Causality violations in realistic nuclear collision simulations

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Based on: arXiv:2103.15889 [accepted as PRC Letter]

See also:
PRC 103 6, 064901 (2021)

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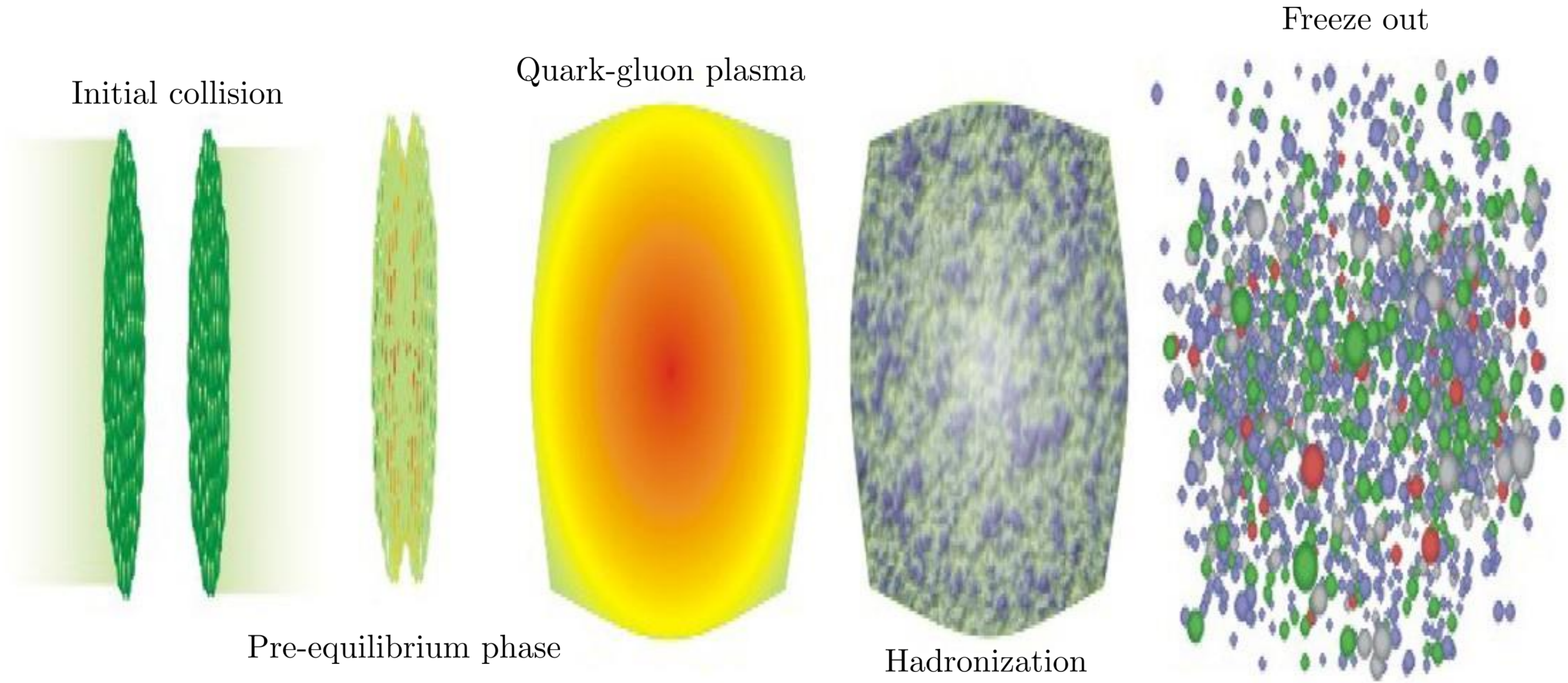
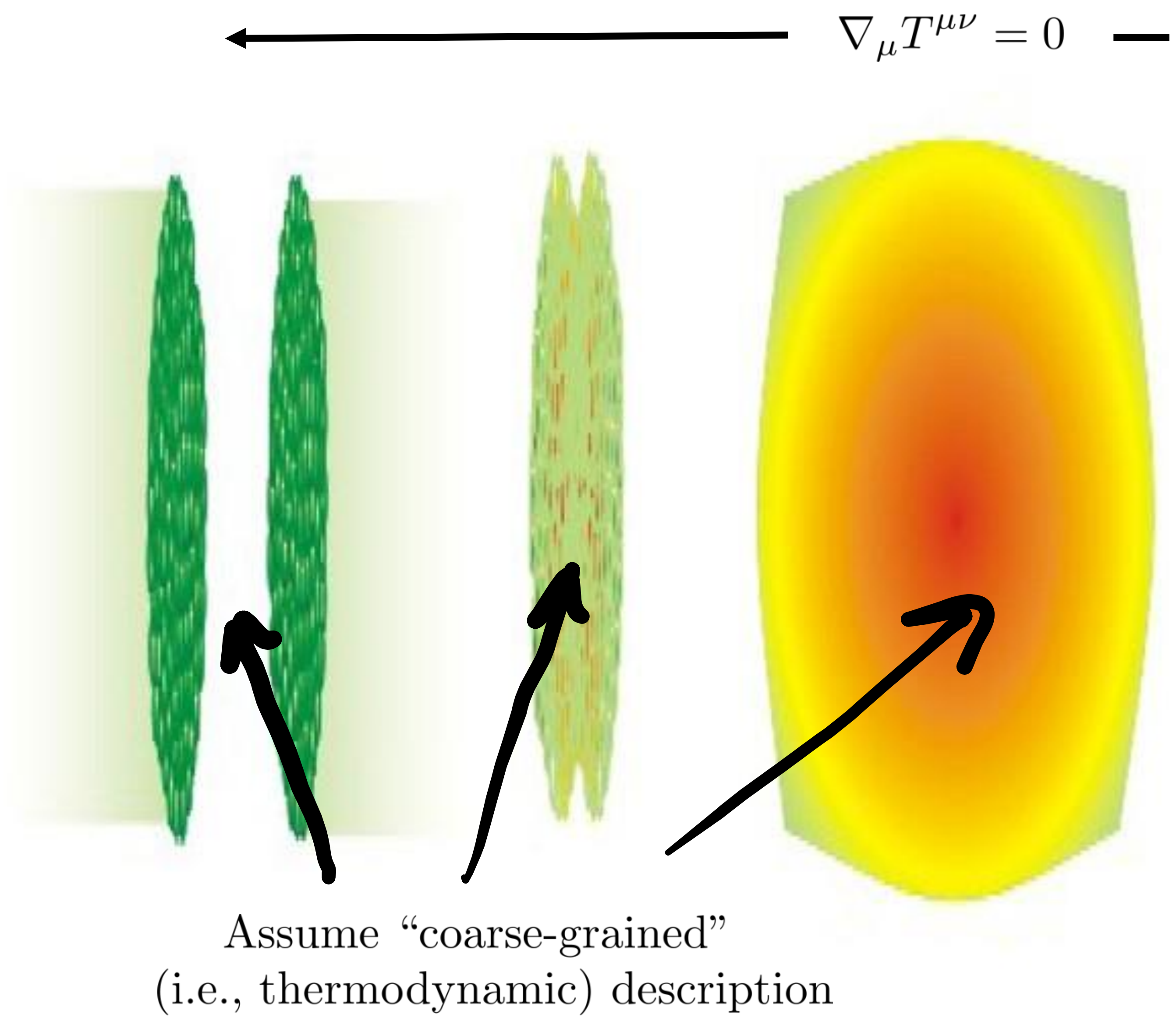
Hydrodynamics and nuclear collisions



Hydrodynamics and nuclear collisions



Relativistic hydrodynamics

Choose:
$T^{\mu\nu} = e u^\mu u^\nu + (P + \Pi)(u^\mu u^\nu - g^{\mu\nu}) + \pi^{\mu\nu}$

- **equilibrium** pieces (e, P, u^μ, ...)
- **non-equilibrium** pieces (Π, $\pi^{\mu\nu}$, ...)

Two basic assumptions:

- Space-time gradients are small
- Non-equilibrium corrections are small

How can we test hydrodynamics?

