Calculating QCD Phase Diagram Trajectories of Nuclear Collisions using a Semi-analytical Model

Todd Mendenhall* and Zi-Wei Lin

East Carolina University

*Speaker

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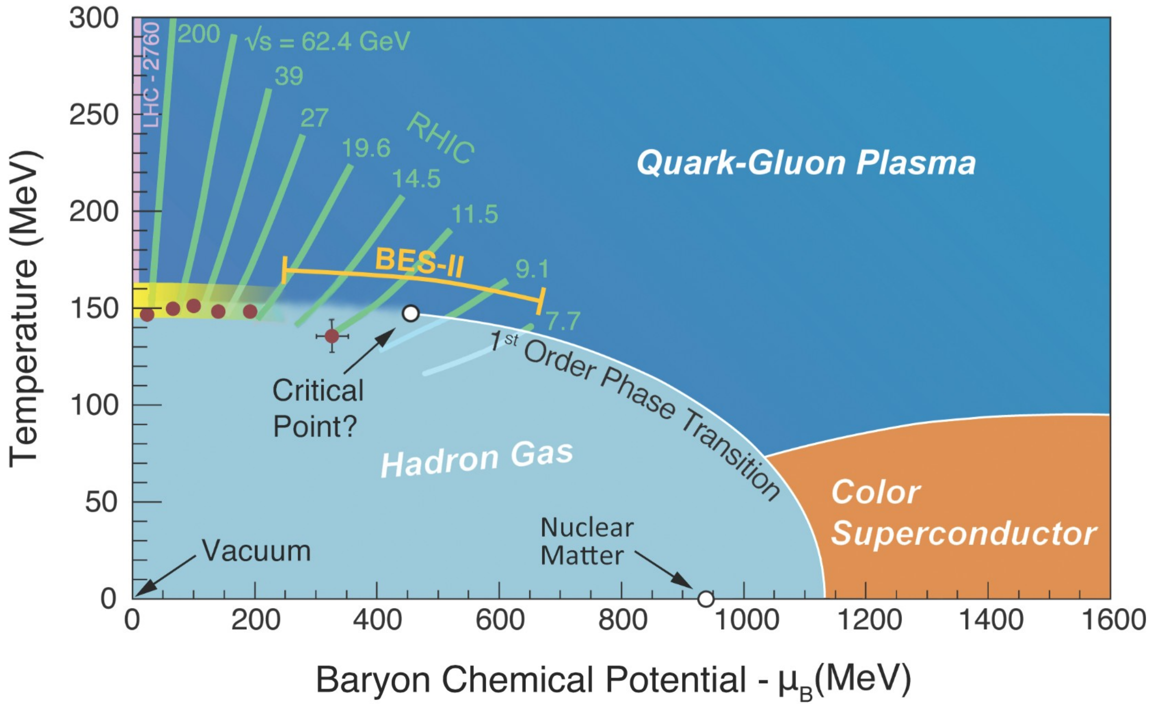


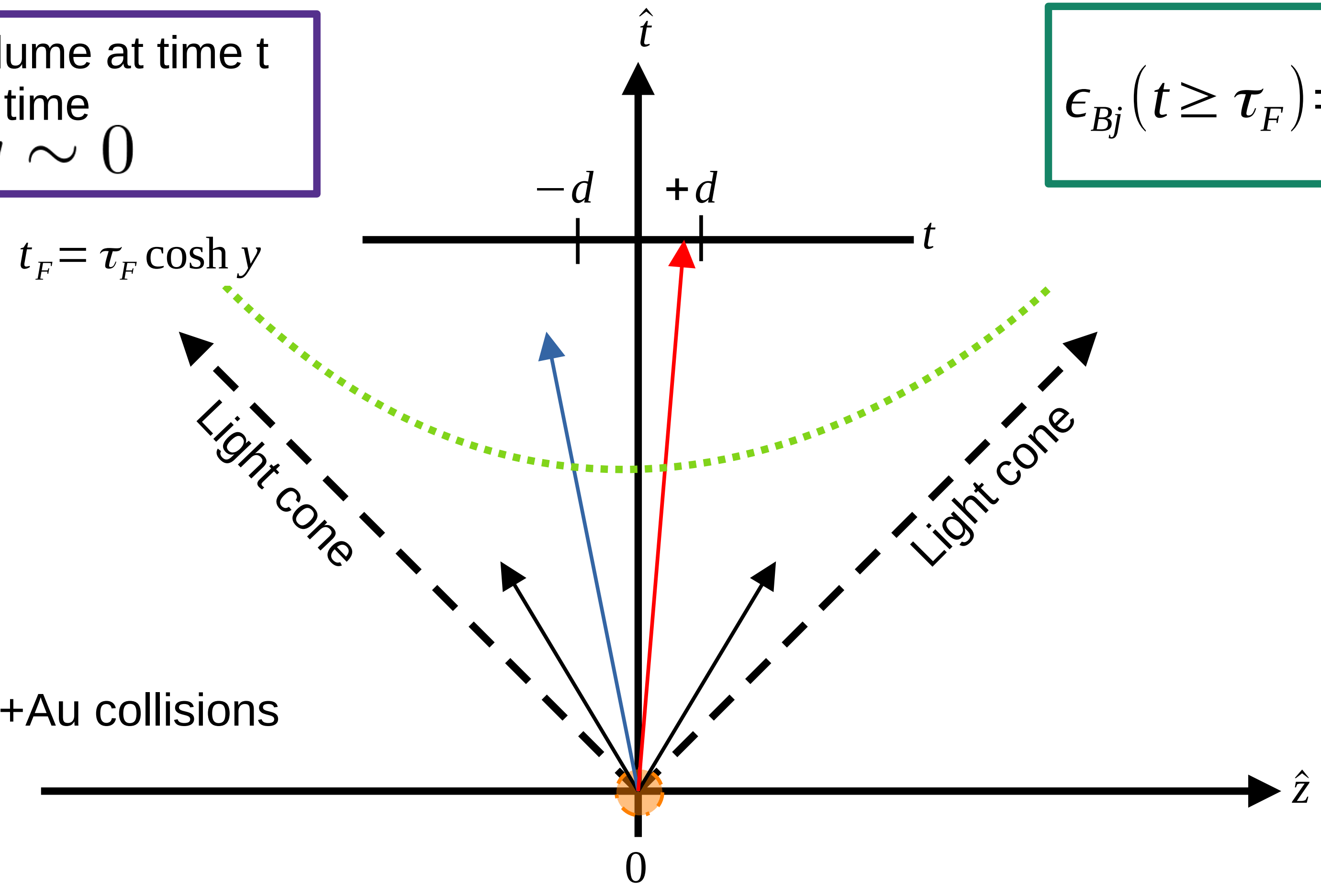
Density calculations – Bjorken ε(t) formula

