Constraints on hadron resonance gas interactions via first-principles Lattice QCD susceptibilities

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In collaboration with:
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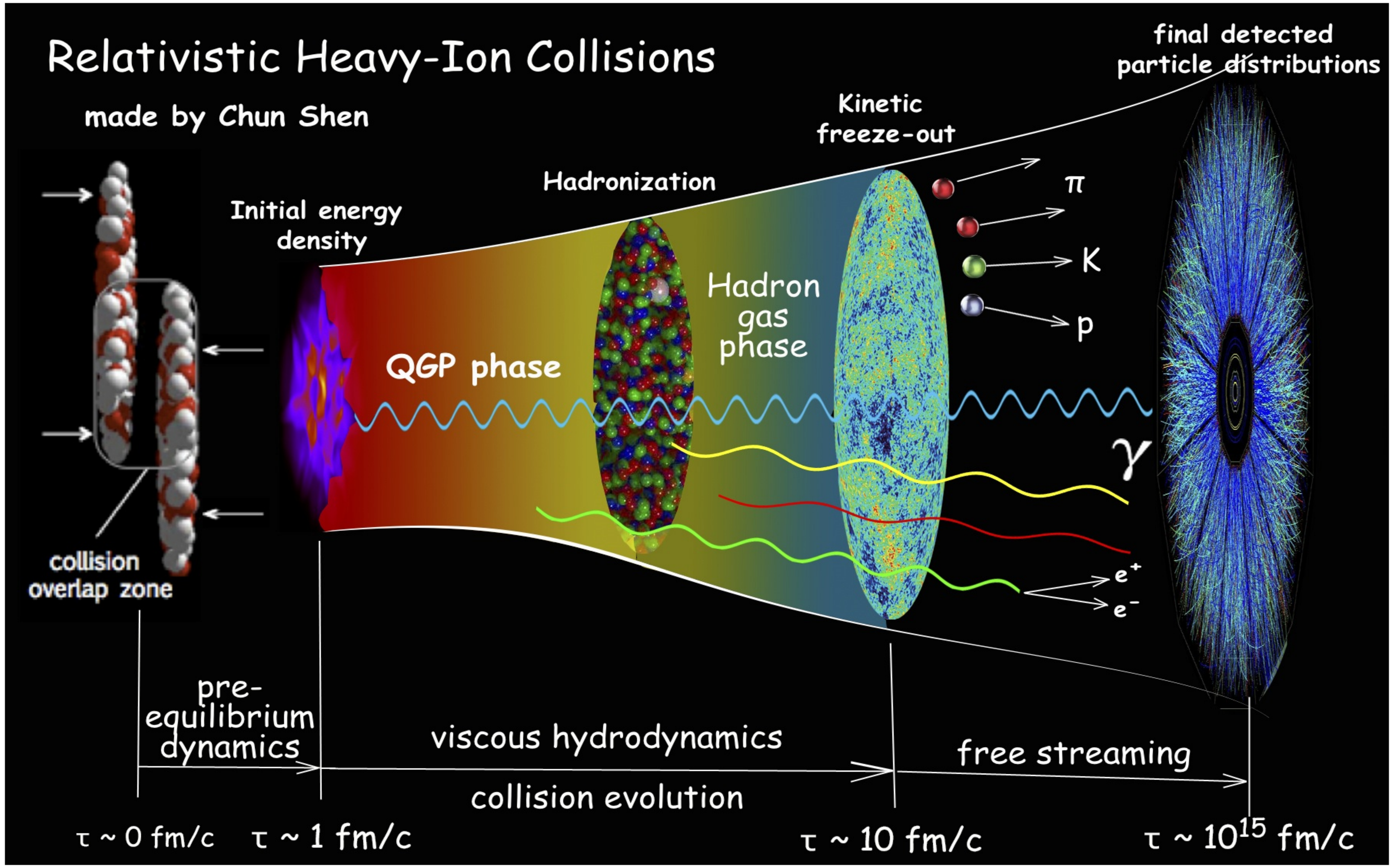
SQM2022
The 20th International Conference on Strangeness in Quark Matter
13-17 June 2022 Busan, Republic of Korea

MIT Physics

Evolution of a Heavy-ion Collision

- Strongly-interacting matter proceeds through several different phases during a collision event → HIC modeling/phenomenology



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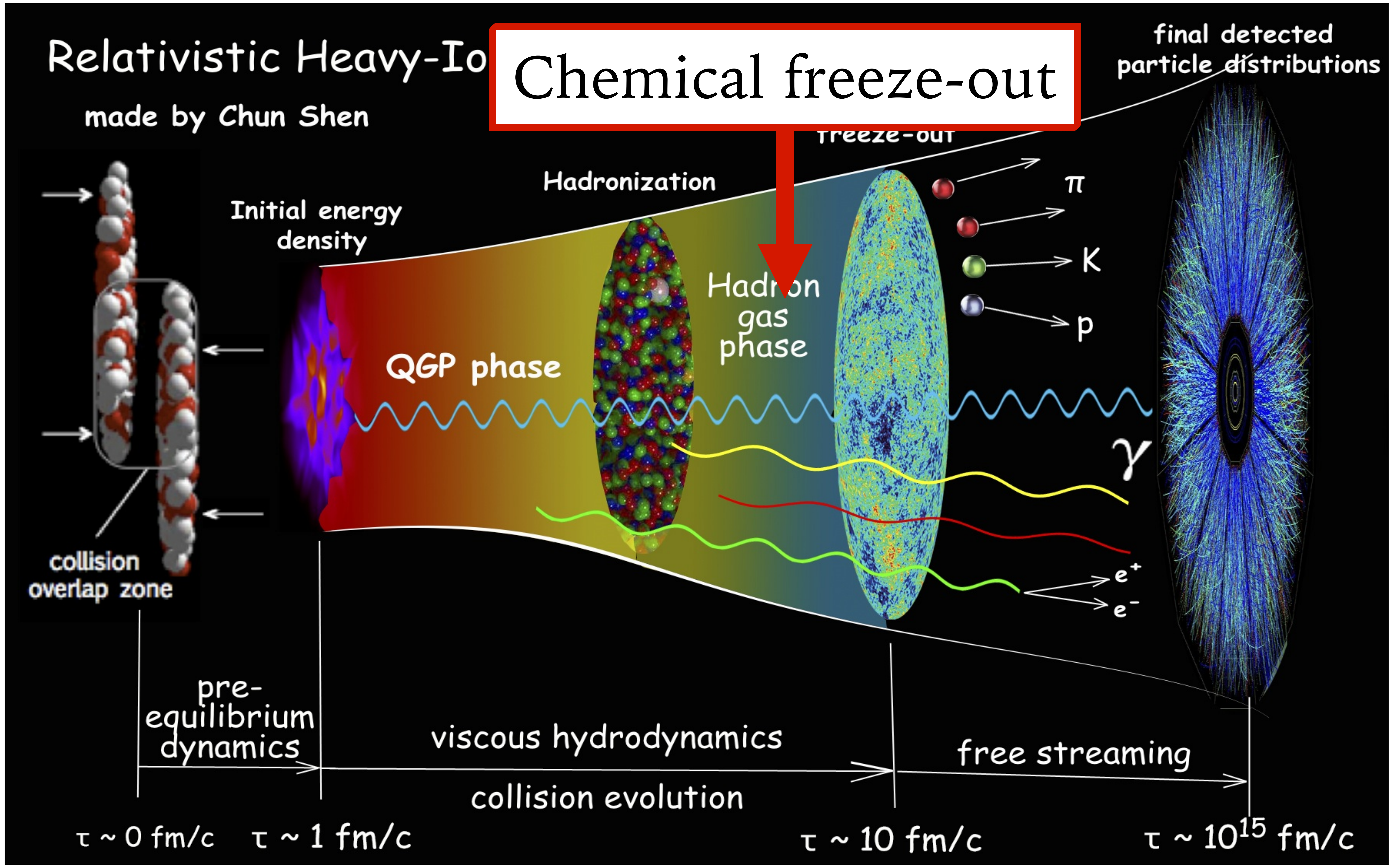
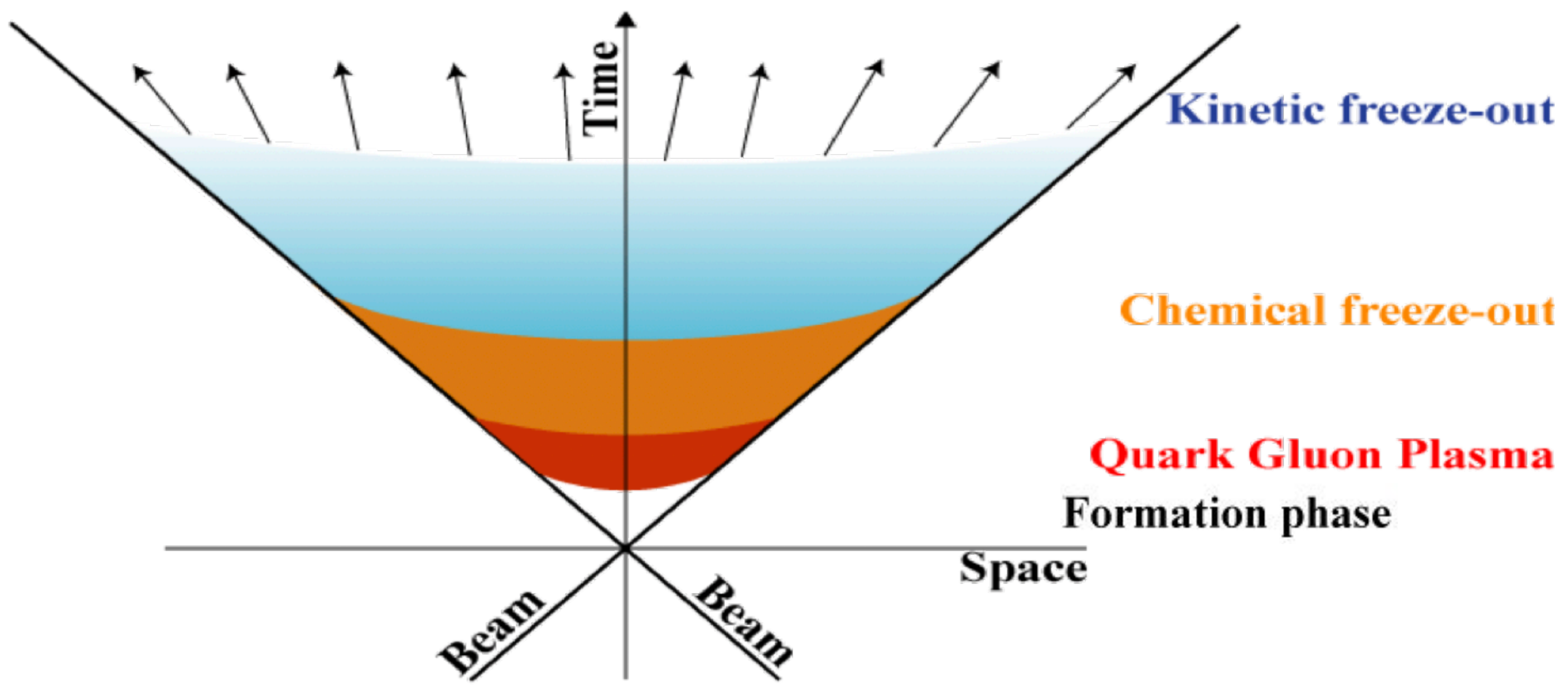


