Anomalous enhancement of dilepton production due to soft modes in dense quark matter

Toru Nishimura
Osaka University
Collaborators : **Masakiyo Kitazawa, Teiji Kunihiro**

SQM 2022 (Jun 15. 2022)

Outline

I. Dilepton production rates due to fluctuations of CSC phase transition

Introduction
Formulation
Results

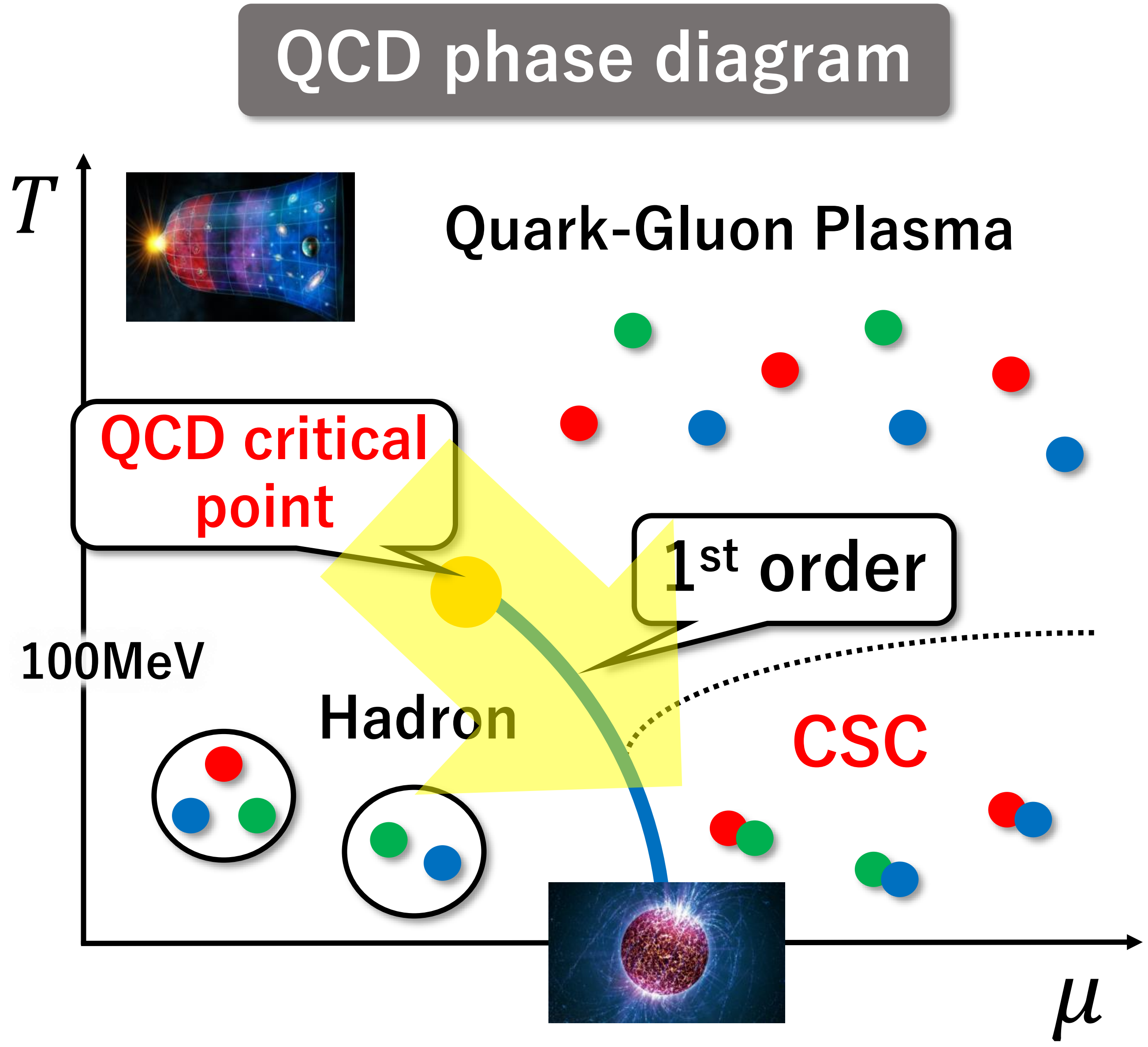
II. Dilepton production rates due to fluctuations of QCD critical point

Introduction
Formulation
Results

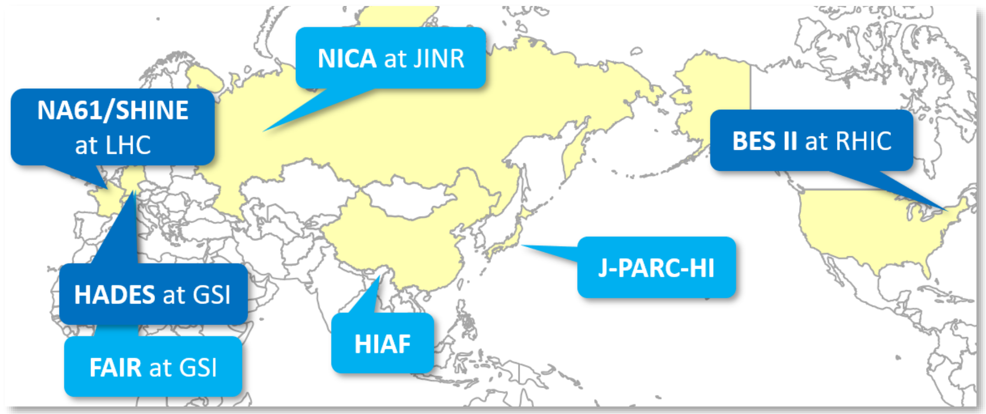
III. Discussion / Summary

Dilepton production rates from soft modes of **CSC phase transition**

High density region in QCD phase diagram



Experiments for high density region with high statistics



Ongoing

- **BES II** at RHIC
- **NA61/SHINE** at LHC
- **HADES** at GSI

Future

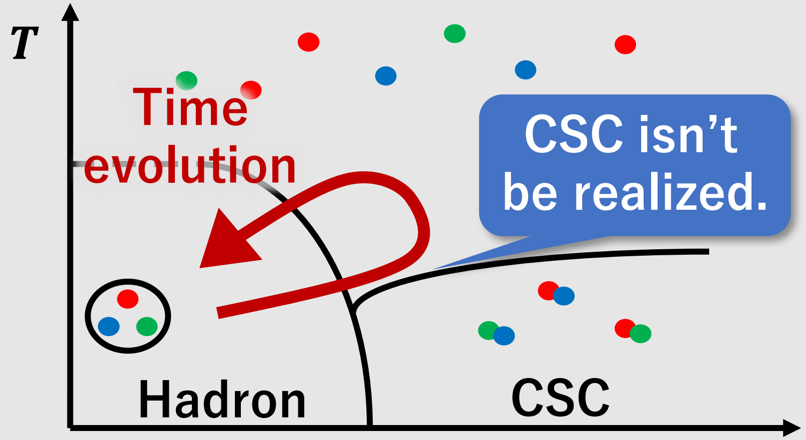
- **FAIR** at GSI
- **NICA** at JINR
- **J-PARC-HI** (planned)

How to observe CSC at HIC?

Probrem I

Matter produced by HIC is high temperature.

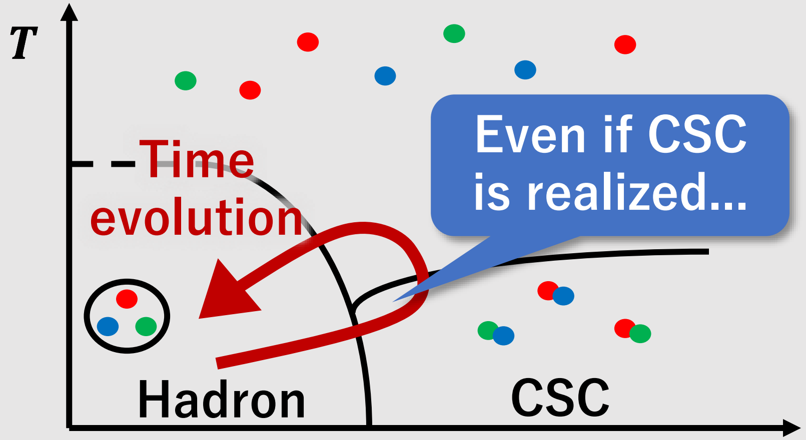
→Is CSC realized?



Probrem II

CSC can exist only in **early stage.**

→Hadrons are bad as probes of CSC.



Solution

Focus on diquark fluctuation···This develops at $T > T_C$

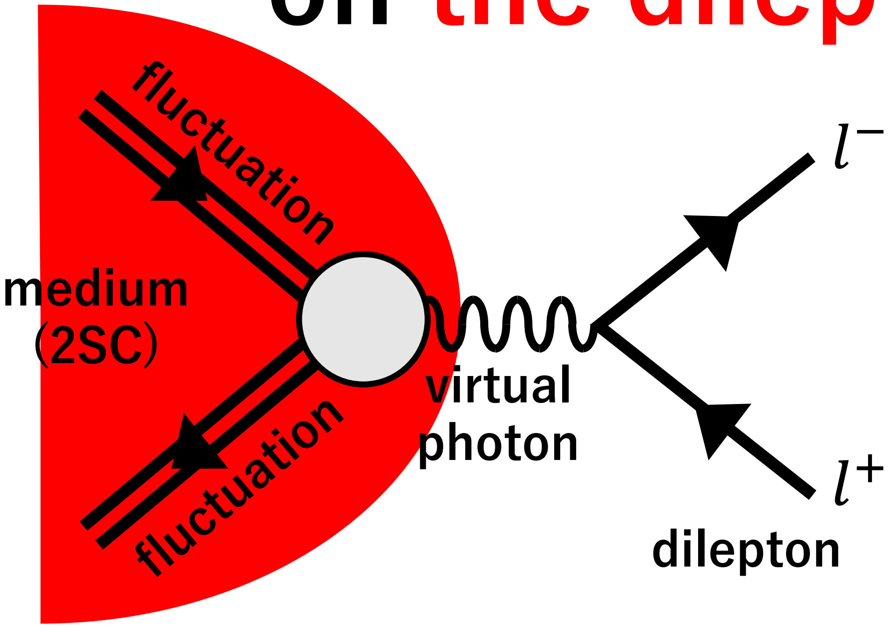
Focus on dilepton···This doesn't interact strongly.

