Fluctuations of conserved charges in hydrodynamics and molecular dynamics

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SQM 2022 - The 20th International Conference on Strangeness in Quark Matter

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Acknowledgements:

M.I. Gorenstein, V. Koch, V.A. Kuznietsov, R. Poberezhnyuk, O. Savchuk, J. Steinheimer, H. Stoecker

QCD phase structure

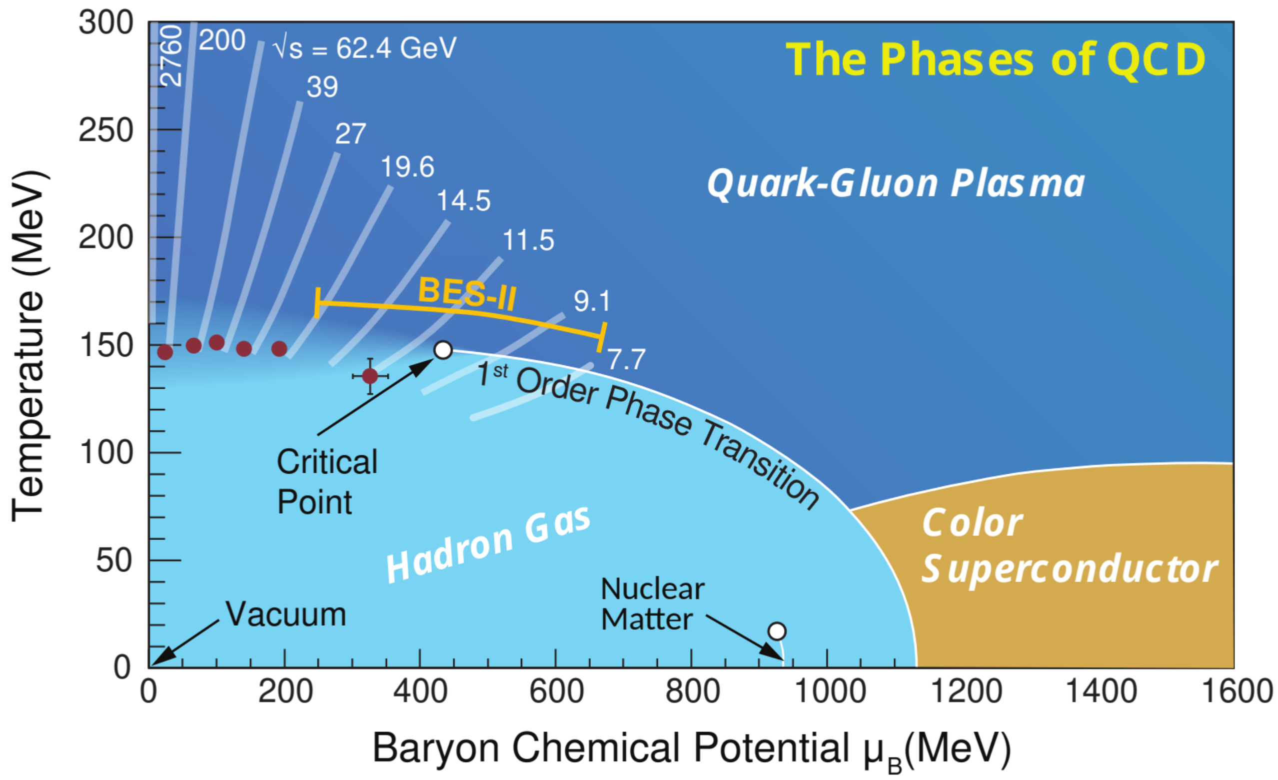


Figure from Bzdak et al., Phys. Rept. '20

- Dilute hadron gas at low T & ρ_B due to confinement, quark-gluon plasma high T & ρ_B
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured

Is there a critical point and how to find it with heavy-ion collisions?

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

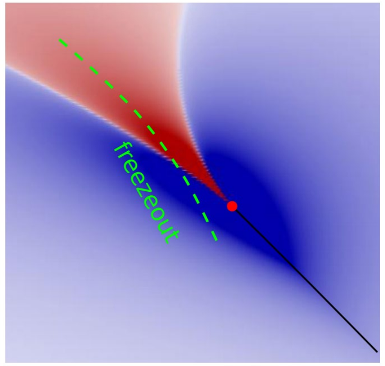
Grand partition function

$$\ln Z^{\mathrm{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\mathrm{ce}}(T,V,N)\right]$$

$$\kappa_n \propto \frac{\partial^n(\ln Z^{\mathrm{gce}})}{\partial \mu^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- **(QCD) critical point** – large correlation length, critical fluctuations of baryon number



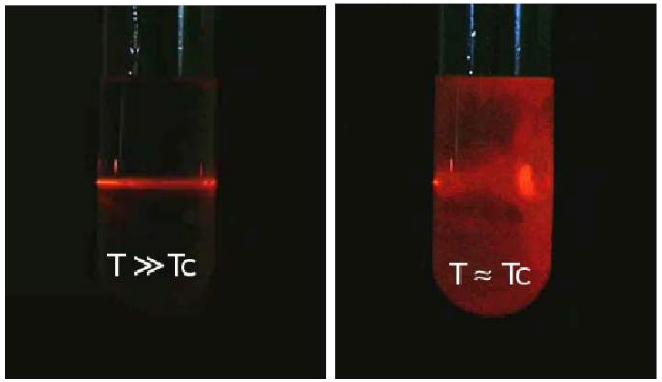
M. Stephanov, PRL '09, '11
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

$$\xi \to \infty$$

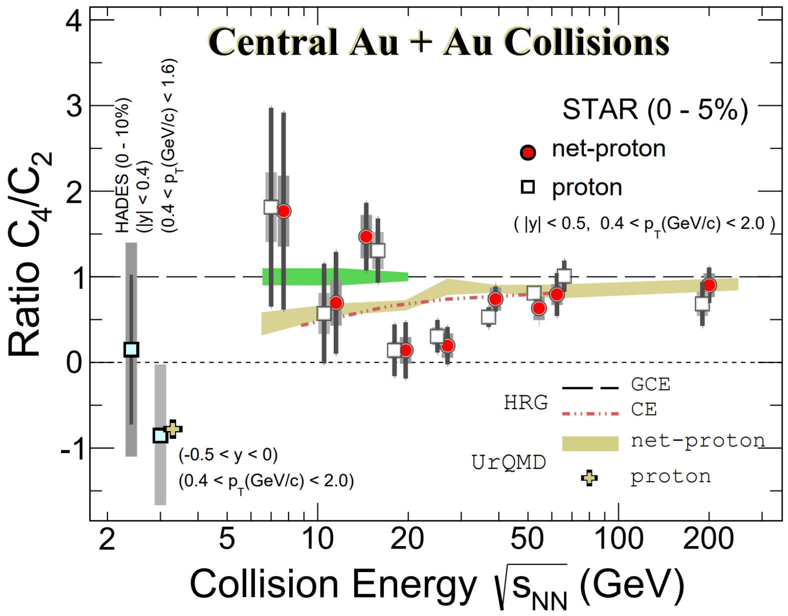
Looking for enhanced fluctuations
and non-monotonicities

Critical opalescence

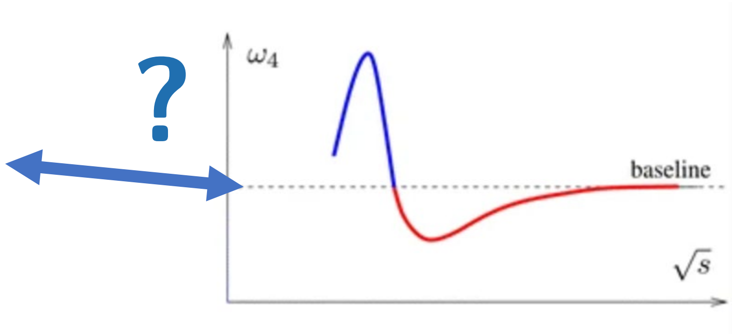


Experimental measurements

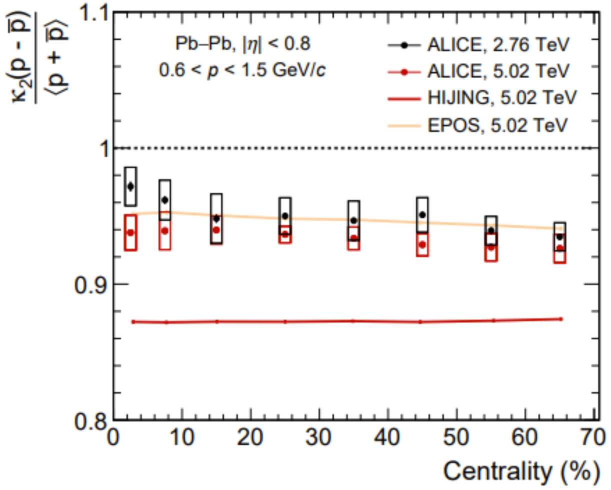
Beam energy scan in search for the critical point (STAR Coll.)



STAR Coll., Phys. Rev. Lett. 126, 092301 (2021); arXiv:2112.00240



M. Stephanov, Phys. Rev. Lett. (2011)



ALICE Coll., arXiv:2206.03343

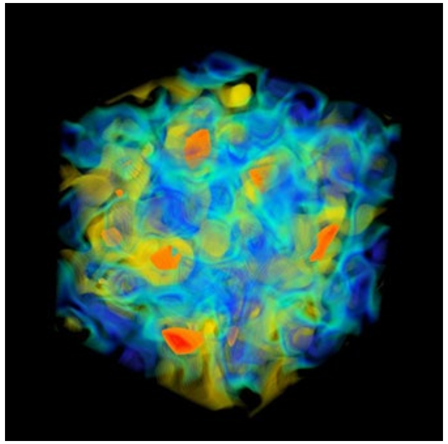
Reduced errors (better statistics), more energies, to come soon from RHIC-BES-II program, STAR-FXT etc.

Can we learn more from the more accurate data available for κ_2 and κ_3?

Other measurements: LHC-ALICE, GSI-HADES & CERN-NA61/SHINE Collaborations

Theory vs experiment: Challenges for fluctuations

Theory



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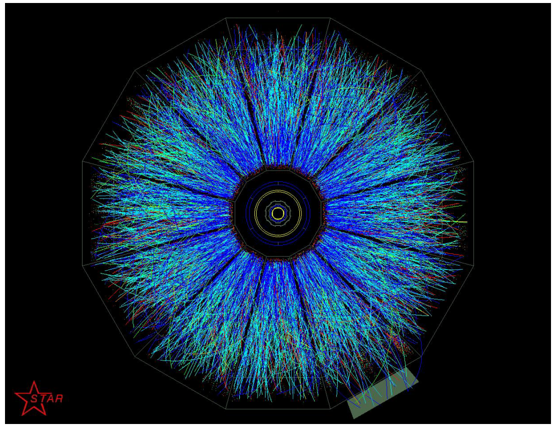
- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

Experiment



STAR event display

- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

Need dynamical description

Theory vs experiment: Challenges for fluctuations

- canonical ensemble effects
  - subensemble acceptance method (SAM) **R. Poberezhnyuk, talk Wed 11:10**

    VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020); JHEP 089(2020); PRC 105, 014903 (2022)
  - ideal gas limit **A. Rustamov, talk Mon 17:35**

    Bzdak, Koch, Skokov, PRC 87, 014901 (2013); Braun-Munzinger et al., NPA 1008, 122141 (2021)

- coordinate vs momentum space

  Ling, Stephanov, PRC 93, 034915 (2016); Ohnishi, Kitazawa, Asakawa, PRC 94, 044905 (2016)

- proxy observables in experiment (net-proton, net-kaon) vs conserved charges in QCD (net-baryon, net-strangeness) **M. Kitazawa, talk Mon 16:45**

  Kitazawa, Asakawa, PRC 85, 021901 (2012); VV, Jiang, Gorenstein, Stoecker, PRC 98, 024910 (2018)

- volume fluctuations

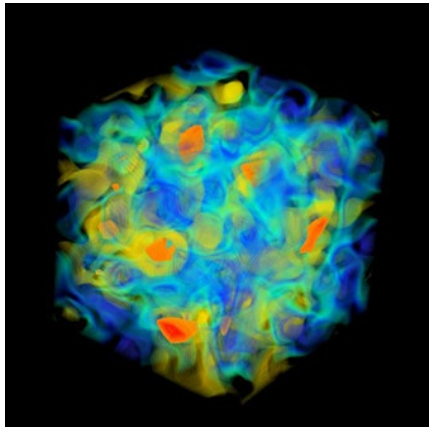
  Gorenstein, Gazdzicki, PRC 84, 014904 (2011); Skokov, Friman, Redlich, PRC 88, 034911 (2013)
  X. Luo, J. Xu, B. Mohanty, JPG 40, 105104 (2013); Braun-Munzinger, Rustamov, Stachel, NPA 960, 114 (2017)

- hadronic phase

  Steinheimer, VV, Aichelin, Bleicher, Stoecker, PLB 776, 32 (2018)
  Savchuk, VV, Koch, Steinheimer, Stoecker, PLB 827, 136983 (2022)

- non-equilibrium (memory) effects

  Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)
  Asakawa, Kitazawa, Müller, PRC 101, 034913 (2020)



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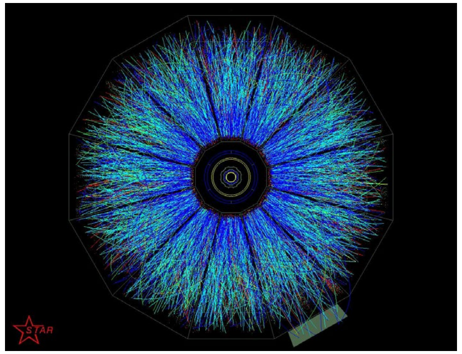


STAR event display

Dynamical approaches to the QCD critical point search

1. **Deviations from precision calculations of non-critical fluctuations**
   - Include essential non-critical contributions to (net-)proton number cumulants
   - Exact baryon conservation + hadronic interactions* (hard core repulsion)
   - Based on realistic hydrodynamic simulations tuned to bulk data

   [VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

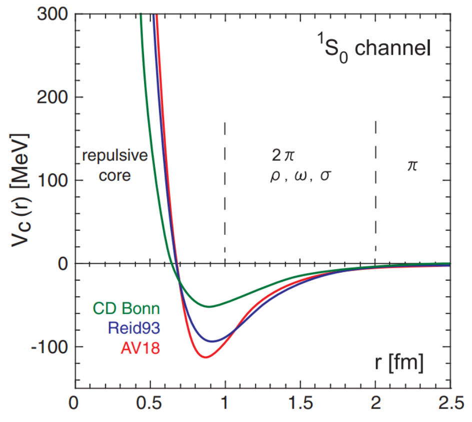


Figure from Ishii et al., PRL '07

2. **Molecular dynamics with a critical point**

   [V.A. Kuznietsov, O. Savchuk, M.I. Gorenstein, V. Koch, VV, Phys. Rev. C 105, 014904 (2022)]

3. Dynamical model calculations of critical fluctuations
   - Fluctuating hydrodynamics
   - Equation of state with tunable critical point [P. Parotto et al, Phys. Rev. C 101, 034901 (2020)]

   Under development within the Beam Energy Scan Theory (BEST) Collaboration

   BEST COLLABORATION [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]

***J. Karthein, talk Tue 09:40**

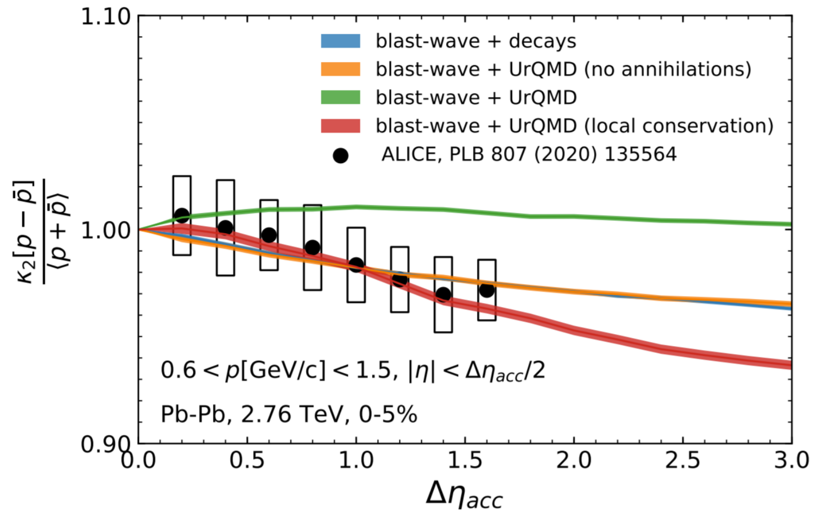
Hydrodynamics based analysis of (net-)particle fluctuations and constraints on the QCD critical point

