SQM2022

The 20th International Conference on Strangeness in Quark Matter

13-17 June 2022 Busan, Republic of Korea

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Busan, Republic of Korea

Quark susceptibilities, transport properties and heavy quark production in an extended Quasi-Particle Model with $N_f = 2+1+1$ flavors

UNIVERSITÀ degli STUDI di CATANIA

DIPARTIMENTO DI FISICA E ASTRONOMIA "ETTORE MAJORANA"

INFN LNS Istituto Nazionale di Fisica Nucleare Laboratori Nazionali del Sud

Maria Lucia Sambataro

Dipartimento di Fisica e Astronomia 'E. Majorana'

Università degli Studi di Catania, INFN-LNS

In collaboration with: S. Plumari, V. Minissale, V. Greco

Outline

❖ **Basic scales of heavy quarks**

QPM standard:

- Catania approach with $N_f = 2+1$ flavors to charm quark dynamics:
  - ➢ $R_{AA}, v_2 \longrightarrow$ Spatial diffusion coefficient $D_s(T)$ of charm.

QPM extension:

- From $N_f = 2+1$ to $N_f = 2+1+1$ flavors including charm.
- Momentum dependence m(p,T) – QPM*
  - ➢ EoS and quark susceptibilities.
  - ➢ Transport coefficients and $D_s(T)$.
- **Preliminary in a static box:**

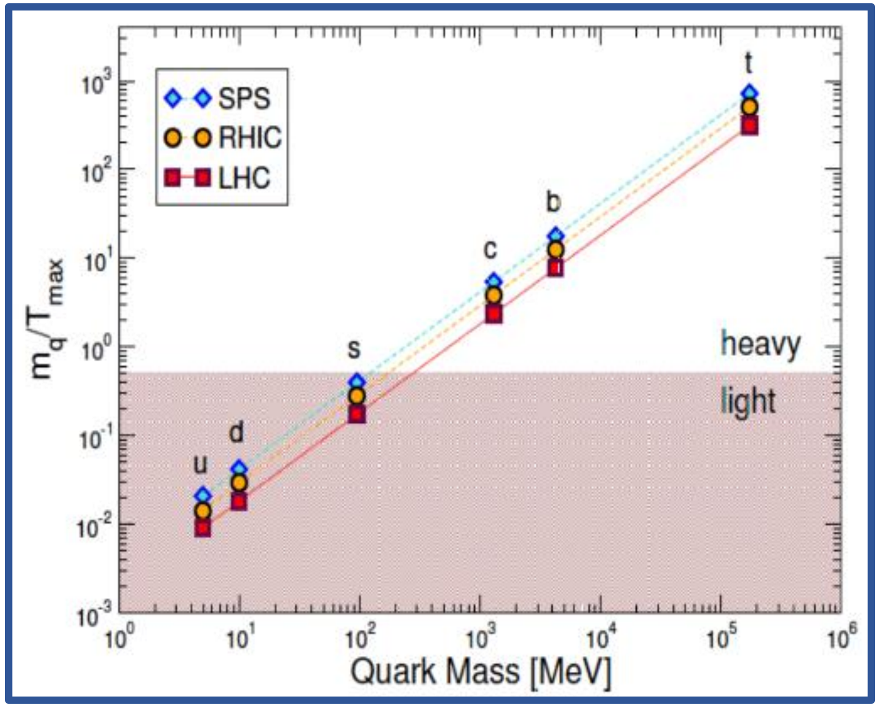
➢ Temporal evolution of charm quark distribution function

➢ Nuclear modification factor R_{AA}

❖ **Conclusions and new perspectives**

Basic scales of charm and bottom quarks

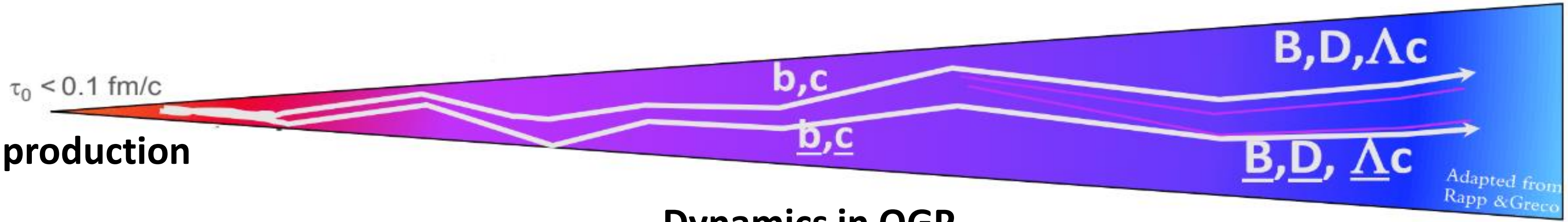
Charm M_c≈1.3 GeV and Bottom M_b≈4.2 GeV



- $m_{c,b} >> \Lambda_{QCD}$
  pQCD initial production
- $m_{c,b} >> T_{RHIC,LHC}$
  negligible thermal production
- $\tau_0 < 0{,}08$ fm/c $<< \tau_{QGP}$
- $\tau_{th} \approx \tau_{QGP} >> \tau_{g,q}$

They experience the full evolution of the QGP.

They carry more informations with respect to their light counterparts.



Initial production

Dynamics in QGP

Hadronization:
Final hadron Spectra and observables

Recent reviews:
1) X.Dong, V. Greco Prog. Part. Nucl. Phys. 104 (2019),
2) A.Andronic EPJ C76 (2016), 3) R.Rapp, F.Prino J.Phys. G43 (2016)

Quasi Particle Model (QPM) fitting lQCD

Parton masses and coupling

Non perturbative dynamics → M scattering matrices (q,g → Q) evaluated by Quasi-Particle Model fit to **lQCD thermodynamics**

N_f=2+1
Bulk:
u,d,s

QPM Standard

no momentum dependence

$$m_g^2(T) = \frac{2N_c}{N_c^2-1} g^2(T)\,T^2$$
$$m_q^2(T) = \frac{1}{N_c} g^2(T)\,T^2$$

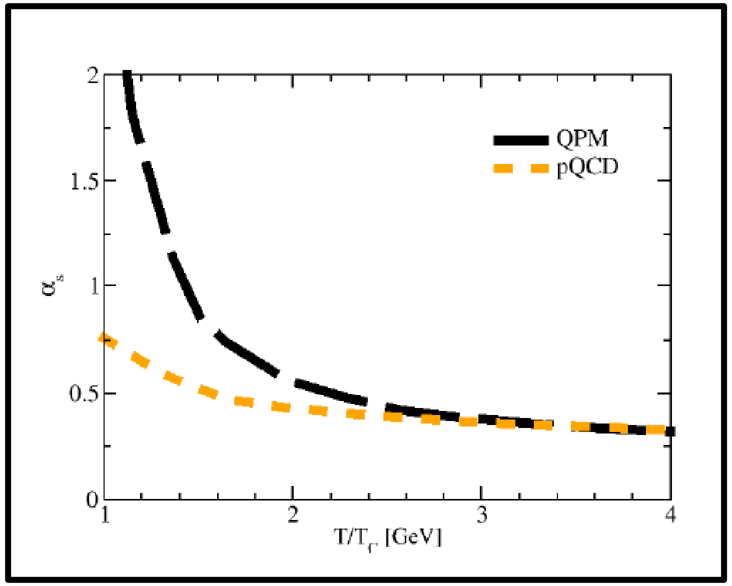
→ **Thermal masses of gluons and light quarks**

