QCD phase diagram in strong magnetic fields: Competition between the magnetic catalysis and the QCD Kondo effect

KH, Suenaga, Suzuki, Yasui, In preparation.
Cf. KH, Itakura, Ozaki, Yasui, 1504.07619

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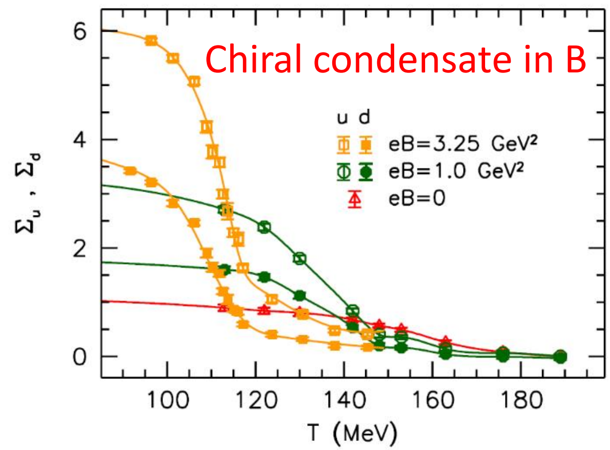
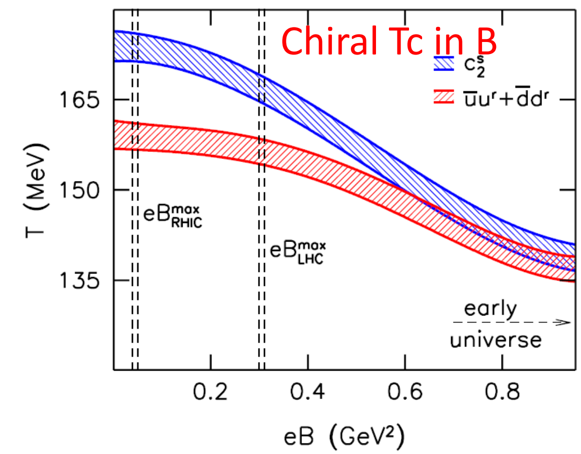
SQM @ Busan, Jun. 12-17, 2022

QCD phase diagram in strong B has been a hot topic.

Numerous data from lattice collaborations

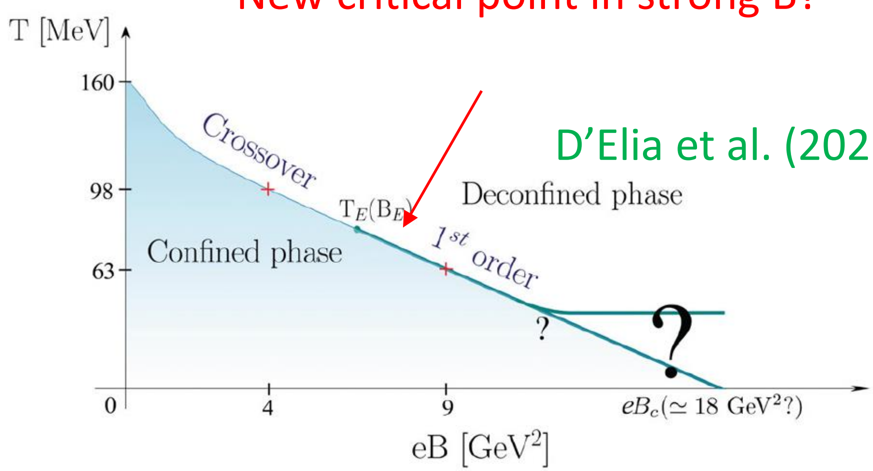
Buividovich, et al. (2010-)
D'Elia, et al. (2012-)
Endrodi, et al. (2012-)
Ding, et al. (2019-)





Endrodi, et al. (2012-

New critical point in strong B?



D'Elia et al. (2022)

More extensions: we discuss the phase diagram in strong B with *heavy flavors.*
→ The QCD Kondo effect

Theoretical foundation:
"Magnetic catalysis" of the chiral symmetry breaking

Chiral condensate increases
in strong (chromo) magnetic fields.

Klevansky, Lemmer (1989)
Tatsumi, Suganuma (1991)
Schramm, Muller, Schramm (1992)

- Moreover, "the chiral symmetry is broken with an infinitesimal attractive interaction, i.e., even in QED."

Gusynin, Miransky, Shovkovy (1995)

Analogy with the BCS theory:
"Superconductivity occurs with an infinitesimal attractive interaction."

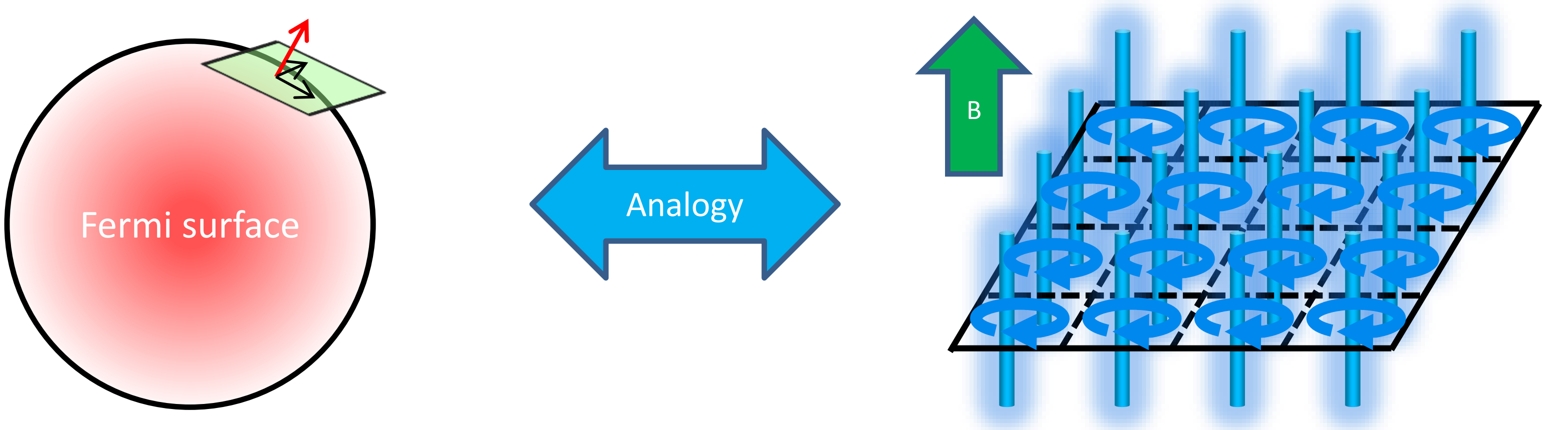
BCS (1957), Polchinski (1992)

Dimensional reduction and dynamical chiral symmetry breaking
by a magnetic field in 3 + 1 dimensions

V.P. Gusynin, V.A. Miransky, I.A. Shovkovy

On the other hand, the dynamics of the fermion pairing in a magnetic field in 3 + 1 dimensions is (1 + 1)-dimensional. We recall that, because of the Fermi surface, the dynamics of the electron pairing in BCS theory is also (1 + 1)-dimensional. This analogy is rather deep. In particular, the expression (20) for m_{dyn} can be

Key physics: "Dimensional reduction" near the Fermi surface and in strong B



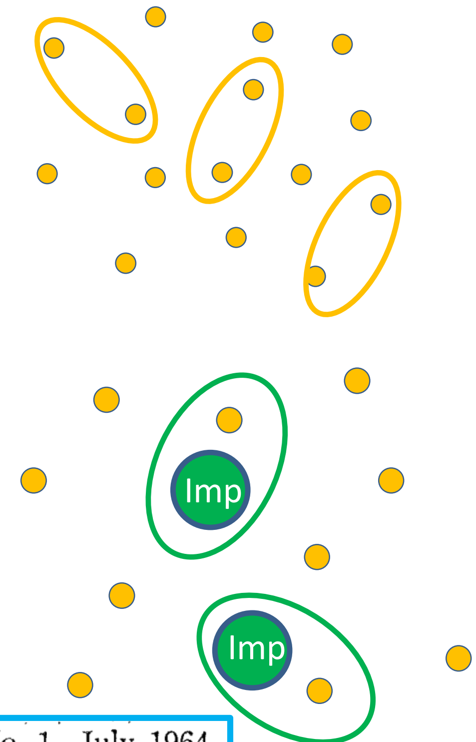
Degeneracy in 2 dim.

