Ab Initio Coupling of Jets to Collective Flow in the Opacity Expansion Approach

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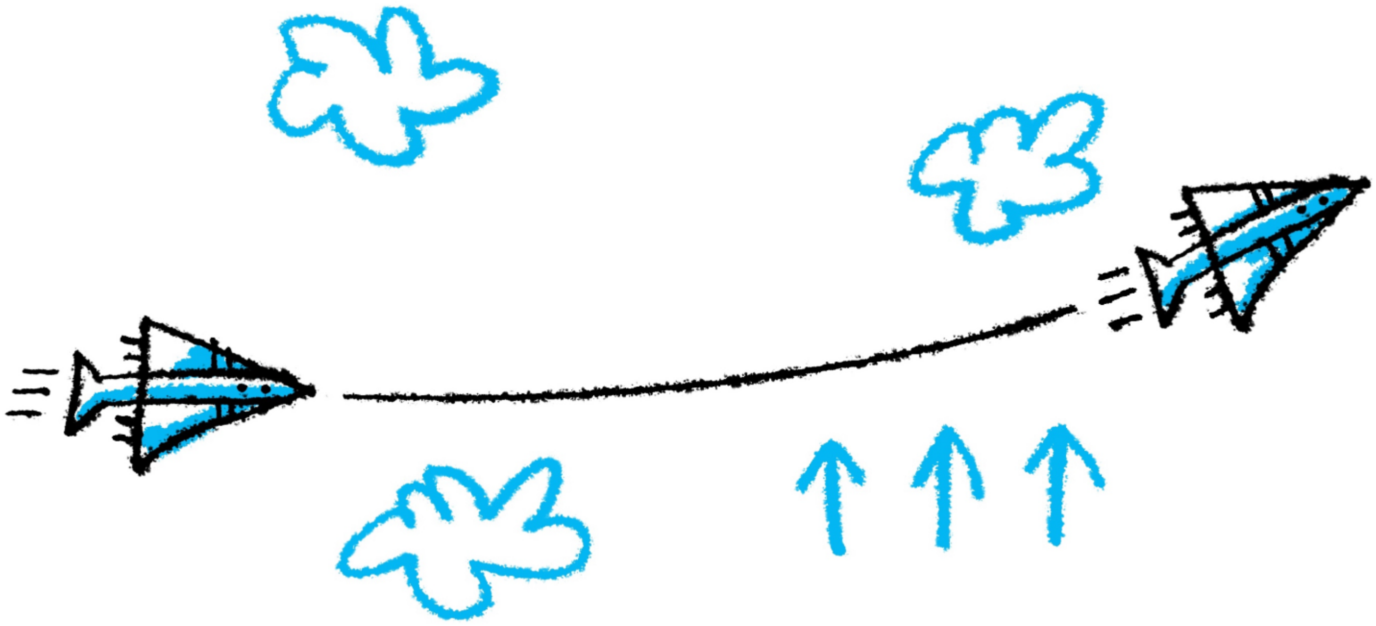
based on 2104.09513

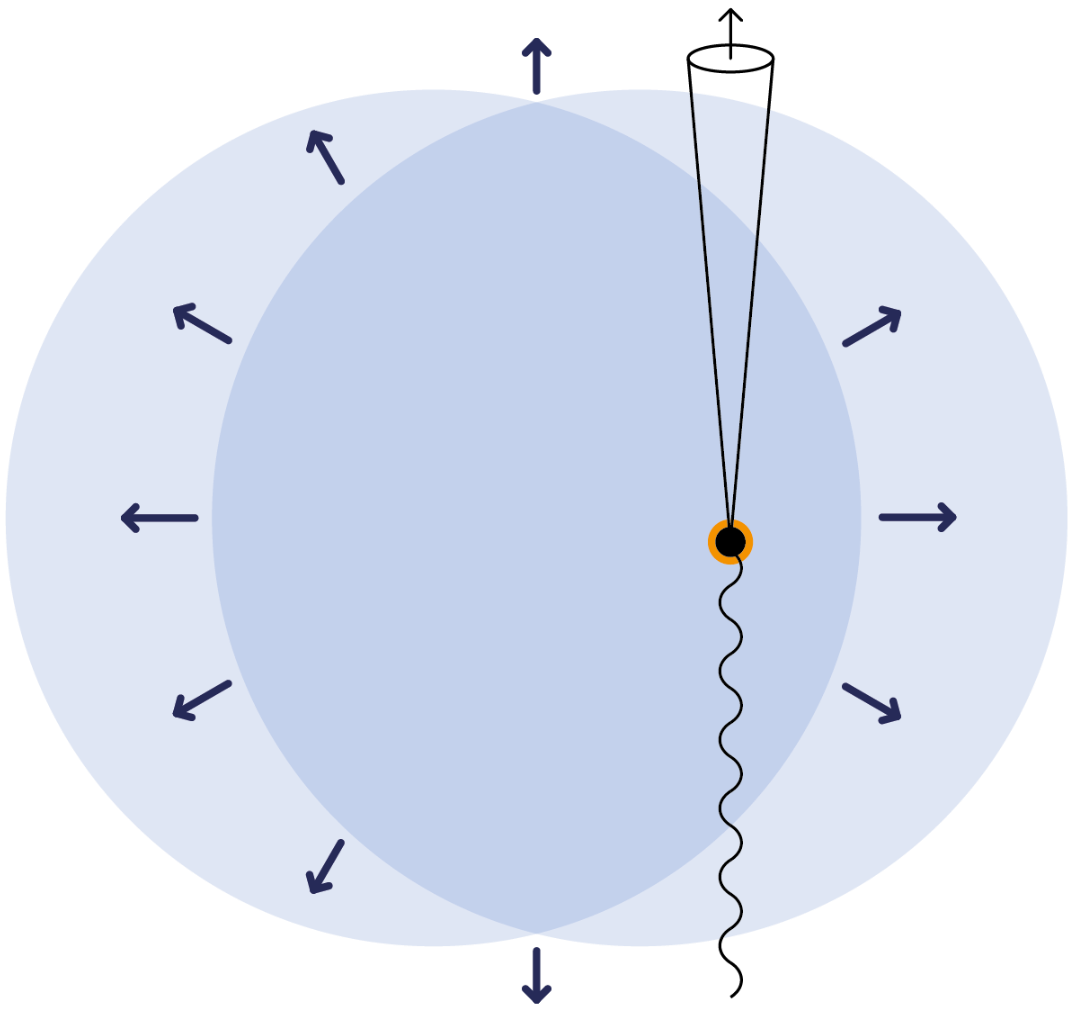
Jet Tomography

- The hot nuclear matter in HIC undergoes multi-phase evolution and its details are hard to access through the soft sector.
- In turn, jets see the matter at multiple scales, and essentially X-ray it;
- However, most approaches to the jet-medium interaction are either empirical or based on multiple simplifying assumptions – static matter, no fluctuations, etc;
- In what follows I will highlight our recent progress on the medium motion effects in the QCD calculations for jet broadening and gluon emission;
- The developed formalism can be also applied to include orbital motion of nucleons and some of in-medium fluctuations (e.g. spatial inhomogeneities) in the DIS context;

Jets

Does a jet feel the flow?





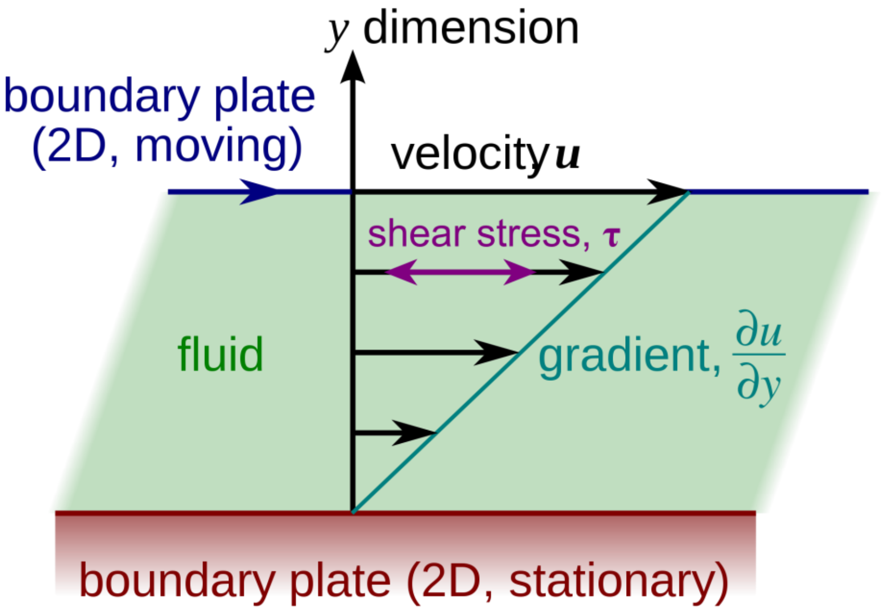
Old questions

$$T^{\mu\nu} = (\epsilon + P)\, u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu} \qquad \tau^{\mu\nu} = \ldots + \mathcal{O}(\partial^n)$$

