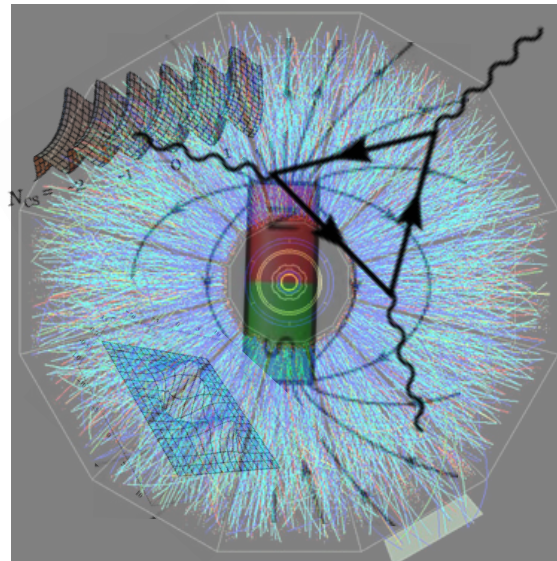


Novel Effects of Rotational Polarization in Relativistic Nuclear Collisions

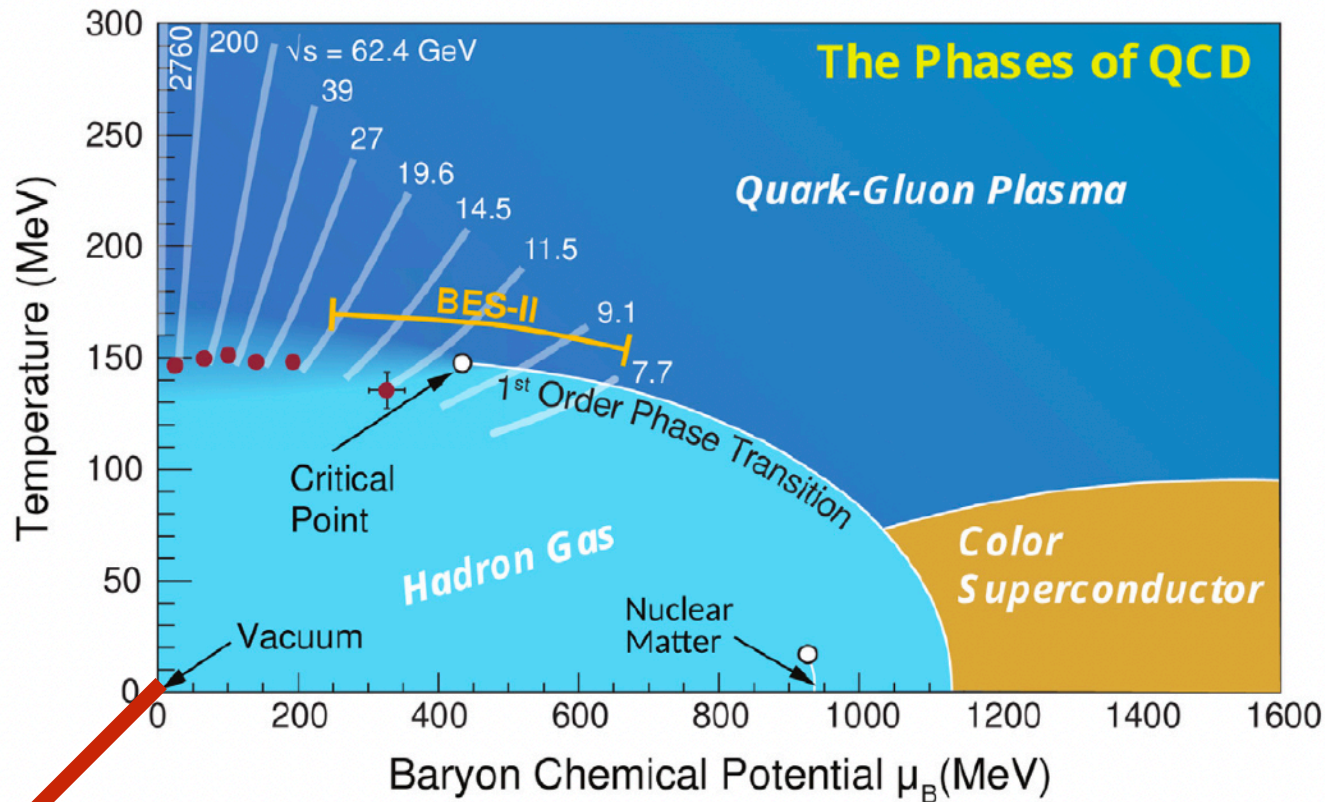


Jinfeng Liao

Indiana University, Physics Dept. & CEEM



Beam Energy Scan & High Baryon Density Region



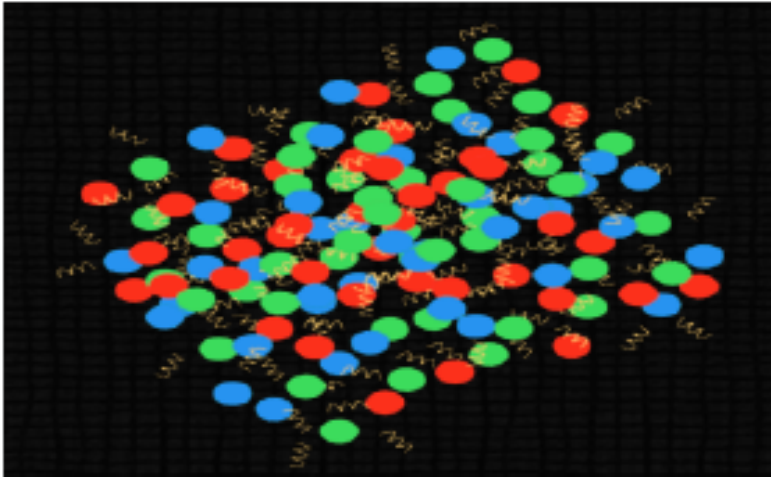
***A wonderful BES “bonus” :
the most vortical fluid!***

***[Recall Columbus’s discovery of
the American continents]***

$\vec{\omega}$

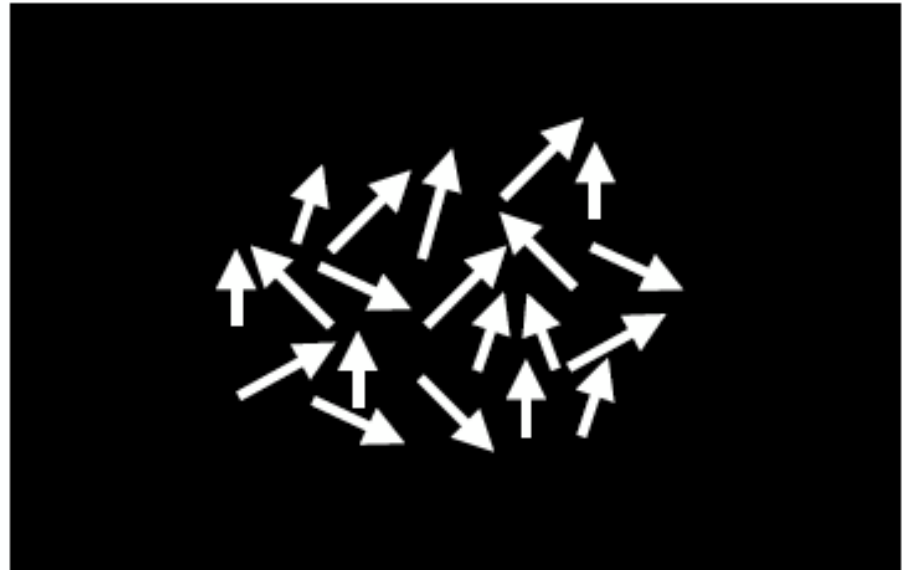
A New “Handle”: Rotation

*A nearly perfect fluid
(of energy-momentum)*



*We now have a new external
control on strong interaction
matter: angular momentum!*

*What happens to the spin
DoF in the fluid???*



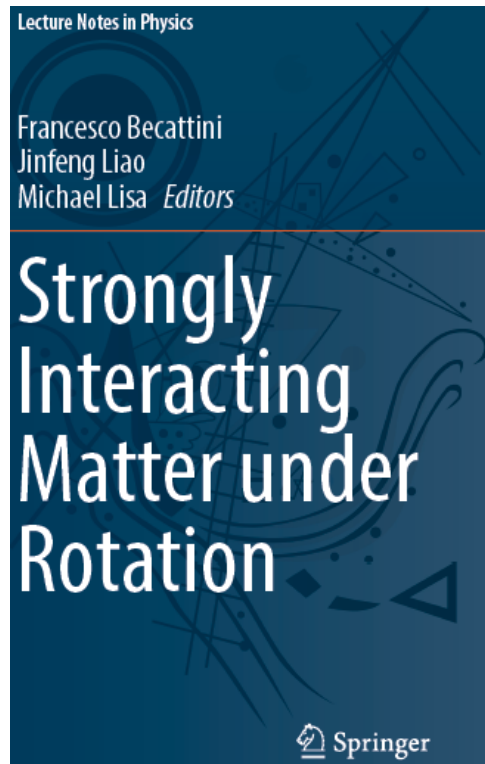
*Angular momentum transport, to
be imaged by spin polarization.*

Strongly Interacting Matter under Rotation

Opening doors for a whole new array of interesting studies:

- *Phase structure change? Equation of state change?*
- *Global and local polarization? Vector mesons?*
- *Spin transport theory? Spin hydrodynamics?*
- *Novel transport processes?*

–



*A recent volume in Springer
Lecture Notes in Physics!*

**Strongly Interacting Matter Under
Rotation: An Introduction**

1

Francesco Becattini, Jinfeng Liao and Michael Lisa

Abstract

Ultrarelativistic collisions between heavy nuclei briefly generate the Quark–Gluon Plasma (QGP), a new state of matter characterized by deconfined partons last seen microseconds after the Big Bang. The properties of the QGP are of intense interest, and a large community has developed over several decades, to produce, measure, and understand this primordial plasma. The plasma is now recognized to be a strongly coupled fluid with remarkable properties, and hydrodynamics is commonly used to quantify and model the system. An important feature of any fluid is its vorticity, related to the local angular momentum density; however, this degree of freedom has received relatively little attention because no experimental signals of vorticity had been detected. Thanks to recent high-statistics datasets from experiments with precision tracking and complete kinetic coverage at collider energies, hyperon spin polarization measurements have begun to uncover the vorticity of the QGP created at the Relativistic Heavy Ion Collider. The injection of this new degree of freedom into a relatively mature field of research represents an enormous opportunity to generate new insights into the physics of the QGP. The community has responded with enthusiasm, and this book represents some of the diverse lines of inquiry into aspects of strongly interacting matter under rotation.

[arXiv:2102.00933; 2010.08937; 2009.04803; 2101.04963; 2004.04050;
2011.09974; 1908.10244; 2007.04029; 2001.00359; 2108.00586; ...]

Spin & Rotational Polarization


Dirac Lagrangian in rotating frame:

$$g_{\mu\nu} = \begin{pmatrix} 1 - \vec{v}^2 & -v_1 & -v_2 & -v_3 \\ -v_1 & -1 & 0 & 0 \\ -v_2 & 0 & -1 & 0 \\ -v_3 & 0 & 0 & -1 \end{pmatrix}$$

$$\vec{v} = \vec{\omega} \times \vec{x}.$$

$$\bar{\gamma}^\mu = e_a^\mu \gamma^a$$

$$\Gamma_\mu = \frac{1}{4} \times \frac{1}{2} [\gamma^a, \gamma^b] \Gamma_{ab\mu}$$



$$\mathcal{L} = \bar{\psi} [i\bar{\gamma}^\mu (\partial_\mu + \Gamma_\mu) - m] \psi$$

Under slow rotation:

$$\mathcal{L} = \psi^\dagger \left[i\partial_0 + i\gamma^0 \vec{\gamma} \cdot \vec{\partial} + (\vec{\omega} \times \vec{x}) \cdot (-i\vec{\partial}) + \vec{\omega} \cdot \vec{S}_{4 \times 4} \right] \psi$$


$$\hat{H} = \gamma^0 (\vec{\gamma} \cdot \vec{p} + m) - \vec{\omega} \cdot (\vec{x} \times \vec{p} + \vec{S}_{4 \times 4}) = \hat{H}_0 - \vec{\omega} \cdot \hat{\vec{J}}$$

Rotational polarization effect!

