Fluctuations in heavy ion collisions and global conservation effects

Roman Poberezhnyuk

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In collaborations with: Volodymyr Vovchenko, Oleh Savchuk, Volker Koch,
Mark Gorenstein, Horst Stoecker

FIAS Frankfurt Institute for Advanced Studies

BERKELEY LAB

Lawrence Berkeley National Laboratory

Fluctuations: Theory vs experiment

- Proxy observables in experiment (net-proton, net-kaon) vs actual conserved charges in QCD (net-baryon, net-strangeness)
- Volume fluctuations
- Non-equilibrium (memory) effects
- Final-state interactions in the hadronic phase
- Accuracy of the grand-canonical ensemble (global conservation laws)

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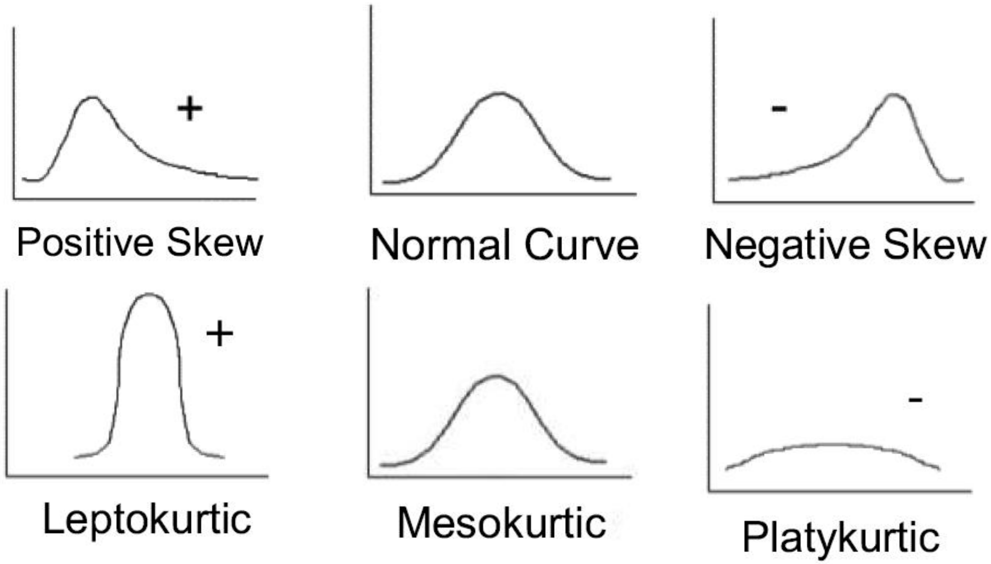
Fluctuations in strongly interacting matter



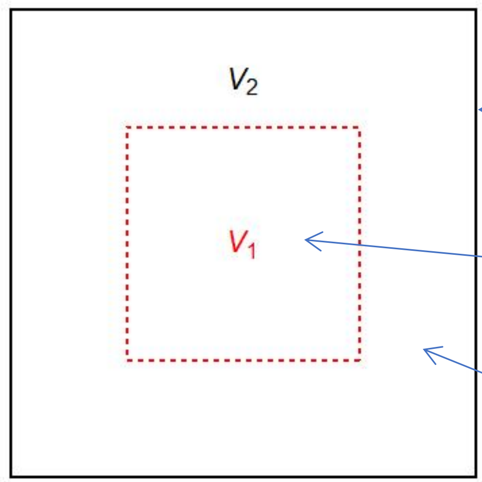
Fluctuations probe finer details of the (QCD) equation of state

Fluctuation measures --- Cumulants (susceptibilities) of distributions --- are sensitive to fine details of interactions, e.g., phase structure

In GCE:

$$\chi^{Q_1 \ldots Q_N}_{l_1 \ldots l_N} = \frac{\partial^{l_1 + \ldots + l_N} (p/T^4)}{\partial(\mu_{Q_1}/T)^{l_1} \, \ldots \, \partial(\mu_{Q_N}/T)^{l_N}} = \frac{{}^{\mathrm{gce}}\kappa^{Q_1 \ldots Q_N}_{l_1 \ldots l_n}}{VT^3}$$

Grand canonical ensemble



coordinate space

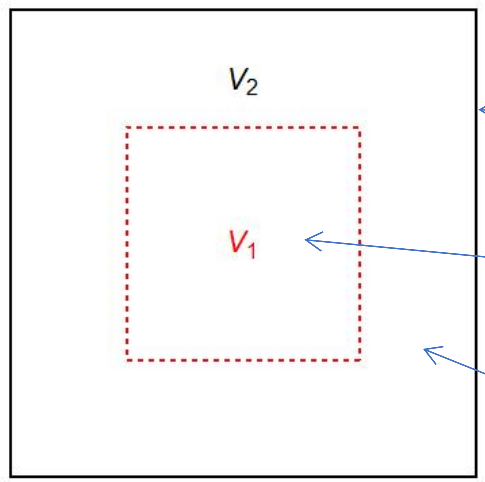
Total system, $V=V_1+V_2$, is in the canonical ensemble (CE)

Subsystem (observed), $V_1 = \alpha V$

Reservoir (thermostat), V_2

$$\alpha = V_1/V$$

Grand canonical ensemble



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Requirements for GCE:

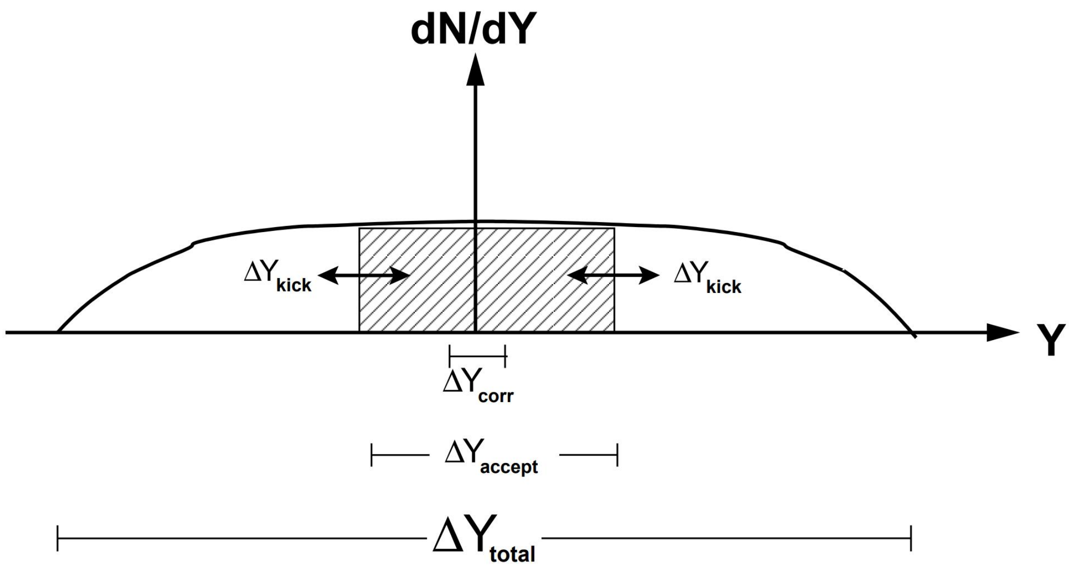
1) $V_1 \gg \xi$ --- (Thermodynamic limit) --- ratios of extensive quantities become V-independent

2) $V \gg V_1$ --- (Subsystem is a small part of the system $\Leftrightarrow \alpha \ll 1$)

$$V \gg V_1 \gg \xi$$

Applicability of the GCE in heavy-ion collisions

Experiments measure fluctuations in a finite momentum acceptance



V. Koch, 0810.2520

GCE applies if $\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}, \Delta Y_{corr}$ and momentum-space correlation is strong (e.g. Bjorken flow)

In practice, difficult to satisfy all conditions simultaneously…

If $\Delta Y_{total} \gg \Delta Y_{accept}$ does not hold, corrections from global conservation appear

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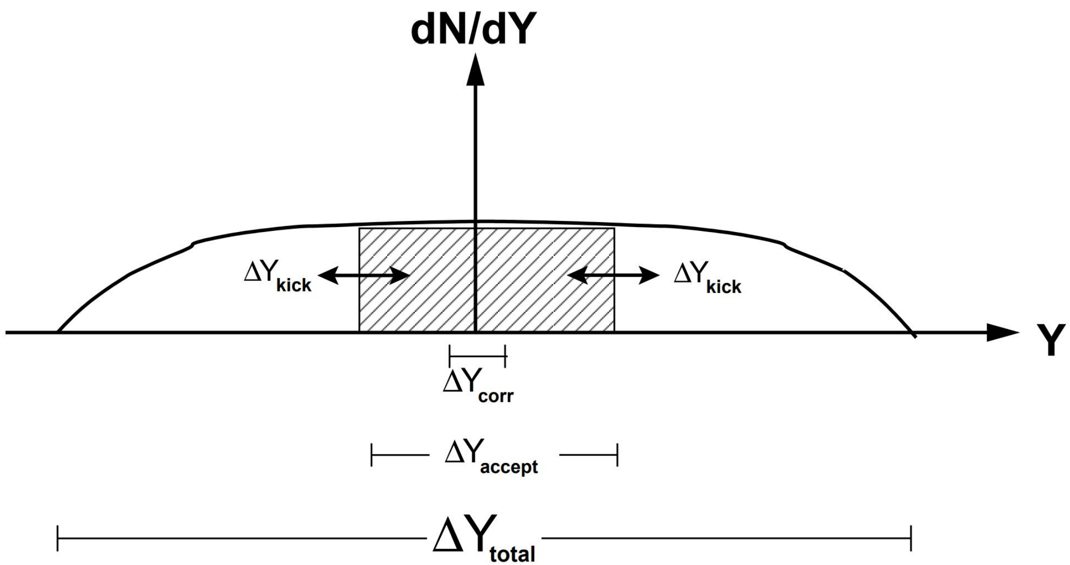

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Subensemble acceptance method allows to account for global conservation effects and provides fluctuation measures insensible to global conservation!