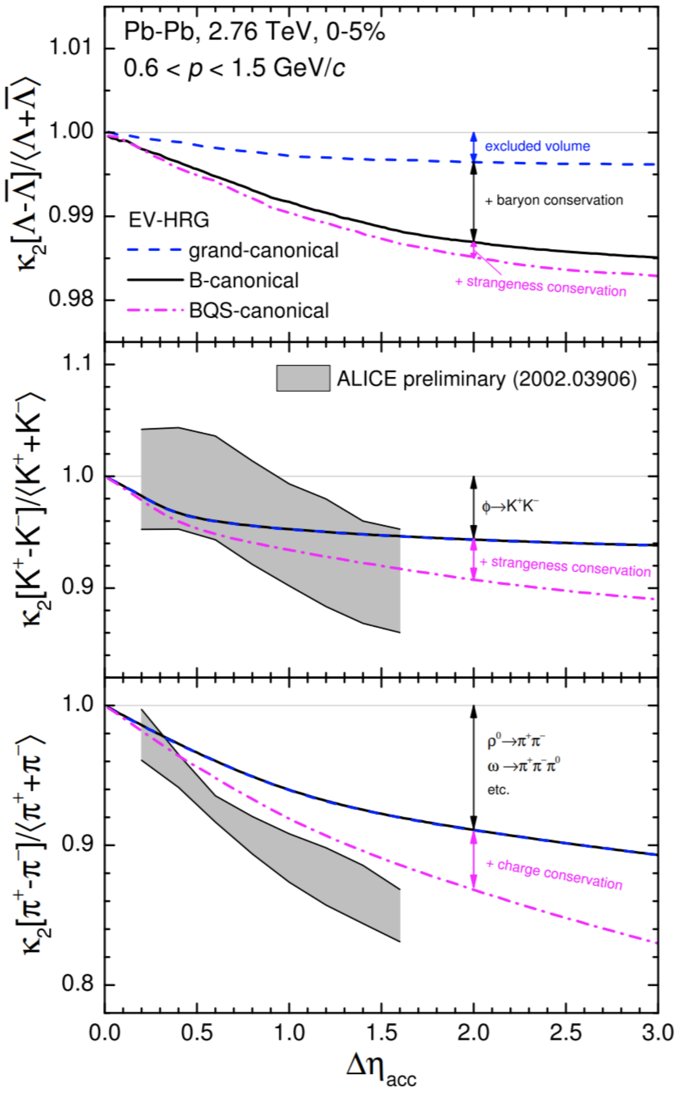
Net-particle fluctuations at the LHC

- Net protons described within errors and consistent with either
  - global baryon conservation without $B\bar{B}$ annihilations

    see e.g. ALICE Coll. arXiv:2206.03343
  - or local baryon conservation with $B\bar{B}$ annihilations

    O. Savchuk et al., Phys. Lett. B 827, 136983 (2022)



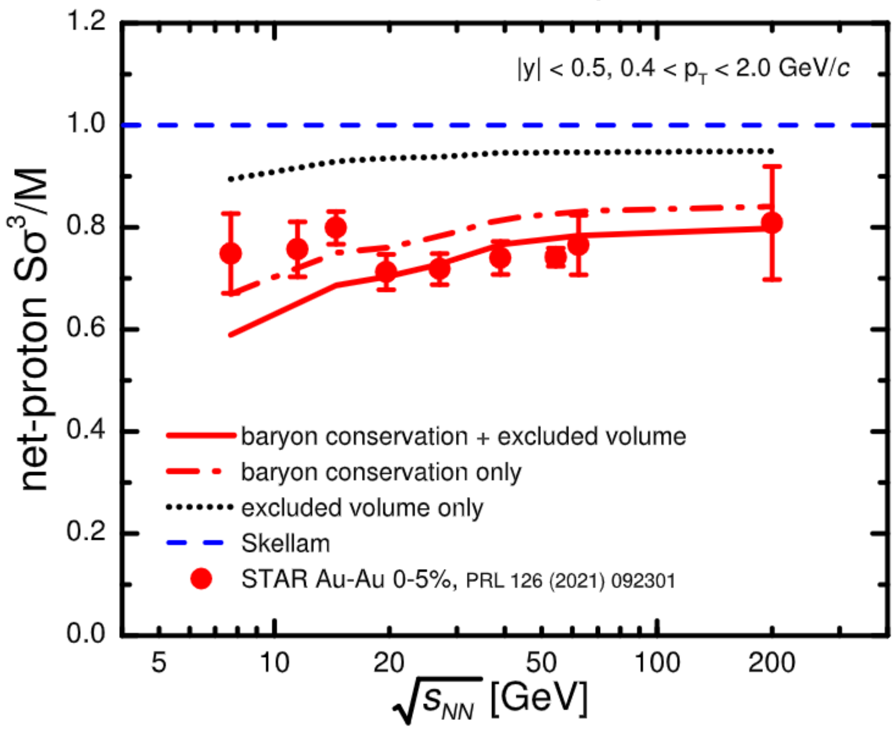


- Large effect from resonance decays for pions and kaons + exact conservation of electric charge/strangeness

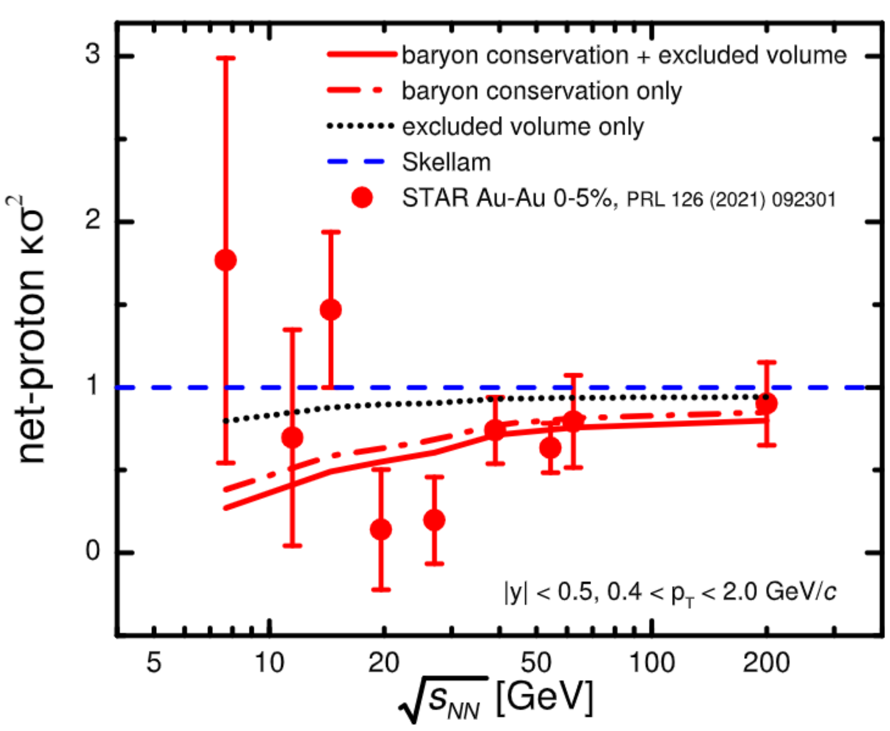
VV, Koch, Phys. Rev. C 103, 044903 (2021)

RHIC-BES: Net proton cumulant ratios

κ_3/κ_1



κ_4/κ_2



- Data at $\sqrt{s_{NN}} \geq 20$ GeV consistent with non-critical physics (baryon conservation and repulsion)
- Effect from baryon conservation is larger than from repulsion
- Excess of skewness in data at $\sqrt{s_{NN}} < 20$ GeV – *hint of attractive interactions?*

RHIC-BES: Net proton cumulant ratios

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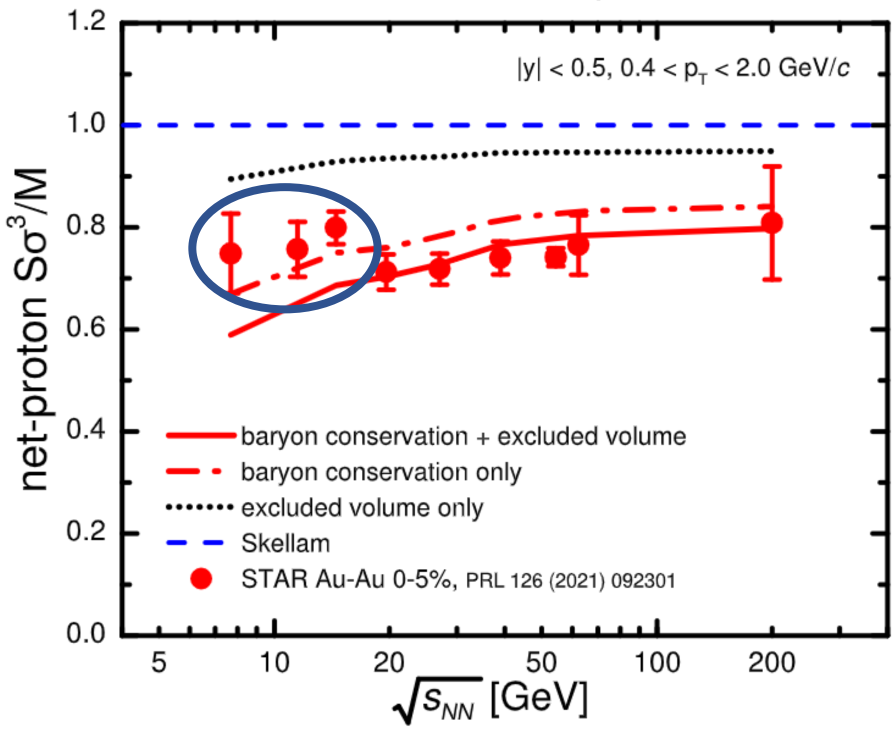
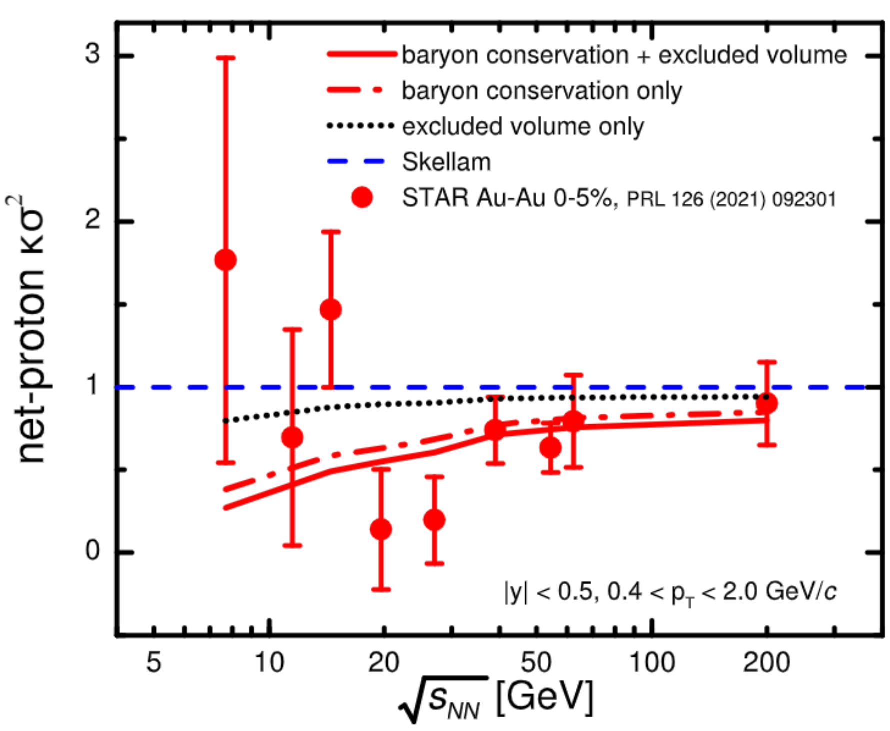


κ_4/κ_2



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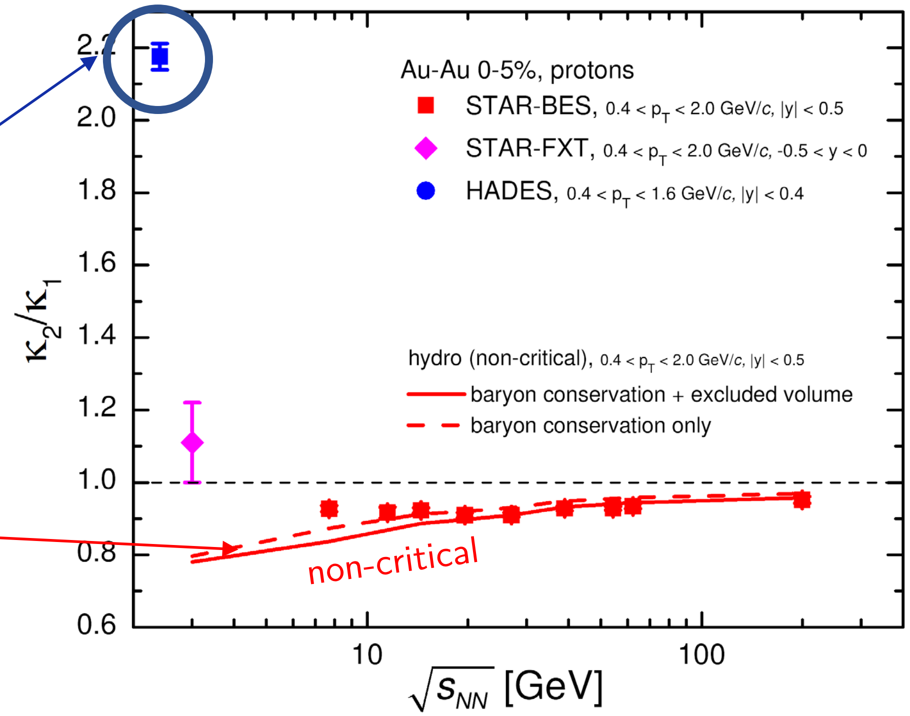
Second order proton cumulants and $\sqrt{s_{NN}} \leq 7.7$ GeV

- Second order cumulants measured with much higher precision
- Intriguing hint from HADES @ $\sqrt{s_{NN}} = 2.4$ GeV: huge excess of two-proton correlations!

  [HADES Collaboration, Phys. Rev. C 102, 024914 (2020)]
- No change of trend in the non-critical hydro
- Additional mechanisms:
  - Nuclear liquid-gas transition
  - Light nuclei formation
- Fill the gap with data from STAR-FXT (e.g. Phys.Rev.Lett. 128 (2022) 202303), future experiments like CBM-FAIR



A closer look at the HADES data

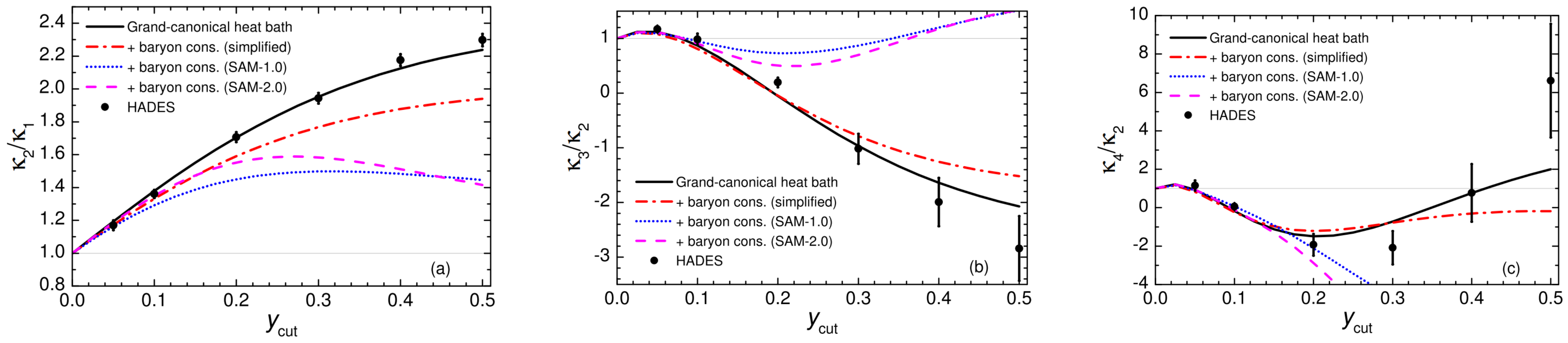
VV, Koch, arXiv:2204.00137

- Fit baryon susceptibilities to data within a fireball model (Siemens-Rasmussen*)

- In the grand-canonical limit (no baryon conservation, small y_{cut}) the data are described well with

$$\frac{\chi_2^B}{\chi_1^B} = 9.35 \pm 0.40, \quad \frac{\chi_3^B}{\chi_2^B} = -39.6 \pm 7.2, \quad \frac{\chi_4^B}{\chi_2^B} = 1130 \pm 488 \quad \text{i.e.} \quad \chi_4^B \gg -\chi_3^B \gg \chi_2^B \gg \chi_1^B$$

- Could be indicative of a *critical point* near the HADES freeze-out at T~70 MeV, μ_B~875 MeV
- However, the results for $y_{cut} > 0.2$ are challenging to describe with baryon conservation included



*Fireball parameters from Harabasz et al., PRC 102 (2020) 054903 and Motornenko et al., PLB 822 (2021) 136703

Critical point particle number fluctuations from molecular dynamics

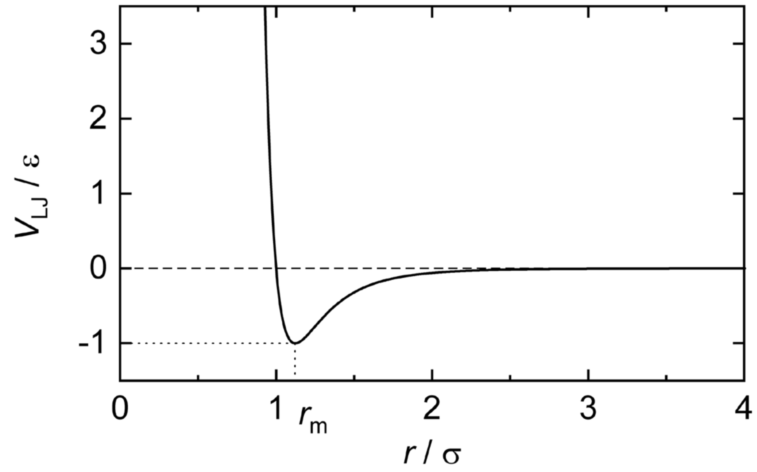
V.A. Kuznietsov, O. Savchuk, M.I. Gorenstein, V. Koch, VV, Phys. Rev. C 105, 044903 (2022)

For non-critical fluctuations in molecular dynamics see Hammelmann et al., arXiv:2202.11417