Spin & Rotational Polarization

Eigenstates of Dirac Hamiltonian in rotating frame:

$$\hat{H} = \gamma^0(\vec{\gamma} \cdot \vec{p} + m) - \vec{\omega} \cdot (\vec{x} \times \vec{p} + \vec{S}_{4 \times 4}) = \hat{H}_0 - \vec{\omega} \cdot \hat{J}$$

 \hat{H} , \hat{p}_z , \hat{p}_t^2 , \hat{J}_z , and $\hat{h}_t \equiv \gamma^5 \gamma^3 \vec{p}_t \cdot \vec{S}_{4 \times 4}$

$$u_{k_z, k_t, n, s} = \sqrt{\frac{E_k + m}{4E_k}} e^{ik_z z} e^{in\theta} \begin{pmatrix} J_n(k_t r) \\ se^{i\theta} J_{n+1}(k_t r) \\ \frac{k_z - isk_t}{E_k + m} J_n(k_t r) \\ \frac{-sk_z + ik_t}{E_k + m} e^{i\theta} J_{n+1}(k_t r) \end{pmatrix},$$

$$E_k \equiv \sqrt{k_z^2 + k_t^2 + m^2}$$

$$E = \pm E_k - (n + 1/2)\omega$$

$$v_{k_z, k_t, n, s} = \sqrt{\frac{E_k + m}{4E_k}} e^{-ik_z z} e^{in\theta} \begin{pmatrix} \frac{k_z - isk_t}{E_k + m} J_n(k_t r) \\ \frac{sk_z - ik_t}{E_k + m} e^{i\theta} J_{n+1}(k_t r) \\ J_n(k_t r) \\ -se^{i\theta} J_{n+1}(k_t r) \end{pmatrix},$$

**Rotational
polarization
energy**

[Yin Jiang, JL, PRL2016]

Rotational Polarization in Thermal System

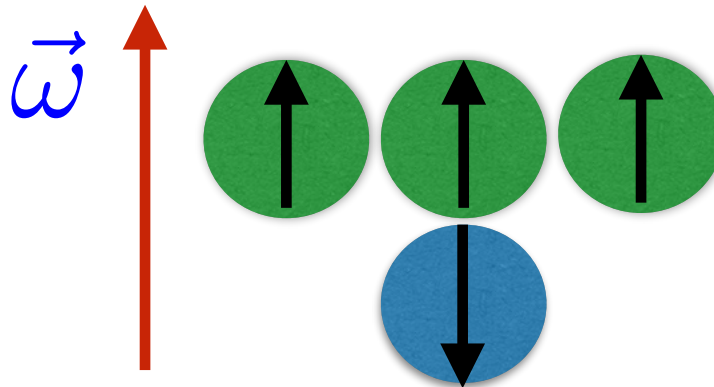
$$\hat{H} = \gamma^0(\vec{\gamma} \cdot \vec{p} + m) - \vec{\omega} \cdot (\vec{x} \times \vec{p} + \vec{S}_{4 \times 4}) = \hat{H}_0 - \vec{\omega} \cdot \hat{J}$$

**Rotational
polarization effect!**



**For thermally produced particles:
“equal-partition” of angular momentum**

$$dN \propto e^{\frac{\vec{\omega} \cdot \vec{J}}{T}}$$



Rotational Polarization in Condensed Matter

Spin hydrodynamic generation

R. Takahashi , M. Matsuo, M. Ono, K. Harii, H. Chudo, S. Okayasu, J. Ieda, S. Takahashi, S. Maekawa & E. Saitoh 

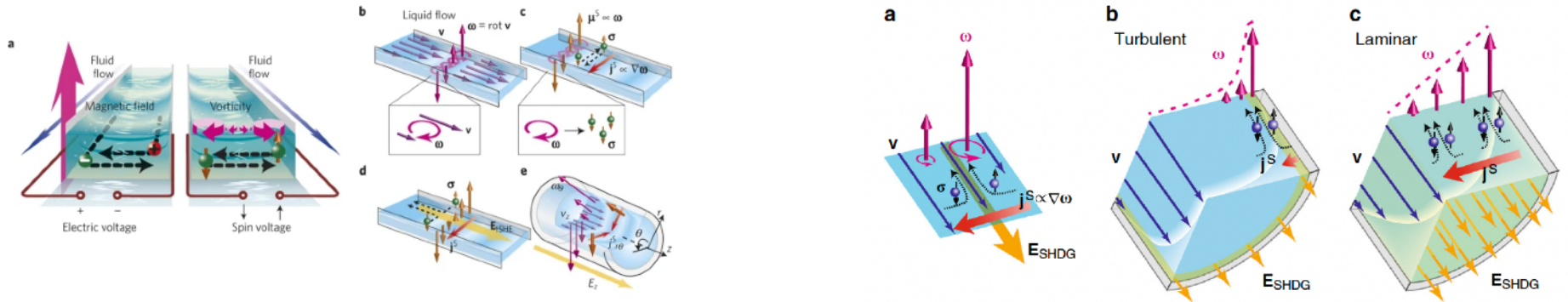
Nature Physics **12**, 52–56(2016) | [Cite this article](#)

**Viscous fluid flow
—> vorticity —>
spin polarization**

Giant spin hydrodynamic generation in laminar flow

R. Takahashi , H. Chudo, M. Matsuo, K. Harii, Y. Ohnuma, S. Maekawa & E. Saitoh

Nature Communications **11**, Article number: 3009 (2020) | [Cite this article](#)

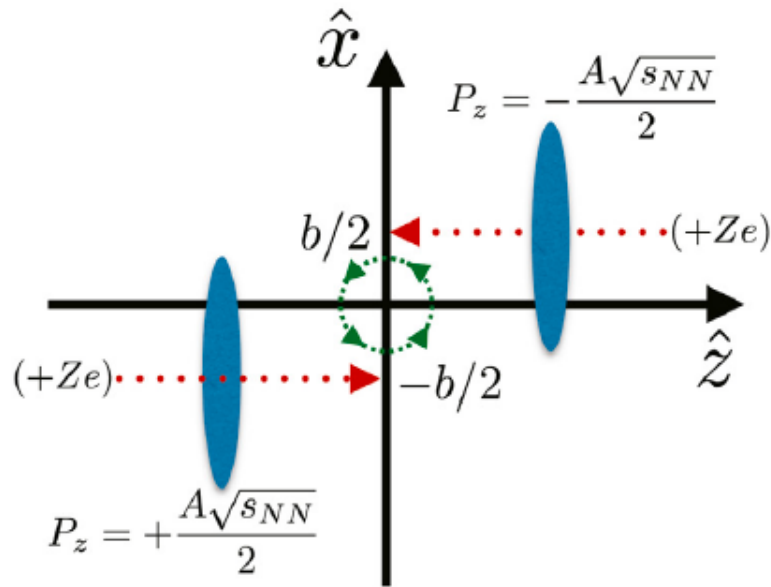


**“Fluid Spintronics”:
Based on spin-fluid-vorticity coupling**

Plan for the Rest of the Talk

- *Phenomenology of rotational polarization in heavy ion collisions: global polarization of hyperons*
- *Nontrivial effects of rotational polarization on the phase structures of matter*
- *Theoretical understanding of angular momentum in hydrodynamic framework*

Angular Momentum in Heavy Ion Collisions



Huge angular momentum for the system in non-central collisions at high energy

$$L_y = \frac{Ab\sqrt{s}}{2} \sim 10^{4\sim 5} \hbar$$

**Liang & Wang ~ 2005: spin-orbital coupling
orbital $L \rightarrow$ spin polarization via partonic collision processes**

Becattini, et al ~ 2008, 2013: A fluid dynamical scenario

$$S^\mu(p) = -\frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma d\Sigma \cdot p \varpi_{\nu\rho} n_F (1 - n_F)}{\int_\Sigma d\Sigma \cdot p n_F} \quad \varpi_{\mu\nu} = \frac{1}{2} \left[\partial_\nu \left(\frac{1}{T} u_\mu \right) - \partial_\mu \left(\frac{1}{T} u_\nu \right) \right]$$

“Rotating” Quark-Gluon Plasma

$$L_y = \frac{Ab\sqrt{s}}{2} \sim 10^{4\sim 5} \hbar$$

PHYSICAL REVIEW C 94, 044910 (2016)

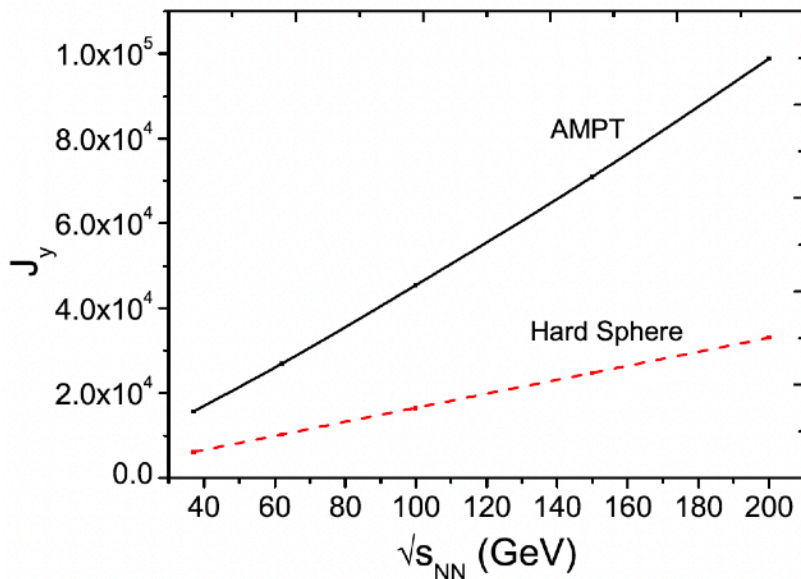
Rotating quark-gluon plasma in relativistic heavy-ion collisions

Yin Jiang,¹ Zi-Wei Lin,² and Jinfeng Liao^{1,3}

¹Physics Department and Center for Exploration of Energy and Matter, Indiana University, 2401 North Milo B. Sampson Lane, Bloomington, Indiana 47408, USA

²Department of Physics, East Carolina University, Greenville, North Carolina 27858, USA

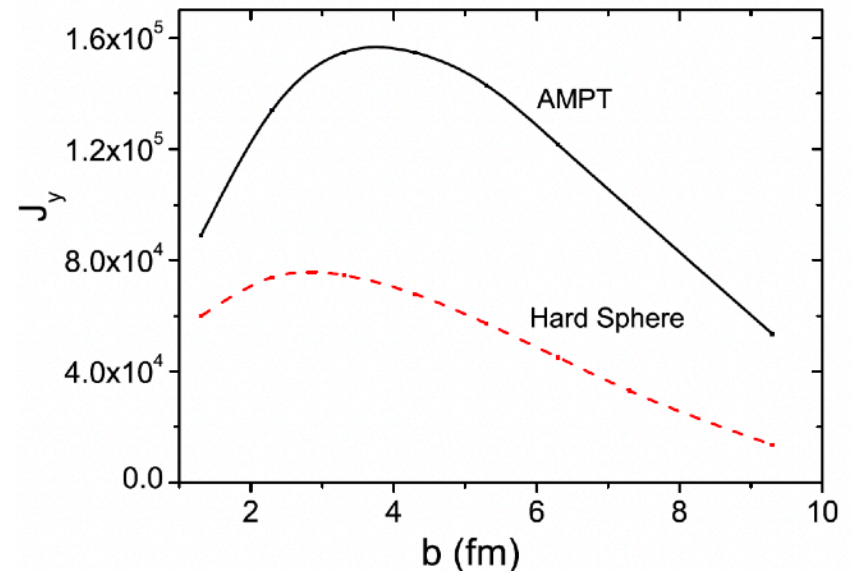
³RIKEN BNL Research Center, Building 510A, Brookhaven National Laboratory, Upton, New York 11973, USA



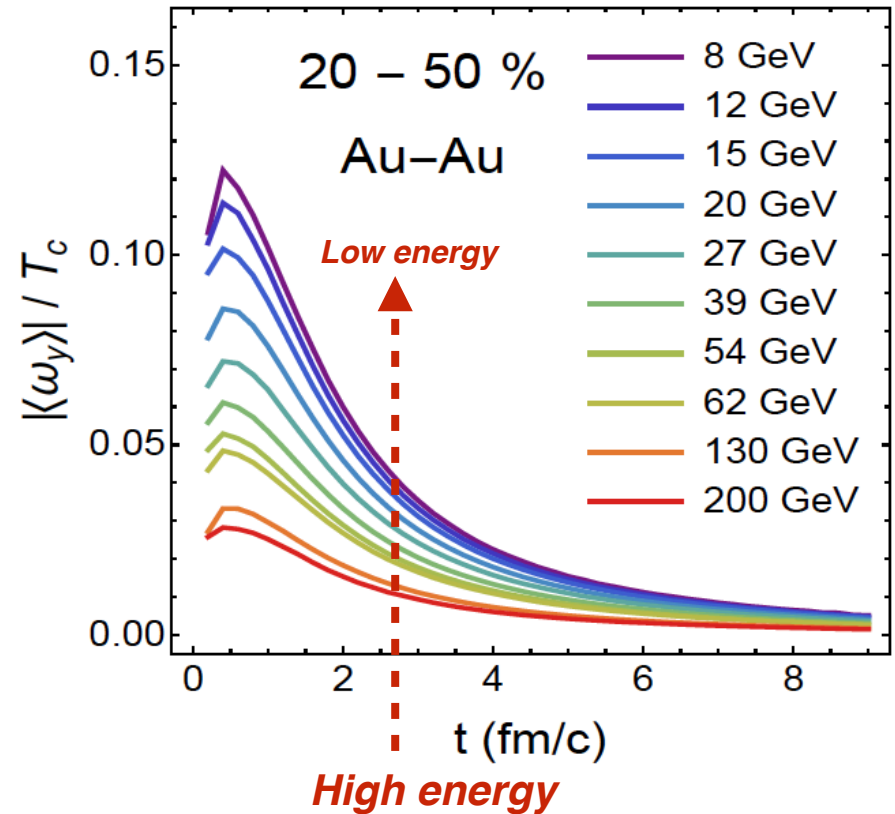
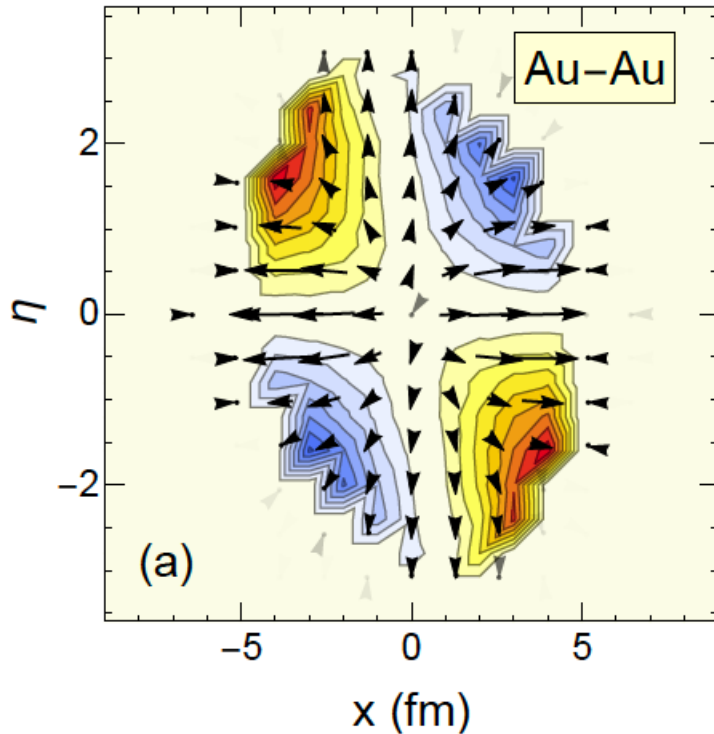
What fraction stays in QGP?
– up to ~20%, depending on collision energy.

