Elliptic and triangular flow of charmonia in heavy ion collisions

The 20th International Conference on Strangeness in Quark Matter

June 15th 2022
Paradise Hotel
Busan, Korea

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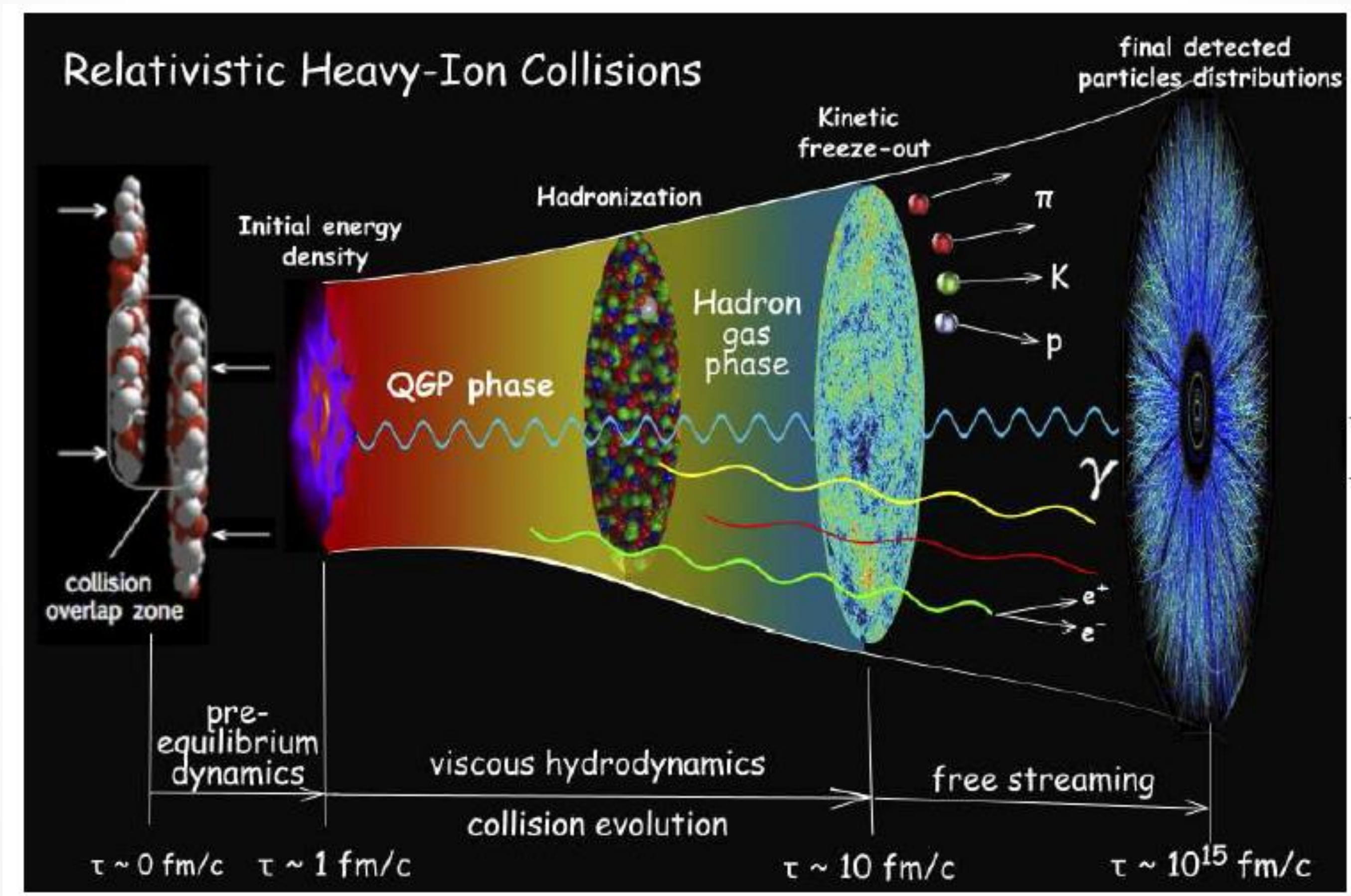
Introduction

– Relativistic heavy ion collisions



U. W. Heinz, J. Phys. Conf. Ser. **455**, 012044 (2013)

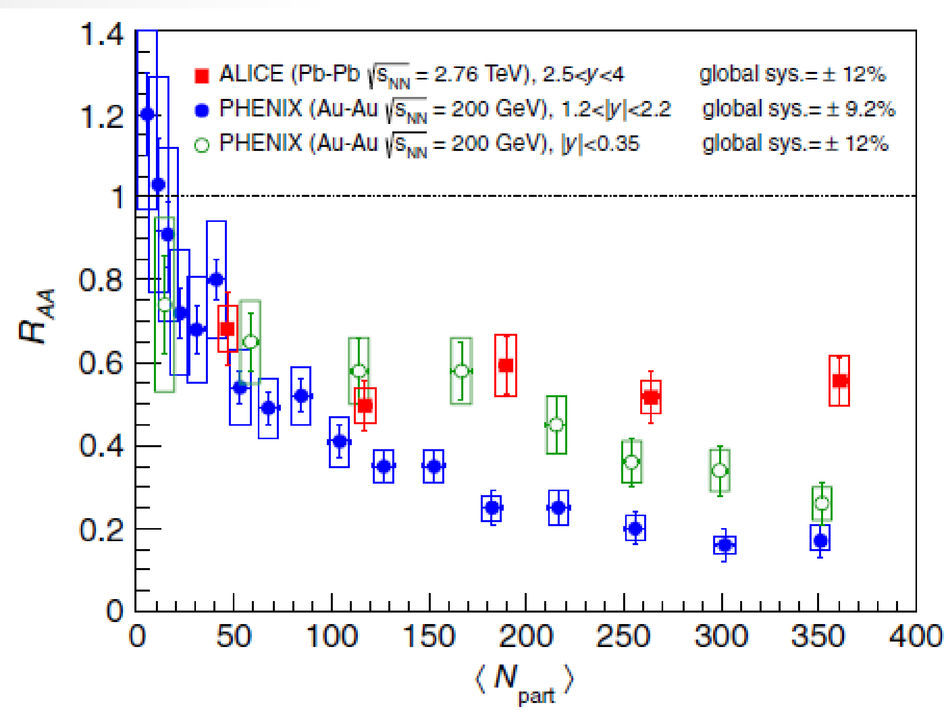
Charmonium states in heavy ion collisions