Freeze-out Stages in HICs

➤ After a heavy-ion collision, the hot, dense system cools and expands, eventually producing color-neutral hadrons that are measured by the detectors



- Chemical freeze-out: inelastic collisions cease; the chemical composition is fixed (particle yields and fluctuations)
- Kinetic freeze-out: elastic collisions cease; spectra and correlations are fixed

The Hadron Resonance Gas Model

➤ The ideal HRG model agrees well with Lattice QCD results on the Equation of State below the cross-over transition temperature:

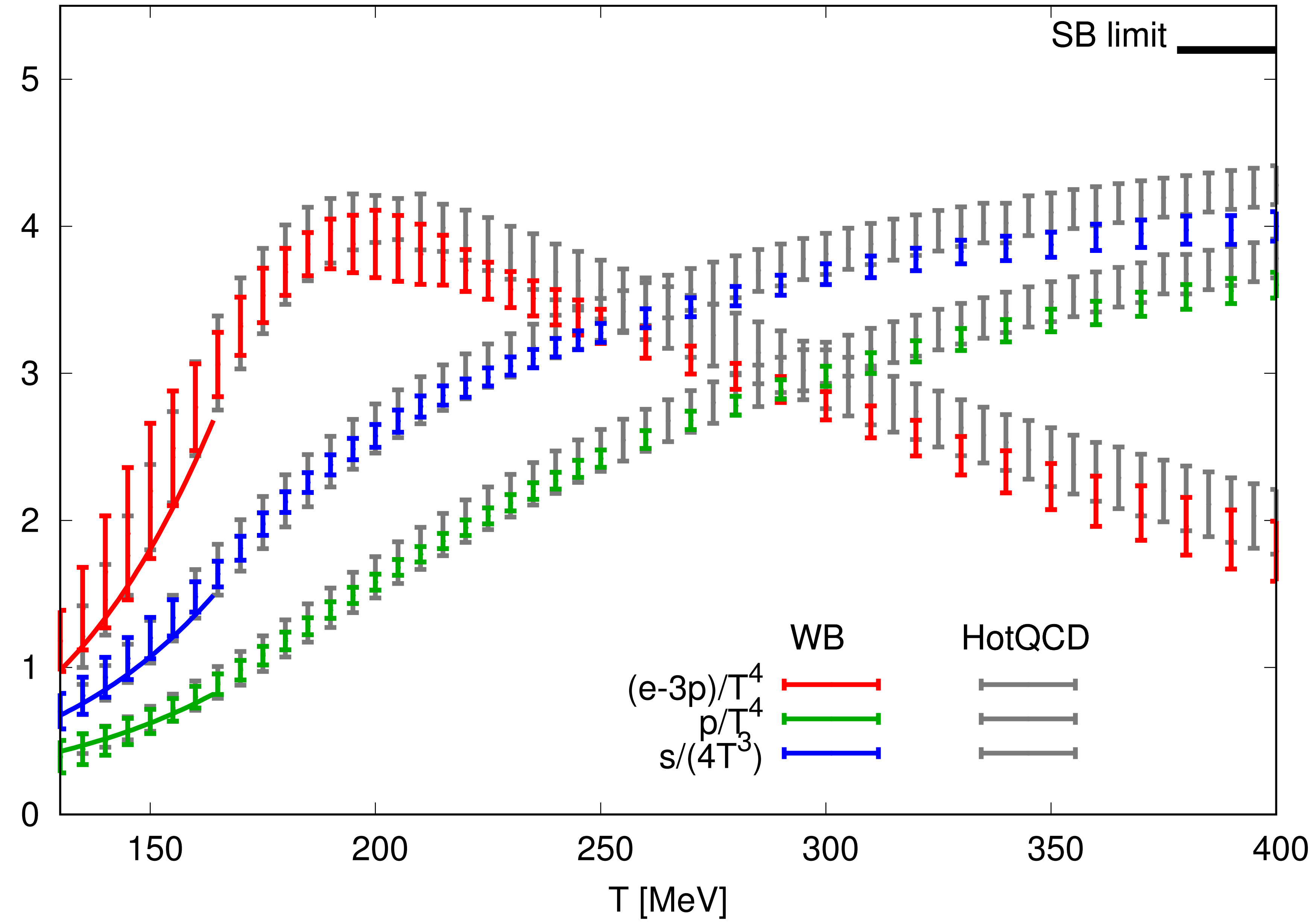
$$\frac{P}{T^4} = \frac{1}{VT^3} \sum_i \ln Z_i(T, V, \vec{\mu})$$

$$\ln Z_i^{M/B} = \mp \frac{V d_i}{(2\pi)^3} \int d^3p \, \ln\left(1 \mp \exp\left[-\left(\epsilon_i - \mu_a X_a^i\right)/T\right]\right)$$

energy $\epsilon_i = \sqrt{p^2 + m_i^2}$

conserved charges $\vec{X}_i = (B_i, S_i, Q_i)$

degeneracy d_i, mass m_i, volume V



S. Borsanyi et al (WB collaboration), PLB (2014),
A. Bazavov et al (HotQCD collaboration), PRD (2014)

Susceptibilities in the HRG Model

- Susceptibilities are fluctuations of conserved charges from a theoretical perspective:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{\,l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}$$

$$\text{mean}: \; M = \chi_1 \qquad \text{variance}: \; \sigma^2 = \chi_2$$

$$\text{skewness}: \; S = \chi_3/\chi_2^{3/2} \qquad \text{kurtosis}: \; \kappa = \chi_4/\chi_2^2$$

$$\sigma^2/M = \chi_2/\chi_1 \qquad S\sigma^3/M = \chi_3/\chi_1$$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