Is this portion conserved?
– YES!

How QGP accommodates this angular momentum?
– Fluid vorticity!



Nontrivial Vorticity Structures



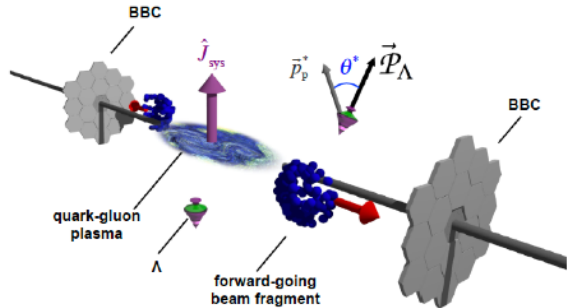
Sizable vorticity in QGP, due to the global orbital angular momentum.

It could be manifested via spin-fluid coupling.

**Jiang, Lin, JL, PRC2016;
Deng, Huang, PRC2016;**

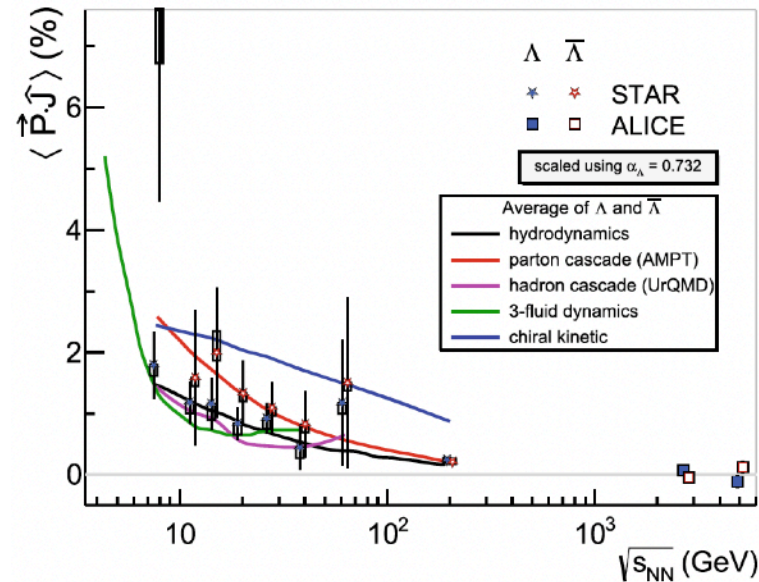
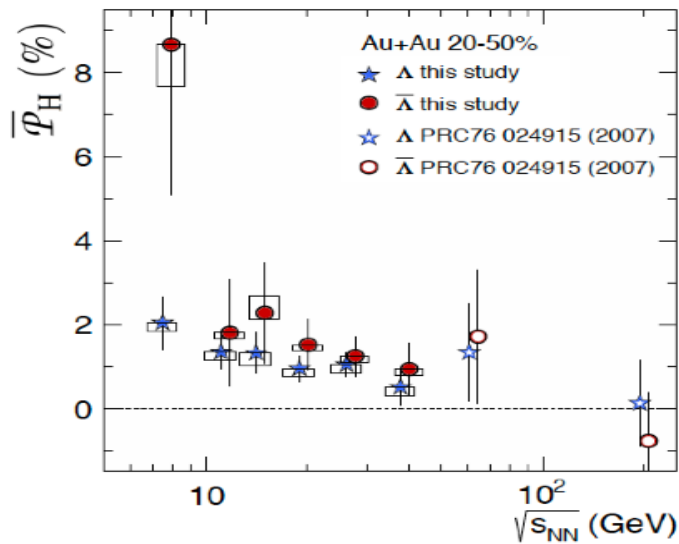
**Vorticity
@ $O(10)$ GeV
>>
Vorticity
@ $O(100)$ GeV**

Spin Polarization in the Subatomic Swirls



**STAR Collaboration,
Nature 2017**

The most vortical fluid!



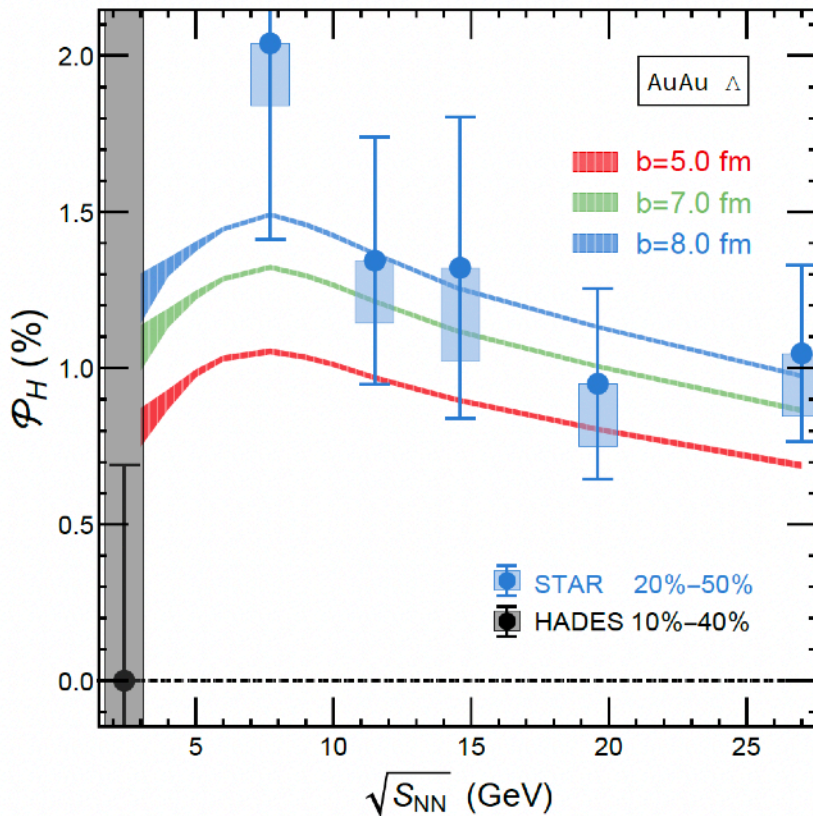
$$\omega \approx (9 \pm 1) \times 10^{21} \text{ s}^{-1}$$

**Many calculations based on
hydro or transport models**

**Many new developments since then,
see e.g. various talks at this conference**

Trend of Global Polarization toward $O(1)$ GeV

The Question: Trend for global hyperon polarization @ $O(1\sim 10)$ GeV ???



*Yu Guo, et al, PRC2021
arXiv:2105.13481*

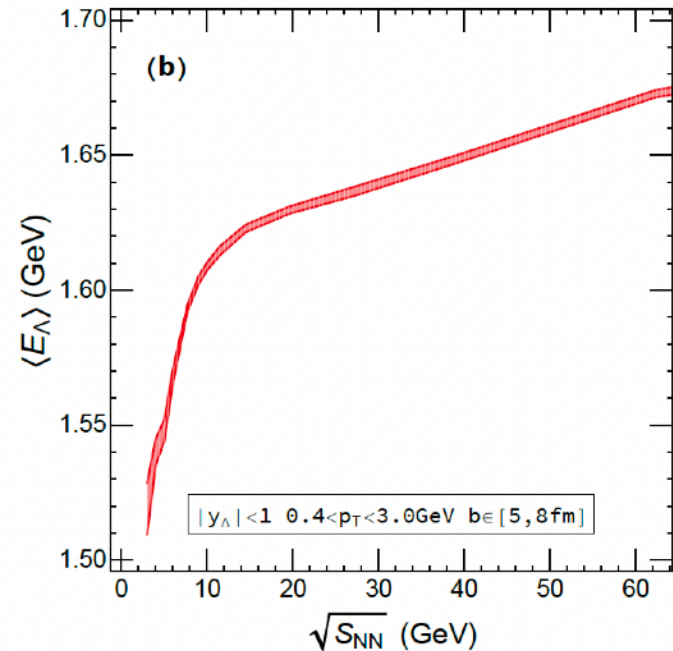
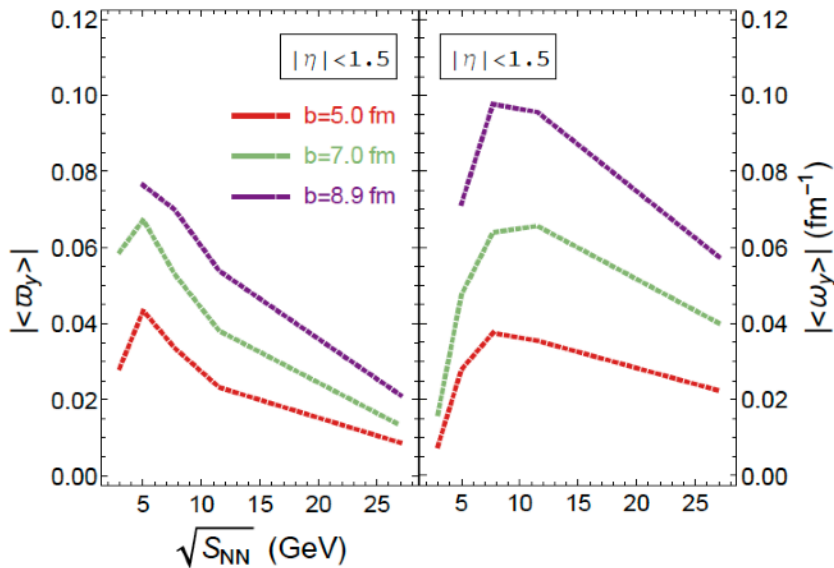
*AMPT calculations predict non-monotonic behavior in the dependence of global polarization on beam energy
—> maximum around 7.7 GeV*

See also results for differential dependence and local polarization in the paper.