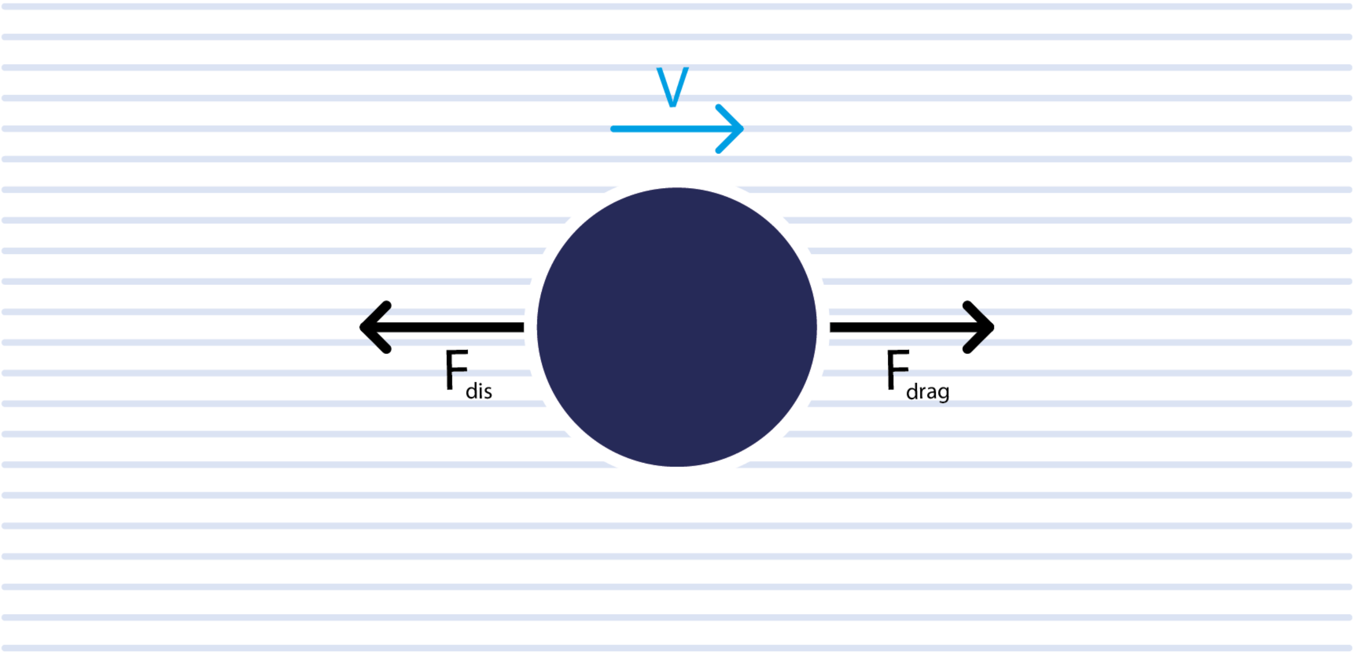
$$J^\mu = n u^\mu + \nu^\mu \qquad \nu^\mu = \ldots + \mathcal{O}(\partial^n)$$

- Hydrodynamic flows are generally non-uniform – the gradients are non-zero (it is not enough to boost the system). **Does this inhomogeneity affects a "bullet" penetrating a flow?**

- We often assume the changes in the flow to be slow in space and time – the hydrodynamic expansion. **Could the same expansion be applied to study the effect of the inhomogeneity on the "bullet"?**