Kitazawa, Koide, Kunihiro, Nemoto, PTP(2005), Kunihiro, Kitazawa, Nemoto, 0711.4429

The purpose of our study

Through **"Diquark fluctuations"** and **"Dilepton"**,
We research the observability of CSC (**2SC**) at HIC.

Calculate the effect of the fluctuations on the dilepton production rate (DPR).



Need the "photon self-energy"

$$\frac{d^4\Gamma}{dk^4} = \frac{\alpha}{12\pi^4}\frac{1}{k^2}\frac{1}{e^{\beta\omega}-1}g_{\mu\nu}\mathrm{Im}\,\Pi^{R\mu\nu}(k)$$

Dilepton production due to diquark fluctuations

TN, Kitazawa, and Kunihiro. *arXiv:2201.01963* (2022).

2-flavor NJL model

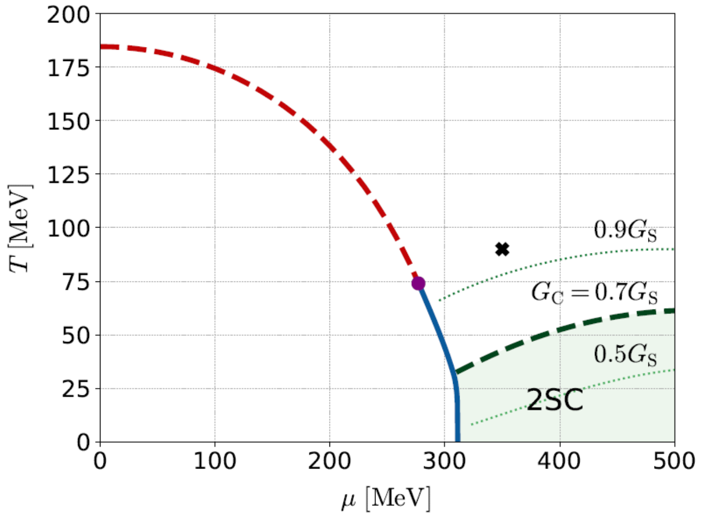
$$\mathcal{L} = \bar{\psi} i\partial\psi + \mathcal{L}_S + \mathcal{L}_C$$

$$\mathcal{L}_S = G_S[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2]$$

$$\mathcal{L}_C = G_C(\bar{\psi}i\gamma_5\tau_2\lambda_A\psi^C)(\bar{\psi}^C i\gamma_5\tau_2\lambda_A\psi)$$

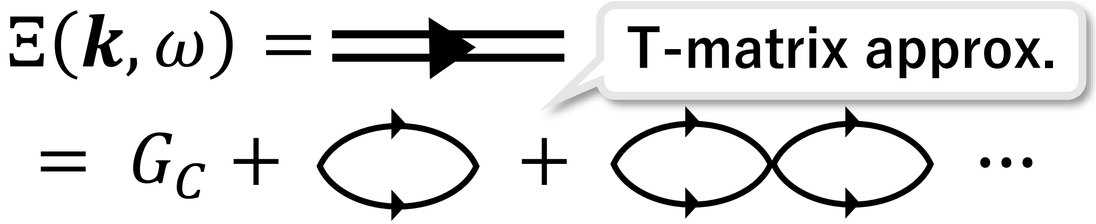
Parameters： $G_S = 5.01\text{MeV}, \Lambda = 650\text{MeV}$

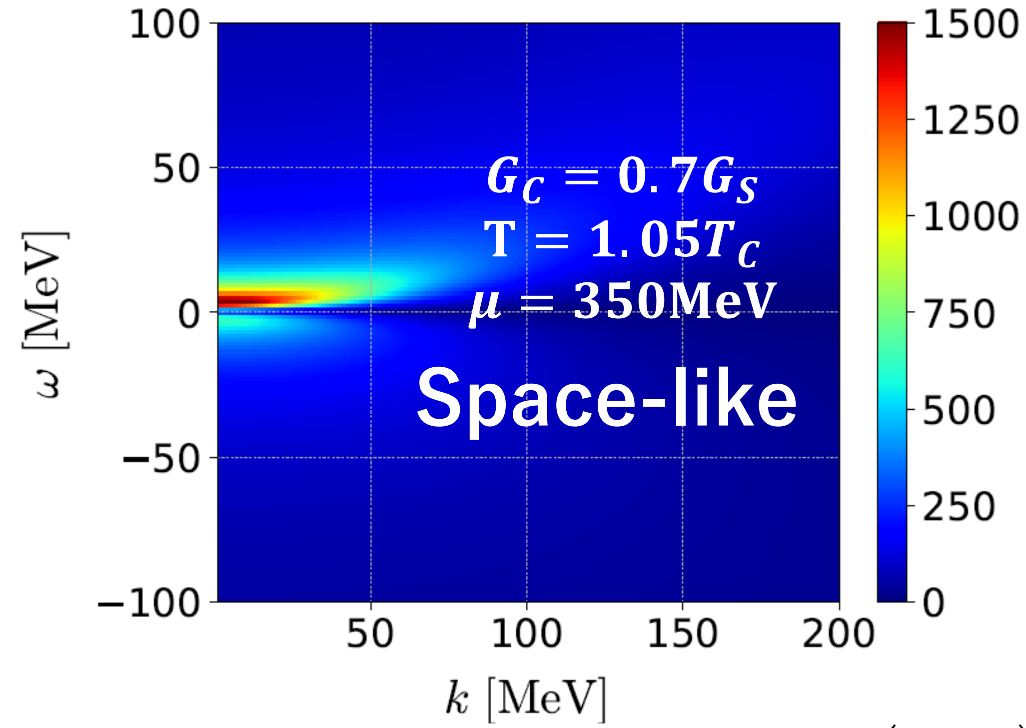
MFA



Kitazawa, Koide, Kunihiro, Nemoto (2002)

Propagator of soft modes

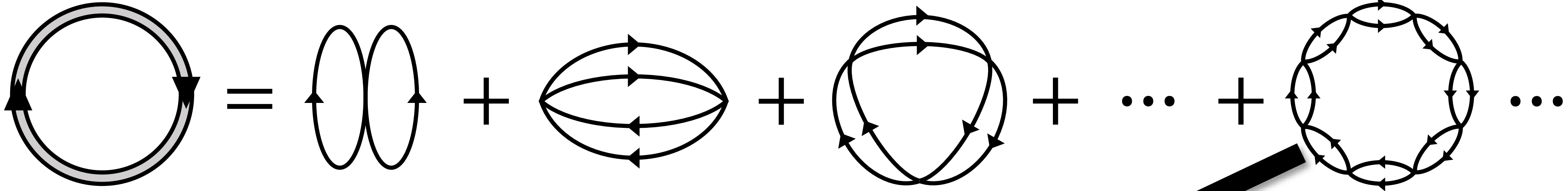




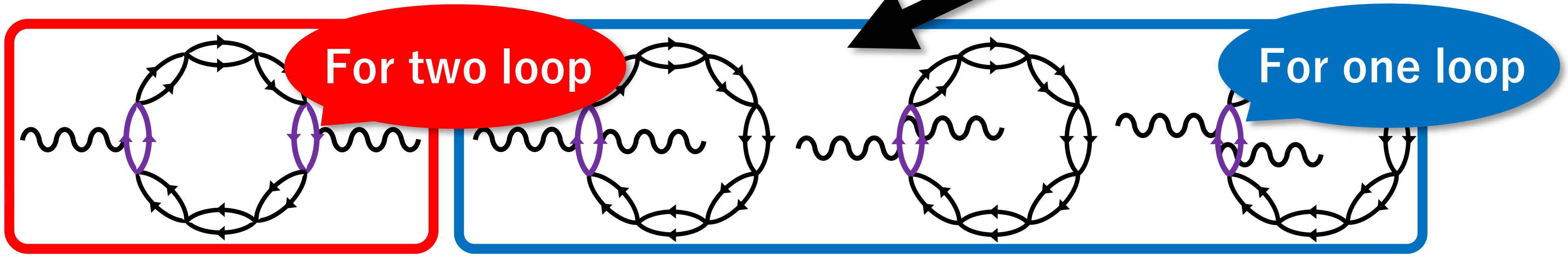
Kitazawa, Koide, Kunihiro, Nemoto (2005)

Construction of photon self-energy

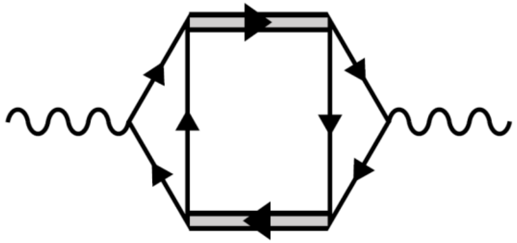
Thermodynamic potential : One loop of diquark fluctuations



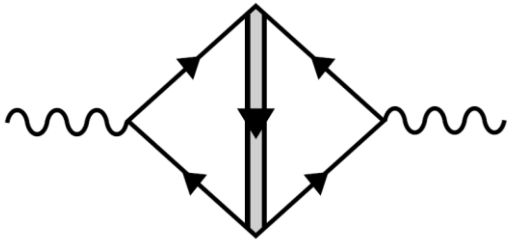
Two photons are attached to the potentials.