Understanding the Trend

“Thermal model” formula for spin d.o.f.

$$S^\mu(p) = -\frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma d\Sigma \cdot p \varpi_{\nu\rho} n_F (1 - n_F)}{\int_\Sigma d\Sigma \cdot p n_F} \quad \varpi_{\mu\nu} = \frac{1}{2} \left[\partial_\nu \left(\frac{1}{T} u_\mu \right) - \partial_\mu \left(\frac{1}{T} u_\nu \right) \right]$$

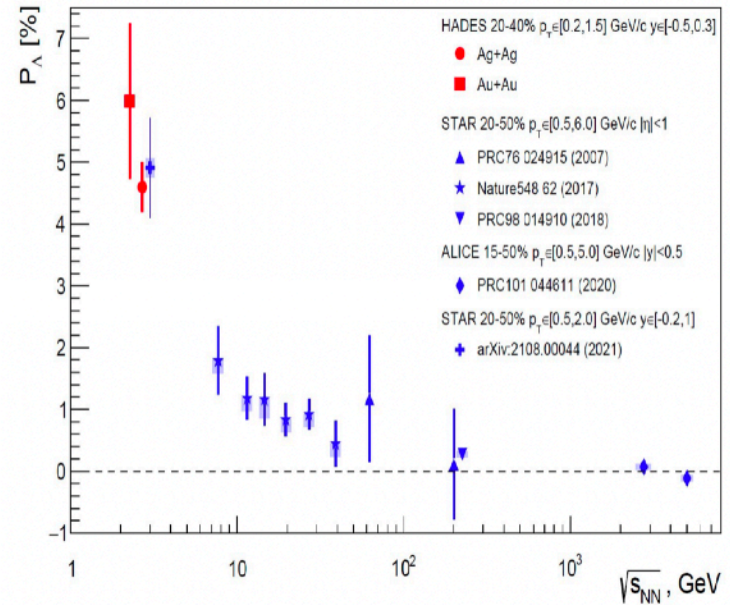
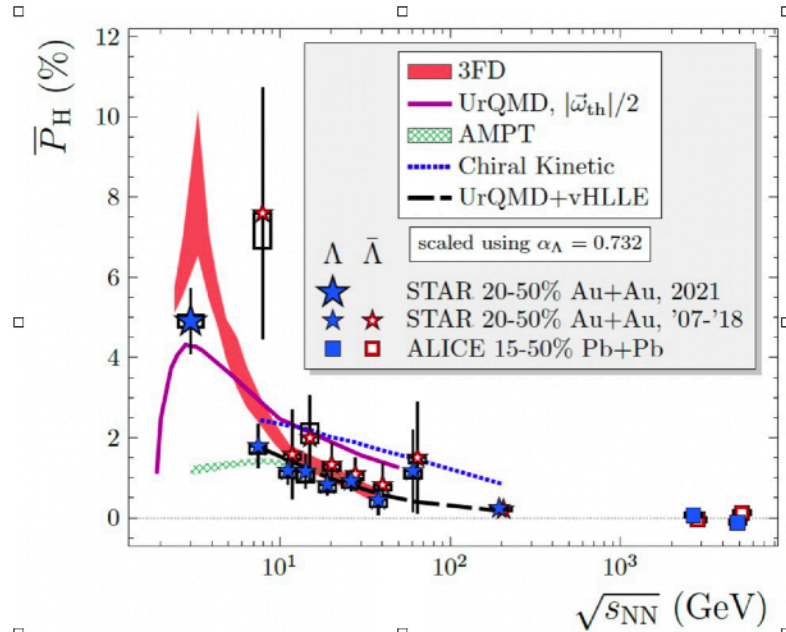


The decrease of polarization toward O(1) GeV region is due to the decrease in both vorticity and the produced hyperon energy.

STAR & HADES New Results

STAR, arXiv:2108.00044

Kornas @ Chirality 2021



Surprisingly large signal at 2.4~3 GeV ...

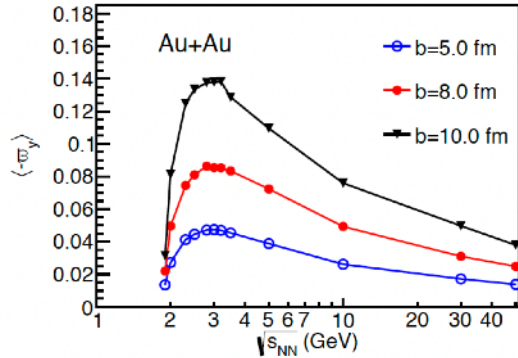
Recall that this regime is VERY close to the threshold:

$$L_y = \frac{1}{2} Ab \sqrt{s} \sqrt{1 - (2M/\sqrt{s})^2}$$

Digesting the New Data

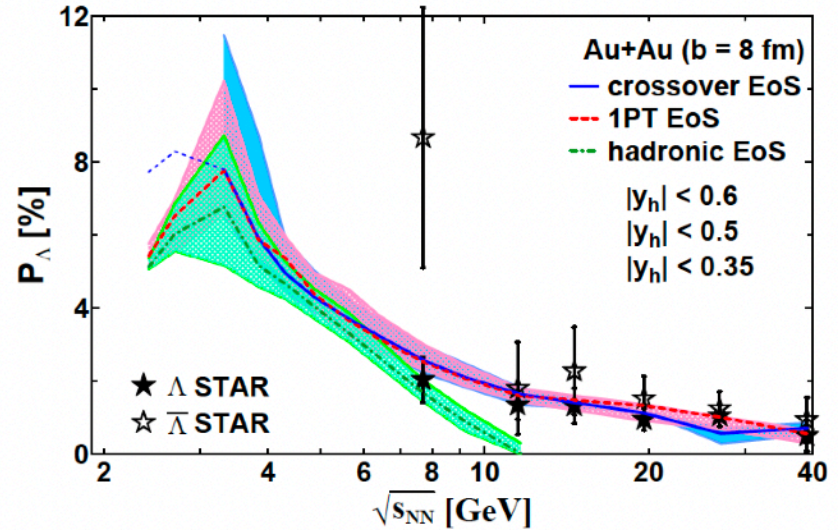
URQMD

Deng, Huang, Ma, Zhang,
arXiv:2001.01371



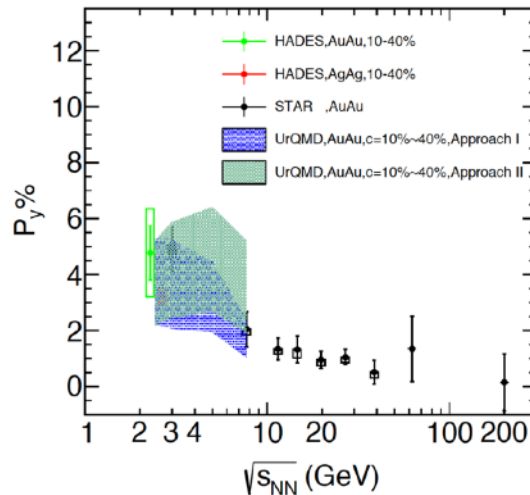
3FD

Ivanov,
arXiv:2012.07597



– URQMD & AMPT
results for vorticity
are similar.

Deng, Huang, Ma,
arXiv:2109.09956



– Many model details could be quite different and need to be understood
– Likely having more spectator-participant interactions and angular momentum transport

Digesting the New Data

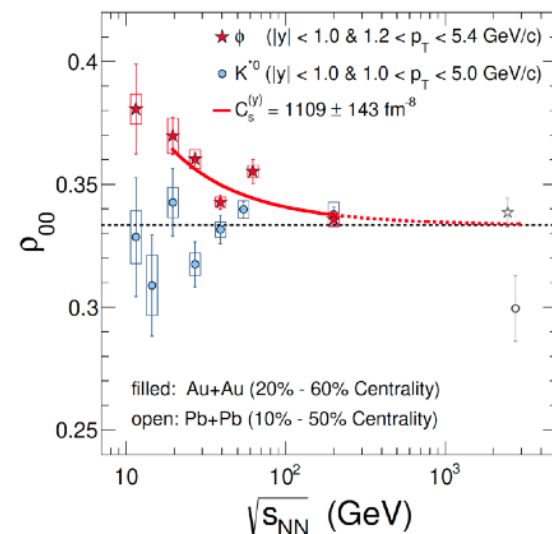
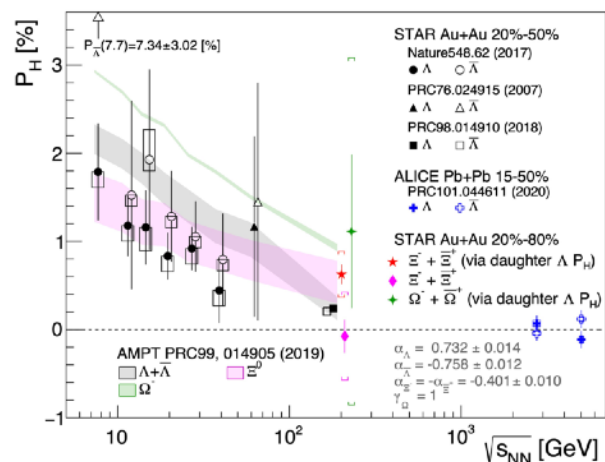
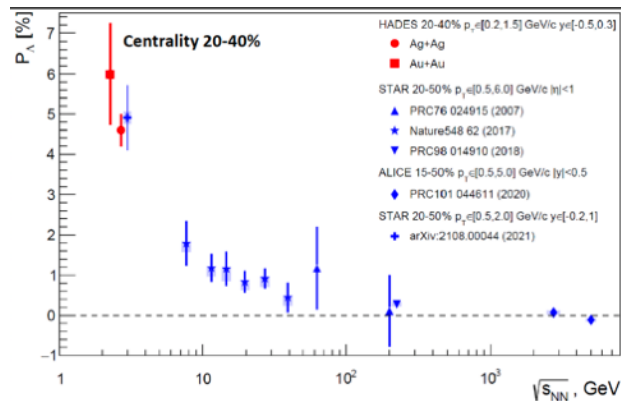
What can go “wrong” in calculating the hyperon global polarization?

$$S^\mu(p) = -\frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_\Sigma d\Sigma \cdot p \varpi_{\nu\rho} n_F (1 - n_F)}{\int_\Sigma d\Sigma \cdot p n_F}$$

- ***complete failure of fluid-vorticity-polarization scenario***
- ***substantial out-of-equilibrium correction***
- ***inaccurate bulk fluid property
(e.g. thermal vorticity)***
- ***particle production mechanism
(e.g. hadronic versus partonic)***

***The low AMPT signal could indicate:
A shift of dominance in hyperon production from
partonic coalescence to direct hadronic reaction.***

Exciting New Regime @ Low Energy: Highly Polarized Strong Interaction Matter



Going from $O(100)\text{GeV}$ down to $O(1)\text{GeV}$:

— the disappearance of partonic collectivity in a nearly perfect QGP fluid

— strong increase of fireball angular momentum, in opposite trend to J_y or J_y/y_{beam}

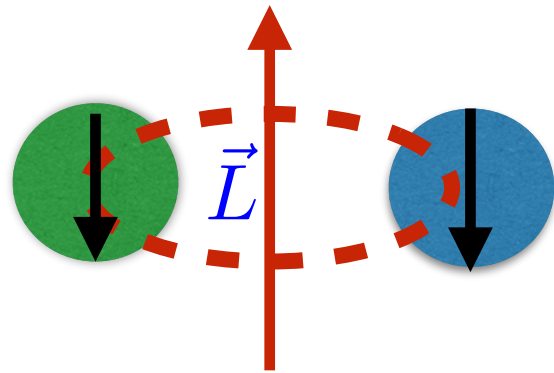
— maybe the appearance of spin collectivity in a high-polarization strong interaction fluid

Phase Structures under Rotation

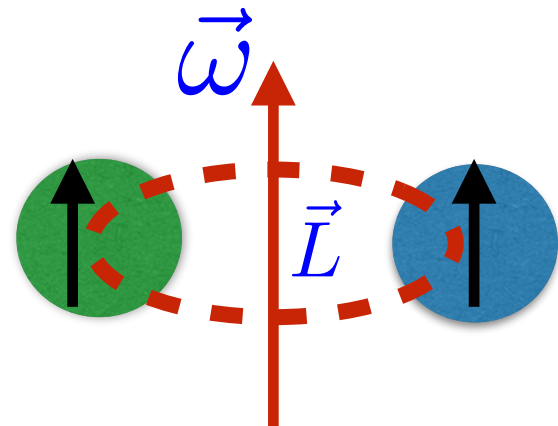
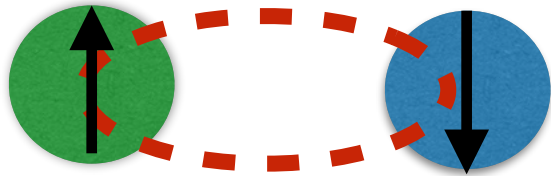
*Let us consider pairing phenomenon in fermion systems.
There are many examples:
superconductivity, superfluidity, chiral condensate, diquark, ...*

First consider scalar pairing state, with $J=0$.

$$\vec{S} = \vec{s}_1 + \vec{s}_2 \quad \vec{J} = \vec{L} + \vec{S}$$

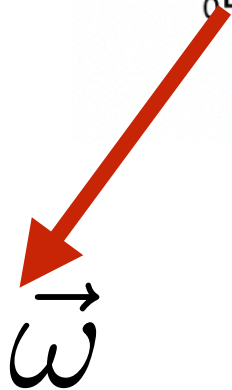
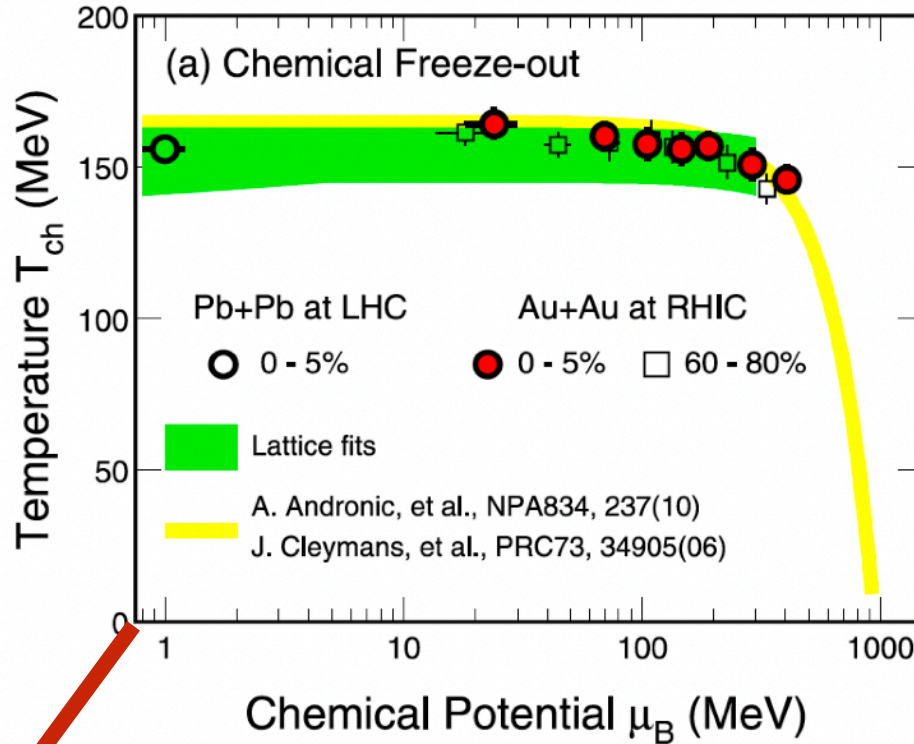