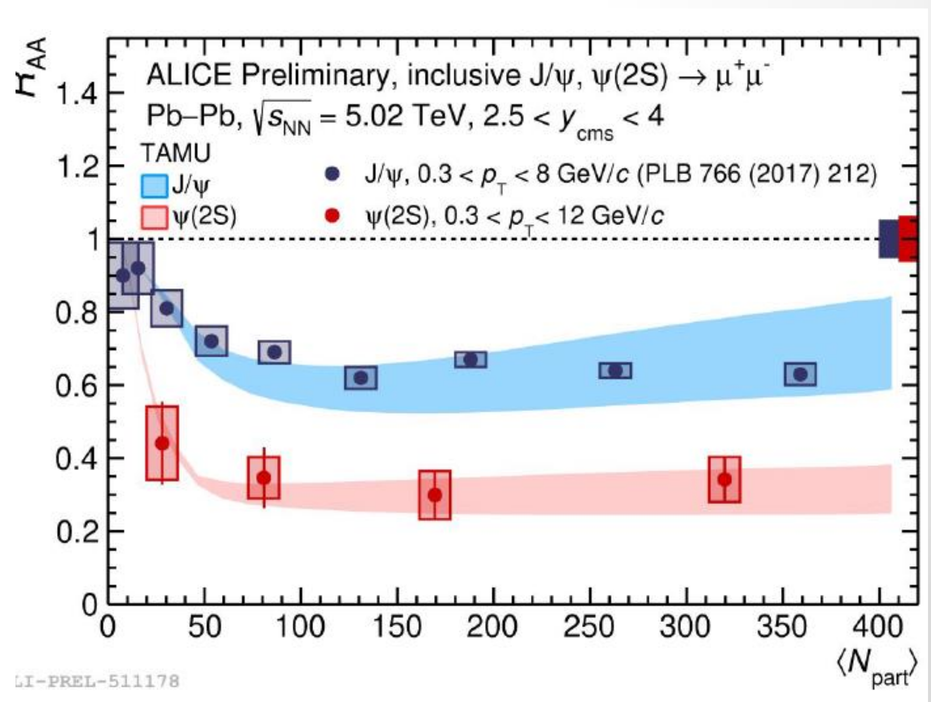
– Regeneration of charmonium states

1) The nuclear modification factor of charmonium states

B. Abelev et al, (ALICE Collaboration), Phys. Rev. Lett. **109**, 072301

Jon-Are Saetre (Univ. of Bergen), Quark Matter 2022, Krakow, April 4-10

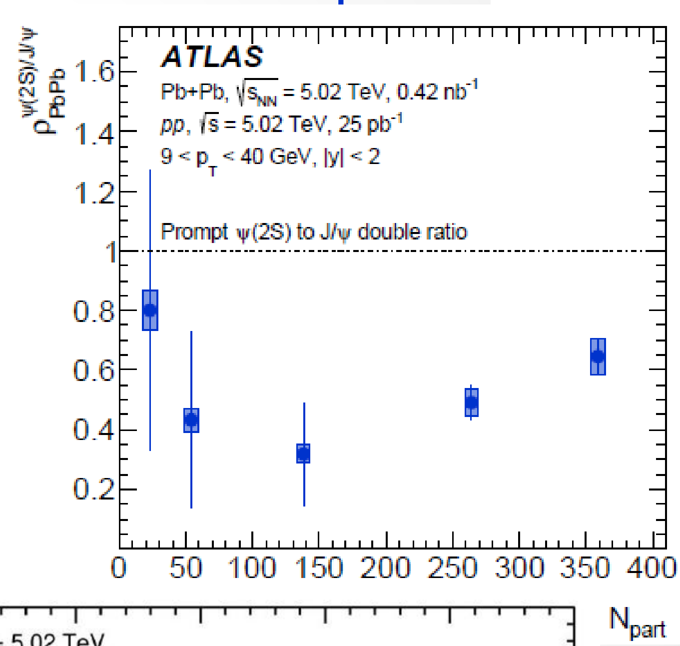


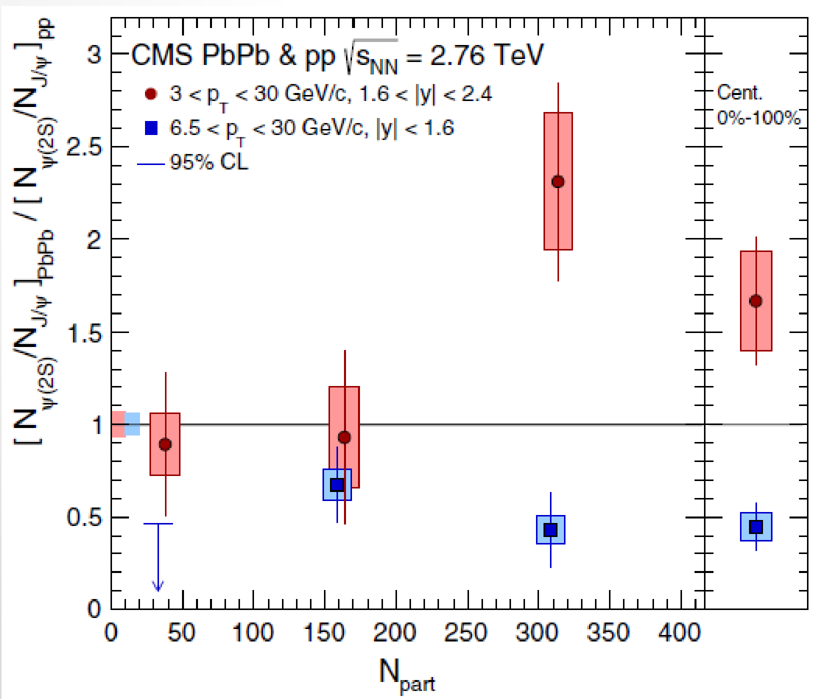


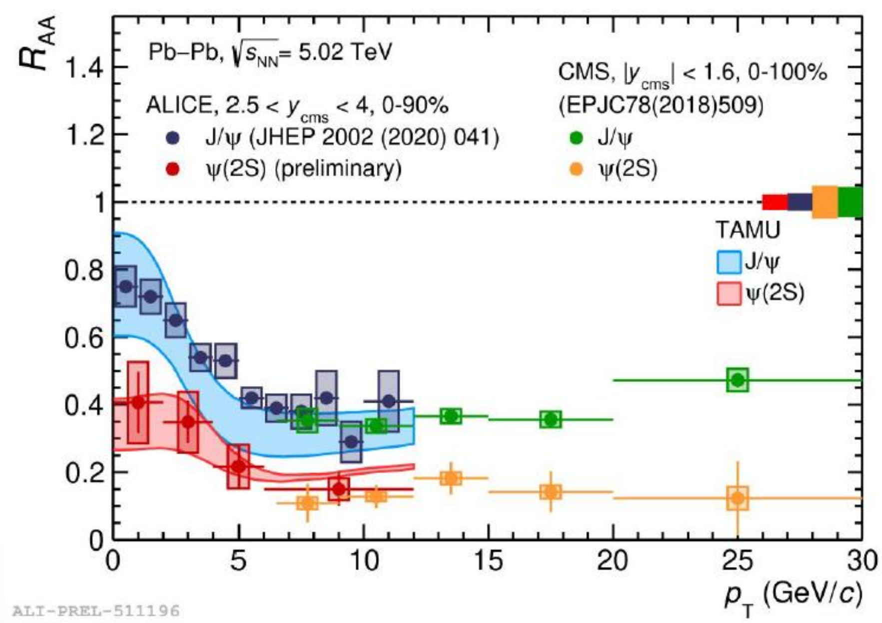
– Nuclear modification factors of the J/ψ and ψ(2S)

1) The nuclear modification factor ratio between the J/ψ and ψ(2S)

