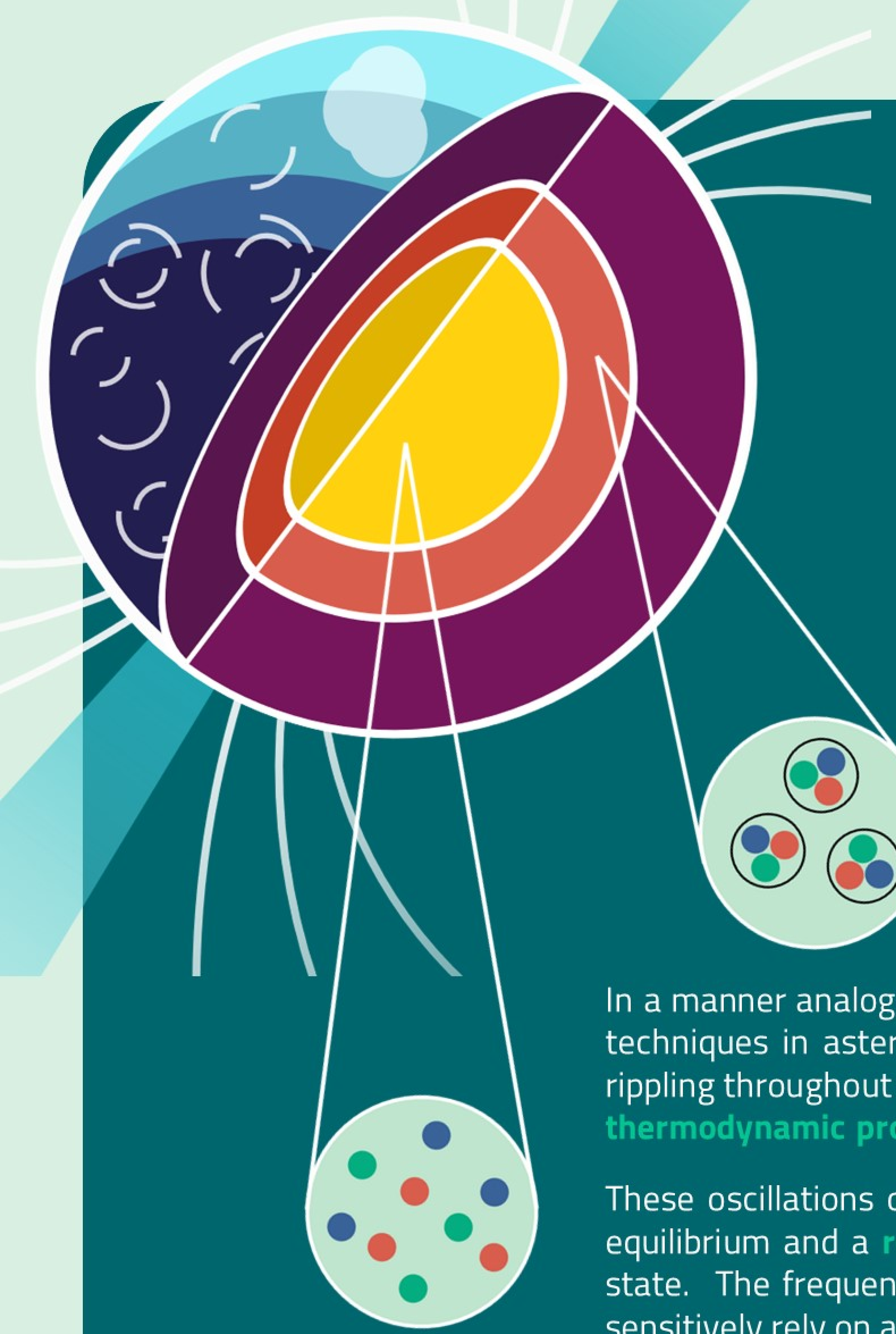
Asteroseismology of compact stars with nucleonic and strange-quark matter cores

Dániel Barta[1], Balázs Kacskovics[1,2]

[1] Wigner Research Centre for Physics, Konkoly-Thege Miklós út 29-33, 1121 Budapest, Hungary
[2] University of Pécs, Vasvári Pál utca 4, 7622 Pécs, Hungary

ELKH Eötvös Loránd Research Network



Quark matter inside neutron stars

In the center of heavy neutron stars, at a few times normal nuclear density (n_0), **quark matter** might appear. These objects are then called **hybrid stars** and consist of several layers.

