Anomalous enhancement of dilepton production due to soft modes in dense quark matter

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Outline

I. Dilepton production rates due to fluctuations of **CSC phase transition**

II. Dilepton production rates due to fluctuations of **QCD critical point**

III. Discussion / Summary
Dilepton production rates from soft modes of CSC phase transition
High density region in QCD phase diagram

QCD phase diagram

Quark-Gluon Plasma

Hadron

CSC

1st order

QCD critical point

100MeV

T

μ

Experiments for high density region with high statistics

Ongoing
- BES II at RHIC
- NA61/SHINE at LHC
- HADES at GSI

Future
- FAIR at GSI
- NICA at JINR
- J-PARC-HI (planned)
How to observe CSC at HIC?

**Problem I**
Matter produced by HIC is high temperature. → Is CSC realized?

**Problem II**
CSC can exist only in **early stage**. → Hadrons are bad as probes of CSC.

**Solution**
- Focus on diquark fluctuation... This develops at $T > T_c$.
- Focus on dilepton... This doesn’t interact strongly.

Kitazawa, Koide, Kunihiro, Nemoto, PTP(2005); Kunihiro, Kitazawa, Nemoto, 0711.4429
The purpose of our study

Through “Diquark fluctuations” and “Dilepton”, we research the observability of CSC (2SC) at HIC. Let's calculate the effect of the fluctuations on the dilepton production rate (DPR).


Need the “photon self-energy”

\[
\frac{d^4\Gamma}{dk^4} = \frac{\alpha}{12\pi^4} \frac{1}{k^2} \frac{1}{e^{\beta\omega} - 1} g_{\mu\nu} \text{Im} \Pi^{\mu\nu}(k)
\]
Dilepton production due to diquark fluctuations

\[ \mathcal{L} = \bar{\psi} i \gamma_5 \psi + \mathcal{L}_S + \mathcal{L}_C \]

\[ \mathcal{L}_S = G_S \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \tau \psi)^2 \right] \]

\[ \mathcal{L}_C = G_C (\bar{\psi} i \gamma_5 \tau_2 \lambda_A \psi^C)(\bar{\psi}^C i \gamma_5 \tau_2 \lambda_A \psi) \]

Parameters: \( G_S = 5.01 \text{MeV}, \Lambda = 650 \text{MeV} \)

2-flavor NJL model

Propagator of soft modes

\[ \Xi(k, \omega) = \]

\[ = G_C + \]

MFA

Kitazawa, Koide, Kunihiro, Nemoto (2002)

Kitazawa, Koide, Kunihiro, Nemoto (2005)
Construction of photon self-energy

Thermodynamic potential: One loop of diquark fluctuations

\[ \begin{array}{c}
\text{Aslamazov-Larkin (AL) term} \\
\text{Maki-Thompson (MT) term} \\
\text{Density of states (DOS) term}
\end{array} \]

Two photons are attached to the potentials.

These terms are well known in condensed matter theory.
These terms are well known in condensed matter theory.
\[ \Xi^R (k, \omega) = \frac{G_C}{1 + G_C Q^R (k, \omega)} = \frac{1}{G_C^{-1} + \alpha(k) + c(k) \omega} \]

\[ 1 + G_C Q^R (0, 0) = 0 \text{ at } T = T_C \]

Thouless criterion

Coefficients are determined by NJL.

\[ \alpha(k) = [\Xi(k, 0)]^{-1}, \quad c(k) = \frac{\partial [\Xi(k, 0)]^{-1}}{\partial \omega} \]

This approximation is valid around \( T_C \) in the low energy-momentum region.
Approximation with Ward Identity (W-I) for vertices

**AL:** \( \Pi^{\mu\nu}_{AL}(k) = \gamma^\mu \gamma^\nu \)

= \( \int \frac{d^4q}{(2\pi)^4} \Gamma^\mu(q, q + k) \Xi(q + k) \Gamma^\mu(q + k, q) \Xi(q) \)

\( \downarrow \) Vertex of AL \( \uparrow \)

**W-I of photon self-energy**

- **time component**
  \( \Pi^{R00}(k) = \frac{k^2 \Pi^{R11}(k)}{k_0^2} \)
- **longitudinal component**

at \( k = (k_0, |k|, 0, 0) \)

**W-I of vertex of AL**

\( k_\mu \Gamma^\mu(q, q + k) = \Xi^{-1}(q + k) - \Xi^{-1}(q) \)

Compare the lowest order terms of \( k \) and \( \omega \).

\( \Gamma^i(q, q + k) \propto \frac{a(q + k) - a(q)}{(q + k)^2 - q^2} (2q + k)^i \)
Approximation with Ward Identity (W-I) for vertices

Approximated vertices are all real.