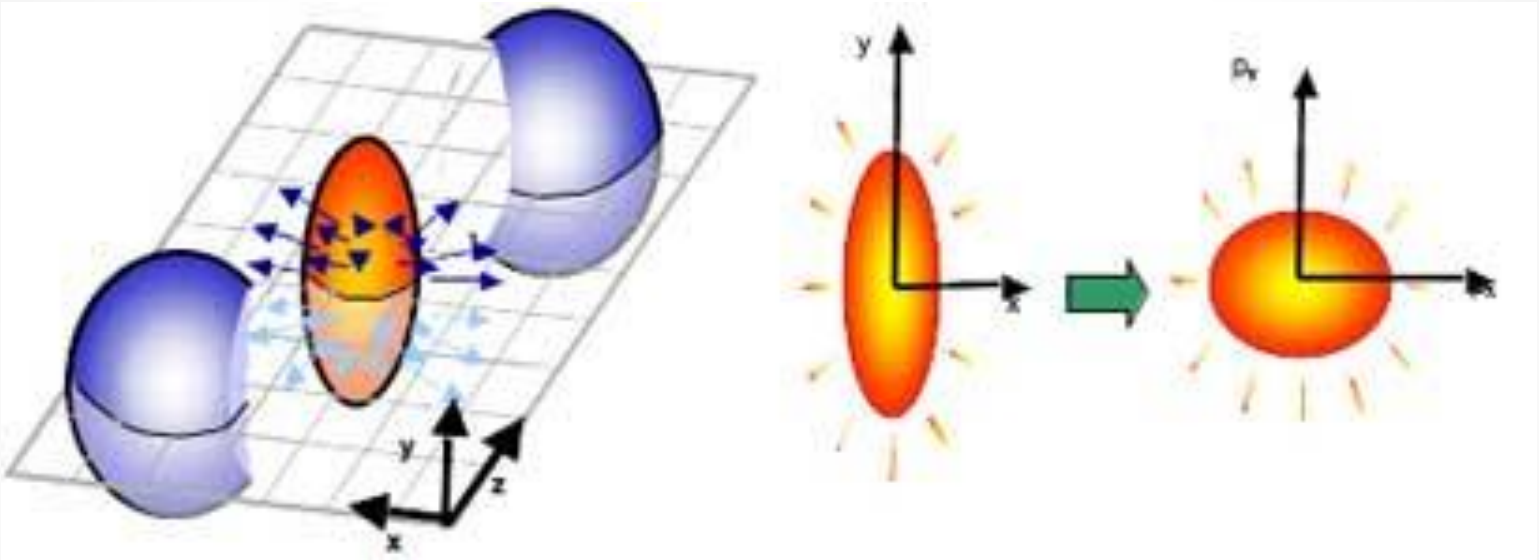
V. Khachatryan et al, Phys. Rev. Lett. **113,** 262301 (2014)
M. Aaboud et al, Eur. Phys. J. C **78,** 762 (2018)







– Non-central collisions, anisotropic flows



$$E\frac{d^3N_q}{d^3p}=\frac{dN_q}{p_T dp_T d\varphi dy}=\frac{1}{2\pi}\frac{dN_q}{p_T dp_T dy}\left[1+\sum_{n=1}2v_{n,q}(p_T)\cos(n\varphi)\right]$$

$$=\frac{1}{2\pi}\frac{dN_q}{p_T dp_T dy}\left[1+2v_{1,q}(p_T)\cos(\varphi)+2v_{2,q}(p_T)\cos(2\varphi)+\cdots\right]$$

A. M. Poskanzer and S. A Voloshin, Phys. Rev. C **58**, 1671 (1998)

1) Quark number scaling of the elliptic flow

D. Molnar and S. A. Voloshin, Phys. Rev. Lett **91**, 092301 (2003)

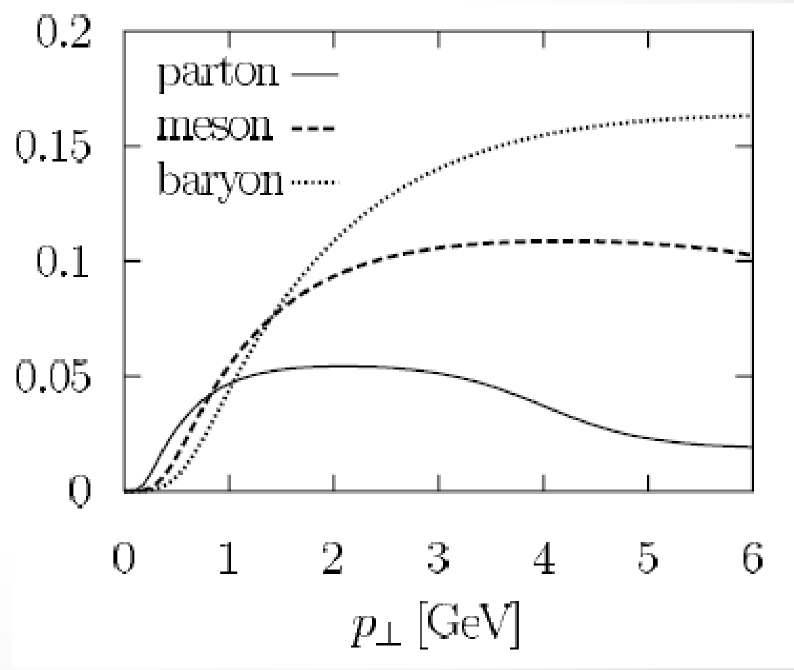
$$v_2(p_T) = \frac{\int d\varphi \cos 2\varphi \frac{d^2N}{dp_T^2}}{\int d\varphi \frac{d^2N}{dp_T^2}} \qquad \frac{dN_q}{p_T dp_T d\varphi} = \frac{1}{2\pi} \frac{dN_q}{p_T dp_T}\left[1 + 2v_{2,q}(p_T)\cos(2\varphi)\right]$$

Coalescence model predictions

$$v_{2,M}(p_T) = \frac{2v_{2,q}(p_T/2)}{1 + 2v_{2,q}^2(p_T/2)}$$

$$v_{2,B}(p_T) = \frac{3v_{2,q}(p_T/3) + 3v_{2,q}^3(p_T/3)}{1 + 6v_{2,q}^2(p_T/3)}$$

$$v_{2,h}(p_T) \approx n v_{2,q}\left(\frac{1}{n} p_T\right)$$



Yields and transverse momentum distributions of charmonium states

– Yields of hadrons in the coalescence model

V. Greco, C. M. Ko, and P. Levai, Phys. Rev. C **68**, 034904 (2003)
R. J. Freis. B. Muller, C. Nonaka, and S. Bass, Phys. Rev. C **68**, 044902 (2003)

$$N^{Coal} = g\int\left[\prod_{i=1}^{n}\frac{1}{g_i}\frac{p_i\cdot d\sigma_i}{(2\pi)^3}\frac{d^3p_i}{E_i}f(x_i,p_i)\right]f^W(x_1,\cdots,x_n:p_1,\cdots,p_n)$$

1) The Wigner function, the coalescence probability function

$$f^W(x_1,\cdots,x_n:p_1,\cdots,p_n) = \int\prod_{i=1}^{n}dy_i e^{p_i y_i}\psi^*\left(x_1+\frac{y_1}{2},\cdots,x_n+\frac{y_n}{2}\right)\psi\left(x_1-\frac{y_1}{2},\cdots,x_n-\frac{y_n}{2}\right)$$

2) A Lorentz-invariant phase space integration of a space-like hyper-surface constraints the number of particles in the system

$$\int p_i\cdot d\sigma_i\frac{d^3p_i}{(2\pi)^3E_i}f(x_i,p_i) = N_i$$

– Production of charmonium states by recombination

S. Cho, Phys. Rev. C **91**, 054914 (2015)

1) Coalescence production of charmonium states

$$N_\psi = g_\psi \int p_c \cdot d\sigma_c p_{\bar{c}} \cdot d\sigma_{\bar{c}} \frac{d^3\vec{p}_c}{(2\pi)^3 E_c} \frac{d^3\vec{p}_{\bar{c}}}{(2\pi)^3 E_{\bar{c}}} f_c(r_c, p_c) f_{\bar{c}}(r_{\bar{c}}, p_{\bar{c}}) W_\psi(r_c, r_{\bar{c}}; p_c, p_{\bar{c}}),$$