- Narrow volume at time t
- Formation time
- Rapidity $y \sim 0$

$$\epsilon_{Bj}(t \geq \tau_F) = \frac{1}{A_T t} \frac{dE_T}{dy}\bigg|_{y=0}$$

$$A_T = \pi R_A^2$$

$$R_A \approx 1.12\, A^{1/3}$$



Central Au+Au collisions

J. D. Bjorken, Phys. Rev. D27, 140 (1983).

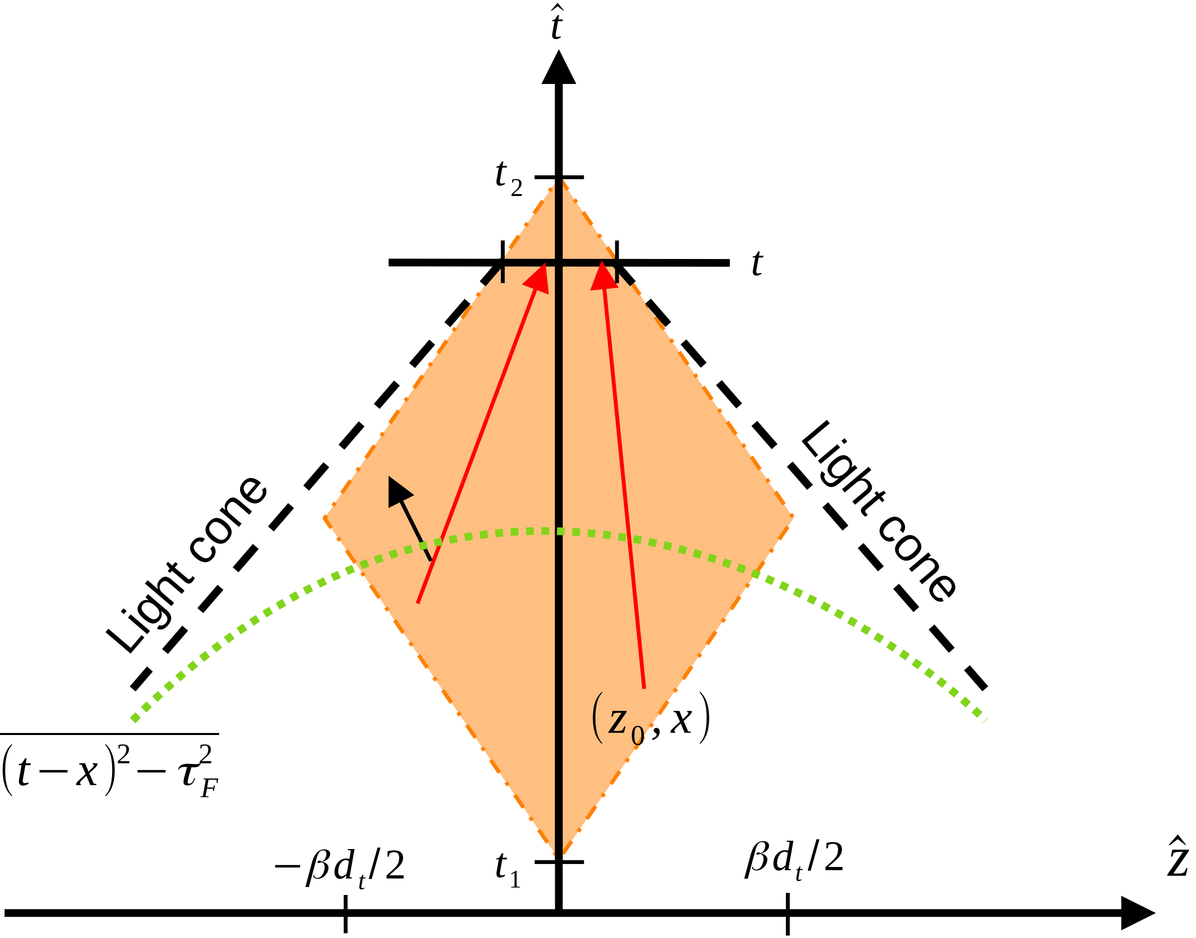
Density calculations – finite thickness

- Integrate transverse mass rapidity density over the production region at time t

$$\frac{d^3 m_T}{dy\, dz_0\, dx}$$

- Factorize the spatial and temporal dependence from the rapidity dependence

- Assume uniform distribution in (z_0, x)



$$z_F(x) = \sqrt{(t-x)^2 - \tau_F^2}$$

Z.-W. Lin, Phys. Rev. C103, 024907 (2018).
T. M. and Z.-W. Lin, Phys. Rev. C103, 024907 (2021).