- Check Knudsen numbers Kn
- Check inverse Reynolds numbers Re^{-1}

Organizing hydrodynamics

– **Denicol-Niemi-Molnar-Rischke** (DNMR) equations of motion

→ Systematic expansion around ideal hydrodynamics in Kn, Re^{-1}

→ Equations obtained by truncating at second-order and matching to kinetic theory

– **Relativistic causality?**

→ Second-order theories (e.g., DNMR) often thought to be automatically relativistically causal

→ This is **not guaranteed**; relativistic causality **must be** checked explicitly

– **New tests for hydrodynamics**

→ **New** constraints recently derived for DNMR equations of motion [PRL 126 (2021), 222301]

→ Causality implies $0 \leq v^2 \leq c^2$, so evolution equations **must**:

(i) be hyperbolic $(v^2 \geq 0)$

(ii) have no superluminal propagation $(v^2 \leq c^2)$

→ Causality conditions (6 necessary and 8 sufficient) can be checked **for each fluid cell**

Causality categorization

– Identify three categories of fluid cells:

1. **Blue** - cells where all sufficient conditions are met (definitely causal)

2. **Purple** - cells where not all sufficient conditions are met but all necessary conditions are met (maybe causal or acausal)

3. **Red** - cells where one or more necessary conditions are violated (definitely acausal)

– Check different models

$\rightarrow$ Trento + free-streaming + iEBE-VISHNU (Bayesian tune)

$\rightarrow$ IP-Glasma + KøMPøST + MUSIC

– Check various collision systems and energies

$\rightarrow$ Small systems:	p+Pb @ 5.02 TeV
$\rightarrow$ Intermediate systems:	O+O @ 5.02 TeV
$\rightarrow$ Large systems:	Pb+Pb @ 2.76 TeV

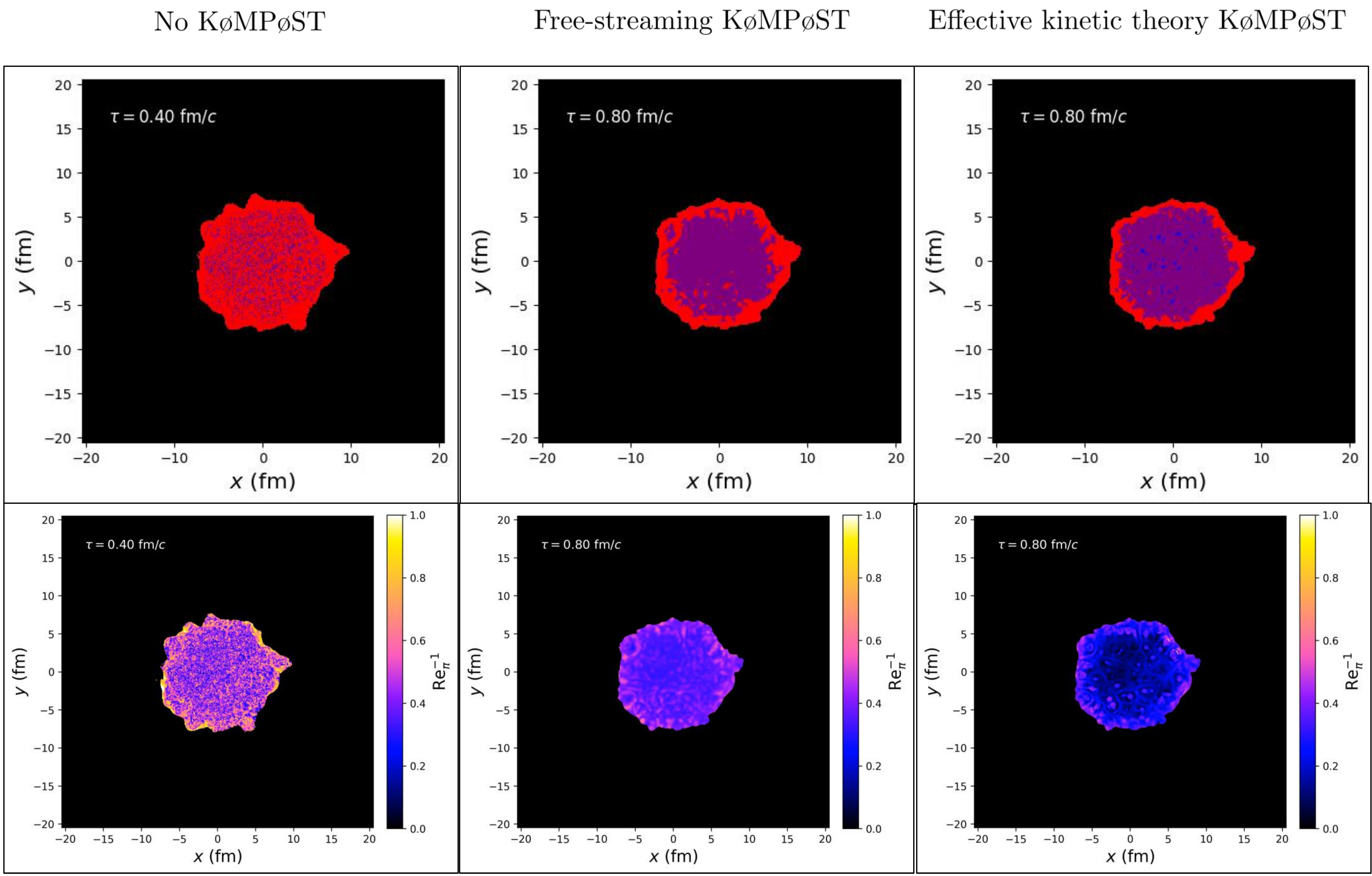
Some results

Pb+Pb @ 2.76 TeV

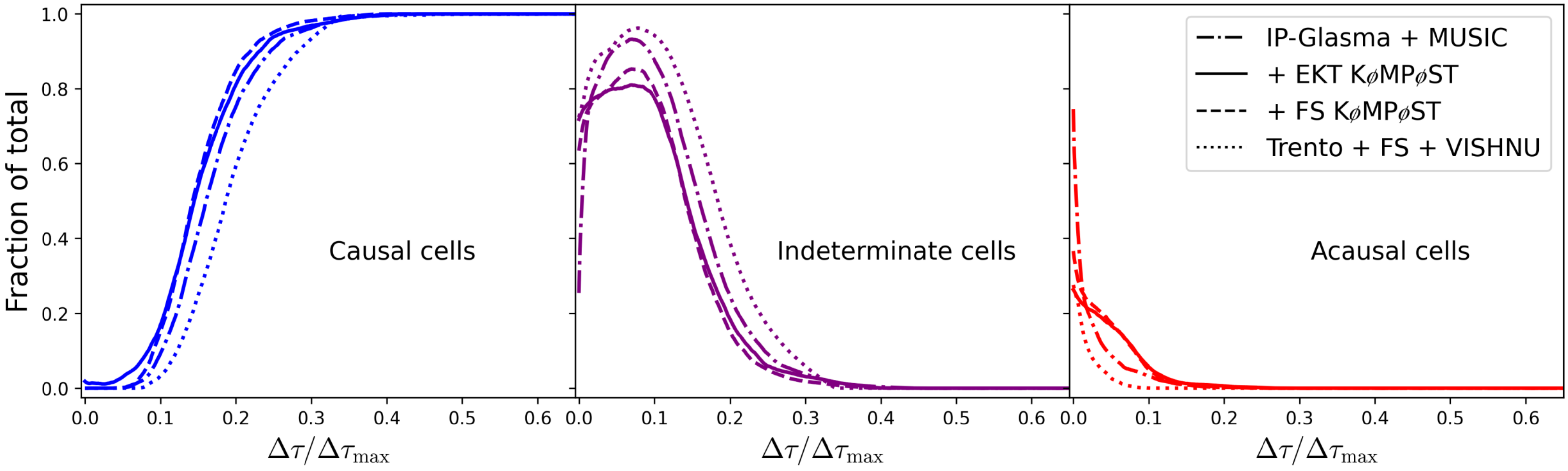
IP-Glasma + MUSIC

Causality analysis

Re_π^{-1}



Cell fractions vs. $\Delta\tau \equiv \tau - \tau_0$



- Most definite causality violations resolved in first 15% of evolution
- 50% of cells definitely causal after 20% of evolution (2-3 fm)
- System complete causal after 40% of evolution (4-5 fm)