The transverse momentum distribution of the charmonium yield

$$\frac{dN_\psi}{d^2\vec{p}_T} = \frac{g_\psi}{V} \int d^3\vec{r} d^2\vec{p}_{cT} d^2\vec{p}_{\bar{c}T} \delta^{(2)}(\vec{p}_T - \vec{p}_{cT} - \vec{p}_{\bar{c}T}) \frac{dN_c}{d^2\vec{p}_{cT}} \frac{dN_{\bar{c}}}{d^2\vec{p}_{\bar{c}T}} W_\psi(\vec{r}, \vec{k})$$

2) Gaussian Wigner functions for different charmonium states

$$W_s(\vec{r}, \vec{k}) = 8 e^{-\frac{r^2}{\sigma^2} - k^2\sigma^2}$$

$$W_p(\vec{r}, \vec{k}) = \left(\frac{16}{3}\frac{r^2}{\sigma^2} - 8 + \frac{16}{3}\sigma^2 k^2\right) e^{-\frac{r^2}{\sigma^2} - k^2}$$

$$W_{\psi_{10}}(\vec{r}, \vec{k}) = \frac{16}{3}\left(\frac{r^4}{\sigma^4} - 2\frac{r^2}{\sigma^2} + \frac{3}{2} - 2\sigma^2 k^2 + \sigma^4 k^4 - 2r^2 k^2 + 4(\vec{r}\cdot\vec{k})^2\right) e^{-\frac{r^2}{\sigma^2} - k^2\sigma^2}.$$

4) Integration of the Wigner function over the spatial coordinates

$$\int d^3\vec{r} W_\psi(\vec{r},\vec{k}) = \begin{cases} (2\sqrt{\pi}\sigma)^3 e^{-k^2\sigma^2} & \psi_s^G; J/\psi \\ \frac{2}{3}(2\sqrt{\pi}\sigma)^3 e^{-k^2\sigma^2}\sigma^2 k^2 & \psi_p^G; \chi_c \\ \frac{2}{3}(2\sqrt{\pi}\sigma)^3 e^{-k^2\sigma^2}\left(\sigma^2 k^2 - \frac{3}{2}\right)^2 & \psi_{10}^G; \psi(2S) \\ 64\pi \frac{a_0^3}{(a_0^2 k^2+1)^4} & \psi_{1S}^C; J/\psi \\ 8\pi a_0^3 \frac{(a_0^2 k^2 - 1/4)^2}{(a_0^2 k^2+1/4)^6} & \psi_{2S}^C; \psi(2S) \end{cases}$$

$$\int d^3\vec{r} W(\vec{r},\vec{k}) = \left|\tilde{\psi}(\vec{k})\right|^2$$

M. Hillery, R. F. O'Connel, M. O. Scully and E. P. Wigner, Phys. Rept. **106**, 121 (1984)

$$\frac{dN_\psi}{d\vec{p}_T} = \frac{g_\psi}{V}\int d\vec{p}_{cT} d\vec{p}_{\bar{c}T}\delta(\vec{p}_T - \vec{p}_{cT} - \vec{p}_{cT})\frac{dN_c}{d\vec{p}_{cT}}\frac{dN_{\bar{c}}}{d\vec{p}_{\bar{c}T}}\left|\tilde{\psi}(\vec{k})\right|^2$$

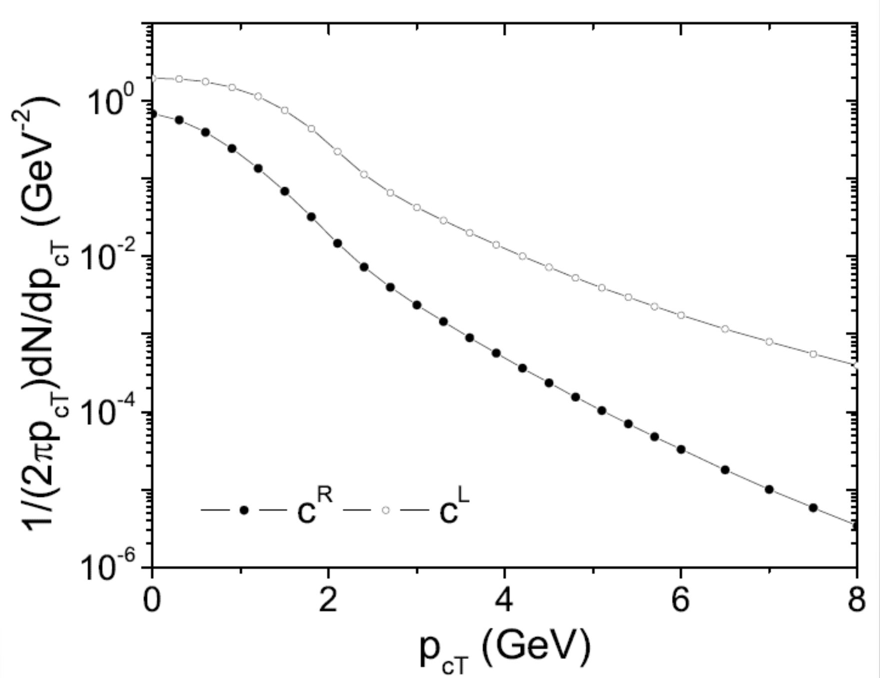
5) Transverse momentum distributions of charm and light quarks

$$\frac{dN_c}{d^2p_T} = \begin{cases} a_0 \exp[-a_1 p_T^{a_2}] & p_T \le p_0 \\ a_0 \exp[-a_1 p_T^{a_2}] + a_3(1+p_T^{a_4})^{-a_5} & p_T \ge p_0 \end{cases}$$

RHIC	a_0	a_1	a_2	a_3	a_4	a_5
$p_T \le p_0$	0.69	1.22	1.57			
$p_T \ge p_0$	1.08	3.04	0.71	3.79	2.02	3.48
LHC	a_0	a_1	a_2	a_3	a_4	a_5
$p_T \le p_0$	1.97	0.35	2.47			
$p_T \ge p_0$	7.95	3.49	3.59	87335	0.5	14.31

