HEAVY-FLAVOR ANISOTROPIC FLOW AT RHIC AND LHC ENERGIES WITHIN A FULL TRANSPORT APPROACH

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few Heavy-Flavor (HF) quarks and antiquarks
CHARM and BOTTOM
produced in relativistic heavy-ion collisions

- $m_{\text{HQ}} \gg \Lambda_{\text{QCD}} \rightarrow$ HQ produced in pQCD initial hard scatterings
- $m_{\text{HQ}} \gg T_{\text{HICs}} \rightarrow$ negligible thermal production of HQs

HQ production points symmetric in the forward-backward hemispheres

- $\tau_0^{\text{HQ}} < 0.08 \text{ fm}/c \ll \tau_0^{\text{QGP}} \rightarrow$ HQ production much earlier than QGP formation
- $\tau_{\text{th}}^{\text{HQ}} \approx \tau_{\text{th}}^{\text{QGP}} \approx 5-10 \text{ fm}/c \gg \tau_{\text{th}}^{\text{QGP}} \rightarrow$ HQ thermalization time comparable to QGP life

HQ final states keep a better memory of both initial stage and QGP evolution

- $q < m_{\text{HQ}}, p_{\text{HQ}}; m_{\text{HQ}} \ll g T_{\text{HICs}} (b \text{ or low momentum } c) \rightarrow$ Brownian motion of HQs in QGP
HEAVY FLAVORS IN RELATIVISTIC HICS

INITIAL PRODUCTION
- pQCD-NLO
- CNM effects
- initial-state fluctuations

DYNAMICS IN QGP
- Boltzmann/Fokker-Planck
- thermalization
- transport coefficients
- collisional & radiative

HADRONIZATION
- recombination and/or fragmentation
- heavy-flavor hadrochemistry

IMPRESSIONS OF STRONG FIELDS
- vorticity
- electromagnetic field
- Glasma phase

INITIAL ENERGY DENSITY
- QGP phase
- HG phase
- Final detected particle distributions
HEAVY FLAVORS IN RELATIVISTIC HICS

IMPACT OF STRONG FIELDS
- vorticity
- electromagnetic field
- Glasma phase

INITIAL PRODUCTION
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HF anisotropic flow coefficients
\[ v_n = \langle \cos(n(\phi - \Psi_r)) \rangle \]

\[ \frac{d^3N}{d^3p} = \frac{1}{2\pi} \int \frac{d^2N}{p_T dp_T dy} \left(1 + 2 \sum_{n=1}^{\infty} v_n(p_T, y) \cos(n(\phi - \Psi_r))\right) \]

\( v_1, v_2, v_3 \)

\( v_2, v_3, v_4 \)

\( \tau \sim 0 \text{ fm/c} \)
\( \tau \sim 1 \text{ fm/c} \)
\( \tau \sim 10 \text{ fm/c} \)
\( \tau \sim 10^{15} \text{ fm/c} \)

Modified from picture of Chun Shen (McGill University, Montreal QC, Canada)
FULL BOLTZMANN TRANSPORT APPROACH

BULK EVOLUTION

\[ p^\mu \partial_\mu f_q(x, p) + m(x) \partial_x m(x) \partial_\mu f_q(x, p) = C[f_q, f_g] \]
\[ p^\mu \partial_\mu f_g(x, p) + m(x) \partial_x m(x) \partial_\mu f_g(x, p) = C[f_q, f_g] \]

Free-streaming Field interaction collision kernel gauged to some \( \eta/s \neq 0 \)

HQ EVOLUTION

\[ p^\mu \partial_\mu f_Q(x, p) = C[f_q, f_g, f_Q](x, p) \]

Non perturbative dynamics:
Extended \( M \) scattering matrices \((q,g \rightarrow Q)\) evaluated by Quasi-Particle Model
fit to IQCD thermodynamics


Boltzmann transport equivalent to viscous hydro at \( \eta/s \approx 0.1 \)


extension of the QPM (e.g., momentum dependence)

talk of M. L. SAMBATARO
FULL BOLTZMANN TRANSPORT APPROACH

**BULK EVOLUTION**

\[
p^{\mu} \partial_{\mu} f_{q}(x, p) + m(x) \partial_{x} m(x) \partial_{p}^\mu f_{q}(x, p) = C[f_{q}, f_{g}]
\]

\[
p^{\mu} \partial_{\mu} f_{g}(x, p) + m(x) \partial_{x} m(x) \partial_{p}^\mu f_{g}(x, p) = C[f_{q}, f_{g}]
\]