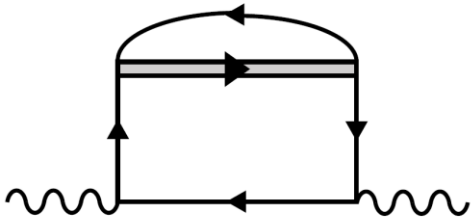
Aslamazov-Larkin (AL) term

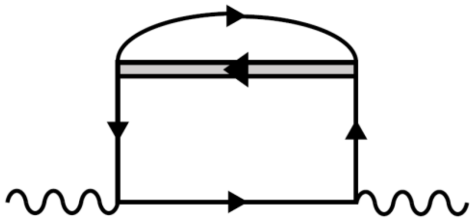


Maki-Thompson (MT) term



Density of states (DOS) term

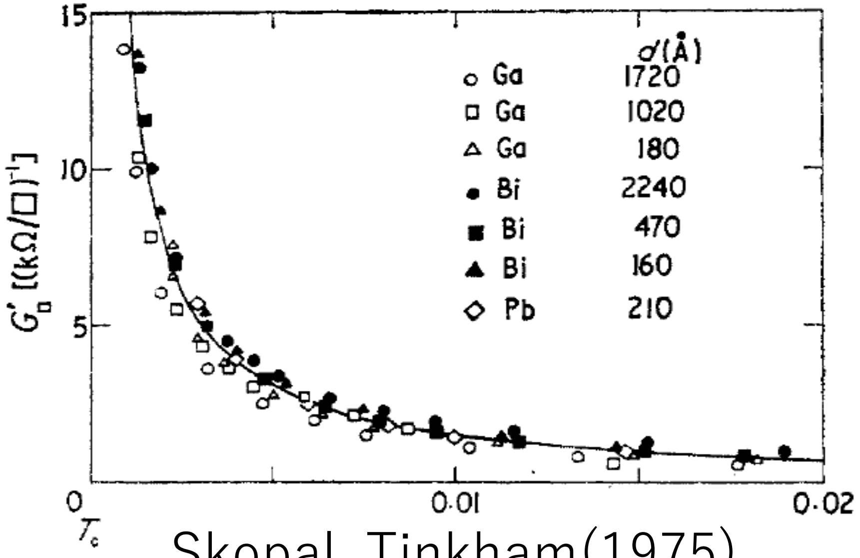




These terms are well known in condensed matter theory.

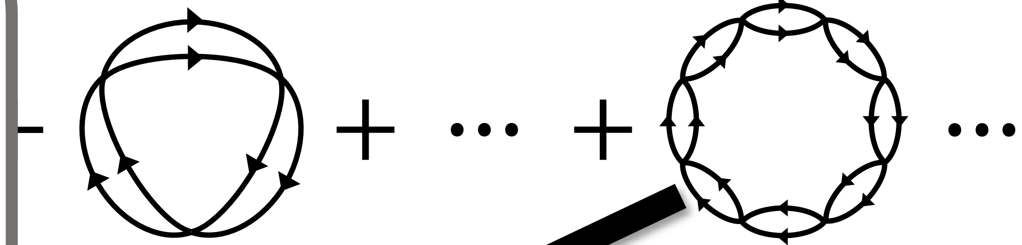
Construction of photon self-energy

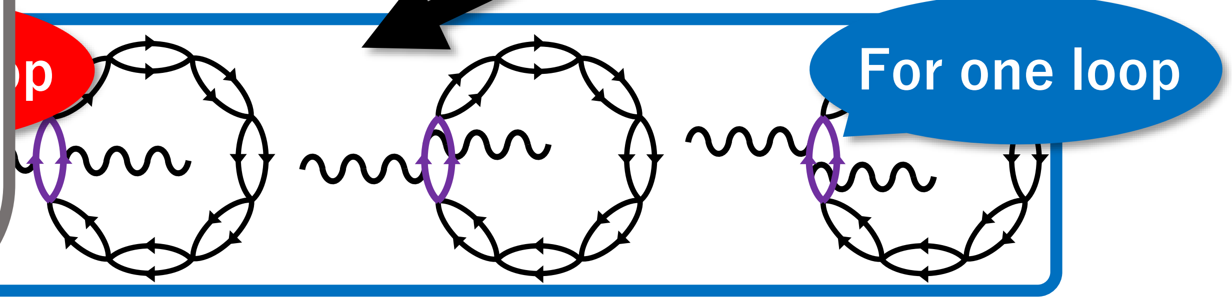
Electric conductivity in metallic SCs



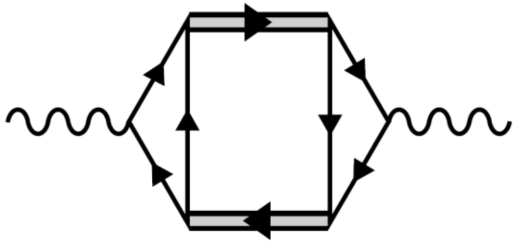
Skopal, Tinkham(1975)

One loop of diquark fluctuations

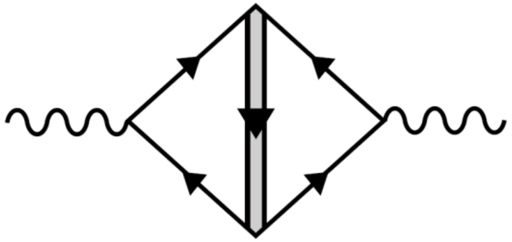




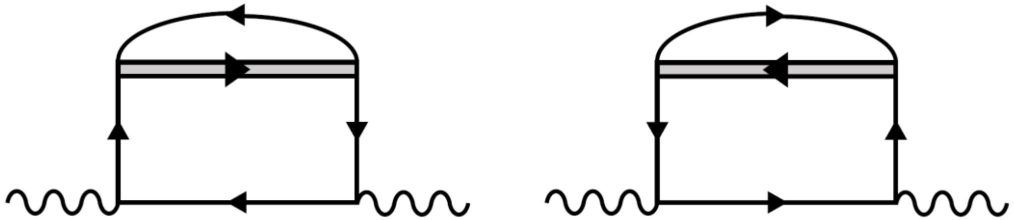
Aslamazov-Larkin (AL) term



Maki-Thompson (MT) term



Density of states (DOS) term



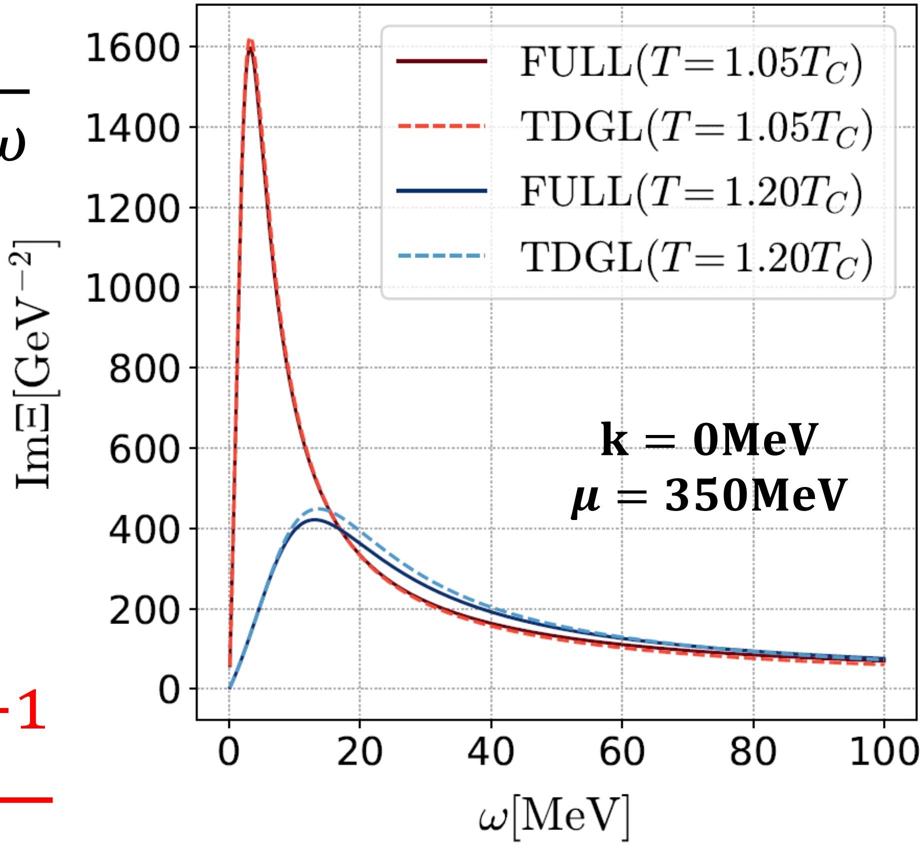
These terms are well known in condensed matter theory.

Time-depending Ginzburg-Landau (TDGL) approx.

$$\Xi^R(\boldsymbol{k},\omega) = \frac{G_C}{1+G_C Q^R(\boldsymbol{k},\omega)} = \frac{1}{G_C^{-1} + a(\boldsymbol{k}) + c(\boldsymbol{k})\omega}$$

$1 + G_C Q^R(\mathbf{0},0) = 0 \ at \ T = T_C$

Thouless criterion

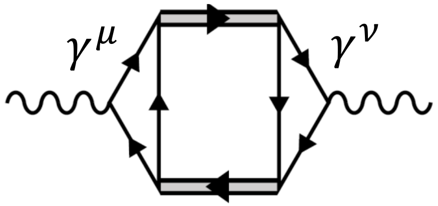


Coefficients are determined by NJL.

$$a(\boldsymbol{k}) = [\Xi(\boldsymbol{k},0)]^{-1}, \ \ c(\boldsymbol{k}) = \frac{\partial[\Xi(\boldsymbol{k},0)]^{-1}}{\partial\omega}$$

This approximation is valid around T_C in the low energy-momentum region.