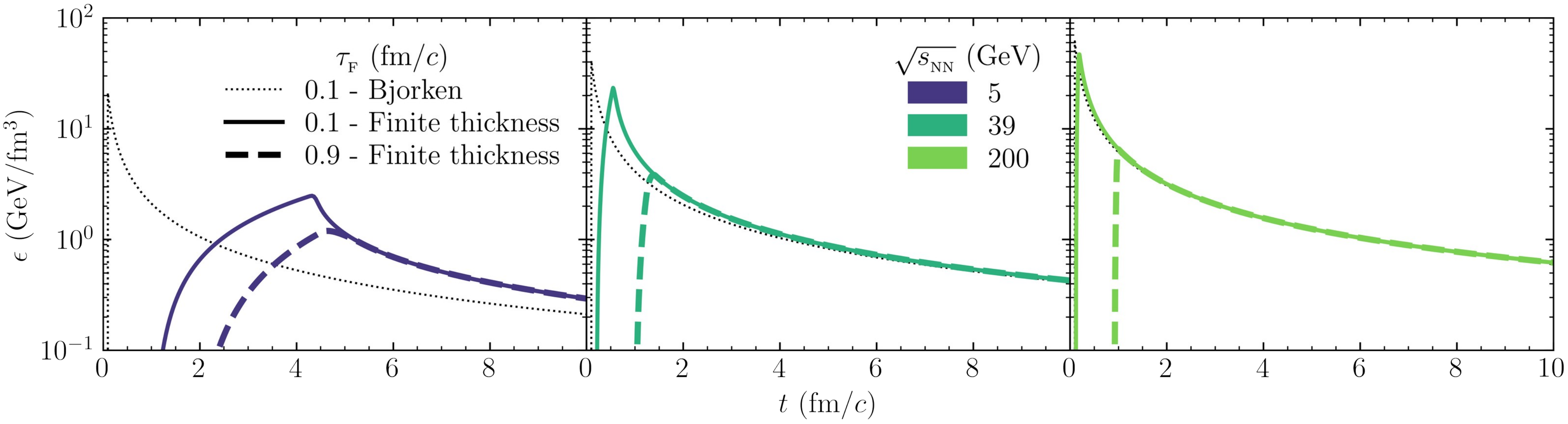
Energy density

- Double Gaussian, (y=0 from proton and net-proton dN/dy)
- Net-baryon number conservation

- Single Gaussian, (y=0 from PHENIX)
- Total energy conservation

$$\frac{dm_T}{dy}=\frac{dE_T}{dy}+m_N\frac{dN_{B-\bar{B}}}{dy}$$

$$\epsilon(t)=\frac{1}{A_T}\iint_{S(t)}\frac{dz_0\,dx}{t-x}\frac{d^3 m_T}{dy\,dz_0\,dx}\cosh^3 y$$



T. M. and Z.-W. Lin, Phys. Rev. C103, 024907 (2021).

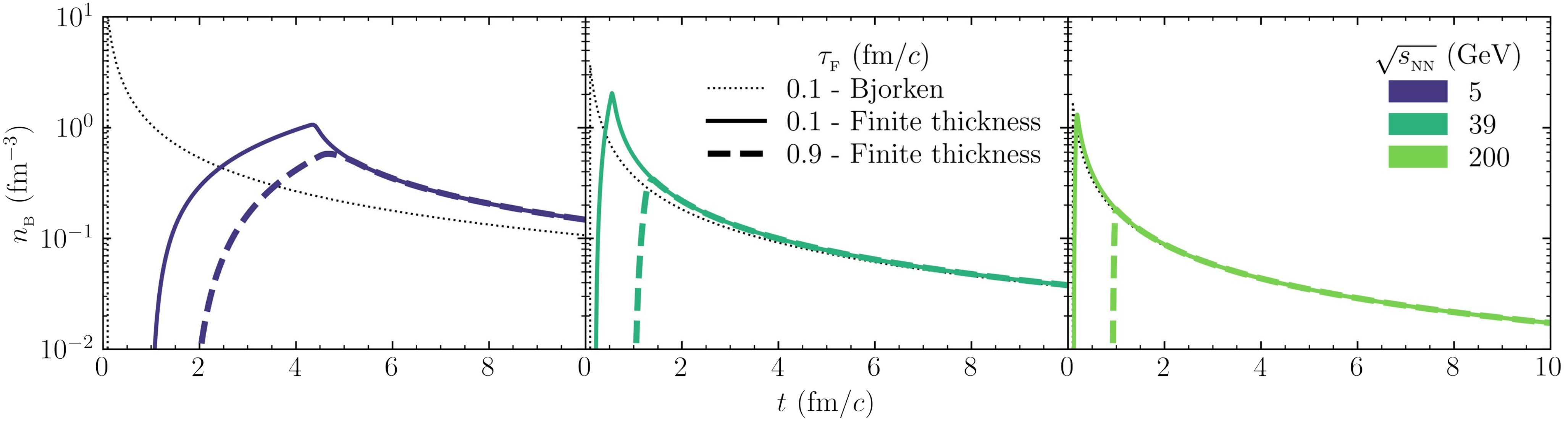
Net conserved-charge densities

- No net strangeness

$$n_B(t)=\frac{1}{A_T}\iint_{S(t)}\frac{dz_0\,dx}{t-x}\frac{d^3N_{B-\bar{B}}}{dy\,dz_0\,dx}\cosh^2 y$$

$$n_Q(t)=n_B(t)\frac{Z}{A}$$

$$n_S(t)=0$$



T. M. and Z.-W. Lin, arXiv:2111.13932 [nucl-th].

Extracting (*T*-μ) from (ε-*n*)

- First, using ideal gas EOS for massless QGP – (*u, d, s*)
  - Quantum & Boltzmann statistics
- Later using a lattice-based EOS