Free-streaming Field interaction collision kernel gauged to some \( \eta/s \neq 0 \)

\( \varepsilon - 3p \neq 0 \)

**HQ EVOLUTION**

\[
p^{\mu} \partial_{\mu} f_{Q}(x, p) = C[f_{q}, f_{g}, f_{Q}](x, p)
\]

Boltzmann transport equivalent to viscous hydro at \( \eta/s \approx 0.1 \)

[Image of Boltzmann transport equivalent to viscous hydro]

**Hybrid hadronization scheme for heavy quarks**

**COALESCEENCE + FRAGMENTATION**

\[
\frac{dN_{\text{Hadron}}}{d^2 p_T} = g_H \int \prod_{i=1}^{n} p_i d\sigma_i \frac{d^3 p_i}{(2\pi)^3} f_q(x_i, p_i) f_W(x_1, \ldots, x_n; p_1, \ldots, p_n) \delta(p_T - \sum_i p_{iT})
\]

\[
\frac{dN_{h}}{d^2 p_h} = \sum_f \int dz \frac{dN_f}{d^2 p_f} D_f \to_h(z)
\]

[Image of hybrid hadronization scheme]


Remarkable impact of coalescence for the description of experimental data (e.g., \( R_{AA}, v_2, v_3 \) for \( D \) mesons, charmed baryon/meson enhancement in pp)

**talk of V. MINISSALE**
INTENSE FIELDS AND HEAVY FLAVOR TRANSPORT

✓ HUGE ANGULAR MOMENTUM GENERATING A STRONG VORTICITY

- tornado cores $\sim 10^{-1} \, s^{-1}$
- Jupiter's spot $\sim 10^{-4} \, s^{-1}$
- He nanodroplets $\sim 10^{7} \, s^{-1}$
- urHICs $\sim 10^{22} - 10^{23} \, s^{-1}$

✓ INTENSE ELECTROMAGNETIC FIELDS

- Earth's field $\sim 1 \, G$
- laboratory $\sim 10^{6} \, G$
- magnetars $\sim 10^{14} - 10^{18} \, G$
- urHICs $\sim 10^{18} - 10^{19} \, G$

Impact on HQ transport coefficients and $D$ meson directed flow

Vorticity $\omega$

Impact on $D$ meson directed flow

Since 2017

Since 2016
Huge **orbital angular momentum** of the colliding system ➢ in ultra-relativistic HICs $J \approx 10^5 - 10^6 \hbar$
➢ dominated by the $y$ component perpendicular to the reaction plane
➢ partly transferred to the plasma


**asymmetry in local participant density from forward and backward going nuclei**

\[
\rho(x_\perp, \eta_s) = \rho_0 \frac{W(x_\perp, \eta_s)}{W(0, 0)} \exp \left[ - \frac{(|\eta_s| - \eta_0)^2}{2\sigma_\eta^2} \right] \\
W(x_\perp, \eta_s) = 2(N_A(x_\perp)f_-(\eta_s) + N_B(x_\perp)f_+(\eta_s))
\]

\[
f_+(\eta_s) = f_-(-\eta_s) = \begin{cases} 
0 & \eta_s < -\eta_m \\
\frac{\eta_s + \eta_m}{2\eta_m} & -\eta_m \leq \eta_s \leq \eta_m \\
1 & \eta_s > \eta_m
\end{cases}
\]

**TILTED FIREBALL**
on the reaction plane

L. Oliva, S. Plumari and V. Greco, JHEP 05, 034 (2021)

huge vorticity in agreement with $\Lambda$ polarization studies

**TILTED FIREBALL**
on the reaction plane

negative slope of charged particle $v_1(\eta)$
Excellent qualitative prediction with LangevinV approach
\( dv_1^D/dy \approx 0.02-0.04 \) (\( \approx 10-15 \) times larger than light charged)

**RHIC ENERGY**

**EXP:** \( dv_1^D/dy = -0.080 \pm 0.017 \) (stat)\( \pm 0.016 \) (syst)
about 30 times larger than that of kaons

**TH:** \( dv_1^D/dy = -0.065 \) (25-30 times larger than ch.)
relativistic BM equations for both QGP and HQs

the slope of \( \langle v_1^D \rangle \) is \( \sim 50 \) times smaller than that at RHIC
(in line with model predictions) and is consistent with 0


**LHC ENERGY**

\( \langle v_1^D \rangle \)

\( \langle v_1^D \rangle \approx 0.02-0.04 \) (\( \approx 10-15 \) times larger than light charged)