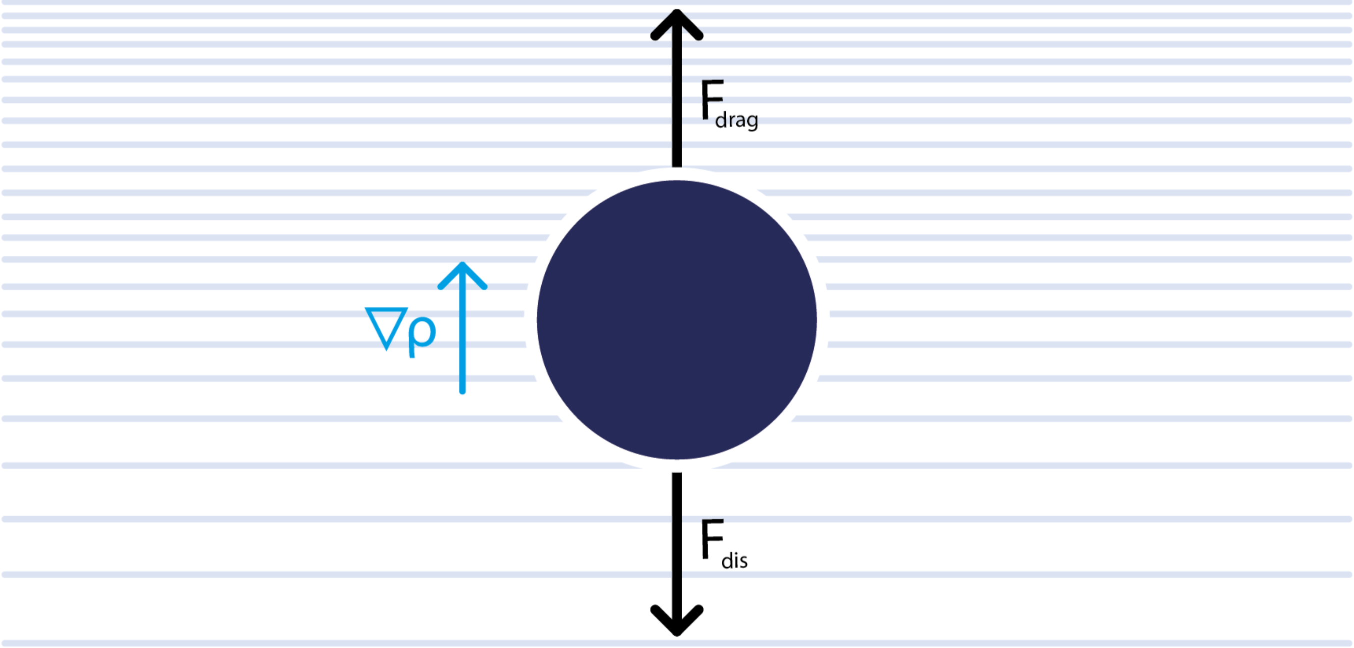
Drag Force



$$\vec{f} \sim T^2 \vec{v}$$

M. Lekaveckas, K. Rajagopal, JHEP, 2014
K. Rajagopal, AS, JHEP, 2017
J. Reiten, AS, JHEP, 2020

Drag Force

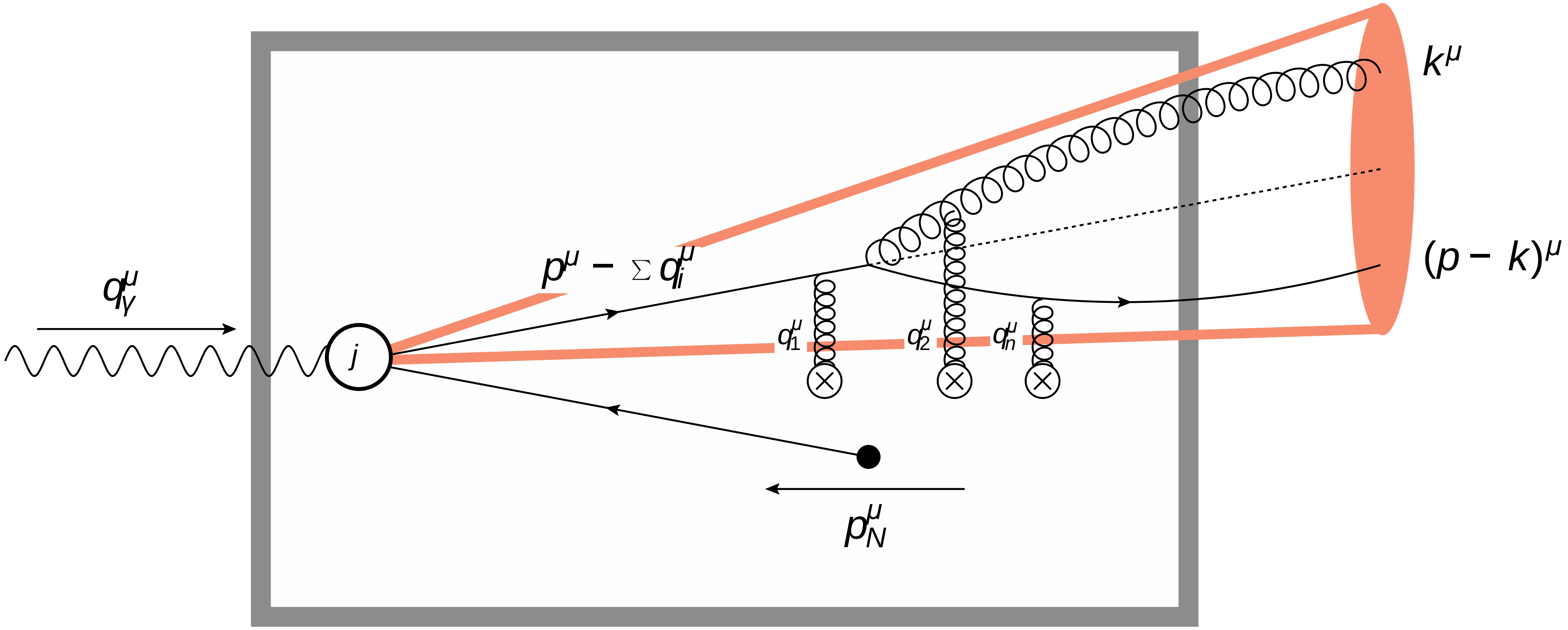


$$\vec{f} \sim \vec{\nabla}\rho$$

R. Baier et al, NPB, 1997
B. G. Zakharov, JETP, 1997
R. Baier et al, NPB, 1998
M. Gyulassy et al, NPB, 2000
M. Gyulassy et al, NPB, 2001
…

Jets

QCD broadening and gluon emission
(GLV/BDMPS-Z) with flow



R.B. Neufeld et al, PRC, 2011
R.B. Neufeld, I. Vitev, PRL, 2012
Y.T. Chien, I. Vitev, JHEP, 2016
Y.T. Chien, I. Vitev, PRL, 2017
Z.B. Kang et al, JHEP, 2017
Z.B. Kang et al, PLB, 2017
…

Jets

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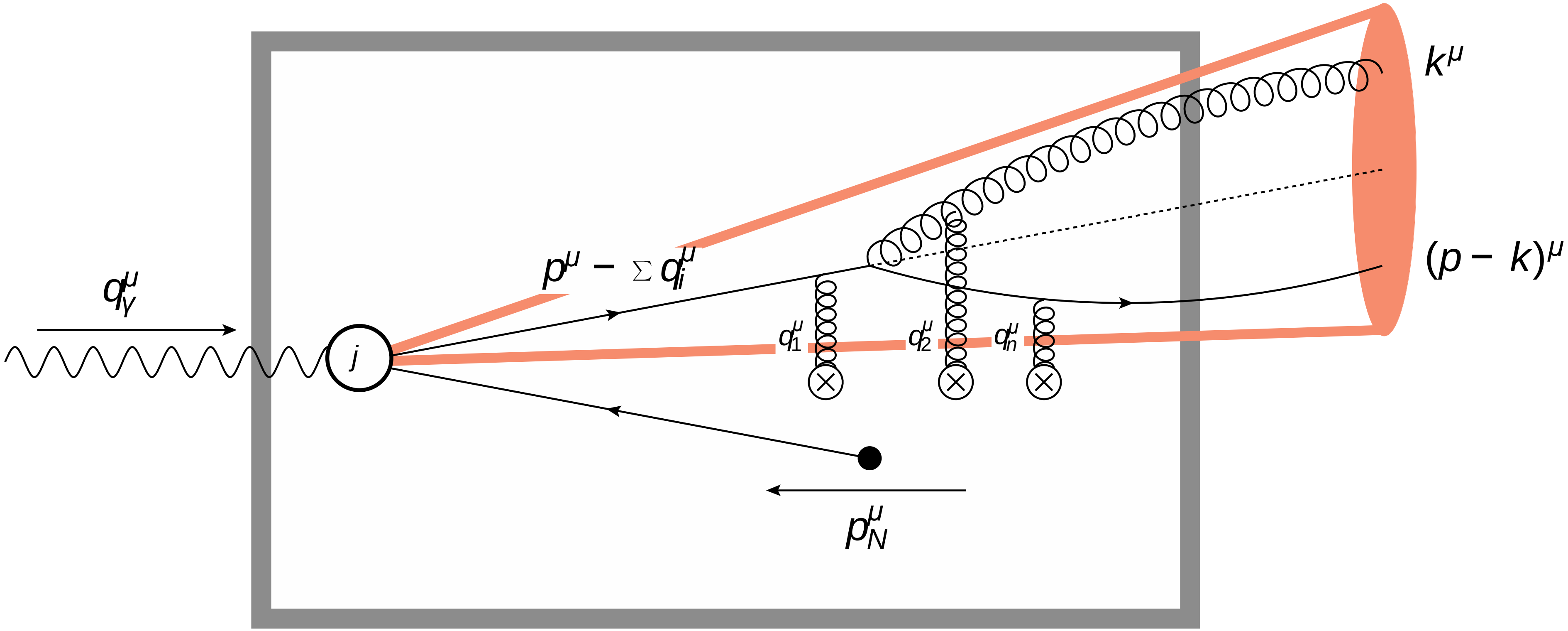
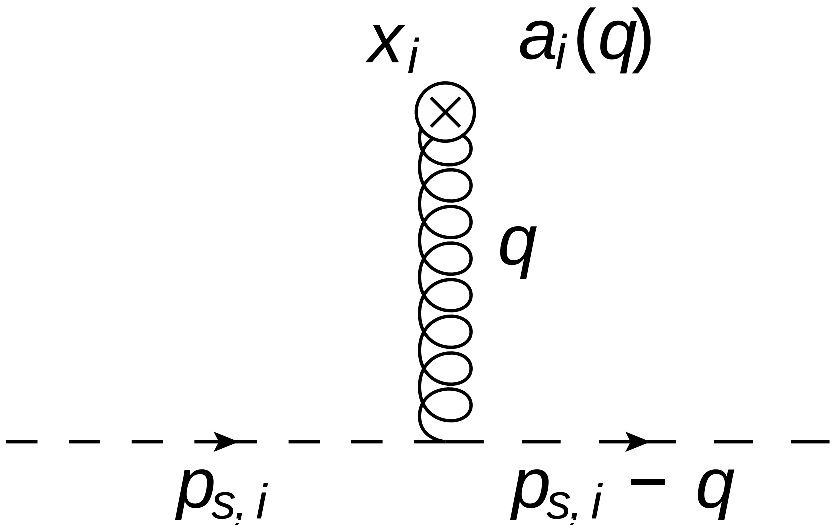