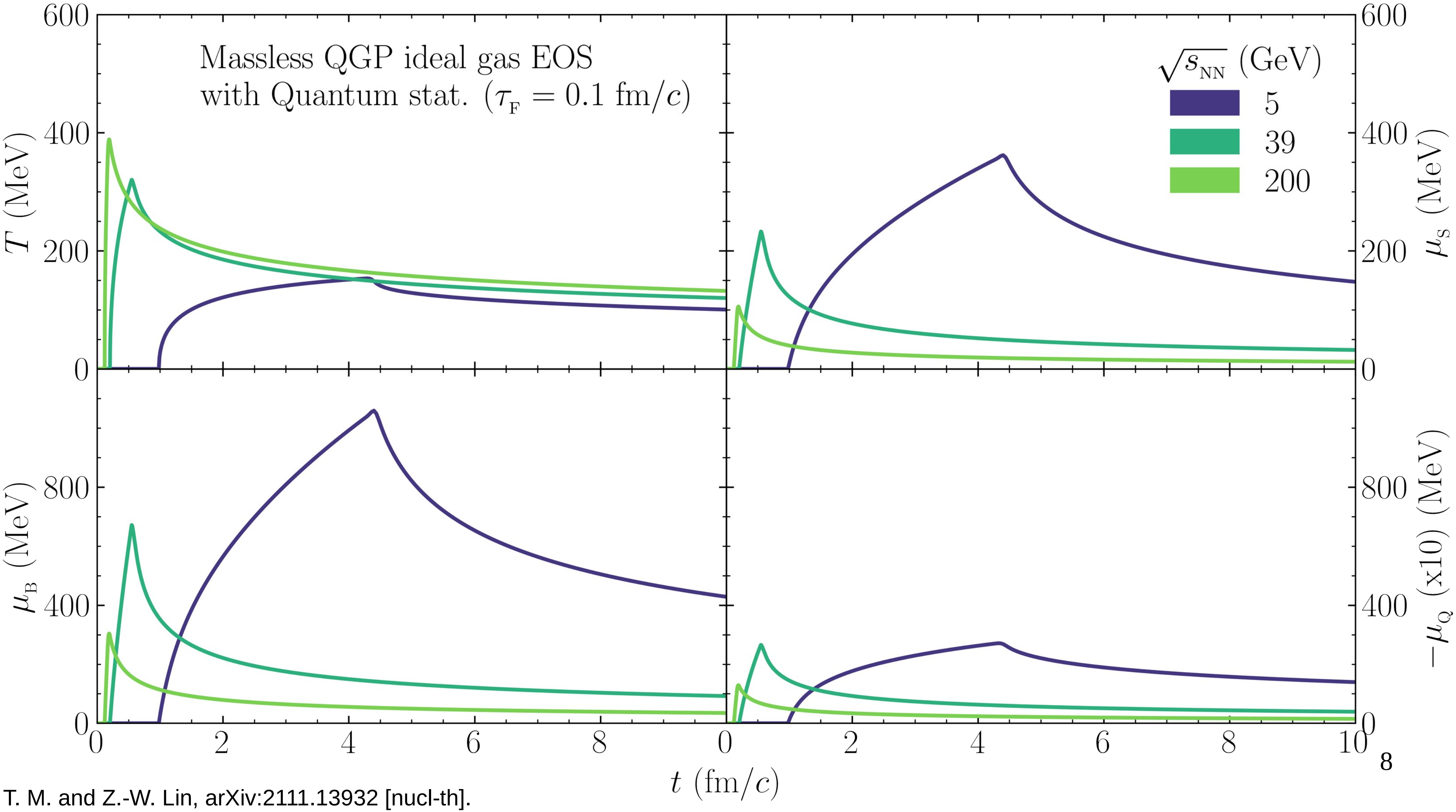
- Quark-antiquark pair production $\mu_q + \mu_{\bar{q}} = 0$

- Chemical potential of parton *i* → $\mu_i = B_i \mu_{\mathrm{B}} + Q_i \mu_{\mathrm{Q}} + S_i \mu_{\mathrm{S}}$

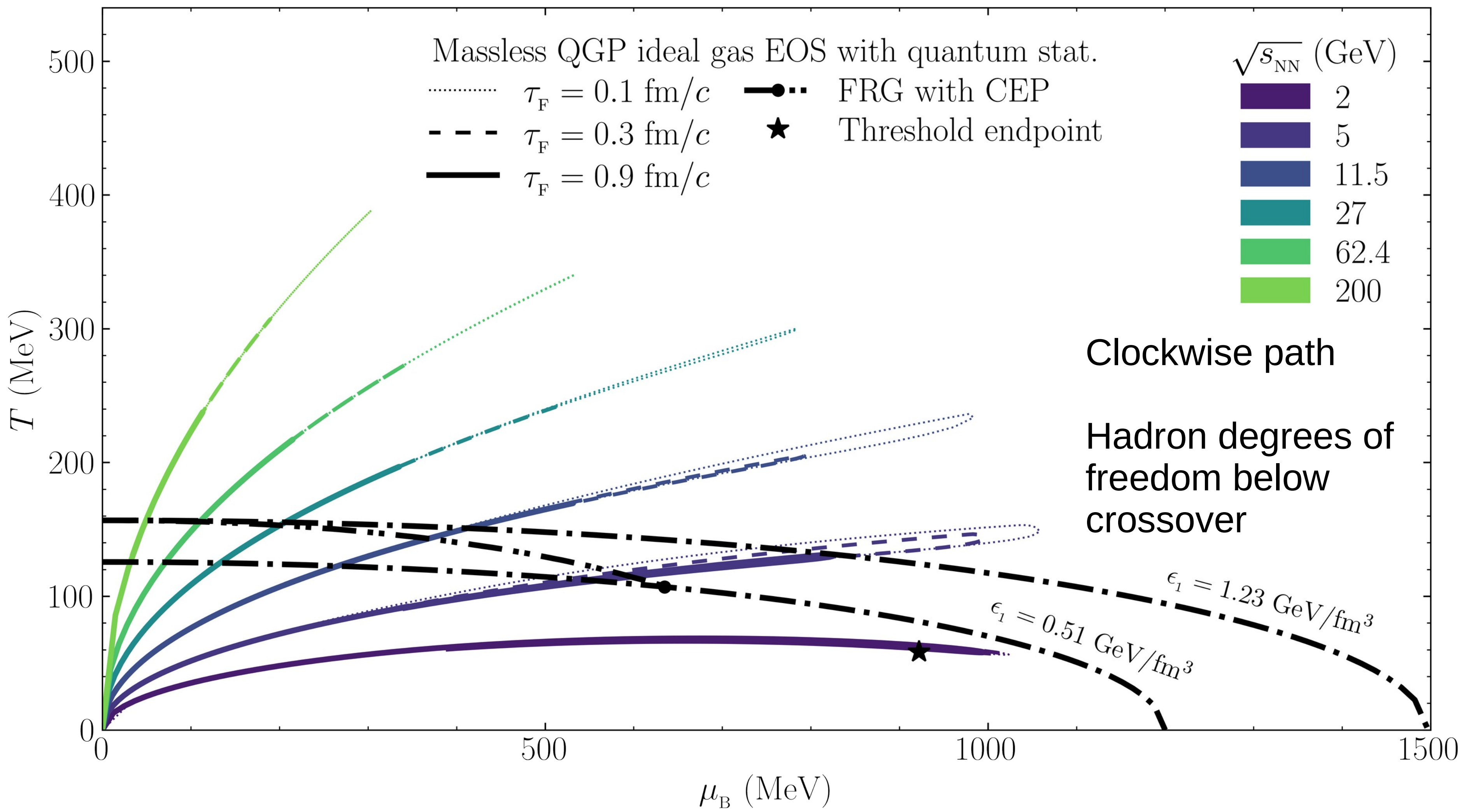
- Strangeness neutrality → $\mu_{\mathrm{B}} - \mu_{\mathrm{Q}} - 3\mu_{\mathrm{S}} = 0$