Pb+Pb @ 2.76 TeV

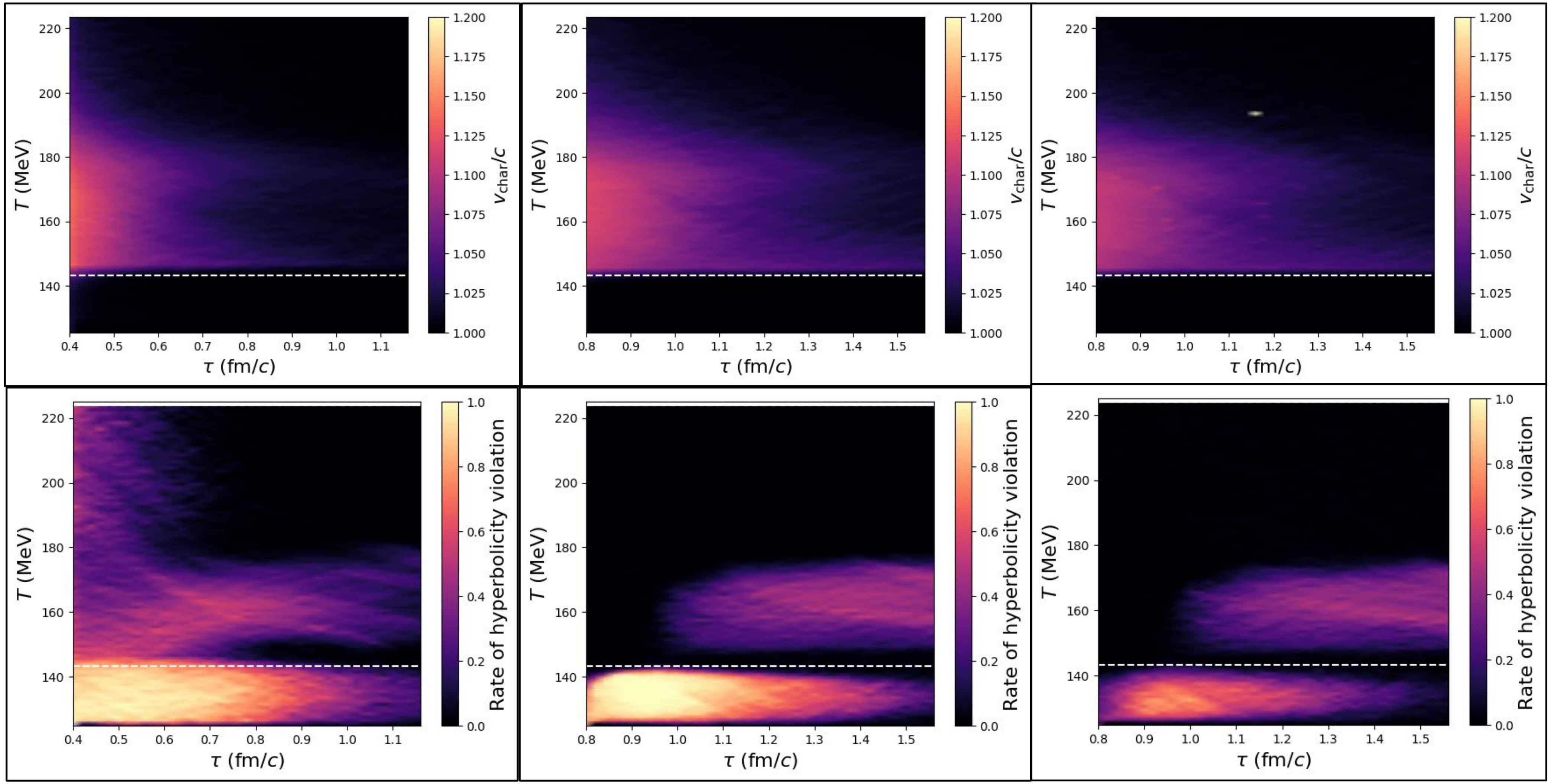
IP-Glasma + MUSIC

Causality conditions distinguish two kinds of violations: superluminal propagation ($v^2 > c^2$) and non-hyperbolicity ($v^2 < 0$)

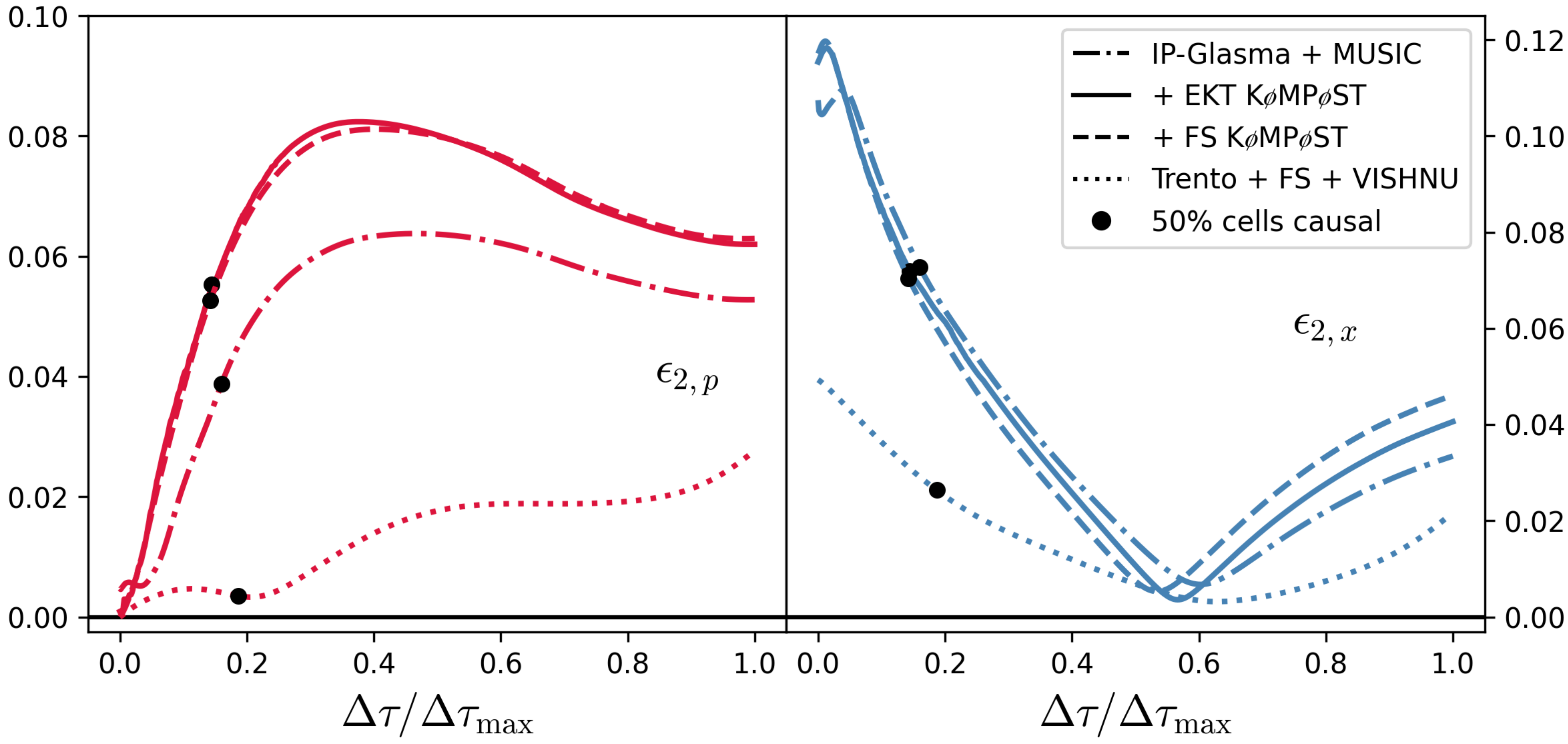
No KøMPøST | Free-streaming KøMPøST | Effective kinetic theory KøMPøST