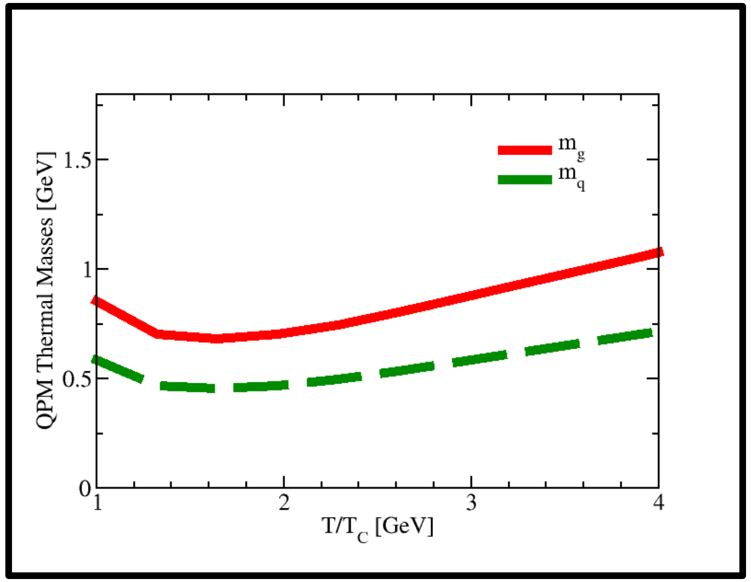
$+\, m_{0s}^2$ for strange quark

g(T) from a fit to ε from lQCD data → good reproduction of P, ε-3P

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[\lambda\left(\frac{T}{T_c} - \frac{T_s}{T_c}\right)\right]^2}$$

λ=2.6
T_s=0.57 T_c





Larger than pQCD especially as T → T_c

S. Plumari et al, *Phys.Rev.D* 84 (2011) 094004
H. Berrehrah,, PHYSICAL REVIEW C **93**, 044914 (2016)
M.L. Sambataro et al. in preparation

Quasi Particle Model (QPM) fitting lQCD

Equation of State and Susceptibilities

Non perturbative dynamics → M scattering matrices (q,g → Q) evaluated by Quasi-Particle Model fit to **lQCD thermodynamics**

N_f=2+1
Bulk:
u,d,s

QPM Standard

no momentum dependence

$$m_g^2(T) = \frac{2N_c}{N_c^2-1}\, g^2(T)\, T^2$$
$$m_q^2(T) = \frac{1}{N_c}\, g^2(T)\, T^2$$

→ **Thermal masses of gluons and light quarks**

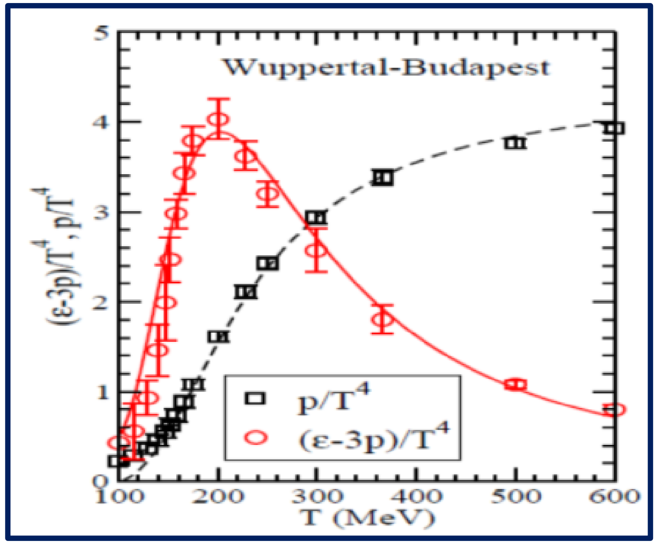
Standard QPM underestimates the **quark susceptibilities**

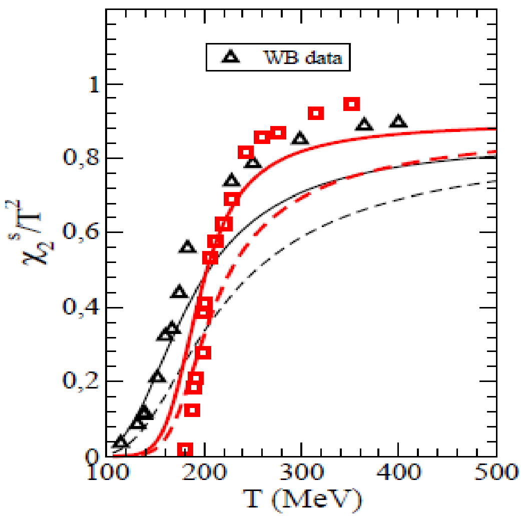
g(T) from a fit to ε from lQCD data → good reproduction of P, ε-3P

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[\lambda\left(\frac{T}{T_c} - \frac{T_s}{T_c}\right)\right]^2}$$

λ=2.6
T_s=0.57 T_c

Larger than pQCD especially as T → T_c





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Relativistic Boltzmann equation at finite η/s

Bulk evolution

$$p^\mu \partial_\mu f_q(x,p) + m(x)\,\partial^x_\mu m(x)\,\partial^\mu_p f_q(x,p) = C[f_q, f_g]$$

$$p^\mu \partial_\mu f_g(x,p) + m(x)\,\partial^x_\mu m(x)\,\partial^\mu_p f_g(x,p) = C[f_q, f_g]$$

Free-streaming

field interaction
$\varepsilon - 3p \neq 0$

Collision term
gauged to some η/s≠ 0

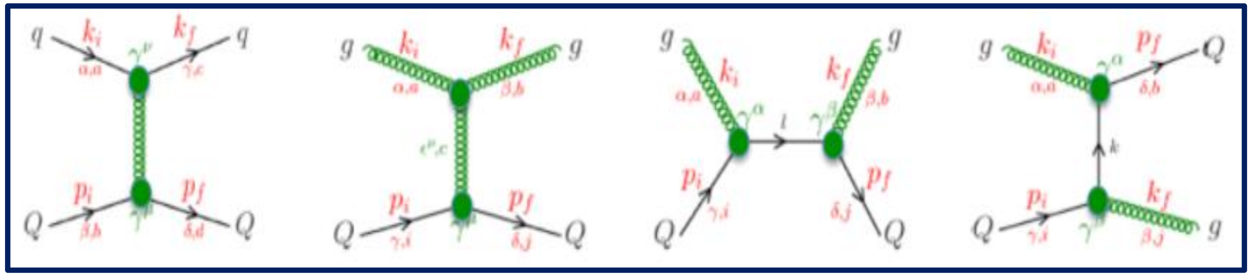
Equivalent to viscous hydro at η/s ≈ 0.1

HQ evolution

$$p^\mu \partial_\mu f_Q(x,p) = C[f_q, f_g, f_Q]$$

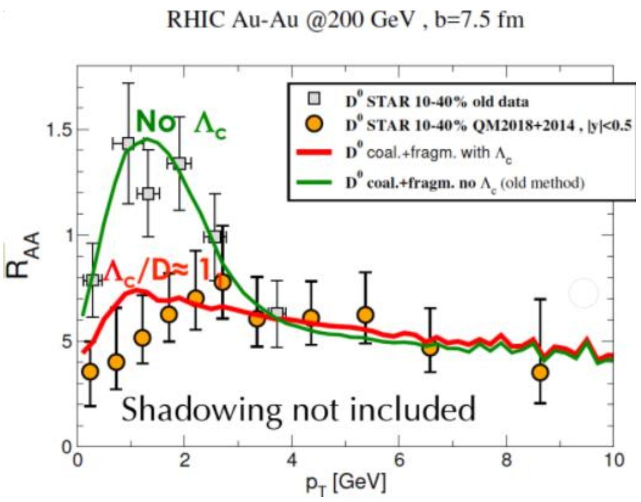
$$C[f_q,f_g,f_Q] = \frac{1}{2E_1}\int \frac{d^3p_2}{2E_2(2\pi)^3}\int \frac{d^3p_1'}{2E_1'(2\pi)^3}$$
$$\times [f_Q(p_1')f_{q,g}(p_2') - f_Q(p_1)f_{q,g}(p_2)]$$
$$\times |M_{(q,g)\to Q}(p_1 p_2 \to p_1' p_2')|$$
$$\times (2\pi)^4 \delta^4(p_1 + p_2 - p_1' - p_2')$$

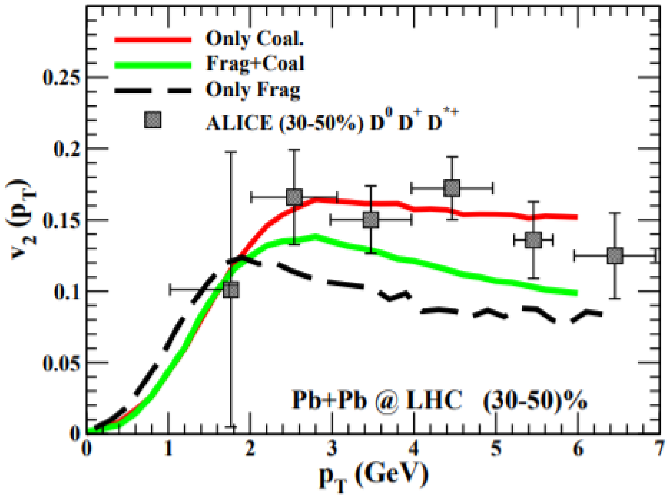
Feynmann diagrams at first order pQCD for HQs-bulk interaction:



Scattering matrices $M_{g,q}$ by QPM fit to lQCD thermodynamics

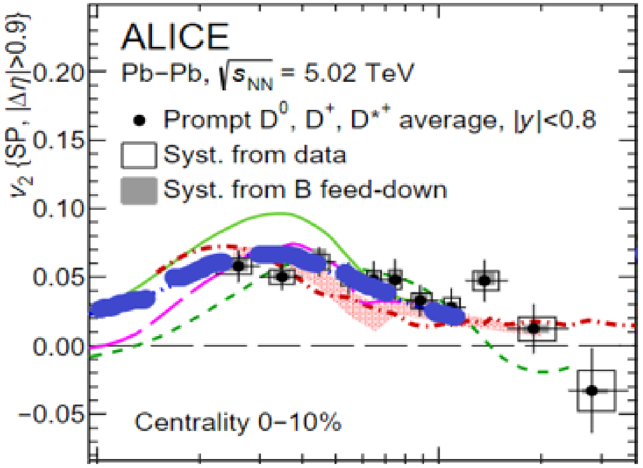
Catania QPM: some prediction for charm…





Scardina et al., PRC 97(2017)

Good description of
R_{AA}, v_2 at RHIC & LHC energies
within error bars



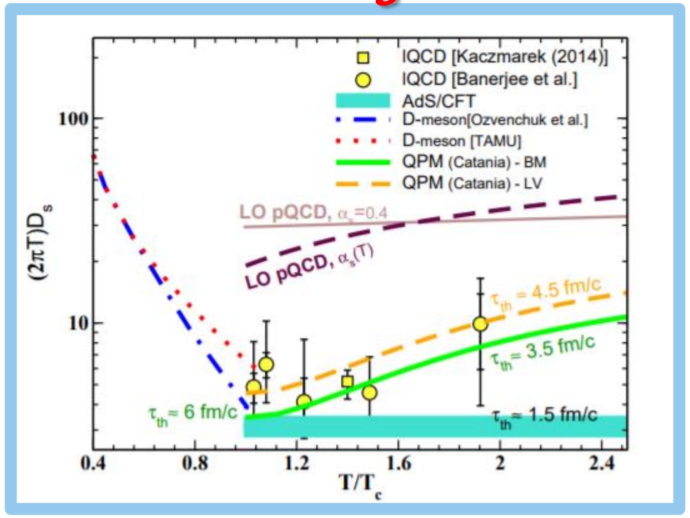
Catania Model

ALICE collaboration, *Phys.Lett.B* 813 (2021) 136054

Spatial Diffusion Coefficient D_s

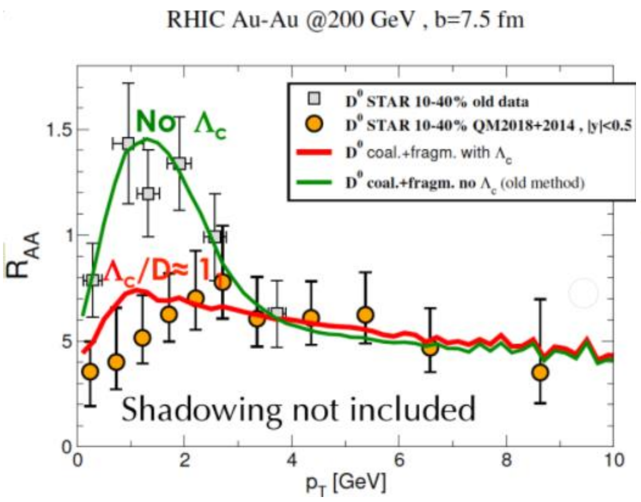
$$\tau_{th} = \frac{M}{2\pi T^2}(2\pi T D_s) \cong 1.8\,\frac{2\pi T D_s}{(T/T_c)^2}\ \mathrm{fm/c}$$

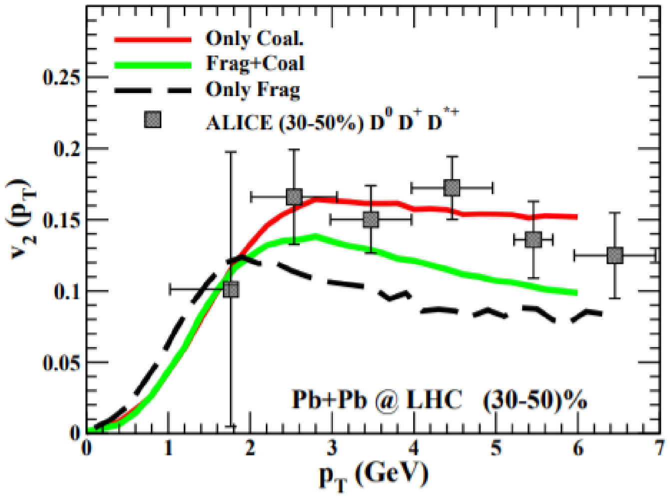
Not a model fit to lQCD data!
Results from R_{AA} (p_T), v_2 (p_T)



__Reviews__:
- F. Prino and R. Rapp, JPG(2019)
- X. Dong and V. Greco, Prog.Part.Nucl.Phys. (2019)
- Jiaxing Zhao et al., arXiv:2005.08277

Catania QPM: some prediction for charm…





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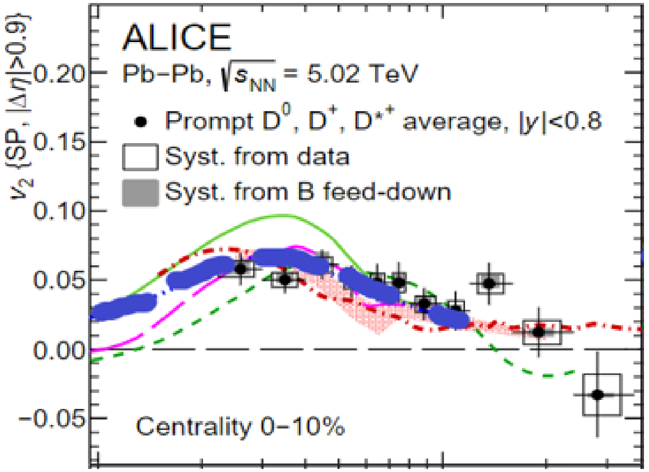
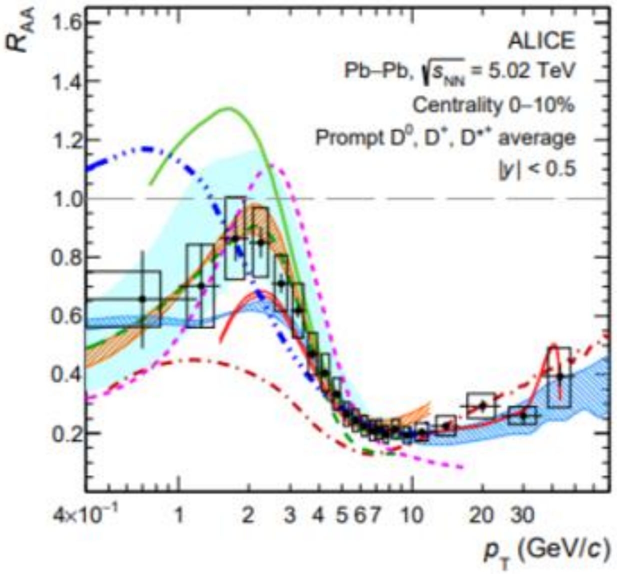
Good description of R_{AA}, v_2 at RHIC & LHC energies within error bars



Catania Model

ALICE collaboration, *Phys.Lett.B* 813 (2021) 136054

Recent access to low p_T



Low p_T → overstimation of ALICE experimetal data especially in most central collision:

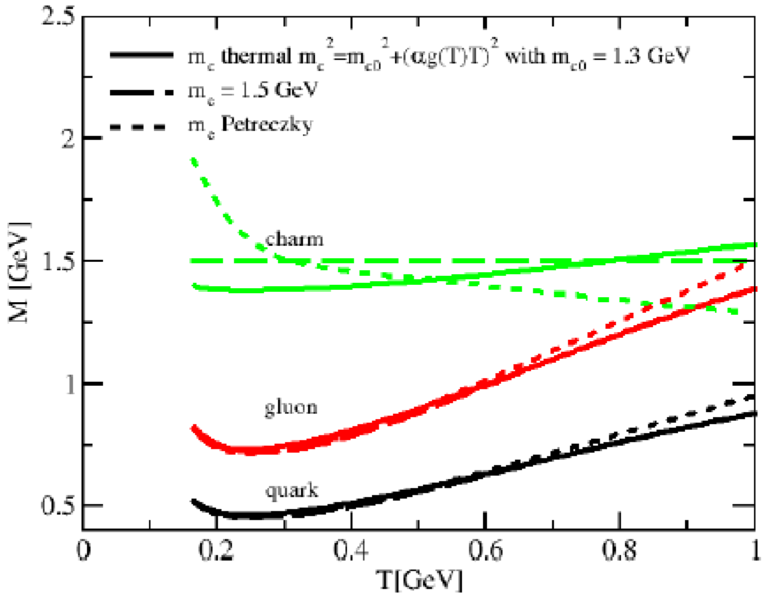
- **Momentum dependent QPM?**

High p_T → good agreement with ALICE experimental data

QPM extension:
$N_f = 2 + 1 + 1$ and momentum dependence

QPM with $N_f = 2+1+1$ including charm

Recently, new lattice results for the equation of state of QCD with 2+1+1 dynamical flavors have become available. Therefore, we extend our QPM approach for $N_f = 2+1$ to $N_f = 2+1+1$ where the charm quark is included.

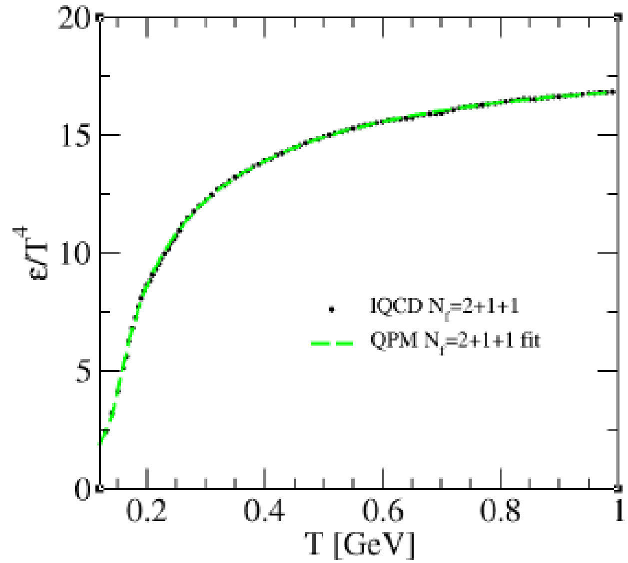


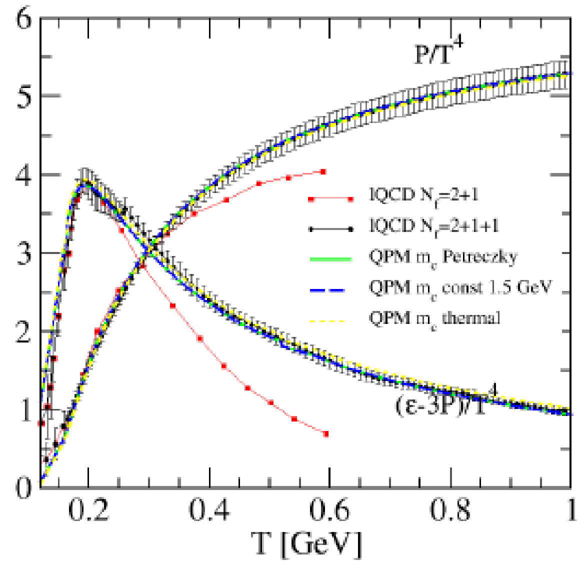
Three cases:

Constant mass $m_c = 1.5\ GeV$

Thermal mass $m_c^2 = m_{c0}^2 + \frac{N_c^2-1}{8N_c} g^2 T^2$

Charm Mass constrained by quark susceptibility from lQCD m_c [Petreczky]





- **Energy density** fit to lQCD data
- Good reproduction of **pressure** and **interaction measure** for three different charm quark masses.

Petreczky, Heavy-Quark Transport from Lattice-QCD, Heavy flavor transport in QCD matter workshop (2021).

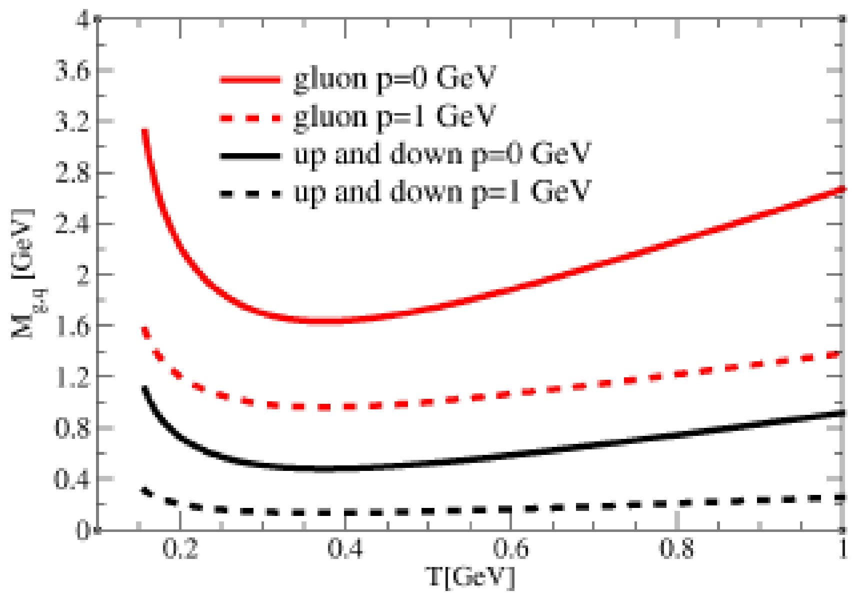
QPM extended – momentum dependence

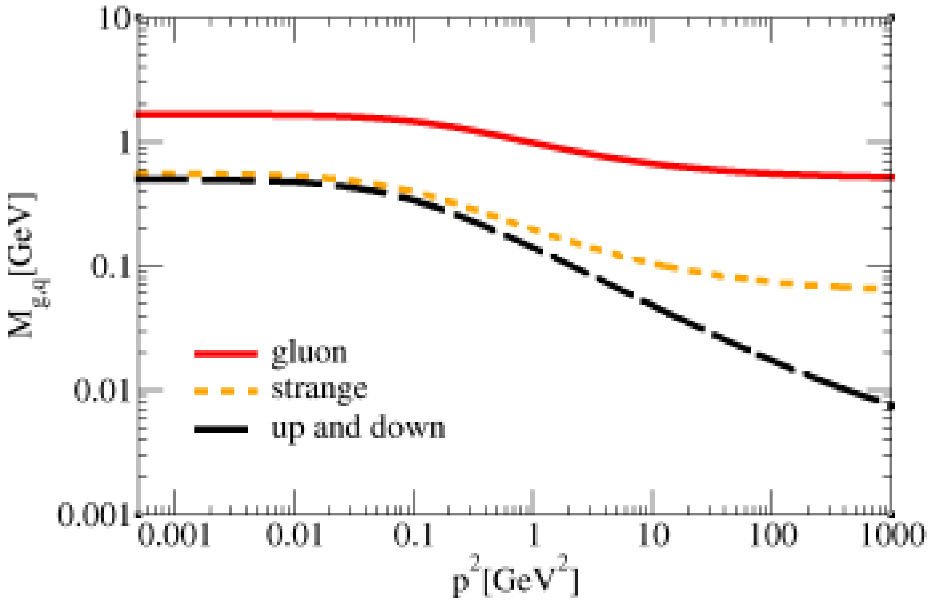
Dyson-Schwinger studies in the vacuum → following the model developed by PHSD group

$$M_g(T,\mu_q,p) = \left(\frac{3}{2}\right)\left(\frac{g^2(T^\star/T_c(\mu_q))}{6}\left[\left(N_c + \frac{1}{2}N_f\right)T^2 + \frac{N_c}{2}\sum_q \frac{\mu_q^2}{\pi^2}\right]\left[\frac{1}{1+\Lambda_g(T_c(\mu_q)/T^\star)p^2}\right]\right)^{1/2} + m_{\chi g}$$

$$M_{q,\bar{q}}(T,\mu_q,p) = \left(\frac{N_c^2-1}{8N_c}g^2(T^\star/T_c(\mu_q))\left[T^2 + \frac{\mu_q^2}{\pi^2}\right]\left[\frac{1}{1+\Lambda_q(T_c(\mu_q)/T^\star)p^2}\right]\right)^{1/2} + m_{\chi q}$$