$$\mathrm{n}_{\mathrm{S}}^{quantum} = -\frac{\mu_{\mathrm{B}} - \mu_{\mathrm{Q}} - 3\mu_{\mathrm{S}}}{3} T^2 - \frac{(\mu_{\mathrm{B}} - \mu_{\mathrm{Q}} - 3\mu_{\mathrm{S}})^3}{27\pi^2}$$

$$\mathrm{n}_{\mathrm{S}}^{Boltzmann} = -\frac{12}{\pi^2} T^3 \sinh\left(\frac{\mu_{\mathrm{B}} - \mu_{\mathrm{Q}} - 3\mu_{\mathrm{S}}}{3T}\right)$$



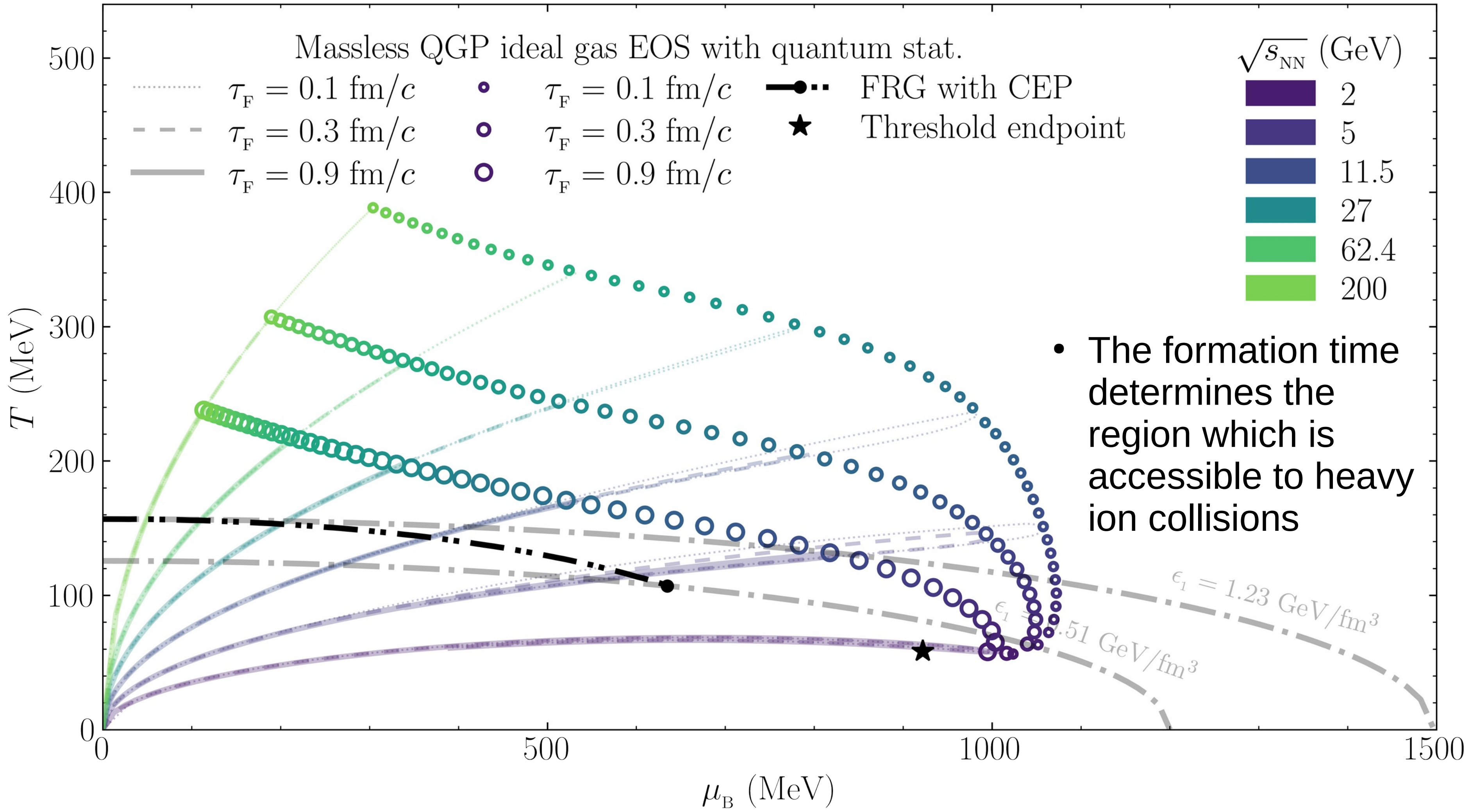
T. M. and Z.-W. Lin, arXiv:2111.13932 [nucl-th].