Subensemble

$$V \not\gg V_1 \not\gg \xi$$

Global conservation effects

Finite size effects

Finite sybsystem in finite reservoir corresponds to Subensemble ---
the generalization of grand canonical and canonical statistical ensembles

V.Vovchenko, O.Savchuk, **R.P.**, M.Gorenstein, V.Koch, PLB, 2020

R.P., O.Savchuk, M.Gorenstein, V.Vovchenko, K.Taradiy, V.Begun, L.Satarov, J.Steinheimer, H.Stoecker PRC, 2020

Relevant for heavy-ion collision experiments

Subensemble acceptance method

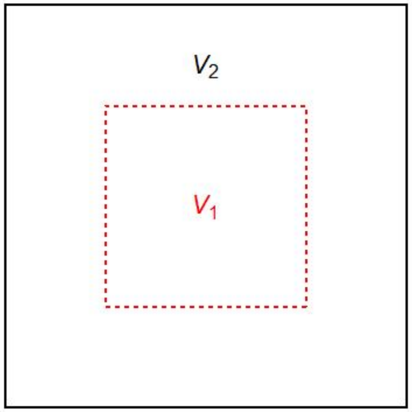
Partition a thermal system with a globally conserved charge *B (canonical ensemble)* into two subsystems which can exchange the charge

Neglect surface effects:

$$\widehat{H} = \widehat{H}_1 + \widehat{H}_2 + \widehat{V}_{1,2} \approx \widehat{H}_1 + \widehat{H}_2$$

The canonical partition function then reads:

$$Z^{\rm ce}(T,V,B) = \mathrm{Tr}\, e^{-\beta \hat{H}} \approx \sum_{B_1} Z^{\rm ce}(T,V_1,B_1)\, Z^{\rm ce}(T,V-V_1,B-B_1)$$



$$V_1 + V_2 = V$$

$$P(B_1) \propto Z^{\rm ce}(T,\alpha V,B_1)\, Z^{\rm ce}(T,(1-\alpha)V,B-B_1), \qquad \alpha \equiv V_1/V$$

The probability to have charge B_1 is:

$$Z^{\rm ce}(T,V,B) \overset{V\to\infty}{\simeq} \exp\left[-\frac{V}{T} f(T,\rho_B)\right]$$

If the canonical partition function known, B_1-cumulants can be calculated explicitly

Subensemble acceptance: Full result up to κ_6

Model-independent!

$$\kappa_1[B_1] = \alpha\, VT^3\, \chi_1^B$$

$$\kappa_2[B_1] = \alpha\, VT^3\, \beta\, \chi_2^B$$

$$\kappa_3[B_1] = \alpha\, VT^3\, \beta\, (1-2\alpha)\, \chi_3^B$$

$$\kappa_4[B_1] = \alpha\, VT^3\, \beta\, \left[\chi_4^B \; - 3\alpha\beta \frac{(\chi_3^B)^2 + \chi_2^B\, \chi_4^B}{\chi_2^B}\right]$$

$$\kappa_5[B_1] = \alpha\, VT^3\, \beta\, (1-2\alpha) \left\{[1-2\beta\alpha]\chi_5^B \; - 10\alpha\beta \frac{\chi_3^B \chi_4^B}{\chi_2^B}\right\}$$

$$\kappa_6[B_1] = \alpha\, VT^3\, \beta\, [1-5\alpha\beta(1-\alpha\beta)]\, \chi_6^B + 5\, VT^3\, \alpha^2\, \beta^2 \left\{9\alpha\beta \frac{(\chi_3^B)^2\, \chi_4^B}{(\chi_2^B)^2} - 3\alpha\beta \frac{(\chi_3^B)^4}{(\chi_2^B)^3} - 2(1-2\alpha)^2 \frac{(\chi_4^B)^2}{\chi_2^B} - 3[1-3\beta\alpha] \frac{\chi_3^B\, \chi_5^B}{\chi_2^B}\right\}$$

$\beta = 1 - \alpha$

$$\chi_n^B = \frac{\partial^n (p/T^4)}{\partial(\mu_B/T)^n}$$ – grand-canonical susceptibilities

Vovchenko., Savchuk, **R.P.**, Gorenstein, Koch, PLB, 2020

Subensemble acceptance: Cumulant ratios

Some common cumulant ratios:

$$\langle B \rangle = \kappa_1, \quad \omega = \frac{\kappa_2}{\kappa_1}, \quad S\sigma = \frac{\kappa_3}{\kappa_2}, \quad \kappa\sigma^2 = \frac{\kappa_4}{\kappa_2}.$$

scaled variance $$\frac{\kappa_2[B_1]}{\kappa_1[B_1]} = (1-\alpha)\,\frac{\chi_2^B}{\chi_1^B},$$

skewness $$\frac{\kappa_3[B_1]}{\kappa_2[B_1]} = (1-2\alpha)\,\frac{\chi_3^B}{\chi_2^B},$$

kurtosis $$\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta\left(\frac{\chi_3^B}{\chi_2^B}\right)^2.$$

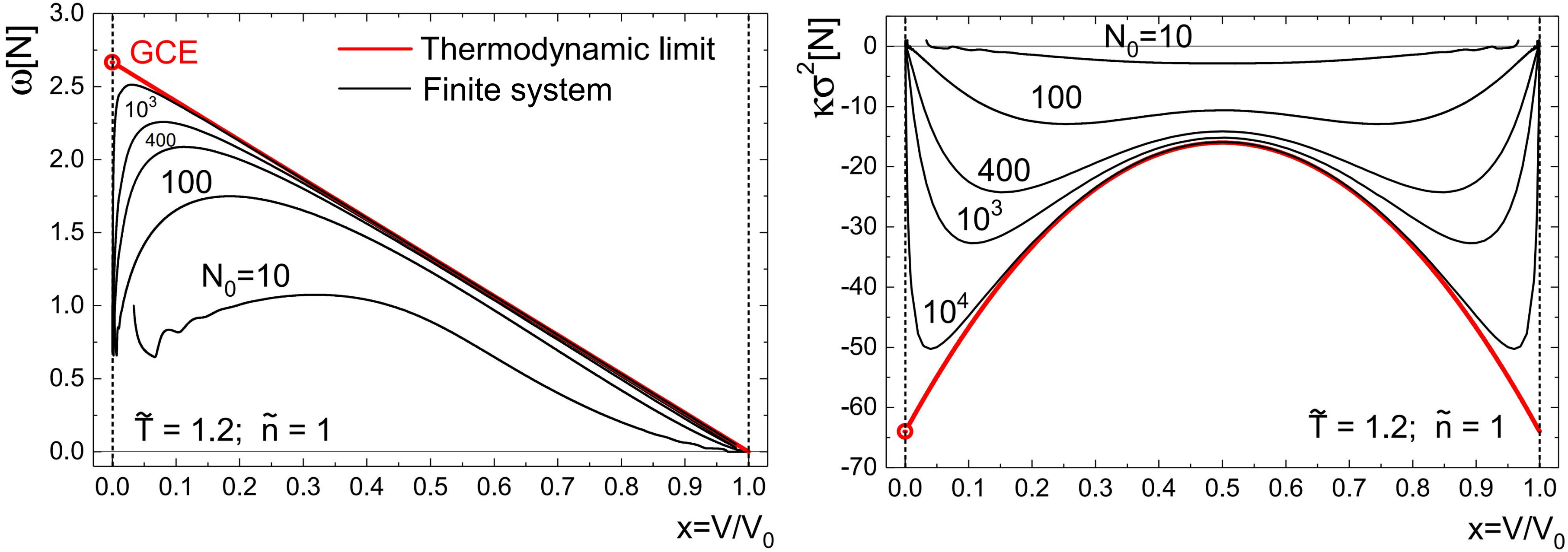
Vovchenko., Savchuk, **R.P.**, Gorenstein, Koch, PLB, 2020

Subensemble acceptance: van der Waals fluid

Calculate cumulants $\kappa_n[N]$ in a subvolume directly from the partition function

$$P(N) \propto Z^{\rm ce}_{\rm vdW}(T, xV_0, N)\, Z^{\rm ce}_{\rm vdW}(T, (1-x)V_0, N_0 - N)$$

and compare with the subensemble acceptance results



Results agree with subsensemble acceptance in thermodynamic limit ($N_0 \to \infty$)

Finite size effects are strong near the critical point: a consequence of large correlation length ξ

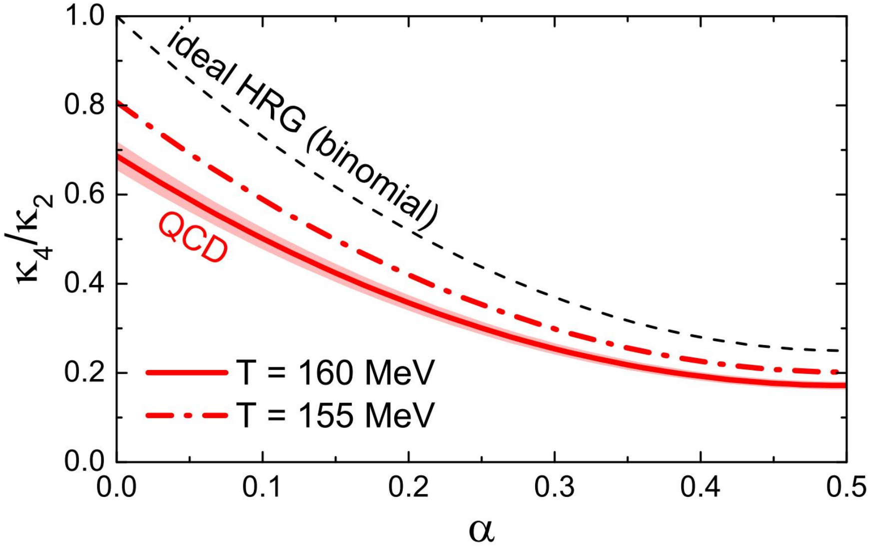
R.P., Savchuk, Gorenstein, Vovchenko, Taradiy, Begun, Satarov, Steinheimer, Stoecker, PRC, 2020

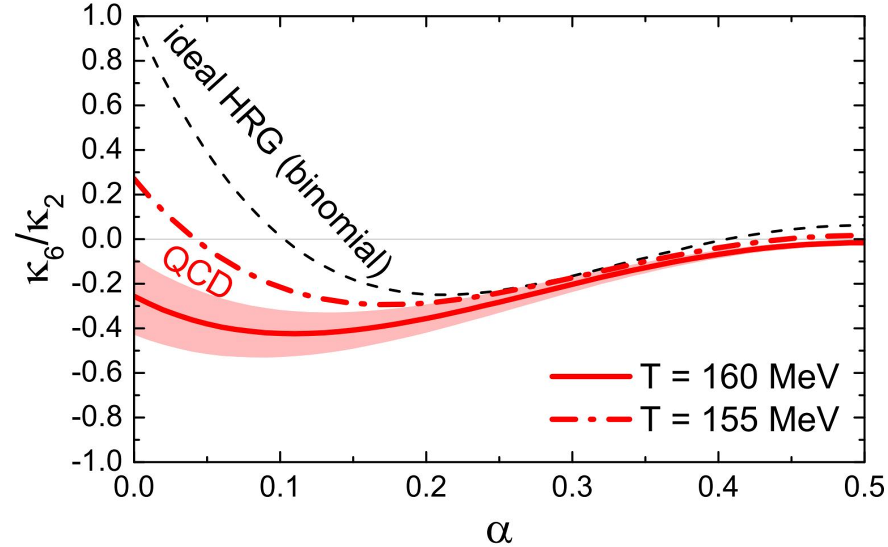
Kuznietsov, Savchuk, Gorenstein, Koch, Vovchenko, PRC, 2022

