(3+1)-D viscous hydrodynamics CLVisc at finite net baryon density across BES energies

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Outline

- Motivation
- CLVisc hydrodynamics framework
- Numerical results
  - Mean transverse momenta
  - Anisotropic flow
  - Flow fluctuations
- Summary and outlook
**Motivation**

LHC & top RHIC collision energy, **Crossover**

Beam energy scan region, 1st order phase transition or **Critical Point**

The collective flow of the QGP fireball converts initial state geometric anisotropy to final state momentum anisotropy.

\[
\frac{dN}{p_T \, dp_T \, dy \, d\phi} = \frac{dN}{2\pi p_T \, dp_T \, dy} \left[ 1 + 2 \sum_n v_n(p_T, y) \cos(n(\phi - \Psi_n(p_T, y))) \right]
\]
Hydrodynamics at high collision energy

Hydrodynamics simulation is successful in describing the collective behavior of QGP fireball at zero chemical potential both in transverse plane and longitudinal direction.

Hydrodynamics at BES energies

- Constrain the longitudinal dynamics


[Chun Shen’s talk@June 14, 10:50 AM]
[Yuuka’s talk@June 15, 10:50 AM]
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Hydrodynamics at BES energies

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- Hydro+
- ...
Initial States: 3D MC Glauber Model

Local entropy density
\[ s(x, y, \eta) \mid \tau_0 = \frac{K}{\tau_0} \left( H^s_P(\eta) s_P(x, y) + H^s_T s_T(x, y) \right) \]

Local baryon density
\[ n(x, y, \eta) \mid \tau_0 = \frac{1}{\tau_0} \left( H^b_P(\eta) s_P(x, y) + H^n_T s_T(x, y) \right) \]
Hydrodynamical Evolution

Energy - momentum conservation and net baryon current conservation:

\[ \partial_{\mu} T^{\mu\nu} = 0 \quad T^{\mu\nu} = e U^{\mu} U^{\nu} - (P + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu} \]
\[ \partial_{\mu} J^{\mu} = 0 \quad J^{\mu} = n U^{\mu} + V^{\mu} \]

Equation of motion of dissipative current:

\[ \Delta^{\mu\nu}_{\alpha\beta} D \pi^{\alpha\beta} = -\frac{1}{\tau_\pi} (\pi^{\mu\nu} - \eta \sigma^{\mu\nu}) - \frac{4}{3} \pi^{\mu\nu} \theta - \frac{5}{7} \pi^{\alpha\mu} \sigma^{\nu\alpha} + \frac{9}{70} \left( \frac{4}{e + P} \pi^{\alpha\nu} \pi^{\nu\alpha} - \kappa_B \nabla^{\mu} \frac{\mu}{T} \right) - V^{\nu} \theta - \frac{3}{10} V^{\nu} \sigma^{\mu\nu} \]

The shear viscosity

\[ \eta = C_\eta \frac{e + P}{T} \]

The baryon diffusion

\[ \kappa_B = C_B \frac{T}{n} \left( \frac{1}{3} \cot \left( \frac{\mu_B}{T} \right) - \frac{n T}{e + P} \right) \]

Equation of state \( P(e, n) \): NEOS (Lattice QCD + hadron gas, crossover)

Particlization and Afterburner

**Cooper-Frye Formula**

\[
\frac{dN}{dY p_T dp_T d\phi} = \frac{g_i}{(2\pi)^3} \int \frac{p_\mu d \Sigma_\mu f^{eq}(1 + \delta f_\pi + \delta f_V)}{1 - Q_f^2}\]

**Out-of-equilibrium corrections**

\[
\delta f_\pi(x, p) = (1 \pm f^{eq}(x, p)) \frac{p_\mu p_\nu \pi^{\mu\nu}}{2T_f^2 (e + P)}
\]

\[
\delta f_V(x, p) = (1 \pm f^{eq}(x, p)) \left( \frac{n}{e + P} - \frac{B}{U^{\mu} p_\mu} \right) \frac{p_\mu V_\mu}{\kappa_B/\tau_V}
\]

**SMASH 1.8: a novel and modern hadronic transport approach**

\[
p_\mu \partial_\mu f + mF^{\mu} \partial_{p_\mu} f = C[f]
\]

*C[f]: elastic collisions, resonance formation and decays, string fragmentation for all mesons and baryons up to mass ~ 2.35 GeV*


Mean transverse Momenta

The CLVisc framework can describe mean transverse momenta of identified particles from STAR. One can clearly see more blue shift effect for more massive particles.
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The initial conditions and pre-equilibrium evolution should improve the model in the future.
One can clearly see more blue shift effect for more massive particles.