Momentum dependent factors





QPM extended – momentum dependence

Dyson-Schwinger studies in the vacuum → following the model developed by PHSD group

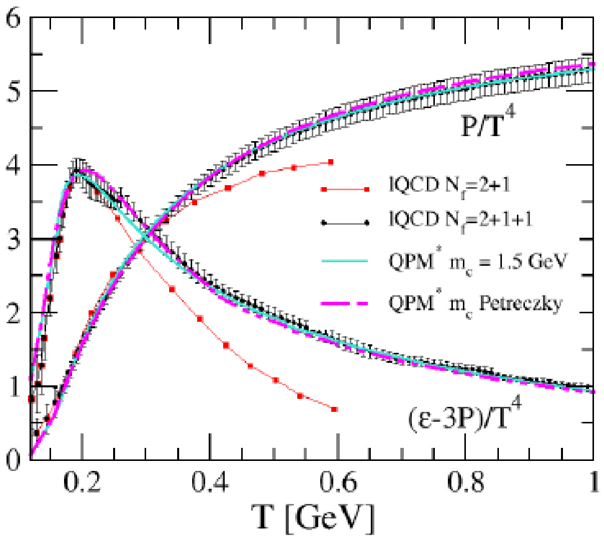
H. Berrehrah, W. et al., Phys.Rev.C 93, 044914 (2016).
C. S. Fischer, J. Phys. G 32, R253 (2006).

$$M_g(T,\mu_q,p) = \left(\frac{3}{2}\right)\left(\frac{g^2(T^\star/T_c(\mu_q))}{6}\left[\left(N_c+\frac{1}{2}N_f\right)T^2+\frac{N_c}{2}\sum_q\frac{\mu_q^2}{\pi^2}\right]\left[\frac{1}{1+\Lambda_g(T_c(\mu_q)/T^\star)p^2}\right]\right)^{1/2}+m_{\chi g}$$

$$M_{q,\bar{q}}(T,\mu_q,p) = \left(\frac{N_c^2-1}{8N_c}g^2(T^\star/T_c(\mu_q))\left[T^2+\frac{\mu_q^2}{\pi^2}\right]\left[\frac{1}{1+\Lambda_q(T_c(\mu_q)/T^\star)p^2}\right]\right)^{1/2}+m_{\chi q}$$

Momentum dependent factors

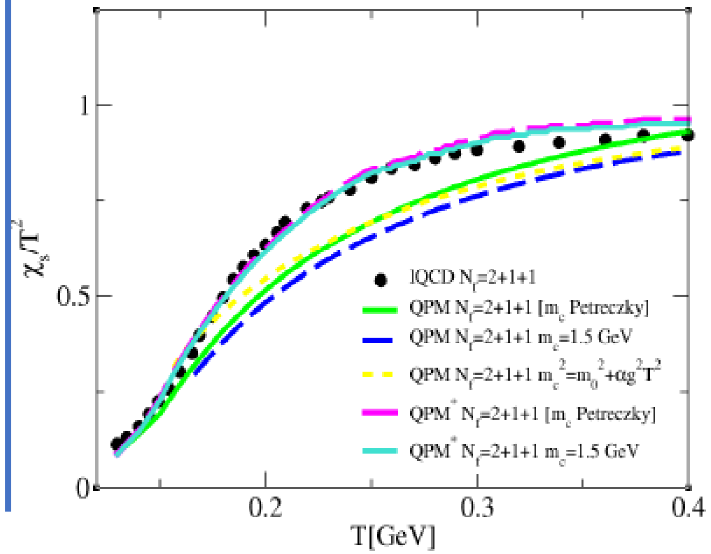
We correctly reproduce both **EoS** and **quark susceptibilities** which are understimated in the standard QPM approach.