L. Oliva, S. Plumari and V. Greco, JHEP 05, 034 (2021)
ORIGIN OF D-MESON DIRECTED FLOW

\( v_1 (HQs) \gg v_1 (QGP) \)

origin of the large directed flow of HQs different from the one of light particles

longitudinal asymmetry leads to pressure push of the bulk on the HQs

L. Oliva, S. Plumari and V. Greco, JHEP 05, 034 (2021)
**ORIGIN OF D-MESON DIRECTED FLOW**

\[ v_1 (HQs) \gg v_1 (QGP) \]

origin of the large directed flow of HQs different from the one of light particles

longitudinal asymmetry leads to pressure push of the bulk on the HQs

effective because the HQ interaction in QGP is largely non-perturbative

strict connection between the magnitude of the D-meson \( v_1 \) and the HQ diffusion coefficient

L. Oliva, S. Plumari and V. Greco, JHEP 05, 034 (2021)


Similar conclusions with POWLANG approach
A. Beraudo et al., JHEP 05, 279 (2021)
Huge magnetic field in the overlap area up to $eB \approx 5-50 \text{ m}_\pi^2$

- mainly produced by spectators protons
- dominated by the y component
- intense electric field generated by Faraday induction
- charged currents induced in the conducting QGP generates a magnetic field pointing towards the initial one

external charge and current produced by a point-like charge in longitudinal motion

\[ J_{\text{ind}} = \sigma_{\text{el}} E \]

Maxwell equations can be solved analytically for a medium with constant electric conductivity


BM eq. with EM interaction term

\[ p^\mu \partial_\mu f(x, p) + q F_{\text{ext}}^{\mu\nu} p_\nu \partial_\mu f(x, p) = C[f] \]
The huge EM fields induce a splitting in the DIRECTED FLOW of particles with the same mass and opposite charge.

\[ \Delta v_1 = v_1^+ - v_1^- \]

- **\( \Delta v_1 \) of light hadrons in AA: \( O(10^{-4} - 10^{-3}) \)**
- **\( \Delta v_1 \) of heavy mesons in AA: \( O(10^{-2}) \)**
- **\( \Delta v_1 \) of light mesons in pA: \( O(10^{-2}) \)**

**Reviews**

The electromagnetic fields induce a large splitting in the directed flow of HEAVY QUARKS.

\[ \Delta v_1(D) = v_1(D^0) - v_1(\bar{D}^0) \]

**DIRECTED FLOW IN A+A AT RHIC ENERGY**

**DIRECTED FLOW OF NEUTRAL D MESONS**

**RHIC ENERGY**

\[ \frac{d(\Delta v_1)}{dy} \big|_{\exp} = -0.011 \pm 0.024 \text{(stat)} \pm 0.016 \text{(syst)} \]

\[ \frac{d(\Delta v_1)}{dy} \big|_{\text{th}} = -0.01 \]

\[ \approx 10 \text{ times larger than charged} \]

in agreement with


**SLOPE TIME EVOLUTION**

\[ \Delta v_1(D) \text{ more sensitive to the early QGP evolution when } T \text{ is higher,} \]

while \[ \Delta v_2(D) \text{ probes more } T \sim T_c \]

\[ \rightarrow \text{ include } v_1(D) \text{ in Bayesian fits} \]

\[ \Delta v_1 \text{ (HQ) } \gg \Delta v_1 \text{ (QGP)} \]

**charm quarks are more sensitive to the EM fields due to the early production**

Exp. data: STAR Coll., PRL. 123 (2019) 162301
**EVENT-BY-EVENT FLUCTUATIONS**

Event-by-event fluctuations in the initial nucleon positions

nth-order spatial eccentricities

\[ e_n = \frac{r_n}{r_\perp} \cos[n(n\phi - \Psi_n)] \]

\[ \Psi_n = \frac{1}{n} \arctan \left( \frac{r_n}{r_\perp} \sin(n\phi) \right) \]

\[ r_\perp = \sqrt{x^2 + y^2} \quad \phi = \arctan(y/x) \]

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**D-MESON \( v_2 \) AND \( v_3 \)**

- \( v_2 \) larger in more peripheral collisions
  - mainly generated by the geometry of overlap region
- \( v_3 \) not much sensitive to the collision centrality
  - mainly driven by the fluctuations of the triangularity of overlap region
- coalescence increases \( v_2 \) and \( v_3 \) at \( p_T > 2 \text{ GeV} \)

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**Figures**

- **Left Diagram:** Elliptic and triangular flow formulas.
- **Right Diagram:** Comparison of experimental data (ALICE Coll.) with theoretical predictions for \( v_2 \) and \( v_3 \) at different collision energies.