Lennard-Jones fluid

S. Stephan, M. Thol, J. Vrabec, H. Hasse, Journal of Chemical Information and Modeling 59, 4248 (2019)

$$V_{\mathrm{LJ}}(r) = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$



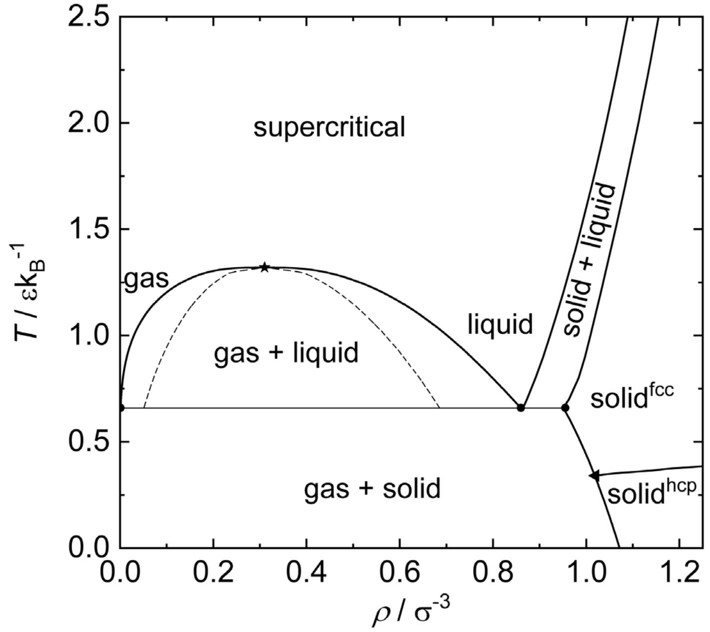
Reduced variables:

$$\tilde{r} = r/\sigma \qquad \tilde{T} = T/(k_B\varepsilon) \qquad \tilde{n} = n\sigma^3$$

Properties:

- Multiple phase transitions, including critical point
- Tractable with molecular dynamics simulations
- Critical point in 3D-Ising universality class at

$$\tilde{T}_c = 1.321 \pm 0.007\ , \quad \tilde{n}_c = 0.316 \pm 0.005$$



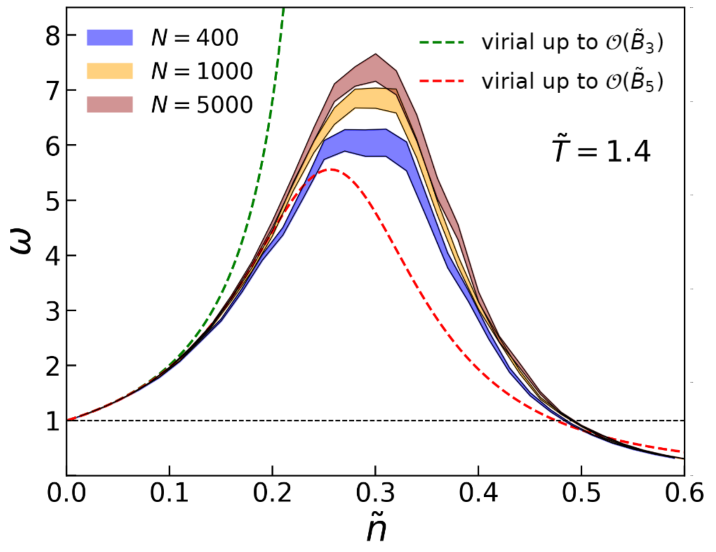
Toy model to study critical point fluctuations microscopically

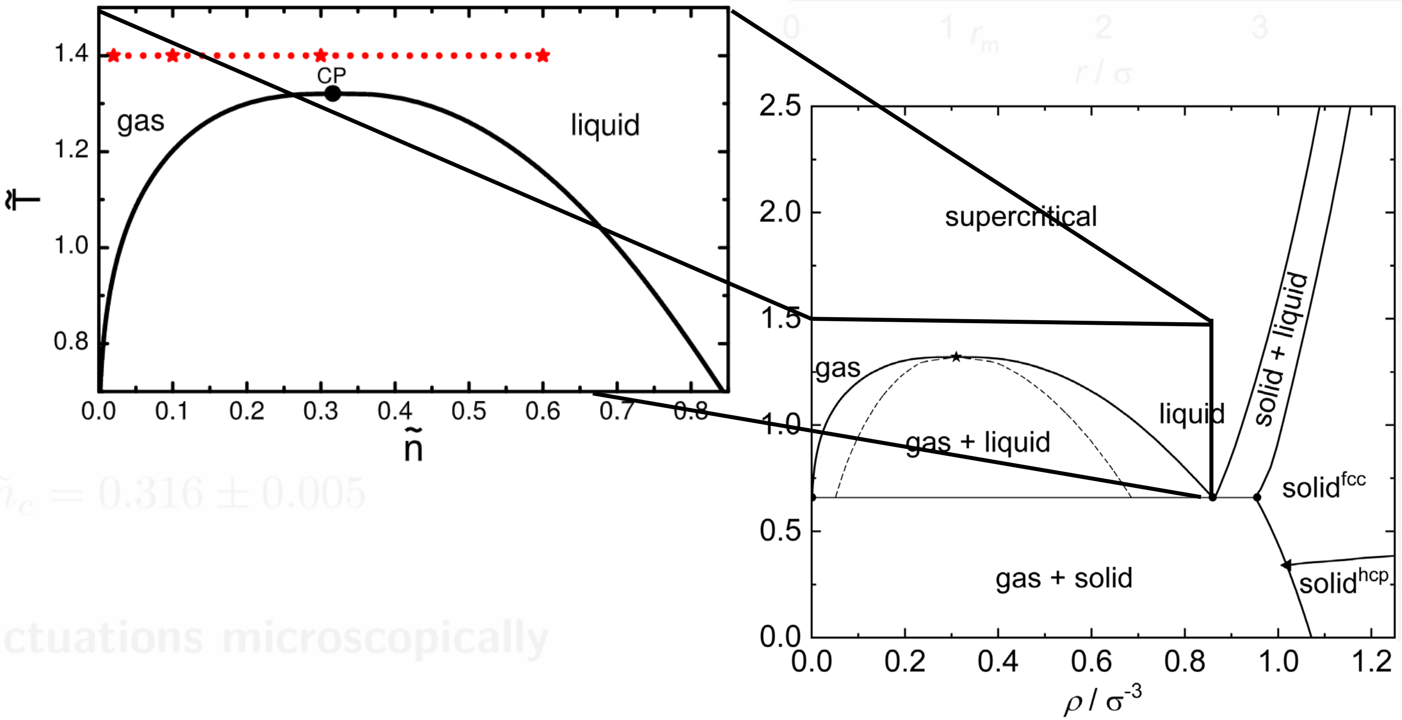
Lennard-Jones fluid

S. Stephan, M. Thol, J. Vrabec, H. Hasse, Journal of Chemical Information and Modeling 59, 4248 (2019)

Study the supercritical isotherm $\tilde{T} = 1.4 = 1.06\,\tilde{T}_C$ in density range $0.05\tilde{n}_C < \tilde{n} < 2\tilde{n}_C$

Large fluctuations



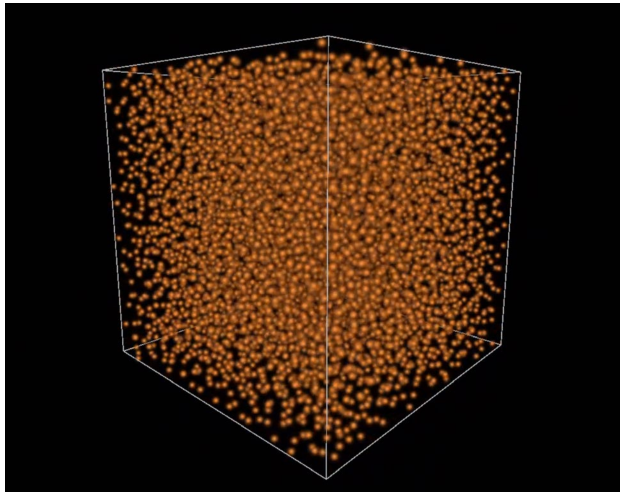


Molecular dynamics setup

- Newton's equations of motion (classical N-body problem)

$$m\ddot{\mathbf{r}}_\mathbf{i} = -\sum_j \nabla_i V_{\mathrm{LJ}}^{ij}(|\mathbf{r}_\mathbf{i} - \mathbf{r}_\mathbf{j}|)$$

- Box simulation
  - Periodic boundary conditions
  - Minimum-image convention
- Microcanonical (UVN) and canonical-like (TVN) ensembles
- Observables as time averages $\langle A \rangle = \frac{1}{\tilde{\tau}} \int_{\tilde{t}_{\mathrm{eq}}}^{\tilde{t}_{\mathrm{eq}}+\tilde{\tau}} A(\{\tilde{\mathbf{r}}_i(\tilde{t}), \tilde{\mathbf{v}}_i(\tilde{t})\}) d\tilde{t}$



Implementation:

Velocity Verlet integration scheme implemented on CUDA-GPU (x100-200 speed-up*)

open source: https://github.com/vlvovch/lennard-jones-cuda

NVIDIA CUDA

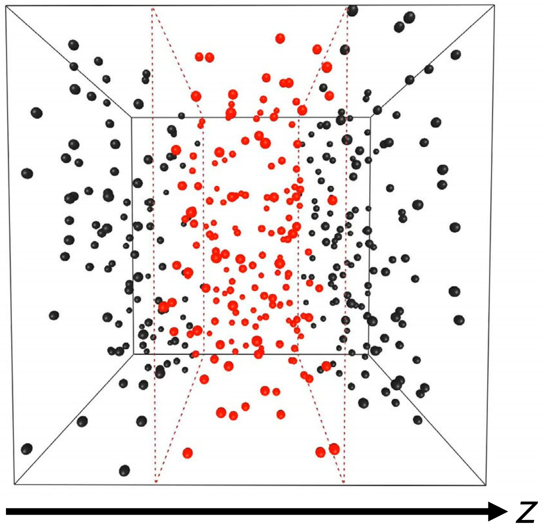
*This research used the Lawrencium computational cluster resource provided by the IT Division at the Lawrence Berkeley National Laboratory

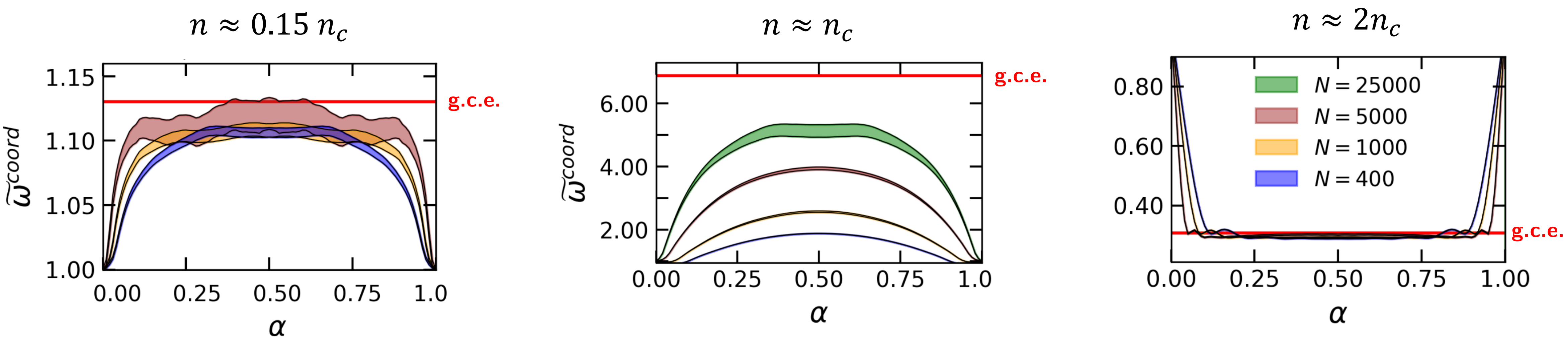
Fluctuations in molecular dynamics

Variance of conserved particle number distribution inside coordinate space subvolume $|z| < z^{max}$ as time average

- $\langle N \rangle$, $\langle N^2 \rangle$ as time averages
- Microcanonical ensemble
- $1 - a$ factor to cancel out global conservation
- $\tilde{\omega}^{coord} \to \omega^{gce}$ expected as $\langle N \rangle \to \infty$

$$\tilde{\omega}^{\text{coord}} = \frac{1}{1-\alpha} \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$





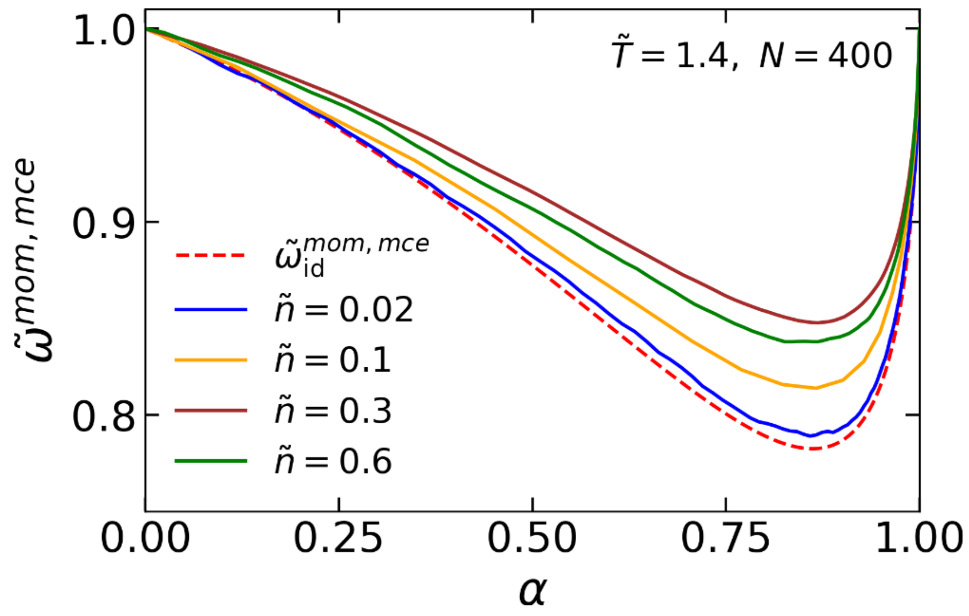
Fluctuations in molecular dynamics: momentum space

Experiments measure momenta, not coordinates → consider momentum space subvolume instead

$|v_z| < v_z^{\rm cut}$ (à la $|y| < y^{\rm cut}$) $\alpha = \langle N^{\rm acc} \rangle / N$

total energy conservation effect

Ideal gas limit: $$\tilde{\omega}_{\rm id}^{\rm mom,mce} = 1 - \frac{2[{\rm erf}^{-1}(\alpha)]^2 e^{-2[{\rm erf}^{-1}(\alpha)]^2}}{3\pi\alpha(1-\alpha)}$$



Large fluctuations near the CP are washed out when momentum cuts imposed instead of coordinates

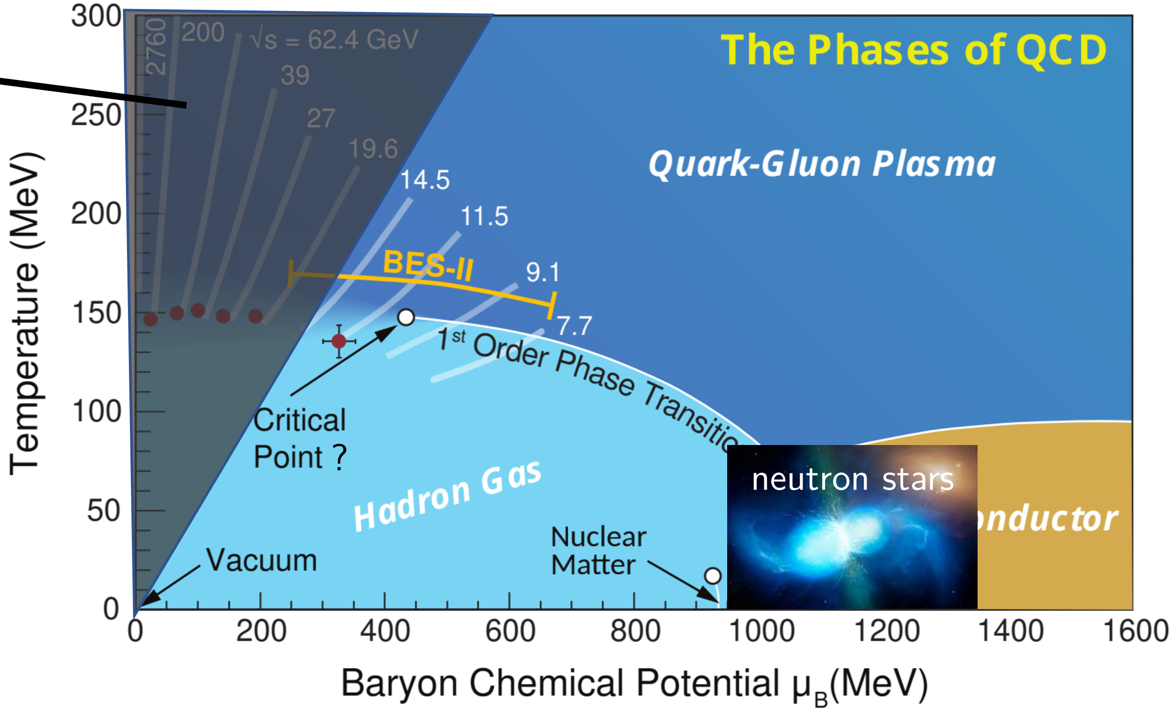
NB: here no collective flow and expansion

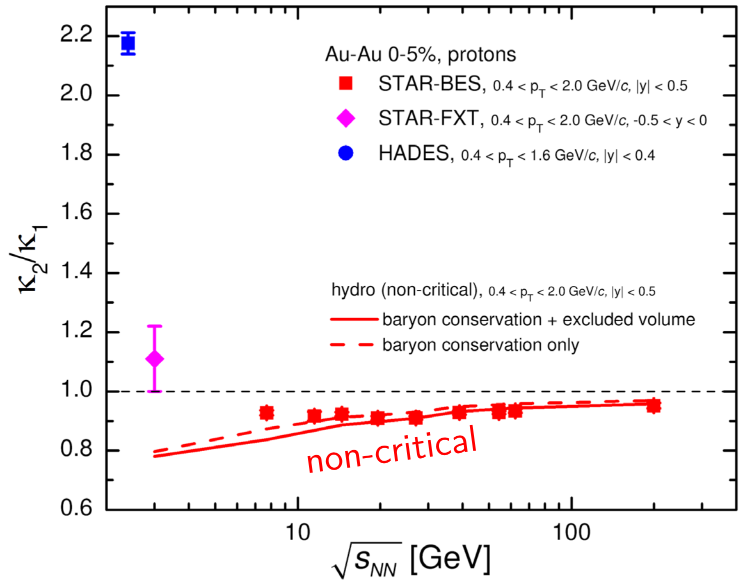
Outlook:

- Collective flow and expansion, clustering
- Ensemble averaging instead of time averaging
- High-order cumulants

Summary: What we learned so far from fluctuations

Critical point disfavored





- Data at high energies ($\sqrt{s_{NN}} \geq 20$ GeV) consistent with "non-critical" physics
- Interesting indications for (multi)-proton correlations at $\sqrt{s_{NN}} \leq 7.7$ GeV
- Critical point: Promising developments in hydrodynamics and molecular dynamics

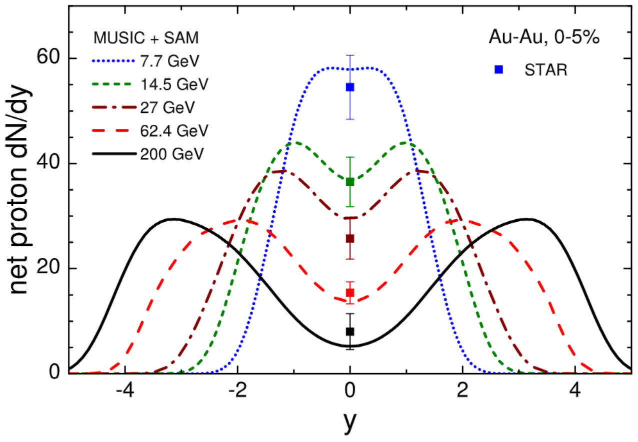
Thanks for your attention!