M. Sievert, I.Vitev, PRD, 2018

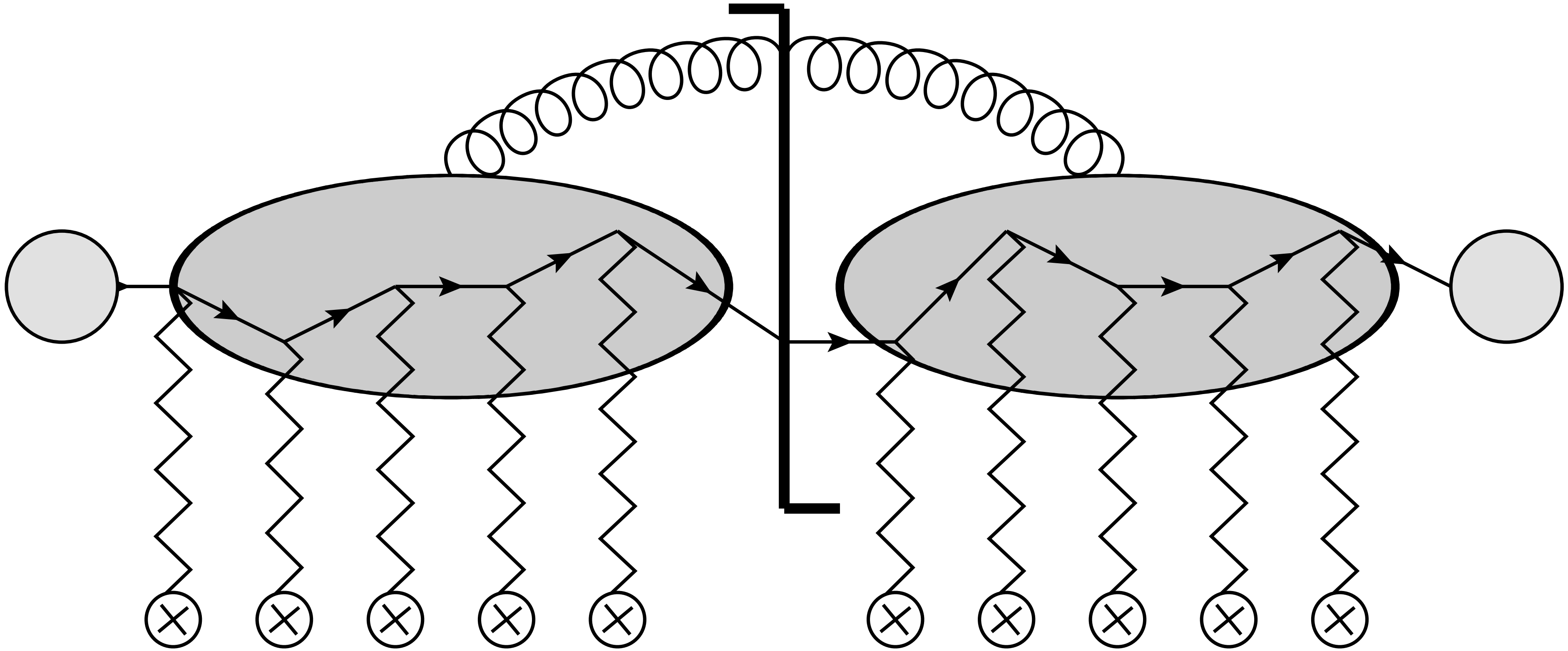
Color Potential



large

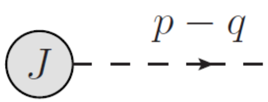
$$a_i^{\mu a}(q) = (ig\, t_i^a)\,(2p_{s\,i} - q)_\nu \left(\frac{-ig^{\mu\nu}}{q^2-\mu^2+i\epsilon}\right)(2\pi)\,\delta\left((p_{s\,i}-q)^2 - M^2\right)$$

$v(q^2)$ -- **the Gyulassy-Wang potential**



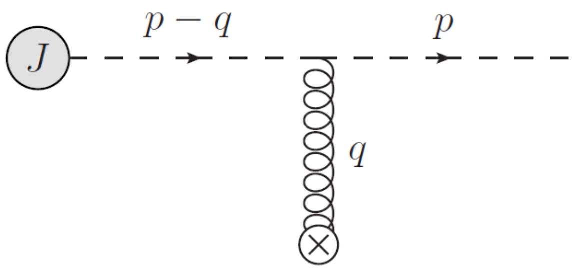
The Opacity Expansion

$$E\frac{dN^{(0)}}{d^3p} = \Bigg|$$

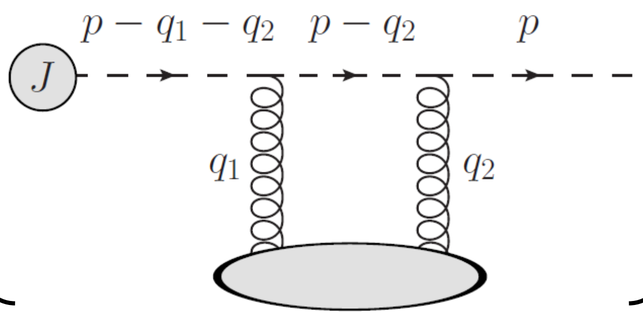


$$\Bigg|^2 = \frac{1}{2(2\pi)^3}\left|J(p)\right|^2$$

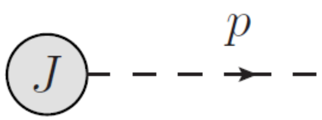
$$E\frac{dN^{(1)}}{d^3p} = \Bigg|$$



$$\Bigg|^2 + \Bigg($$

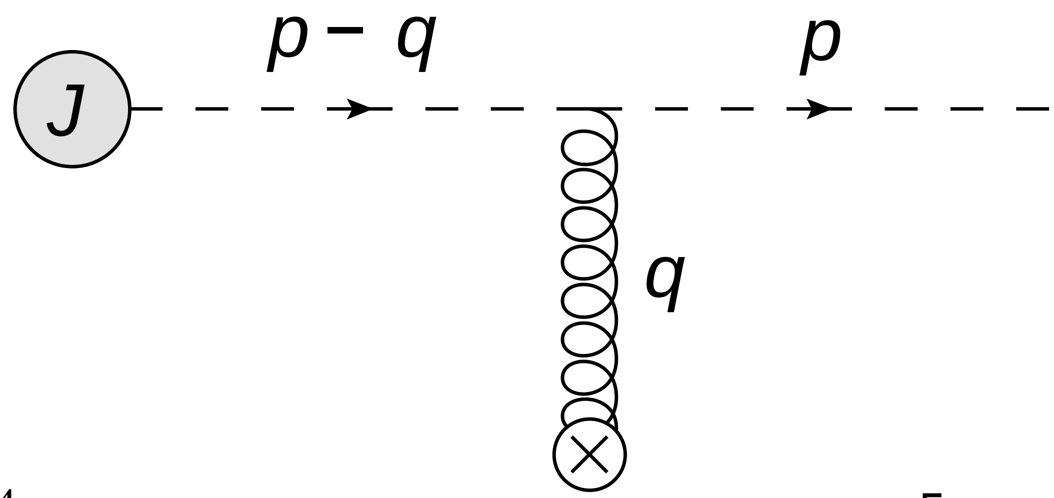


$$\Bigg)\Bigg($$



$$\Bigg)^* + c.c.$$

Jet Broadening



$$iM_1(p) = \int \frac{d^4q}{(2\pi)^4} \left[ig\, t^a_{\rm proj} A^{\mu a}_{\rm ext}(q)\, (2p-q)_\mu \right] \left[\frac{i}{(p-q)^2 + i\epsilon} \right] J(p-q)$$

the four-vector of the fluid non-relativistic velocity

$$g A^{\mu a}_{ext}(q) = \sum_i e^{iq\cdot x_i}\, t^a_i\, u^\mu_i\, v_i(q)\, (2\pi)\, \delta\left(q^0 - \vec{u}_i \cdot \vec{q}\right)$$

$$|\vec{q}| \ll M$$

Jet Broadening

Eikonal approximation -- $E \to \infty$

$$\frac{(2p-q)_\mu A^\mu_{ext}(q)}{(p-q)^2 + i\epsilon} \longrightarrow \frac{2u_\mu p^\mu}{(1-u_z^2)\left(Q^+ - Q^-\right)} = 1 - \frac{\vec{u}_\perp \cdot (\vec{p}-\vec{q})_\perp}{E(1-u_z)} + \mathcal{O}\left(\frac{p_\perp^2}{E^2}\right)$$

$$Q^+_{p-q} = \frac{2E}{1+u_{i\,z}}\left[1 - \frac{\vec{u}_{i\,\perp}\cdot\vec{q}_\perp}{2E} + \mathcal{O}\left(\frac{p_\perp^2}{E^2}\right)\right],$$

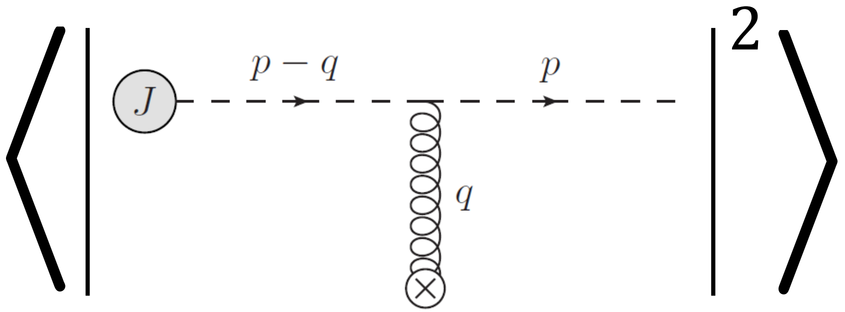
$$Q^-_{p-q} = \frac{\vec{u}_{i\,\perp}\cdot\vec{q}_\perp}{1-u_{i\,z}} + \frac{(p-q)_\perp^2 - p_\perp^2}{2E(1-u_{i\,z})} + \mathcal{O}\left(\frac{p_\perp^2}{E^2}\right)$$

modified LPM phase

$$v(q)$$

$$\mu \ll E \qquad \mu z \gg 1$$

Jet Broadening



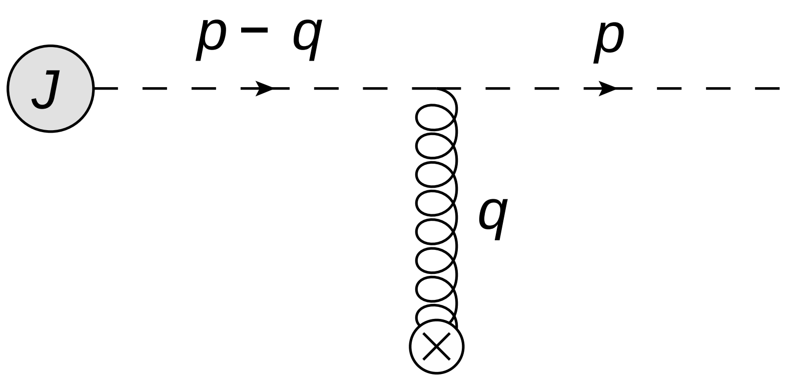
$$\langle t_i^a t_j^b \rangle = \mathcal{C}\,\delta_{ij}\delta^{ab}$$