Net baryon fluctuations at LHC and top RHIC

$$\left(\frac{\kappa_4}{\kappa_2}\right)_{LHC} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B}$$

$$\left(\frac{\kappa_6}{\kappa_2}\right)_{LHC} = [1-5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$





Lattice data for χ_4^B/χ_2^B and χ_6^B/χ_2^B from Borsanyi et al., 1805.04445

For $\alpha > 0.2$ difficult to distinguish effects of the EoS and baryon conservation in χ_6^B/χ_2^B,
$\alpha \leq 0.1$ is a sweet spot where measurements are mainly sensitive to the EoS

Estimates: $\alpha \approx 0.1$ corresponds to $\Delta Y_{acc} \approx 2(1)$ at LHC (RHIC)

Vovchenko, Savchuk, **R.P.**, Gorenstein, Koch, PLB, 2020

Vovchenko, Koch, 2012.09954

Multiple conserved charges: Special cases

$$\beta \equiv 1 - \alpha$$

Two conserved charges: B,Q

$$\kappa_4[B^1] = \alpha V T^3 \beta \left[(1-3\alpha\beta)\,\chi_4^B - 3\,\alpha\,\beta \, \frac{(\chi_3^B)^2 \chi_2^Q - 2\chi_{21}^{BQ}\chi_{11}^{BQ}\chi_3^B + (\chi_{21}^{BQ})^2\chi_2^B}{\chi_2^B\chi_2^Q - (\chi_{11}^{BQ})^2} \right].$$

Three conserved charges: B,Q,S

$$\kappa_4[B^1] = \alpha V T^3 \beta \left[(1-3\alpha\beta)\,\chi_4^B - \frac{3\alpha\beta}{D[\hat{\chi}_2]} \times \right.$$
$$\left\{ (\chi_3^B)^2[\chi_2^Q\chi_2^S - (\chi_{11}^{QS})^2] + (\chi_{21}^{BQ})^2[\chi_2^B\chi_2^S - (\chi_{11}^{BS})^2] + (\chi_{21}^{BS})^2[\chi_2^B\chi_2^Q - (\chi_{11}^{BQ})^2] \right.$$
$$\left.\left. -2\chi_3^B\chi_{21}^{BQ}(\chi_2^S\chi_{11}^{BQ} - \chi_{11}^{BS}\chi_{11}^{QS}) - 2\chi_3^B\chi_{21}^{BS}(\chi_2^Q\chi_{11}^{BS} - \chi_{11}^{BQ}\chi_{11}^{QS}) \right\} \right].$$

The fact that electric (strange) charge is fixed has an effect on observables which do not involve explicitly the electric charge.

But only for fluctuations of non-conserved quantities and for higher order fluctuations of conserved charges

Vovchenko, **R.P.**, Koch, JHEP, 2020

General formulas

$$\hat{\kappa}_{i_1}[\hat{Q}^1] = \alpha V T^3 \, \hat{\chi}_{i_1}$$

$$\hat{\kappa}_{i_1 i_2}[\hat{Q}^1] = \alpha V T^3 \, \beta \, \hat{\chi}_{i_1 i_2}$$

$$\hat{\kappa}_{i_1 i_2 i_3}[\hat{Q}^1] = \alpha V T^3 \, \beta \, (1 - 2\alpha) \, \hat{\chi}_{i_1 i_2 i_3}$$

$$\hat{\kappa}_{i_1 i_2 i_3 i_4}[\hat{Q}^1] = \alpha V T^3 \, \beta \left[(1 - 3\alpha\beta) \, \hat{\chi}_{i_1 i_2 i_3 i_4} - \frac{\alpha\beta}{2!\,2!\,2!} \sum_{\sigma \in S_4} \hat{\chi}^{-1}_{b_1 b_2} \, \hat{\chi}_{i_{\sigma_1} i_{\sigma_2} b_1} \, \hat{\chi}_{i_{\sigma_3} i_{\sigma_4} b_2} \right]$$

For results up to the 6-th order see Vovchenko, **R.P.**, Koch, JHEP, 2020

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Cumulants up to 3-d order have the same simple α-dependence as in the case of single conserved charge

$$\hat{\kappa}_{i_1 i_2 i_3 i_4}[\hat{Q}^1] = \alpha V T^3\, \beta \left[(1-3\alpha\beta)\, \hat{\chi}_{i_1 i_2 i_3 i_4} - \frac{\alpha\beta}{2!\,2!\,2!} \sum_{\sigma \in S_4} \hat{\chi}^{-1}_{b_1 b_2}\, \hat{\chi}_{i_{\sigma_1} i_{\sigma_2} b_1}\, \hat{\chi}_{i_{\sigma_3} i_{\sigma_4} b_2} \right]$$

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Cumulants up to third order have the same simple α-dependence as in the case of single conserved charge

$$\hat{\kappa}_{i_1 i_2 i_3 i_4}[\hat{Q}^1] = \alpha V T^3 \, \beta \left[(1-3\alpha\beta) \, \hat{\chi}_{i_1 i_2 i_3 i_4} - \frac{\alpha\beta}{2!\,2!\,2!} \sum_{\sigma \in S_4} \hat{\chi}^{-1}_{b_1 b_2} \, \hat{\chi}_{i_{\sigma_1} i_{\sigma_2} b_1} \, \hat{\chi}_{i_{\sigma_3} i_{\sigma_4} b_2} \right]$$

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The global conservation effects cancel out in any ratio of second order cumulants and in any ratio of third order cumulants

$$\frac{\kappa_2^Q}{\kappa_2^B} = \frac{\chi_2^Q}{\chi_2^B}, \quad \frac{\kappa_3^Q}{\kappa_3^B} = \frac{\chi_3^Q}{\chi_3^B}, \quad \frac{\kappa_2^{BQ}}{\kappa_2^S} = \frac{\chi_2^{BQ}}{\chi_2^S}, \ldots$$

Ensemble-independent fluctuatio measures, not sensitive to global conservation

Vovchenko, **R.P.**, Koch, JHEP, 2020

non-conserved quantities come later...

Strongly intensive fluctuation measures

$$\Delta[Q_a, Q_b] = C_\Delta^{-1} \left\{ \kappa_1[Q_b] \frac{\kappa_2[Q_a]}{\kappa_1[Q_a]} - \kappa_1[Q_a] \frac{\kappa_2[Q_b]}{\kappa_1[Q_b]} \right\},$$

$$\Sigma[Q_a, Q_b] = C_\Sigma^{-1} \left\{ \kappa_1[Q_b] \frac{\kappa_2[Q_a]}{\kappa_1[Q_a]} + \kappa_1[Q_a] \frac{\kappa_2[Q_b]}{\kappa_1[Q_b]} - 2\,\kappa_{1,1}[Q_a, Q_b] \right\}.$$

M.Gorenstein, M.Gazdzicki, PRC, 2011

$$\Delta[Q_a, Q_b] = C_\Delta^{-1}\, V T^3\, \alpha(1-\alpha) \left\{ \chi_1^{Q_b} \frac{\chi_2^{Q_a}}{\chi_1^{Q_a}} - \chi_1^{Q_a} \frac{\chi_2^{Q_b}}{\chi_1^{Q_b}} \right\},$$

$$\Sigma[Q_a, Q_b] = C_\Sigma^{-1}\, V T^3\, \alpha(1-\alpha) \left\{ \chi_1^{Q_b} \frac{\chi_2^{Q_a}}{\chi_1^{Q_a}} + \chi_1^{Q_a} \frac{\chi_2^{Q_b}}{\chi_1^{Q_b}} - 2\, \chi_{1,1}^{Q_a Q_b} \right\}.$$

$$\frac{\Sigma[Q_a, Q_b]}{\Delta[Q_a, Q_b]} = \frac{\chi_1^\Delta}{\chi_1^\Sigma} \, \frac{\chi_1^{Q_b} \frac{\chi_2^{Q_a}}{\chi_1^{Q_a}} + \chi_1^{Q_a} \frac{\chi_2^{Q_b}}{\chi_1^{Q_b}} - 2\, \chi_{1,1}^{Q_a Q_b}}{\chi_1^{Q_b} \frac{\chi_2^{Q_a}}{\chi_1^{Q_a}} - \chi_1^{Q_a} \frac{\chi_2^{Q_b}}{\chi_1^{Q_b}}}.$$

Both strongly intensive and ensemble independent (insensitive to global conservation)

Vovchenko, **R.P.**, Koch, JHEP, 2020

Non-conserved quantities (net-proton, pion, kaon,...)

Example: net proton number $$Z(T,V,\hat{Q}) = \sum_{N_p} W(T,V,\hat{Q};N_p)\ .$$

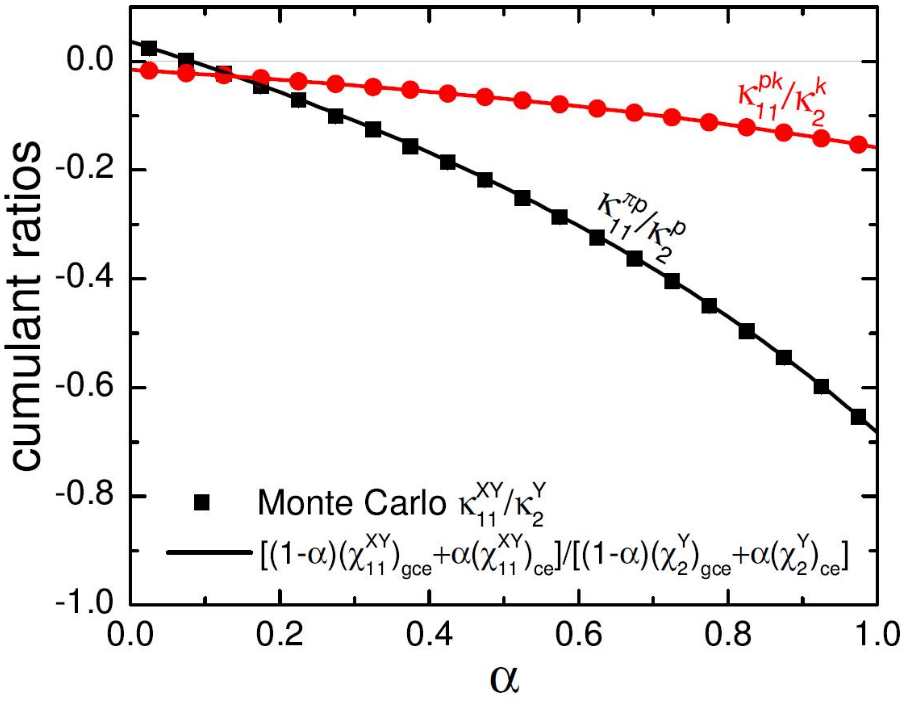
Variance of non-conserved quantity:

$$\breve{\chi} = \begin{pmatrix} \chi_{\hat{Q}_i\hat{Q}_j} & \chi_{\hat{Q}_i p} \\ \chi_{p\hat{Q}_j} & \chi_{pp} \end{pmatrix}, \qquad i,j = 1,\dots,N.$$

$$\kappa_{pp} = \alpha V T^3 \left[(1-\alpha)\chi_{pp} + \alpha \frac{\det \breve{\chi}}{\det \chi} \right]$$