- On the Fermi surface (rotational of the surface)
- In the transverse plane in B (translation of the cyclotron motion)

⇒ Kinematics is only determined by the residual (1+1) dim. dynamics.

Bound states inevitably emerge in the effective (1+1) dim. → Cooper pairs (and thus independently of the coupling strength).

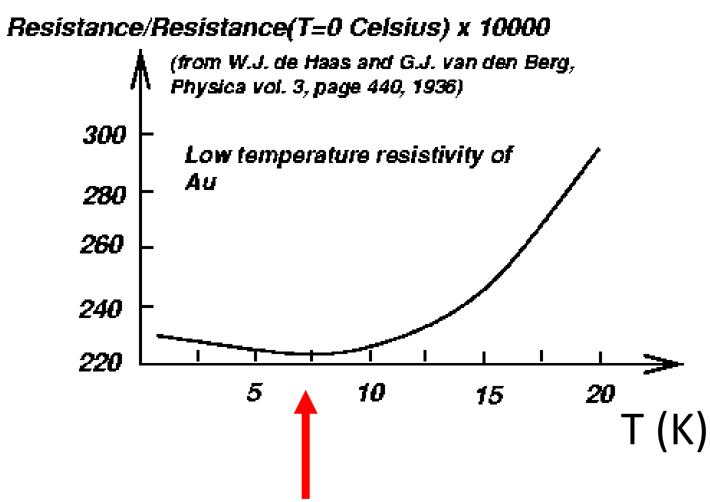
The Kondo effect



Pairing near the Fermi surface

- Copper pairing: electron-electron scattering
- Kondo condensate or Kondo cloud: Bound states in impurity scattering

The formation of the Kondo condensate traps conduction electrons near an impurity and induces an anomalous increase of resistivity in low T.



Progress of Theoretical Physics, Vol. 32, No. 1, July 1964

Resistance Minimum in Dilute Magnetic Alloys

Jun KONDO

The key process is the second-order spin exchange interactions.

- Fermi surface (dim. reduction)
- Quantum effect from loop diagrams
- Non-Abelian (Pauli) matrix

T_K: Kondo temperature (Location of the minimum)

Prof. Kondo passed away this March.

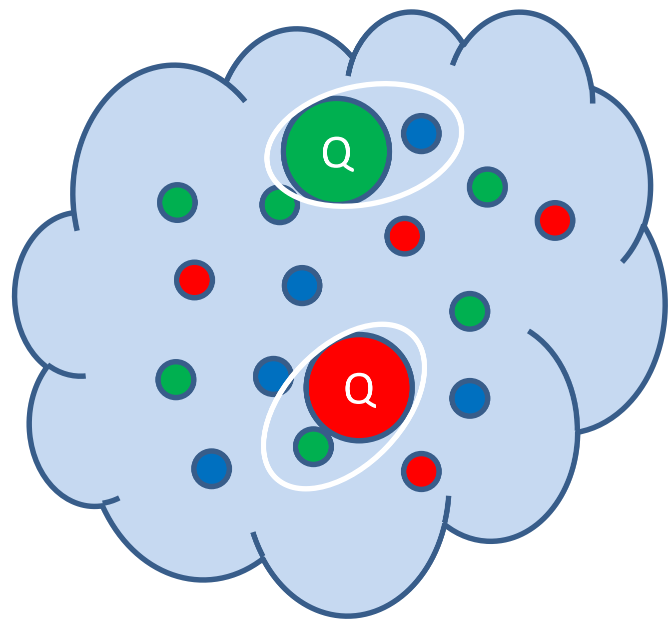
The QCD Kondo effect

Heavy-quark impurities in dense quark matter

- Color-exchange interactions with the Gell-mann Matrix

Yasui, Sudoh, 1301.6830
KH, Itakura, Ozaki, Yasui, 1504.07619

The QCD Kondo condensate
= Heavy-light pairing in quark matter



Analogy in strong B (w/o density):
"Magnetically induced Kondo effect"

Ozaki, Itakura, Kuramoto (2015)

More about the QCD Kondo effect

Finite density

- Renormalization-group analysis, KH, Itakura, Ozaki, Yasui (2015)
- Higher-loop effects on the fixed-point, Kanazawa, Uchino, (2016)
- Mapping to CFT near the fixed-point, Kimura, Ozaki (2017)

Conformal Field Theory

■ Phase diagram from mean-field analysis, Yasui, Suzuki, Itakura (2016)

Consequences and more exotic effects:

✓ The Kondo effect in the 2SC phase, KH, Huang, Pisarski (2019)
✓ The Kondo effect induced by the chirality imbalance, Suenaga, Suzuki, Araki, Yasui (2020)
✓ Catalysis of the chiral separation effect, Suenaga, Araki, Suzuki, Yasui (2021)
Cf. Two-color lattice study, Talk by Buividovich, "Hard Problems of Hadron Physics (2021)."
✓ HQ Spin polarization, Suenaga, Araki, Suzuki, Yasui (2022)

In magnetic fields

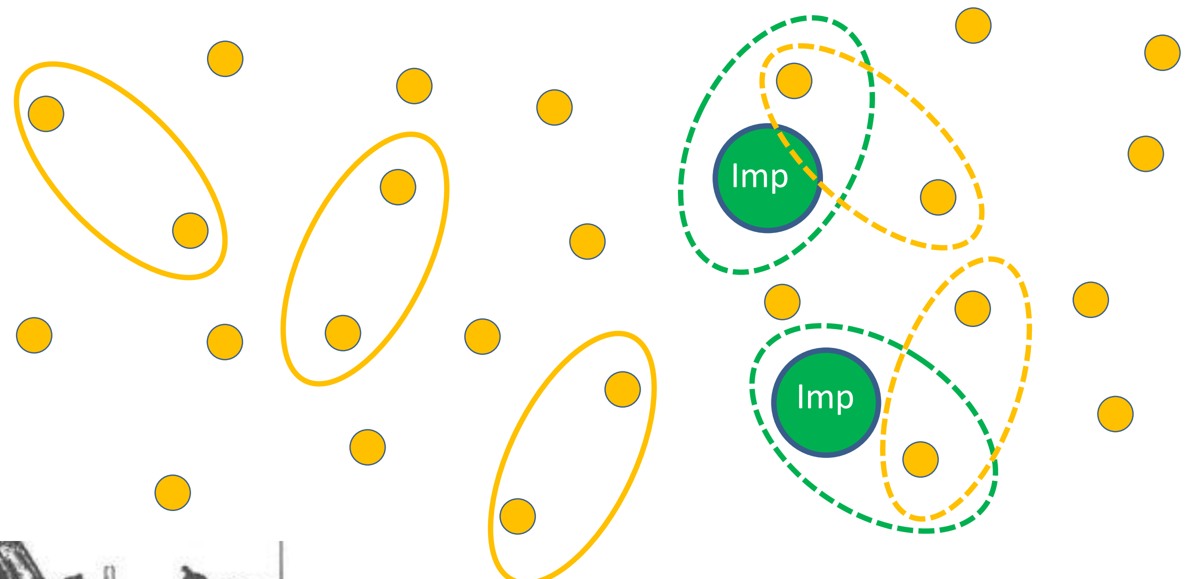
- RG analysis, Ozaki, Itakura, Kuramono (2015)
- Mapping to CFT, Kimura, Ozaki (2019)

Magnetic catalysis vs Magnetic Kondo effect

- Competition between the two strong statements.