The mean momenta of $\pi^+$, $K^+$ and $P$ increase mildly with the collision energy due to larger radial flow.
Anisotropic flows

Our results are in good agreement with the experimental data from STAR:

◆ $v_2(2)$: typical non-monotonic centrality dependences due to the combined effect of the elliptic geometry, geometrical fluctuations and the system size.

◆ $v_3(2)$: weak centrality dependence due to the initial state geometrical fluctuations.
We find that both elliptic and triangular flows increase slightly with the beam energy.

- the eccentricities $\varepsilon_2$ and $\varepsilon_3$ have very weak dependence on collision energy
- the increase of radial flow due to the increase of initial energy density
Flows fluctuations

The multi-particle cumulant ratio $v_2^4/v_2^2$ first increases, and then decreases with centrality increased.

- initial collision geometry dominates in mid-central collisions.
- the fluctuations dominate in central and peripheral collisions.

The multi-particle cumulant ratio $v_2^4/v_2^2$ has weak collision energy dependence.
Summary

We developed the (3+1)-D CLVisc hydrodynamic framework to include fluctuation initial condition, 2nd viscous hydrodynamics with baryon charge using the equation of state from NEOS, and hadronic transport SMASH afterburner.

Our calculation provides a benchmark for understanding the RHIC-BES data.

Elliptic and triangular flows increase slightly with the beam energy.

The multi-particle cumulant ratio $v_2(4)/v_2(2)$ has weak collision energy dependence.

Outlook

Hydro+, critical fluctuations
Back Up
**CLVisc at non-zero $\mu_B$**

Initial condition: MC Glauber model

**Local entropy density**

$$s(x, y, \eta)|_{\tau_0} = \frac{K}{\tau_0} (H^s_P(\eta) s_p(x, y) + H^s_T(\eta) s_T(x, y))$$

**Local baryon density**

$$n(x, y, \eta)|_{\tau_0} = \frac{1}{\tau_0} (H^n_P(\eta) s_p(x, y) + H^n_T(\eta) s_T(x, y))$$

where

$$s_{P/T}(x, y) = \sum_{i=1}^{N_{P/T}} \frac{1}{2\pi\sigma_r^2} \exp \left( -\frac{(x-x_i)^2 + (y-y_i)^2}{2\sigma_r^2} \right), \quad \tau_0 = \tau_{\text{overlap}} = \frac{2R}{\sqrt{\gamma^2 - 1}}$$

**Longitudinal profile**

$$H^s_{P/T} = \theta(\eta_{\max} - |\eta|) \left( 1 \pm \frac{\eta}{y_{\text{beam}}} \right) \left[ \theta(|\eta| - \eta_0^s) \exp \left( -\frac{(|\eta| - \eta_0^s)^2}{2\sigma_s^2} \right) + \theta(\eta_0^s - |\eta|) \right]$$

$$H^n_{P/T} = \frac{1}{N} \left[ \theta(\eta - \eta_0^{n,P/T}) \exp \left( -\frac{(\eta - \eta_0^{n,P/T})^2}{2\sigma_{P/T}^2} \right) + \theta(\eta_0^{n,P/T} - \eta) \exp \left( -\frac{(\eta - \eta_0^{n,P/T})^2}{2\sigma_{T/P}^2} \right) \right]$$

Set-up

Shear viscosity $C_\eta = 0.08$
Baryon diffusion coefficient $\tilde{C}_B = 0.4$
Freezeout energy density $E_{frz} = 0.4 \text{ GeV/fm}^3$

Regulation:
\[
\max(|\pi^{\mu\nu}|) > T_{\text{ideal}}^{\tau\tau} \quad \pi^{\mu\nu} = 0
\]
\[
\max(|V^\mu|) > J_{\text{ideal}}^{\tau} \quad V^\mu = 0
\]