Backup slides

Hydrodynamic description

- Collision geometry based 3D initial state [Shen, Alzhrani, PRC '20]
  - Constrained to net proton distributions

VV, C. Shen, V. Koch, in preparation



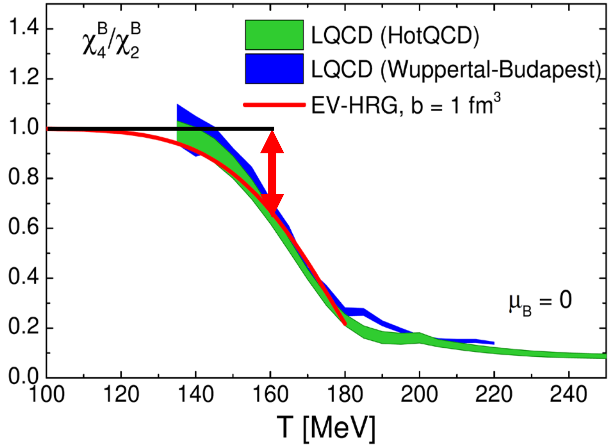
- Viscous hydrodynamics evolution – MUSIC-3.0
  - Energy-momentum and baryon number conservation
  - NEOS-BSQ equation of state [Monnai, Schenke, Shen, PRC '19]
  - Shear viscosity via IS-type equation

- Cooper-Frye particlization at $\epsilon_{sw} = 0.26\ \mathrm{GeV/fm^3}$

$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x)\, p^\mu \, \frac{d_j\, \lambda_j^{\rm ev}(x)}{(2\pi)^3} \exp\left[\frac{\mu_j(x) - u^\mu(x) p_\mu}{T(x)}\right].$$

- Particlization includes QCD-based baryon number distribution
  - Here incorporated via baryon excluded volume

[VV, Pasztor, Fodor, Katz, Stoecker, PLB 775, 71 (2017)]



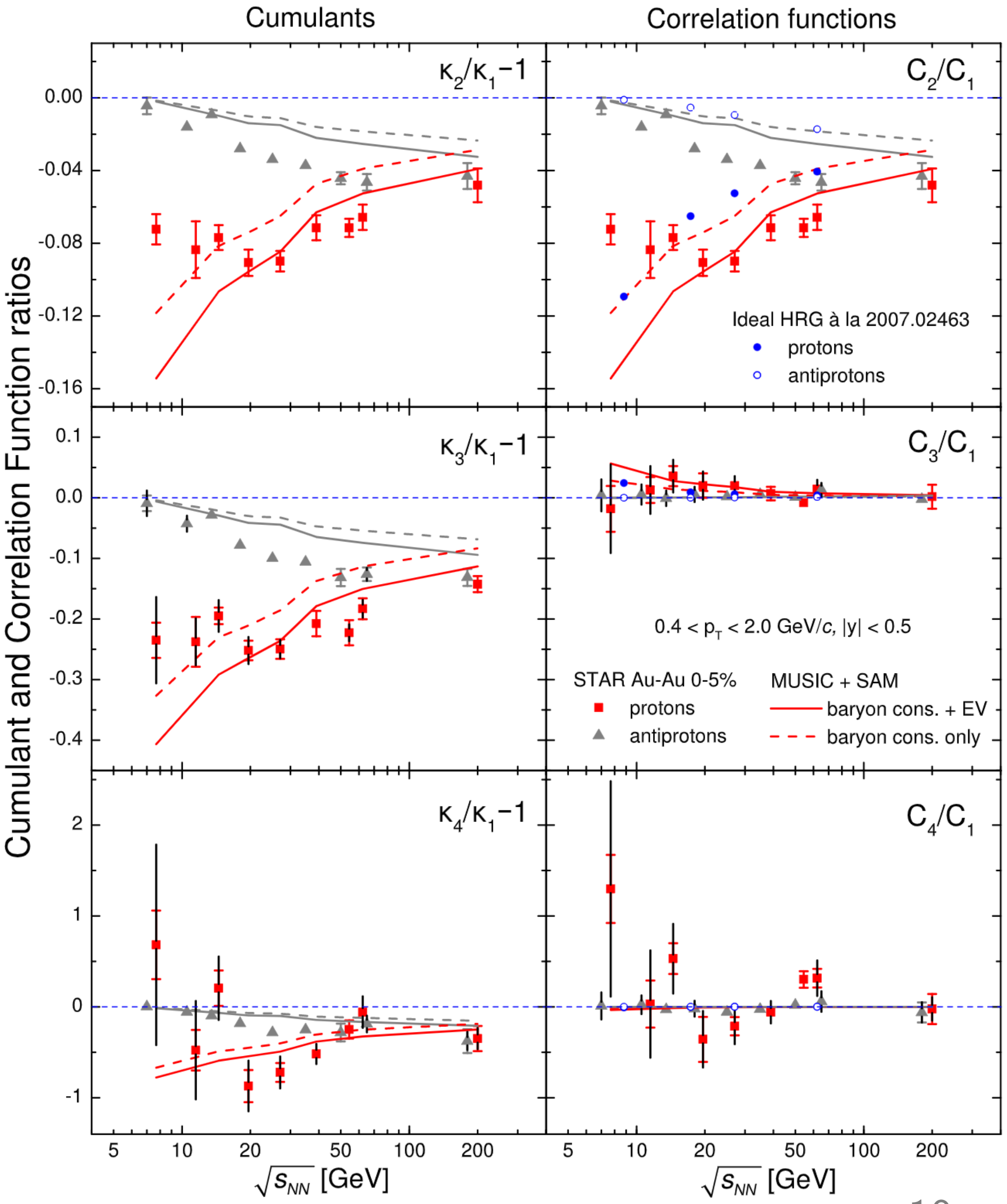
Cumulants vs Correlation Functions

- Analyze genuine multi-particle correlations via **factorial cumulants** [Bzdak, Koch, Strodthoff, PRC '17]

$$\hat{C}_1 = \kappa_1, \qquad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$
$$\hat{C}_2 = -\kappa_1 + \kappa_2, \qquad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$

- Three- and four-particle correlations are small
  - Higher-order cumulants are driven by two-particle correlations
  - Small positive $\hat{C}_3/\hat{C}_1$ in the data is explained by baryon conservation + excluded volume
  - Strong multi-particle correlations would be expected near the critical point [Ling, Stephanov, 1512.09125]

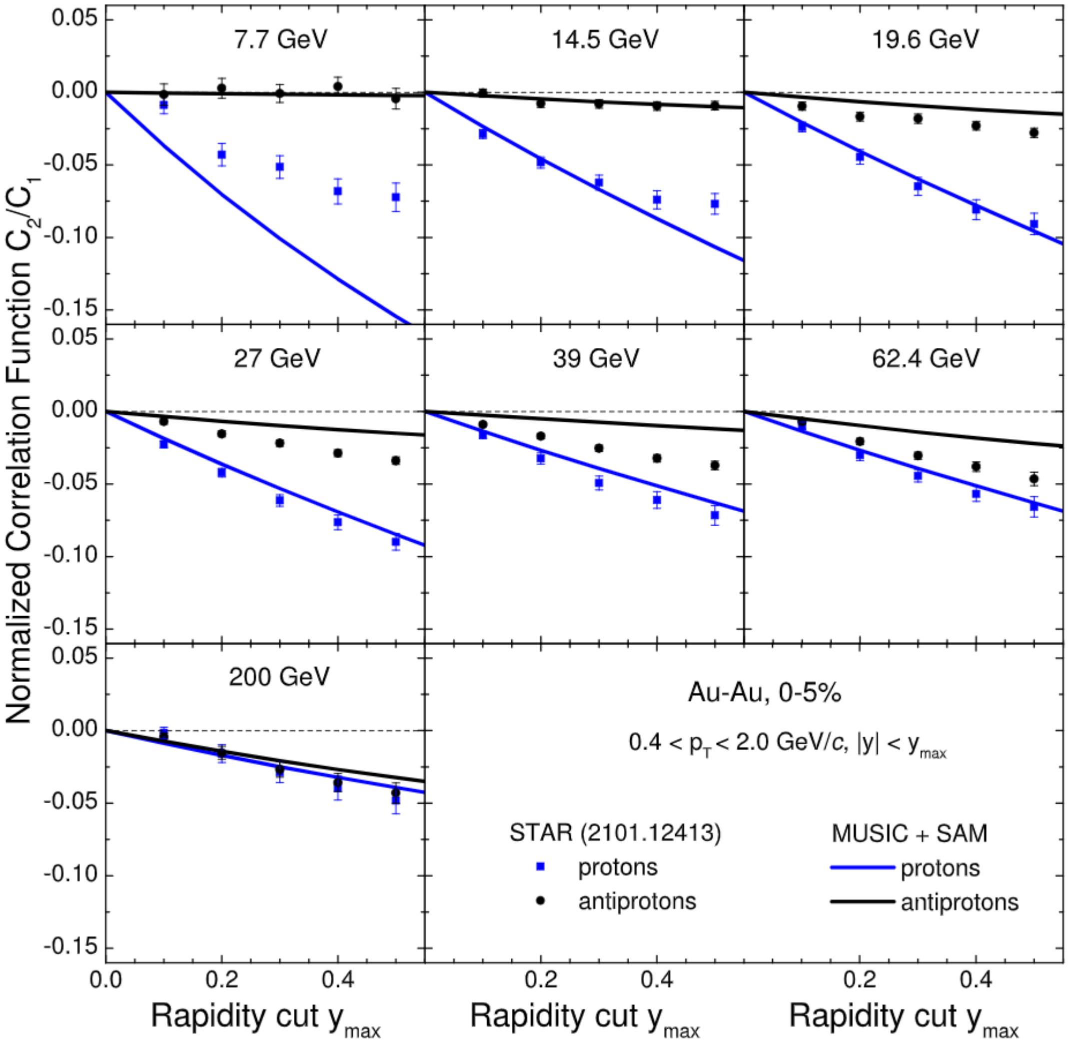
- Two-particle correlations are negative
  - Protons at $\sqrt{s_{NN}} \leq 14.5$ GeV overestimated
  - Antiprotons at $19.6 \leq \sqrt{s_{NN}} \leq 62.4$ GeV underestimated



*We use the notation for (factorial) cumulants from Bzdak et al., Phys. Rept. '20. This is different from STAR's 2101.12413 where it is reversed

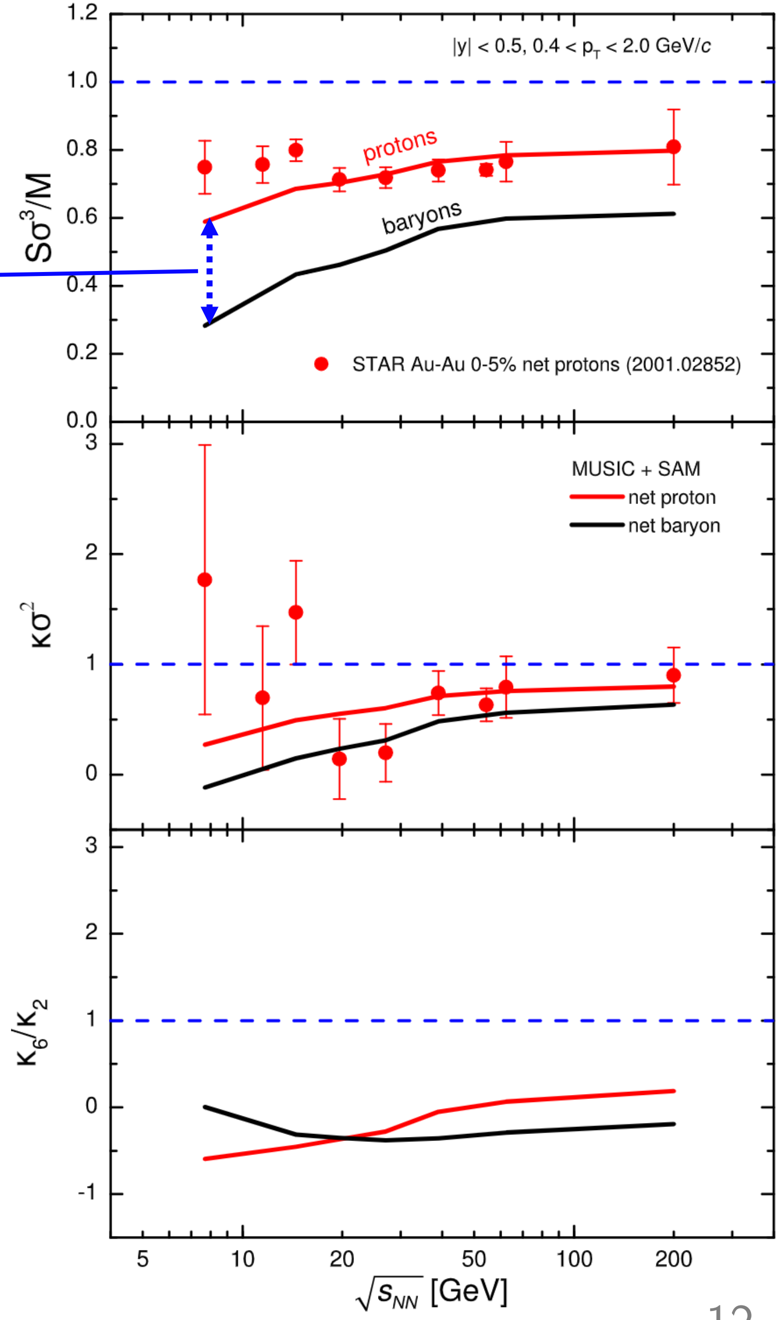
Acceptance dependence of two-particle correlations

- Qualitative agreement with the STAR data
- Data indicate a changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV
- Volume fluctuations? [Skokov, Friman, Redlich, PRC '13]
  - Can improve low energies but spoil high energies?
- Exact electric charge conservation?
  - Worsens the agreement at $\sqrt{s_{NN}} \leq 14.5$, higher energies virtually unaffected (see backup)
- Attractive interactions?
  - Could work if baryon repulsion switches to attraction in the high-μ_B regime

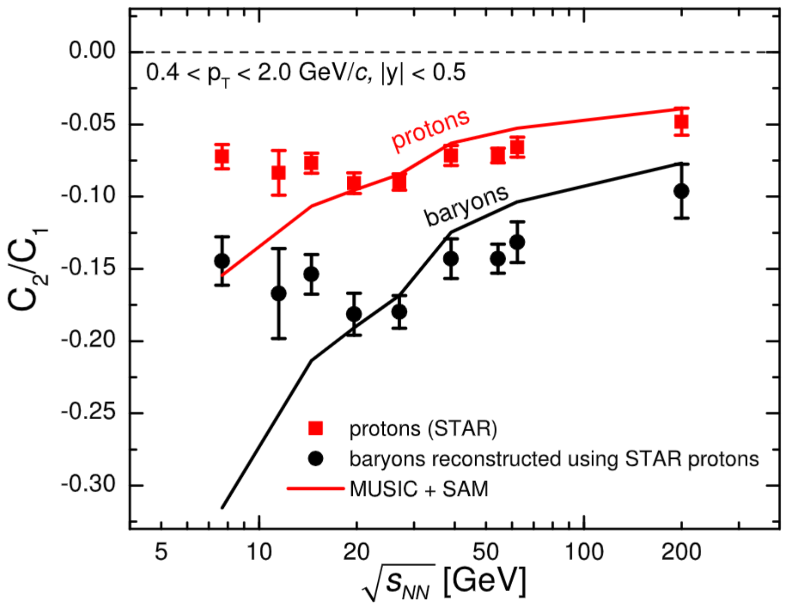


Net baryon vs net proton

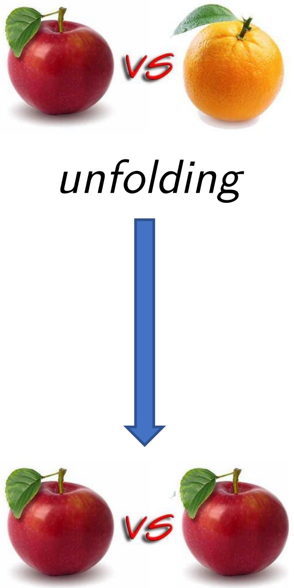
- **net baryon** $\neq$ **net proton**



- Baryon cumulants can be reconstructed from proton cumulants via binomial (un)folding based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]
  - Requires the use of joint factorial moments, only experiment can do it model-independently



