Calculating QCD Phase Diagram Trajectories of Nuclear Collisions using a Semi-analytical Model

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Outline

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  – Densities
  – Quantum ideal gas EOS
  – Boltzmann ideal gas EOS
  – Lattice EOS
  – Transverse expansion
• Summary
Density calculations – Bjorken $\epsilon(t)$ formula

- Narrow volume at time $t$
- Formation time
- Rapidity $y \sim 0$

$$t_F = \tau_F \cosh y$$

$$\epsilon_{Bj}(t \geq \tau_F) = \frac{1}{A_T t} \left. \frac{dE_T}{dy} \right|_{y=0}$$

$A_T = \pi R_A^2$

$R_A \approx 1.12 A^{1/3}$

Central Au+Au collisions

Density calculations – finite thickness

- Integrate transverse mass rapidity density over the production region at time $t$

$$\frac{d^3 m_T}{dy \, dz_0 \, dx}$$

- Factorize the spatial and temporal dependence from the rapidity dependence

- Assume uniform distribution in $(z_0, x)$
Energy density

- Double Gaussian, \((y=0\) from proton and net-proton \(dN/dy\))
- Net-baryon number conservation

\[
\epsilon(t) = \frac{1}{A_T} \int_{S(t)} \frac{dz_0 dx}{t-x} \frac{d^3 m_T}{dy dz_0 dx} \cosh^3 y
\]

\[
\frac{dm_T}{dy} = \frac{dE_T}{dy} + m_N \frac{dN_{B-B}}{dy}
\]

- Single Gaussian, \((y=0\) from PHENIX)
- Total energy conservation

\(\tau_F\) (fm/c)
- 0.1 - Bjorken
- 0.1 - Finite thickness
- 0.9 - Finite thickness

\(\sqrt{s_{NN}}\) (GeV)
- 5
- 39
- 200

Net conserved-charge densities

\[ n_B(t) = \frac{1}{A_T} \iint_{S(t)} dz_0 dx \frac{d^3 N_{B-\bar{B}}}{t-x} \cosh^2 y \]

\[ n_Q(t) = n_B(t) \frac{Z}{A} \]

\[ n_S(t) = 0 \]

- No net strangeness

Extracting \((T-\mu)\) from \((\varepsilon-n)\)

- First, using ideal gas EOS for massless QGP – \((u, d, s)\)
  - Quantum & Boltzmann statistics
  - Later using a lattice-based EOS

- Quark-antiquark pair production \(\mu_q + \mu_{\bar{q}} = 0\)

- Chemical potential of parton \(i\)

- Strangeness neutrality

\[
\mu_i = B_i \mu_B + Q_i \mu_Q + S_i \mu_S
\]

\[
\mu_B - \mu_Q - 3\mu_S = 0
\]

\[
\begin{align*}
\mathbf{n}_s^{\text{quantum}} &= \frac{-\mu_B - \mu_Q - 3\mu_S}{3} T^2 - \frac{(\mu_B - \mu_Q - 3\mu_S)^3}{27\pi^2} \\
\mathbf{n}_s^{\text{Boltzmann}} &= -\frac{12}{\pi^2} T^3 \sinh \left( \frac{\mu_B - \mu_Q - 3\mu_S}{3T} \right)
\end{align*}
\]
Massless QGP ideal gas EOS with Quantum stat. ($\tau_F = 0.1$ fm/c)

$\sqrt{s_{NN}}$ (GeV)
- 5
- 39
- 200

$T$ (MeV)
- $\mu_n$ (MeV)
- $\mu_s$ (MeV)

$t$ (fm/c)

Massless QGP ideal gas EOS with quantum stat.

- $\tau_F = 0.1 \text{ fm/c}$
- $\tau_F = 0.3 \text{ fm/c}$
- $\tau_F = 0.9 \text{ fm/c}$

$\sqrt{s_{NN}}$ (GeV):
- 2
- 5
- 11.5
- 27
- 62.4
- 200

Clockwise path

Hadron degrees of freedom below crossover

$\epsilon_i = 0.51 \text{ GeV/fm}^3$
$\epsilon_i = 1.23 \text{ GeV/fm}^3$

The formation time determines the region which is accessible to heavy ion collisions.
Larger maximum $\mu_B$ in quantum statistics results can be understood using the Pauli exclusion principle.
Lattice EOS with densities from our semi analytical model agree with RHIC chemical freezeout data
Transverse Expansion

\[ R_T(t) = R_A + \beta_T(t) (t - t_1) \]

\[ \beta_T(t) = \begin{cases} 
0, & \text{for } 0 \leq t < t_1 \\
\frac{1-e^{-(t-t_1)/a}}{1-e^{-(t_k-t_1)/a}} \beta_{T,f}, & \text{for } t_1 \leq t < t_k \\
\beta_{T,f}, & \text{for } t_k \leq t 
\end{cases} \]

- \( a \): model-based, collision energy dependent timescale for development of average freezeout velocity
- \( t_k \): time to reach the freezeout velocity \( t_k = t_1 + t_{QGP} + t_{Had} \)

Transverse expansion has small effect on the endpoint of trajectories.

There is a weak dependence on $t_k - t_1$. 

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QGP Lifetime

Use $T(t)$ and $\mu_B(t)$ to find the times when the trajectory intersects the FRG crossover curve.

Transverse expansion has an effect of almost a factor $\sim 2$ over all energies.

Summary

• We have developed a **semi-analytical** model to calculate the trajectories of Au+Au collisions in the QCD phase diagram.
  - We have made a web application to calculate trajectories using quantum or Boltzmann ideal gas equations of state: [http://myweb.ecu.edu/linz/densities/](http://myweb.ecu.edu/linz/densities/)

• The finite thickness and EOS have **large** effects on the calculated trajectories.

• Transverse expansion has a **small** effect on the trajectory path, but a **large** effect on the QGP lifetime.

• The lattice EOS with our semi-analytical densities agrees with the RHIC chemical freezeout data.