S. Ejiri, F. Karsch, K. Redlich, PLB (2006),
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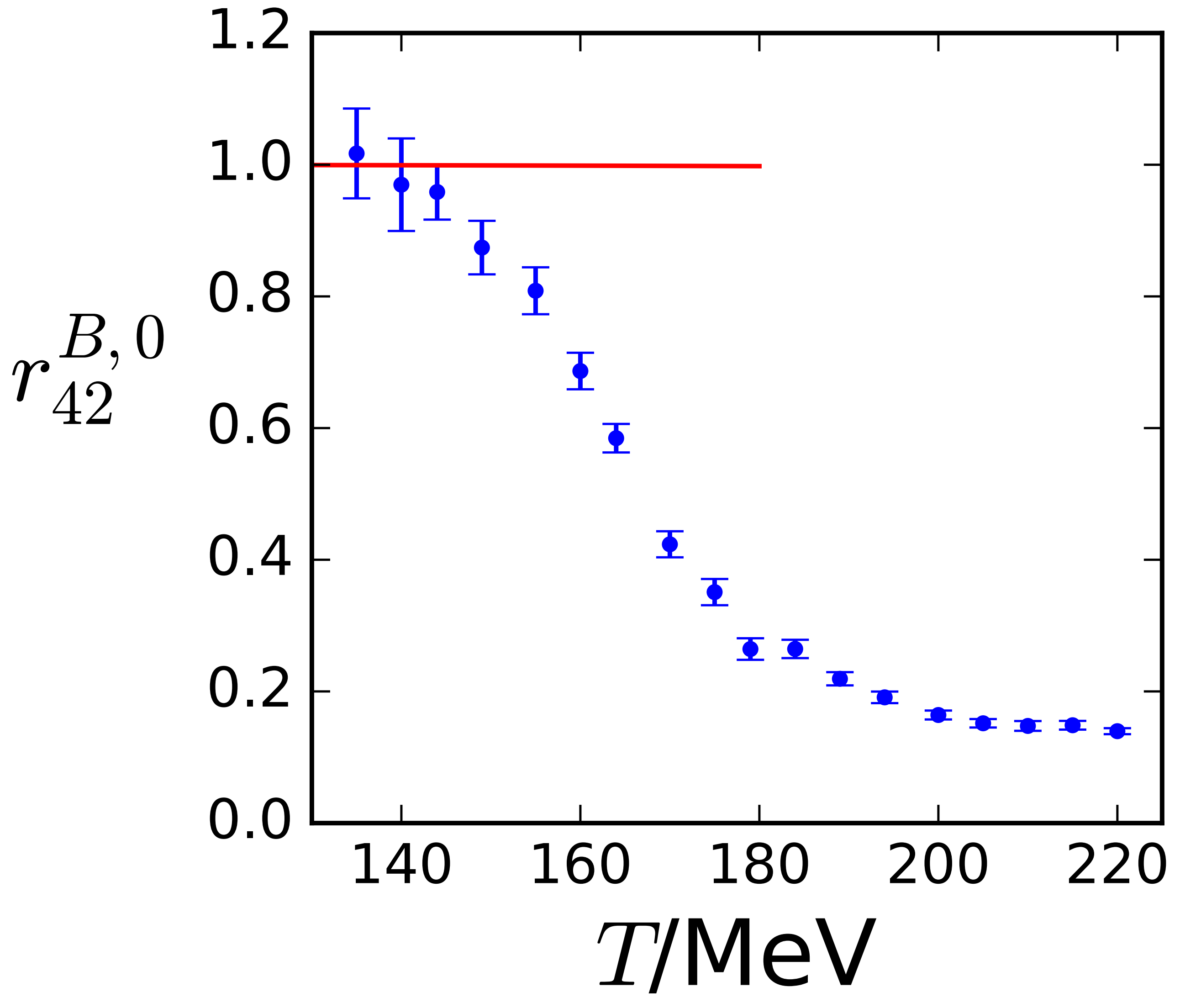
$$\kappa_B \sigma_B^2 = \frac{\chi_4^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)} = r_{42}^{B,0} + \dots$$



Even-order-ratios predicted to be unity in ideal HRG

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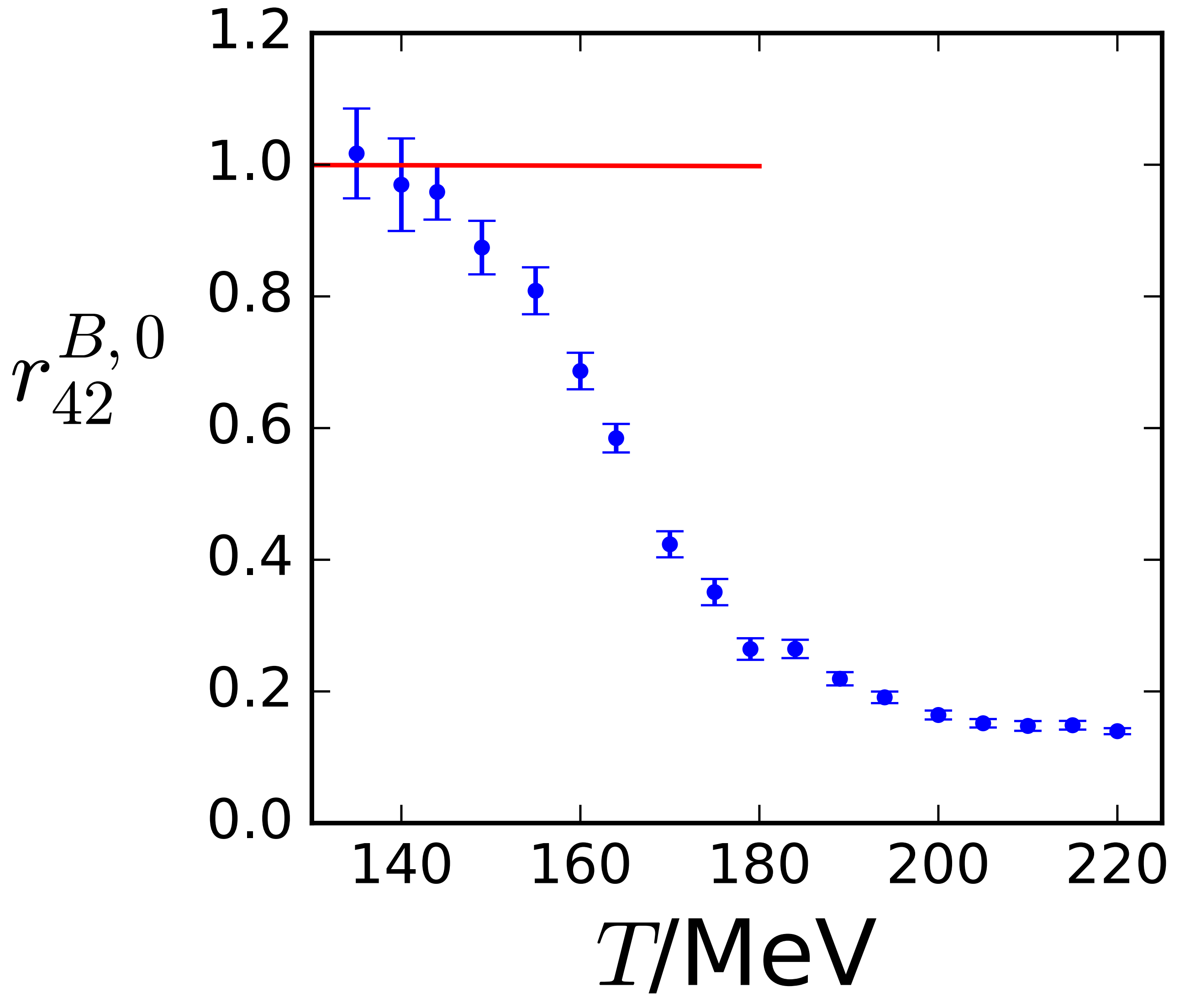

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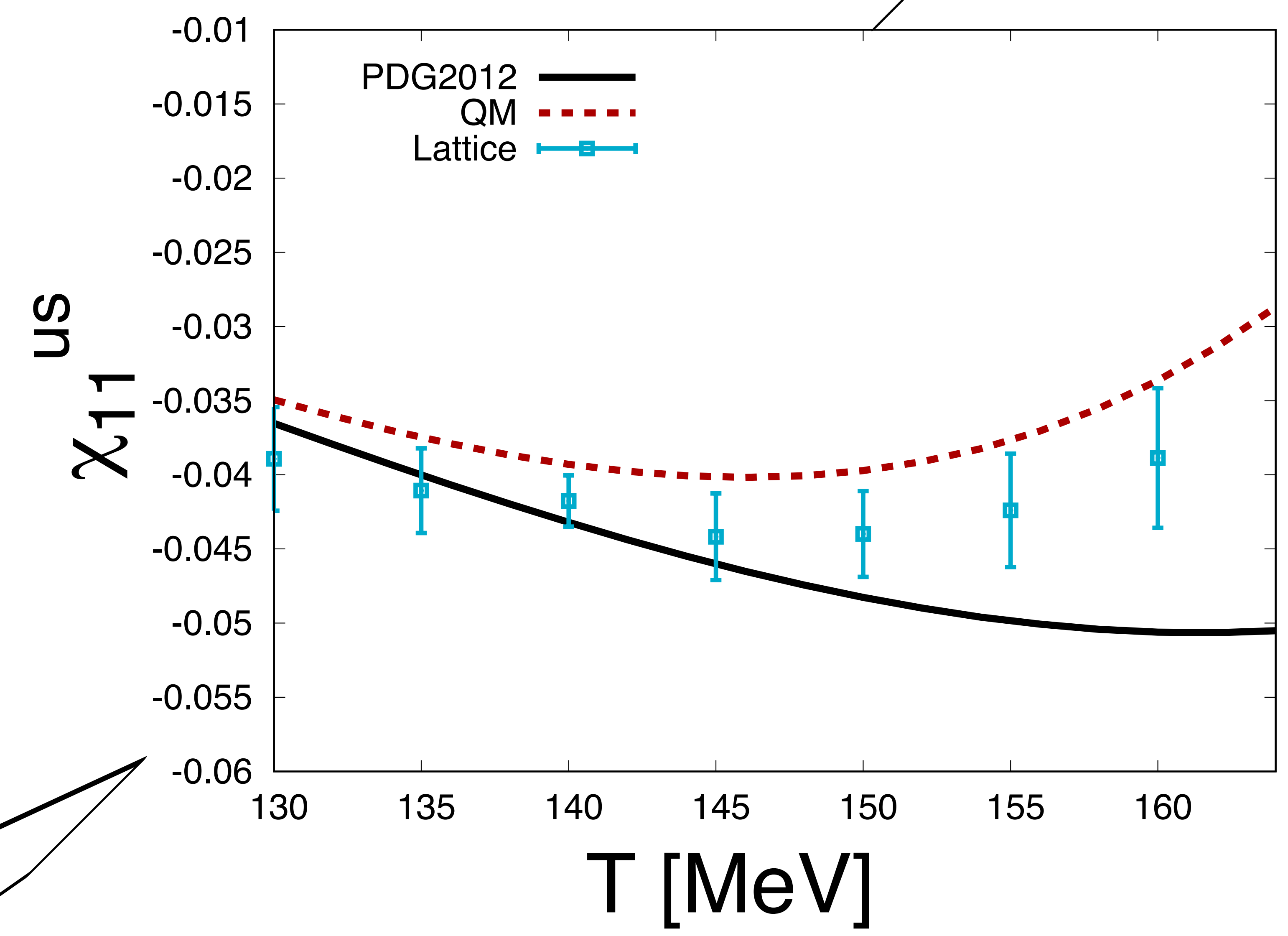