Magnetic catalysis:
Inevitable formation of $< \bar{q}q >$.

The Kondo effect:
Inevitable formation of $< \bar{Q}q >$.



"Spear and shield" from an episode in ancient China:
A weapon merchant advertising his "strongest spear" and "strongest shield" in the world.

Cf. Competition at finite density, Suzuki, Yasui, Itakura (2017)

Model

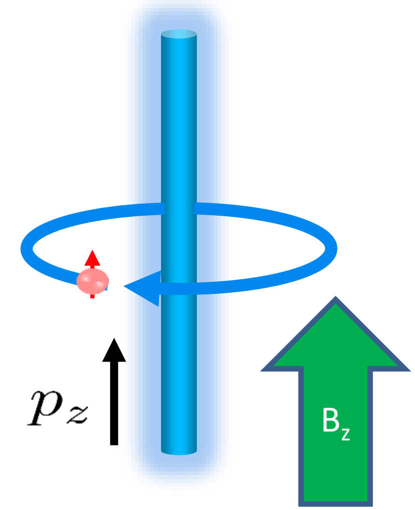
One light flavor ψ + One heavy flavor Ψ with N colors

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/_{\parallel} - m_l)\psi + \frac{G_{ll}}{2N}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2\right]$$

Light-quark kinetic term in B
(the lowest Landau level,
(1+1) dim. along B)

Light-light interactions

Landau quantization



$$+\sum_{c=\pm}\left[c\bar{\Psi}_v^c i\partial_0\Psi_v^c + \frac{G_{hl}}{N}\left\{(\bar{\psi}\Psi_v^c)(\bar{\Psi}_v^c\psi) + (\bar{\psi}i\gamma_5\Psi_v^c)(\bar{\Psi}_v^c i\gamma_5\psi)\right\}\right]$$

Heavy quark
+ heavy antiquark

Heavy-light interactions

Heavy-quark kinetic term in the LO heavy-quark
expansion (Infinite mass, no spin rotation).

Mean-field analysis

Chiral condensate

$$\langle\bar{\psi}\psi\rangle_{\mathrm{LLL}} \equiv -\frac{N}{G_{ll}}M,$$

Kondo condensate

$$\langle\bar{\psi}\Psi_v^{\pm}\rangle_{\mathrm{LLL}} \equiv \frac{N}{G_{hl}}\Delta$$

Color $\bar{3}$ Kondo condensate is suppressed in the large Nc, Yasui (2017).

■ Exactly diagonalization of the MF Lagrangian with M and Δ

Four eigenmodes:

$$E_{\pm}(p) \equiv \frac{1}{2}\left(E_p \pm \sqrt{E_p^2 + |2\Delta|^2}\right)$$

$$\tilde{E}_{\pm}(p) \equiv \frac{1}{2}\left(-E_p \pm \sqrt{E_p^2 + |2\Delta|^2}\right)$$

$$E_p \equiv \sqrt{p_z^2 + (m_l + M)^2}.$$

■ Thermodynamic potential

$$\tilde{\Omega}(M,\Delta) = \frac{1}{2\tilde{G}_{ll}}M^2 + \frac{1}{2\tilde{G}_{hl}}|2\Delta|^2 - \sum_i \int_{-\Lambda}^{\Lambda} \frac{dp_z}{2\pi}\left[\frac{1}{2}E_i + \frac{1}{\beta}\ln(1+e^{-\beta E_i})\right]$$

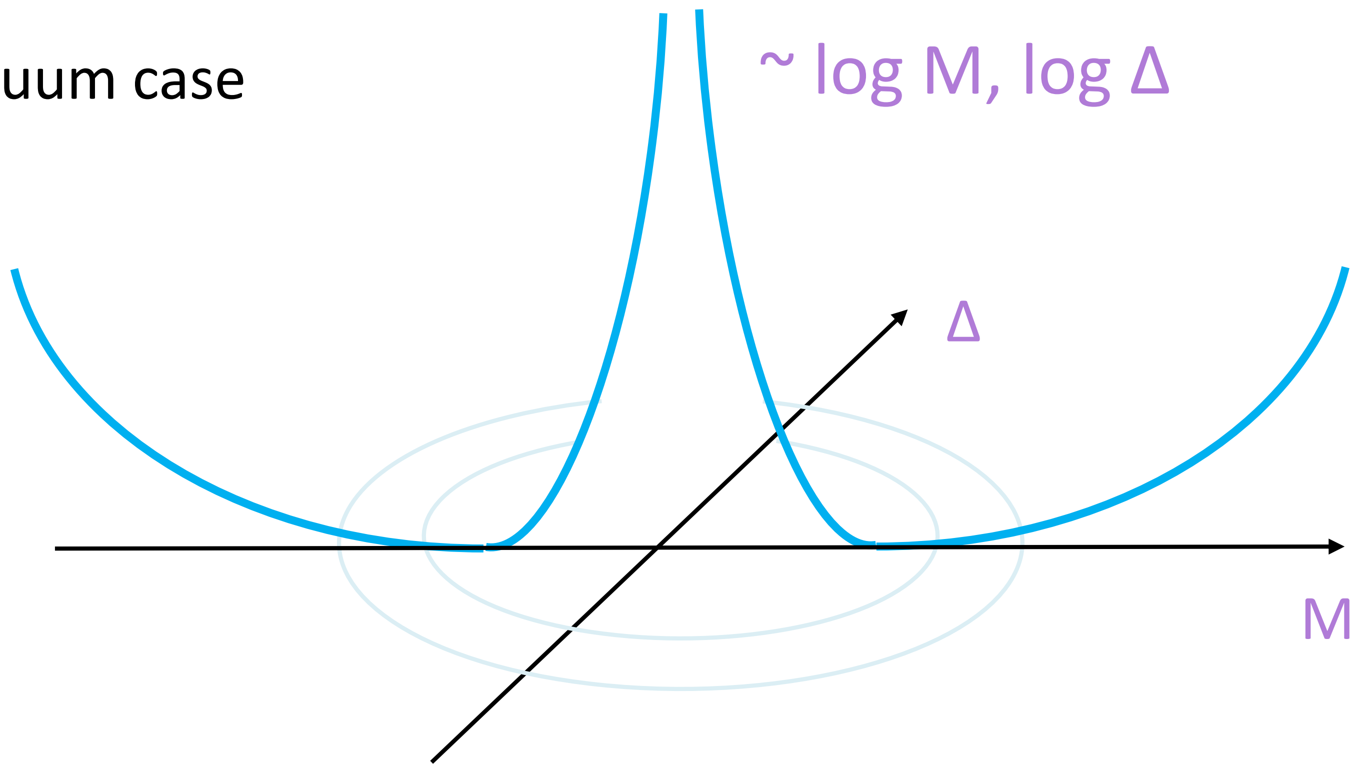
Vacuum (→ $\frac{1}{2}E_i$)

Finite T (→ $\frac{1}{\beta}\ln(1+e^{-\beta E_i})$)

Dim. reduction: Momentum integral only in the B direction.

“Log repulsion” at the origin of the potential

Vacuum case



Nonzero condensates are always favored in the (1+1) dim.