Approximation with Ward Identity (W-I) for vertices

AL : $\Pi^{\mu\nu}_{\mathrm{AL}}(k) =$ 

$$= \int \frac{d^4q}{(2\pi)^4} \Gamma^\mu(q, q+k)\Xi(\mathrm{q}+\mathrm{k})\Gamma^\mu(q+k\ q)\Xi(\mathrm{q})$$

Vertex of AL

W-I of photon self-energy

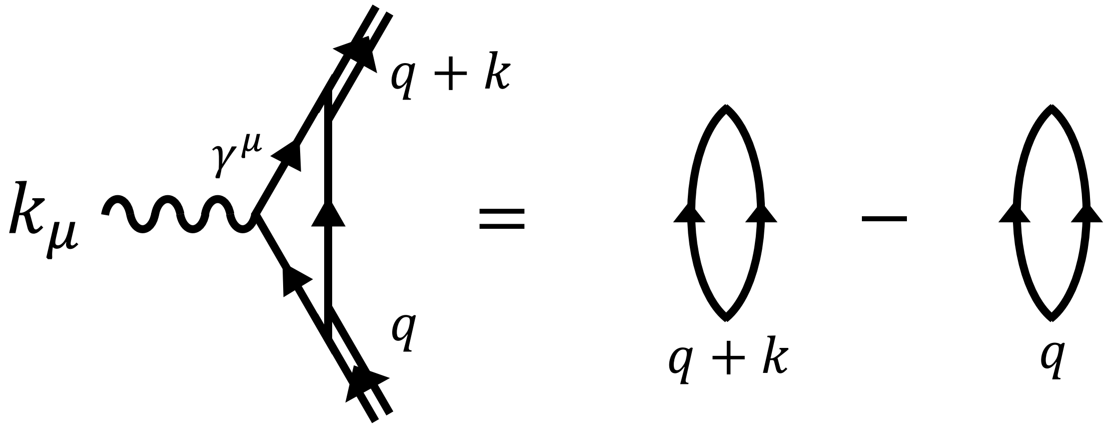
time component — longitudinal component

$$\Pi^{R00}(k) = \frac{k^2 \Pi^{R11}(k)}{k_0^2}$$

at $\mathrm{k} = (\mathrm{k}_0, |\mathrm{k}|, 0, 0)$

Need only space components of vertices Γ^i.

W-I of vertex of AL



$$k_\mu \Gamma^\mu(q, q+k) = \Xi^{-1}(q+k) - \Xi^{-1}(q)$$

Compare the lowest order terms of $\boldsymbol{k}$ and $\boldsymbol{\omega}$.

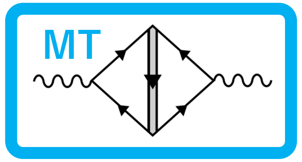
$$\Gamma^i(q, q+k) \propto \frac{a(\boldsymbol{q}+\boldsymbol{k}) - a(\boldsymbol{q})}{(\boldsymbol{q}+\boldsymbol{k})^2 - \boldsymbol{q}^2}(2q+k)^i$$

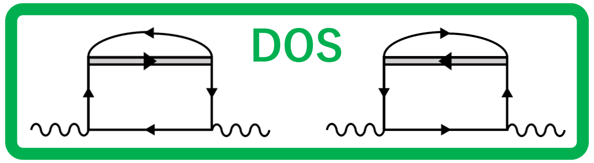
Approximation with Ward Identity (W-I) for vertices

Approximated vertices are all real.

Need $g_{\mu\nu}\mathrm{Im}\,\Pi^{\mu\nu}(k)$ for the DPR : $\dfrac{d^4\Gamma}{dk^4} = \dfrac{\alpha}{12\pi^4}\dfrac{1}{k^2}\dfrac{1}{e^{\beta\omega}-1}g_{\mu\nu}\mathrm{Im}\,\Pi^{R\mu\nu}(k)$

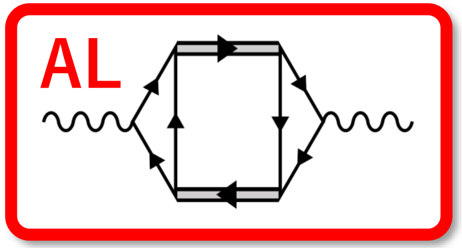
Imaginary part of MT and DOS term cancel.
Consistent with the metallic SC !!!