Strange susceptibilities also show discrepancies

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Susceptibilities beyond the Ideal HRG Model

Outline

I. Extensions of the ideal HRG model

- ➤ Excluded volume repulsive interactions
- ➤ Additional states in the hadronic spectrum than those that are well-established by the Particle Data Group

II. Confronting EV-HRG Susceptibilities with Lattice QCD

- ➤ Excluded volume interaction sensitive susceptibility ratios
- ➤ Hadronic spectrum specific susceptibility ratios
- ➤ Strangeness susceptibilities

I. Extensions of the ideal HRG model

Modified HRG Model

- Improved HRG description of higher order cumulants with repulsive interactions
  - Minimalistic extension
- Complementary yet distinct effects can be constrained separately with specific susceptibility ratios:
  - Constrain the hard-core radius
    - Fourth-order cumulants and excluded volume: $\chi_4^B/\chi_2^B, \chi_{31}^{BQ}/\chi_{11}^{BQ}, \chi_{31}^{BS}/\chi_{11}^{BS}$
  - Constrain hadronic spectrum
    - Extra states from baryon correlators: $\chi_{11}^{BQ}/\chi_2^B, \chi_{11}^{BS}/\chi_2^B$

V. Vovchenko et al, PLB (2017)
JMK, V. Koch, C. Ratti, V. Vovchenko, PRD (2021)

Excluded Volume HRG Model

- Include repulsive interactions for baryons & antibaryons, leading to a sum of pressures for each type of hadron: $P = P_M^{\mathrm{id}} + P_B^{\mathrm{ev}} + P_{\bar{B}}^{\mathrm{ev}}$
- The pressure in the excluded volume model yields a transcendental equation:

$$P_{B(\bar{B})}^{\mathrm{ev}} = \sum_{i \in B} \frac{m_i^2 T^2}{2\pi^2} K_2(m_i/T) \exp(\pm\mu_i/T) \exp\left(\frac{-b\, P_{B(\bar{B})}^{\mathrm{ev}}}{T}\right),$$

which can be solved by making use of the Lambert W function:

$$P_{B(\bar{B})}^{\mathrm{ev}} = \frac{T}{b} W[b \sum_{i \in B} \frac{m_i^2 T^2}{2\pi^2} K_2(m_i/T) \exp(\pm\mu_i/T)].$$

V. Vovchenko et al, PLB (2017)
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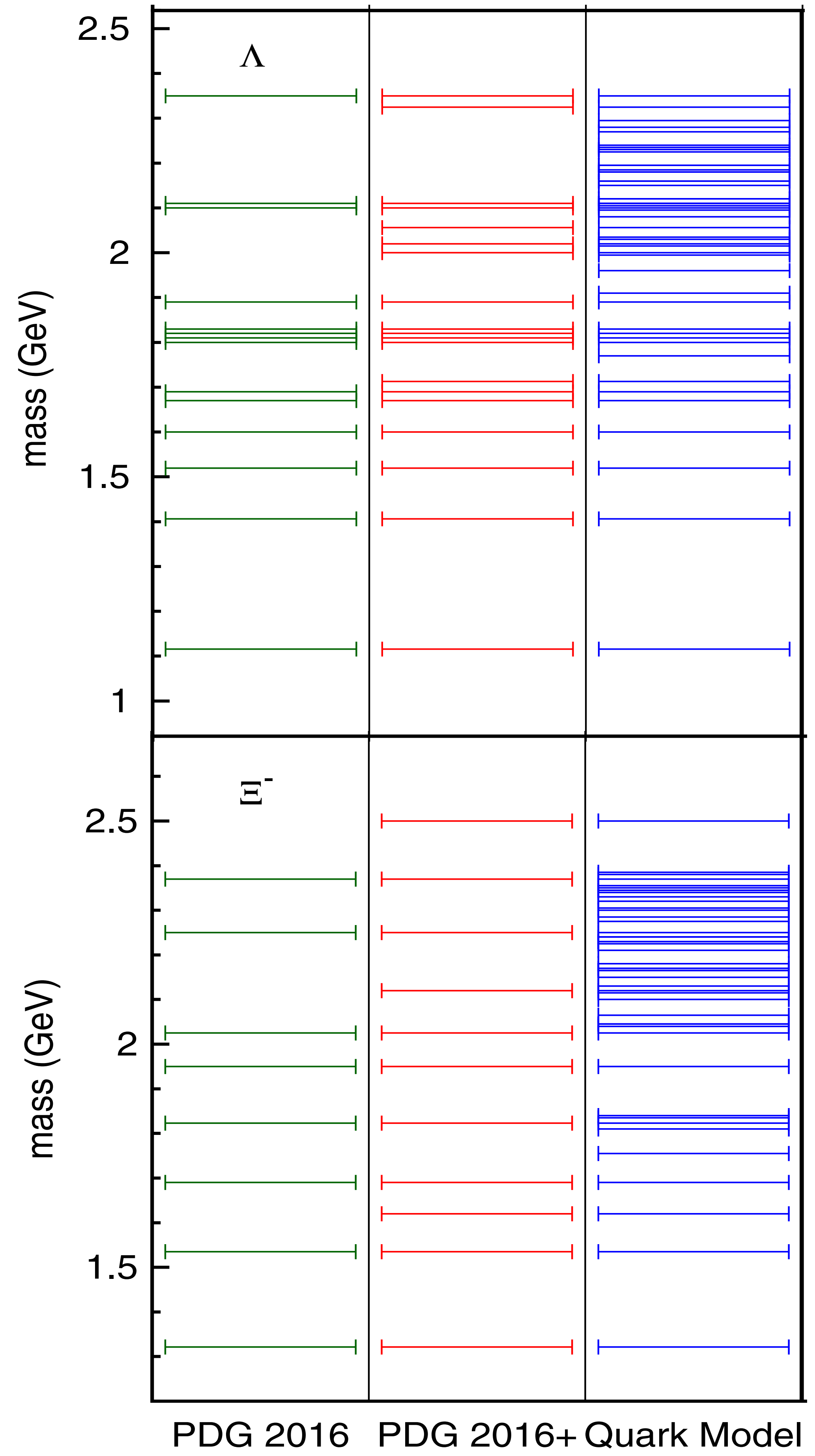
The Hadronic Spectrum

- Pressure in HRG model depends on resonances included in the calculation:

$$\frac{P}{T^4} = \frac{1}{VT^3} \sum_i \ln Z_i(T, V, \vec{\mu})$$

- **PDG2016**: 608 species
- **PDG2016+**: 738 species (all experimentally observed particles)
- **QM**: 1485 species (all states predicted by the Quark Model, updated in this work)

p	$1/2^+$	****
n	$1/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$\Delta(1232)$	$3/2^+$	****
$\Delta(1750)$	$1/2^+$	*

