has nonzero solution $w_\alpha^{(6)} \to w_\alpha^{(6)} + \lambda \bar{w}_\alpha^{(6)}, \lambda > 0$	Yes: Salient cone -> ERs always exist ER not splittable -> unique w.
Zero coefs. rule out all BSM	No: $\lambda \bar{w}_\alpha^{(6)}, \ \lambda \in \mathbf{R}^+$	Yes: 0 is an extreme point of a salient cone.
Finite uncertainty; upper bound on w.	No: $w_\alpha^{(6)} \to w_\alpha^{(6)} + \lambda \bar{w}_\alpha^{(6)}, \lambda > 0$	Yes. $\vec{C}(\lambda) \equiv \vec{C} - \lambda \vec{g}_k = \sum_{i \neq k} w_i \vec{g}_i + (w_k - \lambda)\vec{g}_k$ $\lambda_M = \max_{\vec{C}(\lambda) \in \mathbf{C}} \lambda \geq w_k$