GCE susceptibility

CE susceptibility

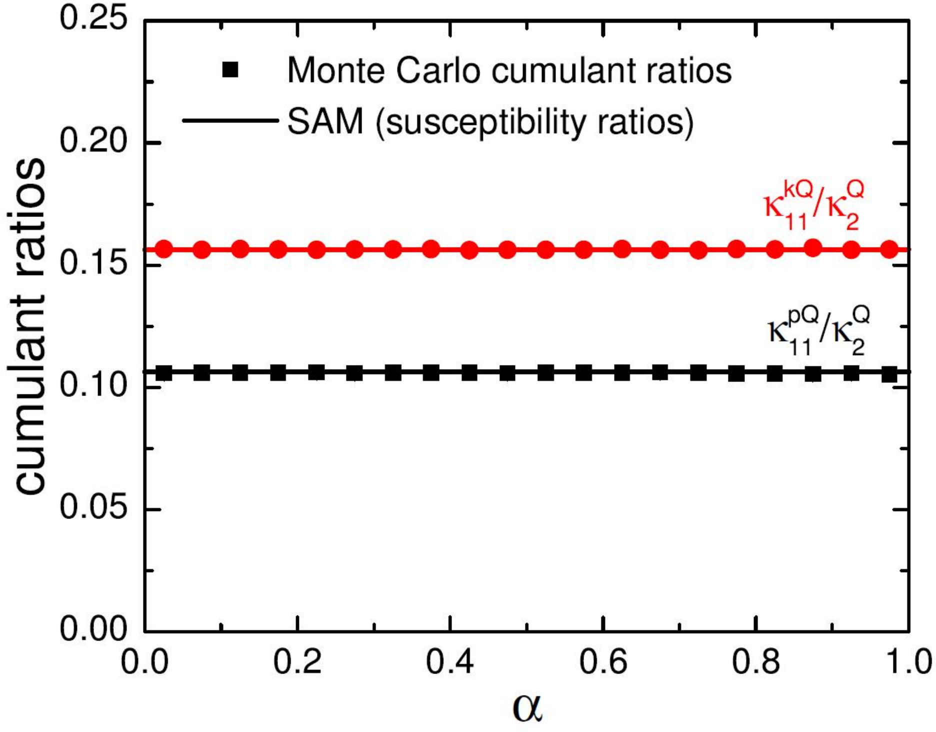


Vovchenko, **R.P.**, Koch, JHEP, 2020

Off-diagonal cumulants involving non-conserved quantity

$$\kappa_{p\hat{Q}_j} = \alpha V T^3 \, \beta \, \chi_{p\hat{Q}_j} \, .$$

$$\frac{\kappa_{p\hat{Q}_j}}{\kappa_{\hat{Q}_i\hat{Q}_j}} = \frac{\chi_{p\hat{Q}_j}}{\chi_{\hat{Q}_i\hat{Q}_j}} \qquad \frac{\kappa_{p\hat{Q}_j}}{\kappa_{k\hat{Q}_i}} = \frac{\chi_{p\hat{Q}_j}}{\chi_{k\hat{Q}_i}} \, .$$



Vovchenko, **R.P.**, Koch, JHEP, 2020

Summary

- Subensemble acceptance method (SAM) provides general formulas to correct cumulants of distributions in heavy-ion collisions for global conservation of all QCD charges
- Formulas connect cumulants measured in the subsystem of the thermal system with grand canonical susceptibilities. The method works for an arbitrary equation of state and sufficiently large systems, such as created in central collisions of heavy ions.
- Some fluctuation measures are insensitive to global conservation and, thus, can be especially convenient for experimental measurement:

$$\frac{\kappa_2^Q}{\kappa_2^B}, \quad \frac{\kappa_3^Q}{\kappa_3^B}, \quad \frac{\kappa_2^{BQ}}{\kappa_2^S}, \quad \frac{\kappa_{p\hat{Q}_j}}{\kappa_{\hat{Q}_i\hat{Q}_j}}, \quad \frac{\kappa_{p\hat{Q}_j}}{\kappa_{p\hat{Q}_i}}, \quad \frac{\kappa_{p\hat{Q}_j}}{\kappa_{k\hat{Q}_i}}, \quad \frac{\Sigma[Q_a, Q_b]}{\Delta[Q_a, Q_b]}$$

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Outlook: Application of method to intermediate collision energies (NA61/SHINE, RHIC-BES,...), net proton higher order fluctuations.
Effects of thermal smearing and "imperfect" space-momentum correlations:
Vovchenko, Koch, PRC, 2021

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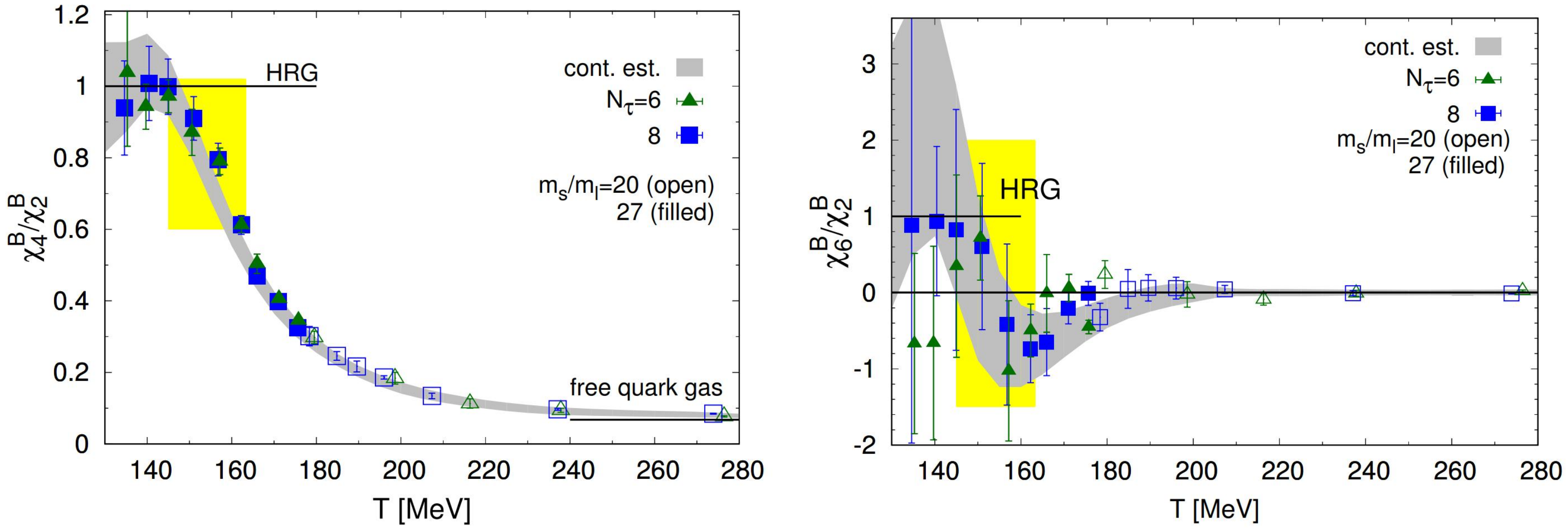
Vovchenko, Koch, PRC, 2021

Thank you for attention!

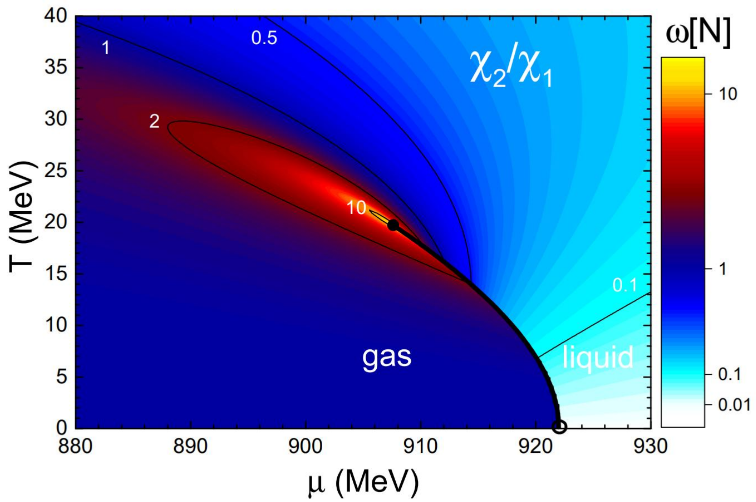
Lattice QCD (fluctuations of conserved charges)



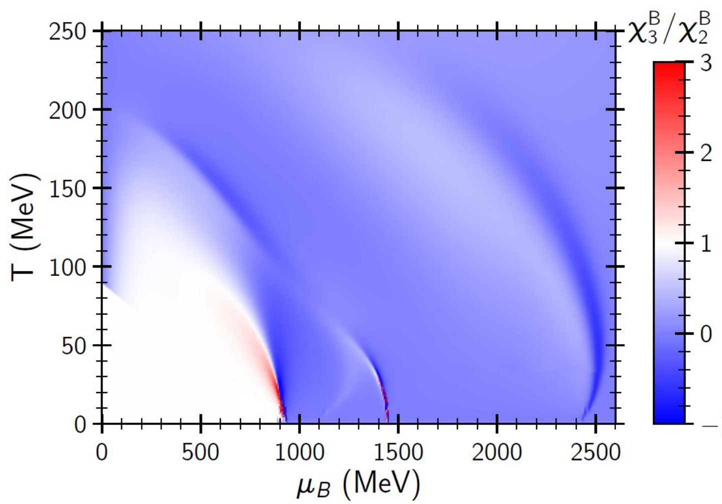
Bazavov et al. (HotQCD), 1701.04325

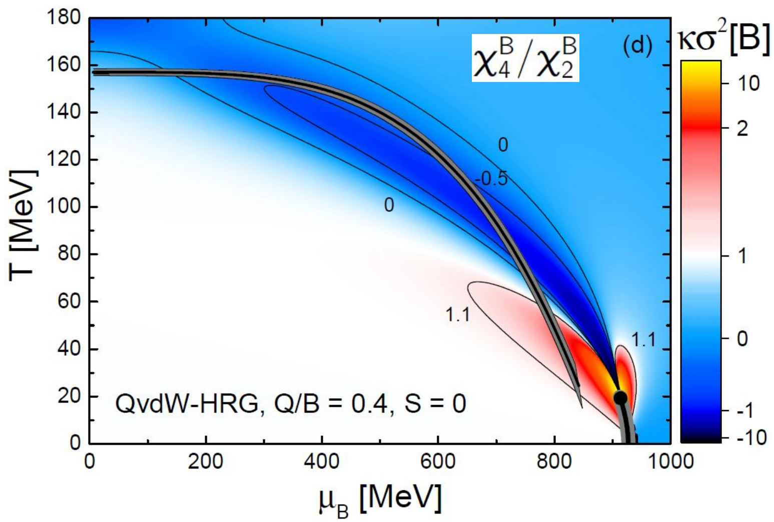
Dip in χ_6^B/χ_2^B down to negative values observed at T_{pc}, possibly related remnants of chiral criticality [Friman, Karsch, Skokov, Redlich, EPJC '11]

Statistical models (HRG, effective QCD,...)