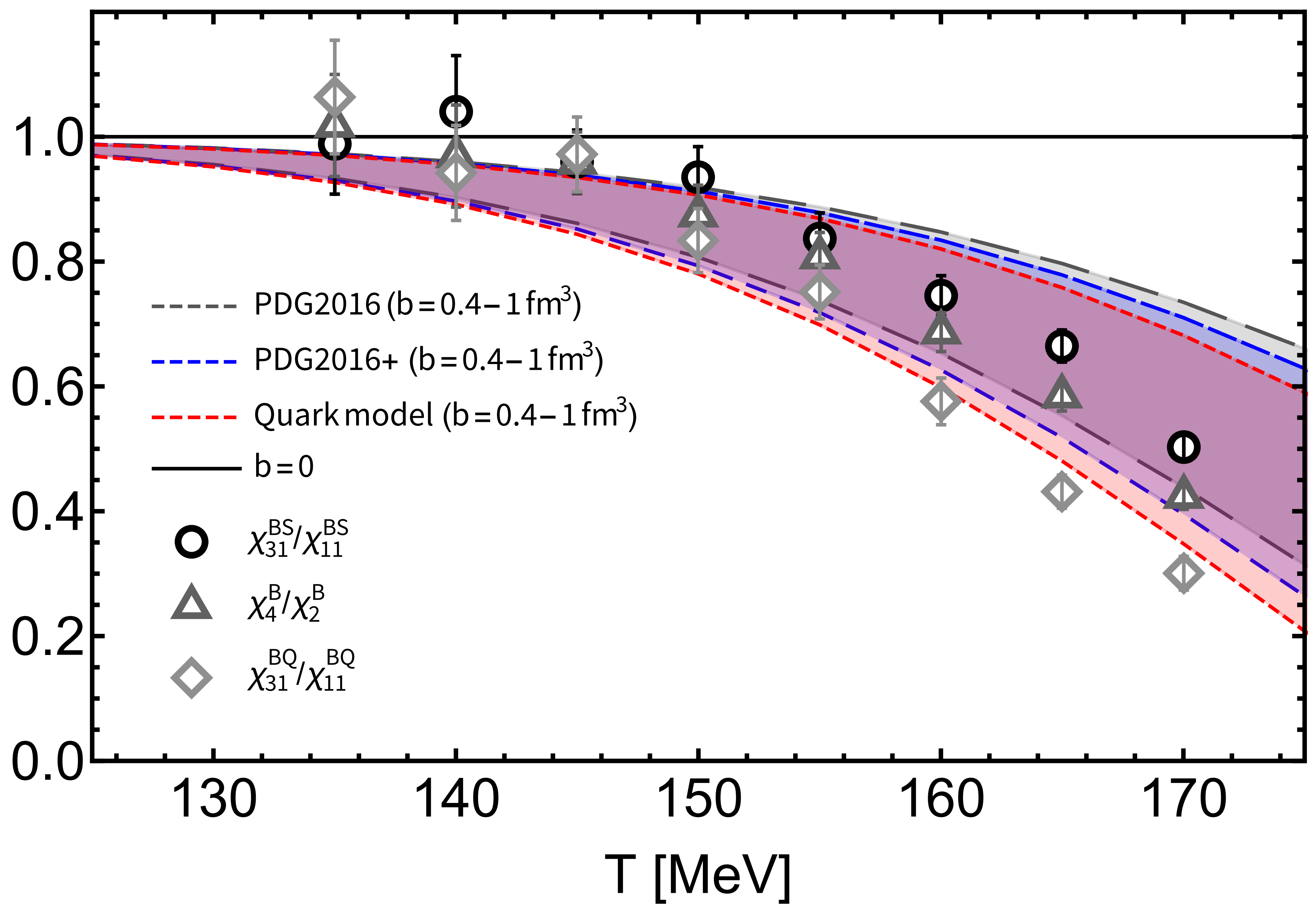


P. Alba et al (WB collaboration), PRD (2017)
JMK, V. Koch, C. Ratti, V. Vovchenko, PRD (2021)

II. Confronting EV-HRG Susceptibilities with Lattice QCD

EV-sensitive Susceptibility Ratios

- In the ideal HRG model, fourth-to-second order susceptibility ratios are predicted to be unity, while the EV-HRG model includes repulsive interaction terms:



4th-to-second-order ratios have:

- weak dependence on particle spectrum

JMK, V. Koch, C. Ratti, V. Vovchenko, PRD (2021)
Lattice data: S. Borsyani et al (WB collaboration), JHEP (2018)
See also: D. Bollweg et al (HotQCD collaboration), PRD (2021)

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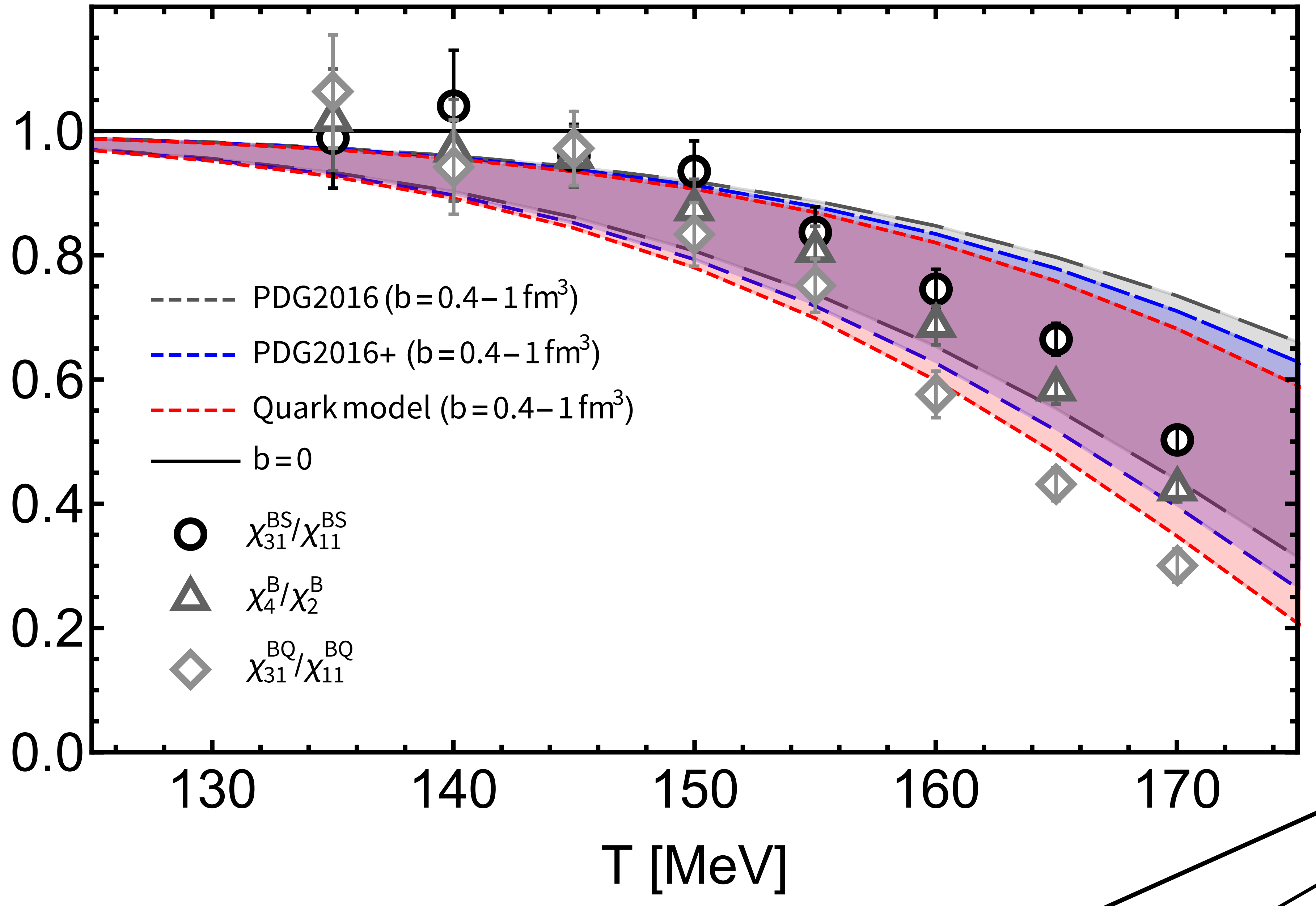

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Sensitive to excluded volume only