Clockwise path

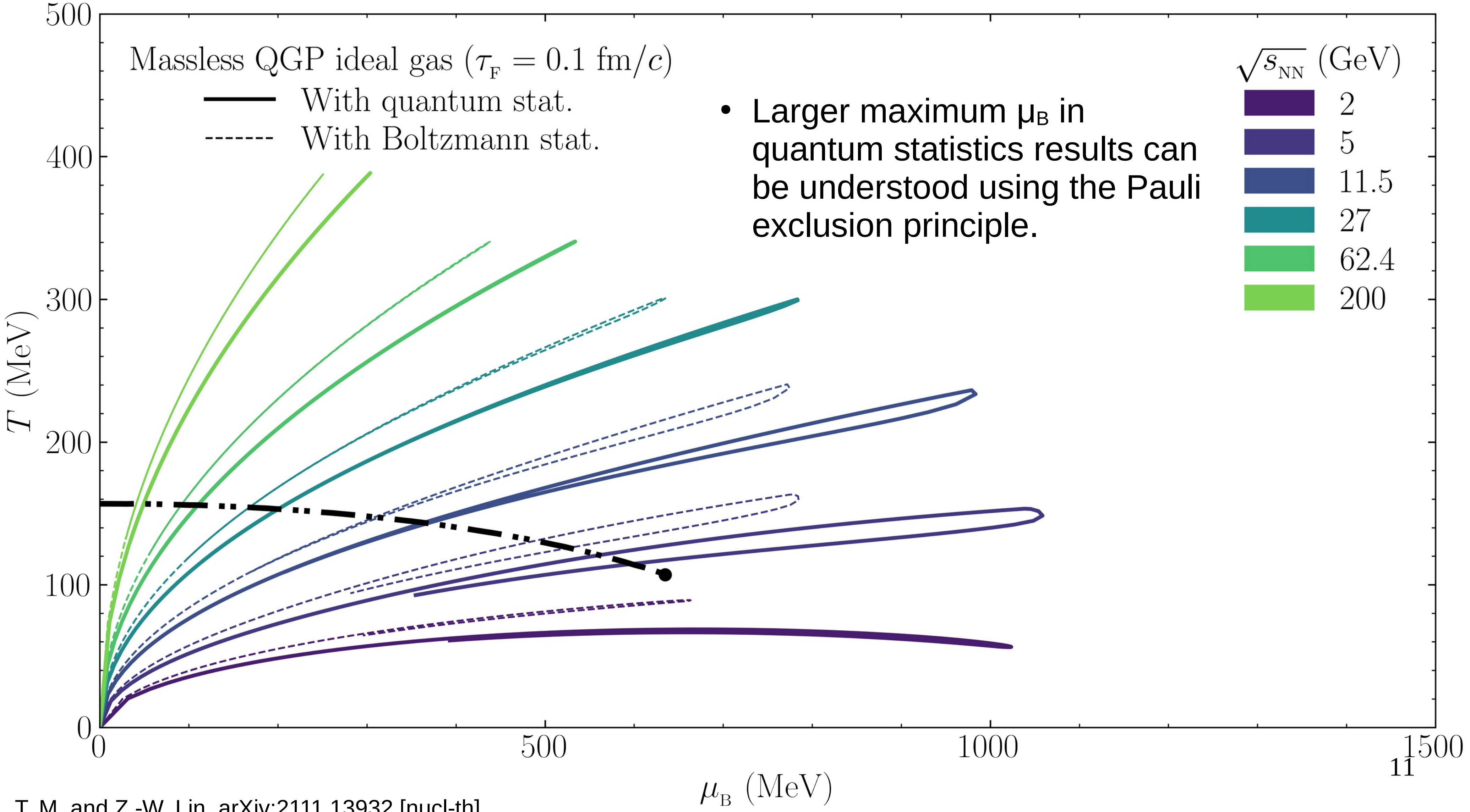
Hadron degrees of freedom below crossover

T. M. and Z.-W. Lin, arXiv:2111.13932 [nucl-th].



- The formation time determines the region which is accessible to heavy ion collisions

T. M. and Z.-W. Lin, arXiv:2111.13932 [nucl-th].



- Larger maximum μ_B in quantum statistics results can be understood using the Pauli exclusion principle.

T. M. and Z.-W. Lin, arXiv:2111.13932 [nucl-th].

Lattice EOS

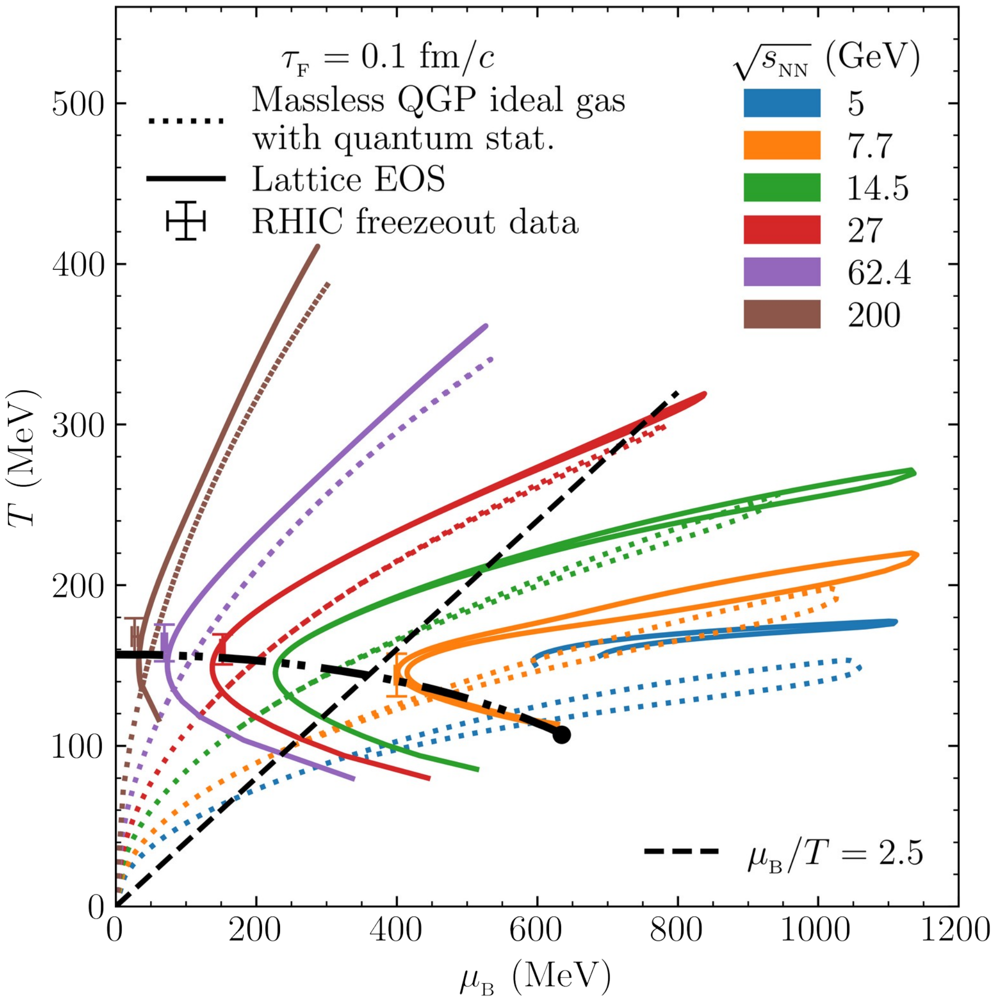
J. Noronha-Hostler, P. Parotto, C. Ratti, and J. M. Stafford
Phys. Rev. C100, 064910 (2019).

