



SQM 2022

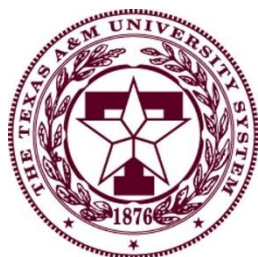
The 20th International Conference on Strangeness in Quark Matter
13-17 June 2022 Busan, Republic of Korea

Light nuclei production from nonlocal many-body scatterings

[arXiv:2106.12742\(2021\)](https://arxiv.org/abs/2106.12742)

Kai-Jia Sun (孙开佳)
(kjsun@tamu.edu)

In collaboration with **Rui Wang**, Che-Ming Ko, Yu-Gang Ma, and Chun Shen



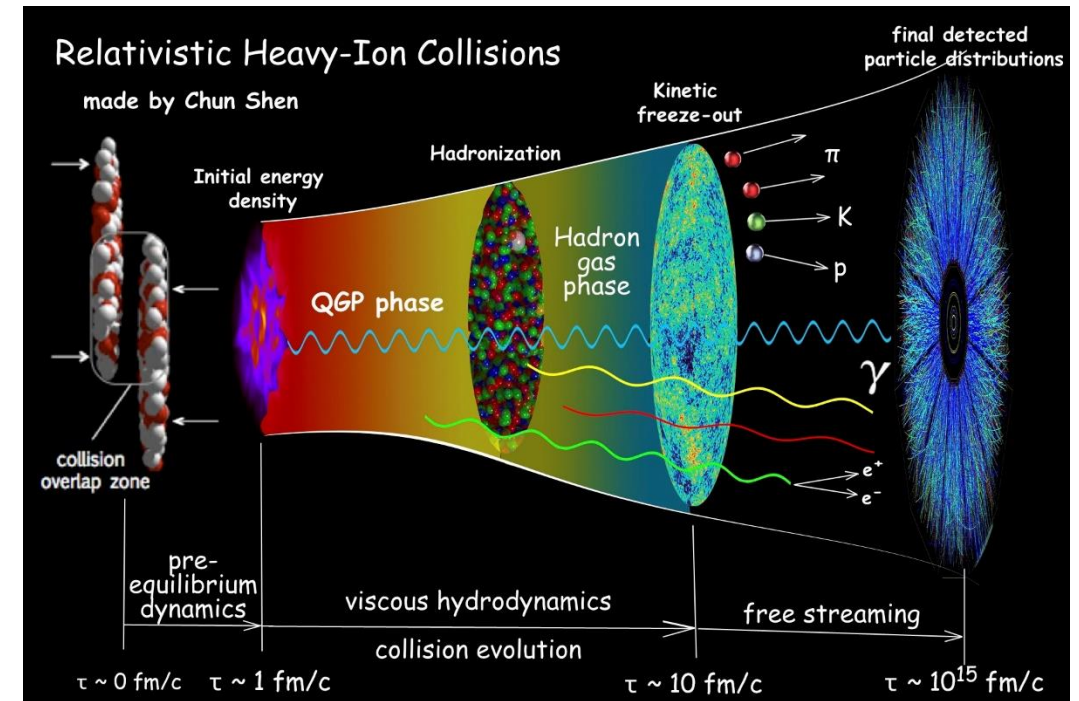
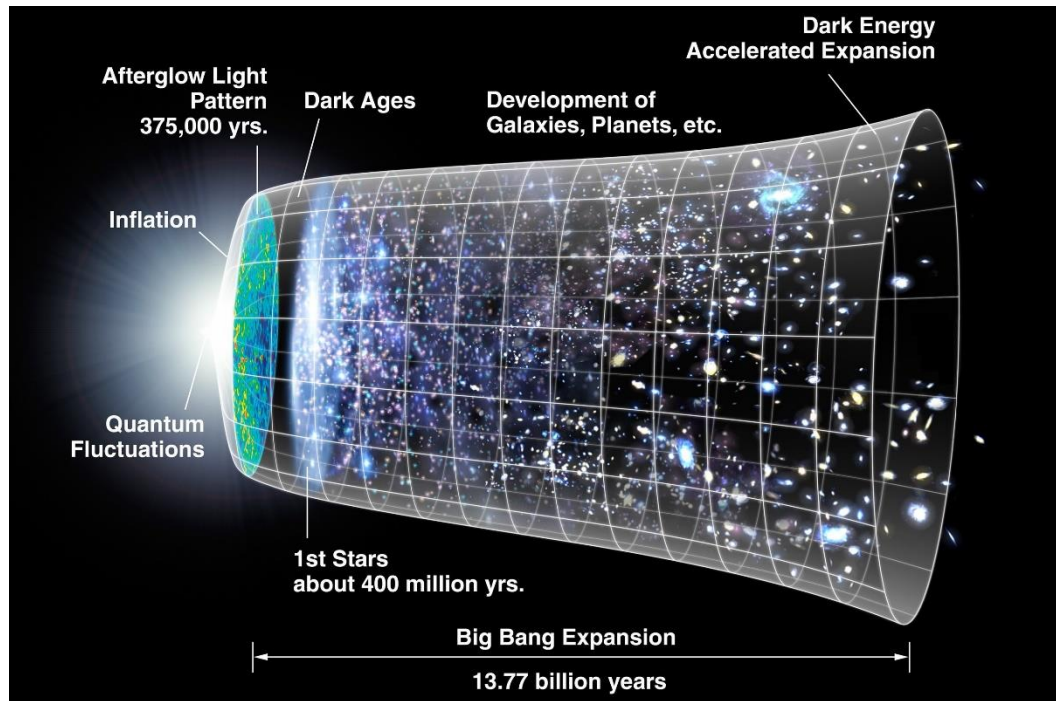
Cyclotron Institute

TEXAS A&M
UNIVERSITY®

Nucleosynthesis in Big Bang and Little Bang (1)

J. Chen et al., Phys. Rep. 760, 1 (2018)

P. Braun-Munzinger and B. Donigus NPA987, 144 (2019)



Big bang nucleosynthesis is responsible for the formation of d , ${}^3\text{He}$, ${}^4\text{He}$ in the Universe. Relativistic high-energy nuclear collisions provide a unique site for antinuclei production.