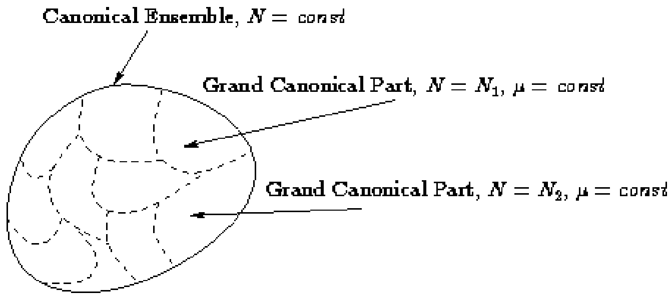
$$\frac{\hat{C}_2^B}{\hat{C}_1^B} \approx 2\,\frac{\hat{C}_2^p}{\hat{C}_1^p}$$



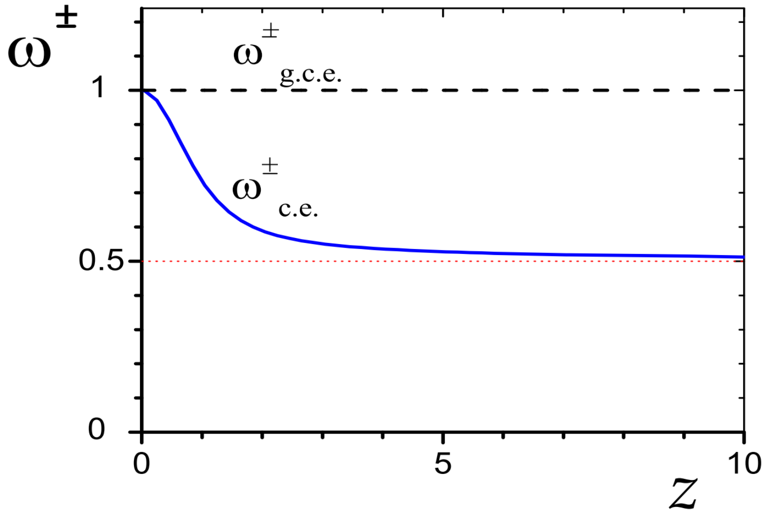
Canonical vs grand-canonical

Grand-canonical ensemble: the system exchanges conserved charges with a heat bath

Canonical ensemble: conserved charges fixed to a same set of values in all microstates



Thermodynamic equivalence: in the limit $V \to \infty$ all statistical ensembles are equivalent wrt to all average quantities, e.g. $\langle N \rangle_{GCE} = N_{CE}$



Begun, Gorenstein, Gazdzicki, Zozulya, PRC '04

Thermodynamic equivalence does *not* extend to fluctuations. The results are ensemble-dependent in the limit $V \to \infty$

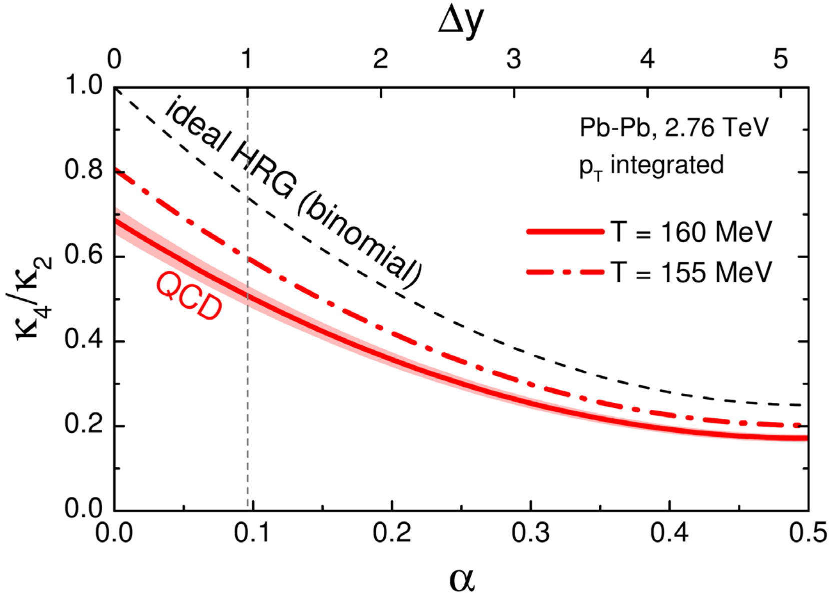
So what ensemble should one use?

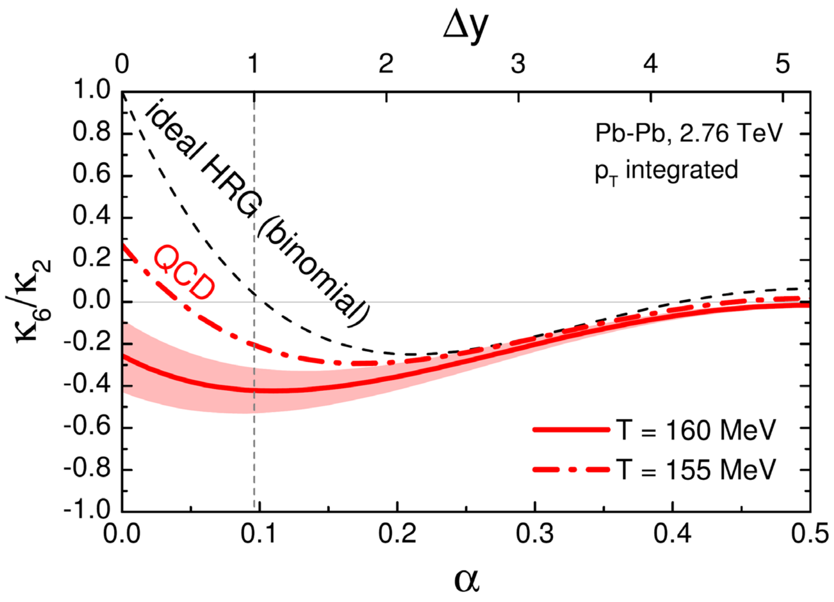
Canonical? Grand-canonical? Something else?

Net baryon fluctuations at LHC ($\mu_B = 0$)

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

$$\left(\frac{\kappa_4}{\kappa_2}\right)_{LHC} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} \qquad \left(\frac{\kappa_6}{\kappa_2}\right)_{LHC} = [1-5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$





Lattice data for χ_4^B/χ_2^B and χ_6^B/χ_2^B from Borsanyi et al., 1805.04445

Theory: negative χ_6^B/χ_2^B is a possible signal of chiral criticality [Friman, Karsch, Redlich, Skokov, EPJC '11]

Experiment: $\alpha \approx \frac{N_{ch}(\Delta y)}{N_{ch}(\infty)} \approx \mathrm{erf}\left(\frac{\Delta y}{2\sqrt{2}\sigma_y}\right)$, for $\Delta y \approx 1$ the κ_6/κ_2 is mainly sensitive to the EoS

Planned measurement in Runs 3 & 4 at the LHC [LHC Yellow Report, 1812.06772]

$N_{ch}(\Delta y)$ measurement: ALICE Collaboration, PLB 726 (2013) 610-622

Net baryon fluctuations at LHC

- Global baryon conservation distorts the cumulant ratios already for one unit of rapidity acceptance

e.g. $\left.\frac{\chi_4^B}{\chi_2^B}\right|_{T=160MeV}^{\text{GCE}} \simeq 0.67$ *"lattice QCD"* $\neq$ $\left.\frac{\chi_4^B}{\chi_2^B}\right|_{\Delta Y_{\text{acc}}=1}^{\text{HIC}} \simeq 0.56$ *experiment*

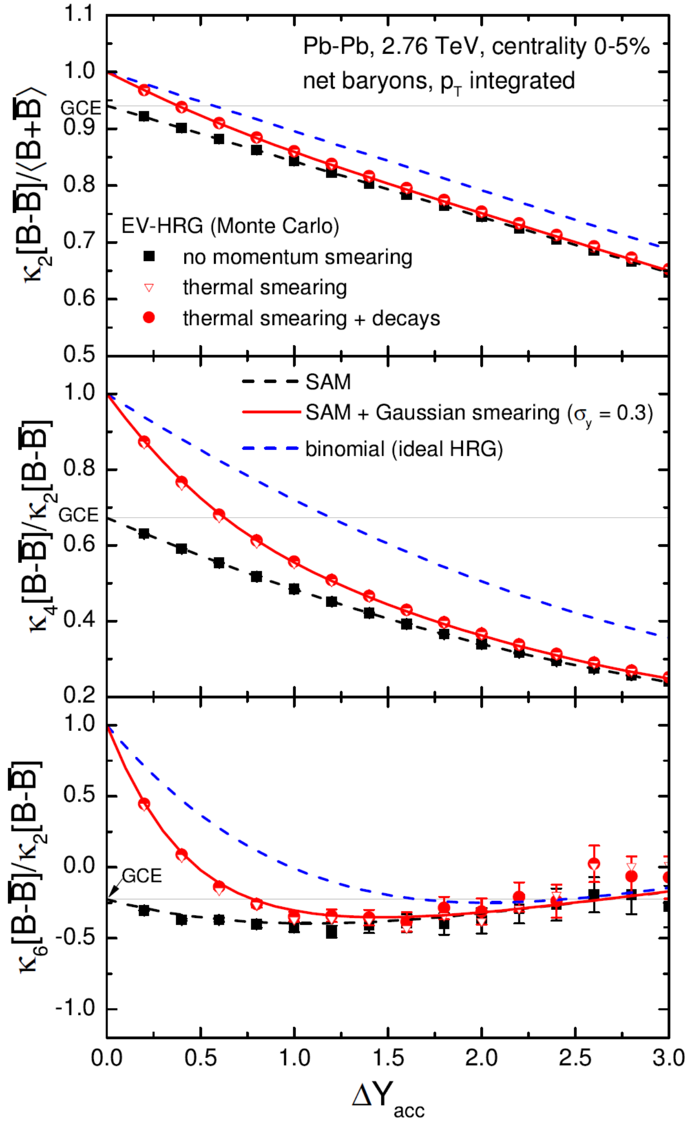
- Neglecting thermal smearing, effects of global conservation can be described analytically via SAM

$$\frac{\kappa_2}{\langle B+\bar{B}\rangle} = (1-\alpha)\frac{\kappa_2^{\text{gce}}}{\langle B+\bar{B}\rangle}, \qquad \alpha = \frac{\Delta Y_{\text{acc}}}{9.6}, \quad \beta \equiv 1-\alpha$$

$$\frac{\kappa_4}{\kappa_2} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B},$$

$$\frac{\kappa_6}{\kappa_2} = [1-5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$

- Effect of resonance decays is negligible



VV, Koch, arXiv:2012.09954

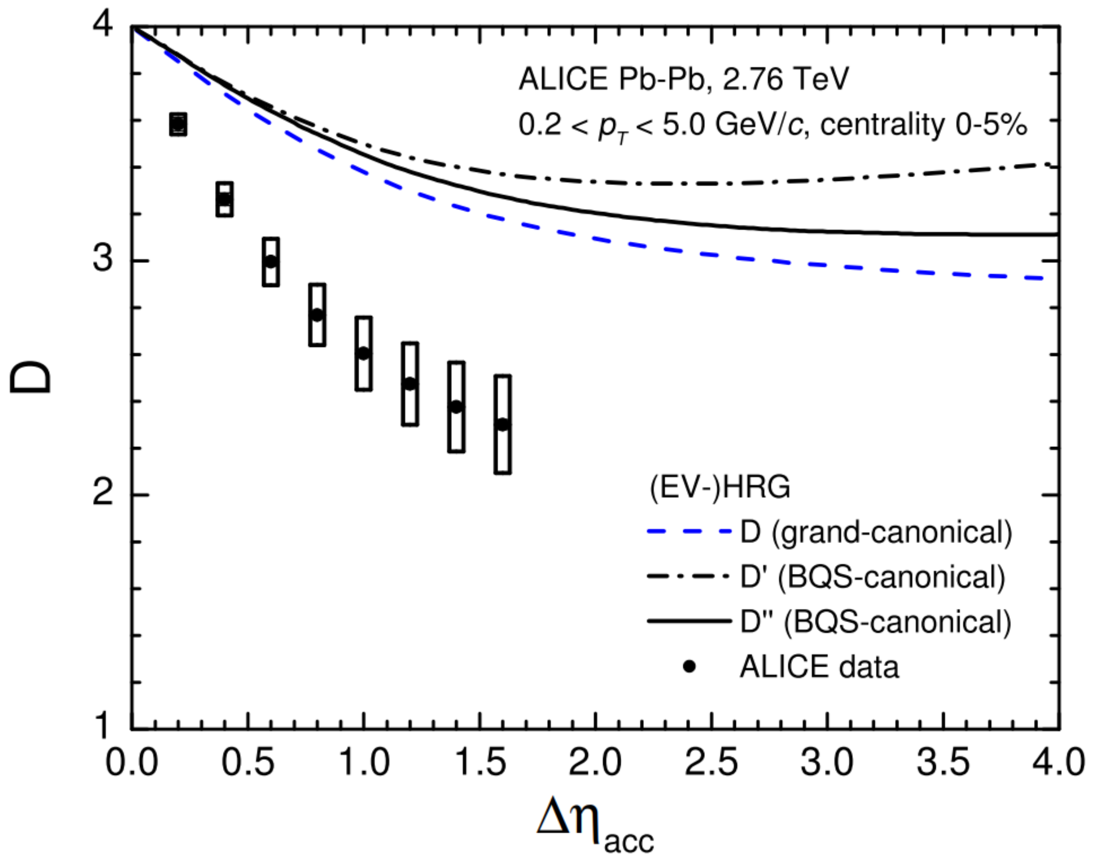
The D-measure

$$D = \frac{\langle \delta Q^2 \rangle}{\langle N_{\mathrm{ch}} \rangle}$$

Jeon, Koch, PRL85, 2076 (2000)

QGP: $D \sim 1 - 1.5$

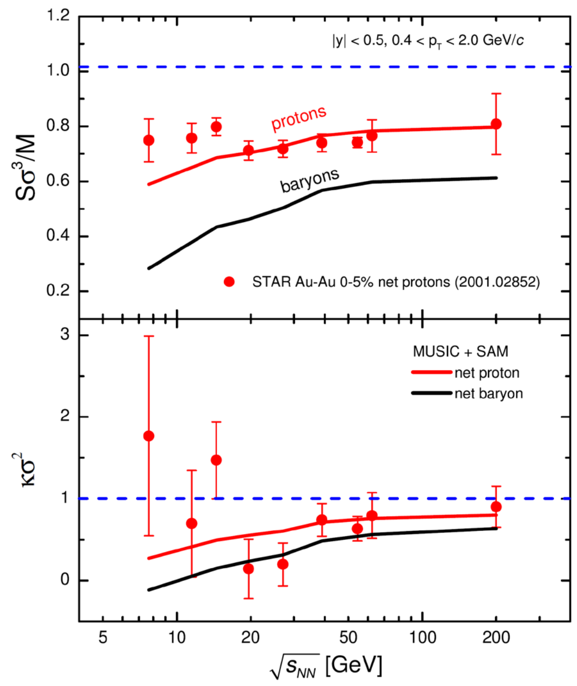
HRG: $D \sim 3 - 4$

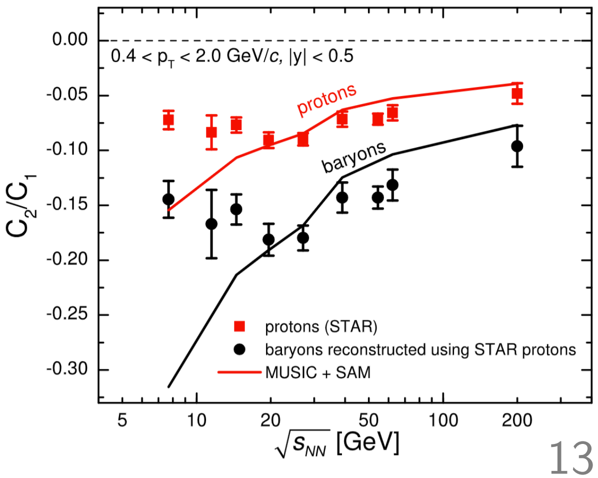


VV, Koch, arXiv:2012.09954

Summary

- (Net-)(anti-) proton cumulants calculated in a hydro description
  - true momentum space acceptance instead of coordinate space
  - simultaneous effects of baryon conservation and repulsive interactions
- Quantitative analysis of Au-Au collisions at $\sqrt{s_{NN}}$=7.7-200 GeV
  - STAR protons are described quantitatively at $\sqrt{s_{NN}} \geq 20$ GeV
  - Significant difference between protons and baryons
- Factorial cumulants carry rich information
  - Small three- and four-particle correlations in absence of critical point effects
  - Possible evidence for attractive proton interactions at $\sqrt{s_{NN}} \leq 14.5$ GeV
  - No quantitative description of antiprotons at $19.6 \leq \sqrt{s_{NN}} \leq 62.4$ GeV





Thanks for your attention!

