Heavy quark transport through viscous quark-gluon plasma

Adiba Shaikh

Indian Institute of Technology Bombay

In collaboration with

M. Kurian (McGill U) | V. Chandra (IIT GN) | S. K. Das (IIT Goa) | S. Dash, B. K. Nandi (IIT Bombay)

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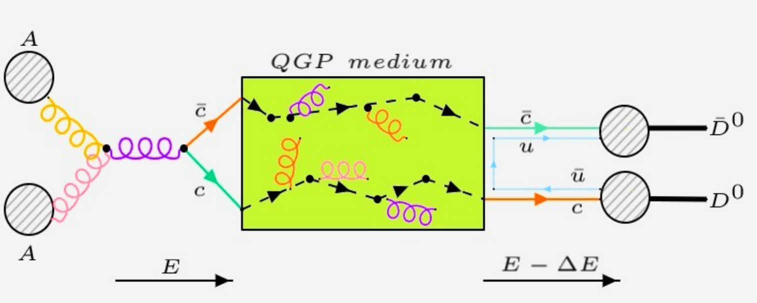
Introduction

- **Heavy quarks (HQ) in Heavy-ion collision:**
  - $c\bar{c}/b\bar{b}$ pairs perturbatively created at early times in HIC experiments.
  - Important probes to study QGP properties ($m_{HQ} >> T$).
  - Undergo energy loss due to collision and soft gluon radiation in medium.
- **HQ transport coefficients:**
  - Sensitive to medium evolution.
  - Drag coefficient: Resistance due to light (anti)quarks and gluons
  - Diffusion coefficient: Transverse and longitudinal in momentum space



HQ transport coefficients

- Boltzmann eqn. + soft scattering approx. $\Rightarrow$ **Fokker-Planck eqn.**

$$\frac{\partial f(\mathbf{p},t)}{\partial t} = \left(\frac{\partial f}{\partial t}\right)_{col} = \frac{\partial}{\partial p_i}\left(A_i(\mathbf{p})f + \frac{\partial}{\partial p_j}[B_{ij}(\mathbf{p})]f\right)$$

- **Drag**: $A_i \equiv \langle (p-p')_i \rangle = p_i A(p)$
- **Diffusion**: $B_{ij} \equiv \langle (p-p')_i (p-p')_j \rangle = \left(\delta_{ij} - \frac{p_i p_j}{p}\right) B_0(p^2) + \left(\frac{p_i p_j}{p^2}\right) B_1(p)$

B. Svetitsky, Phys. Rev. D 37, 9 (1988)

- **Collision (Elastic)**: $HQ(p) + lq/l\bar{q}/g(q) \rightarrow HQ(p') + lq/l\bar{q}/g(q')$

$$\langle F(p)_{col} \rangle = \frac{1}{16(2\pi)^5 E_p \gamma_{HQ}} \int \frac{d^3q}{E_q} \int \frac{d^3q'}{E_{q'}} \int \frac{d^3p'}{E_{p'}} \sum |\mathcal{M}|^2_{2\rightarrow 2}\, \delta(p+q-p'-q') \times f(E_q)\,(1 \pm f(E'_q))\; F(p)$$

- **Radiation (Inelastic)**: $HQ(p) + lq/l\bar{q}/g(q) \rightarrow HQ(p') + lq/l\bar{q}/g(q') + g(k')$

$$\langle F(p)_{rad} \rangle = \langle F(p)_{col} \rangle \times \int \frac{d^3k'}{(2\pi)^3 E_{k'}} \frac{12 g_s^2}{{k'_\perp}^2} \left(1 + \frac{m_{HQ}^2}{s} e^{2y_{k'}}\right)^{-2} \delta(p+q-p'-q'-k') \times (1+f(E_{k'}))\; \theta(E_p - E_{k'})\; \theta(\tau - \tau_f)$$

S. Mazumdar *et al.*, Phys. Rev. D 89, 014002 (2014)

Thermal medium interaction : EQPM

- **EQPM**: **E**ffective fugacity **Q**uasi-**P**article **M**odel (lattice QCD EoS based)

