Dynamics of the QCD matter in heavy ion collisions and binary neutron star mergers

Strangeness in Quark Matter, Busan, Republic of Korea, 15 June 2022

Anton Motornenko
Thanks to:
M. Bleicher, V. Dexheimer, M. Hanauske, A. Heger, P. Jakobus, E. Most, B. Müller, Y. Nara, M. Omana Kuttan, L. Rezzolla, J. Steinheimer, and H. Stöcker

2201.01622 [nucl-th]
2201.13150 [nucl-th]
2204.10397 [astro-ph.HE]
QCD phase diagram

\[ \mathcal{L}_{QCD} = \sum_{i,j} \bar{\psi}_i \left( i \gamma^\mu \left( \partial_\mu \delta_{ij} - \frac{i}{s} g A^a_\mu \lambda_{a,ij} \right) - m_i \delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} \]

\[ G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \]

How to map the well established QCD theory to its phase diagram?
Beyond the lattice data

First-principle lattice QCD data can be used at \( \mu_B > 0 \) by Taylor expansion:

\[
P = P_0 + T^4 \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{i,j,k}^{B,Q,S} \left( \frac{\mu_B}{T} \right)^i \left( \frac{\mu_Q}{T} \right)^j \left( \frac{\mu_S}{T} \right)^k
\]

With limited radius of convergence (see recent progress at P. Parotto talk on Wednesday).

At finite densities the transition to fermion-dominated matter changes isobaric lines.

There is need for phenomenological models!

Lines of CMF constant pressure. The grey shaded regions: mixed phases by nuclear liquid-vapor and chiral phase transitions. Black dots — critical endpoints.
Bulletpoints from theory overview:

(see Joseph Kapusta talk on Monday)

- To describe the matter created at RHIC beam energies and below, knowledge is required of the equation of state as a function of $T$, $\mu_B$, $\mu_Q$, and $\mu_S$ to conserve energy, baryon number, electric charge, and strangeness. This is nontrivial when there is critical behavior in the phase diagram.

- Such an equation of state is also needed for modeling neutron star mergers and closely related to the cold dense matter comprising neutron stars.

There is need for phenomenological models!
An approach for QCD EOS: CMF model

Chiral Mean Field model is a single framework for QCD thermodynamics, can be used for:

- analysis of lattice QCD data
- description of nuclear matter
- modeling of heavy ion collisions
- as well as neutron star description

Papazoglou, Schramm, Schaffner-Bielich, Stoecker, Greiner, nucl-th/9706024
Papazoglou, Zschiesche, Schramm, Schaffner-Bielich, Stoecker, Greiner, nucl-th/9806087
Dexheimer, Schramm, 0901.1748
Steinheimer, Schramm, Stoecker 1009.5239
AM, Steinheimer, Vovchenko, Schramm, Stoecker, 1905.00866

The online service CompOSE provides data tables for different state of the art equations of state (EoS) ready for further usage in astrophysical applications, nuclear physics and beyond.

The cold neutron star EoS tables can be used directly within LOBNE to obtain models of (rotating/magnetised) neutron stars, see the eos_compose class.

If you make use of the tables provided in CompOSE, please cite the publications describing the respective EoS models (available on the CompOSE web pages for each the model) together with a reference to the CompOSE website (https://compose.obspm.fr) and/or the original CompOSE publications:


Data tables, associated software and the manual can be freely downloaded. Log in is required if you wish to use further utilities, such as graphics and online computations. Please contact "develop.compose(at)obspm.fr" if you wish to have an account.
Separate Universal EOS for astrophysics and HIC

CMF $n_B=0$

CMF $T=0$
Separate Universal EOS for astrophysics and HIC

CMF — single equation of state for full hydrodynamic modeling of both heavy ion collisions and neutron star mergers!

Anton Motornenko SQM 2022
HIC vs BNSM: comparison

In the following we use ideal relativistic hydrodynamics, no viscosity and dissipations

**Binary Neutron Star Mergers**

- General-relativistic magneto- hydrodynamics
  - Frankfurt/IllinoisGRMHD (FIL) code
  - with Einstein Toolkit

  *Etienne, Paschalidis, Haas, Mosta, and Shapiro, 1501.07276 [astro-ph.HE]*
  *Most, Papanfort, and Rezzolla, 1907.10328 [astro-ph.HE]*
  *F. Löffler et al., 1111.3344 [gr-qc].*

**Heavy Ion Collisions**

- Relativistic flux-corrected SHASTA code

  *Rischke, Bernard, and Maruhn, arXiv:nucl-th/9504018*

Both codes require equation of state as an input
Heavy ion collision evolution

Au+Au collision at $E_{\text{lab}}=450$ MeV
Nuclei are initialised with Woods-Saxon distributions:

$$n_{WS} = \gamma_{CM} \frac{n_0}{1 + \exp \left( \frac{\Delta r - R}{a} \right)}$$
HIC vs BNSM: comparison

Geometry and scales are drastically different. Thermodynamic conditions are similar!