Ideally, we would have a single model describing all the components, but in reality we use separate **hadronic** and **quark** models. Here we consider four EoSs of cold nucleonic matter (pure npeμ matter) and three EoSs of non-nucleonic matter models.

Asteroseismology of compact stars

In a manner analogous to terrestrial seismology, observational methods and techniques in asteroseismology are using the frequency of **seismic waves** rippling throughout stars with the aim of **probing** their **internal structure** and **thermodynamic properties**.

These oscillations occur when a star is perturbed away from its dynamical equilibrium and a **restoring force** tries to return it back to that equilibrium state. The frequencies appearing in the spectrum of NS oscillations, which sensitively rely on accurate stellar models, are **matched with observations**.

Radial oscillations in f-mode

Among the various types of oscillation modes, this paper focuses on the three lowest-frequency modes (**fundamental** and the **next two higher modes**) of radial oscillations where the **pressure** provides the **dominant restoring force** that produces oscillations.

In this mode the Lagrangian displacement of fluid elements in the star is **purely radial** and the spherical symmetry is preserved.

Obtaining the spectra

In his pioneering paper [17], Chandrasekhar introduced a **variational method** to impose a sufficient criterion for the dynamical stability of radial and non-radial stellar oscillations. The **identification of stable modes** has been in the focus of interest ever since and methods for obtaining their spectra.

The first extensive analysis of radial modes [2] for stellar models with various **zero-temperature EoSs** was computed for six EoSs.

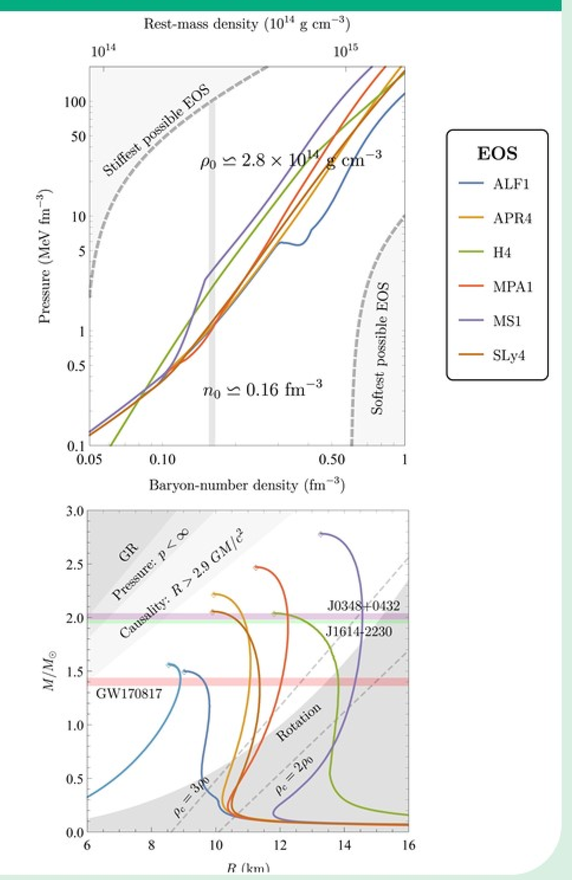
Mass–radius relations with tabulated EoS models

The **quark EoS** greatly influences the mass–radius relation of hybrid stars:

- Hybrid stars can have **larger masses** than both pure hadronic and quark stars.
- The underlying reason is that ρ, p, etc. and other functions contain sharp bends as the matter undergoes phase transitions for increasing pressure these **sharp phase transitions** from hadronic state into another (e.g. hyperons, pion or kaon condensates) change the state and the stiffness of matter quite abruptly, together with the adiabatic index Γ_1.
- The stiffening of EoS is crucial: the maximal mass increases as the EoS gets **stiffer at high densities** [3].