  V. Chandra *et al.*, **Phys. Rev. C 76, 054909 (2007)**

- In-medium interactions of QGP encoded into particle: **quasiparticle**
- Introduction of temperature dependent effective fugacity z_k in the distribution functions of quasiparticle $k \equiv (lq, l\bar{q}, g)$.

$$f_k^0 = \frac{\mathbf{z_k}\ e^{-E_k/T}}{1 \pm \mathbf{z_k}\ e^{-E_k/T}}$$

- Quasiparticle dispersion relation: $\tilde{q}_k{}^{\mu} = q_k^{\mu} + \delta\omega_k\, u^{\mu}$
- Collective excitations of quasipartons: $\delta\omega_k = T^2\, \partial_T\{\ln(z_k)\}$
- Effective strong coupling constant $\alpha_{s(eff)}$ is introduced through EQPM based Debye mass.

$$\alpha_{s(eff)}(T) = \alpha_s(T) \frac{\left\{\frac{2N_c}{\pi^2}\mathrm{PolyLog}[2, z_g] - \frac{2N_f}{\pi^2}\mathrm{PolyLog}[2, -z_q]\right\}}{\left\{\frac{N_c}{3} - \frac{N_f}{6}\right\}}$$

S. Mitra *et al.*, **Phys. Rev. D 96, 094003 (2017)**

Viscous hydrodynamic corrections

- Leading order shear and bulk viscous corrections to the (anti)quark and gluon distribution function obtained by solving the effective kinetic theory.

S. Bhadury *et al.*, **J. Phys. G 47 (2020) 8, 085108**

- Energy-momentum tensor for the dissipative (viscous) hydrodynamics,

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

- Quasiparticle distribution function near local thermal equilibrium,

$$f_k = f_k^0 + \delta f_k \text{ where } \delta f_k / f_k^0 << 1$$

- Boost-invariant Bjorken (longitudinal) expansion of the fluid.
- LO viscous corrections to the quasiparticle thermal distribution function.
  - By solving the relativistic Boltzmann equation with RTA using Chapman-Enskog method,

$$\delta f_k = f_k^0 (1 \pm f_k^0)\{\phi_k(bulk)^{(1)} + \phi_k(shear)^{(1)}\}$$

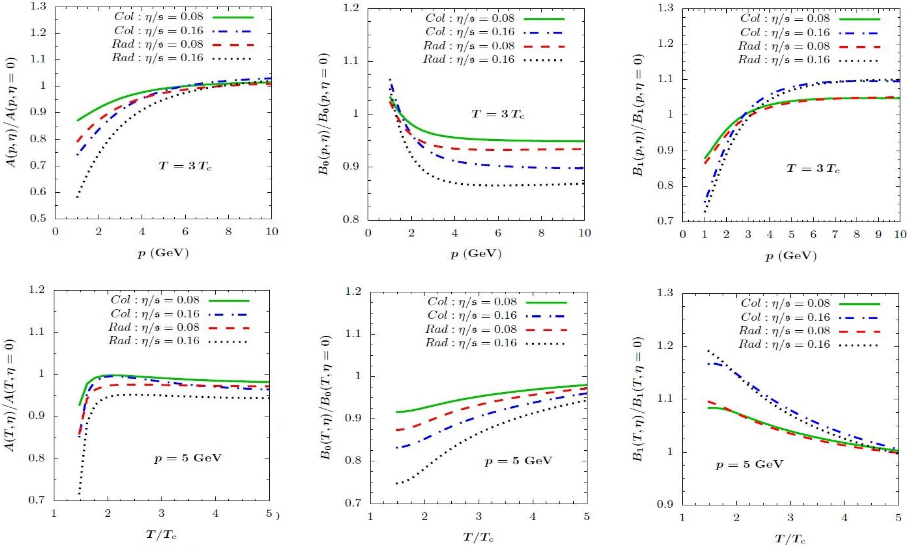
$$\phi_k(bulk)^{(1)} = \frac{s}{\beta_\Pi \omega_k T\tau}\left(\frac{\zeta}{s}\right)\left[\omega_k^2 c_s^2 - \frac{|\overrightarrow{q_k}|^2}{3} - \omega_k \delta\omega_k\right]$$

$$\phi_k(shear)^{(1)} = \frac{s}{\beta_\pi \omega_k T\tau}\left(\frac{\eta}{s}\right)\left[\frac{|\overrightarrow{q_k}|^2}{3} - (q_k)_z^2\right]$$