Study of the QCD phase diagram with heavy-ion collisions

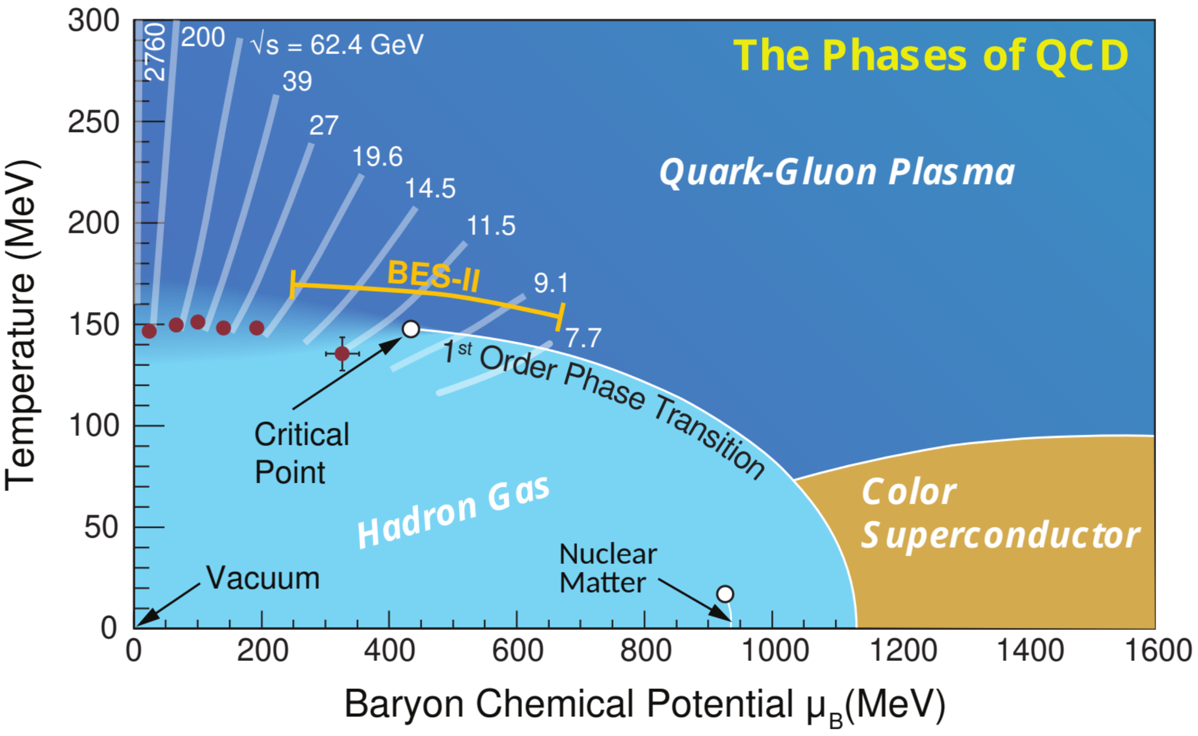
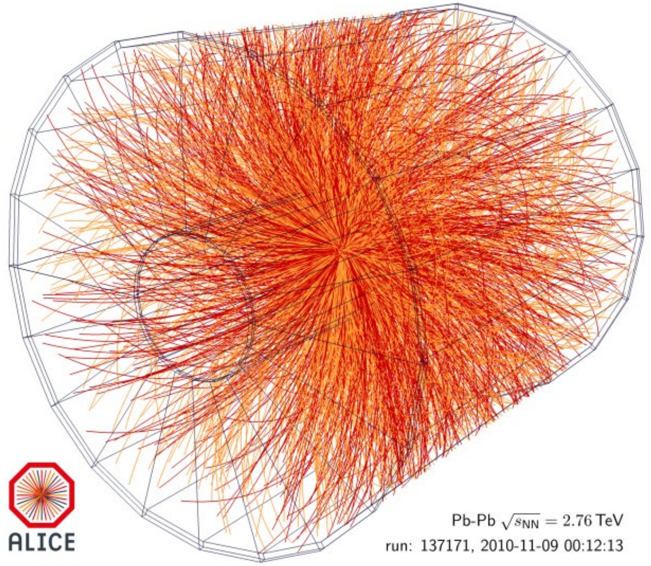


Figure from Bzdak et al., Phys. Rept. ‘20



ALICE event display

Thousands of particles created in relativistic heavy-ion collisions

↓

Apply concepts of statistical mechanics

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

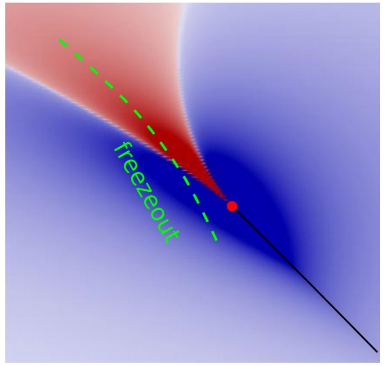
Grand partition function

$$\ln Z^{\text{gce}}(T, V, \mu) = \ln \left[\sum_N e^{\mu N/T} Z^{\text{ce}}(T, V, N) \right]$$

$$\kappa_n \propto \frac{\partial^n (\ln Z^{\text{gce}})}{\partial \mu^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- QCD critical point
- Test of (lattice) QCD at $\mu_B \approx 0$
- Freeze-out from fluctuations



M. Stephanov, PRL '09
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

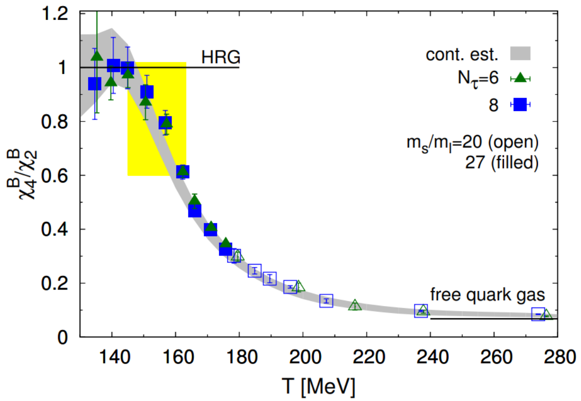
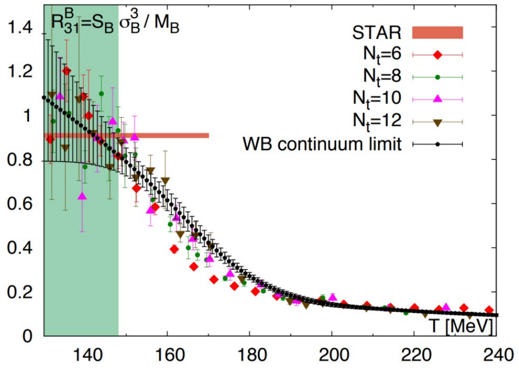


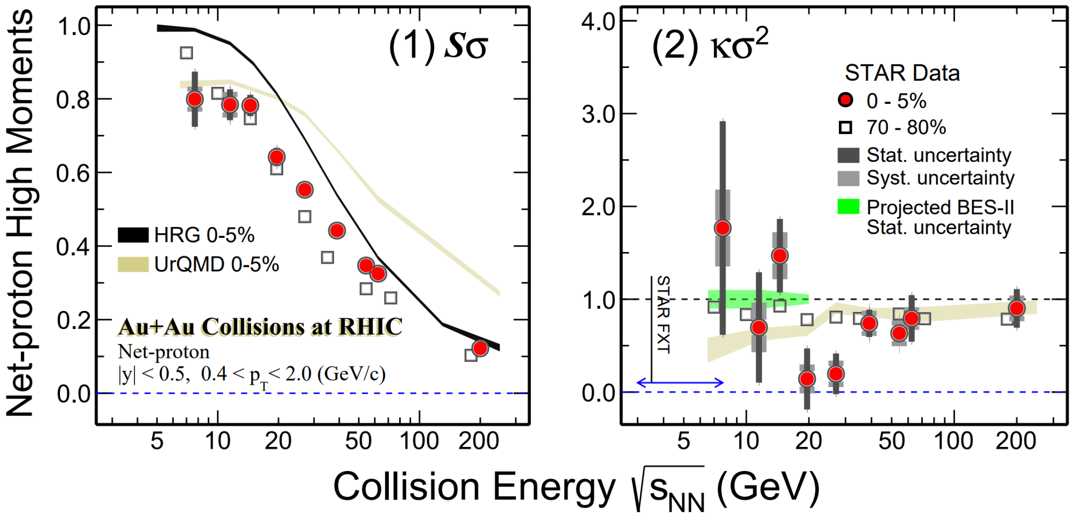
Figure from Bazavov et al. PRD 95, 054504 (2017)
Probed by LHC and top RHIC



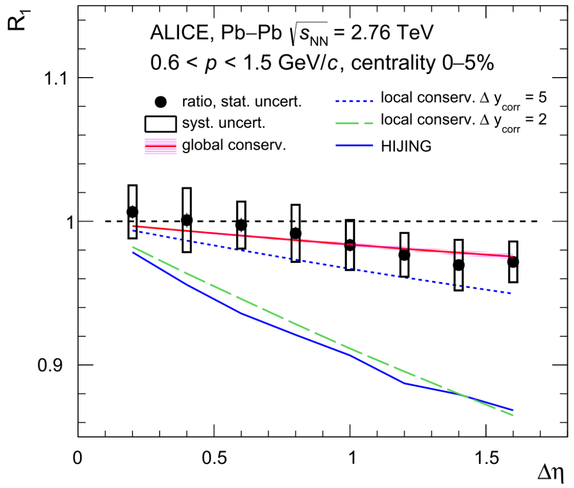
Borsanyi et al. PRL 113, 052301 (2014)
Bazavov et al. PRL 109, 192302 (2012)
...

Experimental measurements

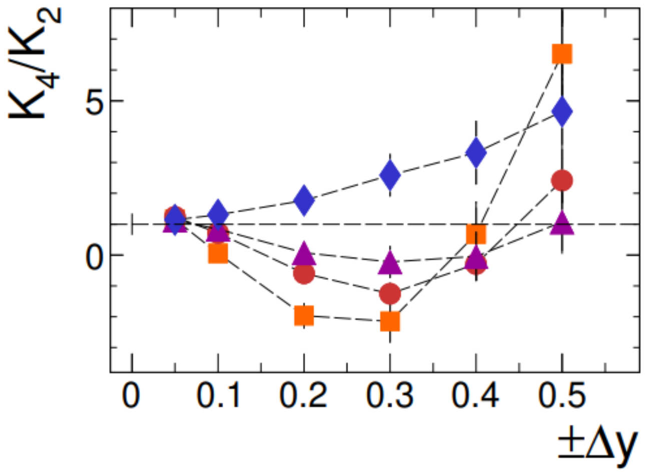
STAR Collaboration, PRL 126, 092301 (2021)



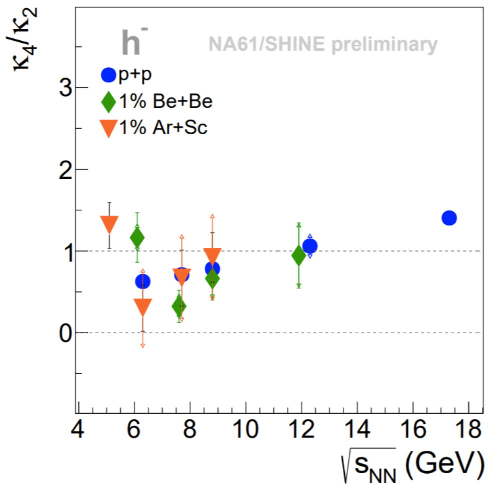
ALICE Collaboration, PLB 807, 135564 (2020)



HADES Collaboration, PRC 102, 024914 (2020)



NA61/SHINE Collaboration, SQM2021



Theory vs experiment: Caveats

- **accuracy of the grand-canonical ensemble (global conservation laws)**
  - **subensemble acceptance method (SAM)**

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

- **coordinate vs momentum space (thermal smearing)**

Ling, Stephanov, PRC 93, 034915 (2016); Ohnishi, Kitazawa, Asakawa, PRC 94, 044905 (2016)

- **proxy observables in experiment (net-proton, net-kaon) vs actual conserved charges in QCD (net-baryon, net-strangeness)**

Kitazawa, Asakawa, PRC 85, 021901 (2012); VV, Jiang, Gorenstein, Stoecker, PRC 98, 024910 (2018)

- volume fluctuations

Gorenstein, Gazdzicki, PRC 84, 014904 (2011); Skokov, Friman, Redlich, PRC 88, 034911 (2013)
X. Luo, J. Xu, B. Mohanty, JPG 40, 105104 (2013); Braun-Munzinger, Rustamov, Stachel, NPA 960, 114 (2017)

- non-equilibrium (memory) effects

Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)

- hadronic phase

Steinheimer, VV, Aichelin, Bleicher, Stoecker, PLB 776, 32 (2018)

Need for ***dynamical description***

Calculating cumulants at particlization

- Strategy:
  1. Calculate proton cumulants in experimental acceptance in the grand-canonical limit*
  2. Apply correction for exact baryon number conservation

First step:

- Sum contributions from each fluid element x_i
  - Cumulants of joint (anti)proton/(anti)baryon distribution
  - Assumes small correlation length $\xi \to 0$

$$\kappa_{n,m}^{B^\pm,p^\pm,\mathrm{gce}}(\Delta p_{\mathrm{acc}}) = \sum_{i\in\sigma} \delta\kappa_{n,m}^{B^\pm,p^\pm,\mathrm{gce}}(x_i; \Delta p_{\mathrm{acc}})$$

- To compute each contribution
  - Grand-canonical susceptibilities $\chi^{B^\pm}(x_i)$ of (anti)baryon number
  - Each baryon ends up in acceptance Δp_{acc} with binomial probability
  - Each baryon is a proton with probability $q(x_i) = \langle N_p(x_i)\rangle / \langle N_B(x_i)\rangle$

[Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]

$$p_{\mathrm{acc}}(x_i; \Delta p_{\mathrm{acc}}) = \frac{\displaystyle\int_{p\in\Delta p_{\mathrm{acc}}} \frac{d^3p}{\omega_p} \delta\sigma_\mu(x_i)\, p^\mu\, f[u^\mu(x_i)p_\mu; T(x_i), \mu_j(x_i)]}{\displaystyle\int \frac{d^3p}{\omega_p} \delta\sigma_\mu(x_i)\, p^\mu\, f[u^\mu(x_i)p_\mu; T(x_i), \mu_j(x_i)]}.$$

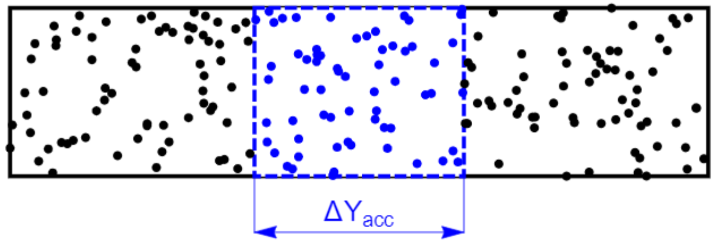
*For similar calculations of critical fluctuations see Ling, Stephanov, 1512.09125 and Jiang, Li, Song, 1512.06164

Correcting for baryon number conservation

- Subensemble acceptance method (SAM)
  - Corrects *any* equation of state for global charge conservation
  - Canonical ensemble cumulants in terms of grand-canonical ones

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, Phys. Lett. B 811, 135868 (2020) [arXiv:2003.13905]

VV, Poberezhnyuk, Koch, JHEP 10, 089 (2020) [arXiv:2007.03850]



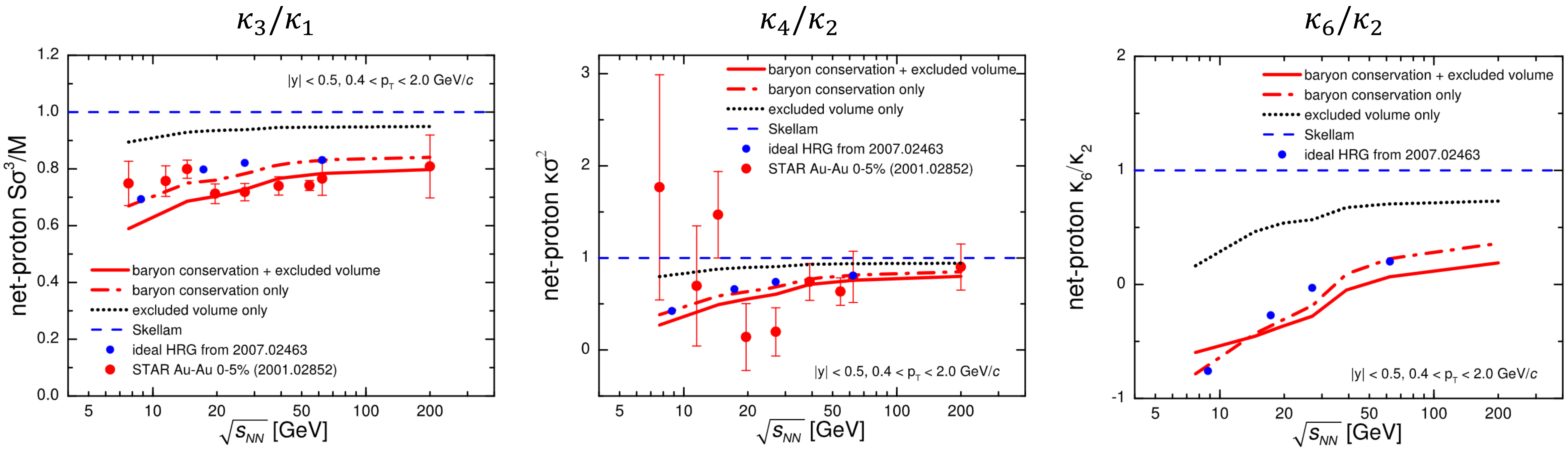
- SAM-2.0
  - Non-conserved quantities (e.g. proton number)
  - Spatially inhomogeneous systems
  - Momentum space
  - Map "grand-canonical" cumulants inside and outside the acceptance to the "canonical" cumulants inside the acceptance

$$\kappa_{p,B}^{\mathrm{in,ce}} = \mathrm{SAM}\left[\kappa_{p,B}^{\mathrm{in,gce}}, \kappa_{p,B}^{\mathrm{out,gce}}\right]$$