Several **astrophysical constraints** can be applied:

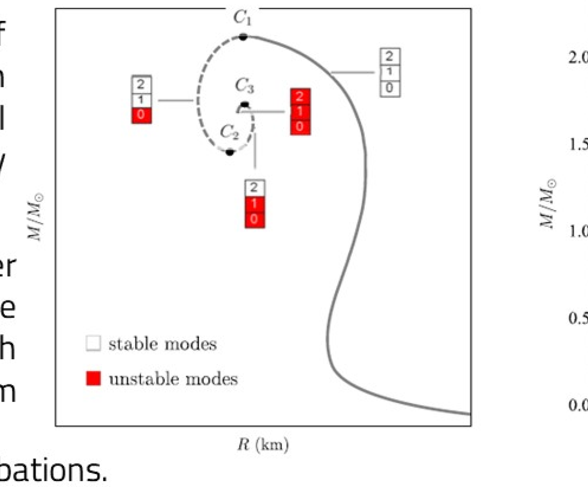
- Lower limit for maximum mass from PSR J0348+0432 [4]
- Upper mass constraint from GW170817 and gamma-ray observation GRB170817a [5]
- Lower and upper radius limits for mid-mass stars from GW170817 [6]

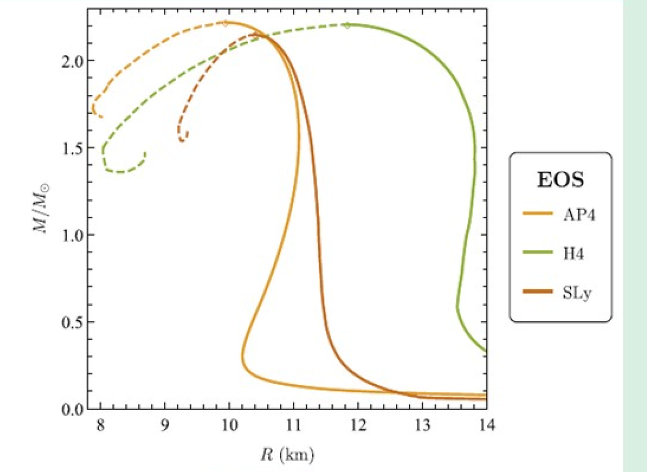


Dynamical stability criteria based on the fundamental mode

The **dynamical stability** of stellar configurations with respect to small radial perturbations is determined by the fundamental mode:

- If $\omega_0^2 > 0$, then all the higher eigenfrequencies of the spectrum are real, which indicates that the equilibrium model is dynamically stable with respect to small perturbations.
- If $\omega_0^2 < 0$, then ω_0 becomes purely imaginary, which indicates that the equilibrium stellar model comes to be dynamically unstable (at least in the f-mode).
- Finally, at $\omega_0 = 0$ the equilibrium stellar model is at neutral stability; the energy density is critical.





The dynamical stability is also inherently related to the mass–radius relations: $dM/d\epsilon_c > 0$ is necessary but not sufficient condition for dynamical stability.

At each critical point where $dR/d\epsilon_c < 0$ holds an even number *n* of radial oscillation mode changes its stability while for $dR/d\epsilon_c > 0$ an odd-numbered mode changes its stability.