$$t \sim 100 \text{ s}, T \sim \text{a few MeV}$$

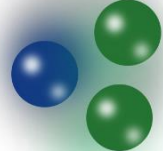
$$t \sim 10^{-22} \text{ s}, T \sim 100 \text{ MeV}$$

Light (anti-)nuclei in high-energy nuclear collisions (2)

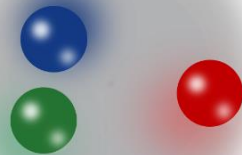
d (np)
B=2.2 MeV



^3He (npp)
B=7.7 MeV



$^3_\Lambda\text{H}$ (npΛ)
 $B_{\Lambda d} = 0.41$ MeV



STAR(Nat. Phys. 16 409(2020))

Deuteron (np)

Triton (nnp)

Helium-3 (npp)

Hypertriton (npΛ)

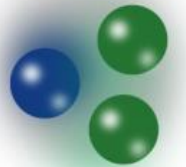
Helium-4 (nnpp)



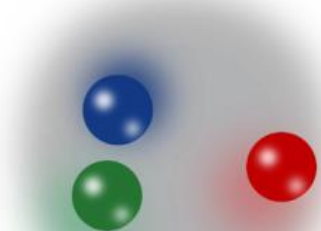
\bar{d}



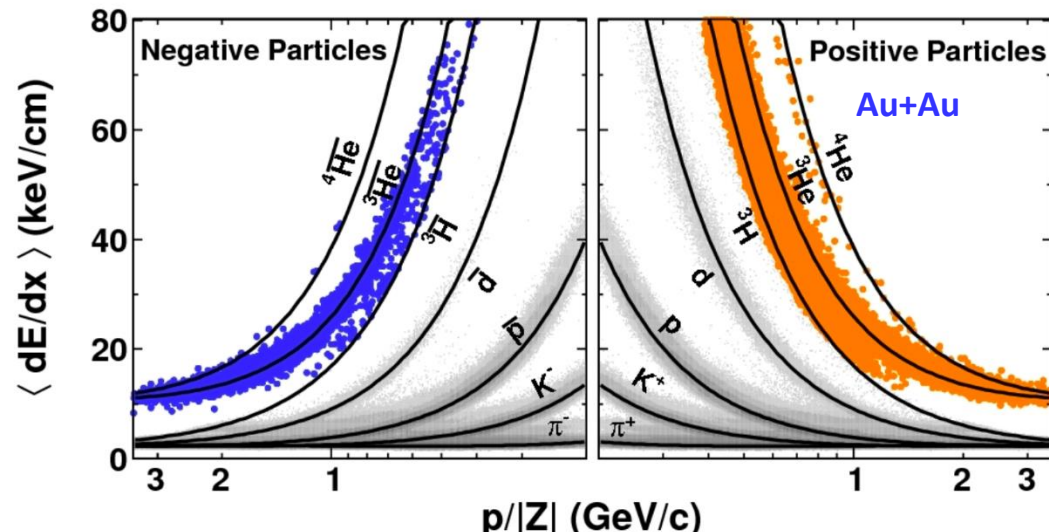
$^3\bar{\text{He}}$



$^3_\Lambda\bar{\text{H}}$



STAR (Nature 473,353(2011)) STAR (Science 328, 58(2010)) ALICE (PRC93,024917(2015))



1. Rarely produced, suppression by $e^{-m_A/T}$
 2. Binding energies (E_B) $\ll T_c$ (~ 154 MeV) $\ll m_N$ (938 MeV)
- The size $r \sim \frac{1}{\sqrt{4\mu E_B}}$, ($r_d \sim 2$ fm, $r_{^3\text{He}} \sim 2$ fm, $r_{^3_\Lambda\text{H}} \sim 5$ fm)