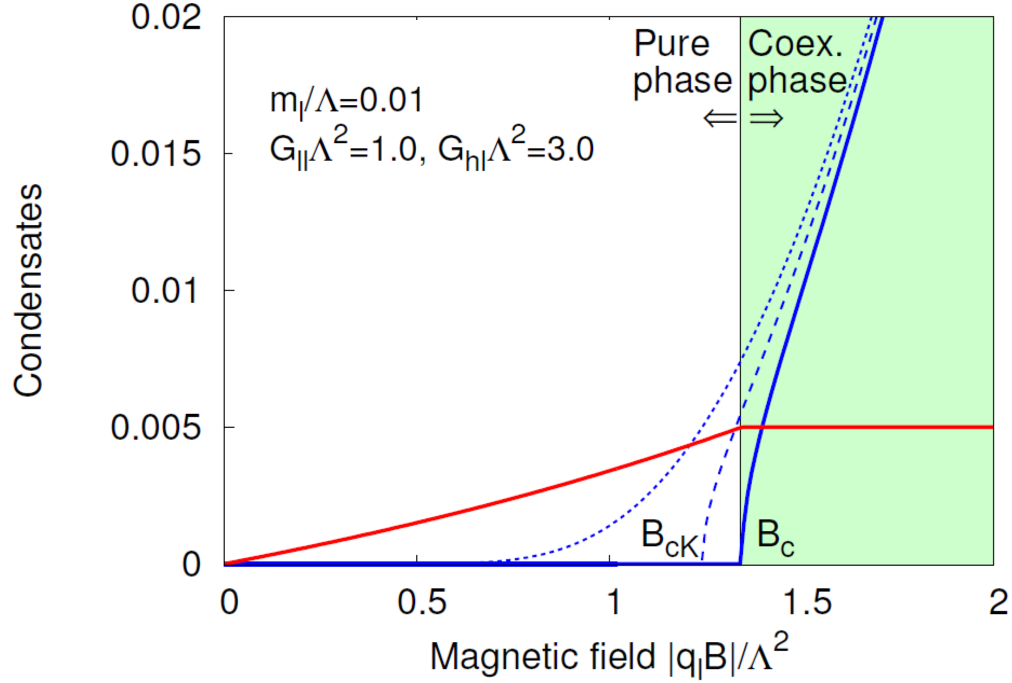
The log dependences only arise from the 1-dim. integral in Ω (previous slide).
- Different dependences in different dimensions.

Finite T contribution yields the same log with the opposite sign $\sim \log \frac{\Lambda_{\rm UV}}{\Lambda_{\rm IR}} \sim \log \frac{\Lambda_{\rm UV}}{M} - \log \frac{T}{M}$

→ Symmetry restoration without the singularity at the origin

Magnetic catalysis vs Kondo effect at zero T

- Results from numerical search of the global stationary point



- Chiral condensate M
- Kondo condensate Δ

Cf. Δ w/o competition (M=0)

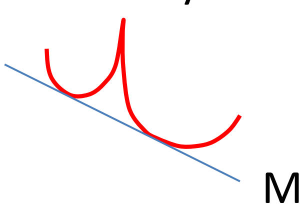
.... Massless limit

- - - - - - Current q mass

Saturation of the magnetic catalysis

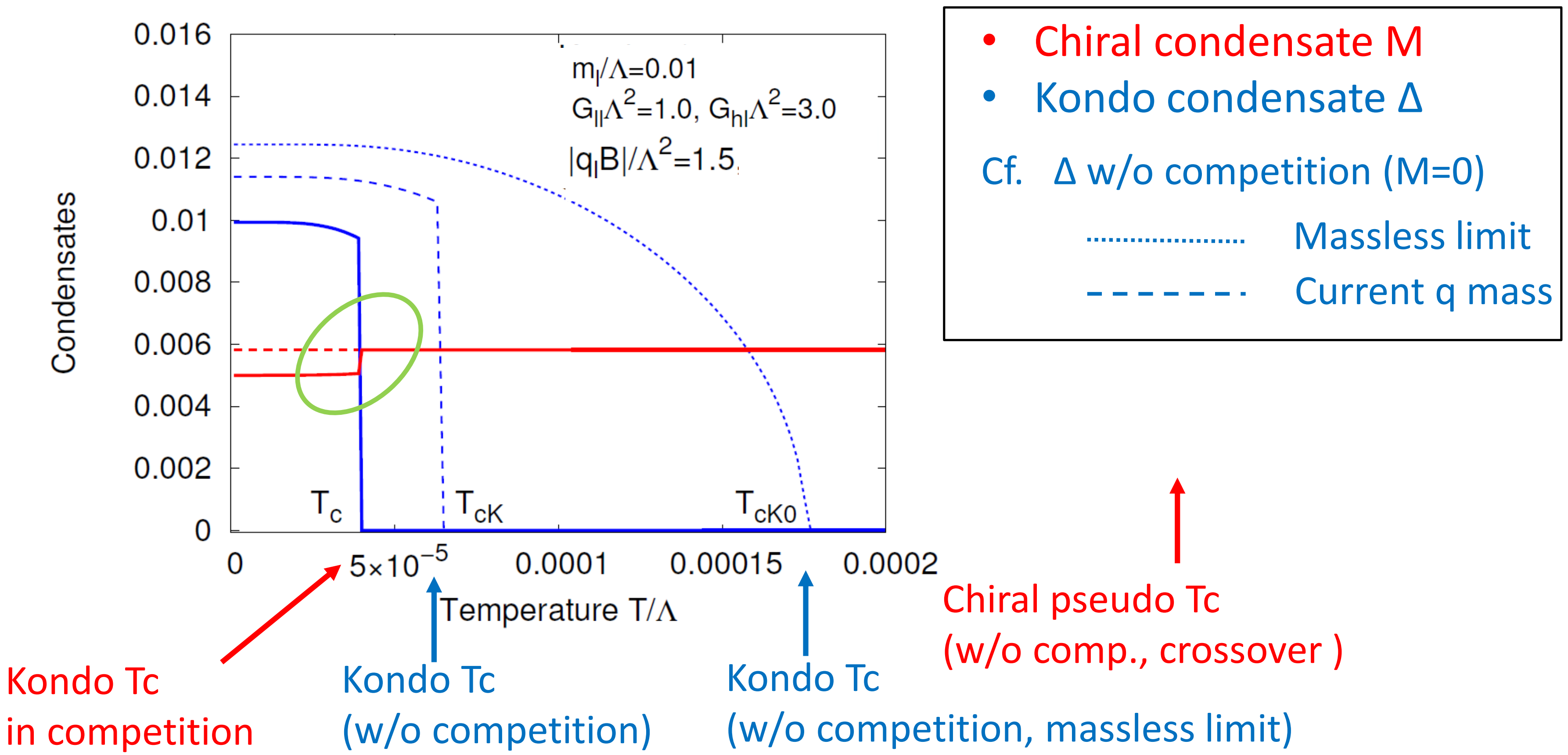
Chiral dominates Kondo due to the tilted potential by the current quark mass.

- Kondo cond. starts growing.
- Chiral cond. saturates at a critical B.



Analytic solution also available in the coexistence phase.

Magnetic catalysis vs Kondo effect at finite T



- Without the competition, the Kondo condensate melts earlier than the chiral condensate.

- The chiral condensate increases as we increase T just after the Kondo cond. melts away.
  → Signature of the end of competition

Summary

Potential signatures of the competition

- Saturation of the magnetic catalysis in strong B due to the growth of the Kondo condensate.
- Anomalous increase of the chiral condensate with T just after the end of competition.