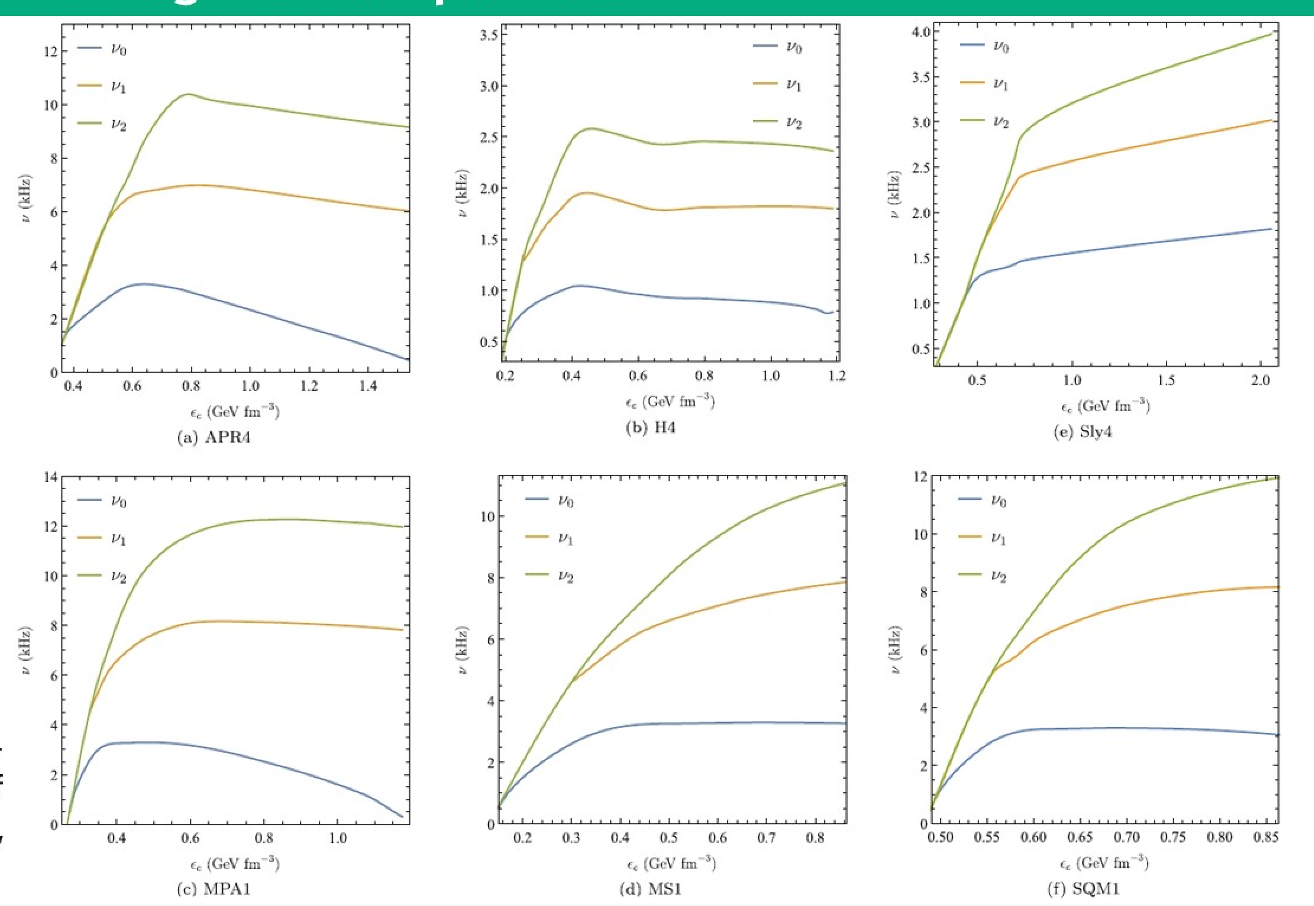
Linear pulsation equation and associated eigenvalue problem

The fundamental equation for radial pulsation together with its boundary conditions constitutes a **Sturm–Liouville eigenvalue problem** for a **discrete set of scalar-valued eigenfunctions** of radial displacement $\{X_0(r), X_1(r), \ldots, X_j(r), \ldots\}$ with their respective eigenvalues $\{\omega_0^2, \omega_1^2, \ldots, \omega_j^2, \ldots\}$ The smallest eigenvalue ω_0^2 is associated with the **fundamental-mode** frequency of radial oscillations which has no nodes between the center and the stellar surface, whereas the first **excited mode** ($j = 1$) has a node, the second one ($j = 2$) has two, and so forth. [7]

We convert the eigenvalue problem associated with radial pulsation into a system of algebraic equations by replacing the derivatives with **finite-difference approximations** in the form of a tridiagonal matrix that can be solved by **matrix algebra techniques**, ideally suited to modern numerical analysis.