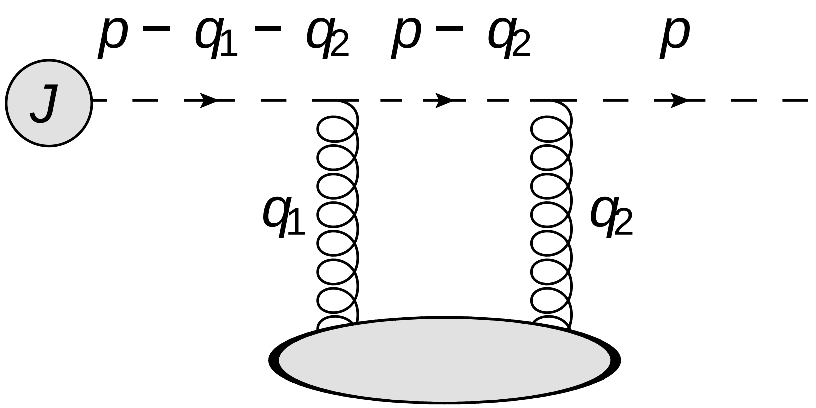
color neutrality
(one sum out of the two)

$$\sum_i = \int d^3x\, \rho(\vec{x})$$

source averaging
(if only the phase depends on $\boldsymbol{x}_\perp$ there is only one momentum integration)

Jet Broadening





the static case:

$$E\frac{dN^{(1)}}{d^3p} = \int dz\, d^2q_\perp\, \rho(z)\, \frac{d\sigma}{d^2q_\perp}\left[\left(E\frac{dN^{(0)}}{d^2(p-q)_\perp\, dE}\right) - \left(E\frac{dN^{(0)}}{d^2p_\perp\, dE}\right)\right]$$

the elastic scattering cross section

$$\frac{d\sigma}{dq_\perp^2} = \frac{1}{(2\pi)^2}\mathcal{C}\left|v\left(q_\perp^2\right)\right|^2$$

the initial distribution

$$E\frac{dN^{(0)}}{d^3p} \equiv \frac{1}{2(2\pi)^3}\left|J(p)\right|^2$$

Jet Broadening

with uniform flow:

$$E\frac{dN^{(1)}}{d^3p} = \int dz\, d^2q_\perp\, \rho(z)\, \frac{d\sigma}{d^2q_\perp}\Bigg[\left(E\frac{dN^{(0)}}{d^2(p-q)_\perp\, dE}\right)\left(1+\vec{u}_\perp(z)\cdot\vec{\Gamma}(\vec{q}_\perp)\right)$$
$$-\left(E\frac{dN^{(0)}}{d^2p_\perp\, dE}\right)\left(1+\vec{u}_\perp(z)\cdot\vec{\Gamma}_{DB}(\vec{q}_\perp)\right)\Bigg] + \mathcal{O}\left(\partial_\perp\right)$$

$$\vec{\Gamma}(\vec{q}_\perp) = -2\frac{\vec{p}_\perp - \vec{q}_\perp}{(1-u_{i\,z})E} + \frac{2\vec{q}_\perp}{(1-u_{i\,z})E}\left(\frac{(p-q)_\perp^2 - p_\perp^2}{v(q_\perp^2)}\right)\frac{\partial v}{\partial q_\perp^2}$$
$$-\frac{\vec{q}_\perp}{1-u_z}\left(\frac{1}{\bar{N}_0(E,\vec{p}_\perp-\vec{q}_\perp)}\frac{\partial \bar{N}_0}{\partial E}\right)$$

Jet Broadening

with flow:

$$\vec{\Gamma}(\vec{q}_\perp) = -2\frac{\vec{p}_\perp - \vec{q}_\perp}{(1-u_{i\,z})E} + \frac{2\vec{q}_\perp}{(1-u_{i\,z})E}\left(\frac{(p-q)_\perp^2 - p_\perp^2}{v(q_\perp^2)}\right)\frac{\partial v}{\partial q_\perp^2}$$
$$- \frac{\vec{q}_\perp}{1-u_z}\left(\frac{1}{\bar{N}_0(E,\vec{p}_\perp - \vec{q}_\perp)}\frac{\partial \bar{N}_0}{\partial E}\right)$$

- The finite collisional energy transfer q^0 to the jet results in a small shift in the energy of the initial jet distribution and in a shift of the transverse momentum spectrum of $\frac{d\sigma}{d^2q}$ leading to the two last terms above;
- The first term in Γ, in turn, appears due to a sub-eikonal correction to the vertex, a penalty for bending the jet, and the modification of the propagator due to the energy transfer, which can increase the scattering amplitude;

Jet Broadening

assuming a model source

$$E\frac{dN^{(0)}}{d^3p} = \frac{1}{2(2\pi)^3}|J(p)|^2 = f(E)\,\delta^{(2)}(\vec{p}_\perp)$$

and ignoring the z-dependence we find

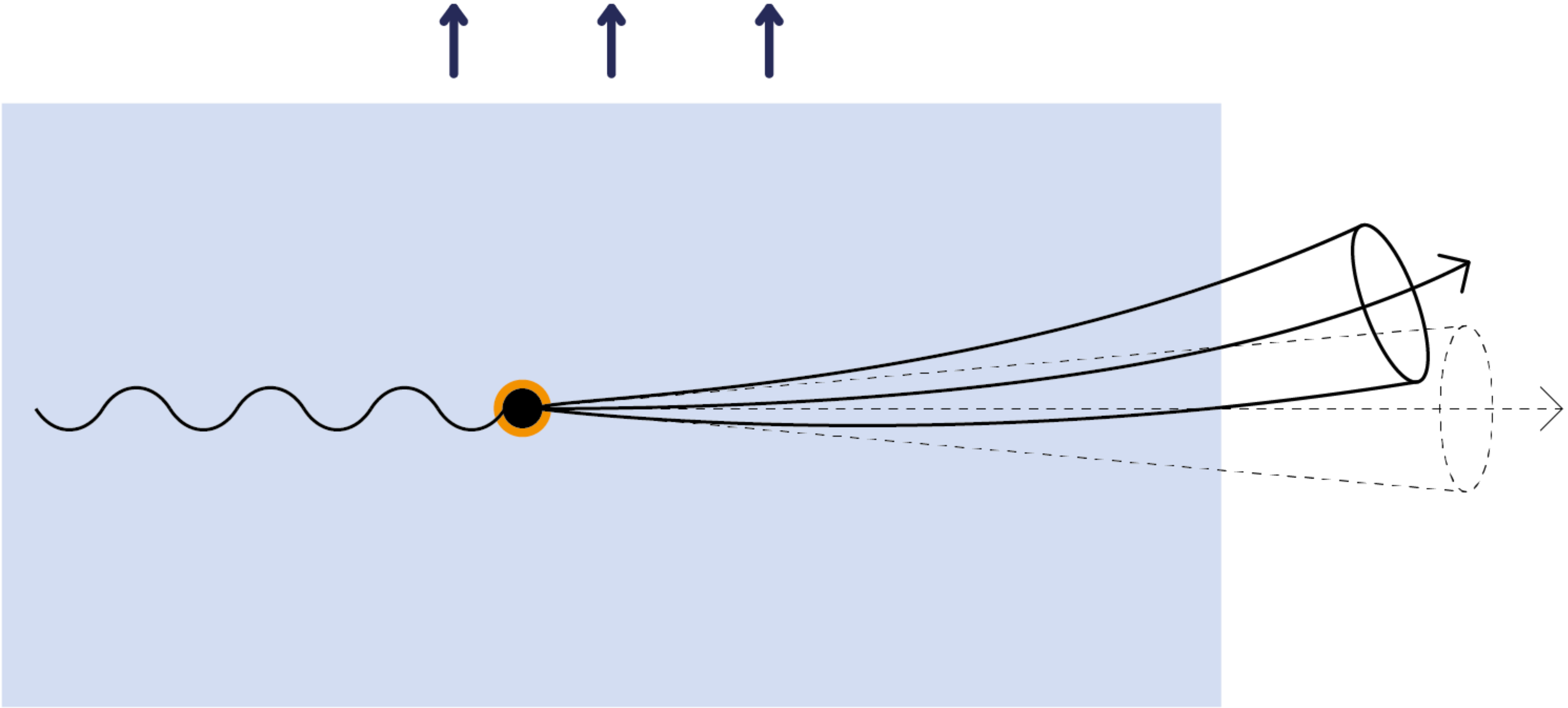
the distribution in energies

$$\langle \boldsymbol{p}_\perp (p_\perp^2)^k\rangle = -\frac{\boldsymbol{u}_\perp}{(1-u_z)}\frac{L}{2\lambda}(\mu^2)^{k+1}\left(-\frac{2}{E}\int_0^\infty d\xi\,\frac{\xi^{k+2}}{(1+\xi)^3} + \frac{1}{f(E)}\frac{\partial f}{\partial E}\int_0^\infty d\xi\,\frac{\xi^{k+1}}{(1+\xi)^2}\right)$$