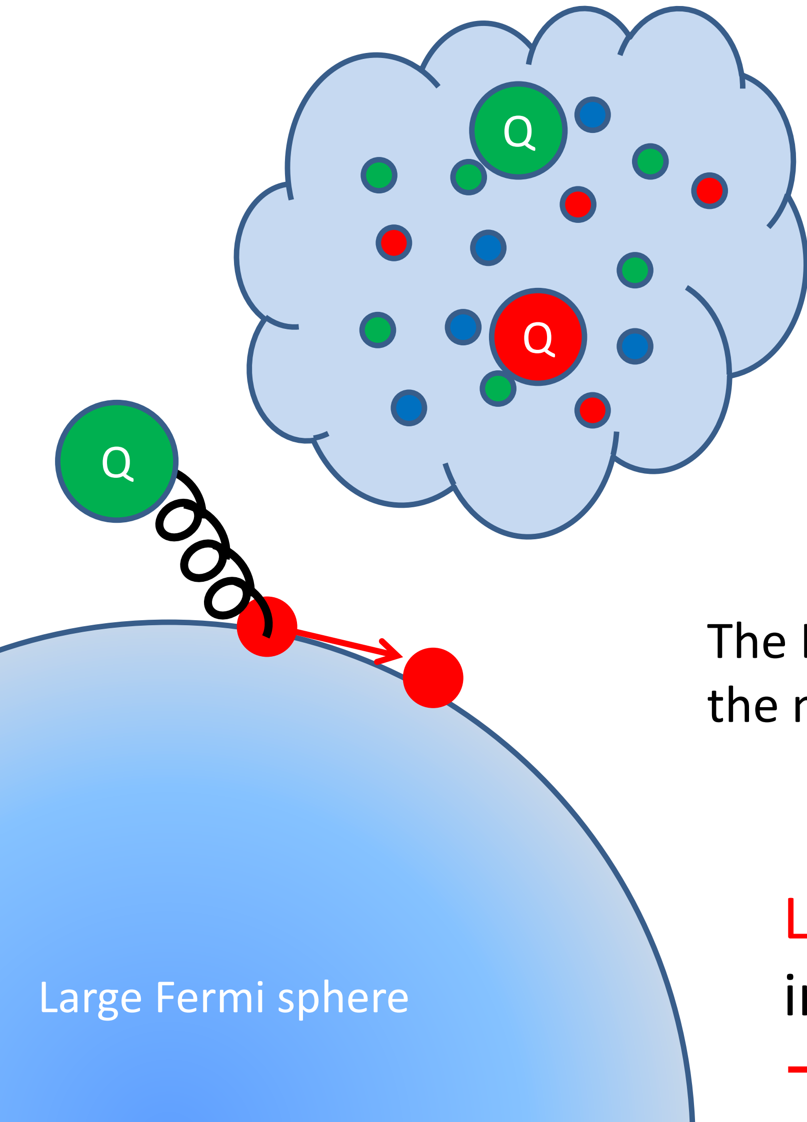
Outlook

- Study by lattice simulations
- Computation of the transport coefficients
- Applications to the Dirac quasi-particles in cond. matt. systems

Back-up slides

Impurity scatterings near a Fermi surface

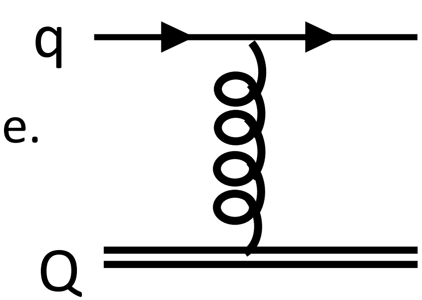
Heavy-quark impurity in light-quark matter



$$G(\Lambda)(\bar{\psi}\psi)(\bar{\Psi}\Psi)$$

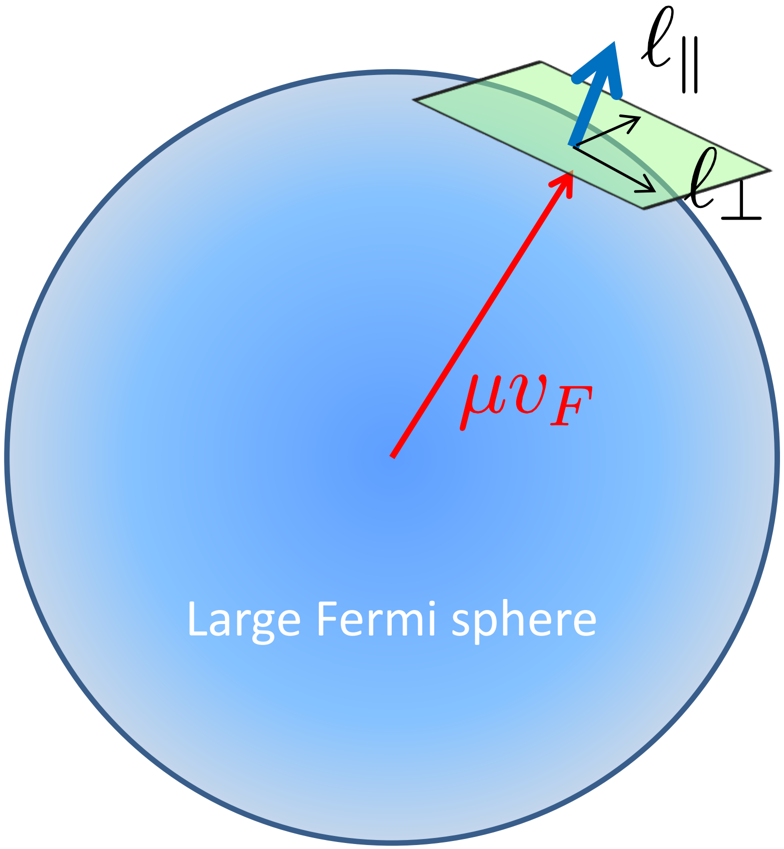
How does the coupling evolve in the IR regime, Λ --> 0?

The LO does not explain the minimum of the resistance.



Logarithmic quantum corrections arise in special kinematics and circumstances.
→ Kondo effect

"Dimensional reduction" in dense systems
-- (1+1)-dimensional low-energy effective theory



+ Low energy excitation along radius [(1+1) D]

$$\epsilon = \pm \ell_\parallel \quad (\ell_\parallel \ll \mu)$$

+ Degenerated states in the tangential plane [2D]

Phase space volume ~ p^{d-1} dp (No suppression for d=1).
Enhanced IR dynamics induces nonperturbative physics.

SC and Kondo effect occur for the dimensional reason, and no matter how weak the attraction is.

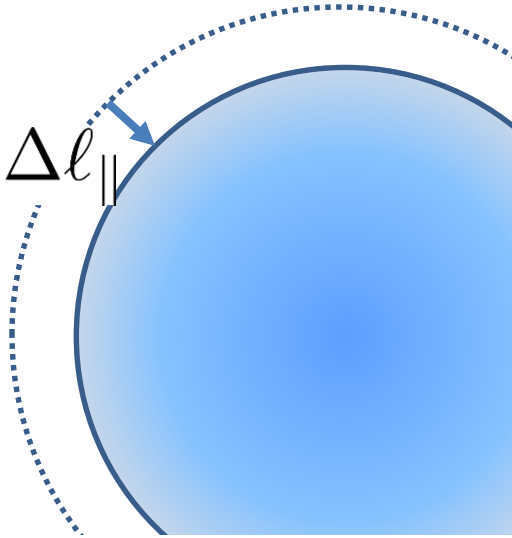
Scaling argument

Scaling dimensions in the IR

Evolution from UV to IR: $\epsilon \to \epsilon - \Delta\epsilon$

$$\ell_{\|} \to \ell_{\|} - \Delta\ell_{\|}$$
$$\ell_{\perp} = \ell_{\perp}$$

$\ell_{\perp}$: Label of the degenerated states (Does not scale)



Scaling dimension of ψ is determined from the kinetic term.

$$\mathcal{S}^{\text{kin}} = \int dt \sum_{\boldsymbol{v}_F} \int \frac{d^2\ell_{\perp} d\ell_{\|}}{(2\pi)^3} \bar{\psi}_{+}(i\partial_t - \ell_{\|})\gamma^0 \psi_{+} + \mathcal{O}(1/\mu)$$

$$0 = \underset{\bar{\psi}\cdot\psi}{2d_{\psi}} + \underset{dt}{(-1)} + \underset{d\ell_{\|}}{1} + \underset{\partial_t}{1} \quad \Rightarrow \quad d_{\psi} = -\frac{1}{2}$$

Spatial dimension = 1

IR scaling dimension for the Kondo effect

Heavy-light 4-Fermi operator

Light quark: $d_\psi = -1/2$
Heavy quark: $d_\Psi = 0$

$$S^{\rm int}_{\rm H-L} = \int dt \left[\int \frac{d^2\ell_\perp d\ell_\parallel}{(2\pi)^3} \right]^2 G[\bar{\psi}^{(3)}_+ t^a \psi^{(1)}_+][\bar{\Psi}^{(4)}_+ t^a \Psi^{(2)}_+]$$

Heavy-quark field (impurity) is a scattering center for light quarks (No scaling).

$$d_{(\psi\Psi)^2} = (-1) + 2(1 + d_\psi) + 2d_\Psi = 0$$

Marginal !! Let us proceed to diagrams.