Rotation tends to polarize ALL angular momentum, both L and S, thus suppressing scalar pairing, E.g. chiral condensate and scalar diquark condensate

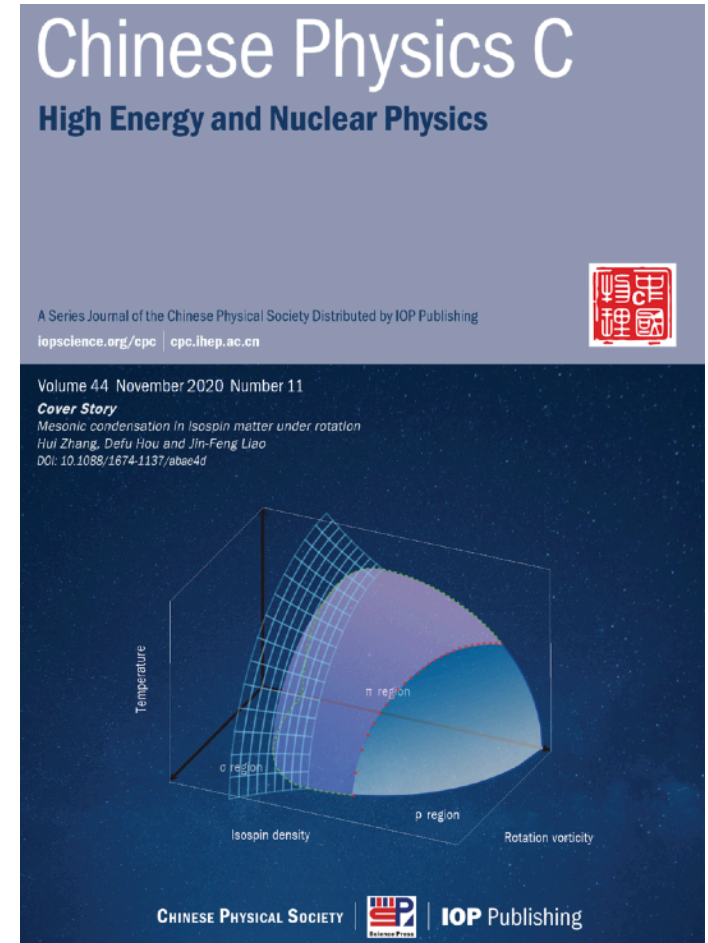


[Yin Jiang, JL, PRL2016]

Isospin Matter under Rotation



Finite isospin density for both low energy collisions and neutron stars



[Hui Zhang, Defu Hou, JL, CPC44(2020)11,111001]

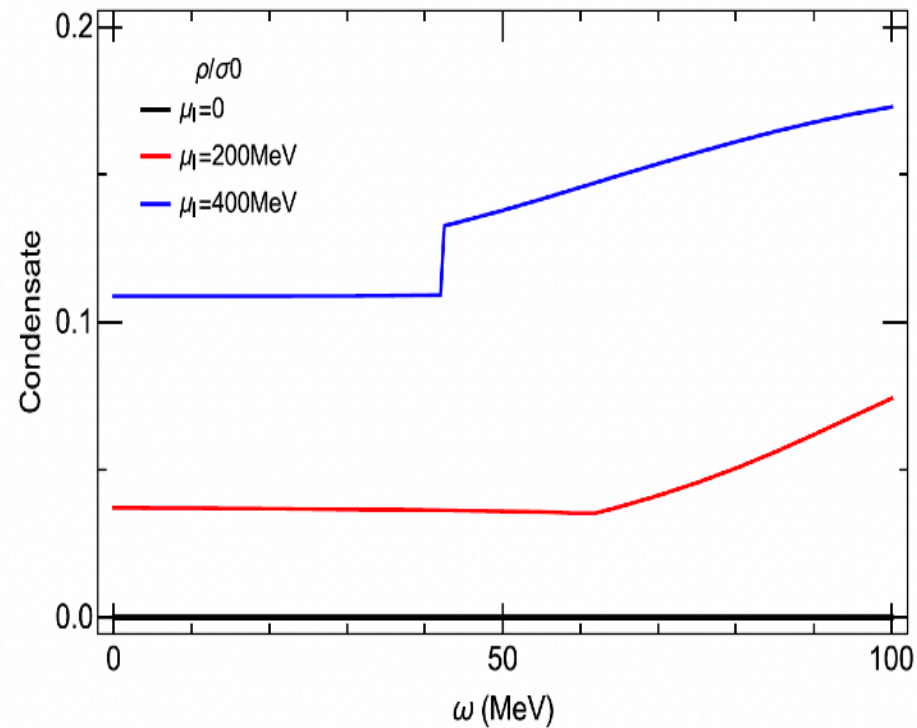
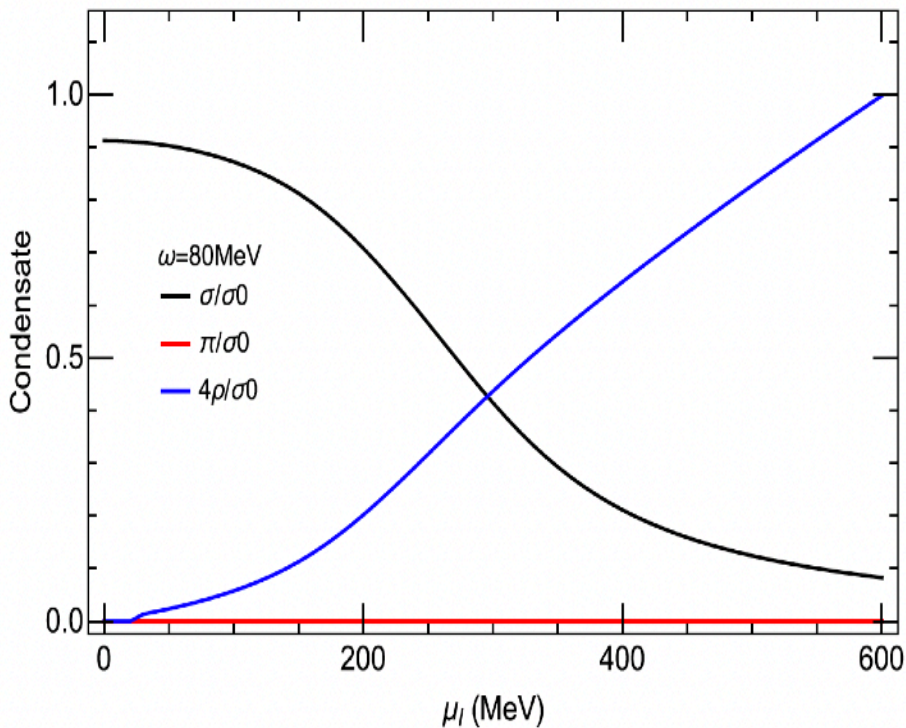
Isospin Matter under Rotation

Vacuum: sigma condensate;

Static isospin matter: pion superfluidity;

Isospin matter under rotation: emergence of rho condensate!

This effect is more significant at high baryon density.



[Hui Zhang, Defu Hou, JL,
CPC44(2020)11,111001]

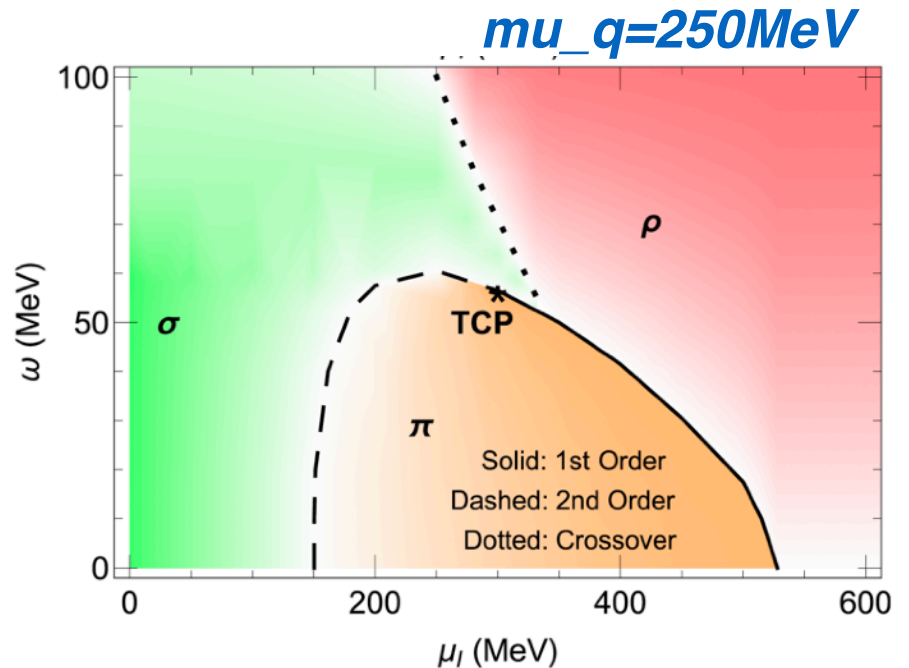
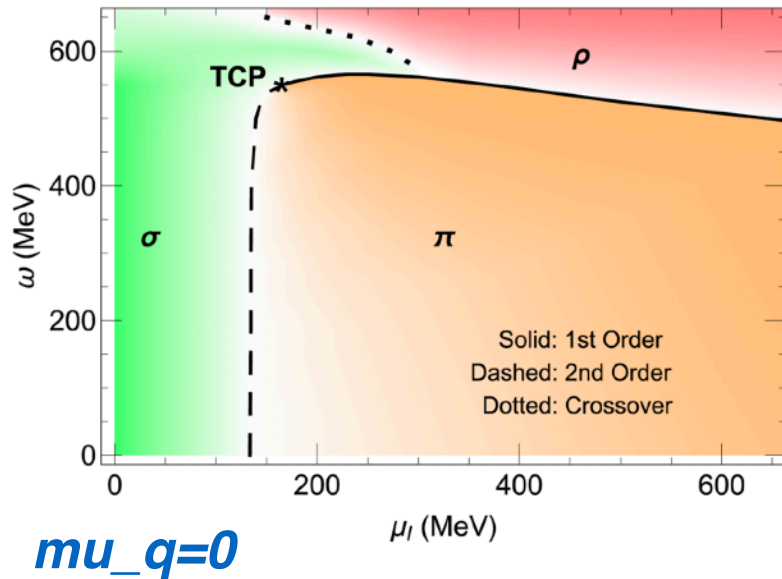
Isospin Matter under Rotation

Vacuum: sigma condensate;

Static isospin matter: pion superfluidity;

Isospin matter under rotation: emergence of rho condensate!

This effect is more significant at high baryon density.



Rich phase structures of isospin matter under rotation;

Relevant to low energy HIC or neutron star matter;

Implications for particle polarization / dileptons?!

[Hui Zhang, Defu Hou, JL, CPC44(2020)11,111001]

Relativistic Viscous Hydro with Ang. Mom.