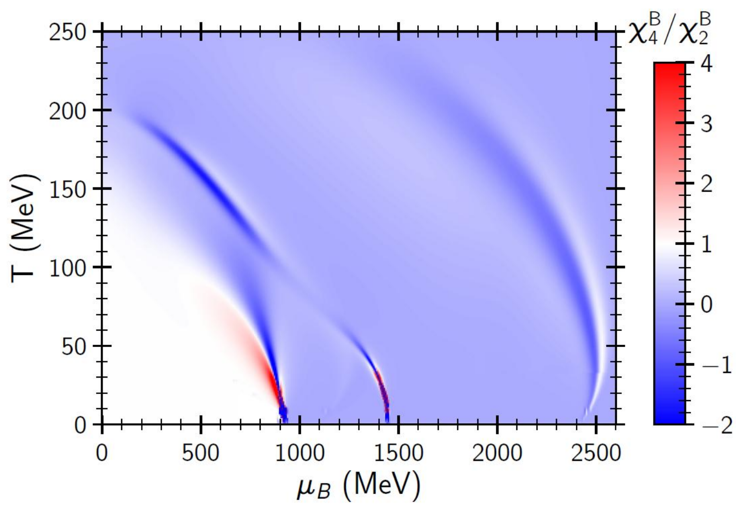


V.Vovchenko et al, PRC 2015





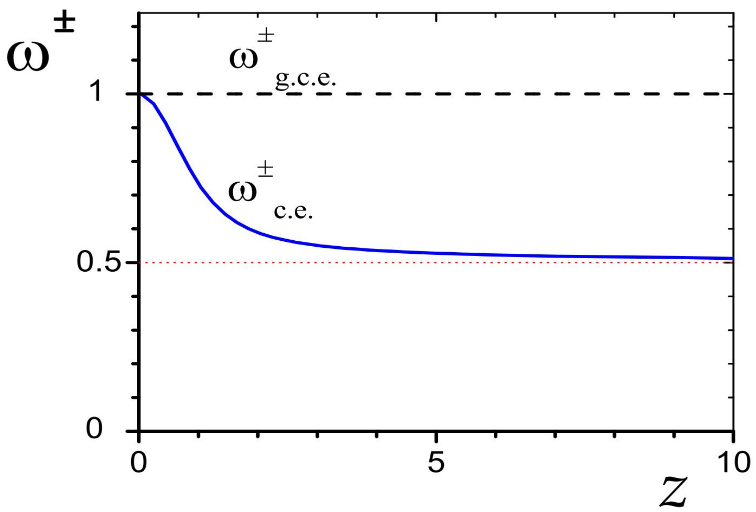
R.P. et al, PRC 2019



A.Motornenko. et al, PRC 2020

Equivalence of ensembles

Thermodynamic equivalence: in the limit $V \to \infty$ all statistical ensembles are equivalent wrt to all average quantities, e.g. $\langle N \rangle_{GCE} = N_{CE}$



Begun, Gorenstein, Gazdzicki, Zozulya, PRC 2004

Thermodynamic equivalence does *not* extend to fluctuations. The results are ensemble-dependent in the limit $V \to \infty$

Theory vs experiment: Caveats

- proxy observables in experiment (net-proton, net-kaon) vs actual conserved charges in QCD (net-baryon, net-strangeness)

  Asakawa, Kitazawa, PRC '12; **V.V.**, Jiang, Gorenstein, Stoecker, PRC '18

- volume fluctuations

  Gorenstein, Gazdzicki, PRC '11; Skokov, Friman, Redlich, PRC '13; Braun-Munzinger, Rustamov, Stachel, NPA '17

- non-equilibrium (memory) effects

  Mukherjee, Venugopalan, Yin, PRC '15

- final-state interactions in the hadronic phase

  Steinheimer, **V.V.**, Aichelin, Bleicher, Stoecker, PLB '18

- accuracy of the grand-canonical ensemble (global conservation laws)

  Jeon, Koch, PRL '00; Bzdak, Skokov, Koch, PRC '13; Braun-Munzinger, Rustamov, Stachel, NPA '17

Subensemble acceptance: Thermodynamic limit

In the thermodynamic limit, $V \to \infty$, Z^{ce} expressed through free energy density

$$Z^{\rm ce}(T,V,B) \overset{V\to\infty}{\simeq} \exp\left[-\frac{V}{T}f(T,\rho_B)\right]$$

B_1 cumulant generating function:

$$\mathrm{G}_{B_1}(t) = \ln\left\{\sum_{B_1} e^{t\,B_1} \exp\left[-\frac{\alpha V}{T} f(T,B_1)\right] \times \exp\left[-\frac{\beta V}{T} f(T,B_2)\right]\right\} + \tilde{C}.$$

B_1 cumulants:

$$\kappa_n[B_1] = \left.\frac{\partial^n G_{B_1}(t)}{\partial t^n}\right|_{t=0} \equiv \tilde{\kappa}_n[B_1(t)]|_{t=0} \qquad \text{or} \qquad \kappa_n[B_1] = \left.\frac{\partial^{n-1}\tilde{\kappa}_1[B_1(t)]}{\partial t^{n-1}}\right|_{t=0}$$

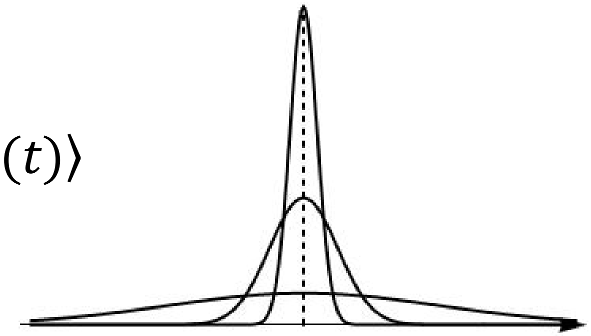
All κ_n can be calculated by determining the t-dependent first cumulant $\tilde{\kappa}_1[B_1(t)]$

V.Vovchenko., Savchuk, **R.P.**, Gorenstein, Koch, 2003.13905

Subensemble acceptance: Thermodynamic limit

$$\tilde{\kappa}_1[B_1(t)] = \frac{\sum_{B_1} B_1 \tilde{P}(B_1;t)}{\sum_{B_1} \tilde{P}(B_1;t)} \equiv \langle B_1(t) \rangle \qquad \text{with} \qquad \tilde{P}(B_1;t) = \exp\left\{ tB_1 - V\frac{\alpha f(T,\rho_{B_1}) + \beta f(T,\rho_{B_2})}{T} \right\}.$$

Thermodynamic limit: $\widetilde{P}(B_1;t)$ *highly peaked* at $\langle B_1(t) \rangle$



$\langle B_1(t) \rangle$ is a solution to equation $d\widetilde{P}/\mathrm{d}B_1 = 0$:

$$t = \hat{\mu}_B[T, \rho_{B_1}(t)] - \hat{\mu}_B[T, \rho_{B_2}(t)]$$

where $\hat{\mu}_B \equiv \mu_B/T, \qquad \mu_B(T,\rho_B) = \partial f(T,\rho_B)/\partial \rho_B$

$t = 0$: $\rho_{B_1} = \rho_{B_2} = B/V$, $B_1 = \alpha B$, i.e. conserved charge uniformly distributed between the two subsystems

V.Vovchenko., Savchuk, **R.P.**, Gorenstein, Koch, 2003.13905

Subensemble acceptance: $\kappa_2[B_1]$

$$t = \hat{\mu}_B[T, \rho_{B_1}(t)] - \hat{\mu}_B[T, \rho_{B_2}(t)] \qquad (*)$$

$$\rho_{B_1}(t) \;=\; \langle B_1(t)\rangle/(\alpha V) \qquad\qquad \rho_{B_2}(t) \;=\; [B - \langle B_1(t)\rangle]/[(1-\alpha)V]$$

$$\frac{\partial(*)}{\partial t}: \qquad 1 = \left(\frac{\partial \hat{\mu}_B}{\partial \rho_{B1}}\right)_T \left(\frac{\partial \rho_{B1}}{\partial \langle B_1\rangle}\right)_V \frac{\partial \langle B_1\rangle}{\partial t} - \left(\frac{\partial \hat{\mu}_B}{\partial \rho_{B2}}\right)_T \left(\frac{\partial \rho_{B2}}{\partial \langle B_2\rangle}\right)_V \frac{\partial \langle B_2\rangle}{\partial \langle B_1\rangle} \frac{\partial \langle B_1\rangle}{\partial t}$$

GCE susceptibilities: $[\partial \hat{\mu}_B(T, \rho_{B_{1,2}})/\partial \rho_{B_{1,2}}]_T \;=\; [T^3\, \chi_2^B(T, \rho_{B_{1,2}})]^{-1}$

Solve the equation for $\tilde{\kappa}_2$:

$$\tilde{\kappa}_2[B_1(t)] = \frac{V\,T^3}{[\alpha\, \chi_2^B(T, \rho_{B_1})]^{-1} + [(1-\alpha)\, \chi_2^B(T, \rho_{B_2})]^{-1}}$$

***t* = 0:**

$$\kappa_2[B_1] = \alpha\,(1-\alpha)\, V\, T^3\, \chi_2^B$$

Connection between cumulant measured in the subsystem and GCE susceptibility

Higher-order cumulants: iteratively differentiate $\tilde{\kappa}_2$ w.r.t. t

V.Vovchenko., Savchuk, **R.P.**, Gorenstein, Koch, 2003.13905

Subensemble acceptance: Cumulant ratios

Some common cumulant ratios: $\langle B \rangle = \kappa_1, \quad \omega = \frac{\kappa_2}{\kappa_1}, \quad S\sigma = \frac{\kappa_3}{\kappa_2}, \quad \kappa\sigma^2 = \frac{\kappa_4}{\kappa_2}.$

scaled variance
$$\frac{\kappa_2[B_1]}{\kappa_1[B_1]} = (1-\alpha)\,\frac{\chi_2^B}{\chi_1^B},$$