Superluminal propagation ($v^2 > c^2$)

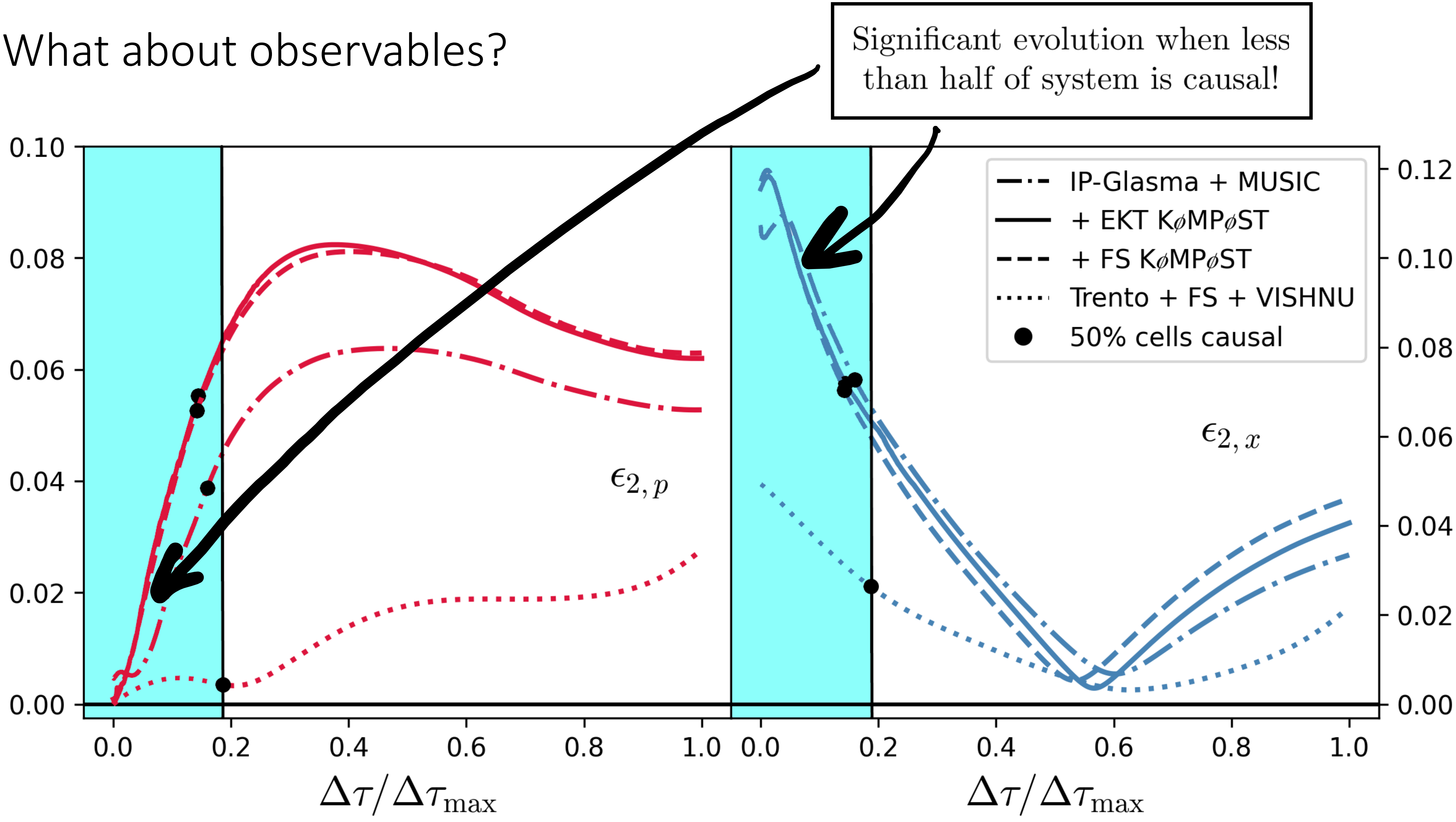
Non – hyperbolicity ($v^2 < 0$)



What about observables?



What about observables?



These results ***do not*** imply that:

- Relativistic causality is ***actually* violated** in nuclear collisions
- Hydrodynamics is **inapplicable** in or **irrelevant** to nuclear collisions

These results ***do*** imply that:

- Enforcing relativistic causality in hydrodynamic simulations will **almost certainly** lead to measurable changes in parameter ranges favored by data
- Theoretical uncertainties induced by violations expected to be O(10%), depending on observables and collision size (**worse in small systems**)
- **Violations predominate at early times** where Knudsen and/or inverse Reynolds numbers are large
- Inclusion of pre-equilibrium phase **significantly reduces severity** of violations

Conclusions

- Causality violations have been observed in realistic fluid dynamical simulations of nuclear collisions
- Violations predominate where hydrodynamic description is breaking down and/or becoming unreliable
- Possible solutions (not mutually exclusive):
  - Delaying the onset of hydrodynamics
  - Improving description of pre-equilibrium dynamics
  - Supplementing with non-hydrodynamic models
- Enforcing relativistic causality offers a **unique opportunity** to better understand transition from initial stages to hydrodynamic evolution

Thank you!

Backup Slides

What about the regulator?

- Up to 75% of system acausal at a given time
- Causality violations closely associated with largeness of inverse Reynolds number for shear
- Pre-equilibrium evolution seems to help significantly

What role does the regulator play?

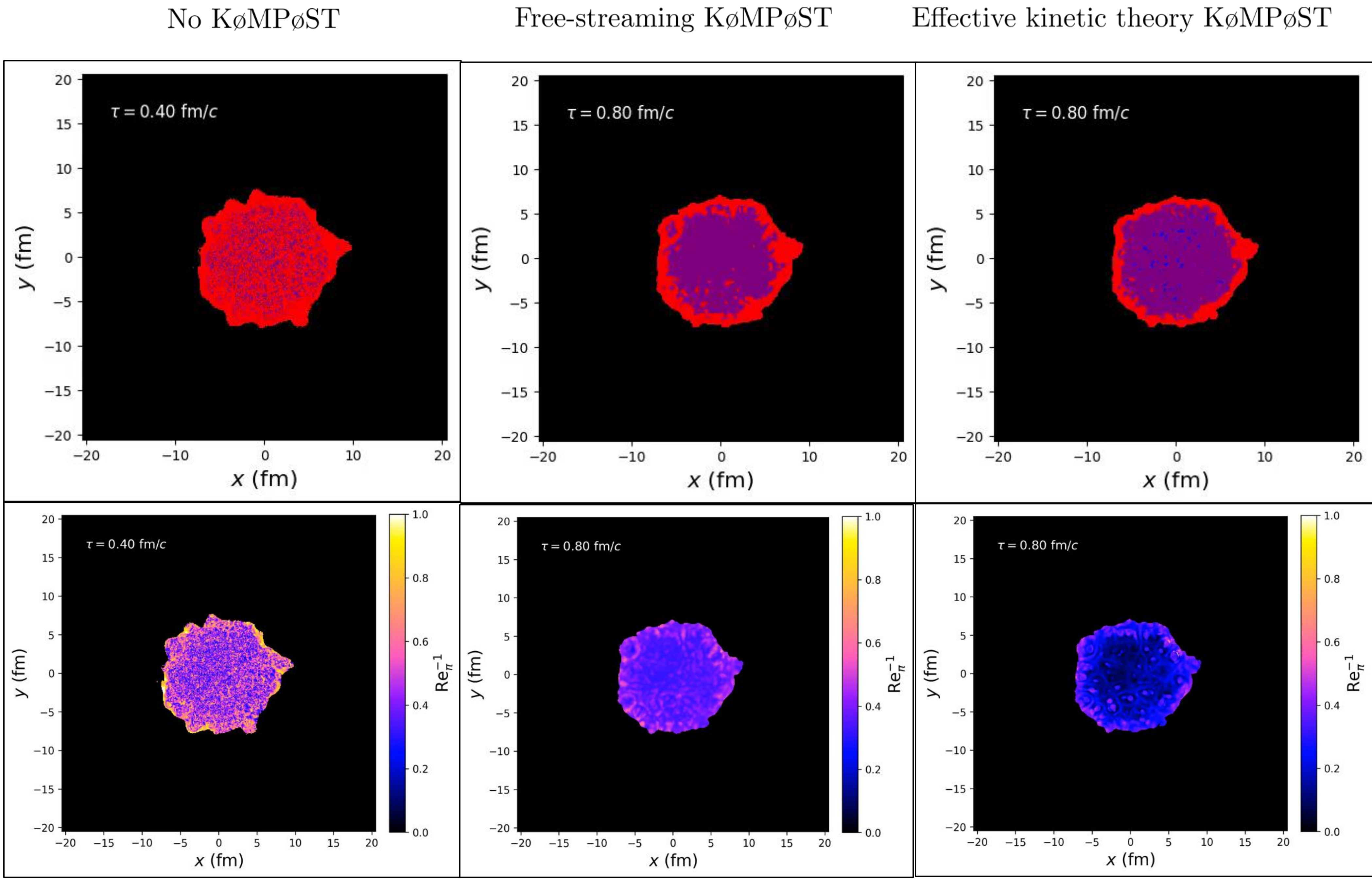
  - Option 1: Default regulator
  - Option 2: Enhanced regulator [cf. PRC102 044905 (2020)]