$\mathrm{Im}($ 

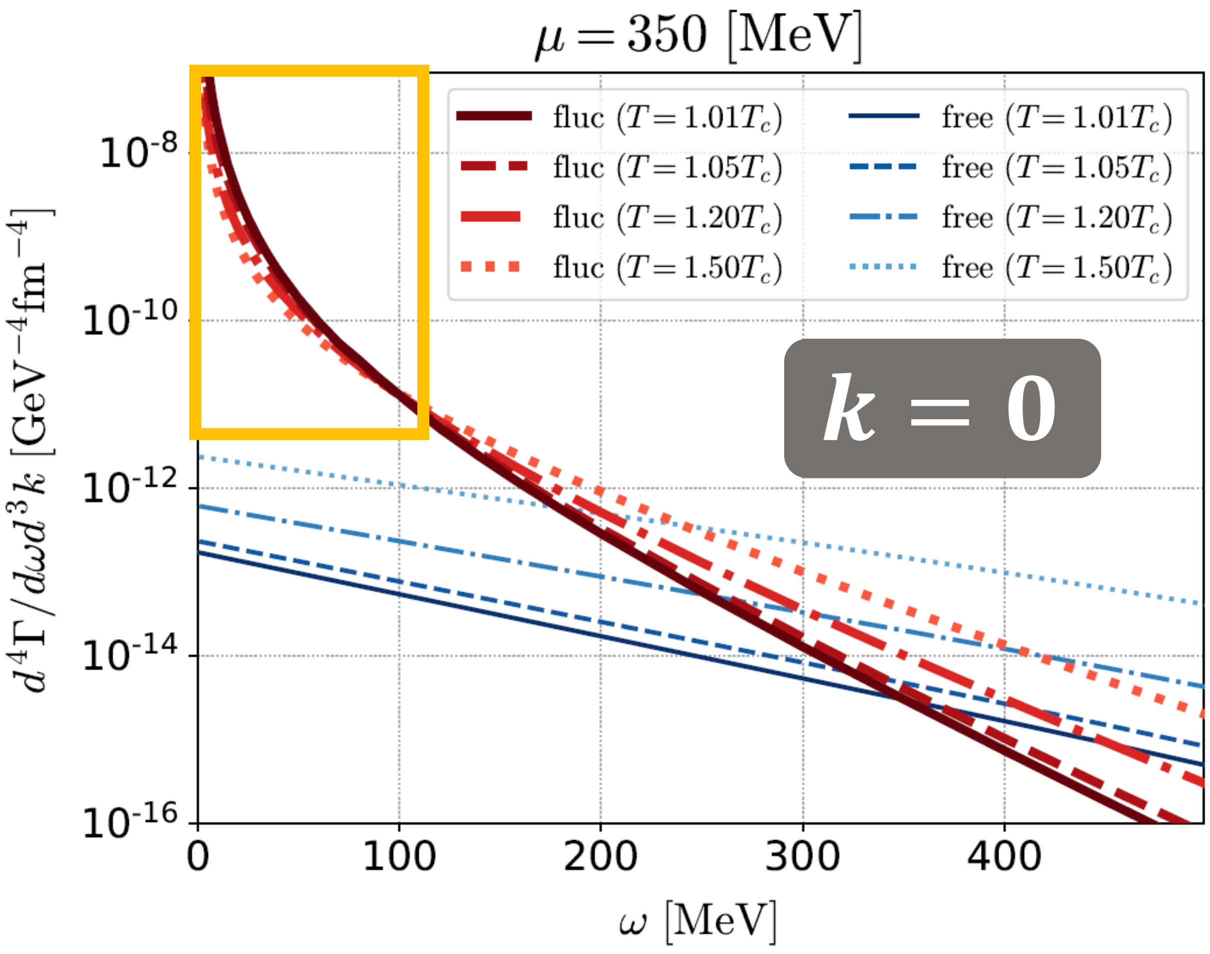
$+$ 

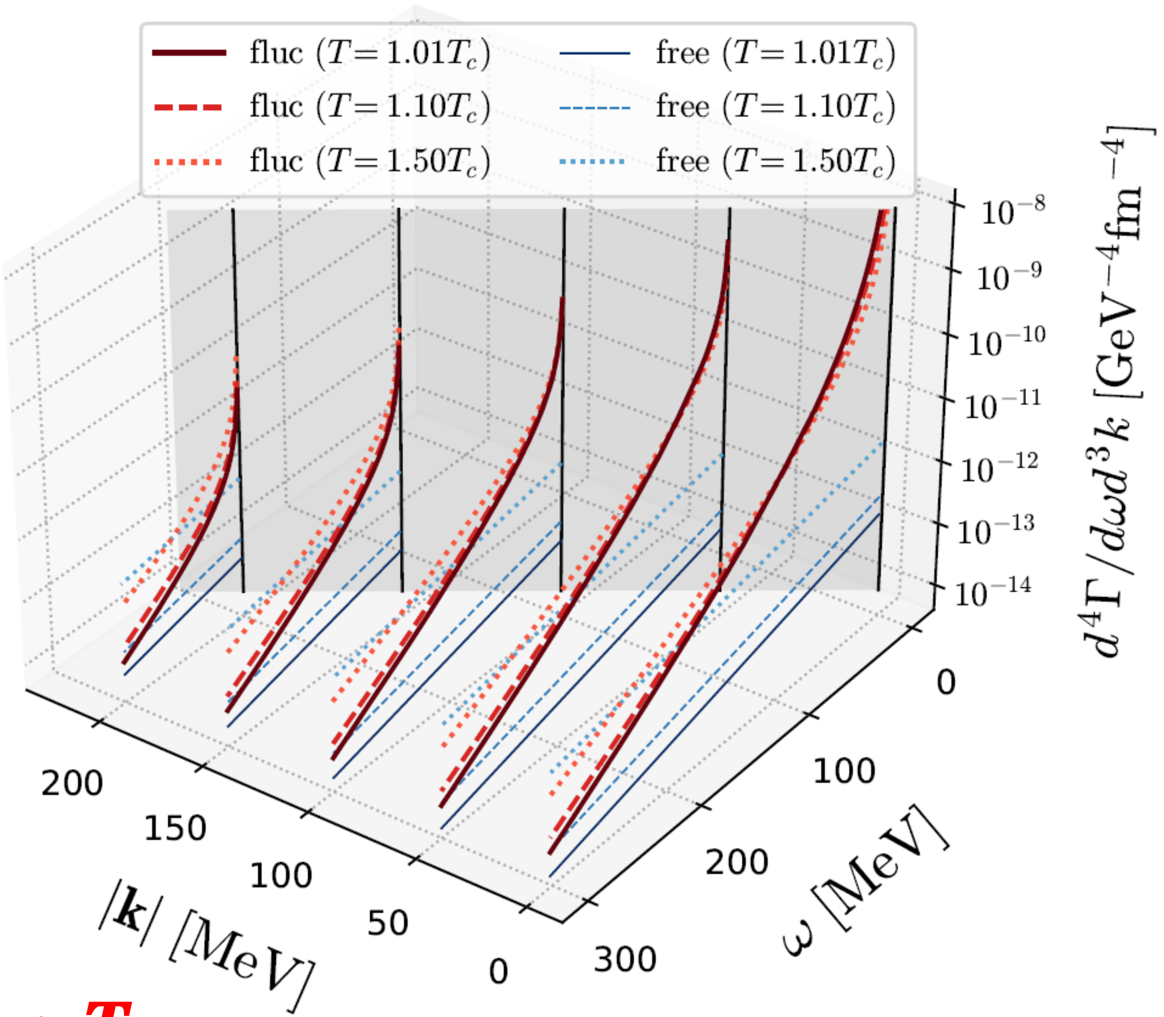
$) = 0$

Only AL term is necessary to calculate the DPR.



Dilepton production rate ($G_C = 0.7G_S$)



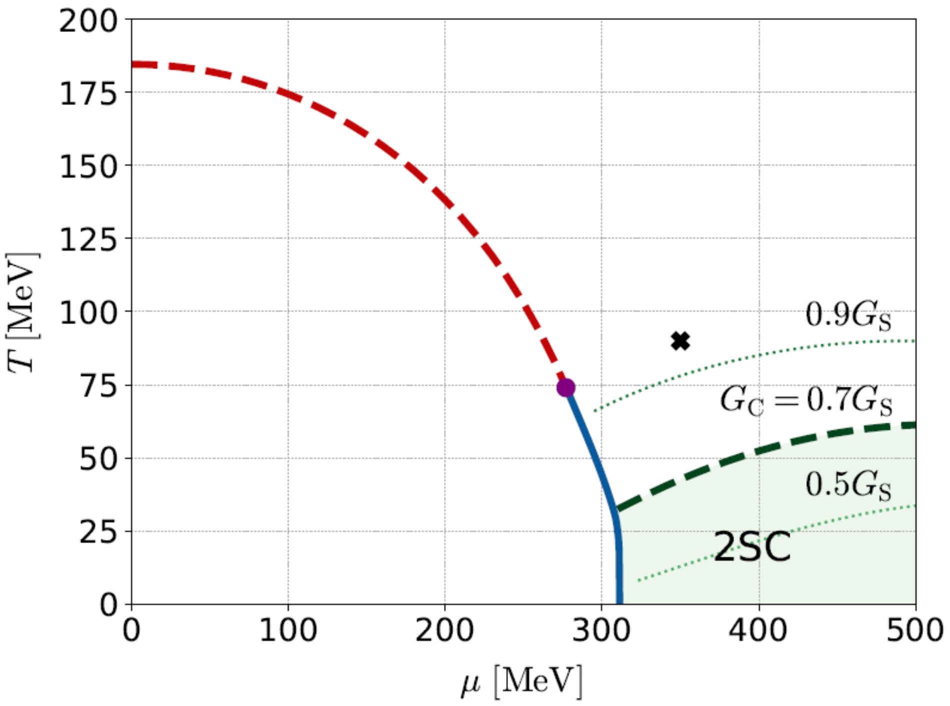


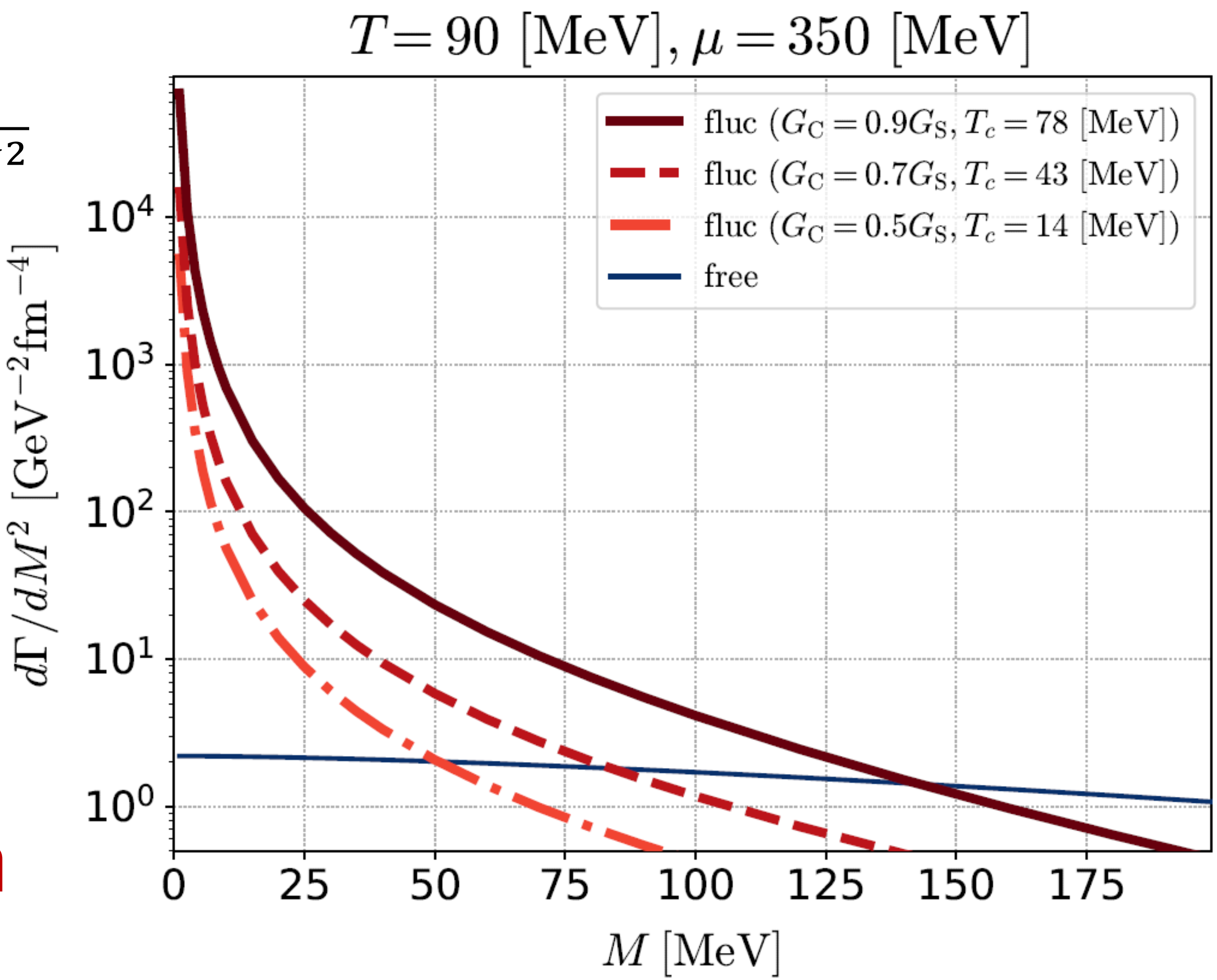
Enhancement at low ω & k as $T \to T_C$

⇨ **Expected from the property of soft modes**

Invariant mass spectrum

$$\frac{d\Gamma}{\mathrm{d}M^2} = \int d^3k \, \frac{1}{2\omega} \frac{d^4\Gamma}{dk^4}\bigg|_{\omega=\sqrt{\boldsymbol{k}^2+M^2}}$$