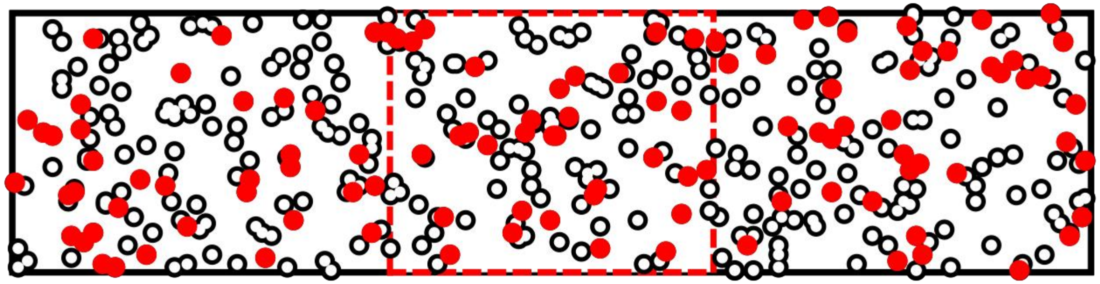
skewness
$$\frac{\kappa_3[B_1]}{\kappa_2[B_1]} = (1-2\alpha)\,\frac{\chi_3^B}{\chi_2^B},$$

kurtosis
$$\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta\left(\frac{\chi_3^B}{\chi_2^B}\right)^2.$$

- Global conservation (α) and equation of state (χ_n^B) effects factorize in cumulants up to the 3rd order, starting from κ_4 not anymore
- $\alpha \to 0$ – GCE limit
- $\alpha \to 1$ – CE limit
- Charge conservation suppresses the magnitude of scaled variance and skewness
- For ideal uncorrelated gas all cumulant ratios equal to 1 and subensemble acceptance is reduced to binomial acceptance

V.Vovchenko, Savchuk, **R.P.**, Gorenstein, Koch, 2003.13905

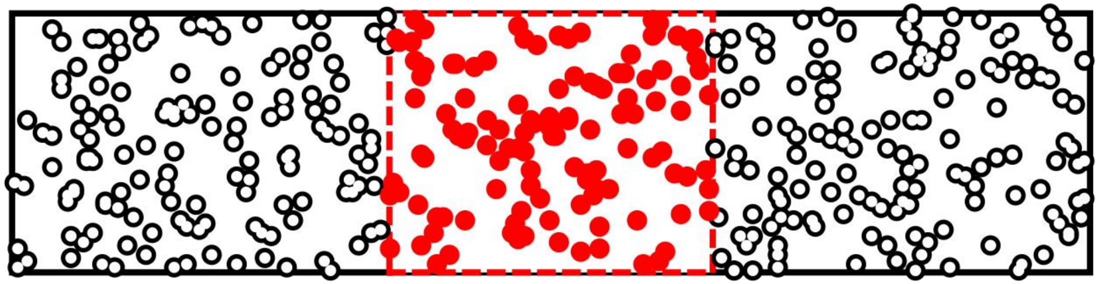
Binomial acceptance vs actual acceptance



Binomial acceptance: accept each particle (charge) with a probability α independently from all other particles

The binomial acceptance does not account for correlations between particles

Subensemble:



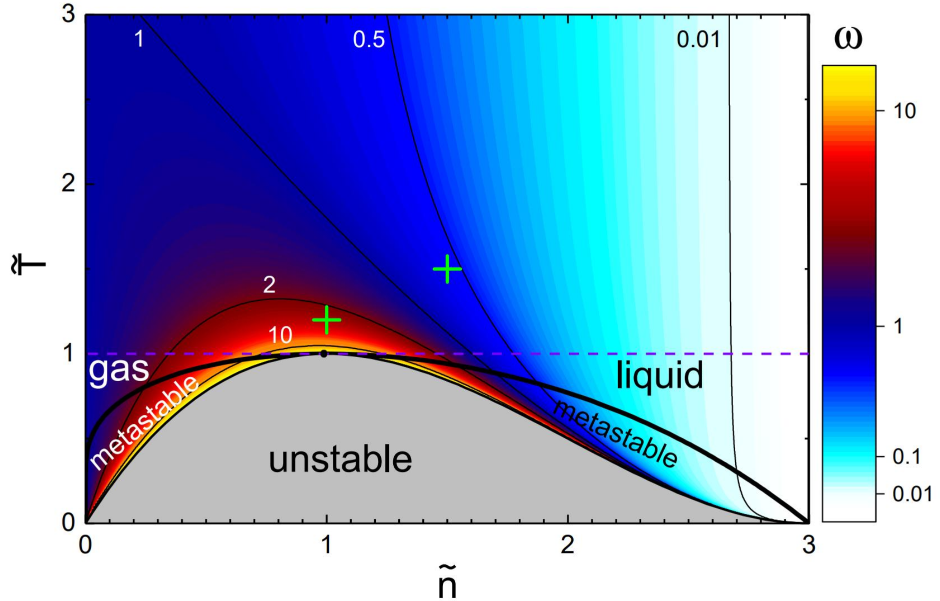
Accounts for correlations, connects measurements with GCE susceptibilities

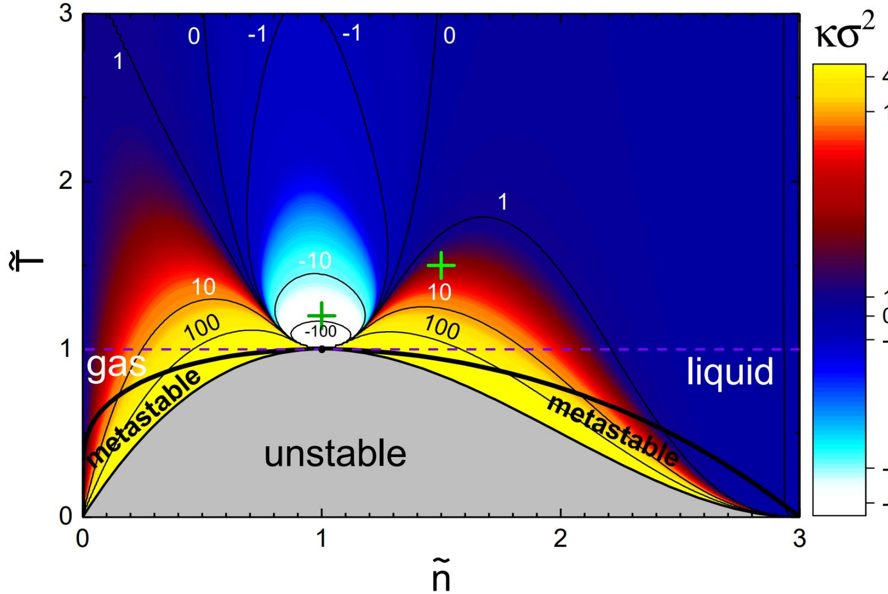
Subensemble acceptance: van der Waals fluid

van der Waals equation of state: first-order phase transition and a critical point

$$Z^{ce}_{vdW}(T, V, N) = Z^{ce}_{id}(T, V - bN, N)\, \exp\left(\frac{aN^2}{VT}\right)\, \theta(V - bN)$$

Rich structures in cumulant ratios close to the CP

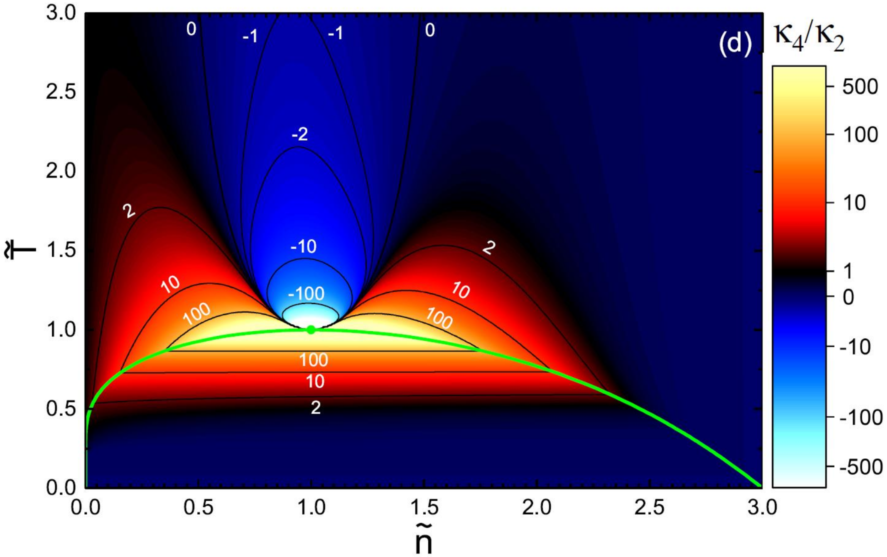




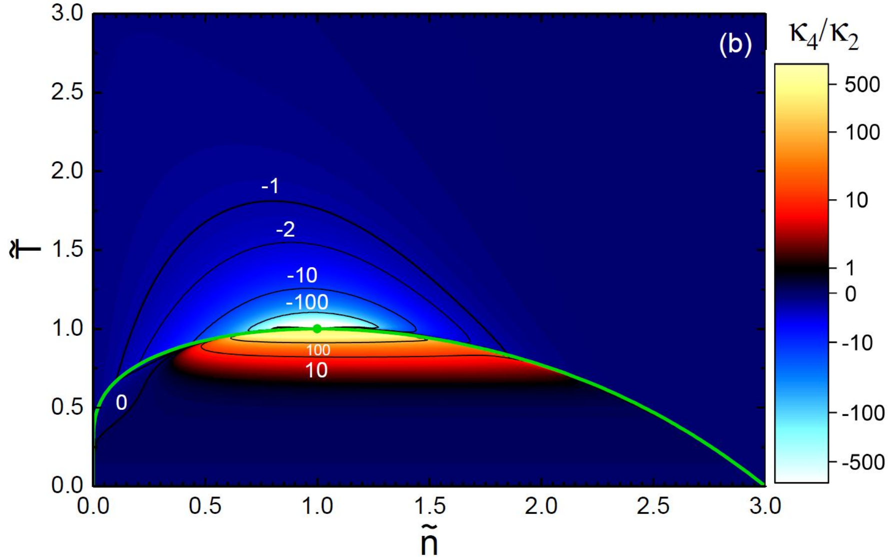
Vovchenko, **R.P.**, Anchishkin, Gorenstein, 1507.06537