$1/\rho\sigma_0$ **the mean free path**

$$\left\langle \frac{\vec{p}_\perp}{p_\perp^2}\right\rangle = \frac{5}{2}\frac{\vec{u}_\perp}{(1-u_z)}\frac{L}{\lambda}\frac{1}{E}$$

while even moments are unmodified



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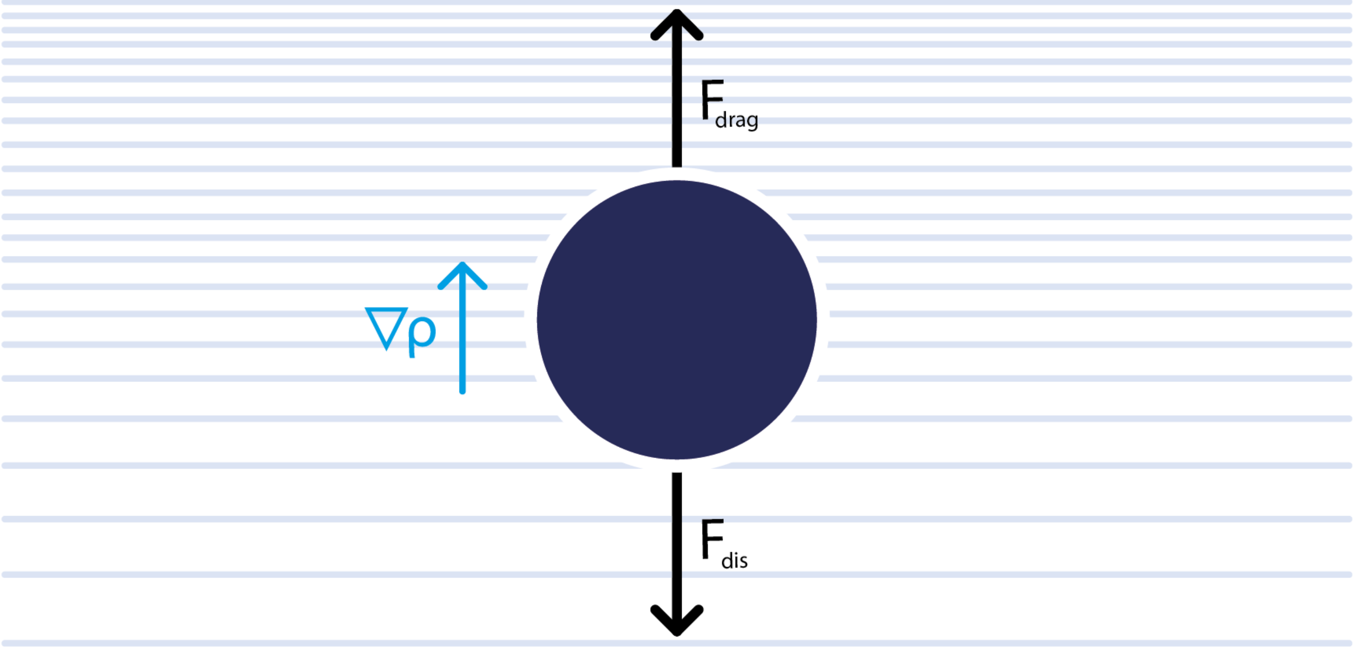


$$\vec{f} \sim \vec{\nabla}\rho$$

Jet Broadening

$$\rho(\vec{x}_\perp, z) \approx \rho_0(z) + \partial^j \rho(z)\, x_\perp^j$$

$$\mu^2(\vec{x}_\perp, z) \approx \mu_0^2(z) + \partial^j \mu^2(z)\, x_\perp^j$$

inhomogeneous matter

$$\sum_i f_i = \int d^3x\, \rho(\vec{x})\, f(\vec{x})$$

$$\int d^2x_\perp e^{-i\left(q_\perp - q'_\perp\right)\cdot x_\perp} = (2\pi)^2 \delta^{(2)}(q_\perp - q'_\perp)$$

$$\int d^2x_\perp\, x_\perp^j e^{-i\left(q_\perp - q'_\perp\right)\cdot x_\perp} = (2\pi)^2\, i \frac{\partial}{\partial(q_\perp - q'_\perp)_j} \delta^{(2)}(q_\perp - q'_\perp)$$

Jet Broadening

gradient expansion at u=0

$$\left(E\frac{dN^{(1)}}{d^3p}\right)^{(\text{linear})} = \int dz \int d^2q_\perp\, \bar{\sigma}(q_\perp^2)\left(\partial^j\rho + \rho\,\frac{1}{\bar{\sigma}(q_\perp^2)}\frac{\partial\bar{\sigma}}{\partial\mu^2}\,\partial^j\mu^2\right)$$
$$\times\left\{\left(E\frac{dN^{(0)}}{d^2(p-q)_\perp\,dE}\right)\left[\frac{(p-q)_\perp^j}{E}z\right] - \left(E\frac{dN^{(0)}}{d^2p_\perp\,dE}\right)\left[\frac{p_\perp^j}{E}z\right]\right\}$$

the leading gradient correction at the first order in opacity

Jet Broadening

gradient expansion

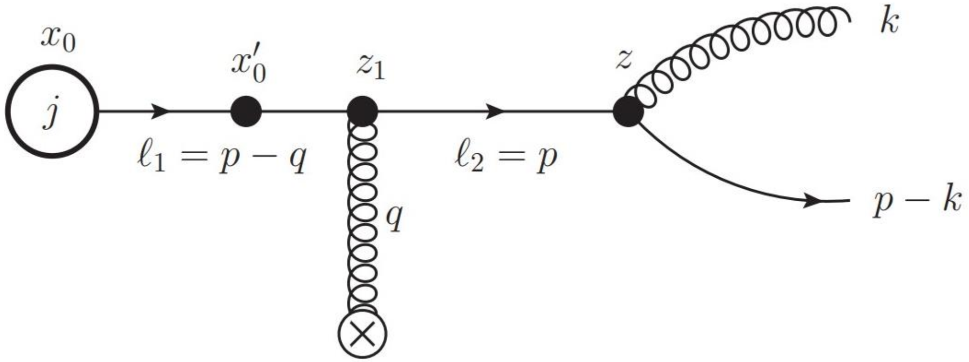
$$E\frac{dN^{(0)}}{d^3p} = \frac{1}{2(2\pi)^3}|J(p)|^2 = \frac{f(E)}{2\pi w^2}\, e^{-\frac{p_\perp^2}{2w^2}}$$

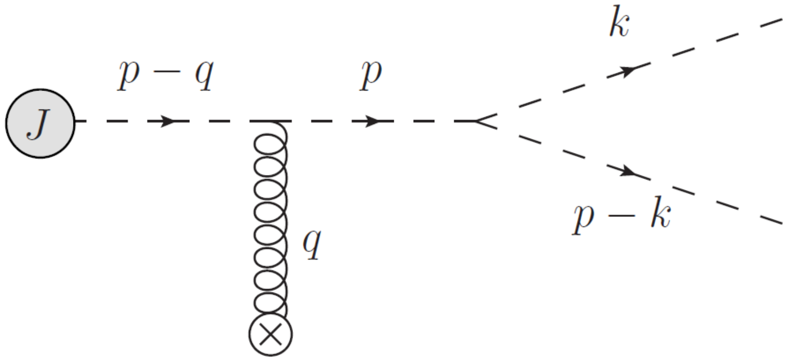
$$\downarrow$$

$$\langle \vec{p}_\perp\, p_\perp^2 \rangle^{(linear)} \simeq \frac{L}{\lambda}\frac{L}{E} w^2 \mu^2 \frac{\vec{\nabla}_\perp \rho}{\rho} \ln\frac{E}{\mu}$$