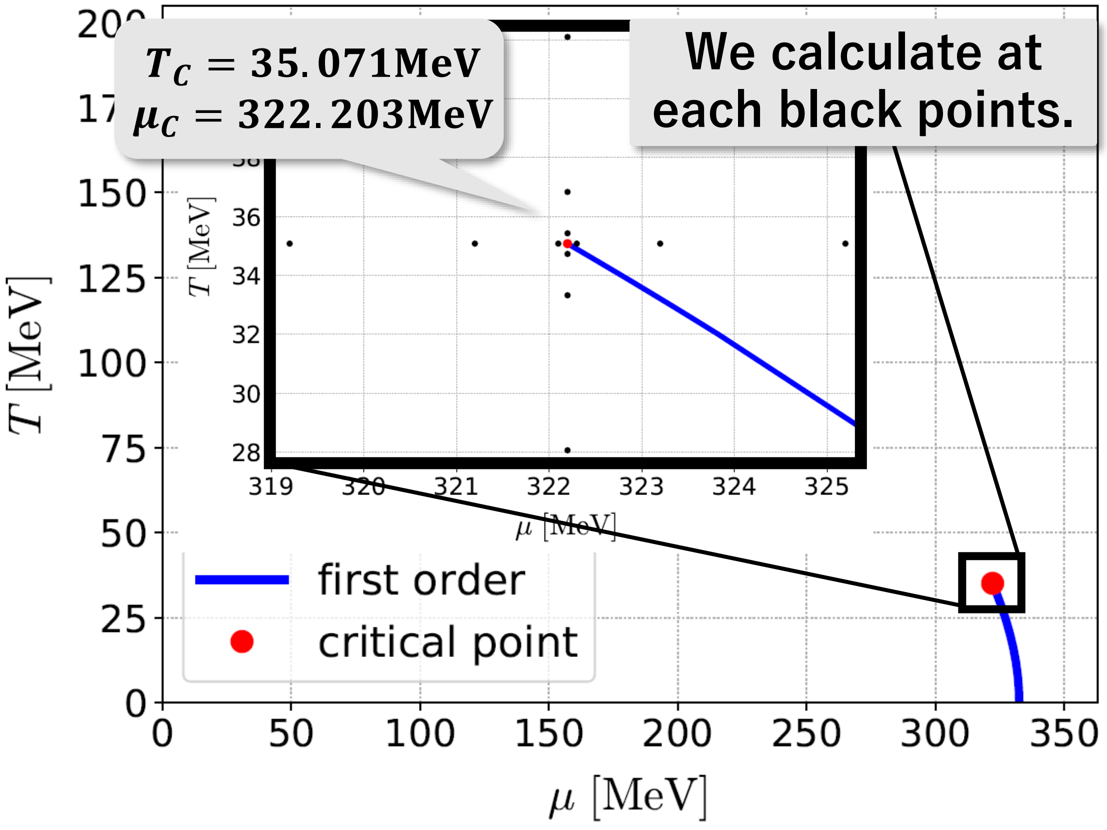
Signal for observation of CSC!?

Dilepton production rates from soft modes of QCD critical point

Phase diagram

$$\mathcal{L} = \bar{\psi} i(\gamma^\mu \partial_\mu - m)\psi$$
$$+ G_S[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \boldsymbol{\tau}\psi)^2]$$

$$\boldsymbol{G_S = 5.01\mathrm{MeV}, \Lambda = 650\mathrm{MeV}, m = 4\mathrm{MeV}}$$



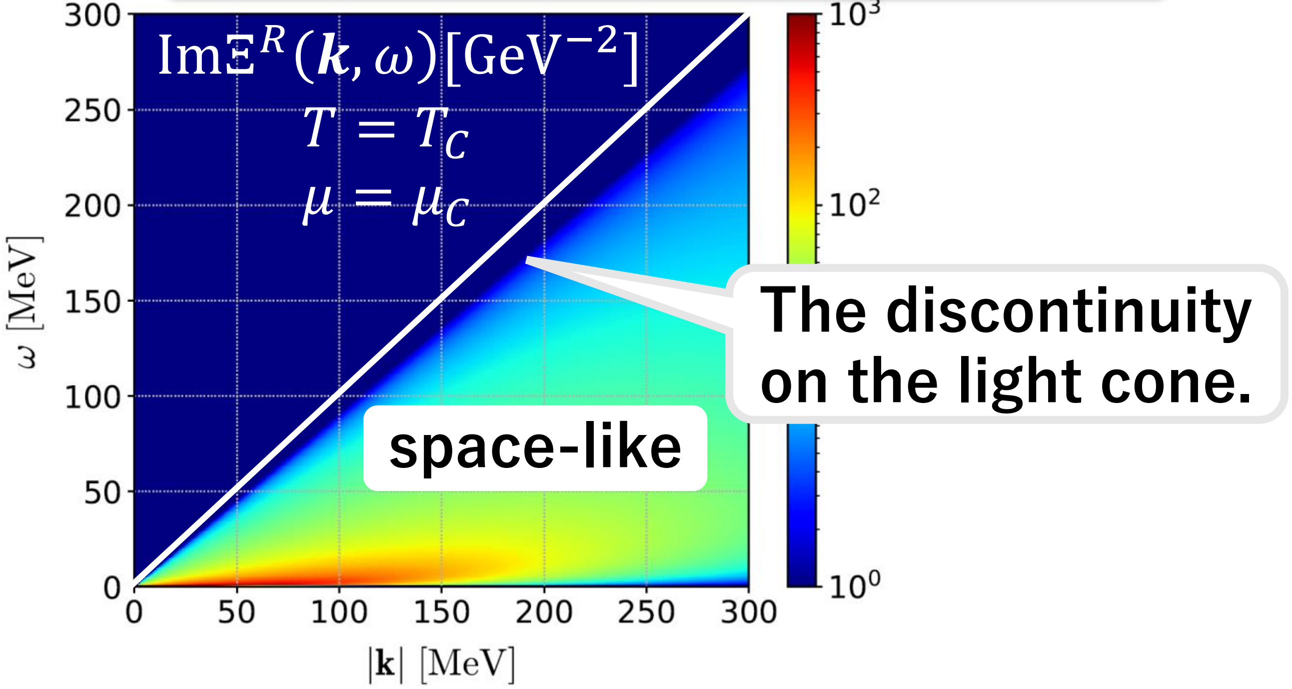
Soft modes

$$\Xi(\boldsymbol{k},\omega) = \Longrightarrow$$
$$= \frac{G_C}{1 + G_C Q^R(\boldsymbol{k},\omega)}$$

T-matrix approx.

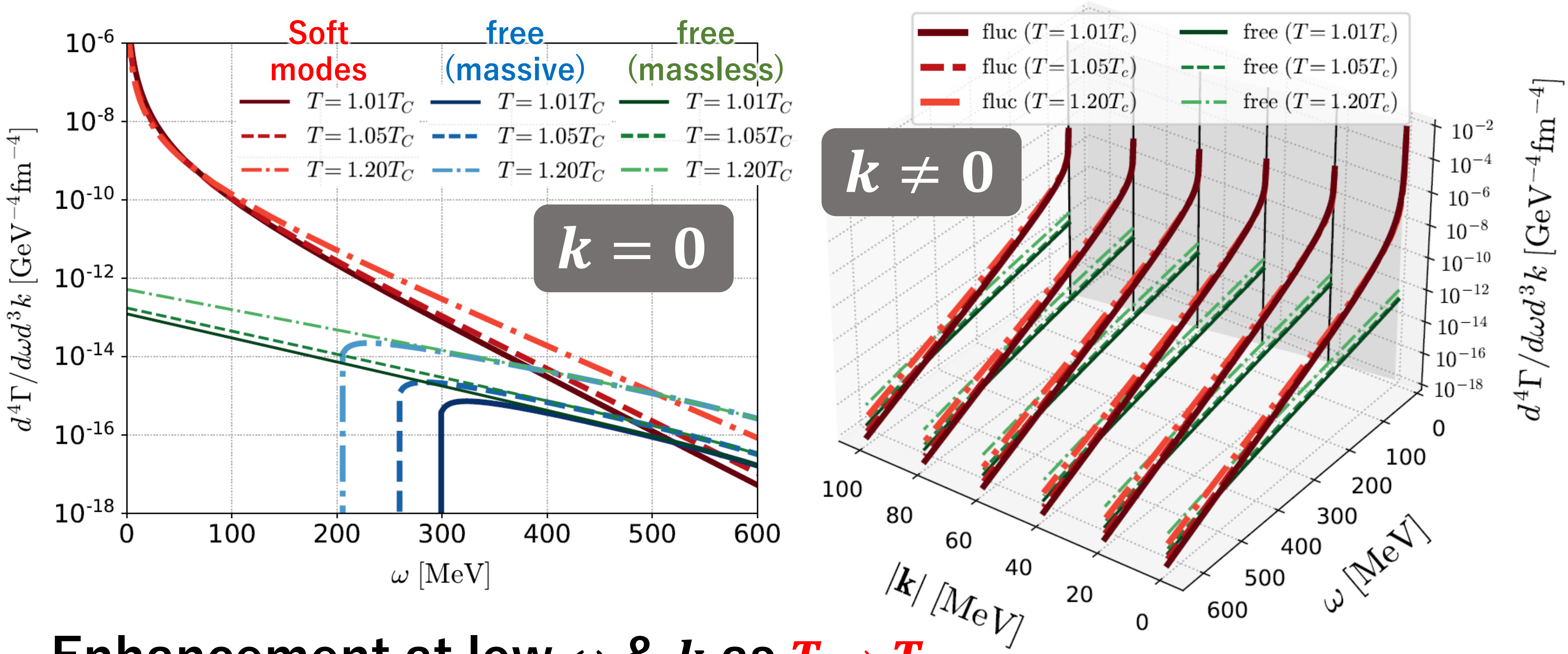
$\bar{q}q$ correlation

$\mathrm{Im}\Xi^R$ in energy-momentum plane



The soft mode in space-like region is the p-h mode. Fujii and Ohtani (2004)