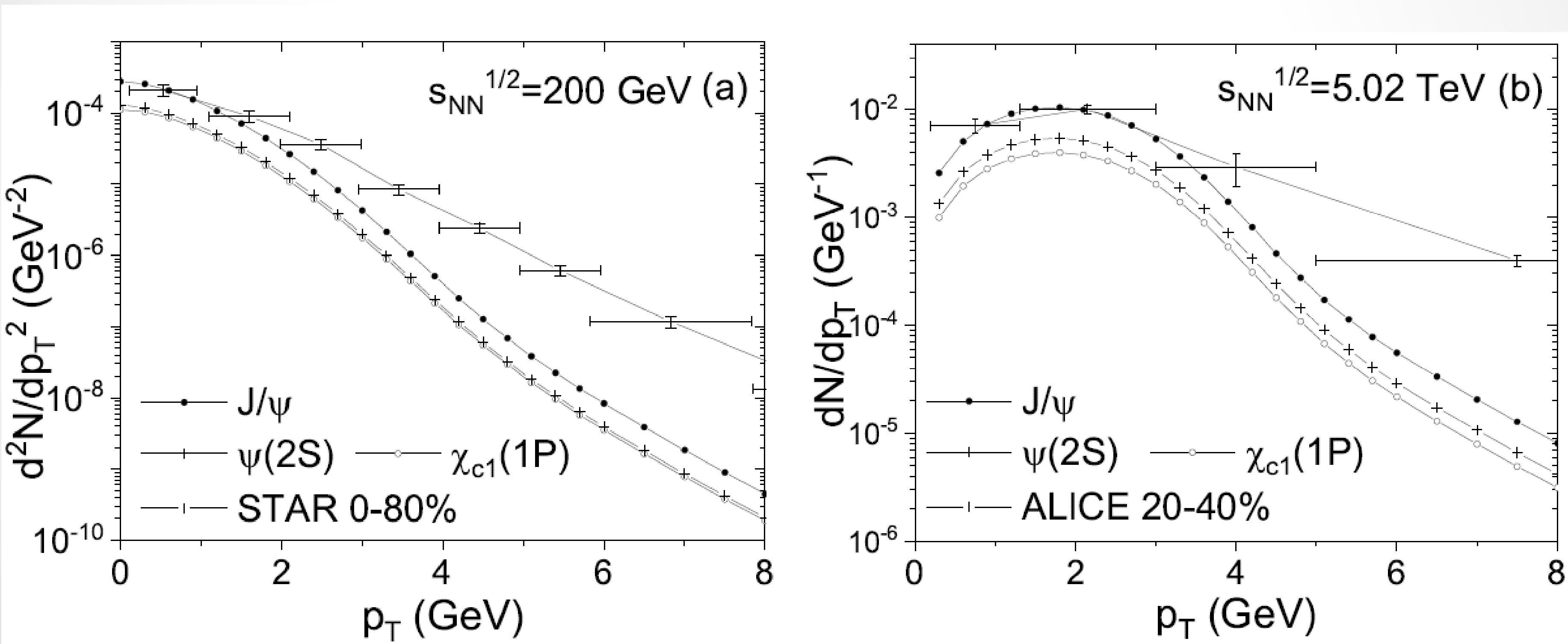
S. Plumari, V. Minissale, S. K. Das, G. Coci and V. Greco, Eur. Phys. J. C **78**:348 (2017)
Y. Oh, C. M. Ko, S.-H. Lee, and S. Yasui, Phys. Rev. C **79** 044905 (2009)
S. Cho *et al*. (EXHIC Collaboration), Prog. Part. Nucl. Phys. **95**, 279 (2017)

	RHIC Sc. 1	RHIC Sc. 2	LHC (2.76 TeV) Sc. 1	LHC (2.76 TeV) Sc. 2
T_H (MeV)	162		156	
V_H (fm^3)	2100		5380	
μ_B (MeV)	24		0	
μ_s (MeV)	10		0	
γ_c	22		39	
γ_b	4.0×10^7		8.6×10^8	
T_C (MeV)	162	166	156	166
V_C (fm^3)	2100	1791	5380	3533
$N_u = N_d$	320	302	700	593
$N_s = N_{\bar{s}}$	183	176	386	347
$N_c = N_{\bar{c}}$	4.1		11	
$N_b = N_{\bar{b}}$	0.03		0.44	



6) Transverse momentum distributions of charmonium states in different centralities at RHIC and LHC

J. Adam et al. [STAR Collaboration], Phys. Lett. B **797**, 134917 (2019).
S. Acharya et al. [ALICE Collaboration], Phys. Lett. B **805**, 135434 (2020).



Elliptic and triangular flow of charmonium states

– Flow harmonics of charmonium states

$$v_n(p_T) = \langle \cos(n(\psi - \Psi_n)) \rangle = \frac{\int d\psi \cos(n(\psi - \Psi_n)) \frac{d^2N}{dp_T^2}}{\int d\psi \frac{d^2N}{dp_T^2}}, \qquad \Psi_n = \frac{1}{n} \tan^{-1} \left(\frac{\langle p_T \cos(n\psi) \rangle}{\langle p_T \cos(n\psi) \rangle} \right),$$

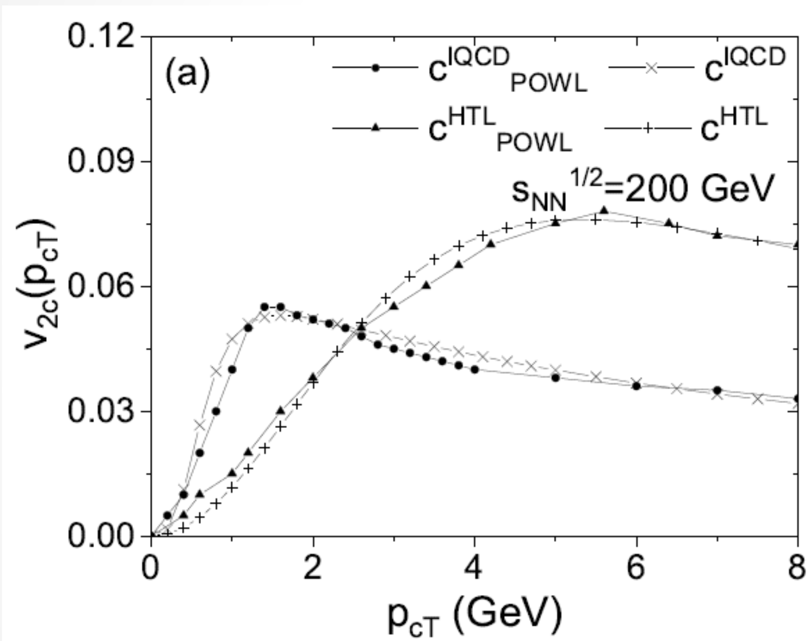
1) Transverse momentum distribution of charm quarks with flow harmonics

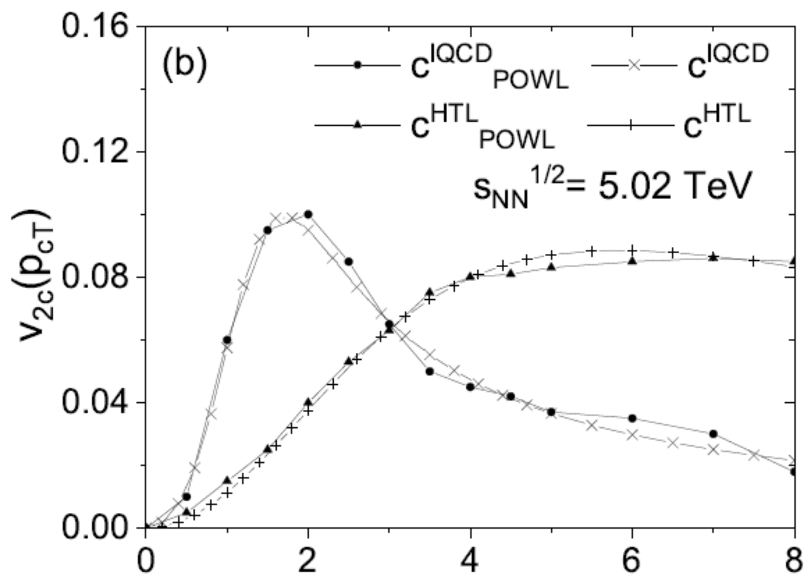
$$\frac{d^2N_c}{dp_{cT}^2} = \frac{1}{2\pi p_{cT}} \frac{dN_c}{dp_{cT}} \left(1 + \sum_{n=1}^{\infty} 2 v_{nc}(p_{cT}) \cos(n(\phi_c - \Psi_n)) \right),$$