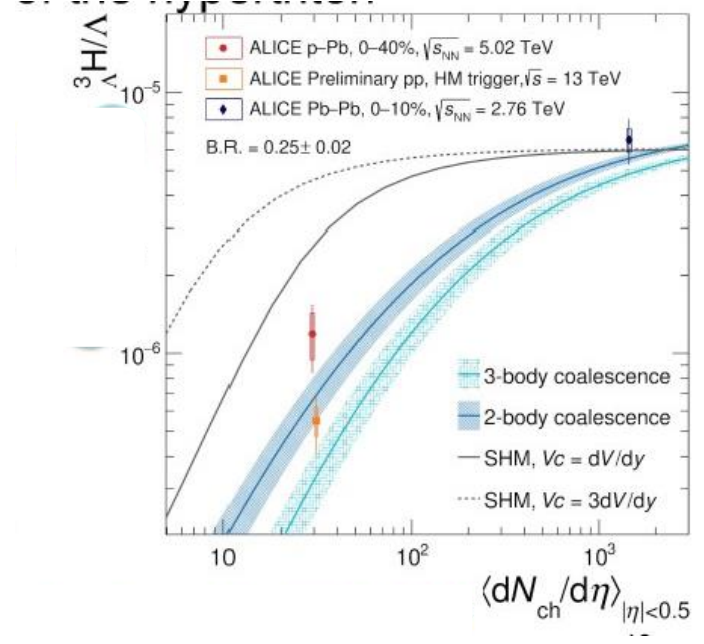
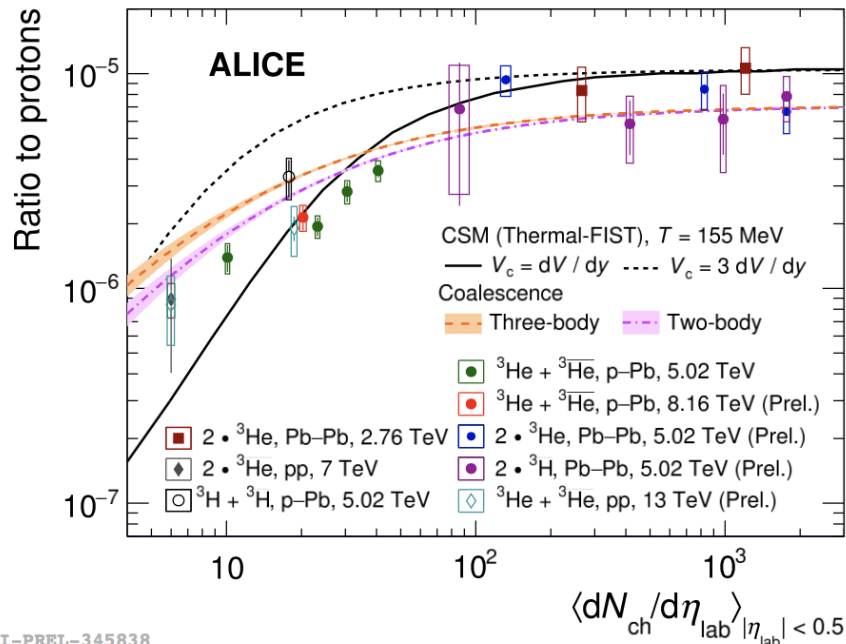
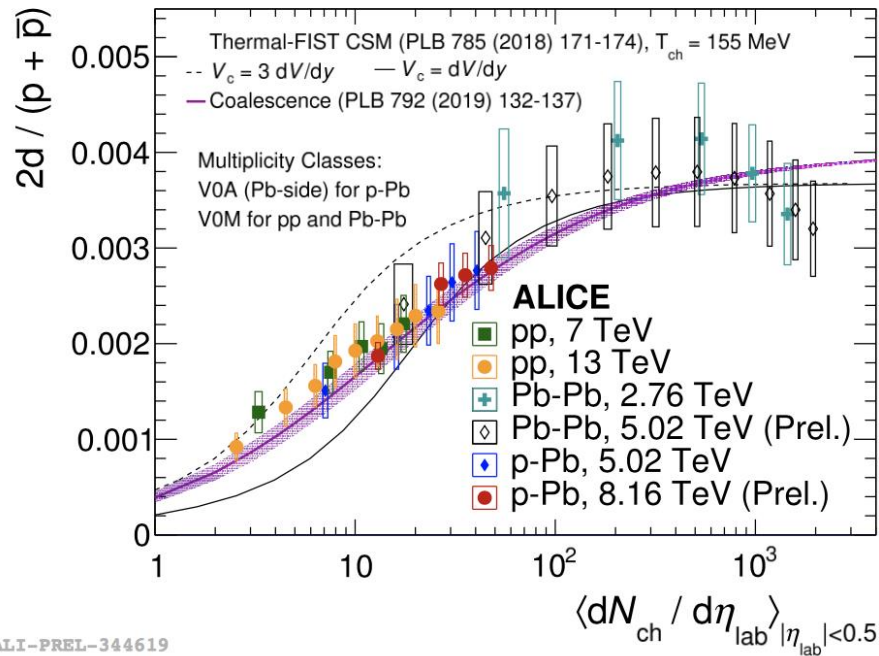
Light nuclei production & their internal structures

(3)

E. Bartsch for ALICE Collaboration, J. Phys. Conf. Ser. 1602, 012022 (2020); arXiv:2107.10627

Coal: Finite nuclei sizes lead to suppression of deuteron and helium-3 yields in collision of small system (better overall description)

SHM: Canonical effects lead to suppression of light nuclei production



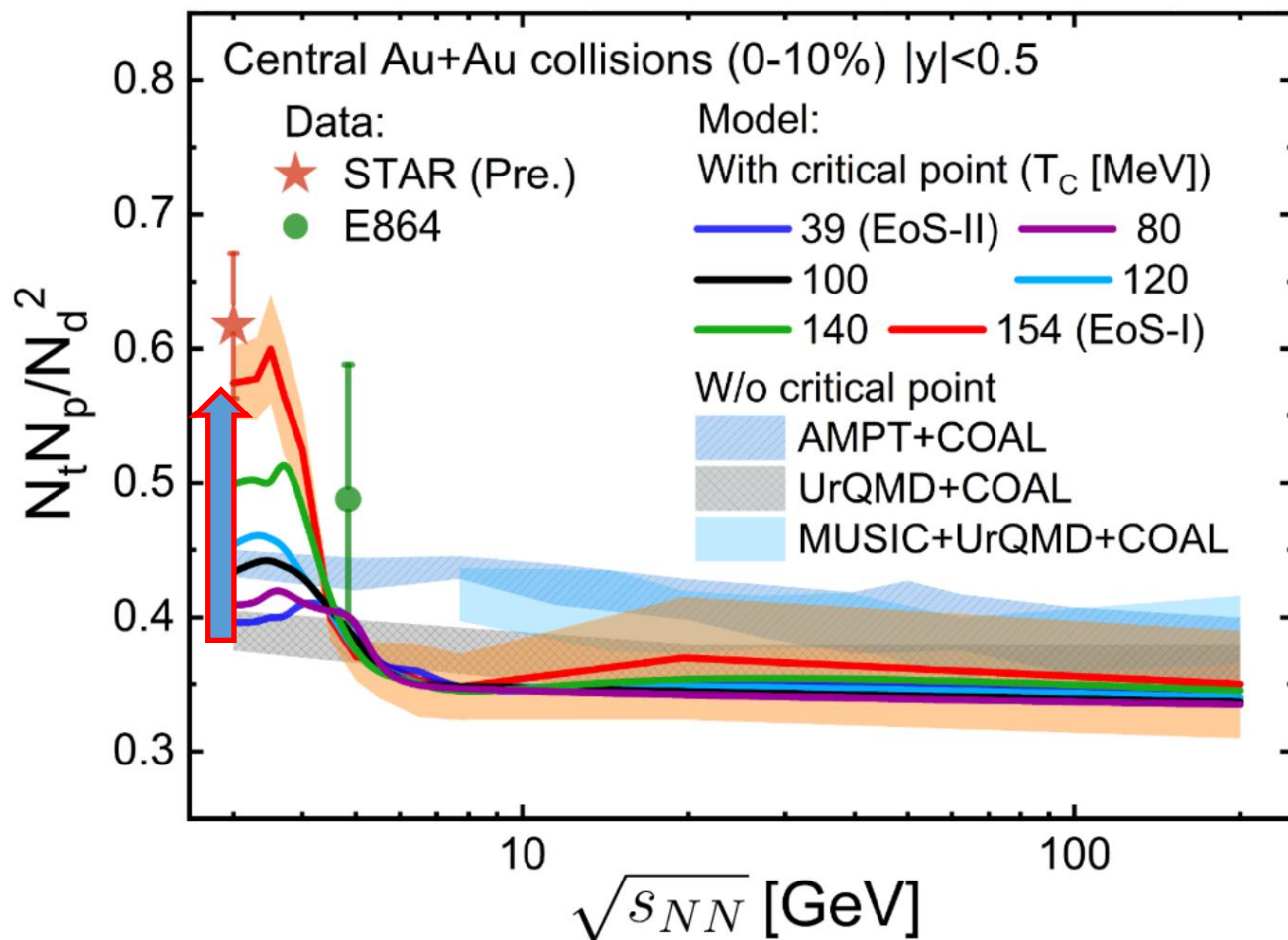
The size effects of loosely bound nuclei may be utilized to decipher internal structure of more exotic QCD molecular states, e.g., X(3872).