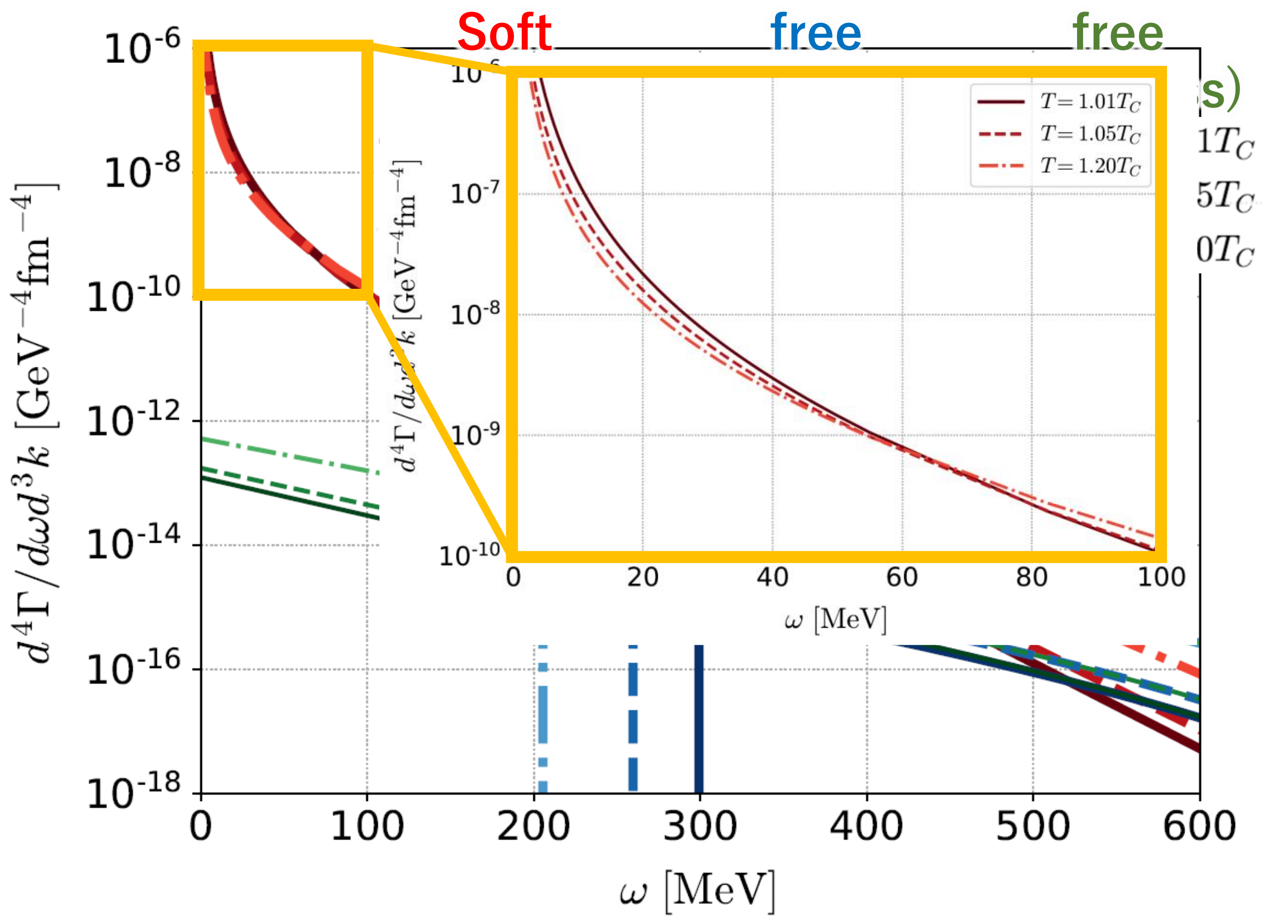
Dilepton production rate ($\mu = \mu_C$)

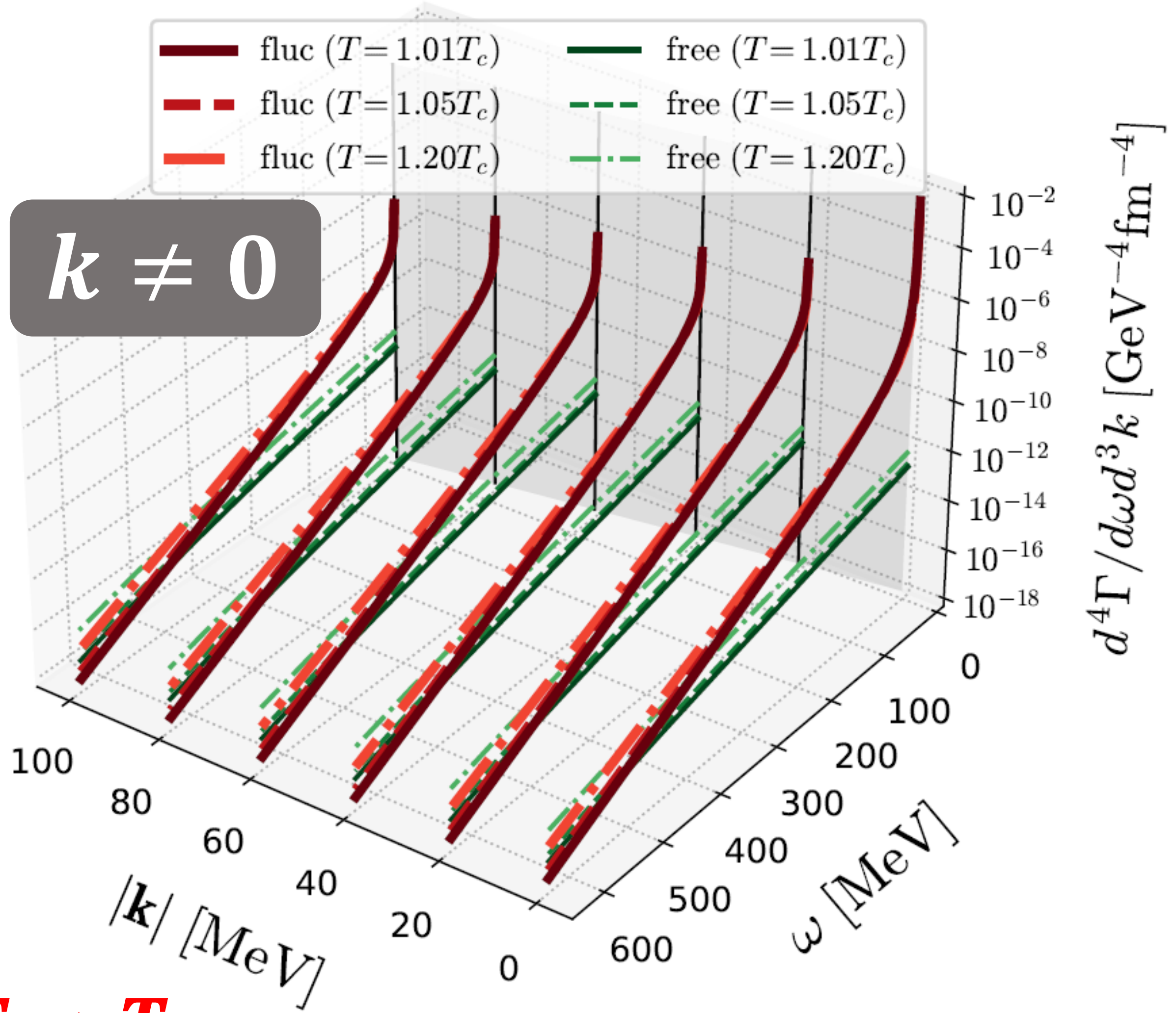


Enhancement at low ω & k as $T \to T_C$

⇨ Expected from the property of soft modes

Dilepton production rate ($\mu = \mu_C$)