Need $g_{\mu\nu} \text{Im} \Pi^{\mu\nu}(k)$ for the DPR:

$$\frac{d^4 \Gamma}{d k^4} = \frac{\alpha}{12 \pi^4} \frac{1}{k^2} \frac{1}{e^{\beta \omega} - 1} g_{\mu\nu} \text{Im} \Pi^{R\mu\nu}(k)$$

Imaginary part of MT and DOS term cancel.
Consistent with the metallic SC !!!

$$\text{Im}(\text{MT term} + \text{DOS term}) = 0$$

Only AL term is necessary to calculate the DPR.
Enhancement at low $\omega$ & $k$ as $T \to T_c$

Expected from the property of soft modes

Dilepton production rate ($G_C = 0.7 G_S$)
Invariant mass spectrum

\[
\frac{d\Gamma}{dM^2} = \int d^3k \left. \frac{1}{2\omega} \frac{d^4\Gamma}{dk^4} \right|_{\omega = \sqrt{k^2 + M^2}}
\]

Signal for observation of CSC!?
Dilepton production rates from soft modes of QCD critical point
Phase diagram

\[ \mathcal{L} = \bar{\psi}i(\gamma^\mu \partial_\mu - m)\psi + G_S[\bar{\psi}\psi + (\bar{\psi}i\gamma_5\tau\psi)^2] \]

\( G_S = 5.01 \text{MeV}, \Lambda = 650 \text{MeV}, m = 4 \text{MeV} \)

\( T_C = 35.071 \text{MeV}, \mu_C = 322.203 \text{MeV} \)

We calculate at each black point.

Soft modes

\[ \Xi(k, \omega) = \frac{G_C}{1 + G_C Q^R(k, \omega)} \]

\( \text{Im} \Xi^R \text{ in energy-momentum plane} \)

The discontinuity on the light cone.

The soft mode in space-like region is the p-h mode. Fujii and Ohtani (2004)
Dilepton production rate ($\mu = \mu_C$)

**Enhancement at low $\omega$ & $k$ as $T \rightarrow T_C$**

Expected from the property of **soft modes**
Dilepton production rate ($\mu = \mu_c$)

Enhancement at low $\omega$ & $k$ as $T \to T_C$

$\sim$ Expected from the property of soft modes
**μ-dependence** \((T = T_C, k = 0)\)

- **μ < μC**
  - **Soft modes**
  - **free (massive)**

  \[ \Gamma_f^\mu = \text{diagram} \]

  \[ \Gamma_f^\mu = 0 \]

- **μ > μC**
  - **Soft modes**
  - **free (massive)**

  \[ \Gamma_f^\mu (q, q + k) \neq 0 \]

  at \( μ \neq 0 \).

**Competition between effects of “soft mode” & “growth of interaction as μ is bigger”**

**Charge conjugation symmetry violation at μ ≠ 0**
Discussion / Summary
Mechanism of enhancement at low $\omega$

One of production processes caused by soft modes

$\omega = \omega_1 - \omega_2$
$|k| = |q_1 - q_2|$

$q_1 = (q_1, \omega_1)$
$q_2 = (q_2, \omega_2)$

$|q_1| > \omega_1$
$|q_2| > \omega_2$
$\omega > |k|$

* Soft modes have strong support in space-like.
* Dileptons are produced in time-like.
* MT&DOS cancel. $\rightarrow$ Scattering in AL.

Scattering process in free quark gases

$q_1 = (q_1, \omega_1)$
$q_2 = (q_2, \omega_2)$

$\omega = \omega_1 - \omega_2$
$|k| = |q_1 - q_2|$

This occurs only in space-like.
$\rightarrow q\bar{q}$ annihilation contributes dilepton production in time-like
Kubo formula:

$$\rho(p, \omega) = \frac{\sigma \omega (\omega^2 - p^2)}{(\tau \omega^2 - D p^2)^2 + \omega^2} + 2 \frac{\sigma \omega}{\tau^2 \omega^2}$$

$$g_{\mu\nu} \text{Im} \Pi^{R\mu\nu}(p, \omega)$$

Diffusion constant

$$\sigma = \frac{1}{3} \frac{\partial}{\partial \omega} \rho(0, \omega) \bigg|_{\omega=0}, \tau = \sqrt{-\frac{1}{18\sigma} \frac{\partial^3}{\partial^3 \omega} \rho(0, \omega) \bigg|_{\omega=0}}$$

For the conductivity,

$$\sigma \propto (T - T_c)^{-\frac{1}{2}} : \text{for CSC}$$

$$\sigma \propto (T - T_c)^{-2} : \text{for QCD CP}$$
Summary

We calculated the effect of soft modes around CSC phase transition and QCD critical point on the DPR.

- Ward Identity for photon self-energies (AL, MT and DOS terms)
- We approximated photon self-energies based on TDGL theory.
- Enhancement of dilepton production rates at low $\omega$ by soft modes
- The mechanism of the enhancement is the scattering process of soft modes.

Outlook

Are the enhancement at low-M observable??

- Apply our result to dynamical model.
- Consider the competition with other dilepton production process. (Dalitz decay etc..)