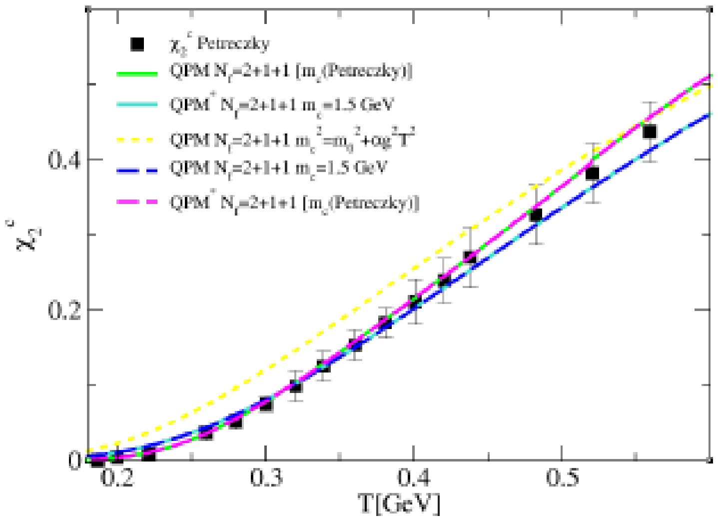


$$\chi_s = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial \mu_s^2}$$

$$\chi_c = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial \mu_c^2}$$

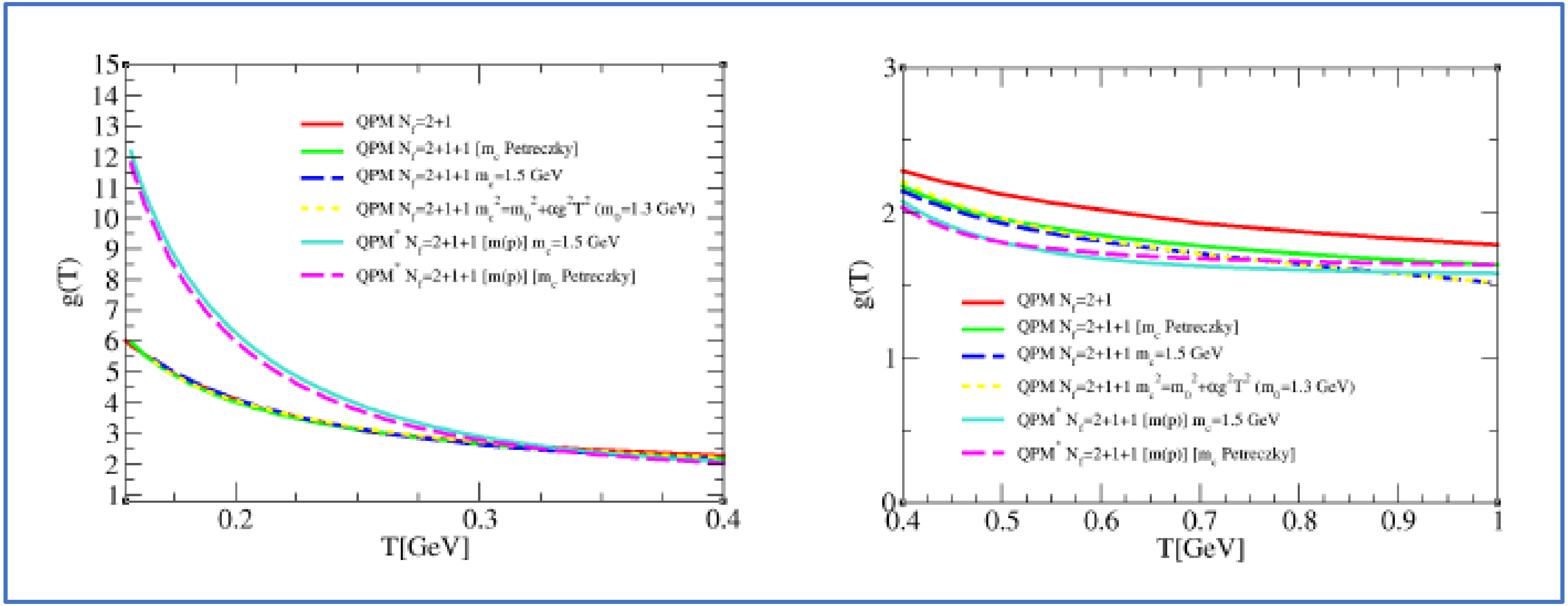
QUARK SUSCEPTIBILITIES





QPM extended – coupling and drag coefficient

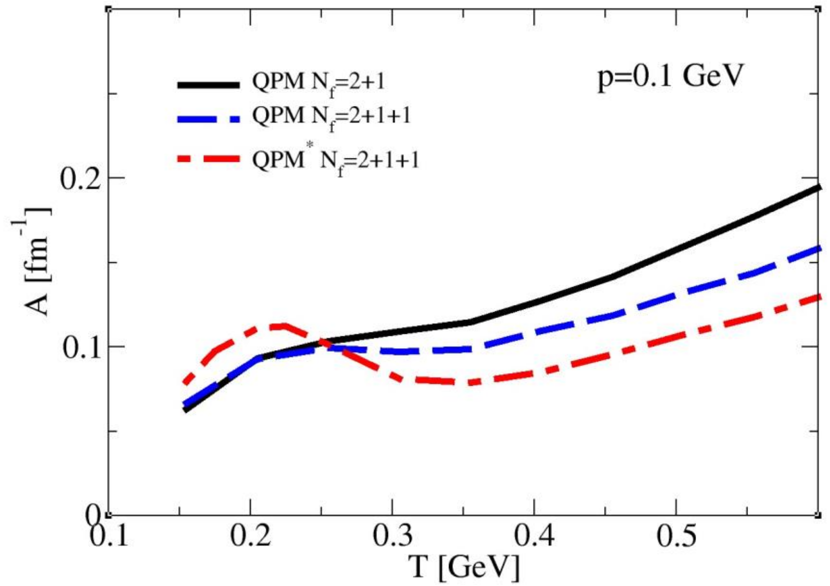
$0{,}155 < T < 0{,}4\ GeV$ $\quad$ $0{,}4 < T < 1\ GeV$

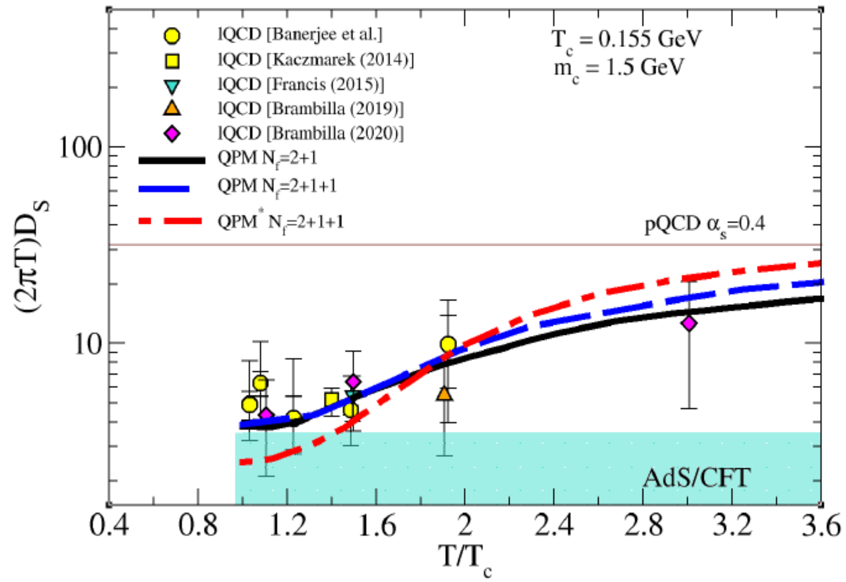


Coupling g(T) → standard QPM vs extended QPM

- Large **enhancement** at low T
- **Comparable value** at high T with a discrepancy of about 10%.

Drag and D_s in QPM extended





Drag coefficient

→ standard QPM
standard QPM including charm
extended QPM

- **Increase** at low T consistent with the large enhancement of the coupling in the same T region
- **Decrease** at high T

Spatial diffusion coefficient D_s

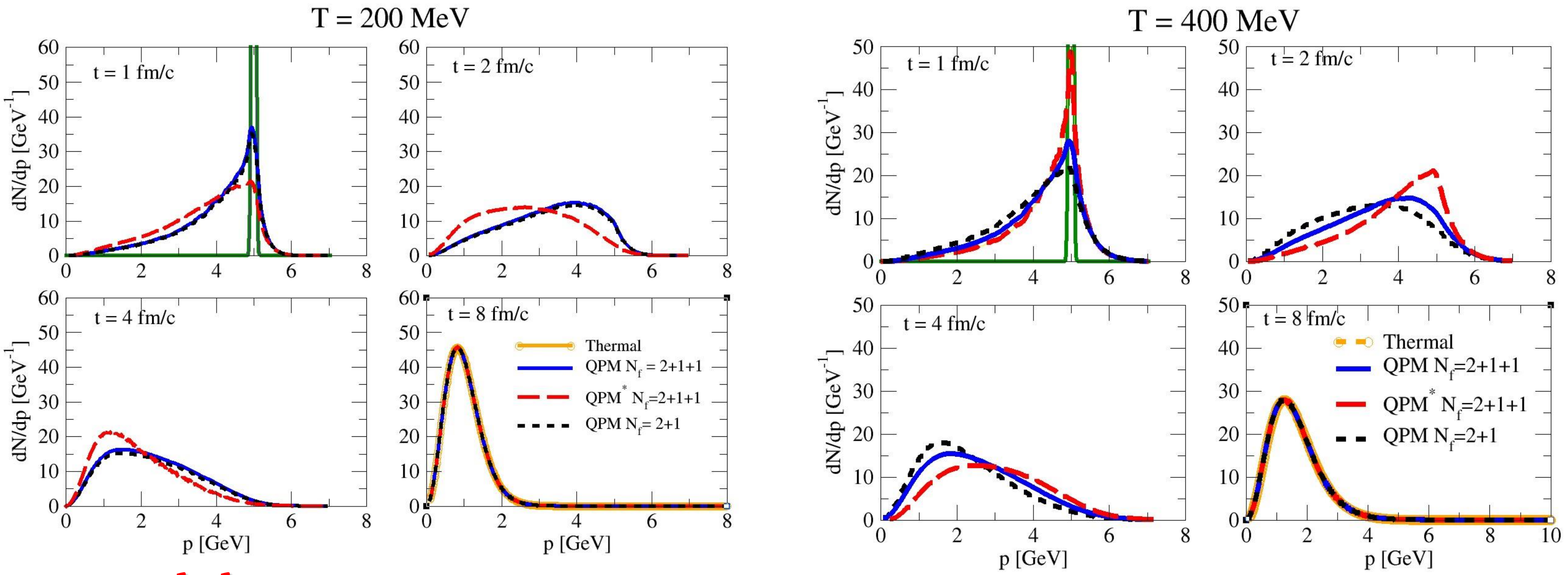
$T/T_c < 2$ → strong non-perturbative behaviour near to T_c.

high T region → the D_s reaches the pQCD limit quickly than the standard QPM.