Relativistic Viscous Hydrodynamics with Angular Momentum

Duan She,^{1,2} Anping Huang,^{2,3} Defu Hou,^{1,*} and Jinfeng Liao^{2,†}

¹*Key Laboratory of Quark and Lepton Physics(MOE) and Institute of Particle Physics, central China Normal University, Wuhan, 430079, China.*

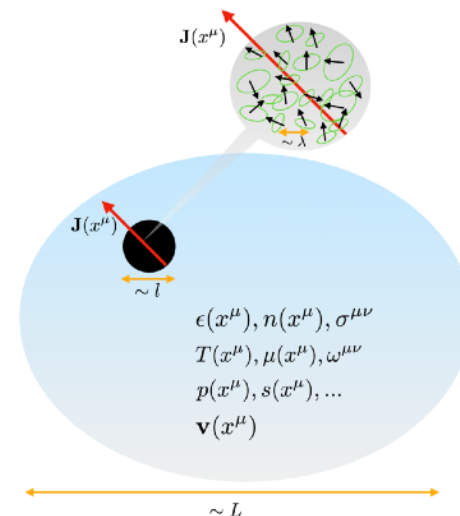
²*Physics Department and Center for Exploration of Energy and Matter, Indiana University, 2401 N Milo B. Sampson Lane, Bloomington, IN 47408, USA.*

³*School of Nuclear Science and Technology, University of Chinese Academy of Sciences, Beijing 100049, China.*

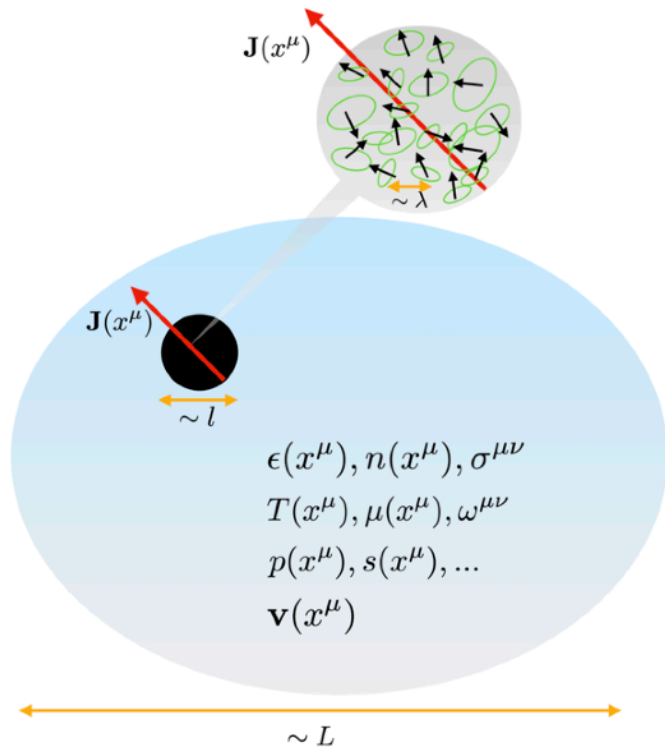
(Dated: February 25, 2022)

Hydrodynamics is a general theoretical framework for describing the long-time large-distance behaviors of various macroscopic physical systems, with its equations based on conservation laws such as energy-momentum conservation and charge conservation. Recently there has been significant interest in understanding the implications of angular momentum conservation for a corresponding hydrodynamic theory. In this work, we examine the key conceptual issues for such a theory in the relativistic regime where the orbital and spin components get entangled. We derive the equations for relativistic viscous hydrodynamics with angular momentum through Navier-Stokes type of gradient expansion analysis and find five new transport coefficients for angular momentum diffusion modes.

[\[arXiv:2105.04060\]](https://arxiv.org/abs/2105.04060)



Goal: Navier-Stokes Program for Ang. Mom.



$\lambda \ll l \ll L$, a coarse-graining process

$$\partial_\mu T^{\mu\nu} = 0,$$

$$\partial_\mu N^\mu = 0,$$

$$\partial_\mu J^{\mu\alpha\beta} = 0,$$

$$J^{\mu\alpha\beta} = (x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha}) + \Sigma^{\mu\alpha\beta}.$$

We choose to deal with only the conserved quantities, i.e. angular momentum.

Local angular momentum current

$$\Sigma^{\mu\alpha\beta}$$

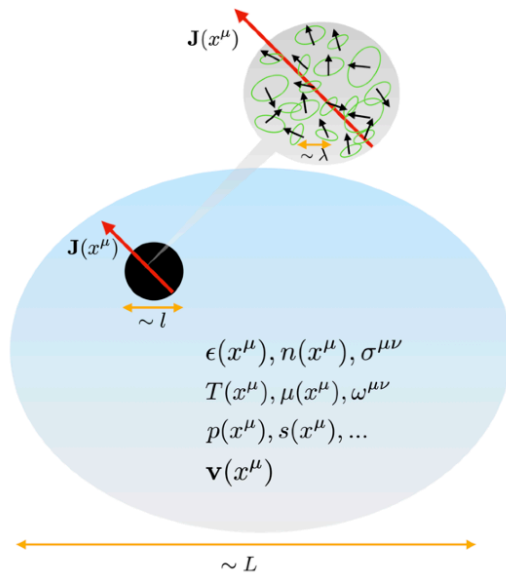
Local angular momentum density

$$\sigma^{\alpha\beta}(x^\mu)$$

Local angular momentum chemical potential

$$\omega_{\alpha\beta}(x^\mu)$$

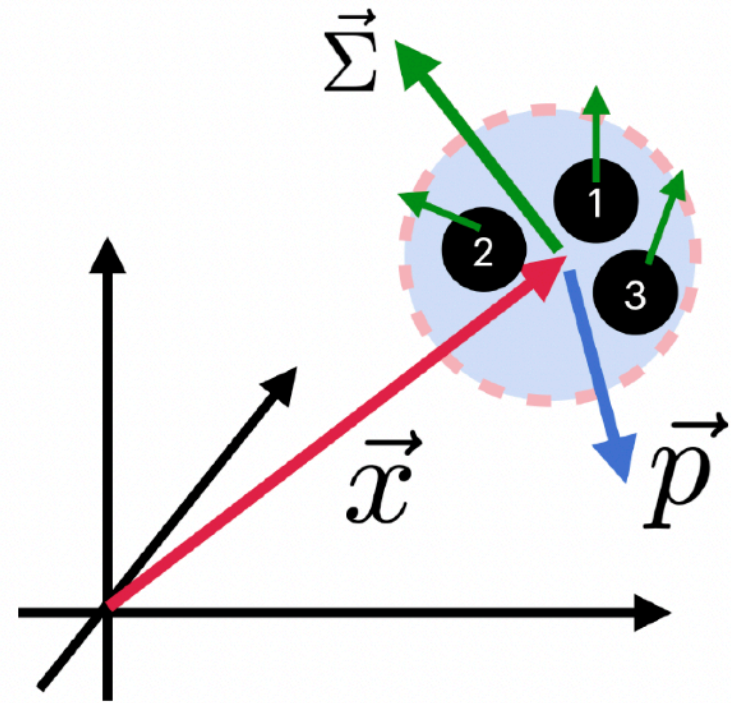
Decomposition of Fluid Cell Angular Momentum



$$J^{\mu\alpha\beta} = (x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha}) + \Sigma^{\mu\alpha\beta}.$$

$\lambda \ll l \ll L$, a coarse-graining process

Conceptually, it may NOT be feasible to further separate the spin part out of the local angular momentum of a coarse-grained fluid cell.



$$\vec{\mathbf{J}} = \vec{\mathbf{x}} \times \vec{\mathbf{p}} + \sum_{i=1,2,3} \left(\vec{\mathbf{x}}'_i \times \vec{\mathbf{p}}'_i + \vec{\mathbf{s}}_i \right).$$

$$\vec{\Sigma} = \sum_{i=1,2,3} \left(\vec{\mathbf{x}}'_i \times \vec{\mathbf{p}}'_i + \vec{\mathbf{s}}_i \right)$$

Viscous Hydro with Ang. Mom.

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - p \Delta^{\mu\nu} + \tilde{T}^{\mu\nu},$$

$$N^\mu = n u^\mu + \tilde{N}^\mu,$$

$$\Sigma^{\mu\alpha\beta} = u^\mu \sigma^{\alpha\beta} + \tilde{\Sigma}^{\mu\alpha\beta},$$

$$S^\mu = s u^\mu + \tilde{S}^\mu.$$

Let us first focus on entropy current:

$$S^\mu = p \beta^\mu + \beta_\nu T^{\mu\nu} - \alpha N^\mu - \beta \omega_{\alpha\beta} \Sigma^{\mu\alpha\beta}$$

Leading order:

$$\partial_\mu S_{(0)}^\mu = 2\beta \omega_{\alpha\beta} \tilde{T}_{(a)}^{\alpha\beta}. \quad \rightarrow \quad \tilde{T}_{(a)}^{\alpha\beta} = 0.$$

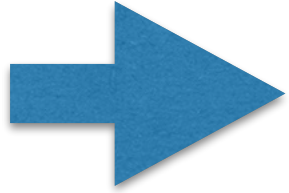
Next order (2nd-gradient):

$$\begin{aligned} \partial_\mu S^\mu &= \partial_\mu (p \beta^\mu + \beta_\nu T^{\mu\nu} - \alpha N^\mu - \beta \omega_{\alpha\beta} \Sigma^{\mu\alpha\beta}) \\ &= \tilde{T}^{\mu\nu} \partial_\mu \beta_\nu - \tilde{N}^\mu \partial_\mu \alpha - \tilde{\Sigma}^{\mu\alpha\beta} \partial_\mu (\beta \omega_{\alpha\beta}), \end{aligned}$$

$$\partial_\mu S^\mu \geq 0.$$

Viscous Hydro with Ang. Mom.: Eckart Frame

$$u_E^\mu = \frac{N^\mu}{\sqrt{N^\nu N_\nu}},$$



Write down all allowed Lorentz structures to the correct order of gradient expansion

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (p + \Pi) \Delta^{\mu\nu} + 2u^{(\mu} q^{\nu)} + \pi^{\mu\nu},$$

$$N^\mu = n u^\mu,$$

$$\begin{aligned} \Sigma^{\mu\alpha\beta} = u^\mu \sigma^{\alpha\beta} + 2u^{[\alpha} \Delta^{\mu\beta]} \Phi \\ + 2u^{[\alpha} \tau_{(s)}^{\mu\beta]} + 2u^{[\alpha} \tau_{(a)}^{\mu\beta]} + \Theta^{\mu\alpha\beta}. \end{aligned}$$

Plug these into the entropy current divergence and look for conditions of positivity:

$$\begin{aligned} \partial_\mu S^\mu &= \partial_\mu (p\beta^\mu + \beta_\nu T^{\mu\nu} - \alpha N^\mu - \beta\omega_{\alpha\beta} \Sigma^{\mu\alpha\beta}) \\ &= \tilde{T}^{\mu\nu} \partial_\mu \beta_\nu - \tilde{N}^\mu \partial_\mu \alpha - \tilde{\Sigma}^{\mu\alpha\beta} \partial_\mu (\beta\omega_{\alpha\beta}), \end{aligned}$$

$$\partial_\mu S^\mu \geq 0.$$

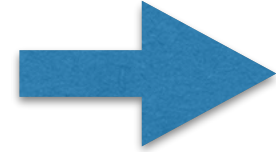
Viscous Hydro with Ang. Mom.: Eckart Frame

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (p + \Pi) \Delta^{\mu\nu} + 2u^{(\mu} q^{\nu)} + \pi^{\mu\nu},$$

$$N^\mu = n u^\mu,$$

$$\Sigma^{\mu\alpha\beta} = u^\mu \sigma^{\alpha\beta} + 2u^{[\alpha} \Delta^{\mu\beta]} \Phi + 2u^{[\alpha} \tau_{(s)}^{\mu\beta]} + 2u^{[\alpha} \tau_{(a)}^{\mu\beta]} + \Theta^{\mu\alpha\beta}.$$

$$\partial_\mu S^\mu \geq 0.$$



$$\Pi = -\zeta\theta,$$

$$\pi^{\mu\nu} = 2\eta \nabla^{(\mu} u^{\nu)},$$

$$q^\mu = \lambda T \left(\frac{\nabla^\mu T}{T} - D u^\mu \right)$$

$$= -\frac{\lambda n T^2}{\epsilon + p} \left[\nabla^\mu \left(\frac{\mu}{T} \right) + \frac{\sigma^{\alpha\beta}}{n} \nabla^\mu \left(\frac{\omega_{\alpha\beta}}{T} \right) \right]$$

Five new positive angular momentum transport coefficients:

$\chi_1, \chi_2, \chi_3, \chi_4$ and χ_5

$$\Phi = -\chi_1 u^\alpha \nabla^\beta \left(\frac{\omega_{\alpha\beta}}{T} \right), \quad (25)$$

$$\tau_{(s)}^{\mu\beta} = -\chi_2 u^\alpha \left[(\Delta^{\beta\rho} \Delta^{\mu\gamma} + \Delta^{\mu\rho} \Delta^{\beta\gamma}) - \frac{2}{3} \Delta^{\mu\beta} g^{\rho\gamma} \right] \nabla_\gamma \left(\frac{\omega_{\alpha\rho}}{T} \right) \quad (26)$$

$$\tau_{(a)}^{\mu\beta} = -\chi_3 u^\alpha (\Delta^{\beta\rho} \Delta^{\mu\gamma} - \Delta^{\mu\rho} \Delta^{\beta\gamma}) \nabla_\gamma \left(\frac{\omega_{\alpha\rho}}{T} \right), \quad (27)$$

$$\Theta^{\mu\alpha\beta} = -\chi_4 (u^\beta u^\rho \Delta^{\alpha\delta} - u^\alpha u^\rho \Delta^{\beta\delta}) \Delta^{\mu\gamma} \nabla_\gamma \left(\frac{\omega_{\delta\rho}}{T} \right) + \chi_5 \Delta^{\alpha\delta} \Delta^{\beta\rho} \Delta^{\mu\gamma} \nabla_\gamma \left(\frac{\omega_{\delta\rho}}{T} \right). \quad (28)$$

Summary

- *A new regime of study for strong interaction matter:
Large angular momentum as external control;*
- *Nontrivial fluid vorticity structures and induced global and local polarization phenomena, especially at low energy;*
- *New and rich phase structures under rotation;*
- *Development of hydrodynamic theoretical framework with angular momentum;*
- *Many more interesting questions to be fully explored!*

BACKUP SLIDES

A Long Story: Barnett Effect

SEPTEMBER 24, 1909]

SCIENCE

413

Lehrbuch der Kristalloptik, by E. B. Wilson; "Notes"; "New Publications."