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**References**

- M. L. Sambataro et al., 2206.03160

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**ESE technique**: selection of events with **same centrality** but **different average bulk flow** on the basis of the magnitude of the $2^\circ$-order harmonic reduced flow vector $q_2$

\[
q_2 = \frac{|\vec{Q}_2|}{\sqrt{M}} = \sum_{j=1}^{M} e^{i2q_j}
\]

- small $q_2$
- large $q_2$

Increasing interest on studying observables with multi-differential methods based on event shape

- Transverse spherocity analysis in small systems

L. Oliva, W. Fan, P. Moreau, S.A. Bass and E. Bratkovskaya, 220404194

- Anti-correlation between $\varepsilon_2$ and $\varepsilon_3$
- Non-linear correlation between $\varepsilon_2$ and $\varepsilon_4$

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**SPATIAL ECCENTRICITIES**

- Pb+Pb@2.76 TeV (30-50)%
ESE: $v_n - v_m$ CORRELATIONS

Correlations between $\epsilon_n$ and $\epsilon_m$ in the initial geometry leads to correlations between $v_n$ and $v_m$

- Good description of experimental data for charged particles
- Prediction of comparable $D$-meson correlations w.r.t. bulk

$\nu_n - \nu_m$ CORRELATIONS


SYMmetric CUMULANT CORRELATOR

Same approach and $D_s(T)$ describing $R_{AA}(p_T) & v_2(p_T)$

M. L. Sambataro et al., 2206.03160
**ESE: $q_2$-SELECTED ELLIPTIC FLOW**


$q_2$-SELECTED $D$-MESON $v_2(p_T)$

$v_2$ (small $q_2$) < $v_2$ (unbiased)

$v_2$ (large $q_2$) > $v_2$ (unbiased)

about 50% difference between $q_2$-selected and unbiased events in both centrality class with no transverse momentum dependence

⇒ ESE selection related to a global property of the events

**D-MESON $v_2(p_T)$ RATIO**

M. L. Sambataro et al., 2206.03160
EXTENSION TO BOTTOM DYNAMICS

CHARM vs BOTTOM FLOW COEFFICIENTS

- Good description of $R_{AA}$ and $v_2$ of electrons from semileptonic $B$ decay
- Prediction of $R_{AA}$ and $v_2$ for $B$ meson

indication for a strong coupling of bottom quarks with collectively expanding bulk

M.L. Sambataro et al., in preparation

NUCLEAR MODIFICATION FACTOR

Exp. data from talk of R. Arnaldi at HP2020

ELLiptic FLOW
Results from $R_{AA}(p_T)$ and $v_2(p_T)$ of B mesons

- $D_s$ is ideally $M$ independent ($M \to \infty$) since from kinetic theory:
  \[
  \frac{\tau_{th}^b}{\tau_{th}^c} \approx \frac{\gamma_c}{\gamma_b} \approx \frac{M_b}{M_c}
  \]

- $D_s$ is a measure of thermalization time:
  \[
  \tau_{th} \approx 1.3 \frac{M}{2\pi T D_s (T/T_c)^2} \text{ fm/c}
  \]

In QPM approach $D_s(c)$ is 30-40% larger than $D_s(b)$

Bottom quarks expected to be fully thermalized @ FCC

M.L. Sambataro et al., in preparation

CHARM vs BOTTOM SPATIAL DIFFUSION COEFFICIENT
Full Boltzmann transport approaches for the description of HQ dynamics in relativistic heavy-ion collisions

QPM for non-perturbative HQ interaction in QGP
coalescence plus fragmentation hadronization scheme
initial-state fluctuations
electromagnetic and vortical fields

✓ The $D^0$-meson $v_1$ gives information on the transport properties of the hot QCD matter: magnitude associated with the HQ diffusion coefficient and splitting connected to the QGP electric conductivity

✓ For $D$ mesons $v_1$ is more sensitive to the early QGP evolution when $T$ is higher, while $v_2$ probes more $T\sim T_c$. Inclusion in Bayesian fit and for $D_s(T)$ estimate?

✓ Spatial diffusion coefficient $D_s(T)$ that reproduces $D$-meson $R_{AA}$ and $v_2$ gives correct predictions for $v_3$ and $q_2$-selected anisotropic coefficients

✓ Prediction for significant $v_n - v_m$ correlation of $D$ mesons, comparable to that of bulk particles

✓ Indication for a strong coupling of bottom quarks with the collectively expanding bulk

Thank you for your attention!