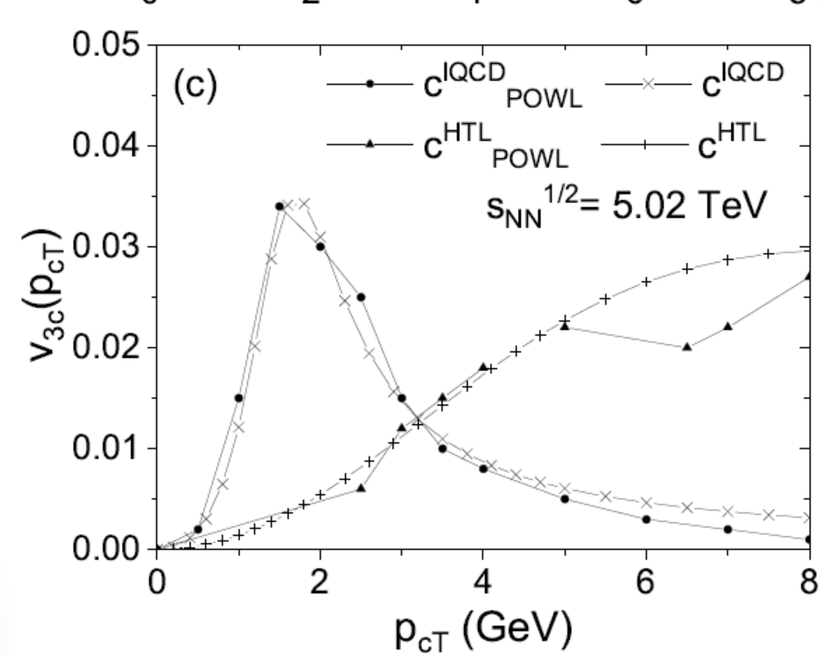
2) Pade approximation of charm quark flow harmonics

$$v_{nc}(p_{cT}) = \frac{a_3 p_{cT}^3 + a_2 p_{cT}^2 + a_1 p_{cT}}{b_4 p_{cT}^4 + b_3 p_{cT}^3 + b_2 p_{cT}^2 + b_1 p_{cT} + 1},$$

3) Charm quark flow harmonics from POWLANG

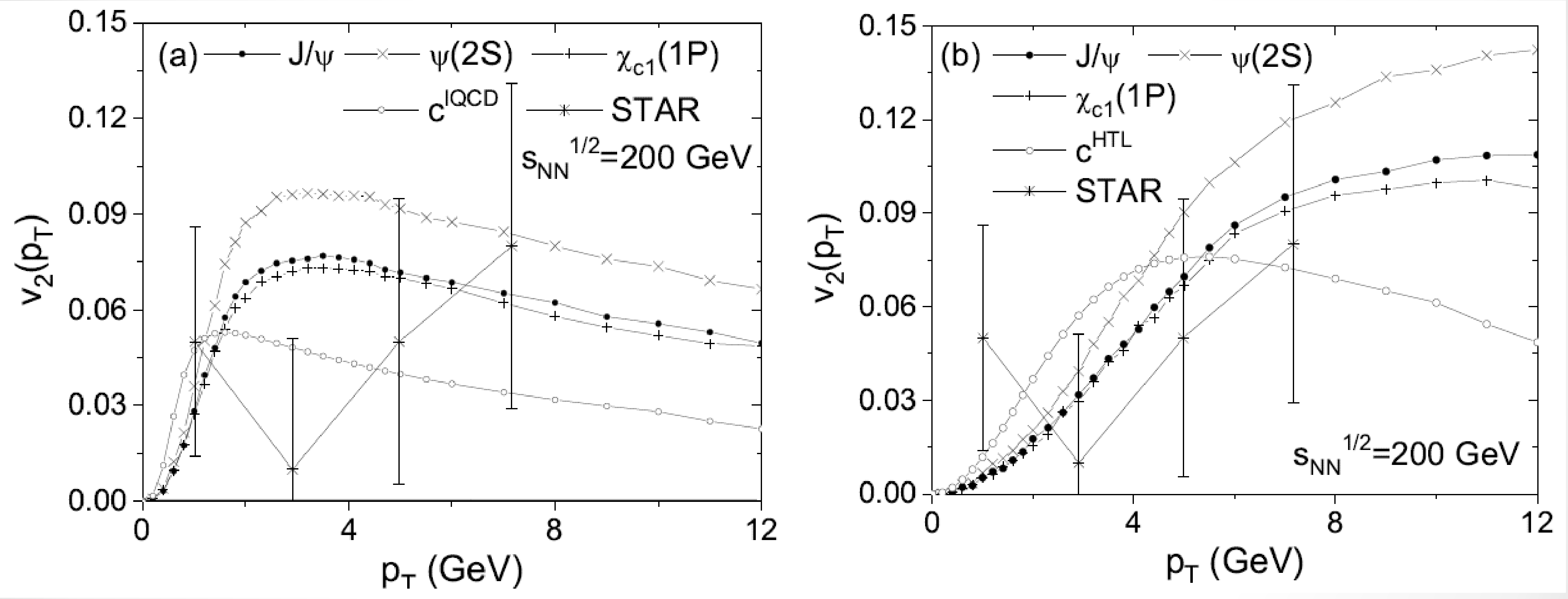






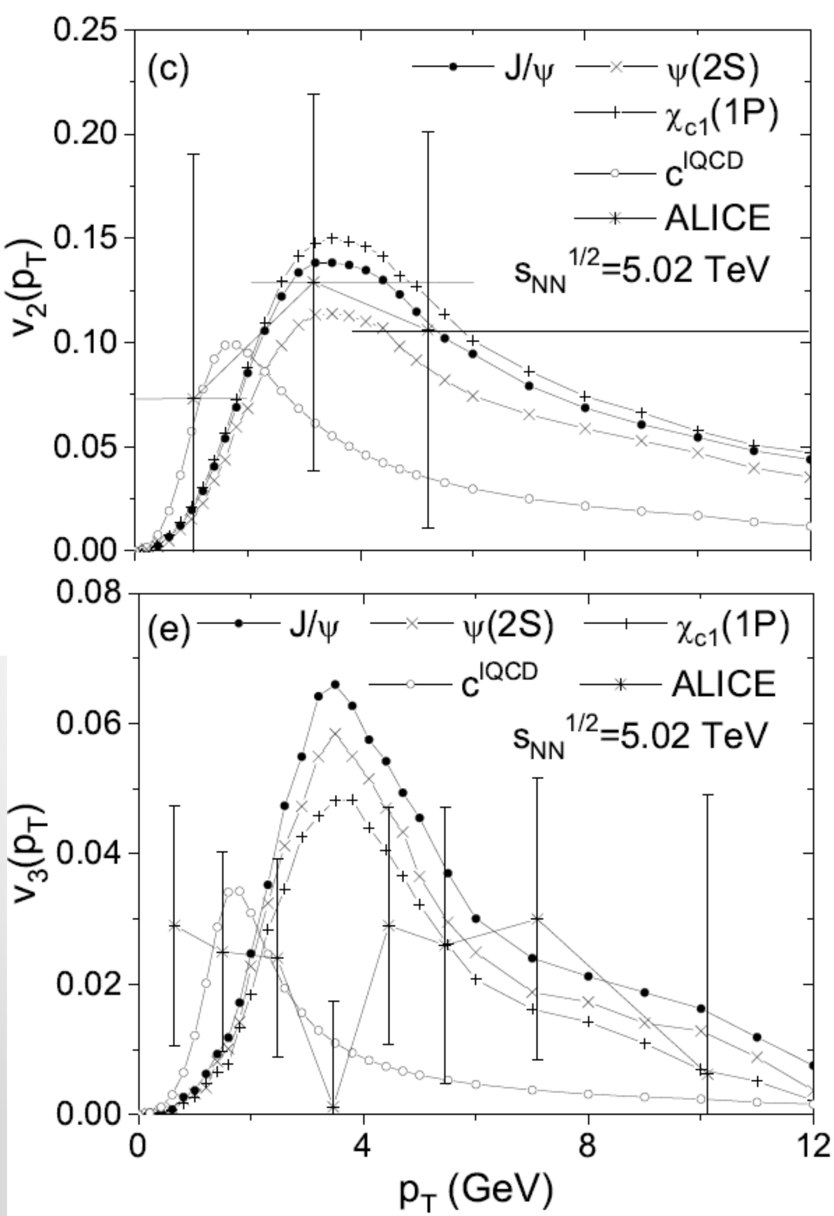
A. Beraudo, A. De Pace, M. Monteno,
M. Nardi andF. Prino, JHEP **02**, 043 (2018).