Entropy per baryon S/A (top colormaps) and temperature T (bottom colormaps) for a BNS merger of mass $M_{\text{tot}} = 2.8 \, M_{\odot}$, and Au + Au HIC at $E_{\text{lab}} = 450$ MeV.

Colored lines mark density contours in units of $n_{\text{sat}}$.

The snapshots refer to $t = -2, 3$ ms before and after merger for the BNS, respectively, and to $t = \pm 5$ fm/c before and after the full overlap for the HIC.

Most, AM, Steinheimer, Dexheimer, Hanauske, Rezzolla, Stoecker
e-Print: [2201.13150](https://arxiv.org/abs/2201.13150) [nucl-th]
HIC vs BNSM: comparison

Spacetime diagrams for the evolution of the temperature and entropy. The green contours correspond to lines of constant entropy per baryon S/A=1.8, 2.2.

HIC:
The whole system expands with approx. constant entropy.

BNSM:
Entropy is localized in a “ring” structure.

Most, AM, Steinheimer, Dexheimer, Hanauske, Rezzolla, Stoecker

e-Print: [2201.13150](https://arxiv.org/abs/2201.13150) [nucl-th]
**HIC:**
The whole system expands with approx. constant entropy

\[ t = 0.16 \text{ fm/c} \]

**BNSM:**
Entropy is localized in a “ring” structure

\[ \sqrt{s_{NN}} = 2.15 \text{ A GeV} \]

**CMF**

**BNSM movie by Elias Most**
Regions of the QCD phase-diagram probed by BNS mergers and by HICs. The colorcode reports the number of cells $N$ in the various spacetimes having a given value of temperature and density. The green lines show contours of constant entropy per baryon. Only cells with density above freeze-out, $n > 1/2 n_{sat}$, are shown for the HIC.
HIC vs BNSM: comparison

Physical conditions of BNSM can be studied in present and future HIC experiments, bridging 18 orders of magnitude in length scale, from microscopic ion collisions to macroscopic astrophysical compact objects.

The colorcode reports the number of cells N in the various spacetimes as a function of temperature and density. The green lines show contours of constant entropy per baryon. Only cells with density above freeze-out, \( n > \frac{1}{2} n_{\text{sat}} \), are shown for the HIC.
Predictions for experiments

How to relate Bulk matter evolution to microscopic picture? So a comparison with experiment can be done...

Cooper-Frye at these low energies is problematic!
Predictions for experiments

How to relate Bulk matter evolution to the microscopic picture? So a comparison with experiment can be done...

Bulletpoints from theory overview:
(see Joseph Kapusta talk on Monday)

- Is the standard model of viscous hydrodynamics coupled with an hadronic afterburner the correct one for lower beam energies?
- Transport theory with hadrons and mean fields is undoubtedly a better description at lower beam energies. Better able to handle longer nuclear transit times.

Cooper-Frye at these low energies is problematic!
Hamilton's equations of motion in a transport model (UrQMD) can be calculated as:

\[
\dot{p}_i = -\frac{\partial H}{\partial r_i} = -\frac{\partial V(n_B)}{\partial n_B} \cdot \frac{\partial n_B(r_i)}{\partial r_i}
\]

Density \( n_B \) is calculated by assuming that each particle can be treated as a Gaussian wave packet:

\[
n_B(r_i) = \left(\frac{\alpha}{\pi}\right)^{3/2} \sum_{j, j \neq i} B_j \exp\left(-\alpha(r_i - r_j)^2\right)
\]

The mean field potential energy per baryon can be related to a density dependent single particle energy:

\[
U_i(n_B) = \frac{\partial(n_B \cdot V_i(n_B))}{\partial n_B}
\]

In the CMF model (or any other EOS) the single nucleon potential at \( T = 0 \), can be calculated from the self energy of the nucleons:

\[
V_{CMF} = \frac{E_{field}}{A} = \frac{E_{CMF}}{A} - \frac{E_{FFG}}{A}
\]
Predictions for experiments: QMD approach

CMF features:
1. A nuclear incompressibility compatible with experimental observations.

2. Stiff at super-saturation densities: explains astrophysical observations.

3. “Softens” at even higher densities due to the slow approach to the high density limit of a free gas quarks.
Comparing QMD approach and hydro: central cell

The bulk evolution of the density in this new description agrees well with a relativistic 1-fluid simulation with the CMF equation of state.