the leading gradient correction at the first order in opacity

“Gluon” Emission



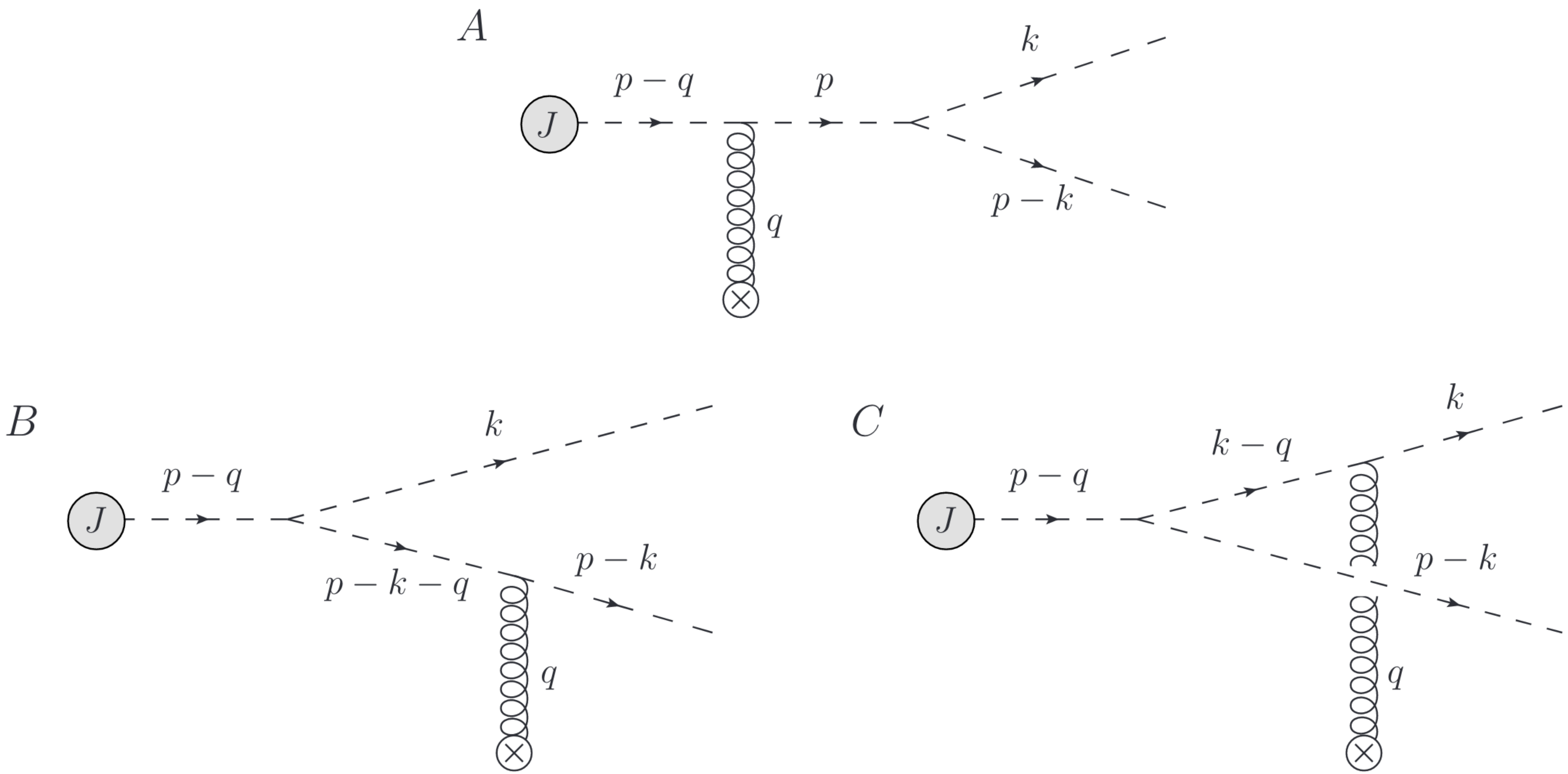


$$\mathcal{L} = \mathcal{L}_{QCD} + g\,\bar{\psi}\,\gamma_\mu A^{\mu\,a}_{ext} t^a\,\psi + \cdots$$

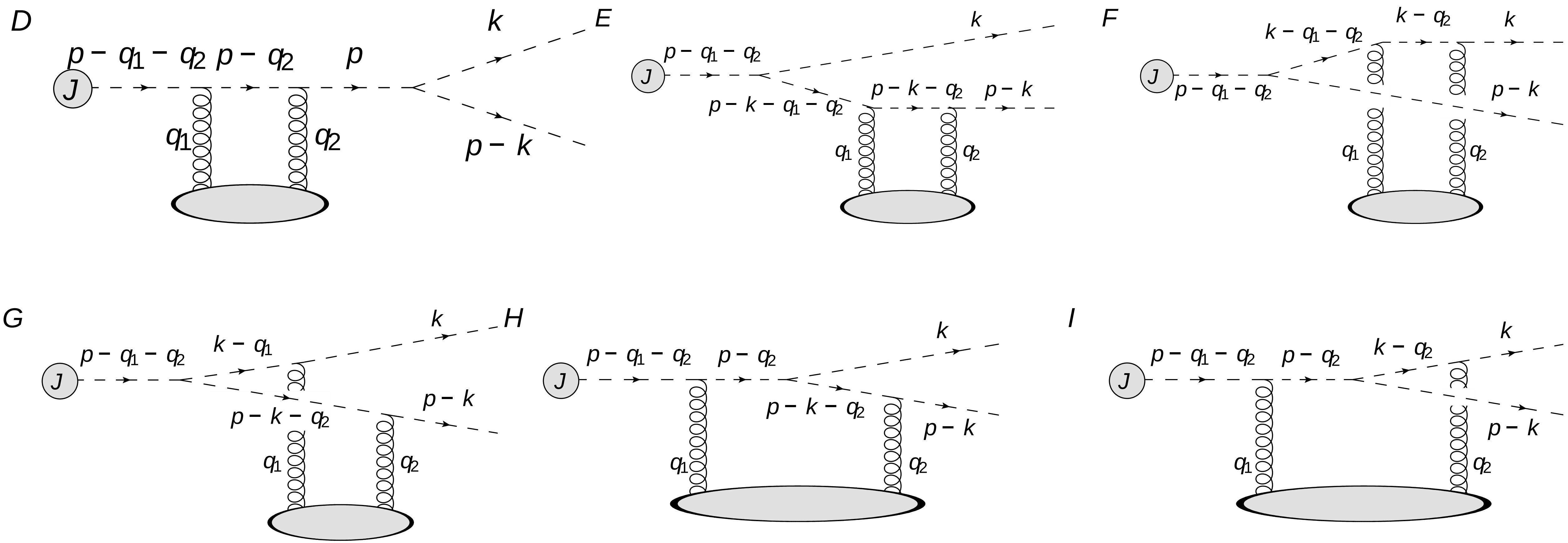
$$\mathcal{L} = \mathcal{L}_{\phi^3} - ig\,(\partial_\mu \phi)\,A^{\mu a}_{ext} t^a\,\phi^c$$

- Interchangeability of light-front wave functions;
- Universality of high-energy scattering;

“Gluon” Emission



“Gluon” Emission



$$
\begin{aligned}
E\frac{dN^{(1)}}{d^2k_\perp\, dx\, d^2p_\perp\, dE} &= \frac{1}{2(2\pi)^3 x(1-x)}\int_0^L dz\,\rho \int d^2q_\perp \frac{d\sigma}{d^2q_\perp}\Bigg\{\left(E\frac{dN^{(0)}}{d^2(p-q)_\perp\, dE}\right) \\
&\times\Bigg[\frac{\mathcal{C}_{(A,A)}}{\mathcal{C}}|\psi_A|^2\left(1+2\vec{u}_\perp\cdot\vec{\Omega}_A\right)+\frac{2\mathcal{C}_{(B,B)}}{\mathcal{C}}|\psi_B|^2\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_{IB}+\vec{\Omega}_{IIB})\right)\left(1-\cos\left((q^-_{p-k-q}-q^-_{p-q})z\right)\right) \\
&+\frac{2\mathcal{C}_{(C,C)}}{\mathcal{C}}|\psi_C|^2\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_{IC}+\vec{\Omega}_{IIC})\right)\left(1-\cos\left((q^-_{k-q}-q^-_{p-q})z\right)\right) \\
&+\frac{2\mathcal{C}_{(A,B)}}{\mathcal{C}}(\psi_A\psi_B^*)\left[\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_A+\vec{\Omega}_{IB})\right)\cos\left((q^-_{p-k-q}-q^-_{p-q})z\right)-\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_A+\vec{\Omega}_{IIB})\right)\right] \\
&+\frac{2\mathcal{C}_{(A,C)}}{\mathcal{C}}(\psi_A\psi_C^*)\left[\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_A+\vec{\Omega}_{IC})\right)\cos\left((q^-_{k-q}-q^-_{p-q})z\right)-\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_A+\vec{\Omega}_{IIC})\right)\right] \\
&+\frac{2\mathcal{C}_{(B,C)}}{\mathcal{C}}(\psi_B\psi_C^*)\Bigg[\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_{IB}+\vec{\Omega}_{IC})\right)\cos\left((q^-_{p-k-q}-q^-_{k-q})z\right)+\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_{IIB}+\vec{\Omega}_{IIC})\right) \\
&-\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_{IB}+\vec{\Omega}_{IIC})\right)\cos\left((q^-_{p-k-q}-q^-_{p-q})z\right)-\left(1+\vec{u}_\perp\cdot(\vec{\Omega}_{IIB}+\vec{\Omega}_{IC})\right)\cos\left((q^-_{k-q}-q^-_{p-q})z\right)\Bigg]\Bigg] \\
&+\left(E\frac{dN^{(0)}}{d^2p_\perp\, dE}\right)\Bigg[-\frac{\mathcal{C}_{(D,0)}}{\mathcal{C}}|\psi_A|^2\cos\left(q^-_p z\right)\left(1+\vec{u}_\perp\cdot\vec{\Gamma}_{DB}\right) \\
&-\frac{\mathcal{C}_{(E,0)}}{\mathcal{C}}|\psi_A|^2\left(1-\cos\left(q^-_p z\right)\right)\left(1+\vec{u}_\perp\cdot\vec{\Gamma}^{(p-k)}_{DB}\right)-\frac{\mathcal{C}_{(F,0)}}{\mathcal{C}}|\psi_A|^2\left(1-\cos\left(q^-_p z\right)\right)\left(1+\vec{u}_\perp\cdot\vec{\Gamma}^{(k)}_{DB}\right) \\
&+\frac{2\mathcal{C}_{(G,0)}}{\mathcal{C}}(\psi_G\psi_A^*)\left(1+\vec{u}_\perp\cdot\vec{\Gamma}_G\right)\left(\cos\left(q^-_p z\right)-\cos\left((q^-_{k+q}+q^-_{p-k-q})z\right)\right)\Bigg]\Bigg\}
\end{aligned}
$$