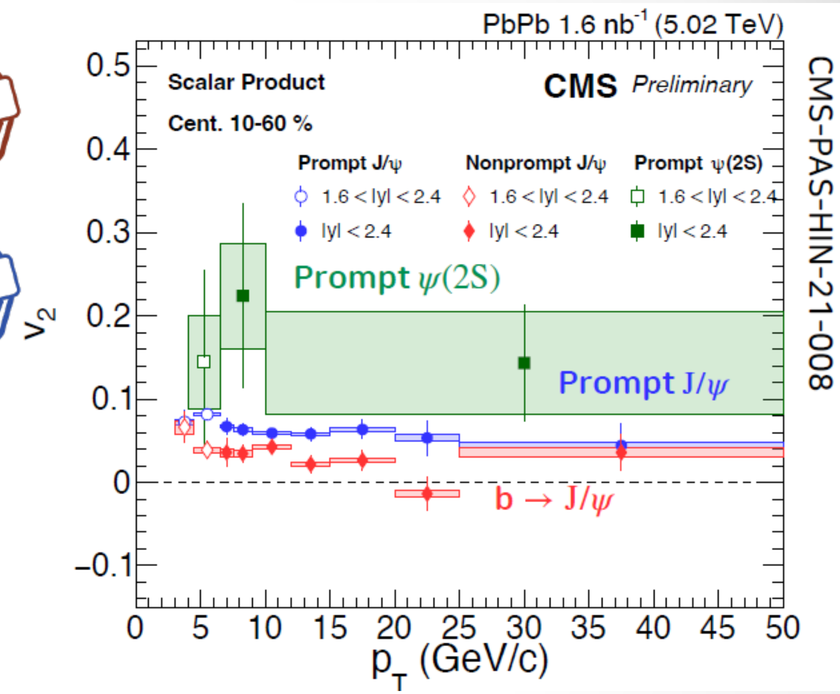
7) Elliptic flow of charmonium states at RHIC



L. Adamczyk et al. [STAR Collaboration], Phys. Rev. Lett. **111**, no. 5, 052301 (2013).

8) Elliptic and triangular flow of charmonium states at LHC





S. Acharya et al. [ALICE Collaboration],
Phys. Rev. Lett. **119**, no. 24, 242301 (2017).
S. Acharya et al. [ALICE Collaboration],
JHEP **2010**, 141 (2020).
G. Bak [CMS Collaboration], Strangeness in
Quark Matter 2022, Busan, June, 13-17

Conclusion

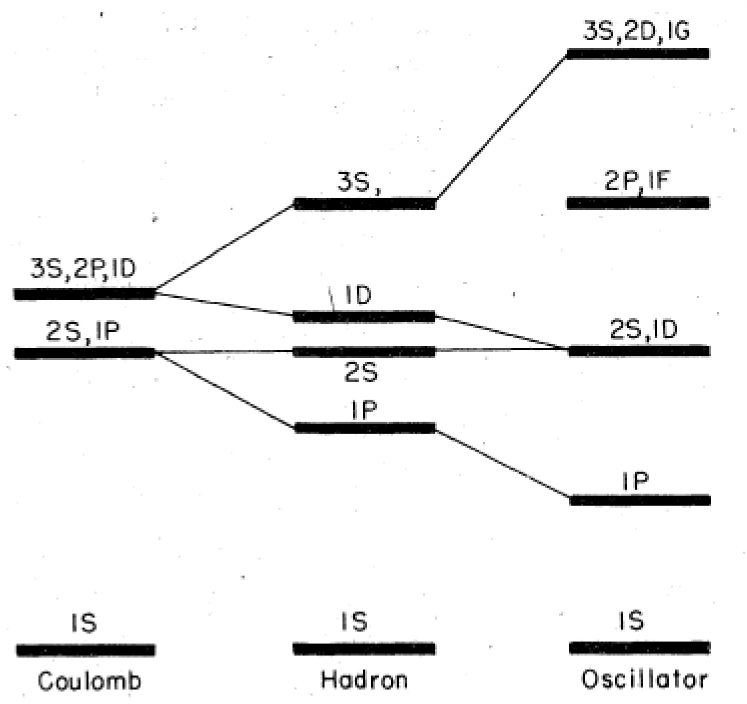
– Elliptic and triangular flow of charmonia in heavy ion collisions

1) The production of heavy quarks hadrons, or charmonium states can be understood in the coalescence model.
2) The transverse momentum distribution is dependent on the internal structure of the hadron
3) The enhanced transverse momentum distribution of $\psi(2S)$ mesons, compared to that of J/ψ mesons, is originated from intrinsic wave function distributions between $\psi(2S)$ and J/ψ mesons.
4) The elliptic and triangular flow of charmonium states are also affected by wave function distributions of charmonium states.
5) Studying charmonium states in heavy ion collisions will help us to understand many aspects in heavy ion collisions experiments.

Thank you for your attention!