A. Shaikh *et al.*, **Phys. Rev. D 104, 034017 (2021)**

Results for shear viscous correction (i. Transport Coefficients)

$$N_c = N_f = 3, \quad m_{lq} = \mu_{lq} = 0, \quad m_c = 1.3\,\text{GeV}, \quad T_c = 170\,\text{MeV}, \quad \tau = 0.25\,\text{fm}$$

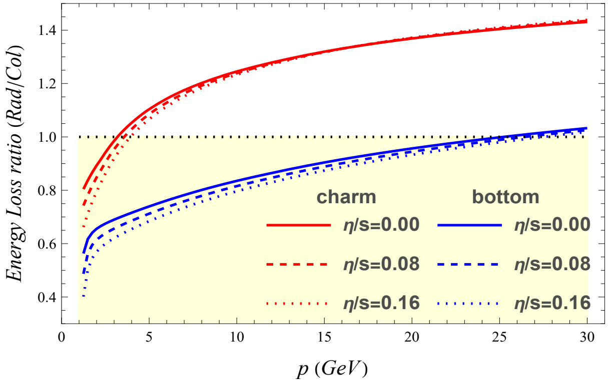


Results for shear viscous correction (ii. Energy Loss)

$$N_c = N_f = 3, \quad m_{lq} = \mu_{lq} = 0, \quad m_c = 1.3\,\text{GeV}, \quad T_c = 170\,\text{MeV}, \quad \tau = 0.25\,\text{fm}$$

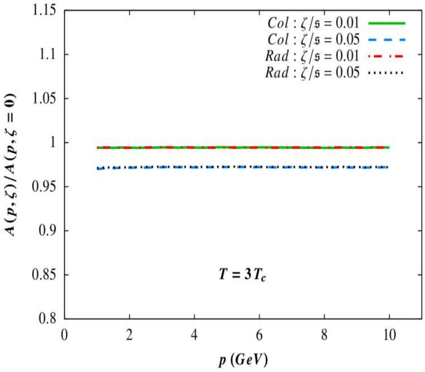
$$m_b = 4.2\,\text{GeV}$$

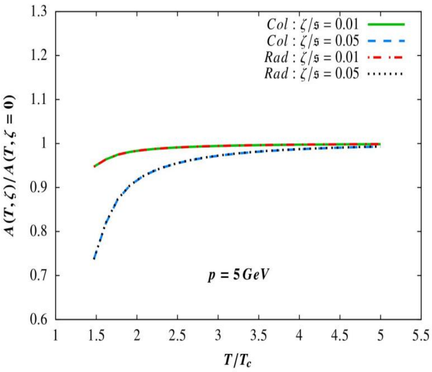
Differential energy loss: $-\dfrac{dE}{dx} = p\,A(p, T)$



Results for bulk viscous correction

$$N_c = N_f = 3, \quad m_{lq} = \mu_{lq} = 0, \quad m_c = 1.3\,\text{GeV}, \quad T_c = 170\,\text{MeV}, \quad \tau = 0.25\,\text{fm}$$