$$q^-_p \equiv \frac{(k-xp)^2_\perp}{2x(1-x)E(1-u_z)}$$

$$q^-_{p-q} \equiv \frac{(k-xp)^2_\perp}{2x(1-x)E(1-u_z)} - \frac{(p-q)^2_\perp - p^2_\perp}{2E(1-u_z)}$$

$$q^-_{p-k-q} \equiv -\frac{(p-k-q)^2_\perp - (p-k)^2_\perp}{2(1-x)E(1-u_z)}$$

$$q^-_{k-q} \equiv -\frac{(k-q)^2_\perp - k^2_\perp}{2xE(1-u_z)}$$

$$\psi_A \equiv \psi(x, \vec{k}_\perp - x\vec{p}_\perp)$$

$$\psi_B \equiv \psi(x, \vec{k}_\perp - x\vec{p}_\perp + x\vec{q}_\perp)$$

$$\psi_C \equiv \psi(x, \vec{k}_\perp - x\vec{p}_\perp - (1-x)\vec{q}_\perp)$$

$$\psi_G \equiv \psi(x, \vec{k}_\perp - x\vec{p}_\perp + \vec{q}_\perp)$$

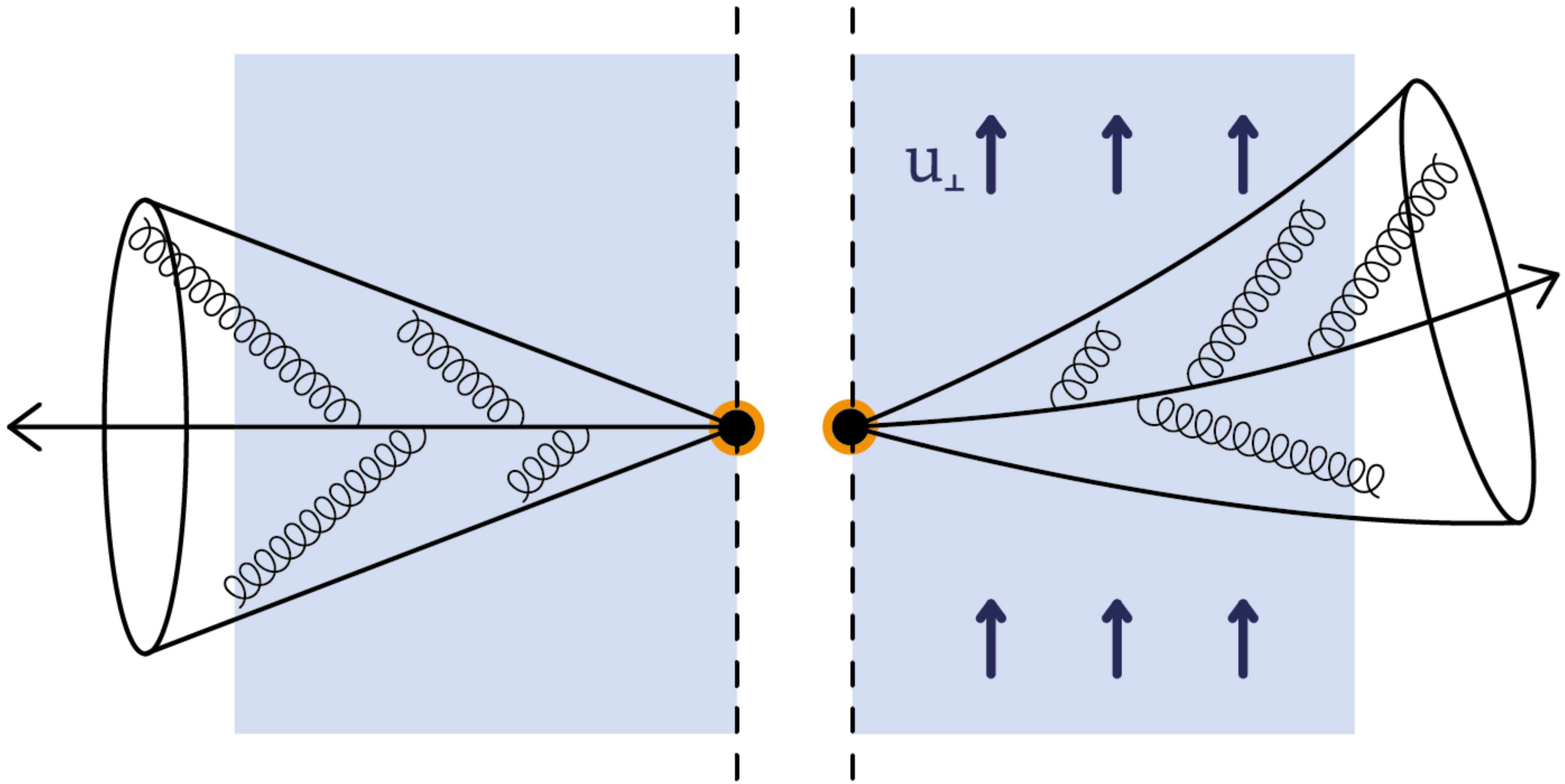
"Gluon" Emission

In the small-x limit, and for broad source, the gluon emission spectrum reduces to

$$E\frac{dN^{(1)}}{d^2k_\perp\,dx\,d^2p_\perp\,dE} = \frac{\alpha_s\,N_c}{\pi^2 x}\left(E\frac{dN^{(0)}}{d^2p_\perp\,dE}\right)\int_0^L dz\,\rho\int d^2q_\perp\bar{\sigma}(q_\perp^2)$$
$$\times\left\{\frac{2\vec{k}_\perp\cdot\vec{q}_\perp}{k_\perp^2(k-q)_\perp^2}\left(1-\cos\left(\frac{(k-q)_\perp^2}{2xE(1-u_z)}z\right)\right)+\frac{q_\perp^2}{k_\perp^2(q_\perp^2+\mu^2)}\frac{\vec{u}_\perp\cdot\vec{k}_\perp}{2(1-u_z)xE}\right\}$$

where the QCD LFWFs were substituted

$$\left\langle\frac{\vec{k}_\perp}{k_\perp^2}\right\rangle = \frac{N_c}{C_F}\frac{L}{\lambda}\frac{\vec{u}_\perp}{8(1-u_z)xE}$$



$$\left\langle \frac{\vec{k}_\perp}{k_\perp^2} \right\rangle = \frac{N_c}{C_F}\frac{L}{\lambda}\frac{\vec{u}_\perp}{8(1-u_z)xE}$$

Summary

- We have constructed a generalization of the GLV approach which includes the medium motion effects. With this tool one can study general flow, temperature, and source density profiles in the context of HIC;

- It is shown that the odd moments of the jet momentum are modified by the medium motion, and the jet is bended by the flow and gradients;

- We have derived the “gluon” emission spectrum in the case of uniformly flowing matter. The resulting answer, while obtained for scalar interactions, can be compared to the QCD results when written in the terms of LFWFs;

- In the context of DIS our formalism can be used to study nucleon orbital motion and spatial inhomogeneities in the system (a relation to GPDs and TMDs?);

- These results open multiple opportunities to include the medium motion and in-medium fluctuation effects into studies of other hard probes of nuclear matter;