The initial compression depends dominantly on the underlying EOS and only marginally on the model used for the dynamical description.

The compression is reproduced and thus predictions for experiment can be done!
Supernova explosions: another application of CMF

Figure 8. Phase diagram with trajectories of the central density and temperature (thick solid lines) of selected CMF models. The background displays the colour-coded adiabatic index at fixed electron fraction $Y_e = 0.25$ for the CMF EoS. The black dashed lines are isentropes for different entropy values (indicated as contour labels).
Supernova explosions: another application of CMF

-85 MeV

Bulletpoints from theory overview:
(see Joseph Kapusta talk on Monday)

- To describe the matter created at RHIC beam energies and below, knowledge is required of the equation of state as a function of $T$, $\mu_B$, $\mu_Q$, and $\mu_S$ to conserve energy, baryon number, electric charge, and strangeness. This is nontrivial when there is critical behavior in the phase diagram.

- Such an equation of state is also needed for modeling neutron star mergers and closely related to the cold dense matter comprising neutron stars.

... and supernova!

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Summary

- CMF + relativistic hydrodynamics = simulations of
- Astrophysical conditions can be probed by low energy nuclear collisions
- Predictions for laboratory experiments can be done by UrQMD with density dependent equation of state

Thank you for the attention!
Backup
Figure 4. Qualitative sketch of a possible QCD phase diagram with a band of approximate chiral spin symmetry terminating at the critical end point of a non-analytic chiral phase transition.

Glozman, Philipsen, Pisarski, e-Print: 2204.05083 [hep-ph]
The CMF phase diagram

Three transitions:
- hadron gas ➜ hadronic liquid ➜ chiral symmetry restoration ➜ quark matter

Two critical points:
- nuclear CP: $T_{CP} \approx 17$ MeV
- chiral CP: $T_{CP} \approx 17$ MeV

Chiral condensate

Quark fraction

The CMF model

\[ \Omega = \Omega_q + \Omega_{\bar{q}} + \Omega_h + \Omega_{\bar{h}} - (U_{sc} + U_{vec} + U_{pol}) \]

**Baryon octet:**

\[ \mathcal{L}_B = \sum_b (\bar{B}_b i \gamma^5 B_b) + \sum_b (\bar{B}_b m_b^* B_b) \]
\[ + \sum_b [\bar{B}_b \gamma_\mu (g_{\omega b} \omega^\mu + g_{\rho b} \rho^\mu + g_{\phi b} \phi^\mu) B_b] \]
\[ \mu_b^* = \mu_b - g_{\omega b} \omega - g_{\phi b} \phi - g_{\rho b} \rho \]
\[ m_{b^\pm} = \sqrt{\left[ (g_{\omega b}^2 (\sigma + g_{\omega b} \xi))^2 + (m_0 + n_s \xi)^2 \right]} \pm g_{\omega b}^2 (\sigma + g_{\omega b} \xi) \]

**Repulsive vector mean fields:**

\[ U_{vec} = -\frac{1}{2} \left( m_{\omega}^2 \omega^2 + m_{\rho}^2 \rho^2 + m_{\phi}^2 \phi^2 \right) \]
\[ - g_4 \left[ \omega^4 + 6 \beta_2 \omega^2 \rho^2 + \rho^4 + \frac{1}{2} \phi^4 \left( \frac{Z_{\phi}}{Z_{\omega}} \right)^2 \right] \]
\[ + 3 (\rho^2 + \omega^2) \left( \frac{Z_{\phi}}{Z_{\omega}} \right) \phi^2 \].

**Attractive scalar fields:**

\[ U_{sc} = V_0 - \frac{1}{2} k_0 I_2 + k_1 I_2^2 - k_2 I_4 + k_6 I_6 + k_4 \ln \frac{\sigma^2 \xi}{\sigma_0^2 \xi_0} - U_{sb} \]
\[ I_2 = (\sigma^2 + \xi^2), \quad I_4 = -(\sigma^4/2 + \xi^4), \quad I_6 = (\sigma^6 + 4 \xi^6) \]
\[ U_{sb} = m_f^2 f_{\pi} \sigma + \left( \sqrt{2} m_K^2 f_K - \frac{1}{\sqrt{2}} m_f^2 f_{\pi} \right) \xi \]

**Hadrons:**

\[ \rho_i = \frac{\rho_i^{id}(T, \mu_i^* - v_i P)}{1 + \sum_{j \in \text{HRG}} \mathcal{V}_j \rho_j^{id}(T, \mu_j^* - v_j P)} \]
\[ \mu_{eff} = \mu_j^* - v_j P, \]
\[ \text{Polyakov loop:} \]

\[ U_{pol}(\Phi, \bar{\Phi}, T) = -\frac{1}{2} a(T) \Phi \bar{\Phi} + b(T) \ln [1 - 6 \Phi \bar{\Phi} + 4 (\Phi^3 + \bar{\Phi}^3) - 3 (\Phi \bar{\Phi})^2], \]
\[ a(T) = a_0 T^4 + a_1 T_0 T^3 + a_2 T_0^2 T^2, \]
\[ b(T) = b_3 T_0^4. \]
Entropy production: hydro vs QMD