Pb+Pb @ 2.76 TeV

IP-Glasma + MUSIC (enhanced regulator)

Causality

Re_π^{-1}

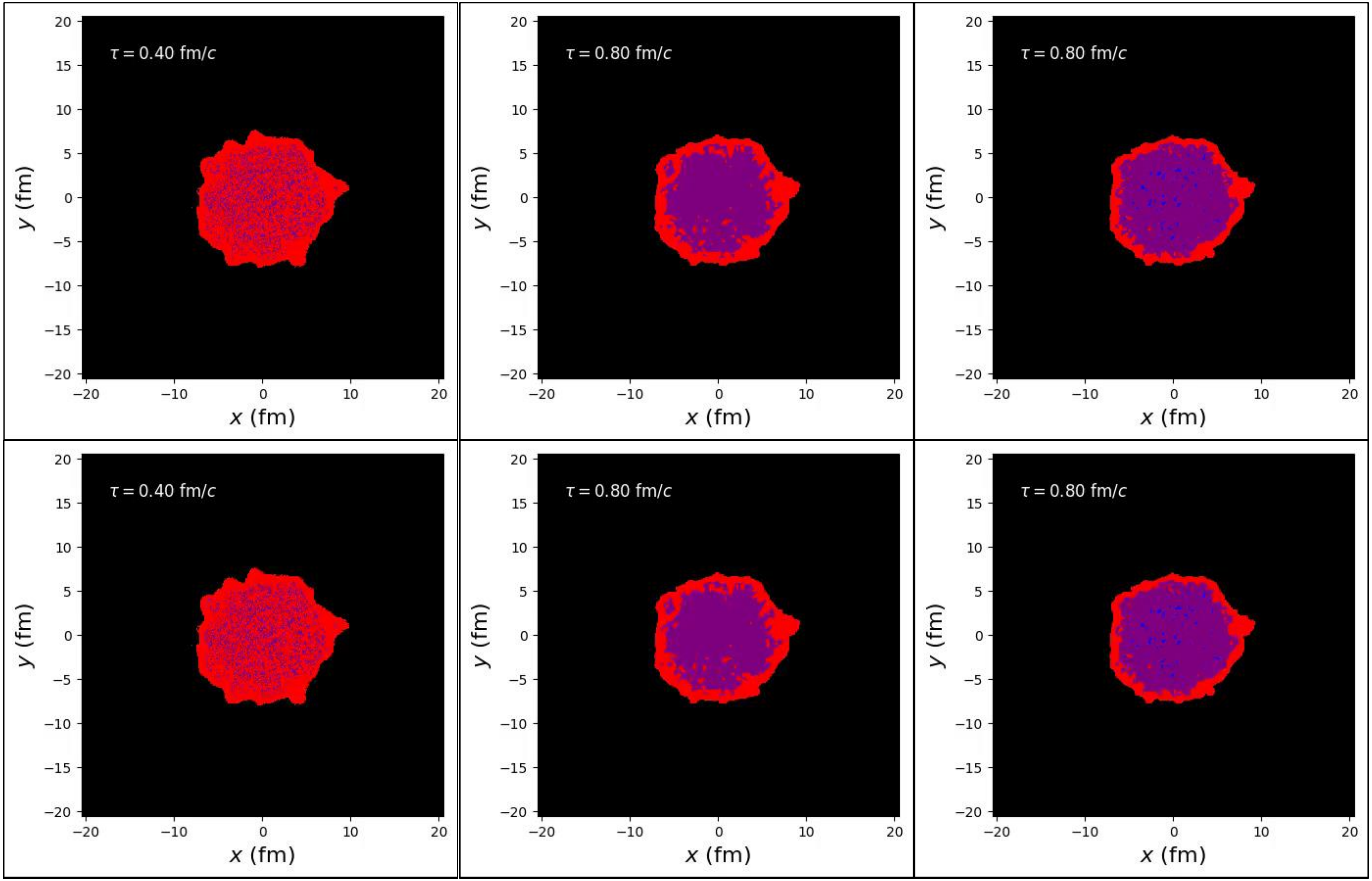


Pb+Pb @ 2.76 TeV

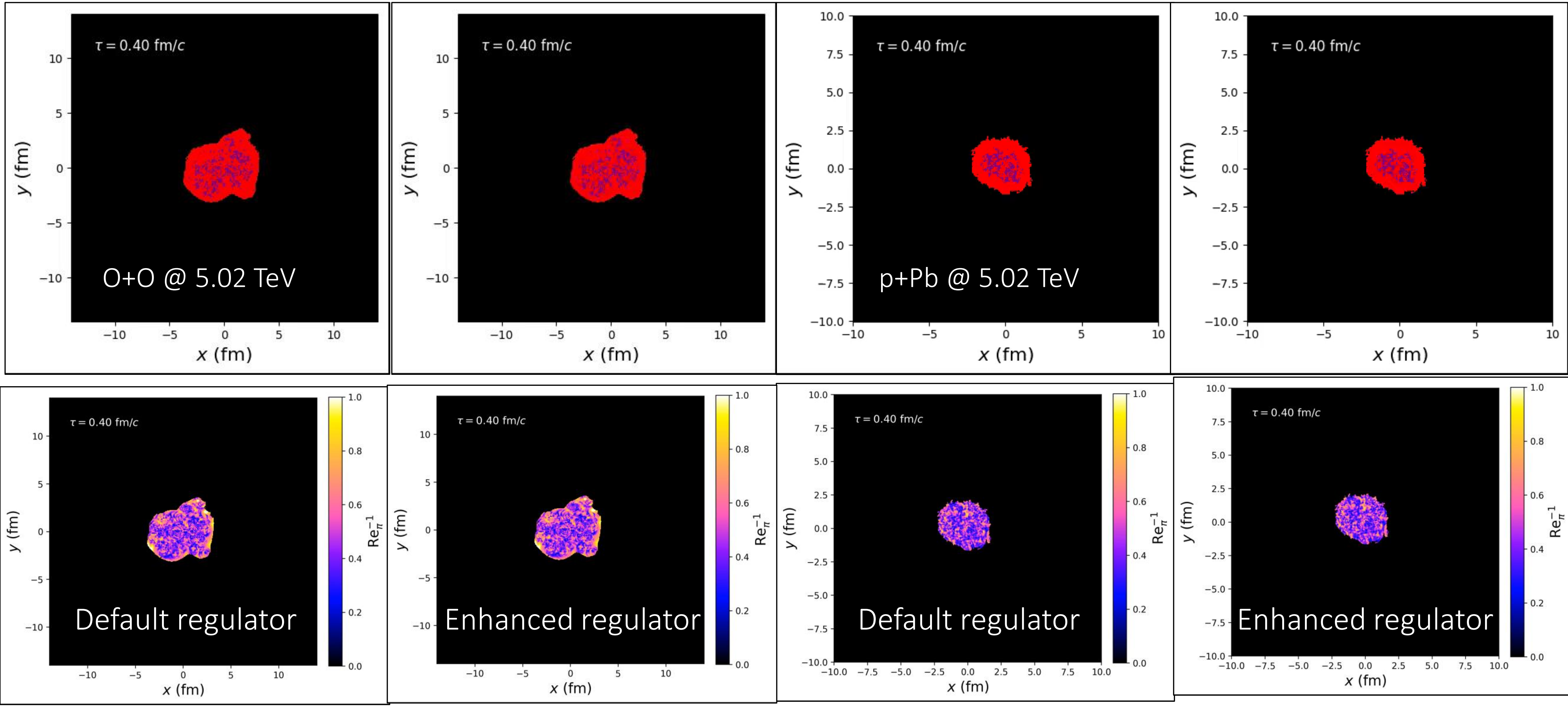
No KøMPøST | Free-streaming KøMPøST | Effective kinetic theory KøMPøST