CSM: V. Vovchenko et al., PLB 785, 171 (2019), PRC 100,054906 (2019)

Coalescence: K. J. Sun, C. M. Ko and B. Donigus, Phys. Lett. B 792, 132 (2019)

Light nuclei production & QCD phase transitions (4)

K. J. Sun, W. H. Zhou, L. W. Chen, C. M. Ko, and F. Li, R. Wang, and J. Xu, arXiv:2205.11010(2022);



See poster session at 17:10 (POS-BLK-02)

1. Without a critical point:
The energy dependence of tp/d^2 is almost flat.
2. With a first-order phase transition:
The spinodal instability induced enhancement of tp/d^2 during the first-order phase transition increases as increasing the critical temperature.

Deuteron probes 2-body spatial correlation
Triton probes 2- and 3-body spatial correlations

Hui Liu (STAR), QM2022
T. A. Armstrong et al. (E864), Phys. Rev. C 61, 064908 (2000).

Main production mechanisms

(5)

⊗ Thermal model or statistical hadronization model

$$N_A \approx g_A V (2\pi m_A T)^{3/2} e^{(A\mu_B - m_A)/T}$$

Light nuclei are produced from hadronization of QGP,
and their yields remain unchanged during hadronic evolution

See talk by A. Andronic on 12/06

A. Andronic, P. Braun-Munzinger, J. Stachel, H. Stöcker, PLB 697, 203 (2011)
A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nature 561, 321 (2018)
V. Vovchenko et al., PLB800, 135131 (2020) (Saha Eq.); T. Neidig et al.,
PLB827,136891(2022)(Rate Eq.);...

⊗ Coalescence model

$$N_A = Tr(\hat{\rho}_s \hat{\rho}_A) = g_c \int d\Gamma \rho_s(\{x_i, p_i\}) \times W_A(\{x_i, p_i\})$$

Light nuclei are produced at the nucleon kinetic freezeout
where nucleons cease interactions.

H. Sato and K. Yazaki, PLB98, 153 (1981);
E. Remler, Ann. Phys. 136, 293 (1981);
M. Gyulassy, K. Frankel, and E. Remler, NPA402,596 (1983);
S. Mrowczynski, J. Phys. G 13, 1089 (1987);
S. Leupold and U. Heinz, PRC50, 1110 (1994);
R. Scheibl and U. W. Heinz, PRC59. 1585(1999);
K. J. Sun, C. M. Ko and B. Donigus, PLB 792, 132 (2019);
F. Bellini et al., PRC 103, 014907(2021);
W. Zhao et al., PLB 820, 136571(2021);
S. Wu et al., arXiv:2205.14302(2022);...

⊗ Kinetic approach

Light nuclei are continuously regenerated and
Dissociated during the hadronic evolution.

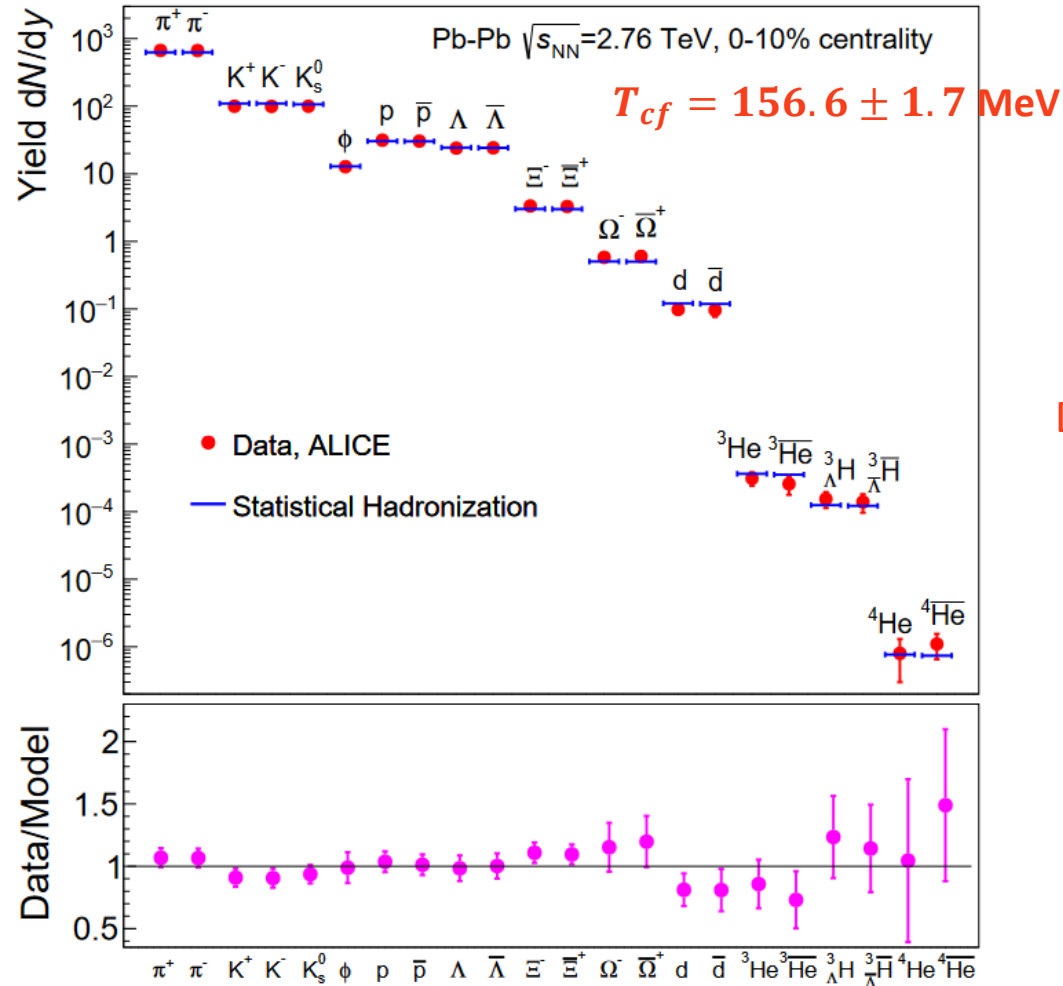
A.Z. Mekjian, PRC17,1051 (1978); P. Danielewicz, G.F. Bertsch, NPA533, 712 (1991);
P. Danielewicz and P. Schuck, PLB274, 268 (1992);
Y. Oh and C. M. Ko PRC76, 054910(2007);PRC80, 064902(2009);
D. Oliinychenko, L. G. Pang, H. Elfner, and V. Koch, PRC99, 044907 (2019);