VV, *to appear*

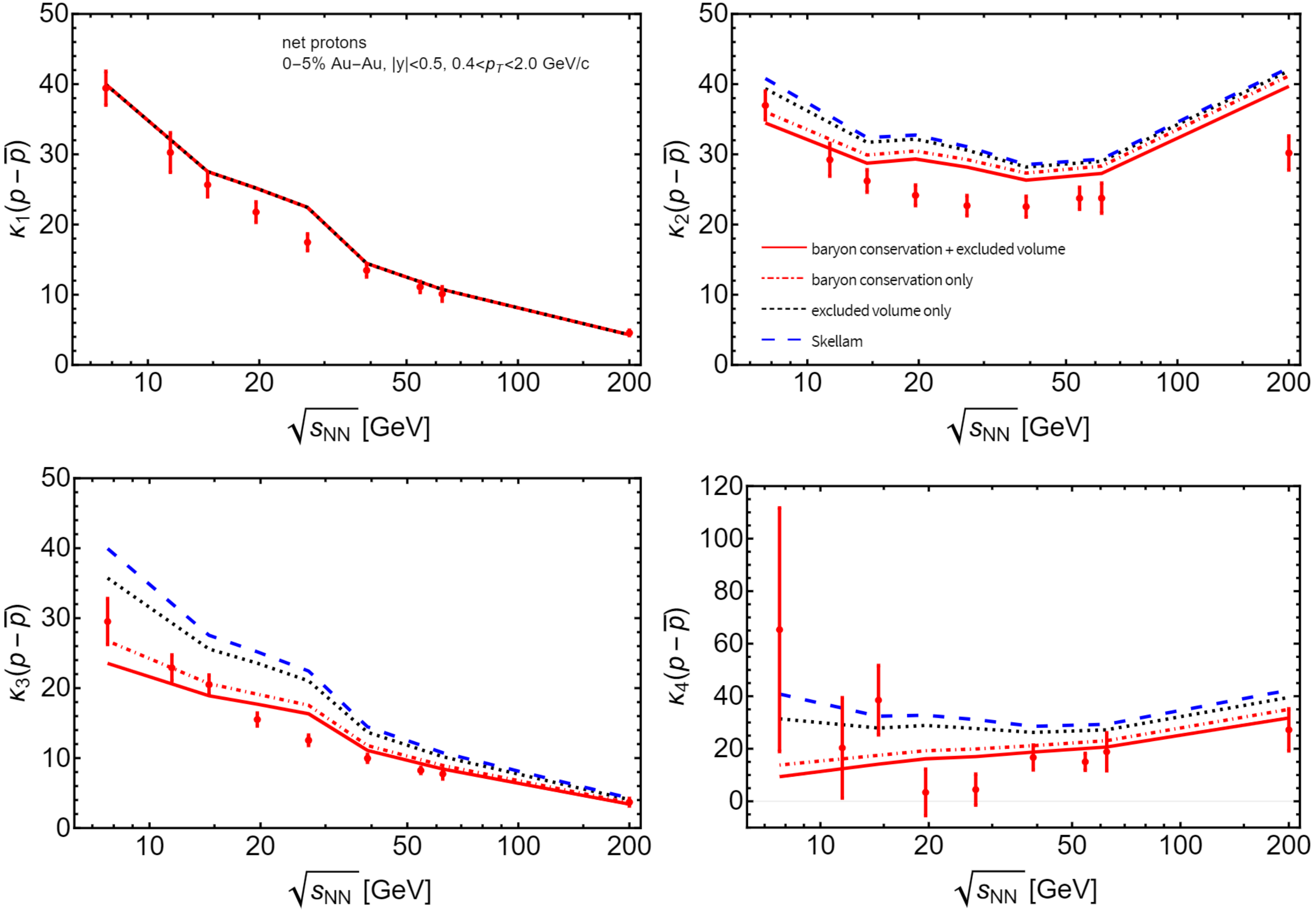
Implementation in the (extended) **Thermal-FIST** package https://github.com/vlvovch/Thermal-FIST

Net proton cumulant ratios

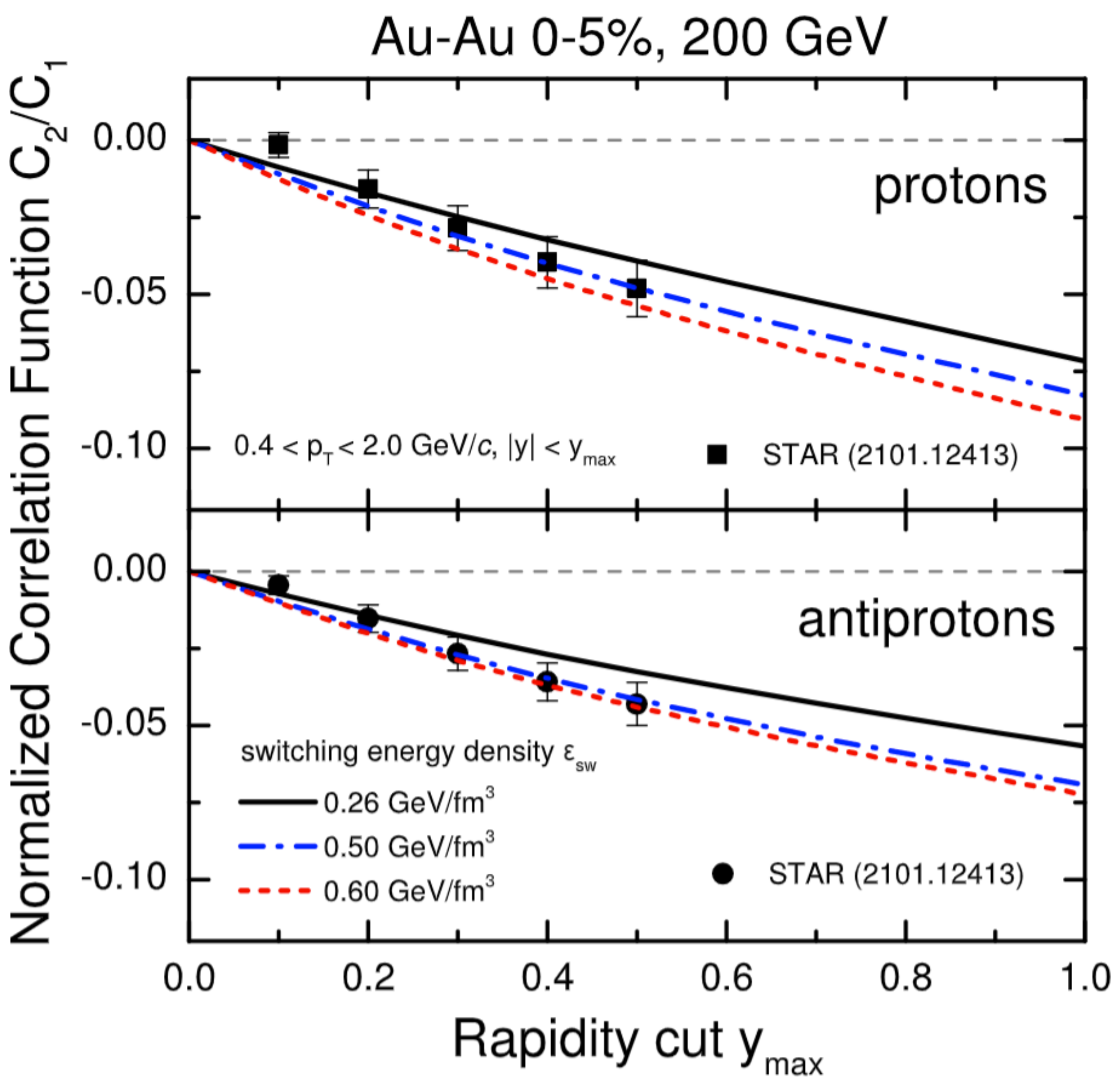


- Both the baryon conservation and repulsion needed to describe data at $\sqrt{s_{NN}} \geq 20$ GeV quantitatively
- Effect from baryon conservation is larger than from repulsion
- Canonical ideal HRG limit is consistent with the data-driven study of [Braun-Munzinger et al., 2007.02463]
- κ_6/κ_2 turns negative at $\sqrt{s_{NN}}$~50 GeV

Net proton cumulants at RHIC

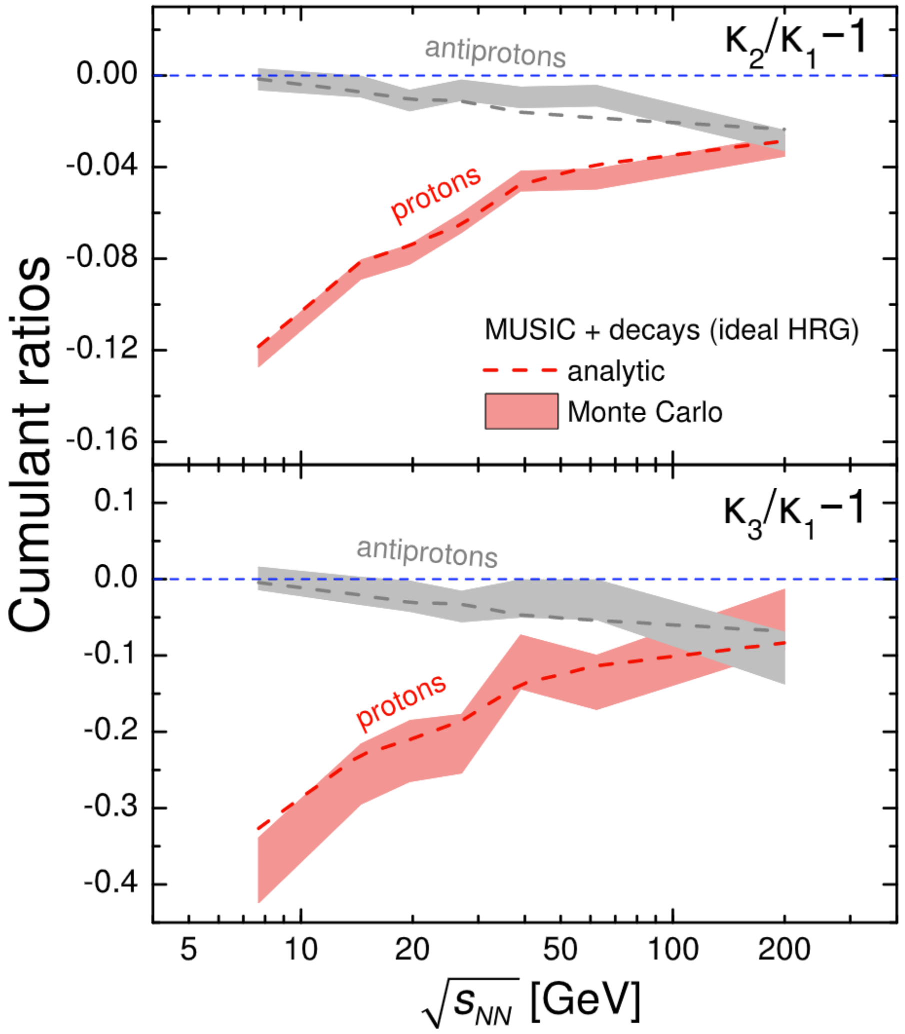


Dependence on the switching energy density



Cross-checking the cumulants with Monte Carlo

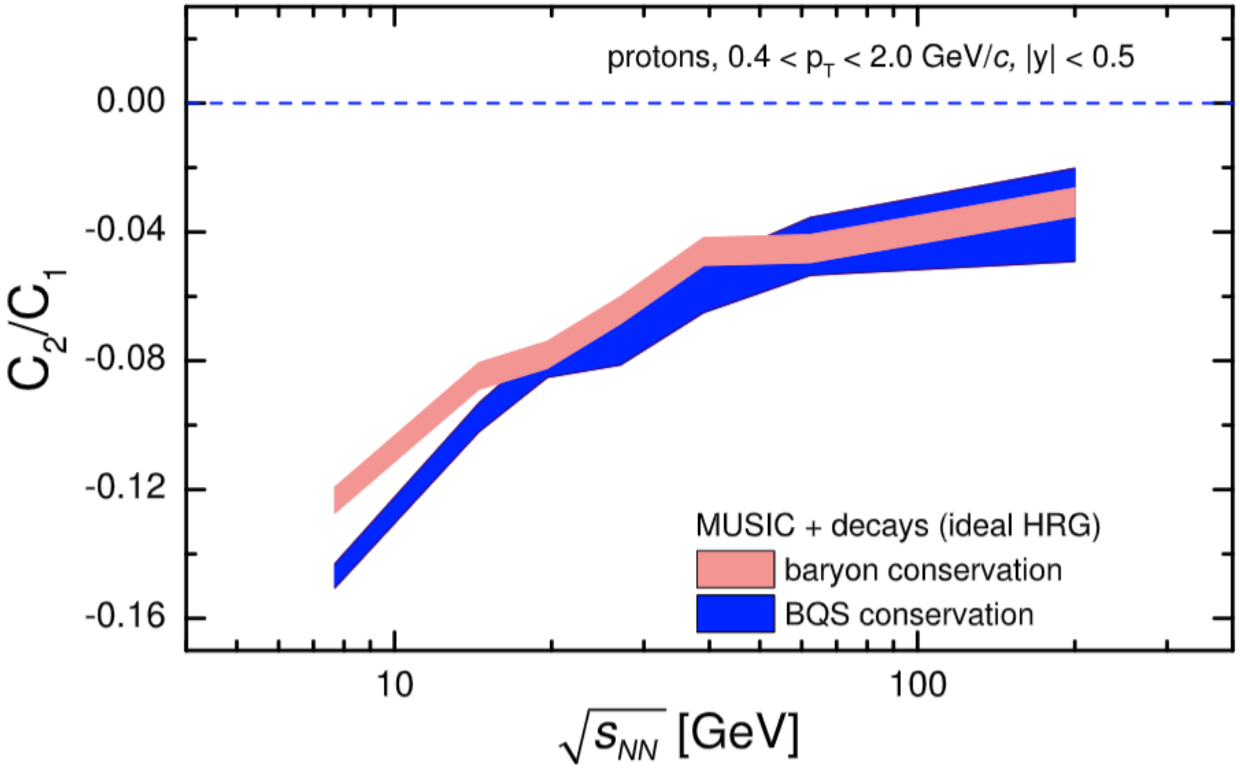
- Sample canonical ideal HRG model at particlization with Thermal-FIST
- Analytic results agree with Monte Carlo within errors



Exact conservation of electric charge

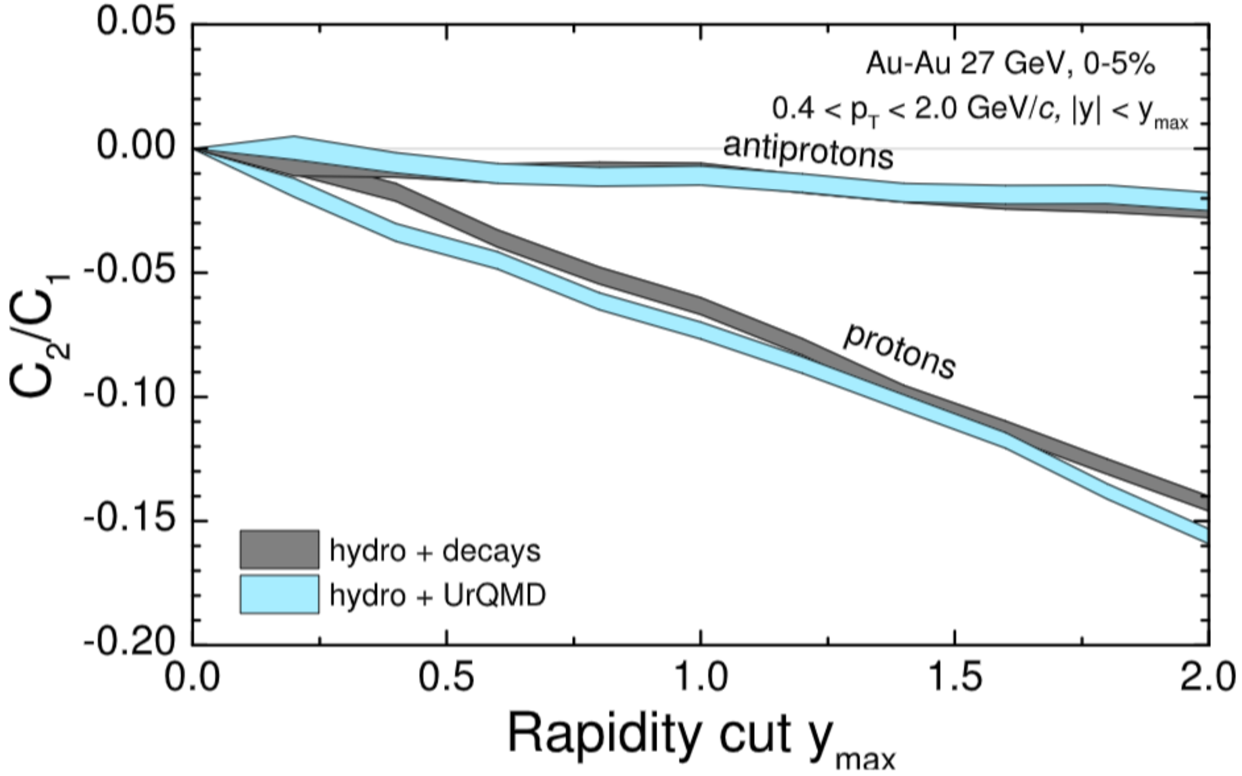
- Sample ideal HRG model at particlization with exact conservation of baryon number, electric charge, and strangeness using Thermal-FIST