Default regulator

Enhanced regulator



Causality in Small Systems



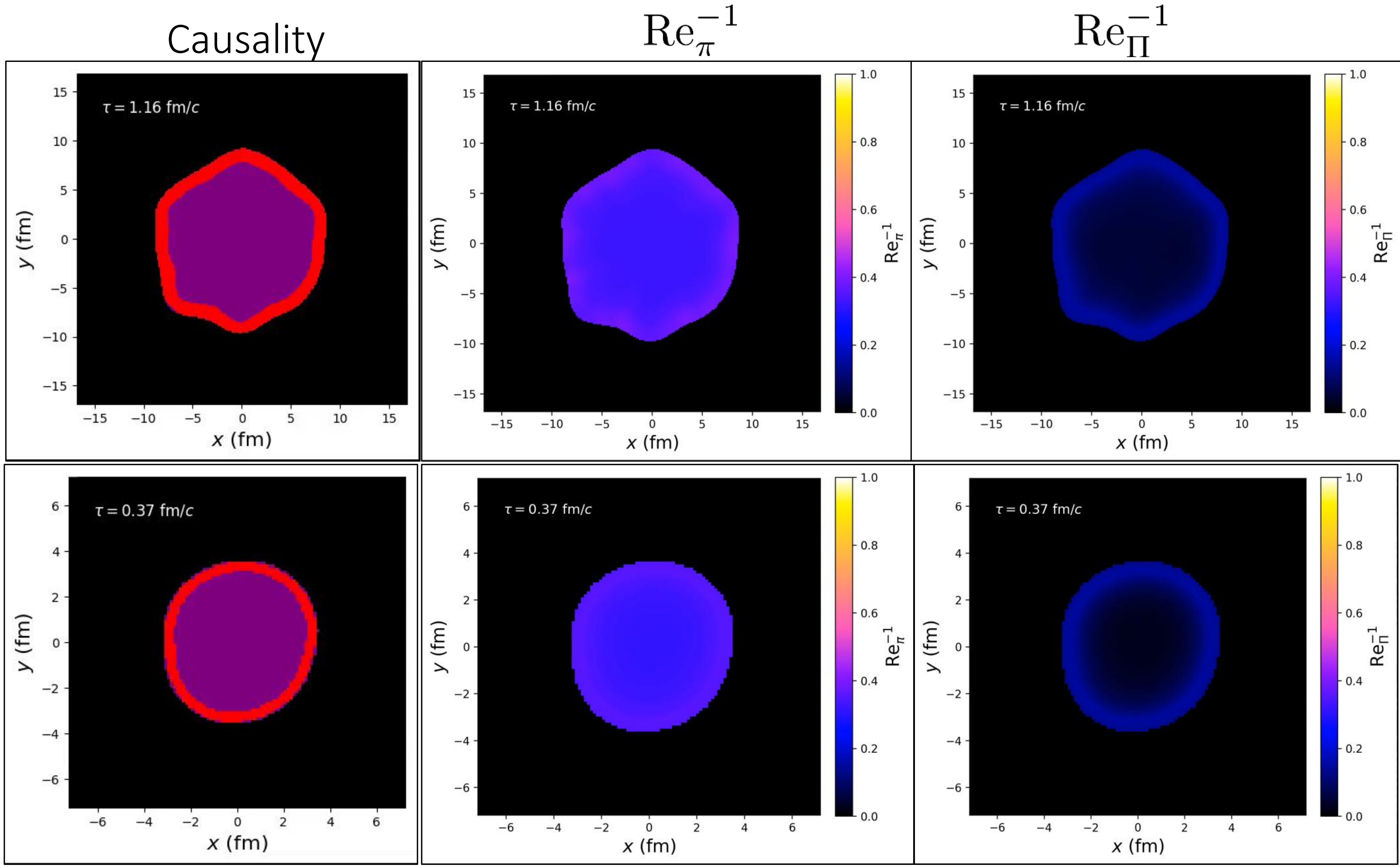
Some observations

- Up to 70% of system acausal at a given time(!)
- Causality violations closely associated with largeness of inverse Reynolds number for shear
- Pre-equilibrium evolution seems to help significantly
- Regulator makes a dramatic difference!
  - Stabilizes simulation
  - Effectively discards causality violations
  - Substantially alters space-time evolution