With finite nuclei sizes: K. J. Sun, R. Wang, C. M. Ko, Y. G. Ma, and C. Shen, arXiv:2106.12742(2021)

The necessity of dynamical modeling

(6)

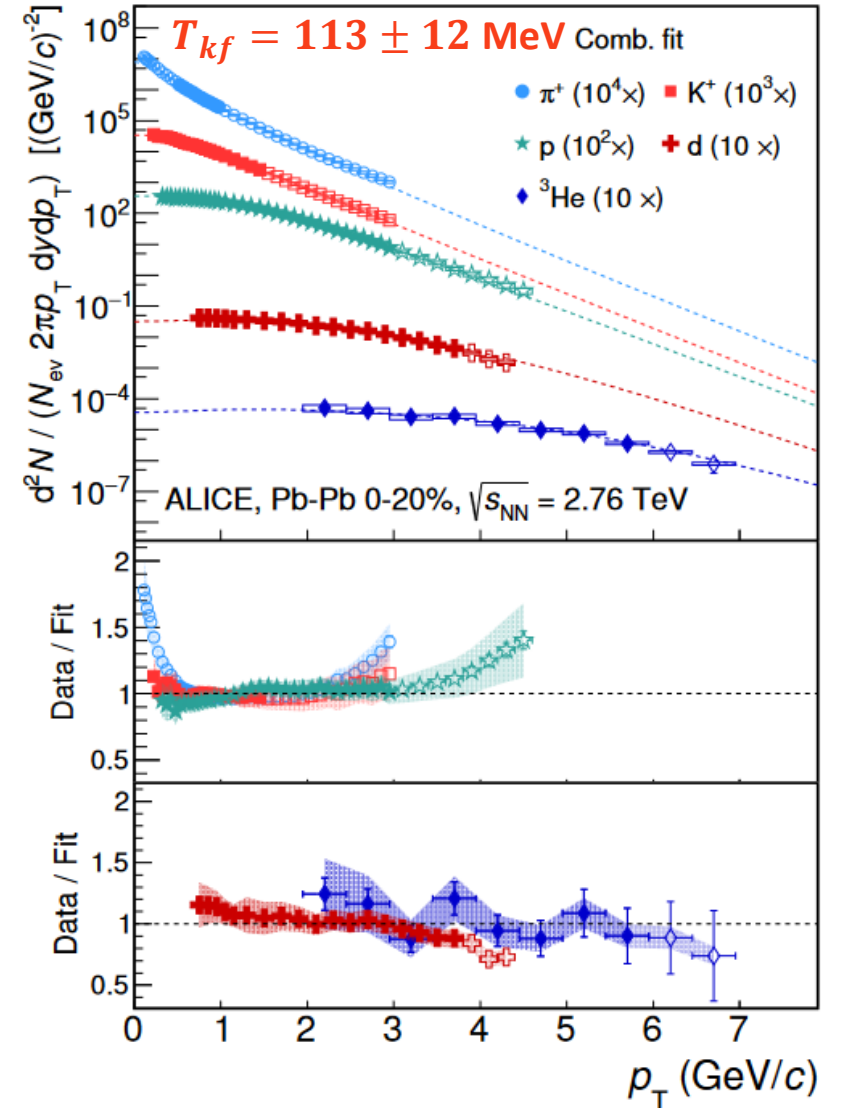
A. Andronic's talk



Dynamics in between?