IR scaling dimensions

$$\text{When } \epsilon \to s\epsilon,\ \ell_\parallel \to s\ell_\parallel. \quad (s<1)$$

Kinetic term

$$\mathcal{S}^{\text{kin}} = \int dt \sum_{\boldsymbol{v}_F} \int \frac{d^2\boldsymbol{\ell}_\perp d\ell_\parallel}{(2\pi)^3} \bar{\psi}_+ (i\partial_t - \ell_\parallel)\gamma^0 \psi_+ + \mathcal{O}(1/\mu)$$

$$0 = \underset{\bar{\psi}\cdot\psi}{2d_\psi} + \underset{dt}{(-1)} + \underset{d\ell_\parallel}{1} + \underset{\partial_t}{1}$$

$$d_\psi = -\frac{1}{2}$$

Four-Fermi operators for superconductivity

Polchinski (1992)

$$\mathcal{S}^{\text{int}} = \int dt \left[\int \frac{d^2\boldsymbol{\ell}_\perp d\ell_\parallel}{(2\pi)^3} \right]^4 G[\bar{\psi}_+^{(4)} \hat{\gamma}_\parallel^\mu \psi_+^{(2)}][\bar{\psi}_+^{(3)} \hat{\gamma}_\mu^\parallel \psi_+^{(1)}] \delta^{(3)}(\boldsymbol{p}^{(1)} + \boldsymbol{p}^{(2)} - \boldsymbol{p}^{(3)} - \boldsymbol{p}^{(4)})$$

In general momentum config.

$$p^{(1)} + p^{(2)} \sim \mu \qquad d_{4-\text{Fermi}} = (-1) + 4(1 - \frac{1}{2}) = +1$$

$$dt \qquad 4(d\ell_\parallel + d_\psi)$$

In the BCS config.

$$p^{(1)} + p^{(2)} \sim \ell_\parallel \ll \mu \qquad d_{4-\text{Fermi}} = (-1) + 4(1 - \frac{1}{2}) - 1 = 0$$

IR scaling dimension for the Kondo effect

Heavy-quark Kinetic term

$$S_H^{\rm kin} = \int dt \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\, \Psi_+^\dagger(\boldsymbol{k})\, i\partial_t \Psi_+(\boldsymbol{k}) + \mathcal{O}(1/m_H)$$

$$d_\Psi = (-1) + 1 = 0$$

Heavy-light four-Fermi operator

$$S_{\rm H-L}^{\rm int} \;=\; \int dt \left[\int \frac{d^2\boldsymbol{\ell}_\perp d\ell_\parallel}{(2\pi)^3}\right]^2 \left[\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\right]^2 G[\bar{\psi}_+^{(3)} t^a \psi_+^{(1)}][\bar{\Psi}_+^{(4)} t^a \Psi_+^{(2)}]$$

$$d_{\rm H-L} = (-1) + 2(1 + d_\psi) + 2d_\Psi = 0$$

Marginal !! Let us proceed to diagrams.

Scaling dimensions in the LLL

When $\epsilon_{\mathrm{LLL}} \to s\epsilon_{\mathrm{LLL}}$, $p_z \to s p_z$. ($\boldsymbol{p}_\perp$ does not scale.)

(1+1)-D dispersion relation → $d_\psi = -1/2$

Four-light-Fermi operator

$$\mathcal{S}^{\mathrm{int}} = \int dt \left[\int \frac{dp_z}{2\pi} \right]^4 G[\bar{\psi}^{(4)}_{\mathrm{LLL}} \hat{\gamma}^\mu_\parallel \psi^{(2)}_{\mathrm{LLL}}][\bar{\psi}^{(3)}_{\mathrm{LLL}} \hat{\gamma}^\parallel_\mu \psi^{(1)}_{\mathrm{LLL}}] \delta(p_z^{(1)} + p_z^{(2)} - p_z^{(3)} - p_z^{(4)})$$

Always marginal thanks to the dimensional reduction in the LLL.

→ Magnetic catalysis of chiral condensate.

Chiral symmetry breaking occurs even in QED.

Gusynin, Miransky, and Shovkovy. Lattice QCD data also available (Bali et al.).

Heavy-light four-Fermi operator

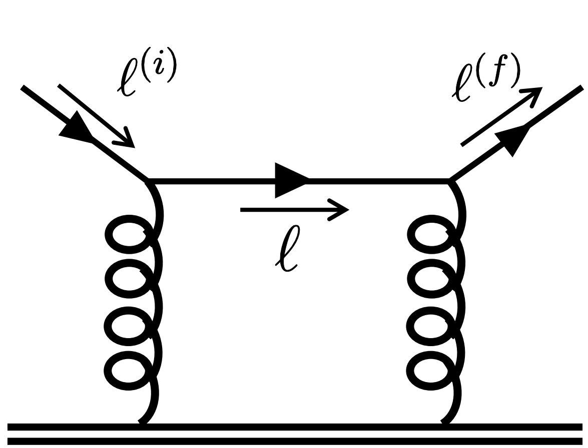
$$S^{\mathrm{int}}_{\mathrm{H-L}} = \int dt \left[\int \frac{dp_z}{2\pi} \right]^2 \left[\int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \right]^2 G[\bar{\psi}^{(3)}_{\mathrm{LLL}} t^a \psi^{(1)}_{\mathrm{LLL}}][\bar{\Psi}^{(4)}_{+} t^a \Psi^{(2)}_{+}]$$

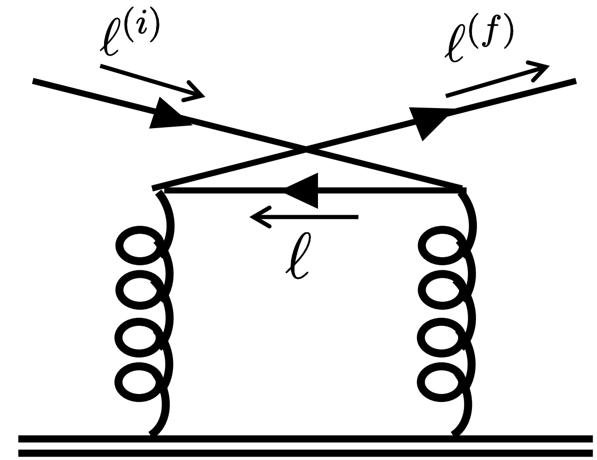
Marginal !! Just the same as in dense matter.