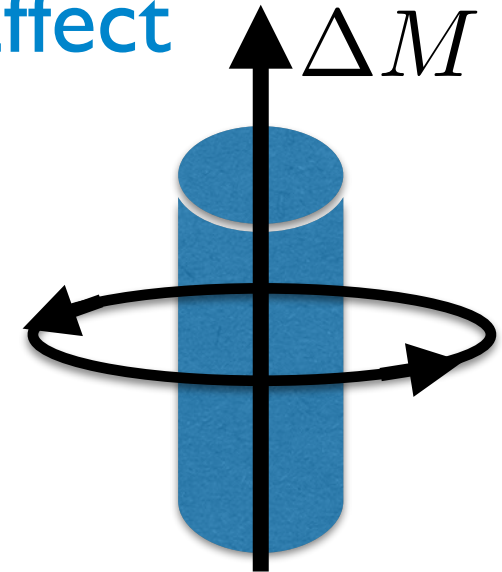
SPECIAL ARTICLES

ON MAGNETIZATION BY ANGULAR ACCELERATION

Some time ago, while thinking about the origin of the earth's magnetism, it occurred to me that any magnetic substance must, according to current theory, become magnetized by receiving an angular velocity.

Thus consider a cylinder of iron or other substance constituted of atomic or molecular systems whose individual magnetic moments

fectly definite and unquestionable, but exceedingly difficult to account for, viz., a magnetization along the rod in a definite direction independent of the direction of rotation and of the direction of the original residual magnetism of the rod. It was not due to the jarring of the cylinder as it was rotated in the earth's field, nor to a possible minute change in the direction of its axis produced by the pull of the motor. In magnitude this effect was several times as great as the other, which became manifest only at the higher of the two speeds used.



Second Series.

October, 1915

Vol. VI., No. 4

**Rotating solid sample
—> magnetization**

$$\Delta J \Rightarrow \Delta M$$

THE PHYSICAL REVIEW.

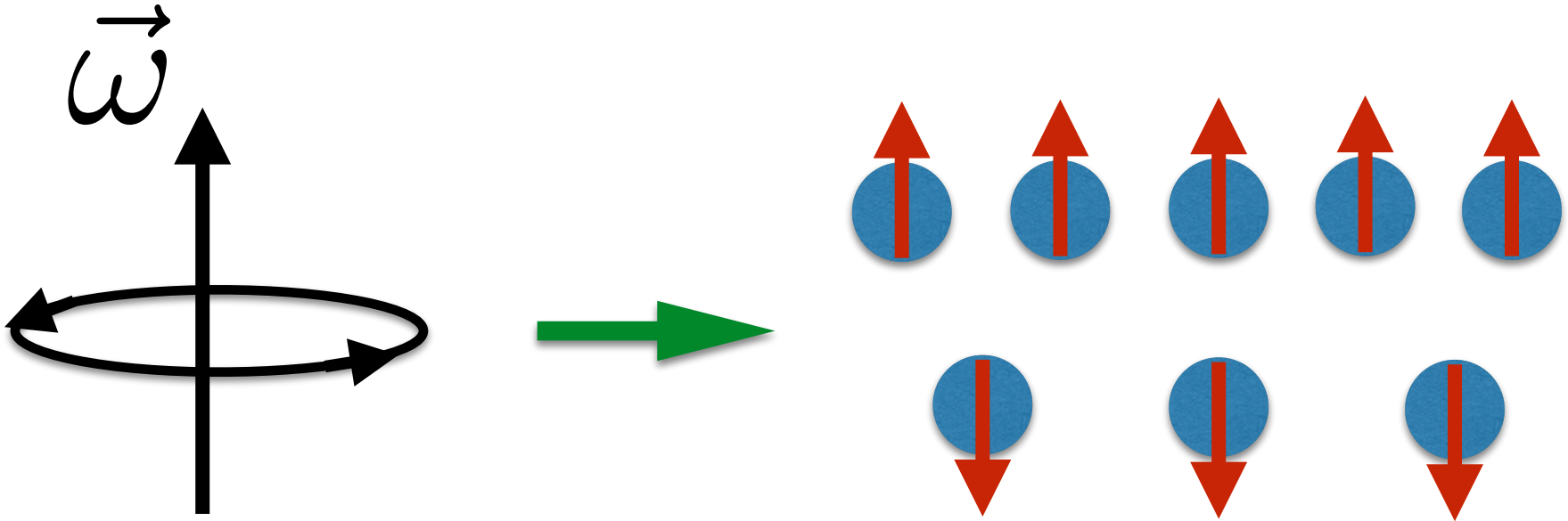
MAGNETIZATION BY ROTATION.¹

By S. J. BARNETT.

§1. In 1909 it occurred to me, while thinking about the origin of terrestrial magnetism, that a substance which is magnetic (and therefore, according to the ideas of Langevin and others, constituted of atomic or molecular orbital systems with individual magnetic moments fixed in magnitude and differing in this from zero) must become magnetized by a sort of molecular gyroscopic action on receiving an angular velocity.

In Short: Rotational Polarization

*Essential assumption underlying the Barnett effect:
rotational polarization*

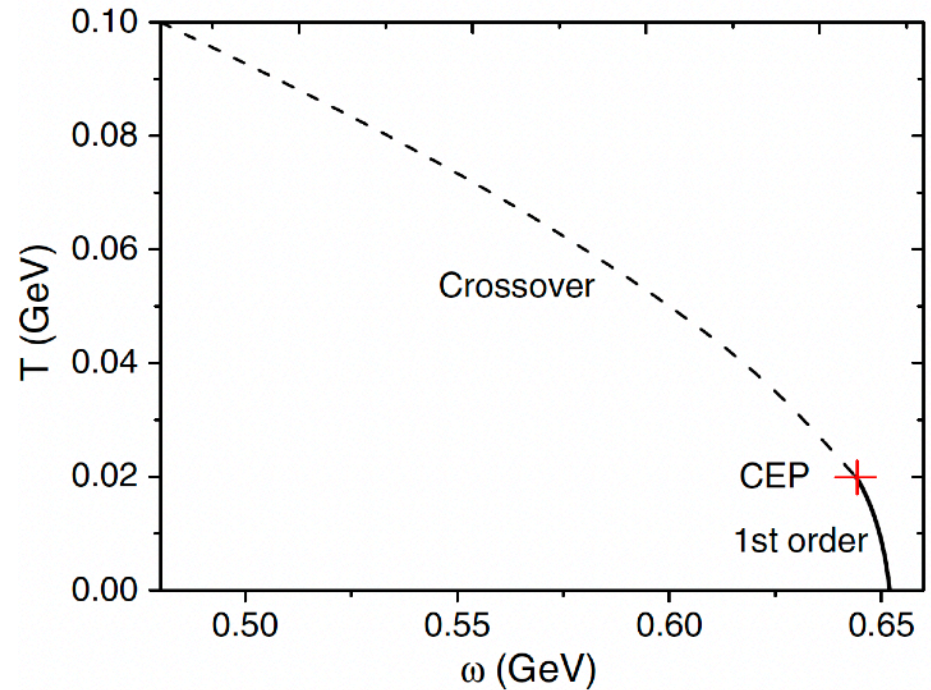
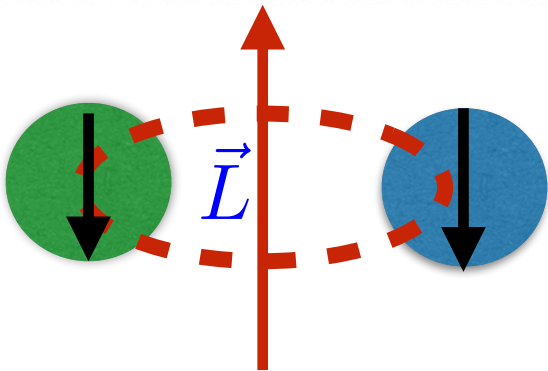
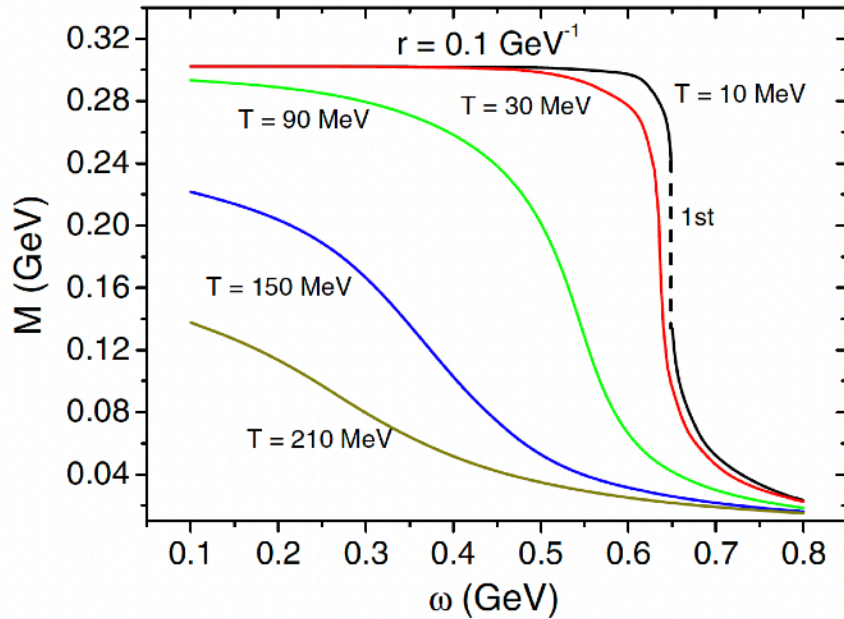


*Macroscopic rotation;
Global angular momentum*

*Microscopic spin
alignment*

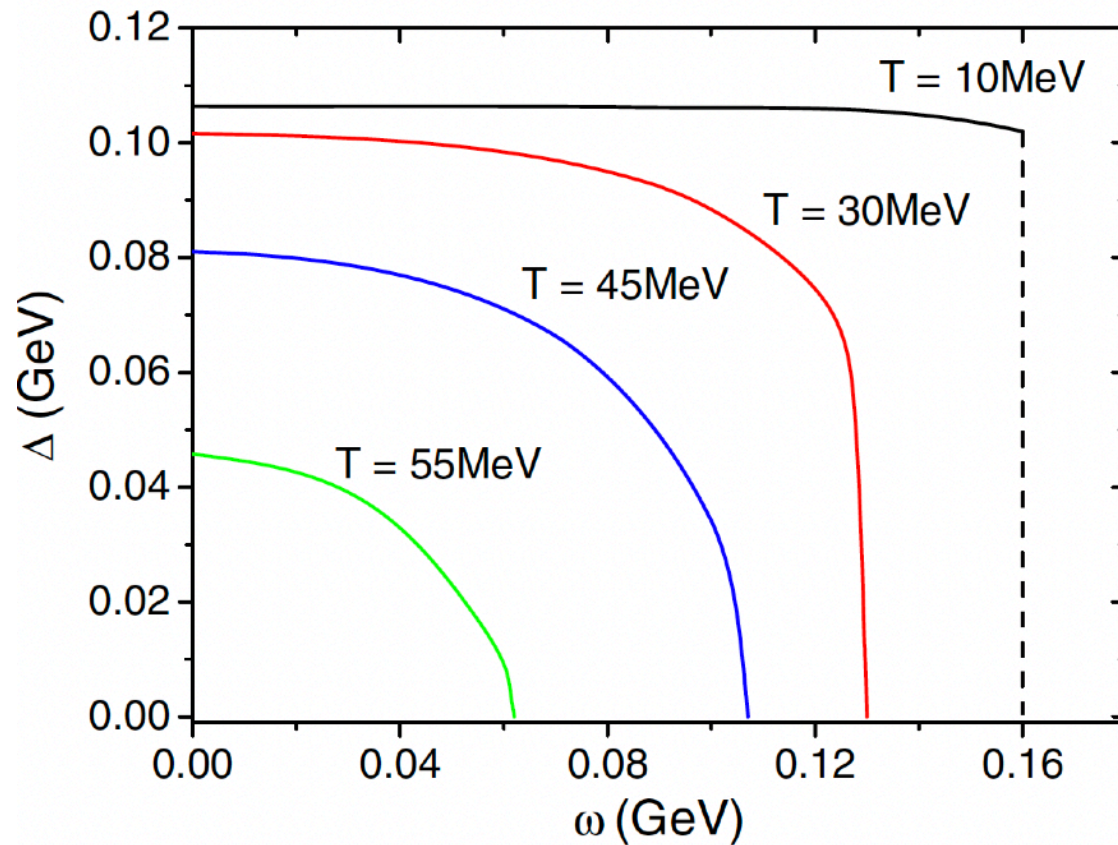
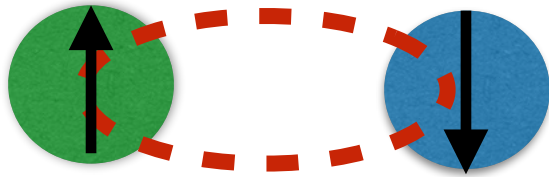
It however is tricky to be directly observed for a flowing fluid.