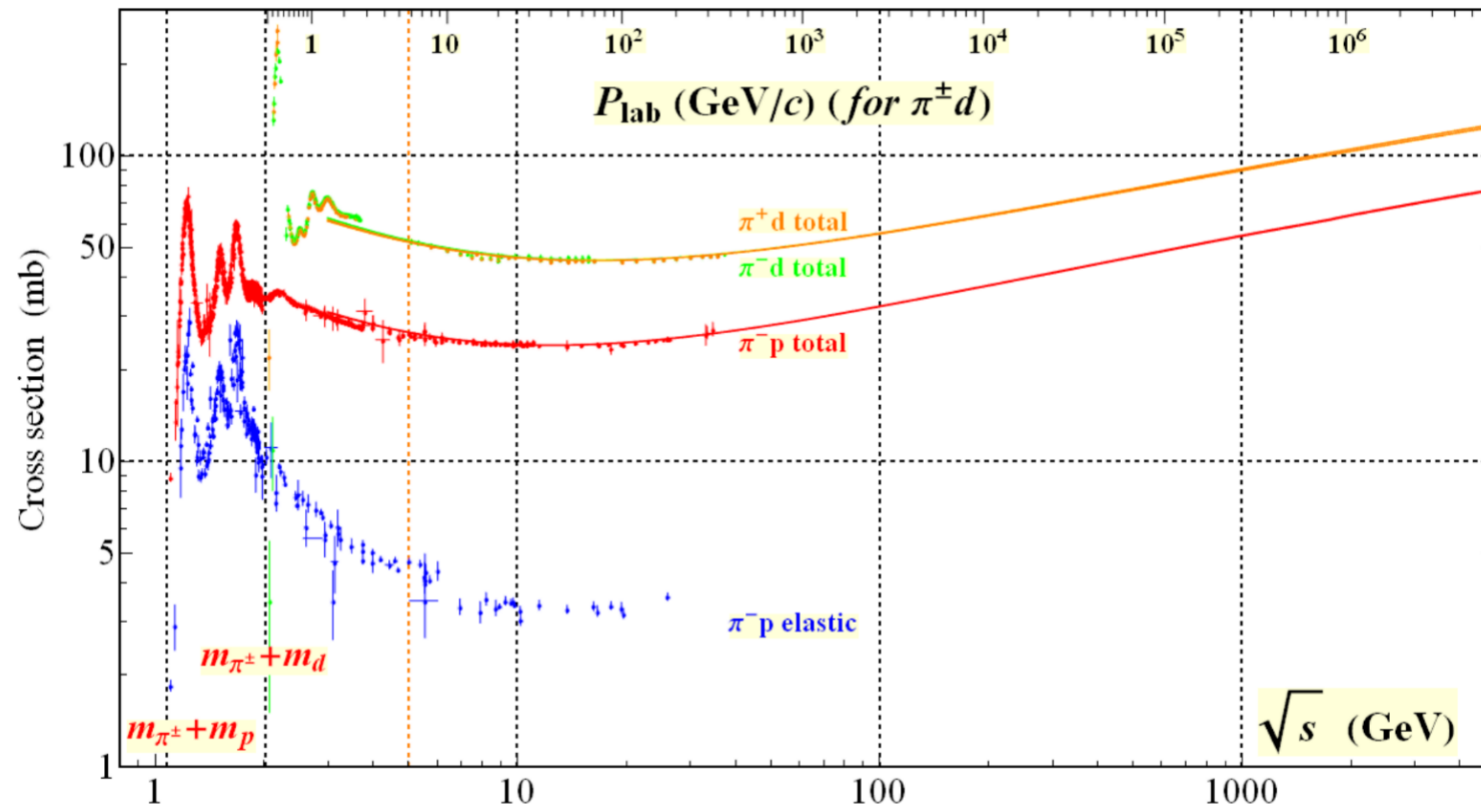
ALICE, Phys. Rev. C93, 024917(2016)



The necessity of dynamical modeling

(6)

PDG 2020



Large inelastic cross section for πd suggests the initially produced deuteron would be quickly disintegrated. The process $\pi d \leftrightarrow \pi NN$ needs to be considered.

A novel approach

(7)

arXiv:2106.12742(2021)

Length/energy scale:

$$\lambda_{thermal} \sim 0.5 \text{ fm} \ll r_{np} \sim 4 \text{ fm}$$

Impulse approximation (IA):

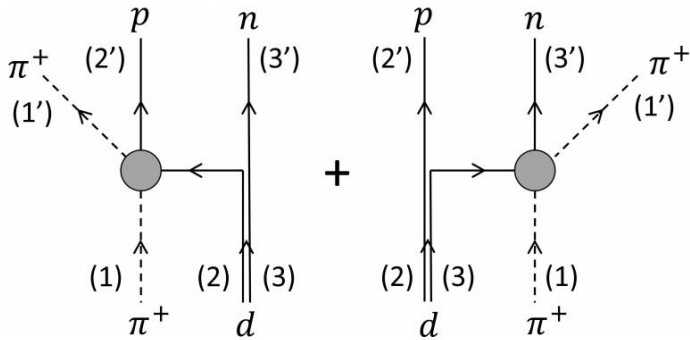


FIG. 1. Diagrams for the reaction $\pi^+ d \leftrightarrow \pi^+ np$ in the impulse approximation. The filled bubble indicates the intermediate states such as a Δ resonance.

Relativistic kinetic equation for $\pi NN \leftrightarrow \pi d$

$$\frac{\partial f_d}{\partial t} + \frac{\mathbf{P}}{E_d} \cdot \frac{\partial f_d}{\partial \mathbf{R}} = -\mathcal{K}^> f_d + \mathcal{K}^<(1 + f_d)$$

with collision integral:

$$\begin{aligned} \text{R.H.S.} = & \frac{1}{2g_d E_d} \int \prod_{i=1'}^{3'} \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i} \frac{d^3 \mathbf{p}_\pi}{(2\pi)^3 2E_\pi} \frac{E_d d^3 \mathbf{r}}{m_d} \\ & \times 2m_d W_d(\tilde{\mathbf{r}}, \tilde{\mathbf{p}}) \left(|\overline{\mathcal{M}}_{\pi+n \rightarrow \pi+n}|^2 + n \leftrightarrow p \right) \\ & \times \left[- \left(\prod_{i=1'}^{3'} (1 \pm f_i) \right) g_\pi f_\pi g_d f_d + \frac{3}{4} \left(\prod_{i=1'}^{3'} g_i f_i \right) \right. \\ & \left. \times (1 + f_\pi)(1 + f_d) \right] \times (2\pi)^4 \delta^4(p_{\text{in}} - p_{\text{out}}) \end{aligned}$$

Nonlocal collision integral to take into account the effects of finite nuclei sizes. W_d denotes deuteron Wigner function.