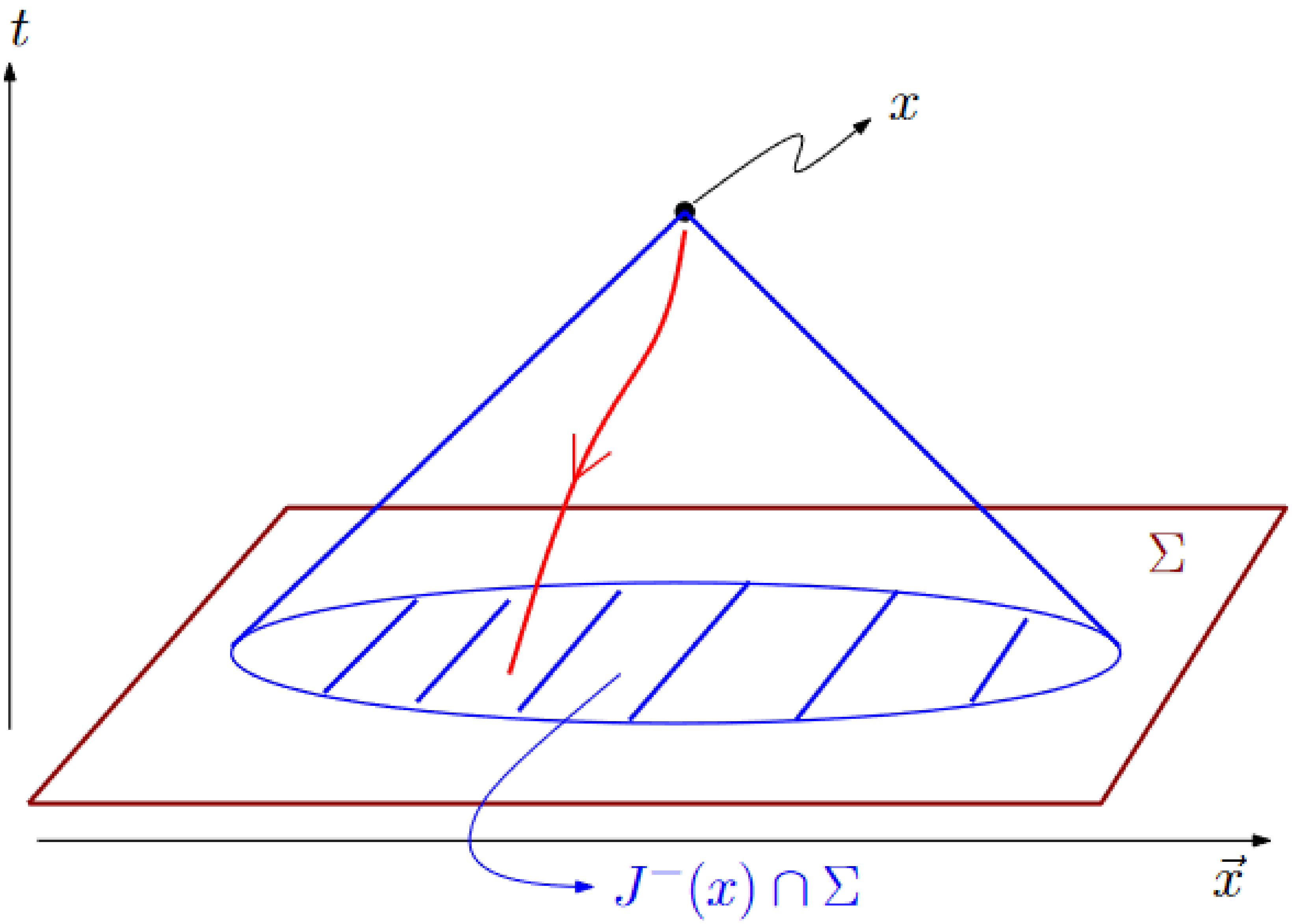
TRENTo
+ free-streaming
+ VISHNU

Pb+Pb @
2.76 TeV

p+Pb @
5.02 TeV



Relativistic Causality



arXiv:2005.11632

DNMR equations

Denicol-Niemi-Molnar-Rischke (DNMR) equations of motion:[1]

$$\begin{aligned}
\tau_\Pi u^\mu \nabla_\mu \Pi + \Pi &= -\zeta \nabla_\mu u^\mu - \delta_{\Pi\Pi} \Pi \nabla_\mu u^\mu - \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu}, \\
\tau_\pi \Delta^{\mu\nu}_{\alpha\beta} u^\lambda \nabla_\lambda \pi^{\alpha\beta} + \pi^{\mu\nu} &= -2\eta\sigma^{\mu\nu} - \delta_{\pi\pi} \pi^{\mu\nu} \nabla_\alpha u^\alpha - \tau_{\pi\pi} \pi_\alpha^{\langle\mu} \sigma^{\nu\rangle\alpha} - \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu},
\end{aligned}$$

Transport coefficients $(\zeta, \eta, \tau_\pi, \tau_\Pi, \tau_{\pi\pi}, \delta_{\pi\pi}, \delta_{\Pi\Pi}, \lambda_{\pi\Pi}, \lambda_{\Pi\pi})$ are (in general) functions of the ten dynamical variables

Energy density ε	Flow velocity u^μ
Bulk pressure Π	Shear stress tensor $\pi^{\mu\nu}$

but not their gradients!

Also define:

$$\sigma^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta} \nabla^\alpha u^\beta, \qquad \Delta^{\mu\nu}_{\alpha\beta} = \left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha\right) - \frac{1}{3} \Delta^{\mu\nu} \Delta_{\alpha\beta}$$

$$A_\lambda^{\langle\mu} B^{\nu\rangle\lambda} = \Delta^{\mu\nu}_{\alpha\beta} A^{\alpha\lambda} B^\beta_\lambda, \qquad \Delta_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$$

[1] Hydrodynamic codes often include an additional term $\varphi_7 \pi_\alpha^{\langle\mu} \pi^{\nu\rangle\alpha}$ which does not affect the causality analysis.

Checking causality: procedure

Step 1: Enforce preconditions for causality analysis

$$\zeta, \eta, \tau_\pi, \tau_\Pi, \tau_{\pi\pi}, \delta_{\pi\pi}, \delta_{\Pi\Pi}, \lambda_{\pi\Pi}, \lambda_{\Pi\pi}, \ldots \text{ are all positive}$$

Step 2: Get eigenvalues of shear stress tensor π^μ_ν, Λ_i:

$$\Lambda_0 = 0, \quad \Lambda_1 \leq \Lambda_2 \leq \Lambda_3 \text{ and } \Lambda_1 + \Lambda_2 + \Lambda_3 = 0$$

(follows from $\pi^\mu_\nu u^\nu = 0$ and $\mathrm{Tr}\,\pi = 0$)

Step 3: Evaluate necessary and sufficient conditions for causality in DNMR

Step 4: Assess hydrodynamic validity using

$$\mathrm{Re}_\pi^{-1} = \sqrt{\pi_{\mu\nu}\pi^{\mu\nu}}/(\varepsilon + P), \qquad \mathrm{Re}_\Pi^{-1} = |\Pi|/(\varepsilon + P)$$

DNMR: necessary conditions for causality

$$(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{1}{2}\tau_{\pi\pi}|\Lambda_1| \geq 0$$

$$\varepsilon + P + \Pi - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{4\tau_\pi}\Lambda_3 \geq 0,$$

$$\frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) + \frac{\tau_{\pi\pi}}{4\tau_\pi}(\Lambda_a + \Lambda_d) \geq 0, \quad a \neq d,$$

$$\varepsilon + P + \Pi + \Lambda_a - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{4\tau_\pi}(\Lambda_d + \Lambda_a) \geq 0, \quad a \neq d$$

$$\frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) + \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_d + \frac{1}{6\tau_\pi}[2\eta + \lambda_{\pi\Pi}\Pi + (6\delta_{\pi\pi} - \tau_{\pi\pi})\Lambda_d] + \frac{\zeta + \delta_{\Pi\Pi}\Pi + \lambda_{\Pi\pi}\Lambda_d}{\tau_\Pi} + (\varepsilon + P + \Pi + \Lambda_d)c_s^2 \geq 0,$$

$$\varepsilon + P + \Pi + \Lambda_d - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_d - \frac{1}{6\tau_\pi}[2\eta + \lambda_{\pi\Pi}\Pi + (6\delta_{\pi\pi} - \tau_{\pi\pi})\Lambda_d] - \frac{\zeta + \delta_{\Pi\Pi}\Pi + \lambda_{\Pi\pi}\Lambda_d}{\tau_\Pi} - (\varepsilon + P + \Pi + \Lambda_d)c_s^2 \geq 0,$$

Total of six necessary conditions: if any conditions are violated, fluid cell is *guaranteed* to be **acausal**

arXiv:2005.11632

DNMR: sufficient conditions for causality

$$(\varepsilon + P + \Pi - |\Lambda_1|) - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_3 \geq 0,$$

$$(2\eta + \lambda_{\pi\Pi}\Pi) - \tau_{\pi\pi}|\Lambda_1| > 0,$$

$$\tau_{\pi\pi} \leq 6\delta_{\pi\pi},$$

$$\frac{\lambda_{\Pi\pi}}{\tau_\Pi} + c_s^2 - \frac{\tau_{\pi\pi}}{12\tau_\pi} \geq 0,$$

$$\frac{1}{3\tau_\pi}[4\eta + 2\lambda_{\pi\Pi}\Pi + (3\delta_{\pi\pi} + \tau_{\pi\pi})\Lambda_3] + \frac{\zeta + \delta_{\Pi\Pi}\Pi + \lambda_{\Pi\pi}\Lambda_3}{\tau_\Pi} + |\Lambda_1| + \Lambda_3 c_s^2$$
$$+ \frac{\frac{12\delta_{\pi\pi} - \tau_{\pi\pi}}{12\tau_\pi}\left(\frac{\lambda_{\Pi\pi}}{\tau_\Pi} + c_s^2 - \frac{\tau_{\pi\pi}}{12\tau_\pi}\right)(\Lambda_3 + |\Lambda_1|)^2}{\varepsilon + P + \Pi - |\Lambda_1| - \frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_3} \leq (\varepsilon + P + \Pi)(1 - c_s^2),$$

$$\frac{1}{6\tau_\pi}[2\eta + \lambda_{\pi\Pi}\Pi + (\tau_{\pi\pi} - 6\delta_{\pi\pi})|\Lambda_1|] + \frac{\zeta + \delta_{\Pi\Pi}\Pi - \lambda_{\Pi\pi}|\Lambda_1|}{\tau_\Pi} + (\varepsilon + P + \Pi - |\Lambda_1|)c_s^2 \geq 0,$$