$$\frac{p(T,\mu_B,\mu_Q,\mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!\,j!\,k!}\chi_{ijk}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

$$\chi_{ijk}^{BQS} = \frac{\partial^{i+j+k}(p/T^4)}{\partial(\mu_B/T)^i\partial(\mu_Q/T)^j\partial(\mu_S/T)^k}\bigg|_{\mu_B,\mu_Q,\mu_S=0}$$

$$\frac{\epsilon}{T^4} = \frac{s}{T^3} - \frac{p}{T^4} + \frac{\mu_B}{T}\frac{n_B}{T^3} + \frac{\mu_Q}{T}\frac{n_Q}{T^3} + \frac{\mu_S}{T}\frac{n_S}{T^3}$$

$$\frac{n_B}{T^3} = \frac{1}{T^3}\frac{\partial p}{\partial \mu_B}\bigg|_{T,\mu_Q,\mu_S} \qquad n_Q = n_B Z/A,\; n_S = 0$$



Lattice EOS with densities from our semi analytical model agree with RHIC chemical freezeout data

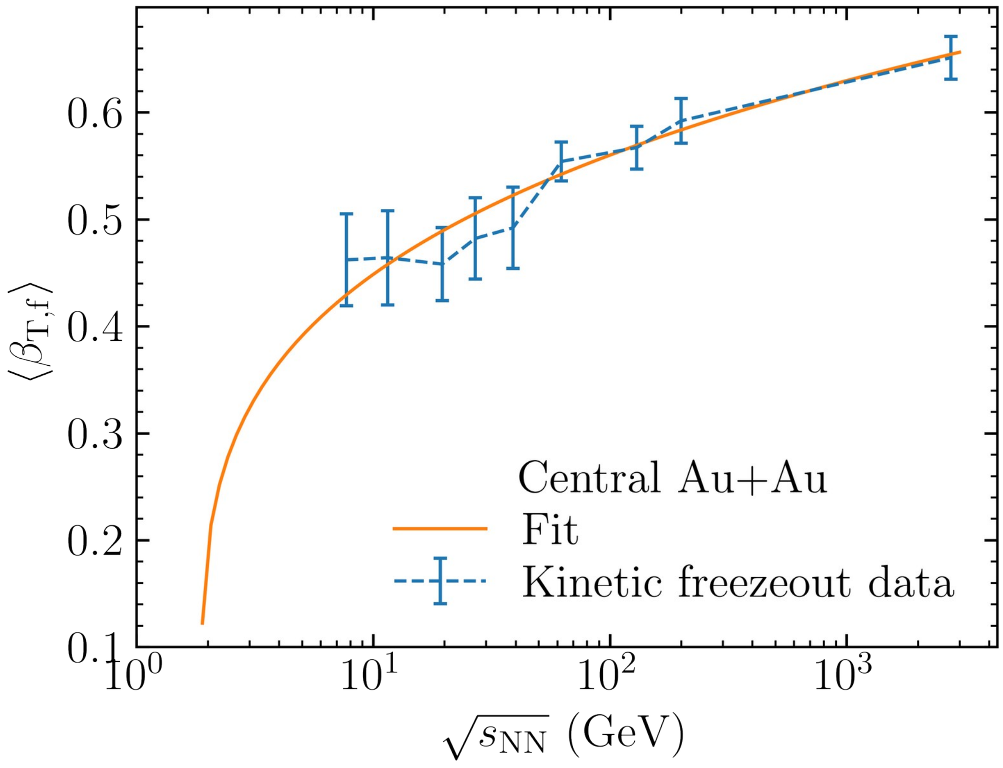
T. M. and Z.-W. Lin, arXiv:2111.13932 [nucl-th]. (to be updated)

Transverse Expansion

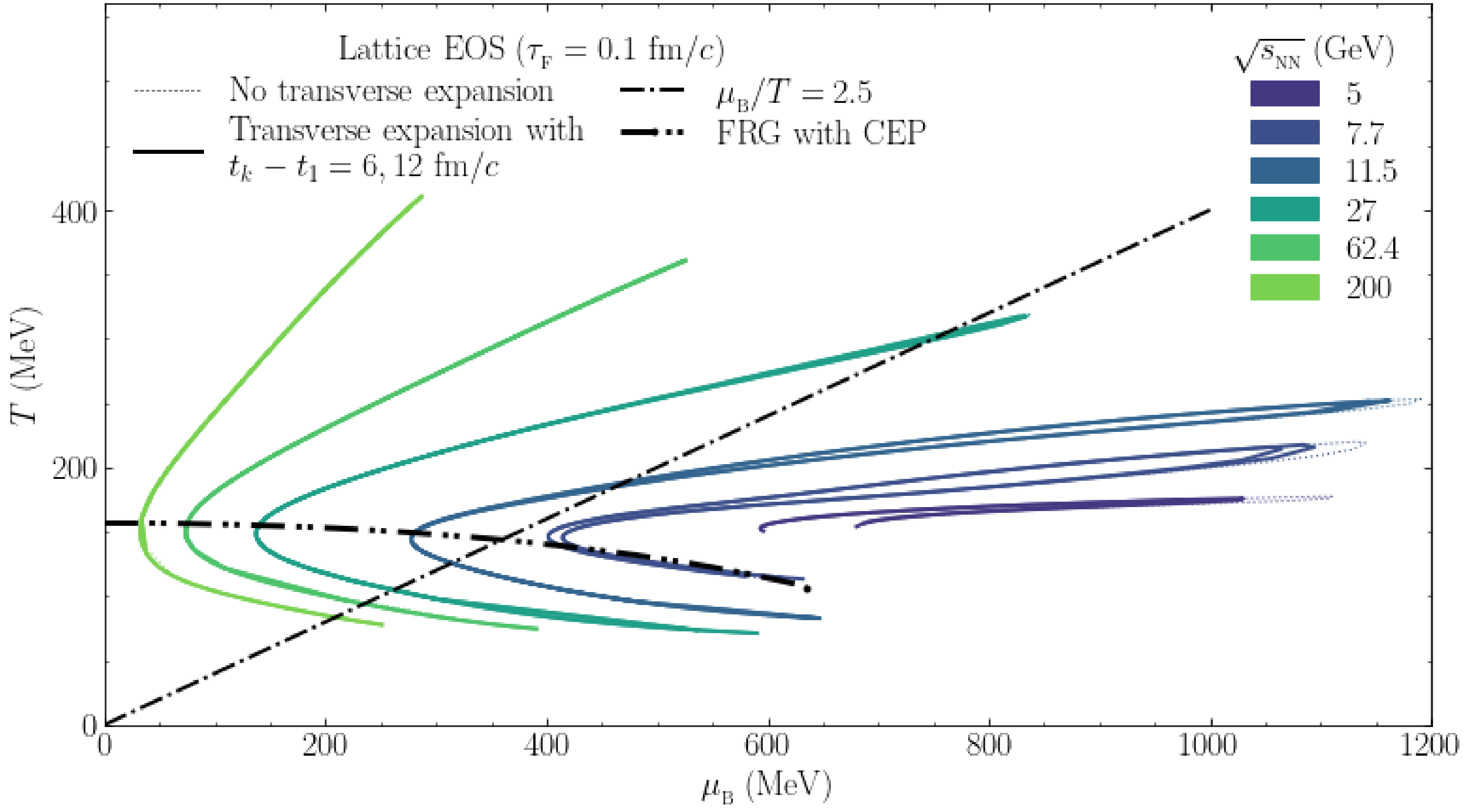
$$\mathrm{R_T}(t) = R_A + \beta_{\mathrm{T}}(t)\,(t - t_1)$$