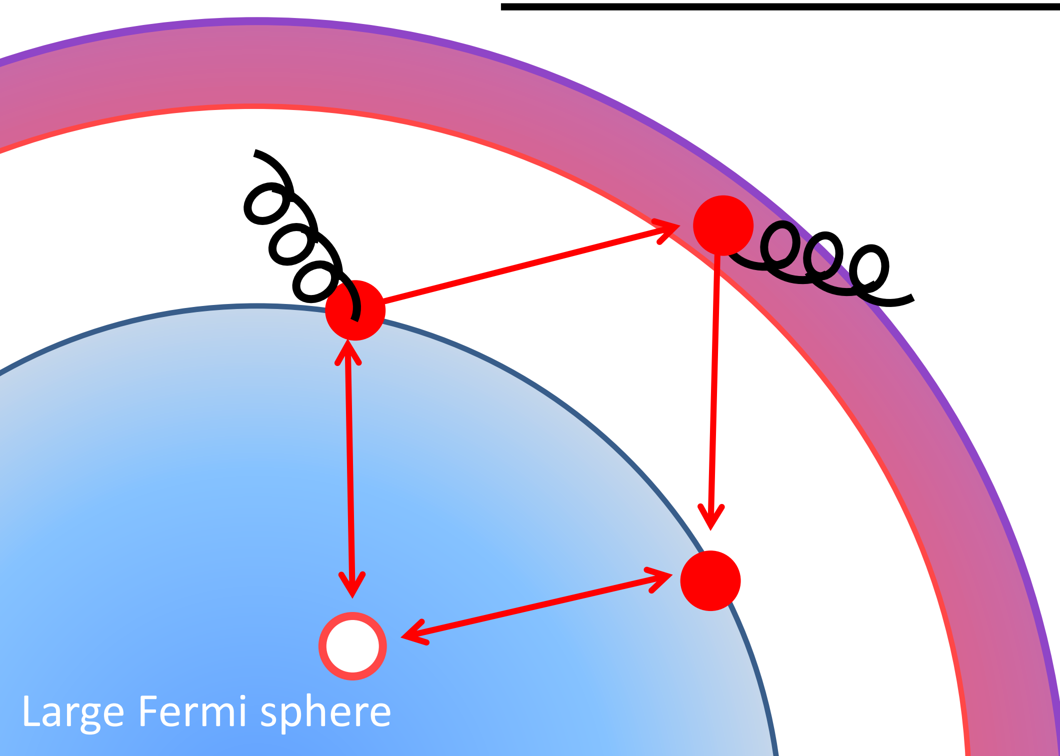
Logarithms from the NLO diagrams

The NLO scattering amplitudes
-- Renormalization in the low energy dynamics

$\mathcal{M} =$





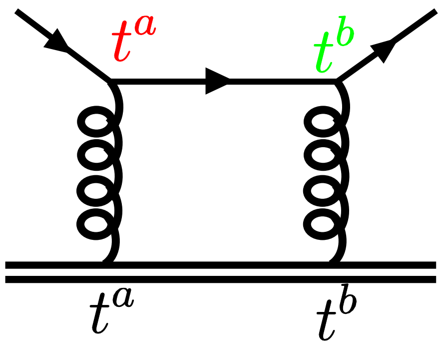


$$\int \frac{d\ell_{\parallel}}{\ell_{\parallel}} \sim \log \Lambda$$

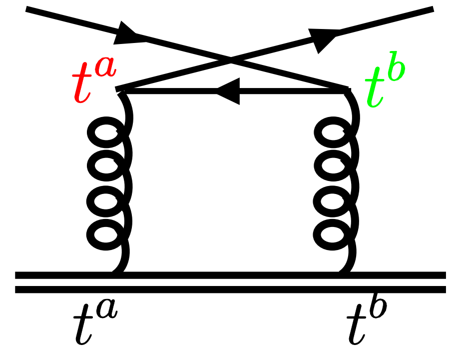
$$\int d^2\ell_{\perp} \sim \rho_F = \frac{\mu^2}{2\pi^2}$$

Density of states

Log correction and color-matrix structures



$$\bar{u}^k (t^b)^{kj} (t^a)^{ji} u^i$$



$$\bar{u}^k (t^a)^{kj} (t^b)^{ji} u^i$$

Particle contribution

$$\int_{\epsilon_F} \frac{d\epsilon}{\epsilon} \sim + \log \frac{\Lambda}{\Lambda - d\Lambda}$$

Hole contribution

$$\int^{\epsilon_F} \frac{d\epsilon}{\epsilon} \sim - \log \frac{\Lambda}{\Lambda - d\Lambda}$$

Logs corrections cancel each other in an Abelian theory (No net effect).

✔ *Incomplete cancellation* due to the color matrices

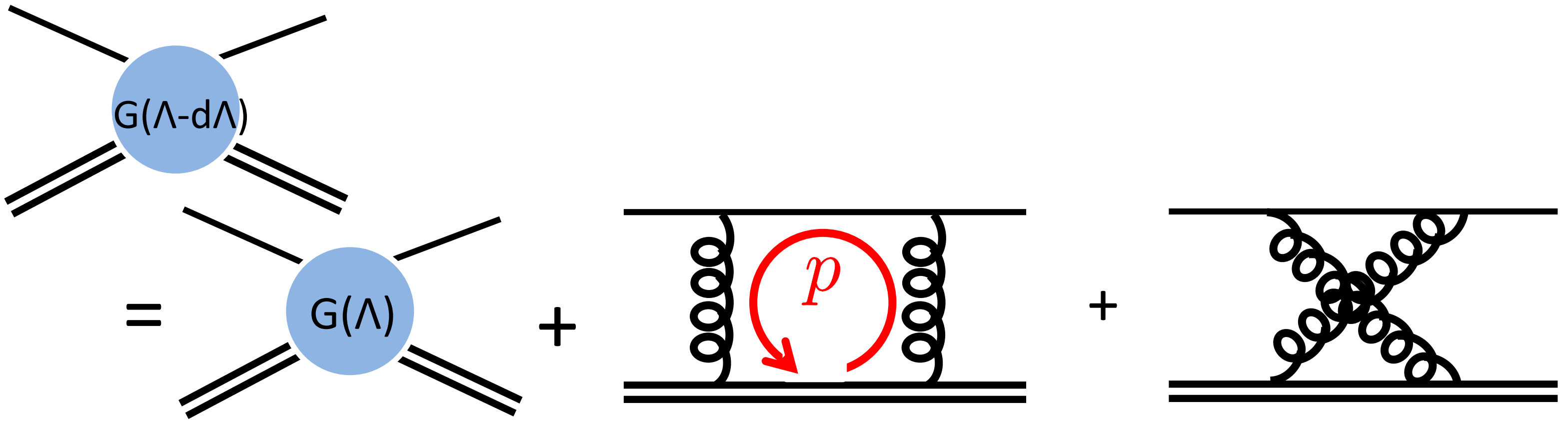
Particle contribution

$$[t^a t^b]_{ij} [t^a t^b]_{k\ell} = c\delta_{ij}\delta_{k\ell} - \frac{1}{n} t^c_{ij} t^c_{k\ell}$$

Hole contribution

$$[t^a t^b]_{ij} [t^b t^a]_{k\ell} = c\delta_{ij}\delta_{k\ell} - \frac{1}{n} t^c_{ij} t^c_{k\ell} + \frac{n}{2} t^c_{ij} t^c_{k\ell}$$

RG analysis for "the QCD Kondo effect"



$\int dp_{\|}/p_{\|} \sim \log \Lambda$

$\int d^2 p_{\perp} \sim \rho_F = \frac{\mu^2}{2\pi^2}$ (Density of states)

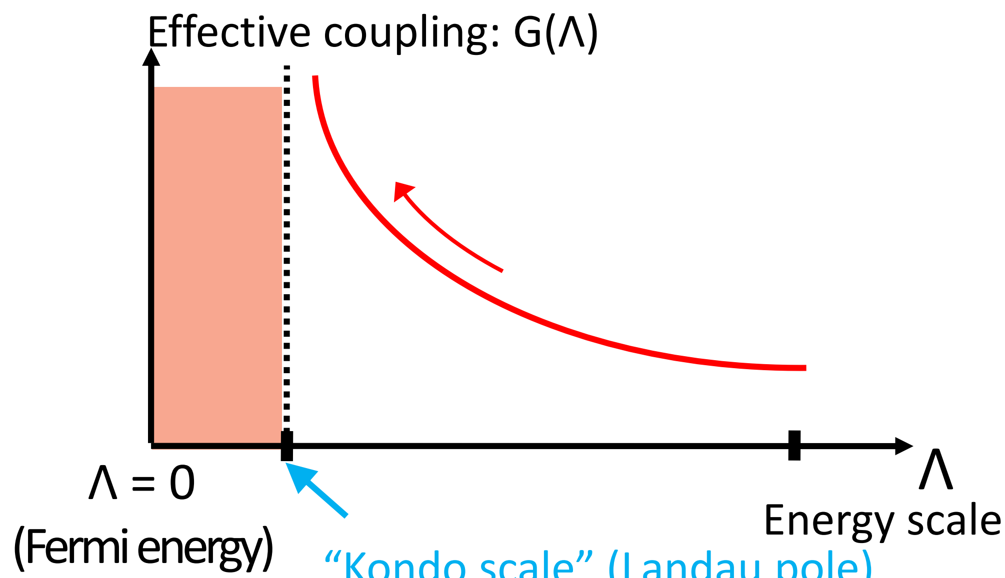
RG equation

$$\Lambda \frac{dG(\Lambda)}{d\Lambda} = -cN_c \cdot \rho_F \cdot G^2(\Lambda)$$

Landau pole in the asymptotic-free solution

$$\Lambda_K \sim \mu \exp(-\frac{c}{N_c g^2})$$

Depends on the interactions.
(Debye screening mass for $A^0 \rightarrow g^2$ dep.)