We use a step function $\theta(f_{eq} + \delta f_\pi + \delta f_V)$ so that the full (equilibrium and nonequilibrium) distribution functions never encounter negative values. (PTM and the maximum-entropy distribution)
The modified Cooper-Frye formula

\[ S^\mu(p) = \int \frac{d \Sigma \cdot p J^\mu_\phi(p, X)}{2m \int d \Sigma \cdot N(p, X)} \]

\( J^\mu_\phi(p, X) \): axial-charge current density
\( N(p, X) \): the number density of fermions

From quantum kinetic theory

\[ S^\mu(p) = S^\mu_{\text{thermal}}(p) + S^\mu_{\text{shear}}(p) + S^\mu_{\text{accT}}(p) + S^\mu_{\text{chemical}}(p) + S^\mu_{\text{EB}}(p) \]

\[ S^\mu_{\text{thermal}}(p) = \int d \Sigma \sigma F_\sigma \epsilon^{\mu\nu\alpha\beta} p_\nu \partial_\alpha \frac{u_\beta}{T} \]

\[ S^\mu_{\text{shear}}(p) = \int d \Sigma \sigma F_\sigma \frac{\epsilon^{\mu\nu\alpha\beta} p_\nu u_\beta}{(u \cdot p) T} \times p^\rho (\partial_\rho u_\alpha + \partial_\alpha u_\rho - u_\rho D u_\alpha) \]

\[ S^\mu_{\text{accT}}(p) = -\int d \Sigma \sigma F_\sigma \frac{\epsilon^{\mu\nu\alpha\beta} p_\nu u_\alpha}{T} \left( D u_\beta - \frac{\partial_\beta T}{T} \right) \]

\[ S^\mu_{\text{chemical}}(p) = 2 \int d \Sigma \sigma F_\sigma \frac{1}{(u \cdot p)} \epsilon^{\mu\nu\alpha\beta} p_\alpha u_\beta \partial_\nu \frac{\mu}{T} \]

\[ S^\mu_{\text{EB}}(p) = 2 \int d \Sigma \sigma F_\sigma \left[ \frac{\epsilon^{\mu\nu\alpha\beta} p_\alpha u_\beta E_\nu}{(u \cdot p) T} + \frac{B^\mu}{T} \right] \]

\[ F^\mu = \frac{\hbar}{8m_\Lambda \Phi(p)} p^\mu f_{eq} (1 - f_{eq}) \]

\[ \Phi(p) = \int d \Sigma^\mu p^\mu f_{eq} \]
CLVisc Hydrodynamics Framework

- Lattice Equation of State (NEOSBQS)
- Initial States (SMASH, AMPT, Glauber)
- (3+1)D 2nd Viscous Hydrodynamics with Baryon Charge
- Cooper Frye
- Hadronic Transport (SMASH or URQMD)
Transverse: Gubser flow

To test the numerical accuracy of new CLVisc code, we compare our numerical results with both analytical solutions of the hydrodynamic equations and other independent codes.

Gubser flow: strong radial flow, longitudinal invariance in conformal system.

\[ \varepsilon(\tau, r) = \frac{\varepsilon}{\tau^4 \left[1 + 2q^2(\tau^2 + r^2) + q^4(\tau^2 - r^2)^2\right]^\frac{4}{3}} \]

\[ n(\tau, r) = \frac{n_0}{\tau^4 \left[1 + 2q^2(\tau^2 + r^2) + q^4(\tau^2 - r^2)^2\right]} \]

with \[ v_{\perp}(\tau, r) = \frac{2q^2\tau r}{1 + (q\tau)^2 + (qr)^2} \]
Longitudinal: (1+1)D Monnai's code

In longitudinal direction, we compare between the CLVisc's numerical results and (1+1) D hydrodynamic code by Monnai.

\[
\Delta^{\mu\nu}DV_{\mu} = -\frac{1}{\tau_v} \left( V^{\mu} - \kappa_B \nabla^{\mu} \frac{\mu}{T} \right)
\]

where \( \kappa_B = \frac{0.2n}{\mu_B} \quad \tau_v = \frac{0.2}{T} \)

Compared with VHLLE hydro model

VHLLE: a (3+1) D hydrodynamics model developed by Iurii Karpenko