JMK, V. Koch, C. Ratti, V. Vovchenko, PRD (2021)
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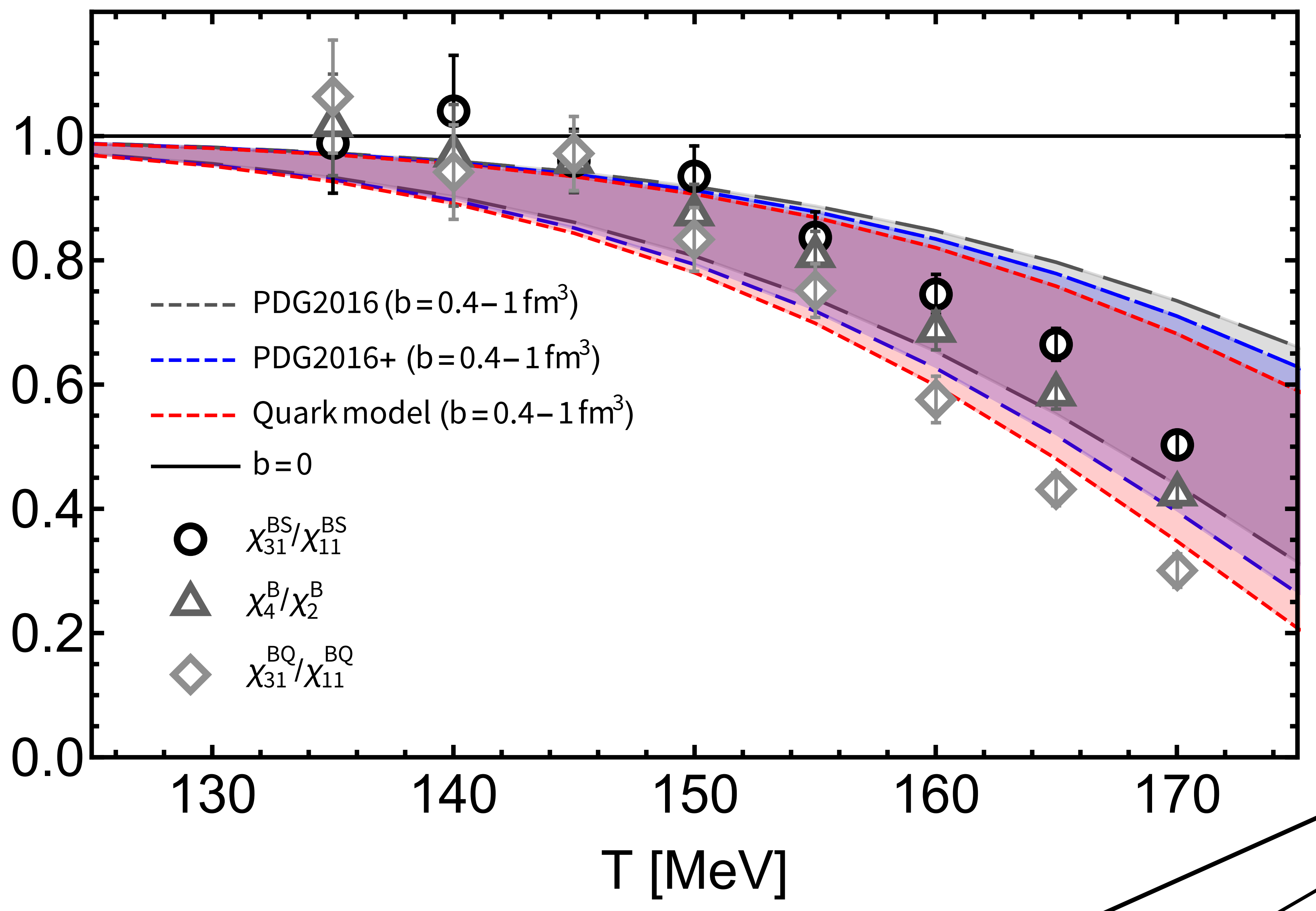

4th-to-second-order ratios have:

- weak dependence on particle spectrum
- the same EV corrections

$$\frac{\chi_4^B}{\chi_2^B} = \frac{\chi_{31}^{BS}}{\chi_{11}^{BS}} = \frac{\chi_{31}^{BQ}}{\chi_{11}^{BQ}} = \frac{1 - 8\,W(\varkappa_B) + 6[W(\varkappa_B)]^2}{[1 + W(\varkappa_B)]^4} = 1 - 12\varkappa_B + O(\varkappa_B^2)$$

where: $\varkappa_{B(\bar{B})}(T, \mu_B, \mu_Q, \mu_S) = b \sum_{i \in B} \tilde{\phi}_i(T) \exp(\pm\mu_i/T)$

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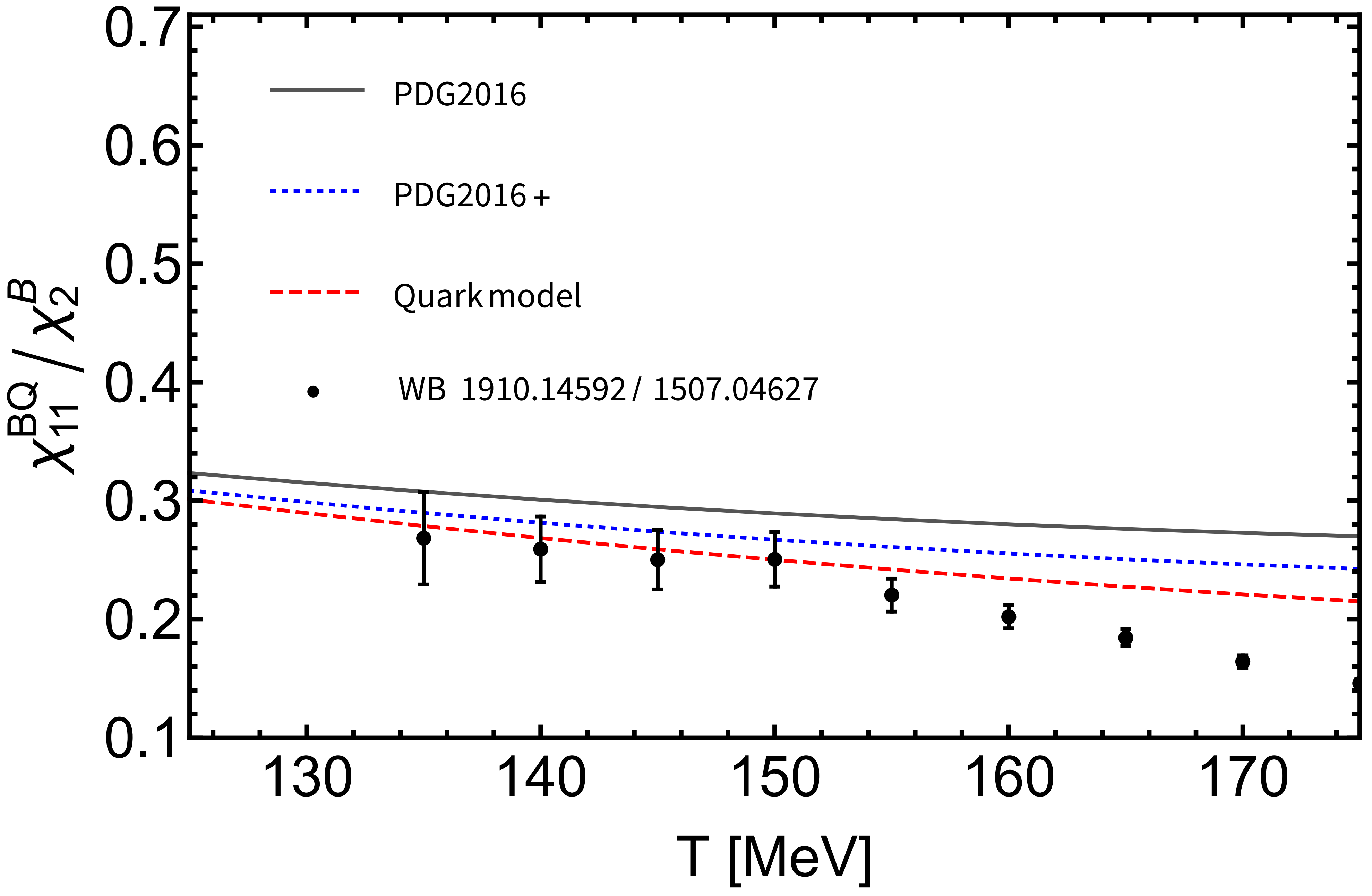
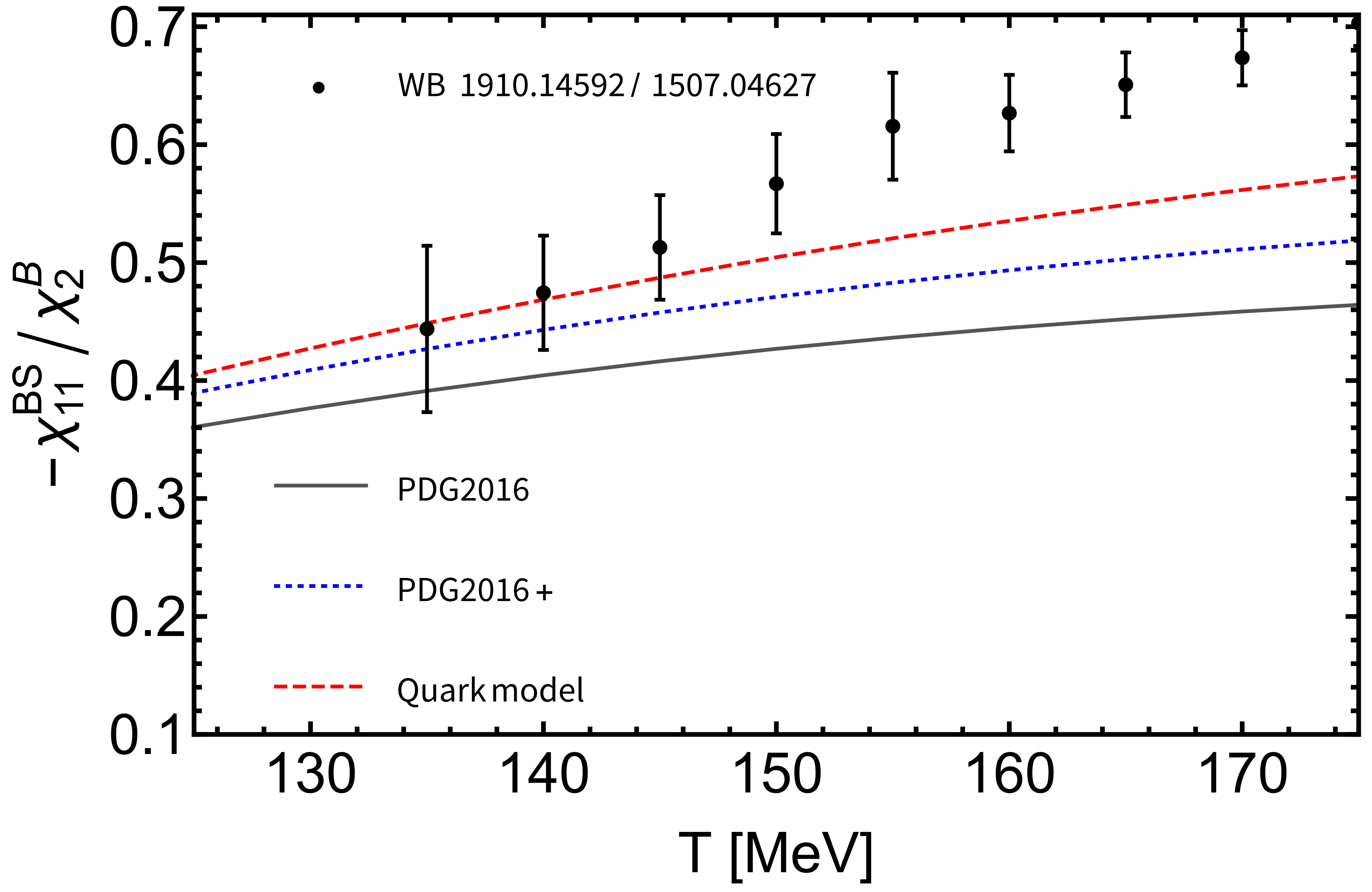
Hadronic-spectrum-sensitive Susceptibility Ratios