Preliminary:

BOX CALCULATION FOR CHARM → static medium

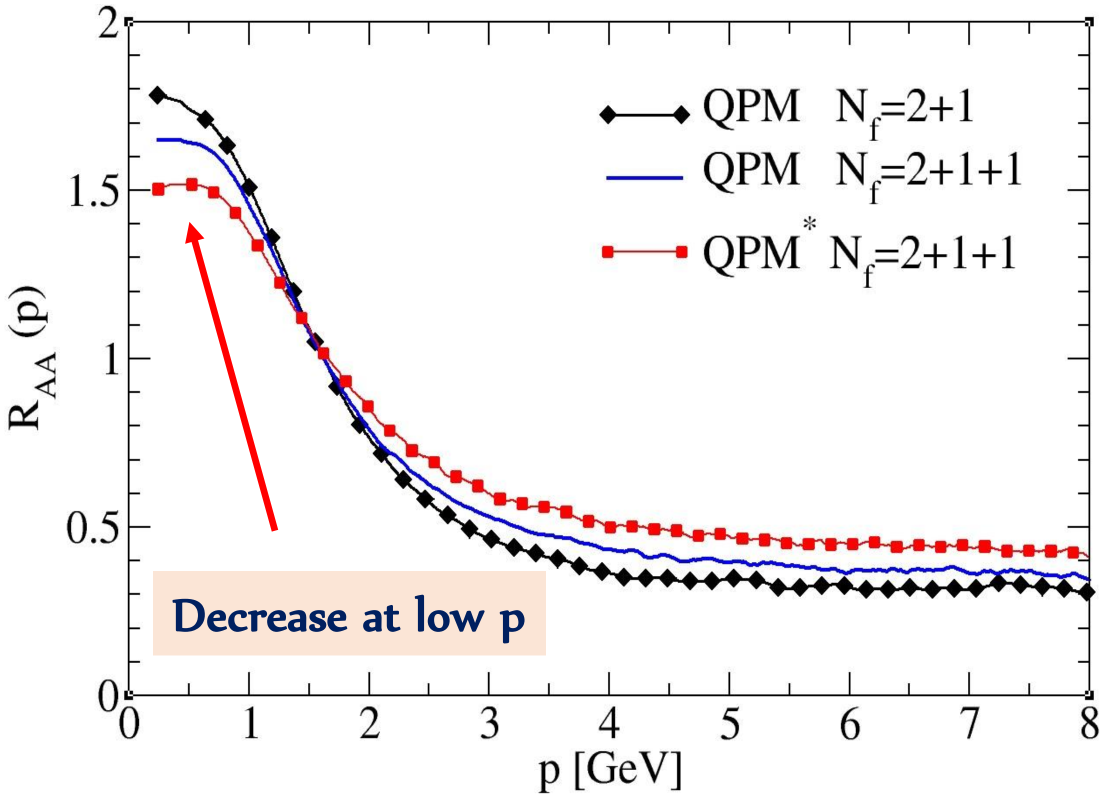
evolution of charm quark distribuction function



Extended QPM:

- $T > T_c$ slower dynamics
- $T \rightarrow T_c$ the increase in the drag coefficient leads to a faster evolution

Preliminary:
Nuclear modification factor R_{AA}



$$R_{AA} \doteq f_C(p, t_f)/f_C(p, t_0)$$

Initial momentum distribution function
→ FONLL for charm quark

Momentum dependent QPM approach

- Better description of recent lQCD data.
- Effects on the global χ^2 coming from the comparison to the experimental data of R_{AA}, v_n?

Conclusions

Standard QPM:

- Good description of EoS but quark susceptibilities understimated
- Good description of R_{AA}, v_2 at RHIC & LHC energies within error bars

  → Recent access to low p_T → overstimation of R_{AA} especially in most central collision at low p_T.

Extended QPM:

- Good reproduction of both EoS and susceptibilities
- Preliminary for a static medium: slower dynamics at high temperature and decrease of R_{AA} at low p

  → Effects on observables for realistic simulation?

Thanks for the attention!

SQM2022

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Back up slides

Relativistic Boltzmann equation at finite η/s

Bulk evolution

$$p^{\mu}\partial_{\mu}f_q(x,p)+m(x)\partial_{\mu}^{x}m(x)\partial_{p}^{\mu}f_q(x,p)=C[f_q,f_g]$$
$$p^{\mu}\partial_{\mu}f_g(x,p)+m(x)\partial_{\mu}^{x}m(x)\partial_{p}^{\mu}f_g(x,p)=C[f_q,f_g]$$

Free-streaming

field interaction
$\varepsilon - 3p \neq 0$

Collision Integral gauged to reproduce viscous Hydro at fixed η/s by means of Chapman-Ensksog

$$\sigma(n(\vec{x}),T)=\frac{1}{15}\frac{\langle p\rangle_0}{g(a)n(\vec{x})}\frac{1}{\eta/s}$$

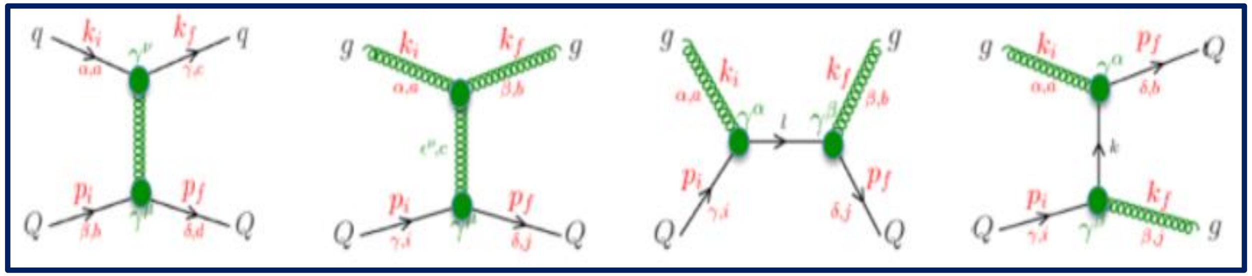
Equivalent to viscous hydro at η/s ≈ 0.1

HQ evolution

$$p^{\mu}\partial_{\mu}f_Q(x,p)=C[f_q,f_g,f_Q]$$

$$C[f_q,f_g,f_Q]=\frac{1}{2E_1}\int\frac{d^3p_2}{2E_2(2\pi)^3}\int\frac{d^3p_1'}{2E_1'(2\pi)^3}$$
$$\times[f_Q(p_1')f_{q,g}(p_2')-f_Q(p_1)f_{q,g}(p_2)]$$
$$\times|M_{(q,g)\to Q}(p_1p_2\to p_1'p_2')|$$
$$\times(2\pi)^4\delta^4(p_1+p_2-p_1'-p_2')$$

Feynmann diagrams at first order pQCD for HQs-bulk interaction:



Scattering matrices $M_{g,q}$ by QPM fit to lQCD thermodynamics

Numerical solution of Boltzmann Equation

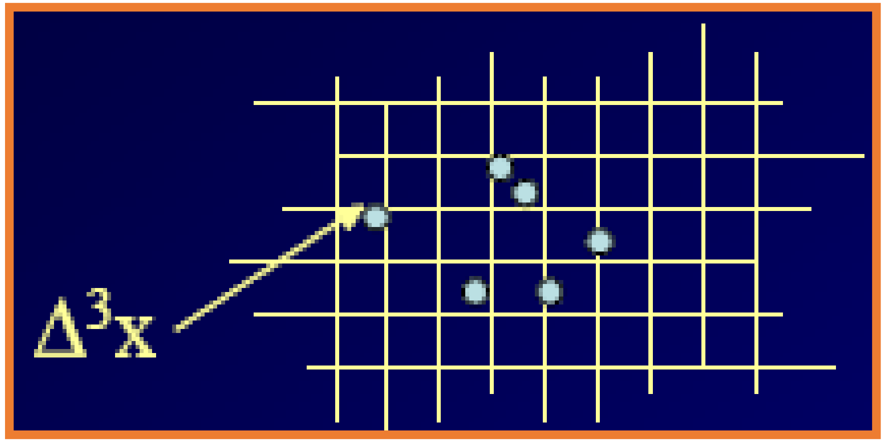
- Use Test-Particle Method to sample the phase space distribution function

$$f(\vec{x},\vec{p},t) = \omega \sum_{i=1}^{N_{test}} \delta^{(3)}(\vec{x}-\vec{r}_i(t))\delta^{(3)}(\vec{p}-\vec{p}_i(t))$$

F_i solution of Boltzmann eq.