Resistivity is enhanced in the strong-coupling regime.

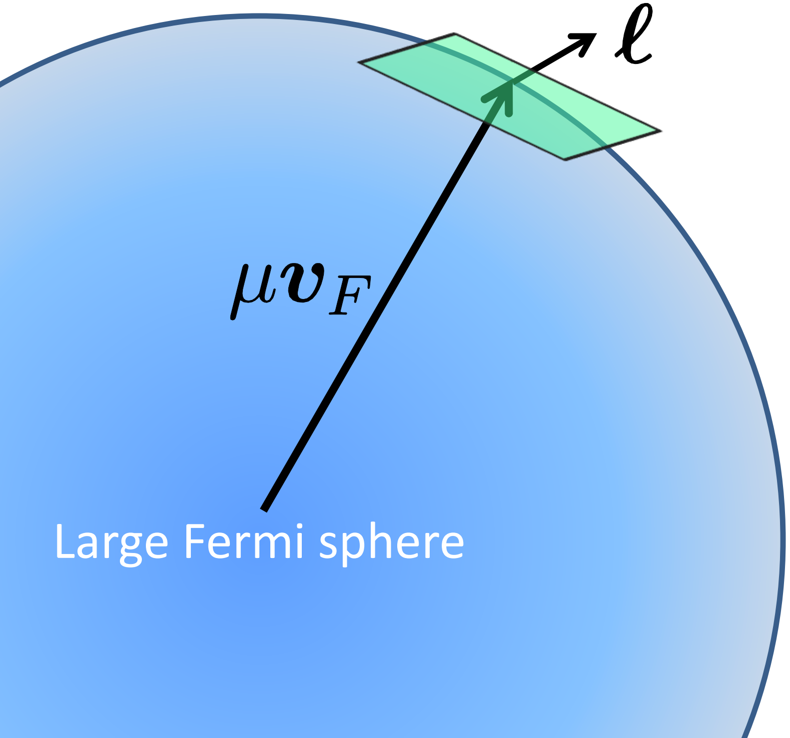
High-Density Effective Theory (LO)

Expansion around the large Fermi momentum

$$p^0 = \ell^0 , \quad \boldsymbol{p}^i = \mu \boldsymbol{v}_F^i + \boldsymbol{\ell}^i$$

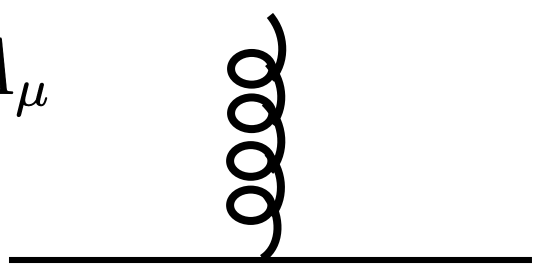
(1+1)-dimensional dispersion relation

$$\ell^0 = \boldsymbol{v}_F \cdot \boldsymbol{\ell} \equiv \ell_{\parallel}$$



Spin flip suppressed
when the mass is small m << μ.

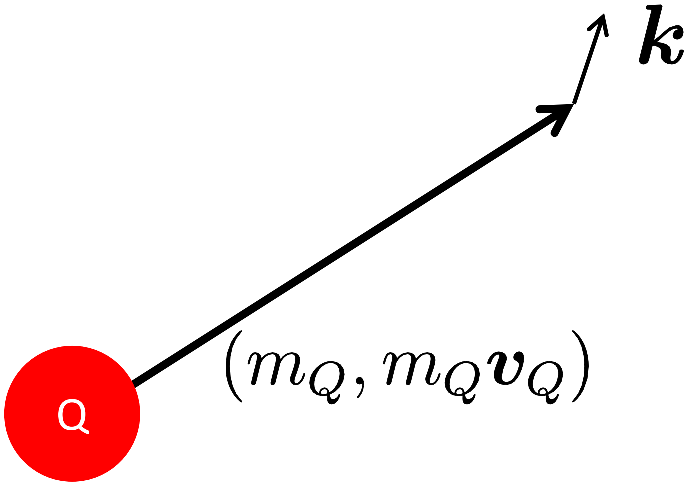
$$\gamma^\mu A_\mu \to \gamma^0 v_F^\mu A_\mu$$



Heavy-Quark Effective Theory (LO)

HQ-momentum decomposition

$$p^\mu = m_Q v_Q^\mu + k^\mu$$



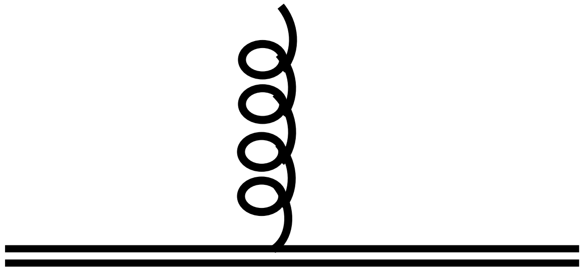
HQ velocity

$$v_Q^\mu = \frac{1}{m_Q} P^\mu \Big|_{P^2 = m_Q^2}$$

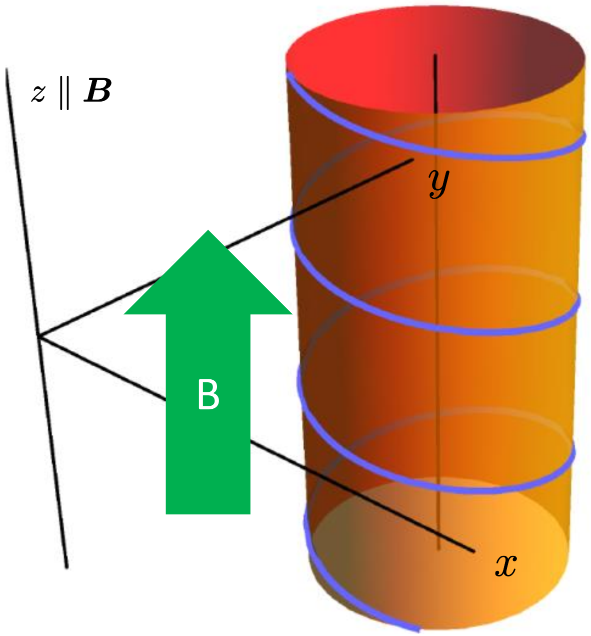
Nonrelativistic magnetic moment suppressed by $1/m_Q$

$$\gamma^\mu A_\mu \to v_Q^\mu A_\mu$$

$$\gamma^\mu A_\mu = A^0 \text{ when } \vec{\boldsymbol{v}}_Q = 0.$$



Landau level discretization due to the cyclotron motion



"Harmonic oscillator" in the transverse plane

Nonrelativistic: $\epsilon_n = \frac{p_z^2}{2m} + (n + \frac{1}{2})\frac{eB}{m}$

Cyclotron frequency

Relativistic: $\epsilon_n = \sqrt{p_z^2 + (2n+1)eB + m^2}$

In addition, there is the Zeeman effect.