➤ In these second order ratios, the EV parameter, *b*, cancels exactly:

$$\frac{\chi_{11}^{BQ}}{\chi_2^B} = \frac{\sum_{j\in\text{sectors}} B_j\, Q_j\, \tilde{\phi}_j(T)}{\sum_{j\in\text{sectors}} B_j^2\, \tilde{\phi}_j(T)}$$





JMK, V. Koch, C. Ratti, V. Vovchenko, PRD (2021)
Lattice data: R. Bellwied et al (WB collaboration), PRD (2015, 2019)
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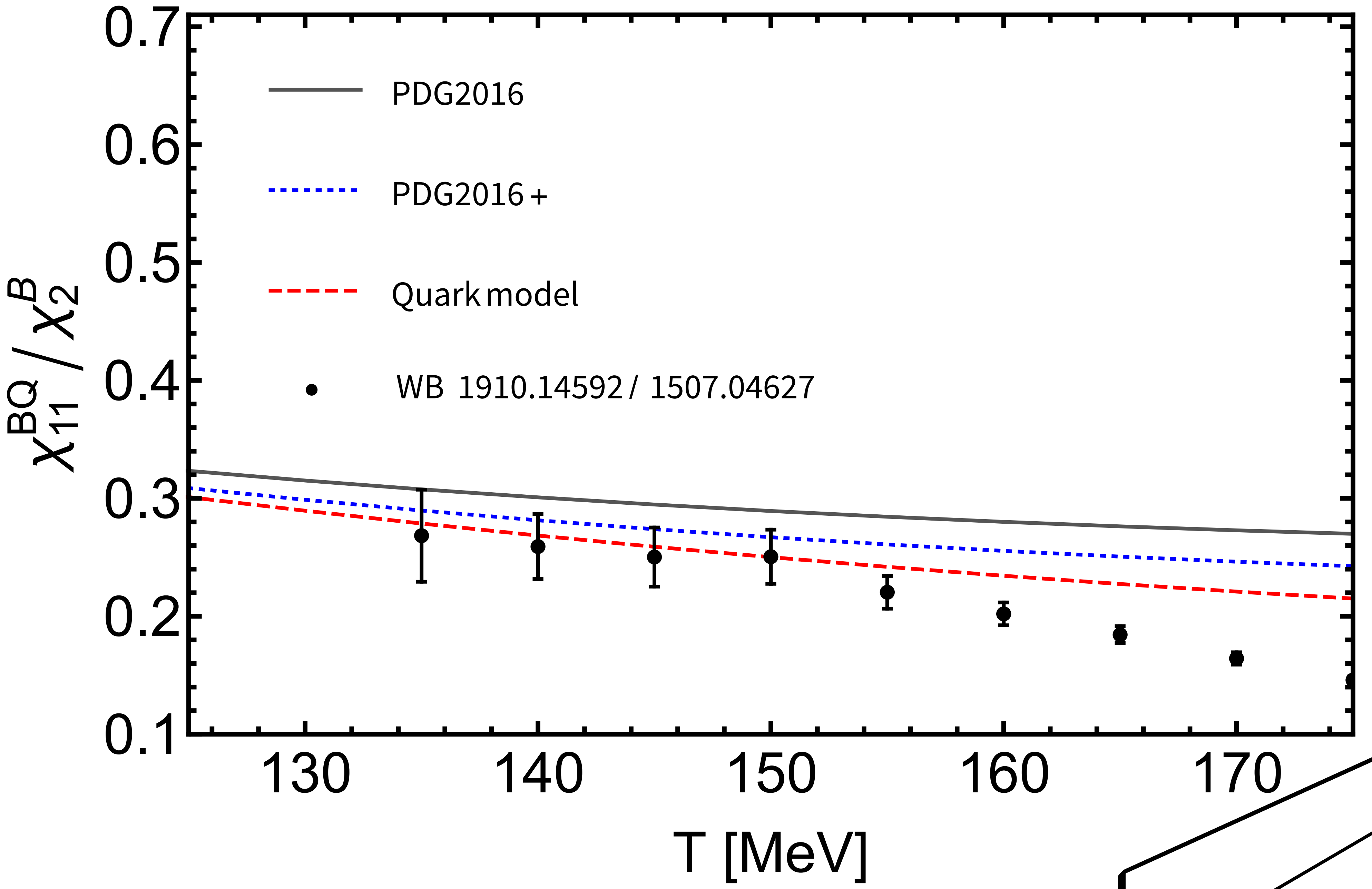