Solving kinetic equations with the stochastic method

(8)

arXiv:2106.12742(2021)

Relativistic kinetic equation for $\pi NN \leftrightarrow \pi d$

$$\frac{\partial f_d}{\partial t} + \frac{\mathbf{P}}{E_d} \cdot \frac{\partial f_d}{\partial \mathbf{R}} = -\mathcal{K}^> f_d + \mathcal{K}^<(1 + f_d)$$

with nonlocal collision integral:

$$\begin{aligned} \text{R.H.S.} = & \frac{1}{2g_d E_d} \int \prod_{i=1'}^{3'} \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i} \frac{d^3 \mathbf{p}_\pi}{(2\pi)^3 2E_\pi} \frac{E_d d^3 \mathbf{r}}{m_d} \\ & \times 2m_d W_d(\tilde{\mathbf{r}}, \tilde{\mathbf{p}}) (|\mathcal{M}_{\pi+n \rightarrow \pi+n}|^2 + n \leftrightarrow p) \\ & \times \left[- \left(\prod_{i=1'}^{3'} (1 \pm f_i) \right) g_\pi f_\pi g_d f_d + \frac{3}{4} \left(\prod_{i=1'}^{3'} g_i f_i \right) \right. \\ & \left. \times (1 + f_\pi)(1 + f_d) \right] \times (2\pi)^4 \delta^4(p_{\text{in}} - p_{\text{out}}) \end{aligned}$$

Stochastic method with test particles

Probability for reaction $\pi d \leftrightarrow \pi NN$ to take place in volume ΔV and time interval Δt are given by

$$\begin{aligned} \longrightarrow P_{23}|_{\text{IA}} & \approx F_d v_{\pi+p} \sigma_{\pi+p \rightarrow \pi+p} \frac{\Delta t}{N_{\text{test}} \Delta V} + (p \leftrightarrow n), \\ P_{32}|_{\text{IA}} & \approx \frac{3}{4} F_d v_{\pi+p} \sigma_{\pi+p \rightarrow \pi+p} \frac{\Delta t W_d}{N_{\text{test}}^2 \Delta V} + (p \leftrightarrow n) \end{aligned}$$

For triton or helium-3:

$$P_{42}|_{\text{IA}} \approx \frac{1}{4} F_t \frac{v_{\pi N} \sigma_{\pi N \rightarrow \pi N} \Delta t}{N_{\text{test}}^3 \Delta V} W_t$$

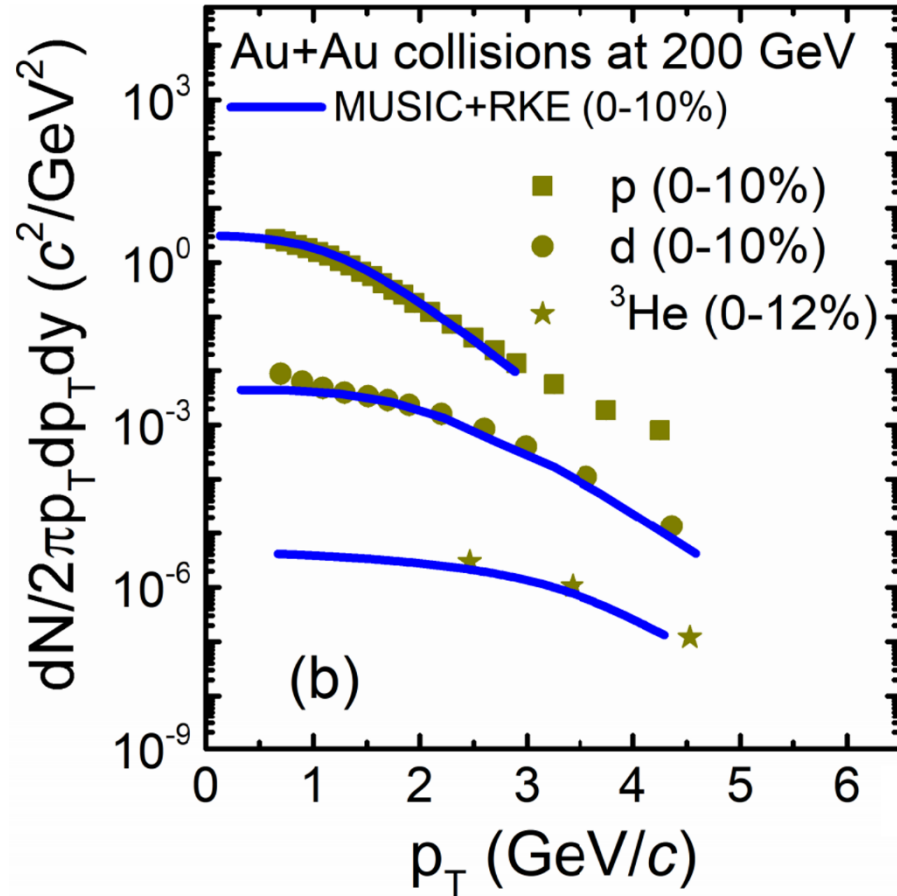
'renormalization' factor F_d, F_t which can be fixed by πd and πt cross sections.