❖ **Lowest-frequency eigenmodes (right panel):**

The frequencies of the fundamental mode (ν_0) and the first two lowest-frequency excited modes (ν_1 and ν_2) of radial oscillation as functions of central density (ϵ_c) for each EoS of nucleonic state (APR4, MPA1, MS1, SLy4) and hybrid nucleon–hyperon–quark state (H4, SQM1).



❖ **Two potential sources of numerical errors**

- **Round-off errors** occure due to inexactness in the floating-point representation of numbers and the arithmetic operations and they are determined by the machine precision (roughly 10^{-15}).
- **Local truncation errors** incur due to the inaccuracy in the approximation for derivatives of grid functions which can be controlled by varying the spacing of the radial grid on which equilibrium stellar models and the eigenfunctions of radial displacement are determined [8].

References

D. Barta Class. Quantum Grav. 38, 185002 (2021), arXiv:1908.02808

[1] S. Chandrasekhar, *Astrophys. J.* **140**, 417 (1964)
[2] E. N. Glass and L. Lindblom, *Astrophys. J. Suppl.* **53**, 93 (1983)
[3] F. Özel F and P. Freire, *Annu. Rev. Astron. Astrophys.* **54**, 401 (2016)
[4] J. Antoniadis, et al., *Science* **340**, 448 (2013)
[5] L. Rezzolla, E. R. Most, L. R. Weih, *Astrophys. J. Lett.* **852**, L25 (2018)
[6] E. Annala, et al., *Phys. Rev. Lett.* **120**, 172703 (2018)
[7] K. D. Kokkotas and J. Ruoff, *Astron. Astrophys.* **366**, 565 (2001)
[8] R. LeVeque: *Local Truncation Error* (2007)

Conclusion

The goal of our study was to compute and interpret the first three radial-mode frequencies as a function of central energy density for each realistic EoS.

❖ **Decaying frequency and dynamical stability**

The **decay** of the lowest-frequency eigenmodes appears to be a general feature, irrespective of the particular EoS. On each panel, the f-mode frequency drops toward zero as the particular stellar model **approaches its dynamical stability limit** which, indeed, is indicated by the presence of **an eigenmode with zero frequency**.

❖ **Higher-order excited radial modes**

The oscillation frequency of higher modes is always larger than that of a lower stable mode and for all modes it **appears to decrease** as the central energy density approches the **smallest possible value ϵ_{min}** of the particular stellar model. As reflected in the decreasing frequency, the **phase transition** causes a **drop of adiabatic index** with respect to stars with MS1 EoS, which **do not contain hyperons**.

❖ **Dynamical aspect of the EoS stiffness**

Stellar models of softer EoSs are generally associated with **more centrally condensed stars with larger average densities**. Having the highest pressure among the considered types of EoSs at a high-density region $\epsilon \lesssim 0.57\ \mathrm{GeV\ fm^{-3}}$, MS1 and H4 are the stiffest EoS models and produce NSs with comparatively low-frequency oscillations.

For more information on the topic, contact the authors: barta.daniel@wigner.hu & kacskovics.balazs@wigner.hu

Acknowledgements

D. B. and B. K. acknowledge financial support by the National Research, Development and Innovation Office (Hungarian abbreviation: NKFIH) fund of Hungary, financed under OTKA grant agreement No. K138277. D. B. also acknowledges partial support provided under the European COST Action CA16214 "PHAROS".

cost EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

PHAROS THE MULTI-MESSENGER PHYSICS AND ASTROPHYSICS OF NEUTRON STARS

NATIONAL RESEARCH, DEVELOPMENT AND INNOVATION OFFICE HUNGARY