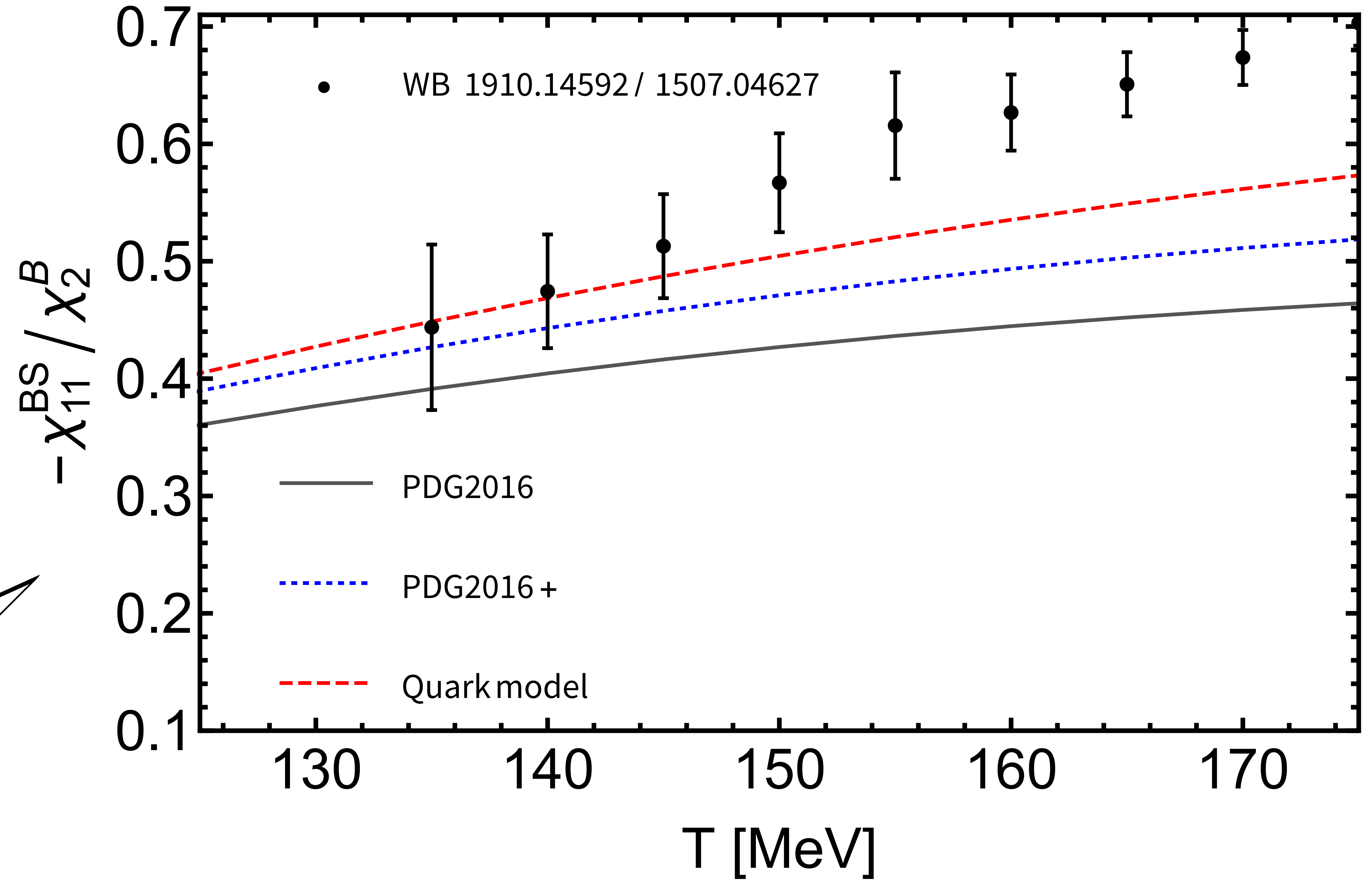

Sensitive to particle spectrum only

JMK, V. Koch, C. Ratti, V. Vovchenko, PRD (2021)
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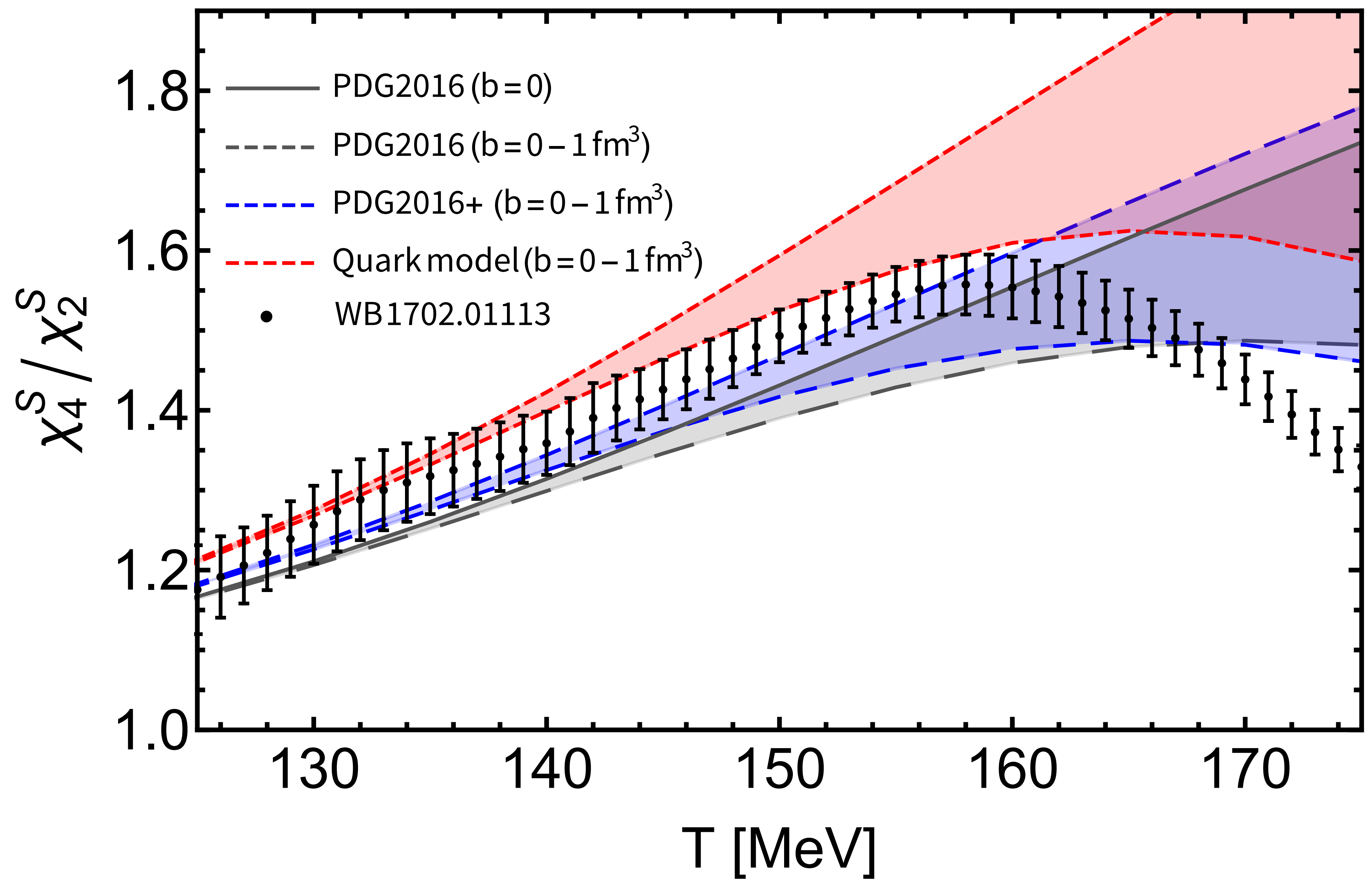
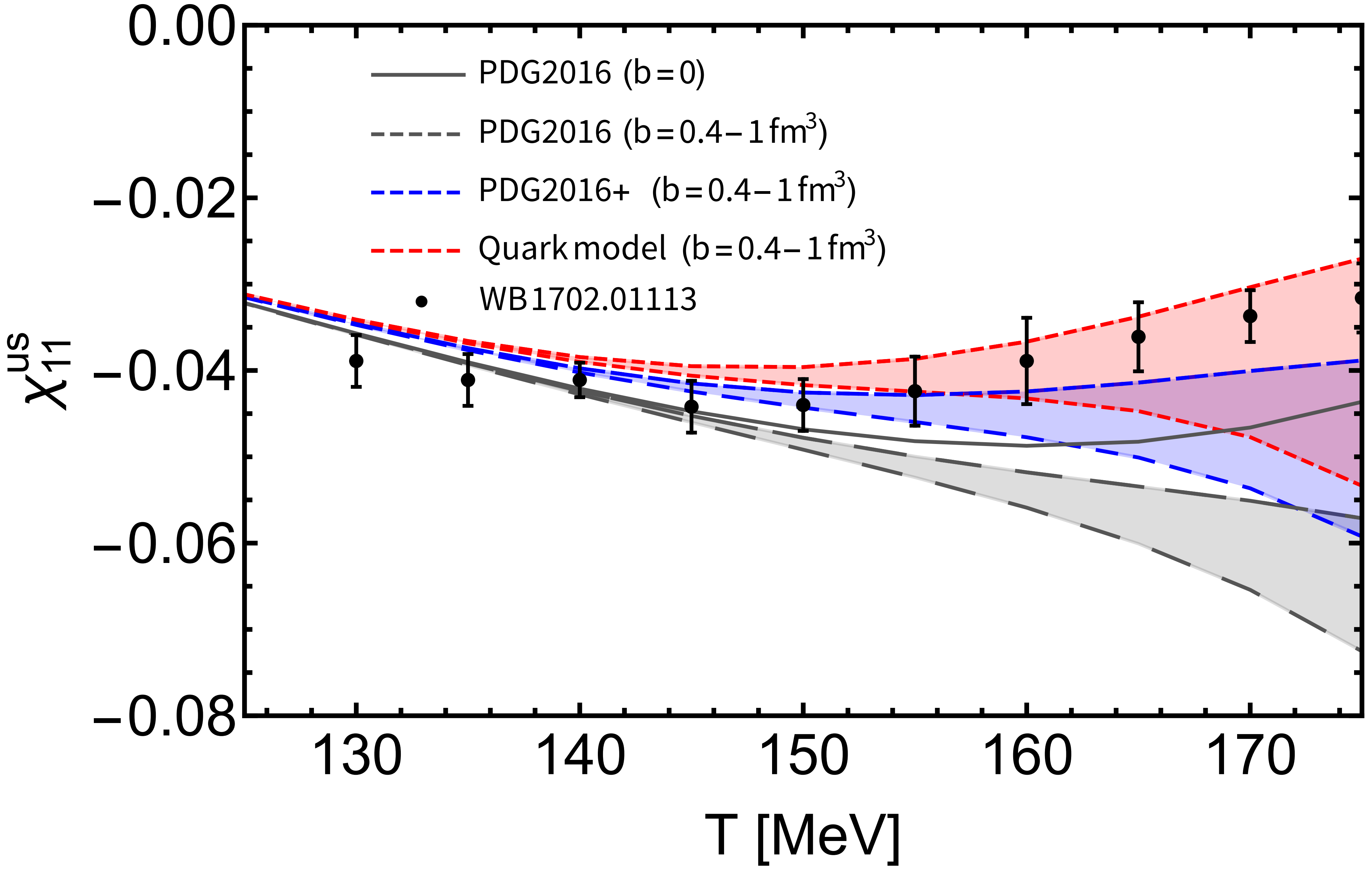
Strangeness Susceptibilities

- ➤ Further investigate strangeness-carrying quantities to study potential flavor hierarchy:
  - ➤ Smaller b value preferred for strangeness susceptibilities with PDG2016+





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Lattice data: P. Alba et al (WB collaboration), PRD (2017)
See also: D. Bollweg et al (HotQCD collaboration) PRD (2021)

Conclusions

- Improved HRG description of higher order cumulants with repulsive interactions
- Minimalistic extension of HRG constrains the excluded volume parameter and disentangles effects of particle spectrum with specific susceptibility ratios
- Best hadronic list: PDG2016+ list with a small b, QM list with a larger b
- Excluded volume parameter, b, shows a quark flavor hierarchy with smaller volumes for strangeness susceptibilities

Back-up Slides

Indications of Flavor Hierarchy