Chiral Condensate under Rotation



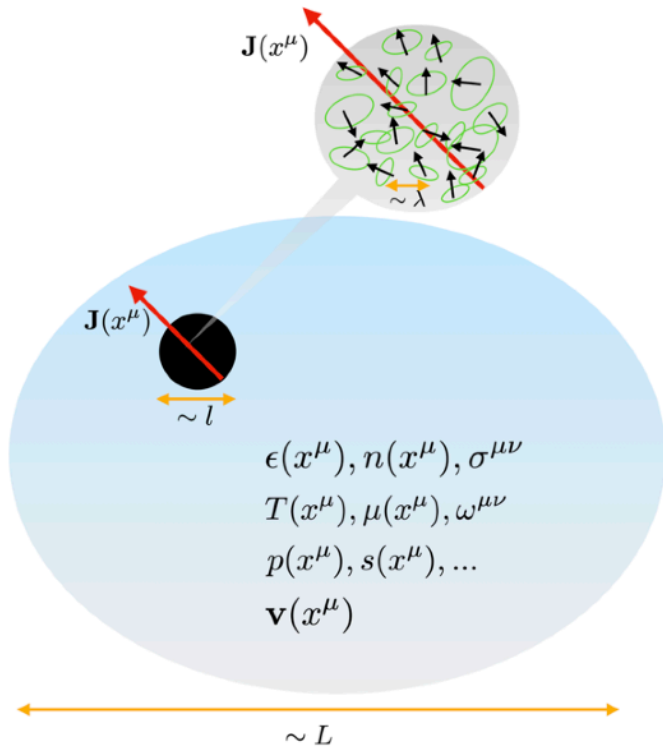
[Yin Jiang, JL, PRL2016]

Color Superconductor under Rotation



[Yin Jiang, JL, PRL2016]

Ideal Hydrodynamics



$\lambda \ll l \ll L$, a coarse-graining process

$$\partial_\mu T^{\mu\nu} = 0,$$

$$\partial_\mu N^\mu = 0,$$

$$T_{(0)}^{\mu\nu} = \epsilon u^\mu u^\nu - p \Delta^{\mu\nu}, \quad N_{(0)}^\mu = n u^\mu.$$

$$\epsilon = -p + Ts + \mu n$$

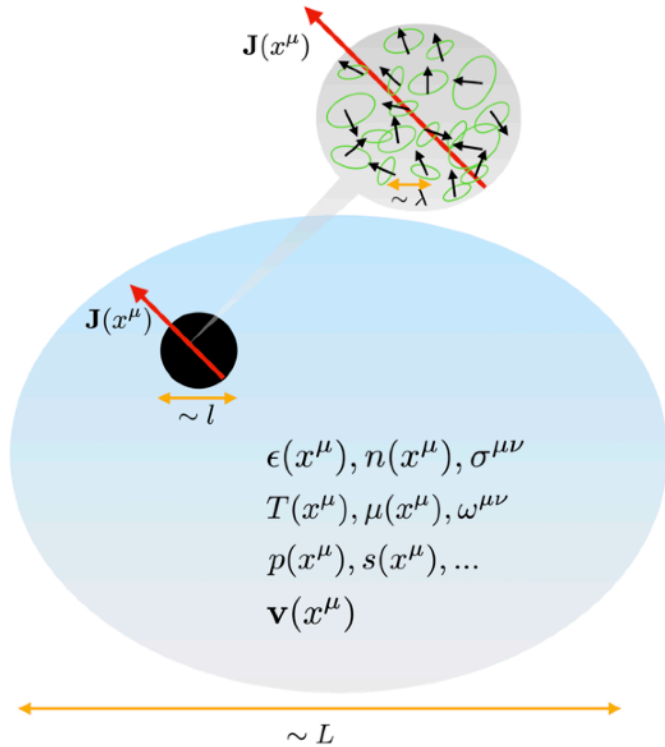
$$S_{(0)}^\mu = s u^\mu$$

$$\partial_\mu S_{(0)}^\mu = \partial_\mu (s u^\mu) = 0$$

Microscopic physics enters via thermodynamic relations (i.e. EOS).

See e.g. Landau and Lifshitz

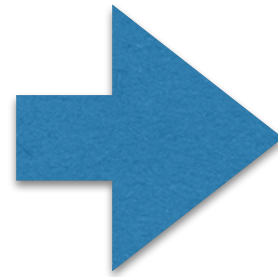
Navier-Stokes Viscous Hydrodynamics



$\lambda \ll l \ll L$, a coarse-graining process

See e.g. Landau and Lifshitz

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= 0, \\ \partial_\mu N^\mu &= 0, \\ T^{\mu\nu} &= \epsilon u^\mu u^\nu - p \Delta^{\mu\nu} + \tilde{T}^{\mu\nu}, \\ N^\mu &= n u^\mu + \tilde{N}^\mu, \\ S^\mu &= s u^\mu + \tilde{S}^\mu. \\ \partial_\mu S^\mu &\geq 0. \end{aligned}$$



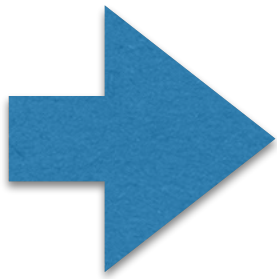
$$\begin{aligned} \Pi &= -\zeta \theta, \\ \pi^{\mu\nu} &= 2\eta \nabla^{\langle\mu} u^{\nu\rangle}, \\ q^\mu &= \lambda T \left(\frac{\nabla^\mu T}{T} - D u^\mu \right) \end{aligned}$$

Microscopic physics also enters via transport coefficients in viscous terms.

Ideal Hydro with Ang. Mom.

$$T_{(0)}^{\mu\nu} = \epsilon u^\mu u^\nu - p \Delta^{\mu\nu}, \quad N_{(0)}^\mu = n u^\mu.$$

$$\Sigma_{(0)}^{\mu\alpha\beta} = \sigma^{\alpha\beta} u^\mu.$$



$$\partial_\mu J_{(0)}^{\mu\alpha\beta} = \sigma^{\alpha\beta} \theta + D\sigma^{\alpha\beta} = 0.$$

Generalized thermodynamics:

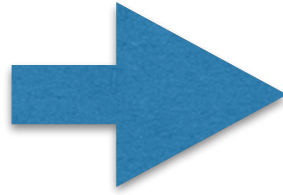
$$\epsilon = -p + Ts + \mu n + \omega_{\alpha\beta} \sigma^{\alpha\beta}$$

It is straightforward to verify: no entropy generation

$$\partial_\mu S_{(0)}^\mu = \partial_\mu (s u^\mu) = 0.$$

Viscous Hydro with Ang. Mom.: Landau Frame

$$u_L^\mu = \frac{T_\nu^\mu u_L^\nu}{\sqrt{u_L^\alpha T_\alpha^\beta T_{\beta\gamma} u_L^\gamma}}$$



$$T^{\mu\nu} = \epsilon u_L^\mu u_L^\nu - (p + \Pi) \Delta_L^{\mu\nu} + \pi^{\mu\nu},$$

$$N^\mu = n u_L^\mu - n \frac{q^\mu}{\epsilon + p},$$

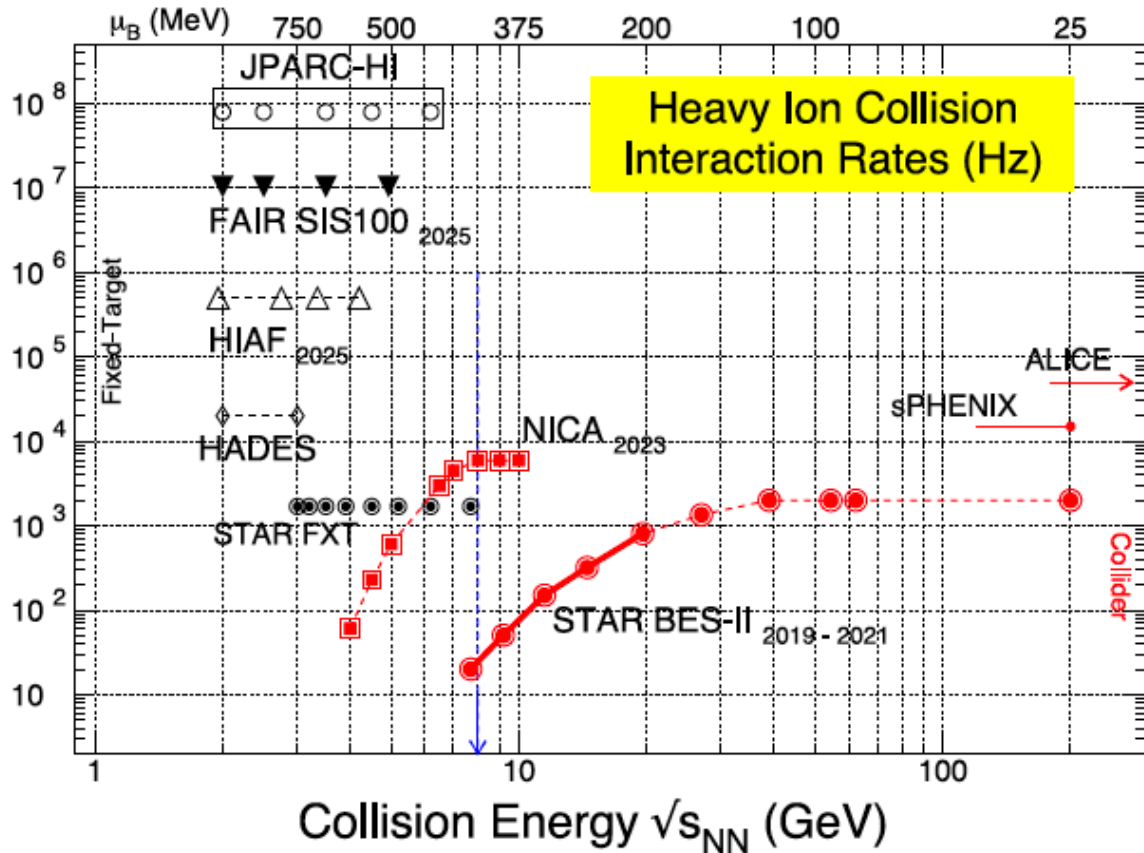
$$\begin{aligned} \Sigma^{\mu\alpha\beta} = u_L^\mu \sigma^{\alpha\beta} - \frac{q^\mu}{\epsilon + p} \sigma^{\alpha\beta} + 2u_L^{[\alpha} \Delta_L^{\mu\beta]} \Phi \\ + 2u_L^{[\alpha} \tau_{(s)}^{\mu\beta]} + 2u_L^{[\alpha} \tau_{(a)}^{\mu\beta]} + \Theta^{\mu\alpha\beta}. \end{aligned}$$

Following the same procedure, one obtains essentially the same consistent results.

High-Polarization Matter in Low Energy Collisions

Relativistic nuclear collisions have been and will continue to be done from $O(1)$ GeV to $O(1000)$ GeV beam energy!

*“Mapping the Phases of Quantum Chromodynamics with Beam Energy Scan”,
Bzdak, Esumi, Koch, JL, Stephanov, Xu, Phys. Rep. 853(2020)1-87.*

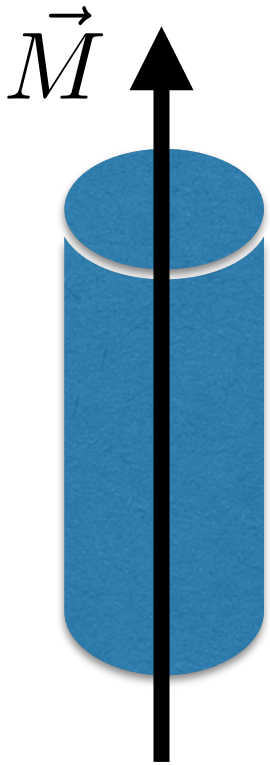


*High Polarization Matter:
Exciting new regime!*

*Potentially a new,
interesting and integral
component of a
rich and diverse
low energy
nuclear collision
physics program.*

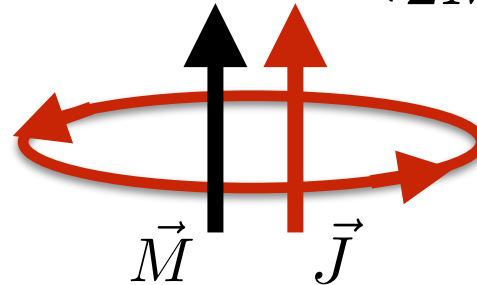
Einstein-de Hass Effect

*Richardson, ~1908; Einstein-de Hass, ~1915:
Change of a free body's magnetic momentum →
Mechanical rotation of the sample*



*Orbital
contribution:*

$$\Delta M = \left(\frac{q}{2M} \right) \Delta J$$



*Spin
contribution:*

$$\Delta M = \left(\frac{2 \times q}{2M} \right) \Delta J$$

Barnett (OSU), ~1915:

*1st correct measurement, supporting the $g \sim 2$,
Indicating dominant spin contributions in magnetization.*