$$\beta_{\mathrm{T}}(t) = \begin{cases} 0, & \text{for } 0 \le t < t_1 \\ \frac{1-e^{-(t-t_1)/a}}{1-e^{-(t_k-t_1)/a}}\beta_{\mathrm{T,f}}, & \text{for } t_1 \le t < t_k \\ \beta_{\mathrm{T,f}}, & \text{for } t_k \le t \end{cases}$$

- *a*: model-based, collision energy dependent timescale for development of average freezeout velocity
- t_k: time to reach the freezeout velocity $\mathrm{t}_k = t_1 + t_{\mathrm{QGP}} + t_{\mathrm{Had}}$



T. M. and Z.-W. Lin, arXiv:2111.13932 [nucl-th]. (to be updated)



Transverse expansion has small effect on the endpoint of trajectories

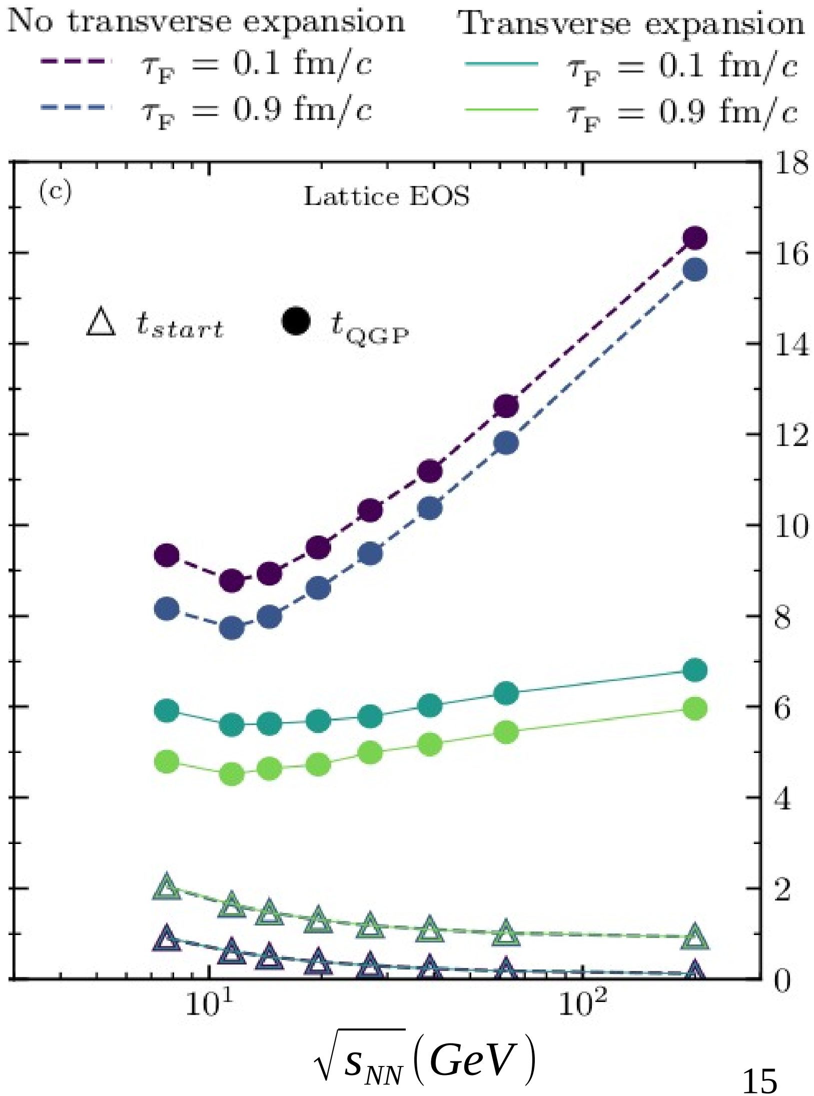
There is a weak dependence on t_k-t_1

T. M. and Z.-W. Lin, arXiv:2111.13932 [nucl-th]. (to be updated)

QGP Lifetime

Use T(t) and μ_B(t) to find the times when the trajectory intersects the FRG crossover curve.

Transverse expansion has an effect of almost a factor ~2 over all energies.



T. M. and Z.-W. Lin, arXiv:2111.13932 [nucl-th]. (to be updated)

Summary

- We have developed a **semi-analytical** model to calculate the trajectories of Au+Au collisions in the QCD phase diagram.
  - We have made a web application to calculate trajectories using quantum or Boltzmann ideal gas equations of state: http://myweb.ecu.edu/linz/densities/
- The finite thickness and EOS have **large** effects on the calculated trajectories.
- Transverse expansion has a **small** effect on the trajectory path, but a **large** effect on the QGP lifetime.
- The lattice EOS with our semi-analytical densities agrees with the RHIC chemical freezeout data.