![Graph showing entropy production vs beam energy]
Hadronic part: QCD matter at low densities

\[ \rho_i = \frac{\rho_{i}^{id}(T, \mu_i^* - v_i p)}{1 + \sum_j v_j \rho_{j}^{id}(T, \mu_j^* - v_j p)} \]

\[ \varepsilon_i = \frac{\varepsilon_{i}^{id}(T, \mu_i^* - v_i p)}{1 + \sum_j v_j \rho_{j}^{id}(T, \mu_j^* - v_j p)} \]

PDG list of known hadrons is included with Excluded Volume interactions. **EV suppress hadrons** at high energy densities.

EV of baryons: 1 fm³
EV of mesons: 1/8 fm³

EV triggers the switch between hadron and quark degrees of freedom: hadron pressure is **suppressed** as function of T and \( \mu_B \) — quarks are dominant at high densities.

V. Vovchenko, D. Anchishkin, M. Gorenstein, 1412.5478
SU(3)$_f$ octet and parity doubling

We include all states of the SU(3)$_f$ baryon octet:

\[
\begin{pmatrix}
    \Sigma^0 + \frac{\Lambda}{\sqrt{6}} \\
-\Sigma^- \\
-\Xi^{-}
\end{pmatrix} + \begin{pmatrix}
    \Sigma^+ \\
    \Sigma^0 + \frac{\Lambda}{\sqrt{6}} \\
-2\frac{\Lambda}{\sqrt{6}}
\end{pmatrix}
\]

and their parity partners (G. Aarts et al., 1710.08294), i.e. states with the same quantum numbers but opposite parity. Those interact within SU(3)$_f$ $\sigma$ model:

\[\mathcal{L}_B = \sum_i (\bar{B}_i i\not{D} B_i) + \sum_i (\bar{B}_i m^*_i B_i)
+ \sum_i (\bar{B}_i \gamma_\mu (g_{\omega\mu} + g_{\rho\mu} + g_{\phi\mu}) B_i)\]

with effective masses generated by chiral fields $\sigma$ and $\zeta$:

\[m^*_i = \sqrt{\left[ (g^{(1)}_{\sigma i} + g^{(1)}_{\zeta i})^2 + (m_0 + n_s m_s)^2 \right] + g^{(2)}_{\sigma i} \sigma \pm g^{(2)}_{\zeta i} \zeta} \]

The ‘+’ stands for positive and ‘−’ for negative parity states.
Including Quarks: PNJL-like approach

Quarks are included within PNJL inspired approach:

\[ \Omega_q = -VT \sum_{i \in Q} \frac{d_i}{(2\pi)^3} \int d^3k \frac{1}{N_c} \left[ \ln \left( 1 + 3\Phi e^{-(E_i^* - \mu_i^*)/T} + 3\bar{\Phi} e^{-2(E_i^* - \mu_i^*)/T} + e^{-3(E_i^* - \mu_i^*)/T} \right) 
+ \ln \left( 1 + 3\bar{\Phi} e^{-(E_i^* + \mu_i^*)/T} + 3\Phi e^{-2(E_i^* + \mu_i^*)/T} + e^{-3(E_i^* + \mu_i^*)/T} \right) \right] \]

Polyakov loop \( \Phi \) — is deconfinement order parameter:

\( \Phi=0 \) — no quarks, \( \Phi=1 \) — free quarks.

\( \Phi \) is controlled by the potential \( U(\Phi) \):

\[
U = -\frac{1}{2} \left( a_0 T^4 + a_1 T_0 T^3 + a_2 T_0^2 T^2 \right) \Phi \Phi^* \\
+ b_3 T_0^4 \log \left[ 1 - 6\Phi \Phi^* + 4(\Phi^3 + \Phi^{*3}) - 3(\Phi \Phi^*)^2 \right]
\]

Fukushima, hep-ph/0310121
Roessner, Ratti, Weise, hep-ph/0609281
The phase transitions are only driven by hadrons. Deconfinement is always smooth.
Importance of HRG list

CMF Isentropic trajectories, lines of constant entropy per baryon S/A with:
- the full particle list (solid);
- only the stable baryons+quarks (dashed), mesons and resonances are neglected.

Note the increase of temperature for the isentropes where the mesons and resonances are neglected.