- Protons are affected by electric charge conservation at $\sqrt{s_{NN}} \leq 14.5$



Effect of the hadronic phase

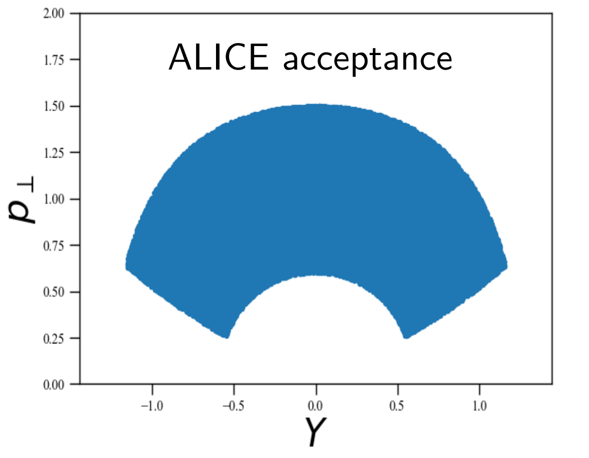
Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD



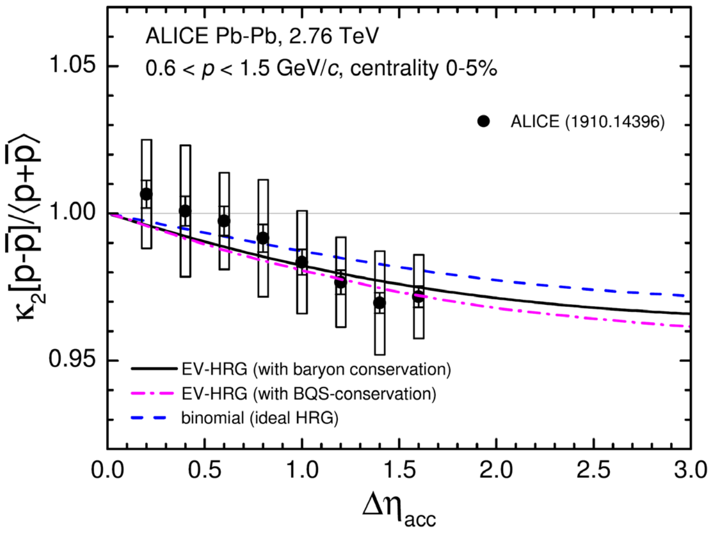
Net-particle fluctuations at the LHC

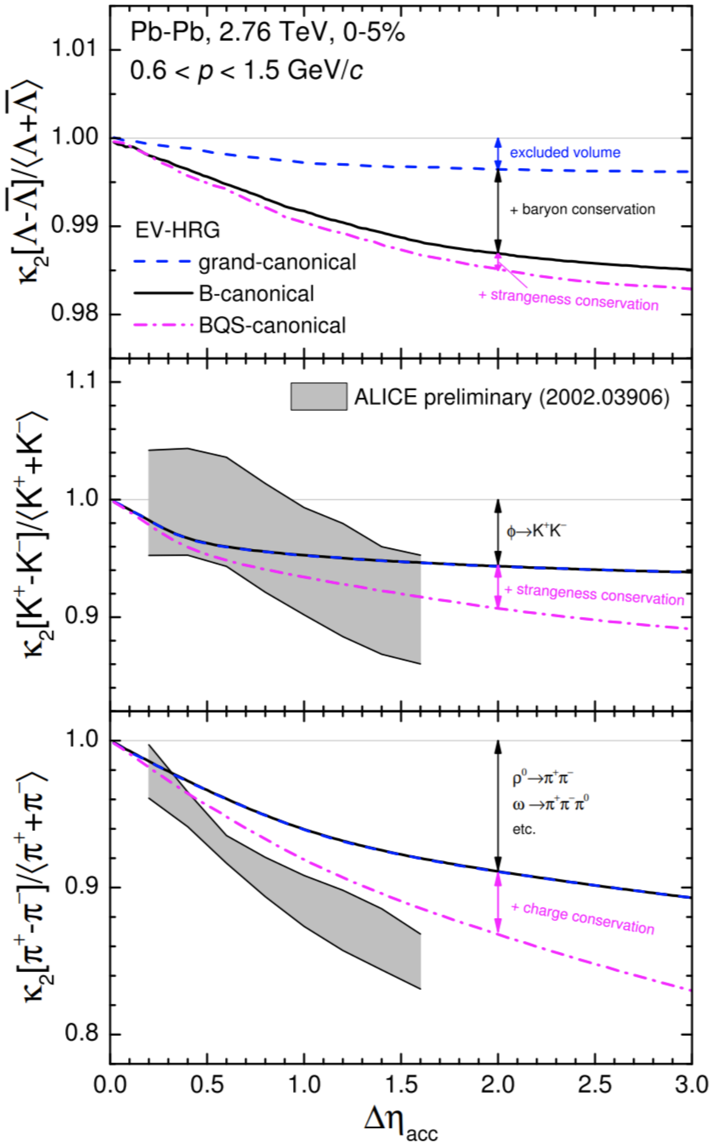
VV, Koch, Phys. Rev. C 103, 044903 (2021)

- Net protons described within errors but not sensitive to the equation of state for the present experimental acceptance
- Large effect from resonance decays for lighter particles
- Future measurements will require larger acceptance



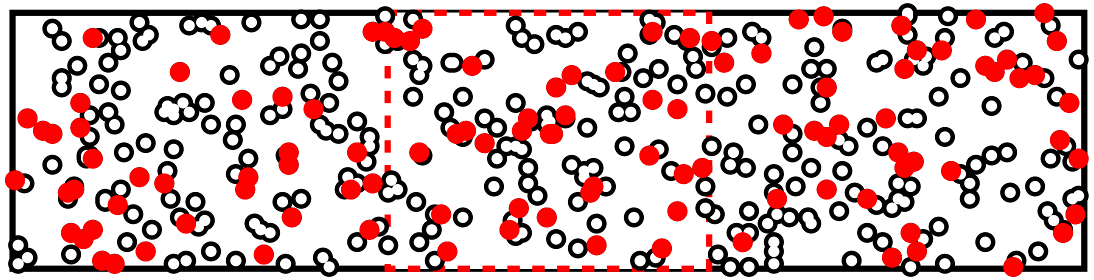
$0.6 < p < 1.5$ GeV$/c$, $\Delta\eta_{acc} = 1.6$



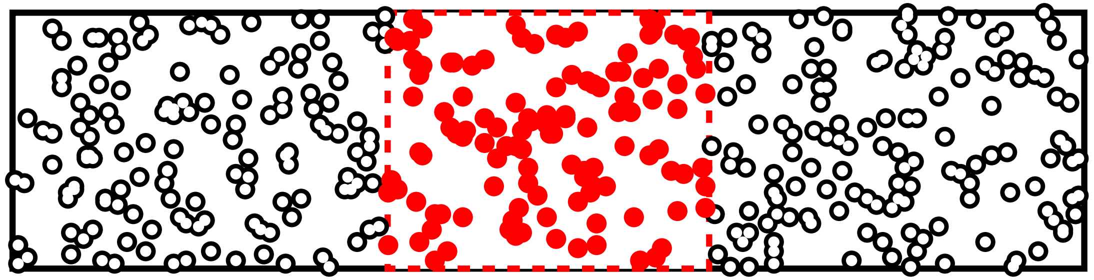


Binomial acceptance vs actual acceptance

Binomial acceptance: accept each particle (charge) with a probability α independently from all other particles



SAM:



SAM for multiple conserved charges (B,Q,S)

VV, Poberezhnyuk, Koch, JHEP 10, 089 (2020)

Key findings:

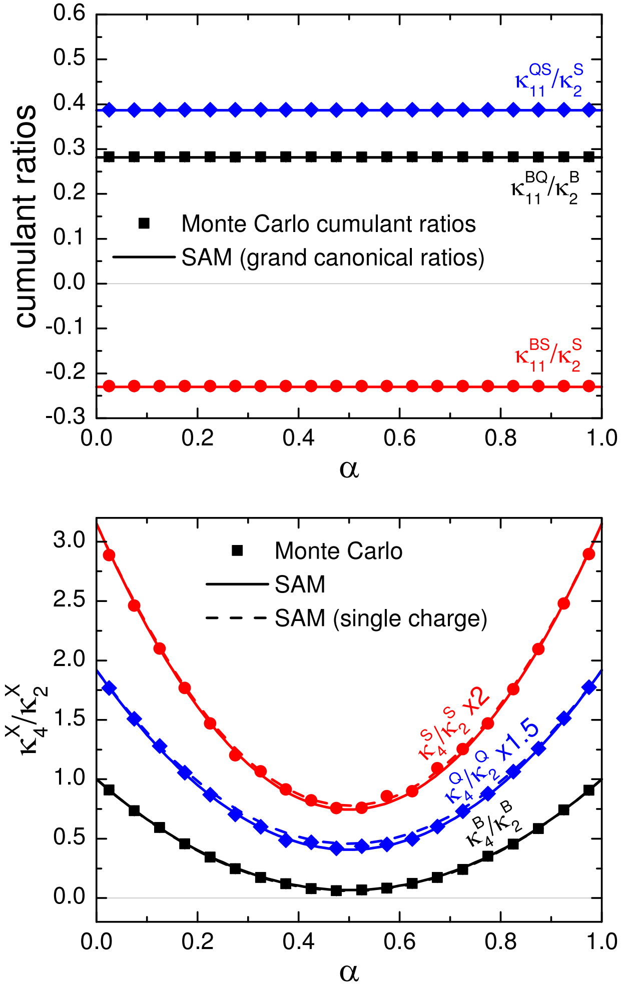
- Cumulants up to 3rd order factorize into product of binomial and grand-canonical cumulants

$$\kappa_{l,m,n} = \kappa^{\rm bino}_{l+m+n}(\alpha) \times \kappa^{\rm gce}_{l,m,n} \,, \qquad l+m+n \leq 3$$

  - Ratios of second and third order cumulants are NOT sensitive to charge conservation
  - Also true for the measurable ratios of covariances involving one non-conserved charge, such as κ_{pQ}/κ_{kQ}

- For order $n > 3$ charge cumulants "mix". Effect in HRG is tiny

$$\kappa_4^B = \kappa_4^{B,\rm gce}\, \beta \left[(1-3\alpha\beta)\, \chi_4^B - 3\,\alpha\,\beta\, \frac{(\chi_3^B)^2 \chi_2^Q - 2\chi_{21}^{BQ}\chi_{11}^{BQ}\chi_3^B + (\chi_{21}^{BQ})^2\chi_2^B}{\chi_2^B\chi_2^Q - (\chi_{11}^{BQ})^2} \right]$$



Experiment: Measurements of the off-diagonal cumulants are in progress, e.g. [STAR Collaboration, arXiv:1903.05370]

Mathematica notebook to express any B,Q,S-cumulant of order $n \leq 6$ in terms of grand-canonical susceptibilities available at https://github.com/vlvovch/SAM

Net baryon fluctuations at LHC

- Global baryon conservation distorts the cumulant ratios already for one unit of rapidity acceptance

e.g. *"lattice QCD"* $\left.\frac{\chi_4^B}{\chi_2^B}\right|_{T=160MeV}^{GCE} \simeq 0.67 \neq$ *experiment* $\left.\frac{\chi_4^B}{\chi_2^B}\right|_{\Delta Y_{acc}=1}^{HIC} \simeq 0.56$

- Neglecting thermal smearing, effects of global conservation can be described analytically via SAM

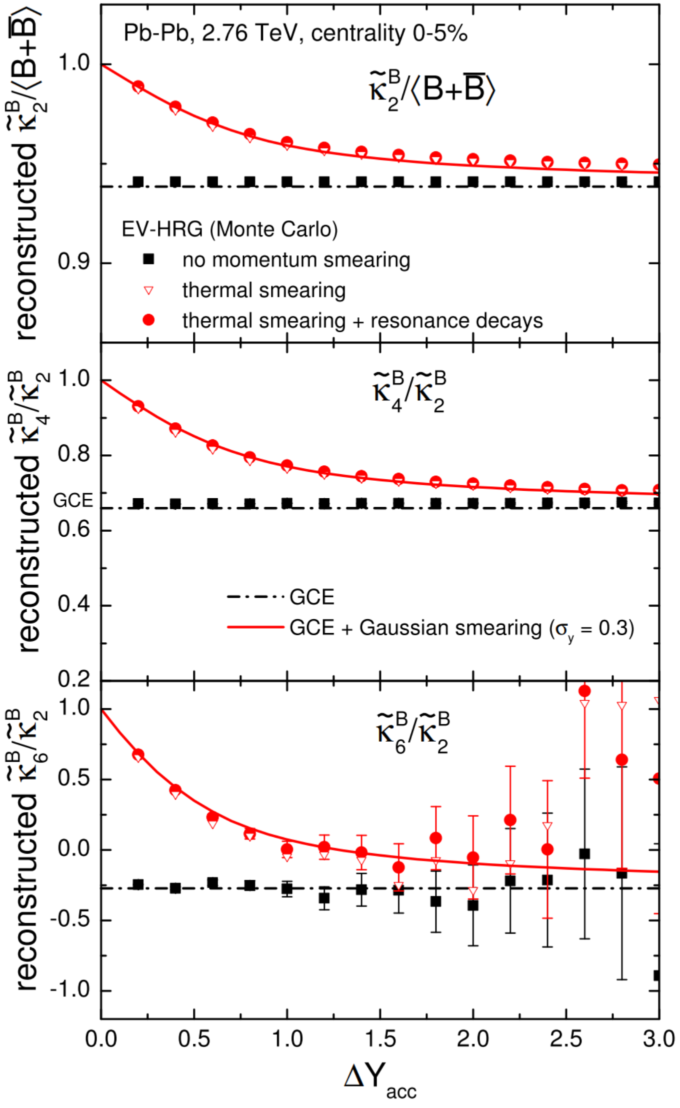
$$\frac{\kappa_2}{\langle B+\bar{B}\rangle} = (1-\alpha)\frac{\kappa_2^{gce}}{\langle B+\bar{B}\rangle}, \qquad \alpha = \frac{\Delta Y_{acc}}{9.6}, \quad \beta \equiv 1-\alpha$$

$$\frac{\kappa_4}{\kappa_2} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B},$$

$$\frac{\kappa_6}{\kappa_2} = [1-5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$

- Effect of resonance decays is negligible

Cumulants corrected for baryon conservation



VV, Koch, arXiv:2012.09954

Volume fluctuations

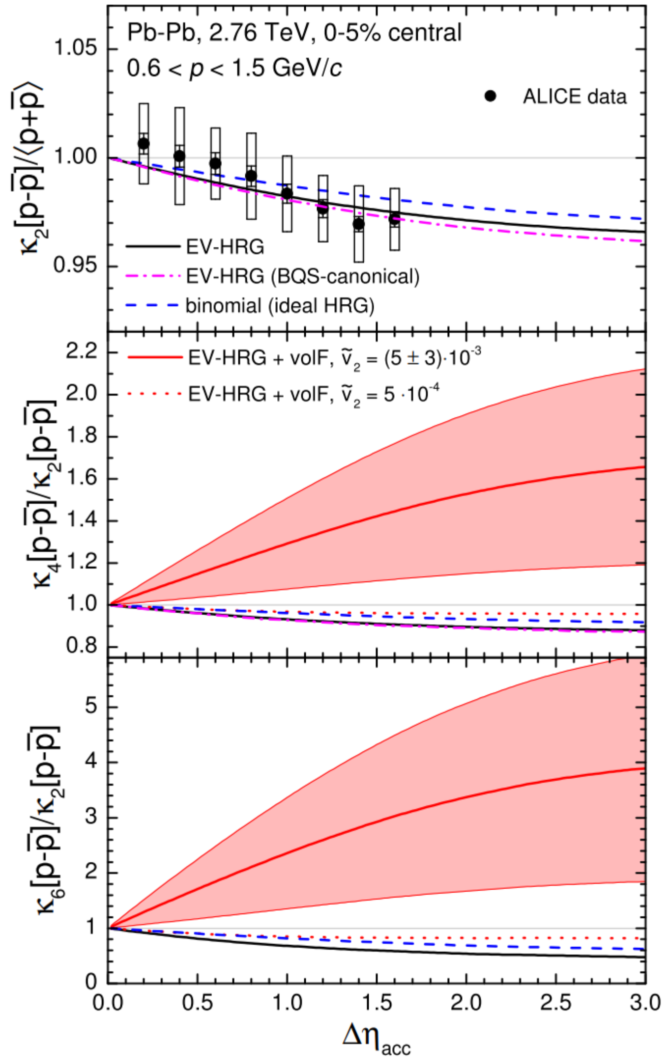
$$\tilde{\kappa}_n = \sum_{l=1}^{n} V_l \, B_{n,l}(\kappa_1/V, \kappa_2/V, \dots, \kappa_{n-l+1}/V)$$

Net-protons at LHC:

$$\tilde{\kappa}_2 = \kappa_2,$$
$$\tilde{\kappa}_4 = \kappa_4 + 3\kappa_2^2 \, \tilde{v}_2,$$
$$\tilde{\kappa}_6 = \kappa_6 + 15\kappa_2 \, \kappa_4 \, \tilde{v}_2 + 15\kappa_2^3 \, \tilde{v}_3 \ .$$

Protons at LHC:

$$\frac{\tilde{\kappa}_2^p}{\langle p \rangle} = \frac{\kappa_2^p}{\langle p \rangle} + \langle p \rangle \, \tilde{v}_2$$



VV, Koch, arXiv:2012.09954