$$1 \geq \frac{\frac{12\delta_{\pi\pi} - \tau_{\pi\pi}}{12\tau_\pi}\left(\frac{\lambda_{\Pi\pi}}{\tau_\Pi} + c_s^2 - \frac{\tau_{\pi\pi}}{12\tau_\pi}\right)(\Lambda_3 + |\Lambda_1|)^2}{\left[\frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) - \frac{\tau_{\pi\pi}}{2\tau_\pi}|\Lambda_1|\right]^2}$$

$$\frac{1}{3\tau_\pi}[4\eta + 2\lambda_{\pi\Pi}\Pi - (3\delta_{\pi\pi} + \tau_{\pi\pi})|\Lambda_1|] + \frac{\zeta + \delta_{\Pi\Pi}\Pi - \lambda_{\Pi\pi}|\Lambda_1|}{\tau_\Pi} + (\varepsilon + P + \Pi - |\Lambda_1|)c_s^2$$
$$\geq \frac{(\varepsilon + P + \Pi + \Lambda_2)(\varepsilon + P + \Pi + \Lambda_3)}{3(\varepsilon + P + \Pi - |\Lambda_1|)}\left\{1 + \frac{2\left[\frac{1}{2\tau_\pi}(2\eta + \lambda_{\pi\Pi}\Pi) + \frac{\tau_{\pi\pi}}{2\tau_\pi}\Lambda_3\right]}{\varepsilon + P + \Pi - |\Lambda_1|}\right\},$$

Total of eight sufficient conditions: if all conditions are satisfied, fluid cell is *guaranteed* to be **causal**

arXiv:2005.11632

Where do we go from here?

Some concrete recommendations:

- Causality violations should be incorporated into all hydrodynamic codes and/or Bayesian analyses of hydrodynamic frameworks
- New/better observables which may be more sensitive to the causality constraints
  - space-time observables (e.g., HBT)
  - momentum-space observables (e.g., principal component analyses)
- More thorough exploration of causality analyses' dependence on model parameters underway
- Some form of pre-equilibrium evolution will likely prove necessary for restoring causality more generally

Basic ingredients for hydrodynamics

Energy-momentum tensor: $T^{\mu\nu} = \varepsilon u^\mu u^\nu + (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$

Conservation laws: $\nabla_\mu T^{\mu\nu} = 0$

Initial conditions:

$$\varepsilon = \varepsilon_0(\tau = \tau_0, x, y, \eta)$$
$$u^\mu = u_0^\mu(\tau = \tau_0, x, y, \eta)$$
$$\pi^{\mu\nu} = \pi_0^{\mu\nu}(\tau = \tau_0, x, y, \eta)$$

Here: IP-Glasma, $\mathrm{T_R ENTo}$

Equation of state: $P = P(T, \{\mu\})$

Pre-equilibrium evolution: KøMPøST? Free-streaming?

→ Still a lot of room for different implementations of hydrodynamics!

Putting hydrodynamics to the test

- Assess hydrodynamic validity using

  $\mathrm{Re}_\pi^{-1} = \sqrt{\pi_{\mu\nu}\pi^{\mu\nu}}/(\varepsilon + P)$

  $\mathrm{Re}_\Pi^{-1} = |\Pi|/(\varepsilon + P)$ (*similar for* Kn_π, Kn_Π)

- Hydrodynamics applies when:

  $\mathrm{Re}_\pi^{-1}, \mathrm{Re}_\Pi^{-1} \ll 1$

  $\mathrm{Kn}_\pi, \mathrm{Kn}_\Pi \ll 1$

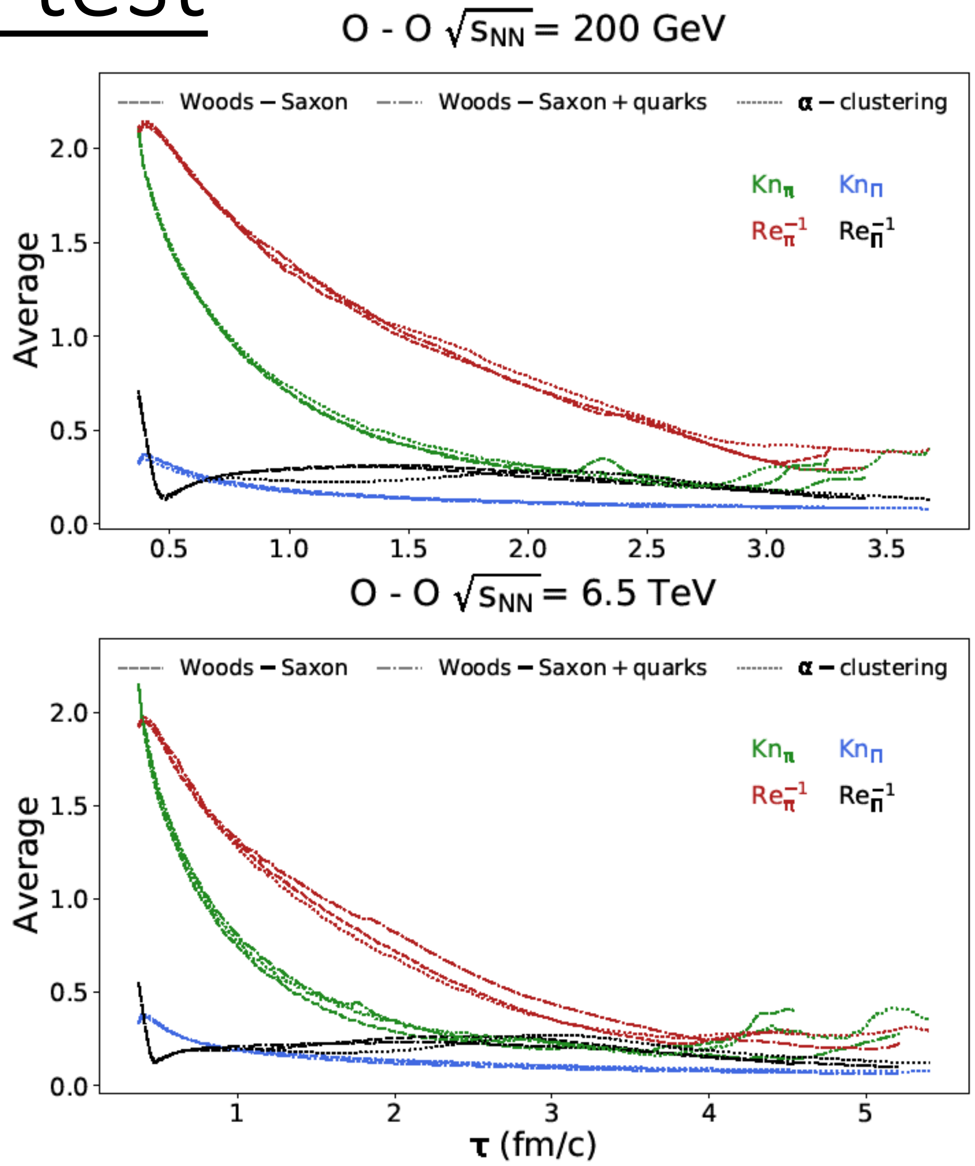
- Test on model $^{16}O^{16}O$ collisions

  TRENTo + free-streaming + iEBE-Vishnu

  Model parameters adjusted to reproduce data

$\Rightarrow$ **Conclusion:**

hydrodynamics is pressed to its limits in small and intermediate systems!



N. Summerfield, B.-N. Lu, CP, D. Lee, J. Noronha-Hostler, and A. Timmins [Phys. Rev. C 104, L041901 (2021)]

Conclusions

- Current hydrodynamic frameworks are pressed to their limits at early times where gradients and non-equilibrium corrections are large, regardless of collision species
- Fluid cells with large gradients and non-equilibrium corrections tend to generate causality violations when evolved hydrodynamically
- Pre-equilibrium evolution mitigates these violations to some extent, but does not eliminate them entirely
- Causality constraints must be incorporated into future hydrodynamic tunes to experimental data

Thank you!