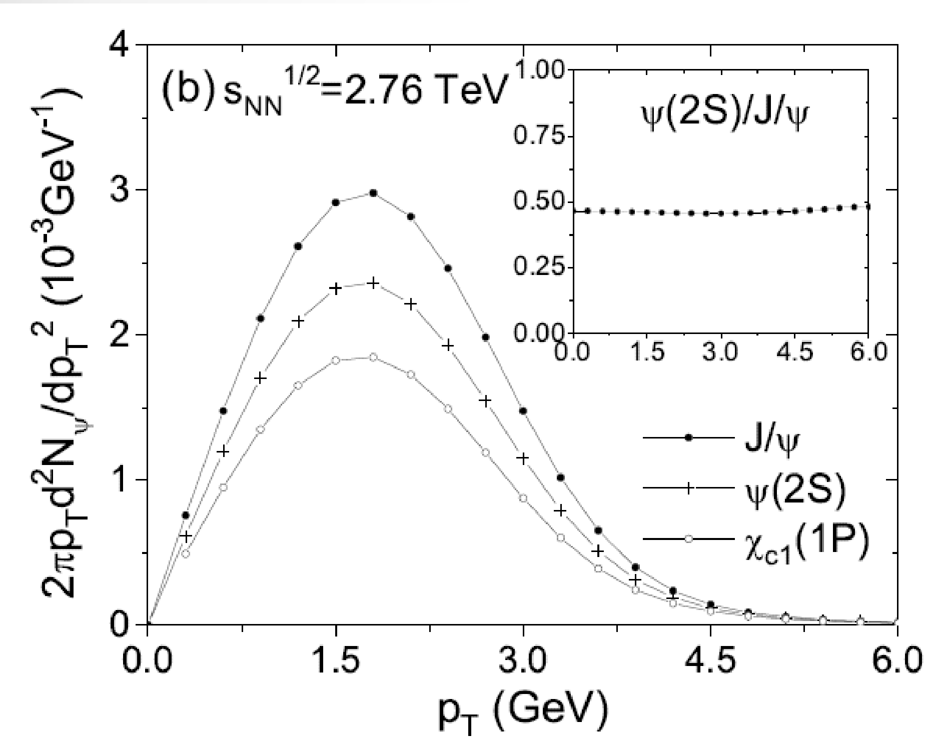
Backup slides

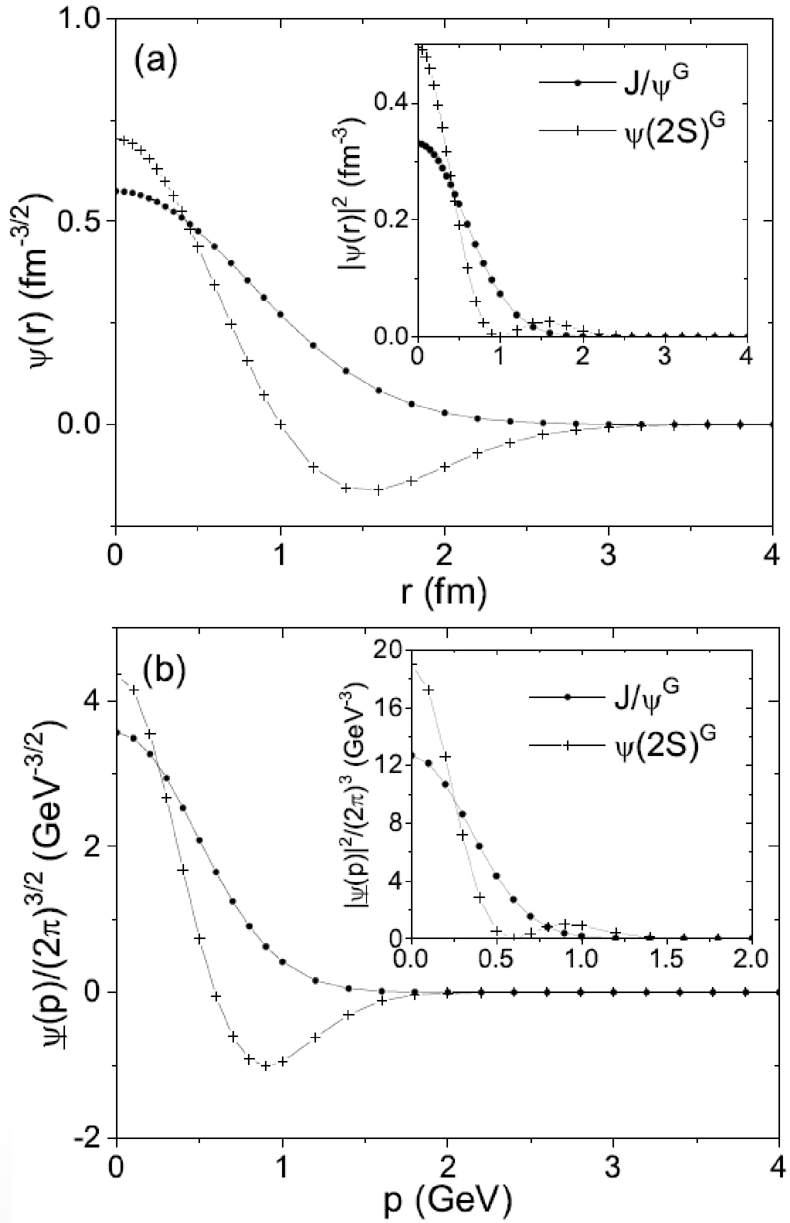
3) Charmonium wave functions



E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, T-M, Yan, Phys. Rev. D **17**, 3090 (1978)

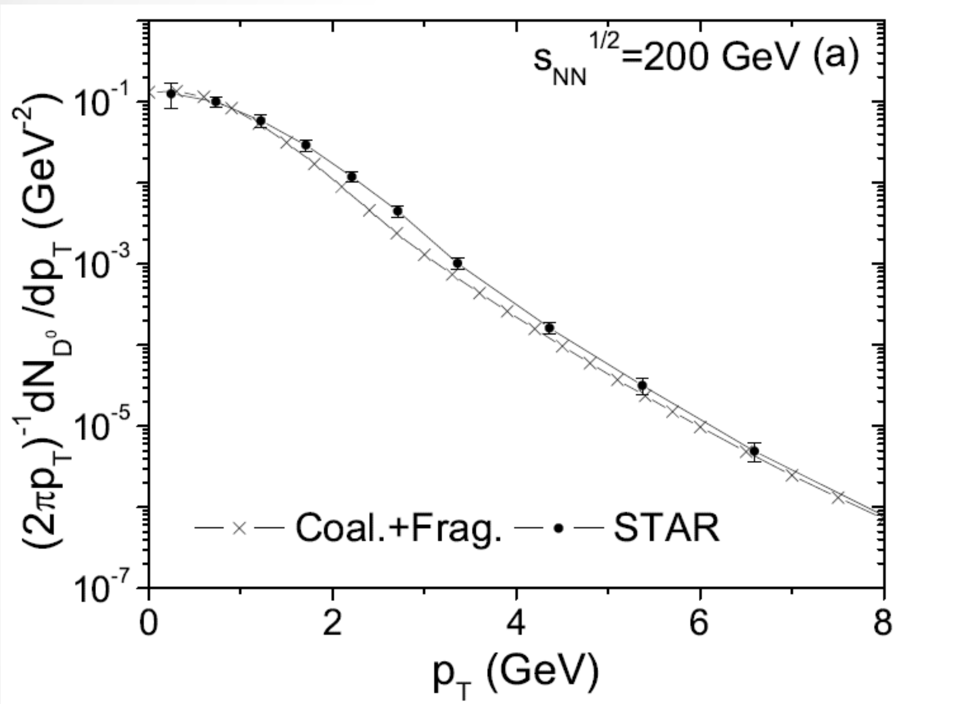
7) Wave function distributions of charmonium states





6) Transverse momentum distributions of charmonium states in 0-10% centralities at RHIC and LHC

S. Cho and S. H. Lee, Phys. Rev. C **101**, 024902 (2020)



J. Adam et al. [STAR Collaboration], Phys. Lett. B **797**, 134917 (2019).
S. Acharya et al. [ALICE Collaboration], Phys. Lett. B **805**, 135434 (2020).