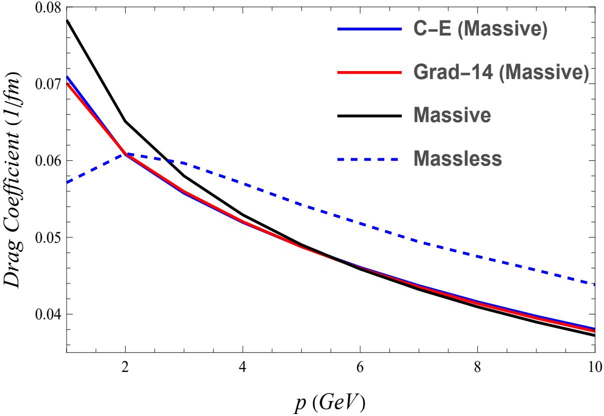


PoS CHARM2020 (2021) 060

Comparison with Grad 14-moment approximation method

$$\delta f_{k\,Grad}(shear) = f_k^0(1 \pm f_k^0)\left\{\frac{2\,s\,\beta_2}{\tau}\left(\frac{\eta}{s}\right)\left[\frac{|\vec{q_k}|^2}{3} - (q_k)_z^2\right]\right\}$$

$N_c = N_f = 3$, $\underline{m_{lq} = 300\,\text{MeV}}$, $m_c = 1.3\,\text{GeV}$, $T_c = 170\,\text{MeV}$, $\tau = 0.25\,\text{fm}$



Second order shear viscous correction

$$\delta f(shear) = f^0(1 \pm f^0)\{\phi^{(1)} + \phi^{(2)}\}$$

General Case

$$\phi^{(2)} = \frac{\beta}{\beta_\pi}\Bigg[\frac{5}{14\beta_\pi(u\cdot p)}p^\alpha p^\beta \pi^\gamma_\alpha \pi_{\beta\gamma} - \frac{\tau_\pi}{(u\cdot p)}p^\alpha p^\beta \pi^\gamma_\alpha \omega_{\beta\gamma} - \frac{(u\cdot p)}{70\beta_\pi}\pi^{\alpha\beta}\pi_{\alpha\beta} + \frac{6\tau_\pi}{5}p^\alpha \dot{u}^\beta \pi_{\alpha\beta} - \frac{\tau_\pi}{5}p^\alpha(\nabla^\beta \pi_{\alpha\beta}) - \frac{\tau_\pi}{2(u\cdot p)^2}p^\alpha p^\beta p^\gamma(\nabla_\gamma \pi_{\alpha\beta}) + \frac{3\tau_\pi}{(u\cdot p)^2}p^\alpha p^\beta p^\gamma \pi_{\alpha\beta}\dot{u}_\gamma - \frac{\tau_\pi}{3(u\cdot p)}p^\alpha p^\beta \pi_{\alpha\beta}\theta + \frac{\beta + (u\cdot p)^{-1}}{4(u\cdot p)^2\beta_\pi}(p^\alpha p^\beta \pi_{\alpha\beta})^2$$

For Bjorken (1D) expansion

$(\omega_{\mu\nu} = \dot{u}_\mu = 0)$

$$\phi^{(2)} = \frac{s^2}{T\beta_\pi^2\tau^2}\left(\frac{\eta}{s}\right)^2\Bigg[-\left(\frac{10}{63}\right)\frac{|\vec{q}|^2+3q_z^2}{E} - \left(\frac{4}{105}\right)E + \left(\frac{4}{15}\right)E - \left(\frac{4}{3}\right)\frac{q_z^2}{E} - \left(\frac{2}{3}\right)\frac{|\vec{q}|^2-3q_z^2}{3E} + \left(\frac{1}{T}+\frac{1}{E}\right)\left(\frac{|\vec{q}|^2-3q_z^2}{3E}\right)^2$$

C. Chattopadhyay *et al.*, Phys. Rev. C 91, 024917 (2015)

Conclusion

1. Heavy quark transport coefficients is studied in a viscous QCD medium for collisional and radiative processes.
2. The thermal medium interactions are incorporated using EQPM and the first order shear and bulk viscous corrections are included to the distribution function of the quasiparticles.
3. Shear viscous corrections are substantial for slow moving HQ ($p \approx 1 - 2$ GeV at $T = 3T_c$) where the increase in η/s decreases the drag coefficient.
4. Bulk viscous corrections are prominent near transition temperature ($T \approx 1.5T_c$).
5. The effect of the second order viscous corrections to the HQ transport coefficients is in progress.

Thank you !