Global conservation effects at α = 0.4

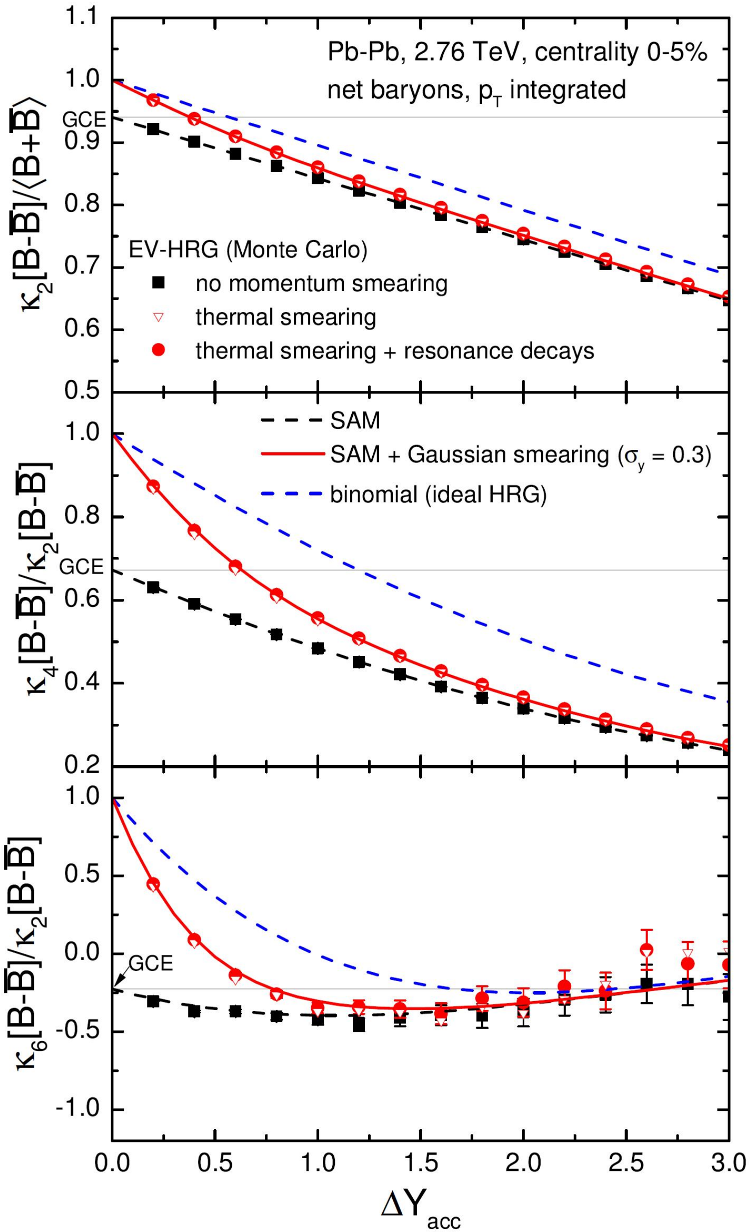
$$\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1 - 3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta\left(\frac{\chi_3^B}{\chi_2^B}\right)^2 .$$



GCE (α → 0)



α = 0.4



Vovchenko, Koch, 2012.09954

Multiple conserved charges

Let us have a vector $Q = (Q_1, \ldots, Q_N)$ of N independent conserved charges in the system.

NOTATIONS

Diagonal and off-diagonal GCE susceptibilities:

$$\chi_{l_1 \ldots l_N}^{Q_1 \ldots Q_N} = \frac{\partial^{l_1 + \ldots + l_N} (p/T^4)}{\partial(\mu_{Q_1}/T)^{l_1} \, \ldots \, \partial(\mu_{Q_N}/T)^{l_N}}, \qquad l_1 + \ldots + l_N = M.$$

Other notation:

$$\hat{\chi}_{i_1 \ldots i_M} = \frac{\partial^M (p/T^4)}{\partial(\mu_{i_1}/T) \, \ldots \, \partial(\mu_{i_M}/T)} = \frac{\hat{\kappa}^{\text{gce}}_{i_1 \ldots i_M}}{VT^3}, \qquad i_j \in 1 \ldots N.$$

$$\chi_4^B \equiv \hat{\chi}_{1111} \, ,$$

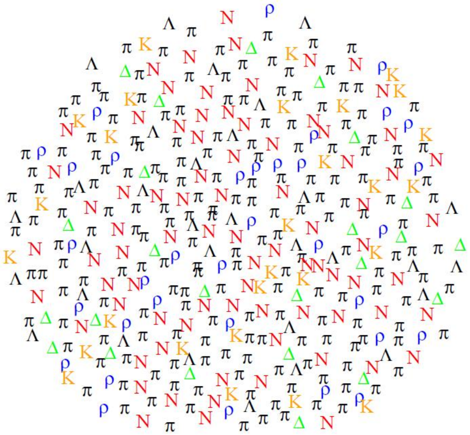
$$\chi_{211}^{BQS} \equiv \hat{\chi}_{1123} = \hat{\chi}_{1132} = \ldots = \hat{\chi}_{3211} \, .$$

https://github.com/vlvovch/SAM

$$Z(T, V, \hat{Q}) = \sum_{\hat{Q}^1} Z(T, \alpha V, \hat{Q}^1) \, Z(T, \beta V, \hat{Q} - \hat{Q}^1)$$

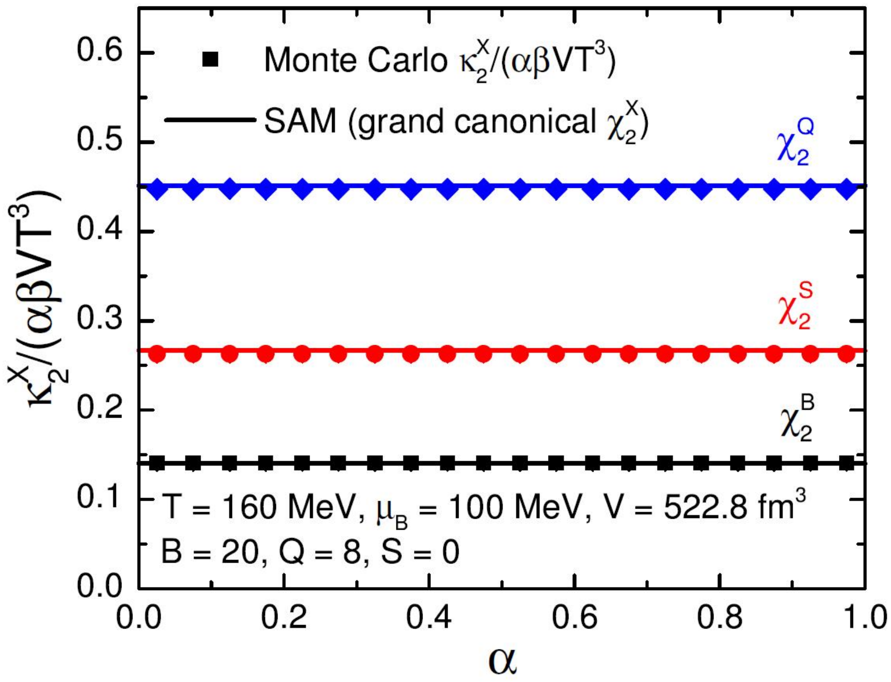
Vovchenko, **R.P.**, Koch, JHEP, 2020

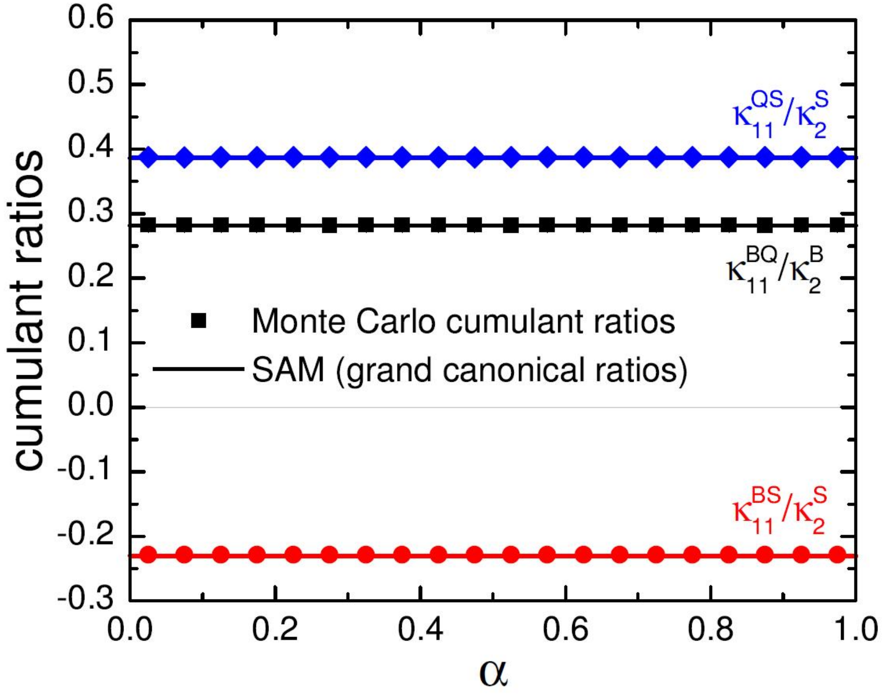
Check in Hadron Resonance Gas



Two sources of correlations:

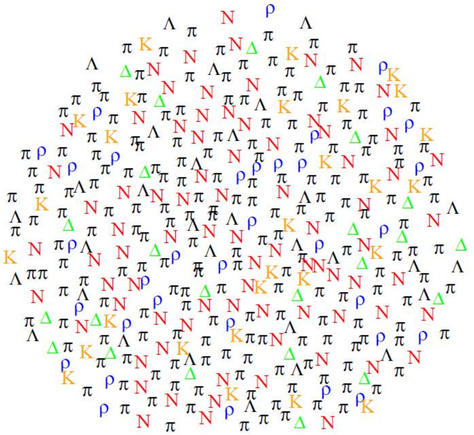
1) Global conservation:different hadron species do carry different values of B,Q,S charges
2) Resonance decays