→ Test particles solve classical Hamilton eq. of motion

$$\begin{cases} \vec{p}_i(t+\Delta t) = \vec{p}_i(t-\Delta t) + 2\Delta t \cdot \left(\frac{\partial \vec{p}_i}{\partial t}\right)_{coll} \\ \vec{r}_i(t+\Delta t) = \vec{r}_i(t-\Delta t) - 2\Delta t \cdot \left[\frac{\vec{p}_i(t)}{E_i(t)}\right] \end{cases}$$



- Collision Integral mapped through a Stochastic Algorithm

Δt → 0 and $\Delta^3 x$ → 0 : exact solution

$$P_{22} = \frac{\Delta N_{coll}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{rel}\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$

Final phase-space of HQ + bulk parton scattering sampled according to $|M_{QCD}|^2$ → code test through simulations in a "box"

[Scardina, Colonna, Plumari, and Greco PLB v.724, 296 (2013)]
[Xu and Greiner PRC v. 71, (2005)]
[P. Danielewicz and G.F. Bertsch, Nucl. Phys. A533 (1991) 712]

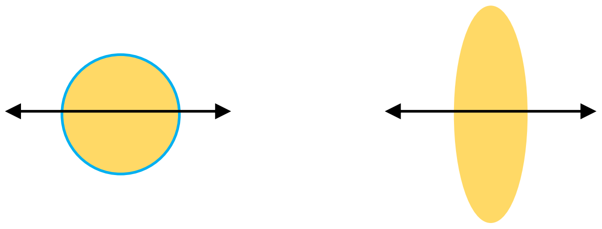
Extension to higher order anisotropic flows $v_n(p_T)$

ESE tecnique and v_n correlations

Selection of events with the same centrality but different initial geometry on the basis of the magnitude of the second-order harmonic reduced flow vector q_2.

$$q_2 = |\vec{Q}_2|/\sqrt{M}$$

$$\vec{Q}_2 = \sum_{j=1}^{M} e^{i2\varphi_j}$$



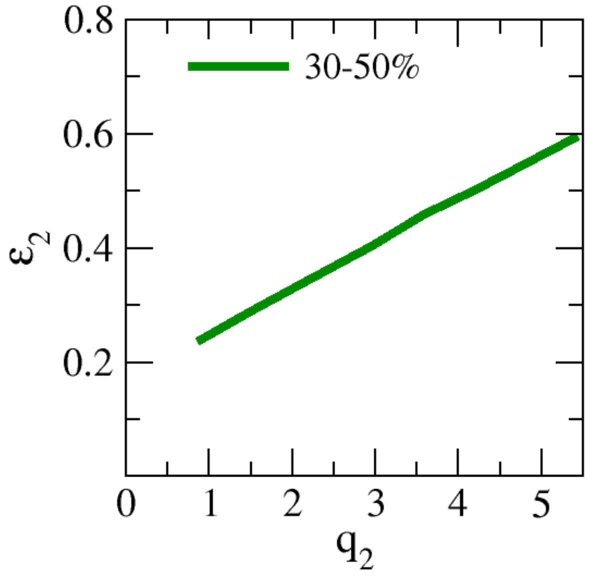
20 % small q_2 20 % large q_2

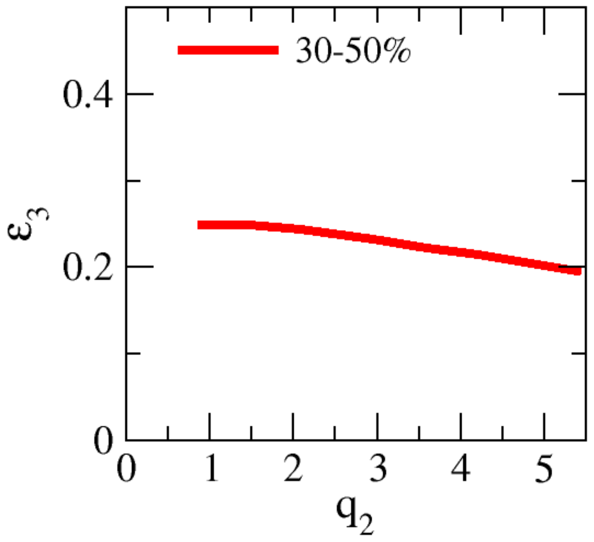
Large $q_2 \rightarrow$ large ε_2

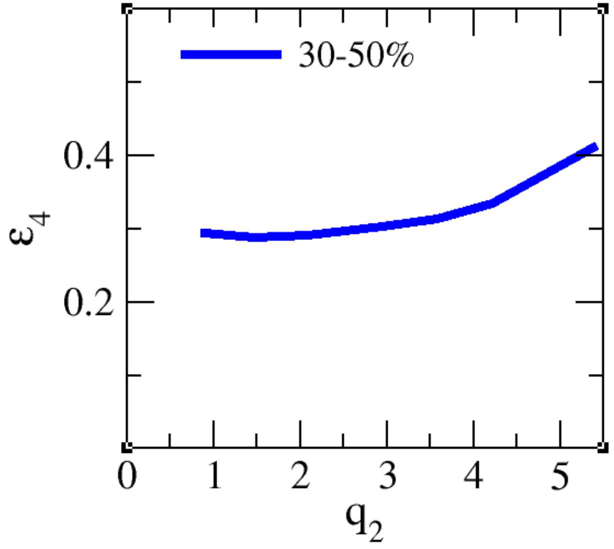
$$\epsilon_n = \frac{\langle r_\perp^n \cos[n(\varphi - \Phi_n)]\rangle}{\langle r_\perp^n \rangle} \qquad \Phi_n = \frac{1}{n}\arctan\frac{\langle r_\perp^n \sin(n\varphi)\rangle}{\langle r_\perp^n \cos(n\varphi)\rangle}$$

$$r_\perp = \sqrt{x^2 + y^2}, \qquad \varphi = \arctan(y/x)$$

Pb-Pb 5,02 TeV







Anti-correlation between ε_2 and ε_3

Non-linear correlation between ε_2 and ε_4

Hybrid Hadronization Model for HQs

✓ COALESCENCE: Formula developed for the light sector [Greco, Ko, Levai PRL 90 (2003)]

$$\frac{dN_H}{d^2\boldsymbol{P}_T} = g_H \int \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 E_i} p_i \cdot d\sigma_i \, f_{q_i}(x_i, p_i) \, f_W(x_1 \dots x_n; p_1 \dots p_n) \, \delta\left(\boldsymbol{P_T} - \sum_i^n p_{T,i}\right)$$

Statistical Factor
Color-spin-isospin symmetry

Parton Distribution Functions
(after Boltzmann evolution)

Wigner Function ≈ Hadron wave function
(parameters fix according to quark model)
[Hwang EPJ C23 (2002)]

✓ FRAGMENTATION: HQs that do not undergo to Coalescence

$$\frac{dN_H}{d^2\boldsymbol{P}_T} = \sum_f \int dz \frac{dN_f}{d^2 p_T} \frac{D_{f\to H}(z)}{z^2}$$

[Plumari, Minissale, Das , Coci , Greco, EPJ C 78 (2018) no.4]

We use Peterson parametrization: $D_H(z) \propto \left[z\left(1 - \frac{1}{z} - \frac{\epsilon_c}{1-z}\right)^2\right]^{-1}$ ✓ [Peterson et al. PRD 27 (1983) 105]

Parameter ε_c tuned to reproduce *D* and *B* meson spectra in pp collisions.

Predictions for D mesons: R_{AA} and elliptic flow v_2

Nuclear modification factor R_{AA}

$$R_{AA}(p_T) \equiv \frac{d^2N_{AA}/d\eta dp_T}{N_{coll} \cdot d^2N_{pp}/d\eta dp_T}$$

HQs propagate and loose energy in the plasma
→ suppression of hadron distributions at high p_T

$R_{AA} \approx 1$ weakly interacting or extremely dilute medium

- Intial p_T-spectrum from FONLL pp @5.02 TeV [**Cacciari et al. JHEP10 (2012)**]

$$R_{AA}^{D^0}(p_T) = \frac{(dN/d^2p_T)^{D^0}_{AAcoal+fragm}}{\frac{N^c_{AA}}{N^D_{pp}}(dN/d^2p_T)^{D^0}_{ppfragm}}$$

$$N^c_{AA} = \int d^2p_T \left(\frac{dN}{d^2p_T}\right)^{charm}_{AA}$$

$$N^D_{pp} = \int d^2p_T \left(\frac{dN}{d^2p_T}\right)^{charm}_{pp} \otimes D_{c\to D}$$