Results

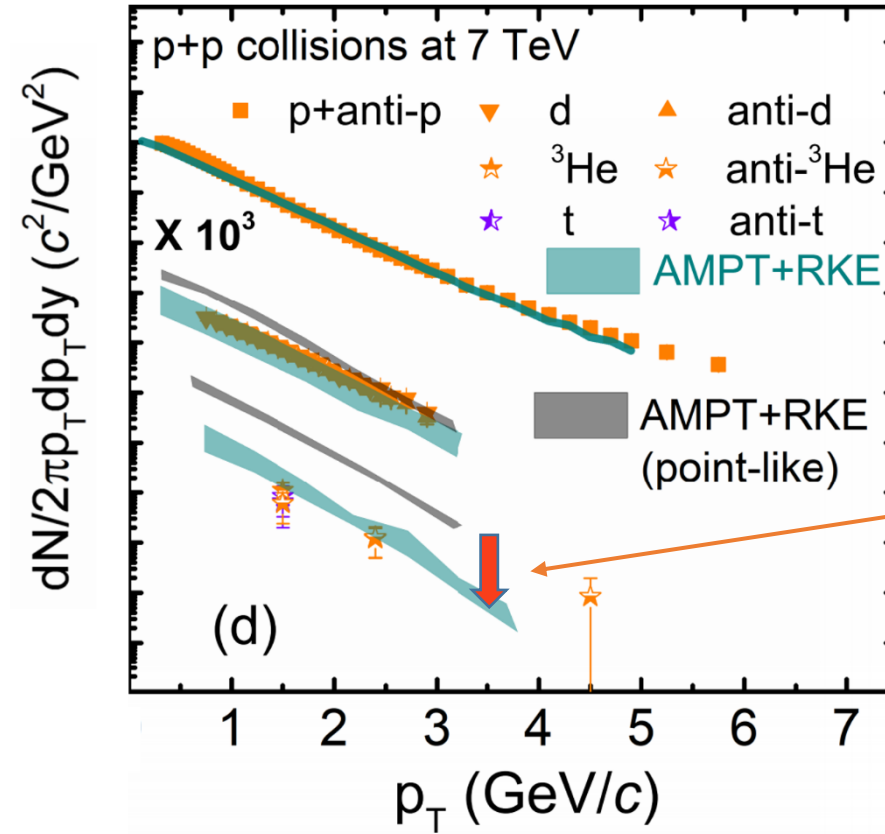
(9)

arXiv:2106.12742(2021)

Large collision system



Small collision system



size effects

Both yields and PT spectra are well reproduced.



SQM 2022

The 20th International Conference on Strangeness in Quark Matter
13-17 June 2022 Busan, Republic of Korea

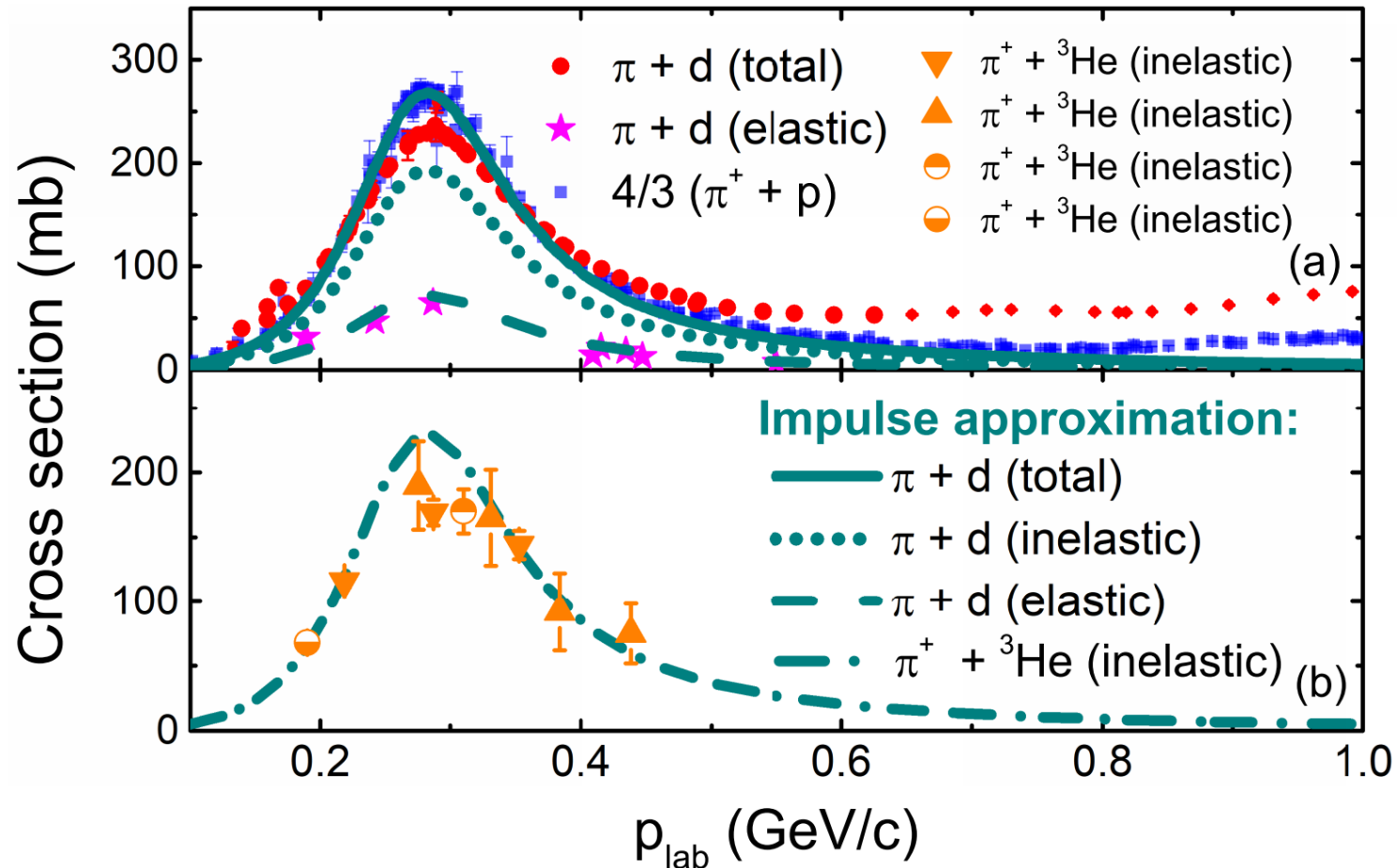
Summary

We have successfully developed a novel kinetic approach to light nuclei production in high-energy nuclear collisions, with the inclusion of many-body scatterings and finite nuclei sizes. This kinetic approach is shown to describe well both the yields and the spectra of deuteron and helium-3 in both large and small systems of collisions at very high energies.

[arXiv:2106.12742\(2021\)](https://arxiv.org/abs/2106.12742)

Cross sections under impulse approximation (IA) (9)

(9)



$$\sigma_{\pi+d \rightarrow \pi+np} = F_d (\sigma_{\pi+n \rightarrow \pi+n} + \sigma_{\pi+p \rightarrow \pi+p}) \quad F_d \approx 0.72$$