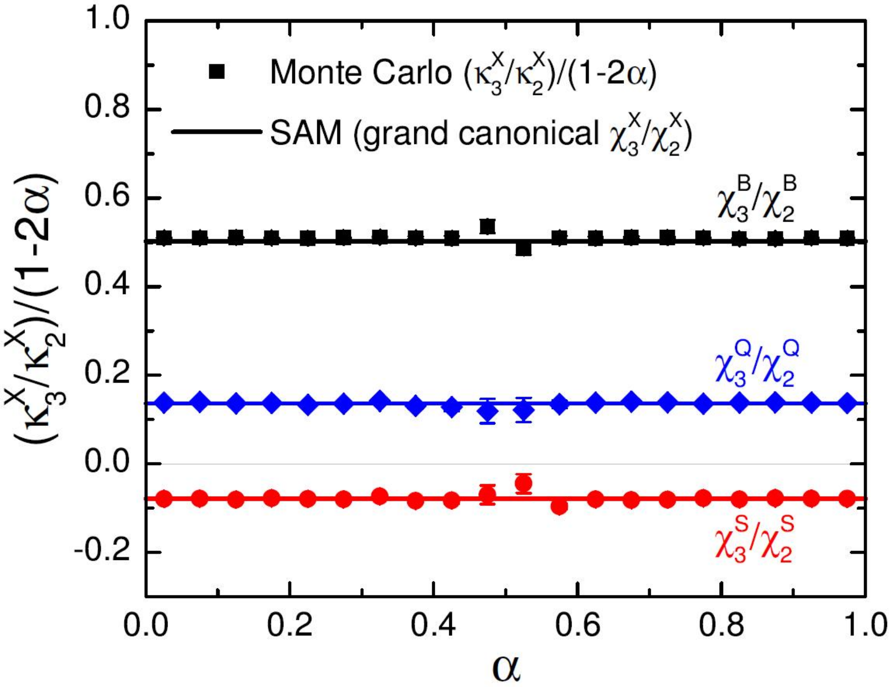
Vovchenko, **R.P.**, Koch, JHEP, 2020

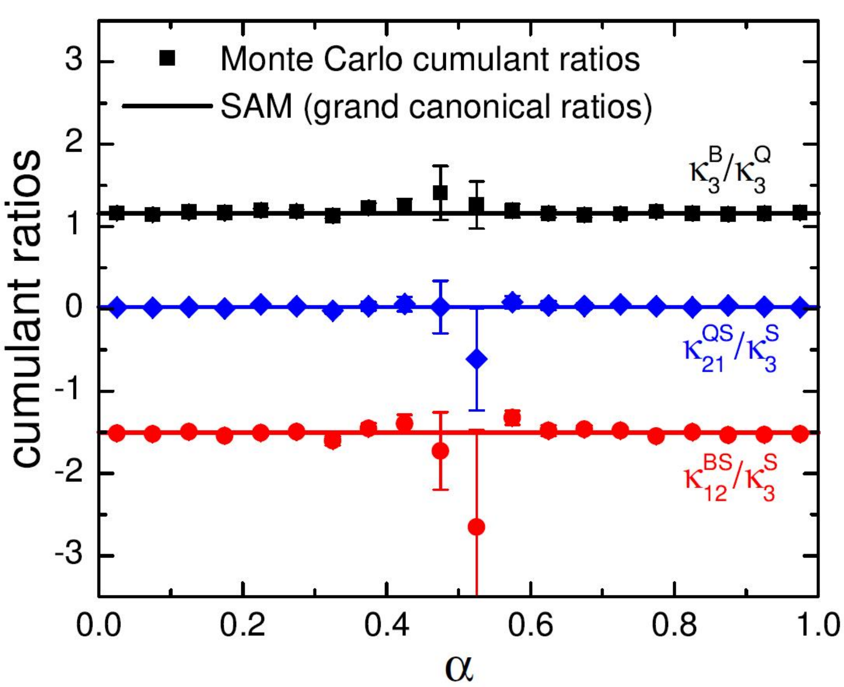
Check in Hadron Resonance Gas (third order)



Two sources of correlations:

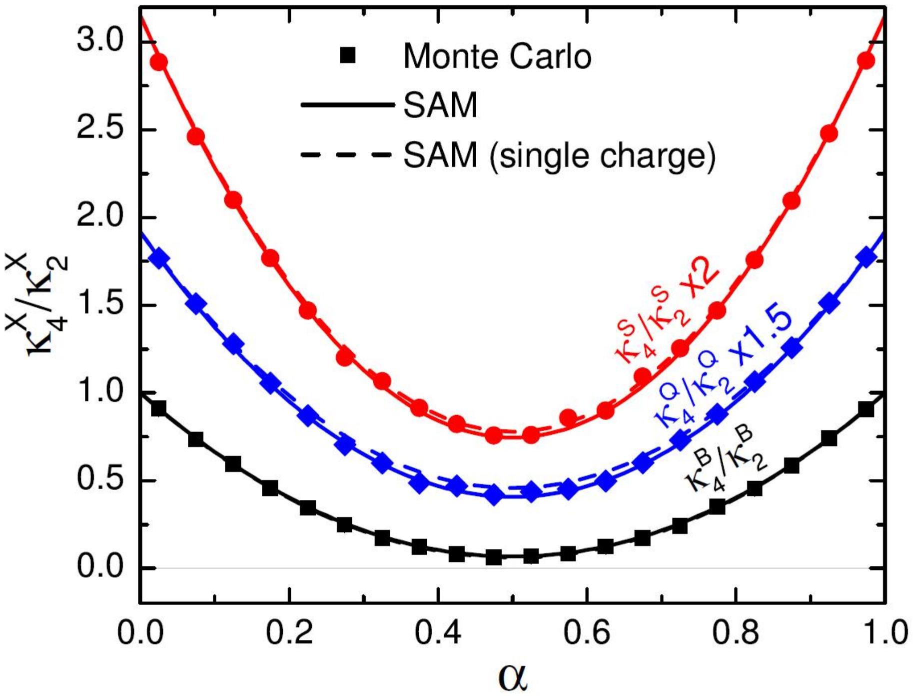
1) Global conservation:different hadron species do carry different values of B,Q,S charges
2) Resonance decays





Vovchenko, **R.P.**, Koch, JHEP, 2020

Check in Hadron Resonance Gas (higher order)



B and S charge conservation do influence the higher order fluctuations of electric charge.

The effect is stronger for non-conserved quantities, even higher cumulants of conserved charges

Vovchenko, **R.P.**, Koch, JHEP, 2020

LHC energies

All odd-order GCE susceptibilities vanish

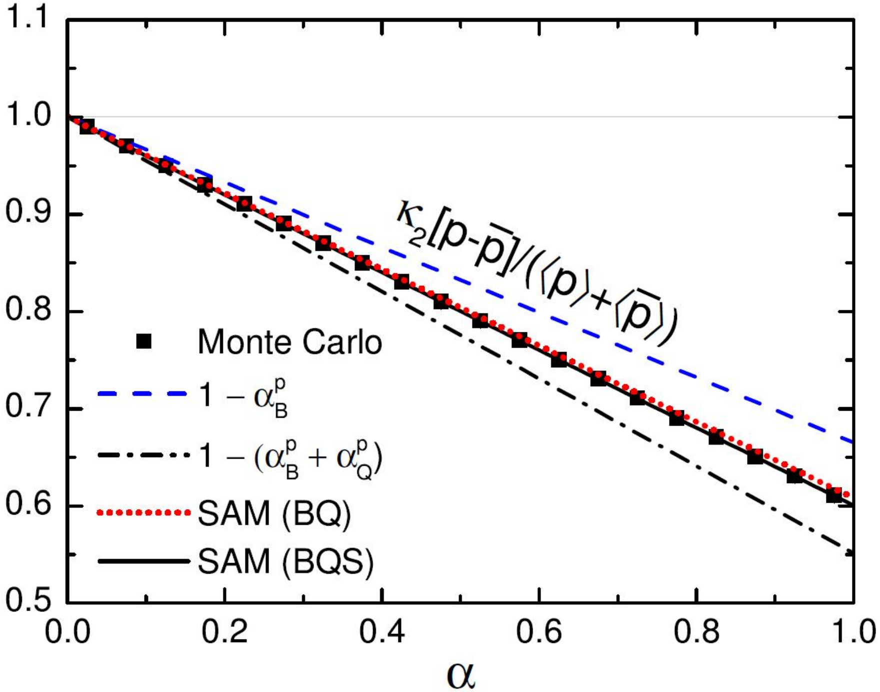
$$\kappa_4[B^1]|_{\hat{\mu}=0} = \alpha V T^3 \beta (1 - 3\alpha\beta) \chi_4^B$$

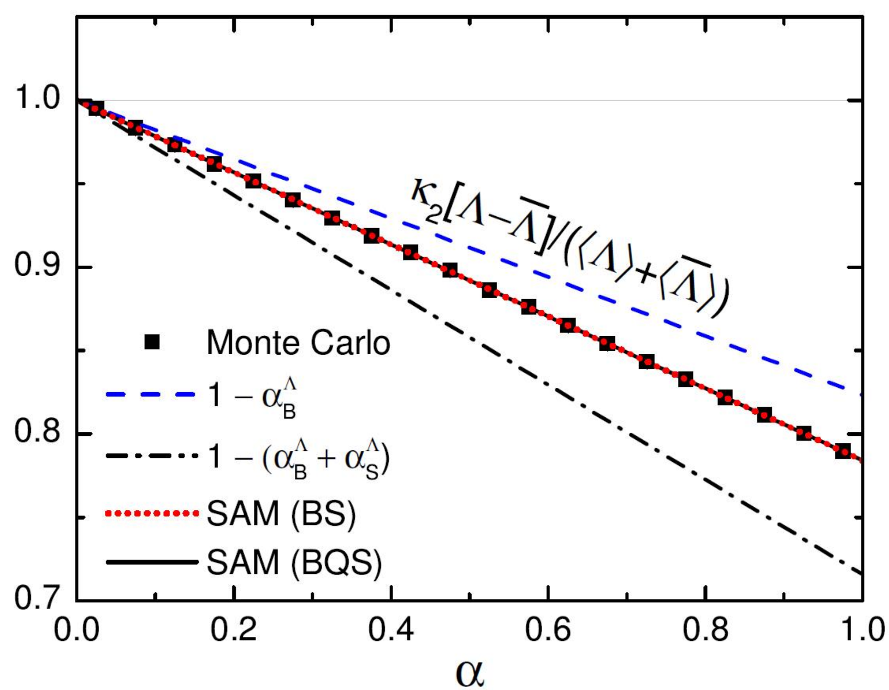
Now cumulants up to fifth order have the same simple α-dependence as for single conserved charge.

$$\begin{aligned}\kappa_6[B^1]\big|_{\hat{\mu}=0} = \alpha V T^3 \beta \Bigg[& (1 - 5\alpha\beta(1-\alpha\beta))\chi_6^B - \frac{10\alpha\beta(1-2\alpha)^2}{D[\hat{\chi}_2]} \times \\ & \Big\{ (\chi_{31}^{BS})^2 [\chi_2^B \chi_2^Q - (\chi_{11}^{BQ})^2] + (\chi_{31}^{BQ})^2 [\chi_2^B \chi_2^S - (\chi_{11}^{BS})^2] \\ & + (\chi_4^B)^2 [\chi_2^Q \chi_2^S - (\chi_{11}^{QS})^2] + 2\chi_{31}^{BS} \chi_{31}^{BQ} (\chi_{11}^{BS} \chi_{11}^{BQ} - \chi_2^B \chi_{11}^{QS}) \\ & + 2\chi_{31}^{BS} \chi_4^B (\chi_{11}^{QS} \chi_{11}^{BQ} - \chi_2^Q \chi_{11}^{BS}) + 2\chi_{31}^{BQ} \chi_4^B (\chi_{11}^{QS} \chi_{11}^{BS} - \chi_2^B \chi_{11}^{BQ}) \Big\} \Bigg] .\end{aligned}$$

Vovchenko, **R.P.**, Koch, JHEP, 2020

Net-proton and net-Λ fluctuations





$$\alpha_B^p = \frac{\langle N_p^{\rm acc} \rangle + \langle N_{\bar{p}}^{\rm acc} \rangle}{\langle N_B^{4\pi} \rangle + \langle N_{\bar{B}}^{4\pi} \rangle}$$

Vovchenko, **R.P.**, Koch, JHEP, 2020