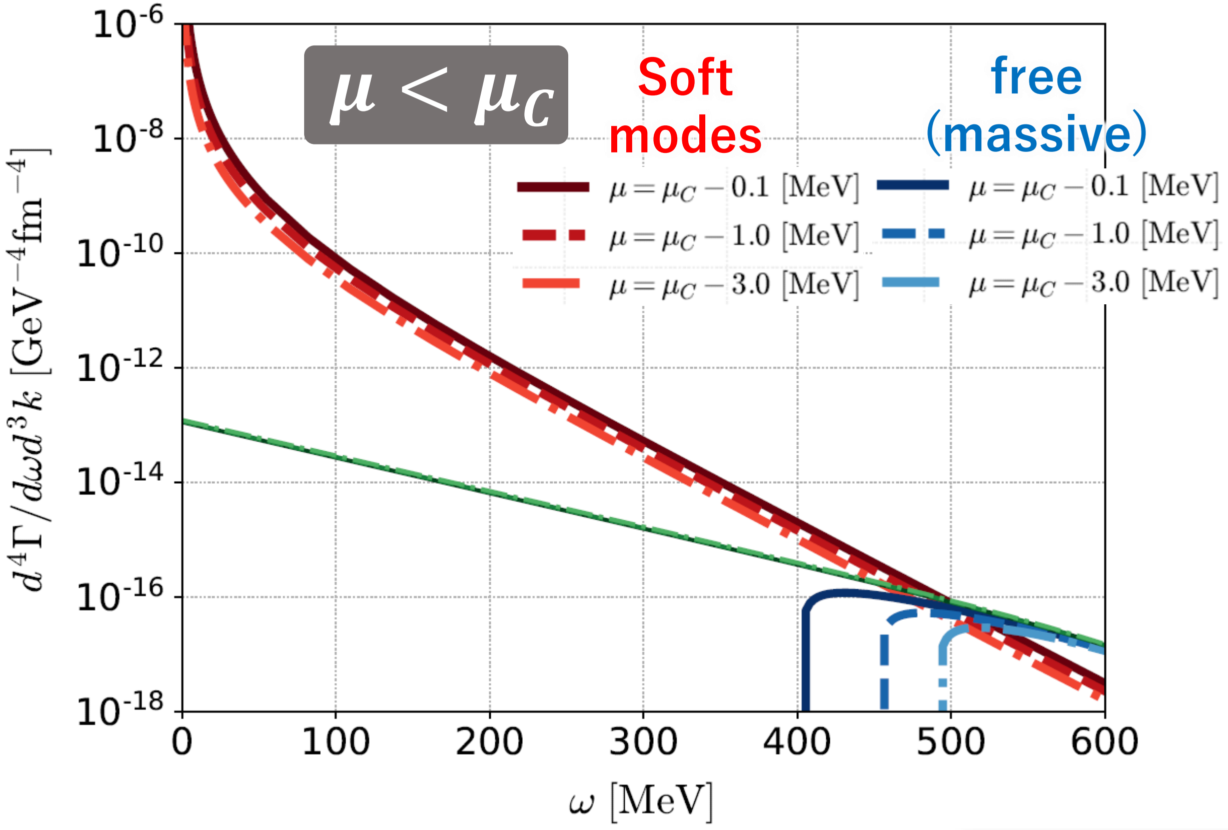


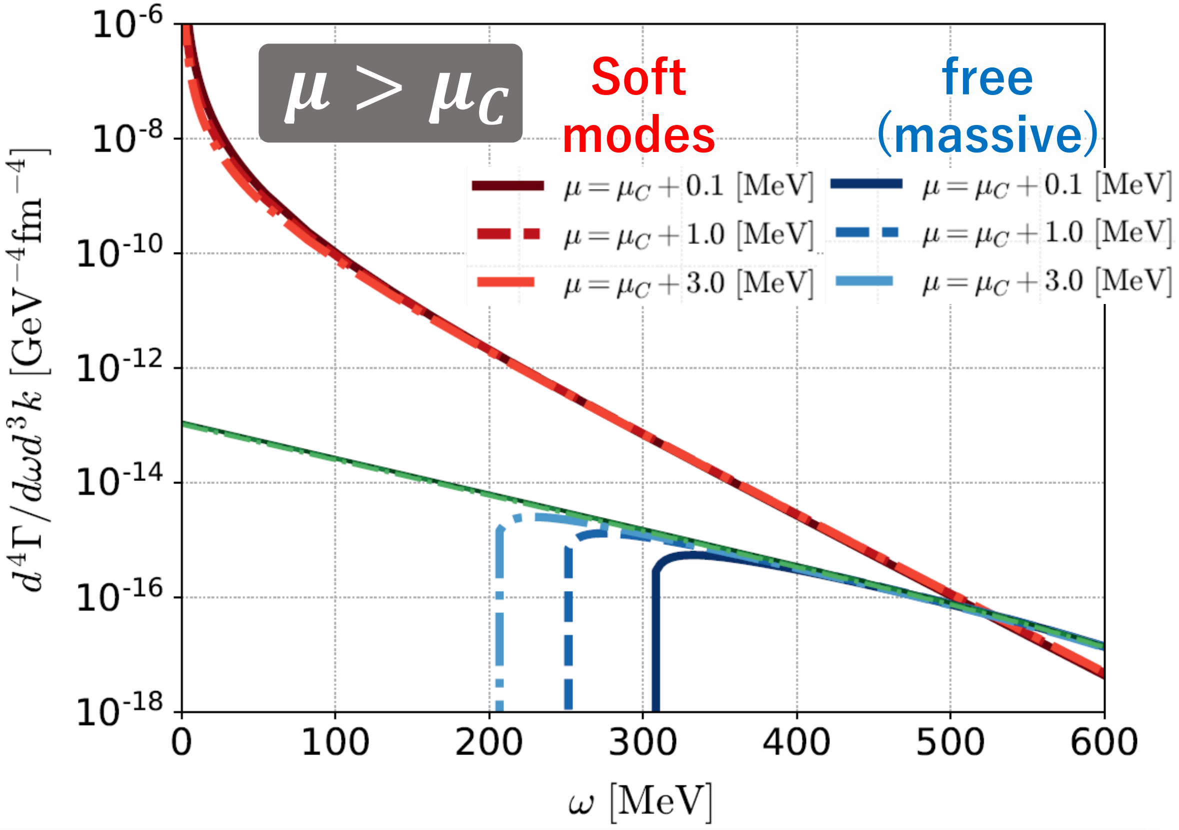


Enhancement at low ω & k as $T \to T_C$

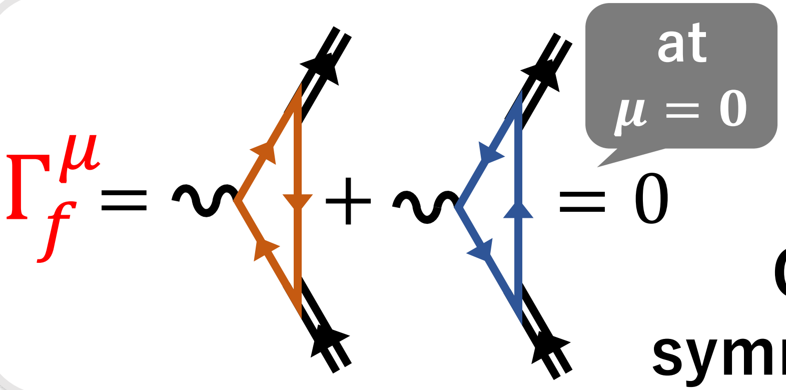
➡ Expected from the property of soft modes

μ-dependence $(T = T_C, k = 0)$





Competition between effects of "soft mode" & "growth of interaction as μ is bigger"



But $\Gamma_f^\mu(q, q+k) \neq 0$ at $\mu \neq 0$.

Charge conjugation symmetry violation at $\mu \neq 0$

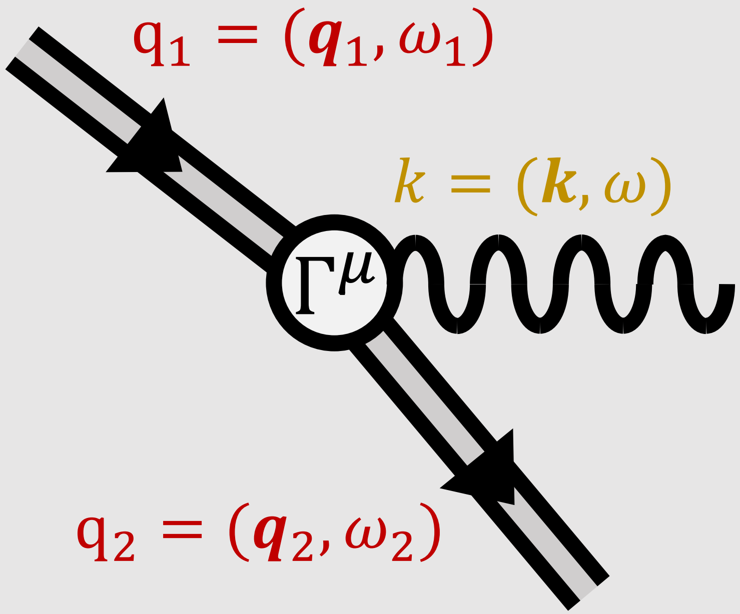
Discussion / Summary

Mechanism of enhancement at low ω

One of production processes caused by soft modes

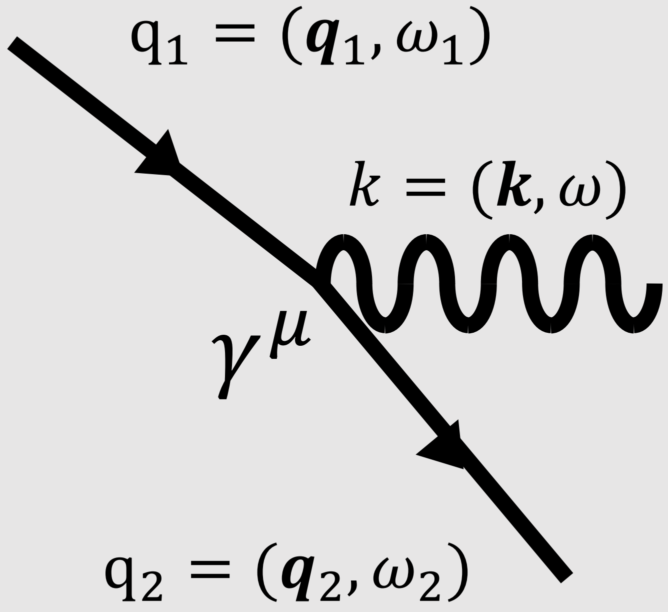
$$\omega = \omega_1 - \omega_2$$
$$|\boldsymbol{k}| = |\boldsymbol{q}_1 - \boldsymbol{q}_2|$$

$$|\boldsymbol{q}_1| > \omega_1$$
$$|\boldsymbol{q}_2| > \omega_2$$
$$\omega > |\boldsymbol{k}|$$



* Soft modes have strong support in space-like.
* Dileptons are produced in time-like.
* MT&DOS cancel. → Scattering in AL.

Scattering process In free quark gases



This occurs only in space-like.
→ $q\bar{q}$ annihilation contributes dilepton production in time-like

Conductivity "σ" and Relaxation time "τ"

Kubo formula: $\rho(\boldsymbol{p},\omega) = \dfrac{\sigma\omega(\omega^2 - \boldsymbol{p}^2)}{(\tau\omega^2 - D\boldsymbol{p}^2)^2 + \omega^2} + 2\dfrac{\sigma\omega}{\tau^2\omega^2}$

$g_{\mu\nu}\mathrm{Im}\Pi^{R^{\mu\nu}}(\boldsymbol{p},\omega)$

Diffusion constant

$$\sigma = \frac{1}{3}\frac{\partial}{\partial\omega}\rho(\boldsymbol{0},\omega)\Big|_{\omega=0}, \quad \tau = \sqrt{-\frac{1}{18\sigma}\frac{\partial^3}{\partial^3\omega}\rho(\boldsymbol{0},\omega)\Big|_{\omega=0}}$$

For the conductivity,

$\sigma \propto (T - T_C)^{-\frac{1}{2}}$: for CSC

$\sigma \propto (T - T_C)^{-2}$: for QCD CP

Summary

We calculated the effect of soft modes around CSC phase transition and QCD critical point on the DPR.

- Ward Identity for photon self-energies (AL, MT and DOS terms)
- We approximated photon self-energies based on TDGL theory.
- Enhancement of dilepton production rates at low ω by soft modes
- The mechanism of the enhancement is the scattering process of soft modes.

Outlook

Are the enhancement at low-M observable??

- Apply our result to dynamical model.
- Consider the competition with other dilepton production process.
  (Dalitz decay etc..)