Charge and heat transport coefficients of a weakly magnetized hot and dense QCD medium

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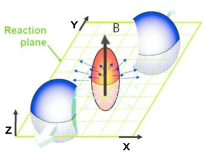
- **Introduction**
- Production of strong magnetic fields in noncentral heavy-ion collisions (10^{18} G to 10^{20} G) ($1 m_\pi^2 \sim 10^{18}$ G)
- **(i)** Strong magnetic field ($|q_f B| \gg T^2$)
  (ii) Weak magnetic field ($T^2 \gg |q_f B|$)
- For an electrically conducting medium, the lifetime of magnetic field gets extended.
- Transport coefficients are important to understand the response of the nonequilibrium system to external fields.



- Quasiparticle mass : $m_{fT}^2 = \frac{g^2 T^2}{6}\left(1 + \frac{\mu_f^2}{\pi^2 T^2}\right)$, where $g^2 = 4\pi\alpha_s$, $\alpha_s\left(\Lambda^2, eB\right) = \frac{\alpha_s\left(\Lambda^2\right)}{1 + b_1 \alpha_s\left(\Lambda^2\right) \ln\left(\frac{\Lambda^2}{\Lambda^2 + eB}\right)}$,
  with $\alpha_s\left(\Lambda^2\right) = \frac{1}{b_1 \ln\left(\Lambda^2/\Lambda_{\overline{\rm MS}}^2\right)}$, $b_1 = \frac{11 N_c - 2 N_f}{12\pi}$, $\Lambda_{\overline{\rm MS}} = 0.176$ GeV and $\Lambda = 2\pi\sqrt{T^2 + \mu_f^2/\pi^2}$.
- **Charge transport properties:**
- The spatial component of the induced current due to the action of an external electric field:
  $J^i = \sum_f g_f \int \frac{d^3\rm p}{(2\pi)^3 \omega_f} p^i [q\delta f_f(x,p) + \bar{q}\delta \bar{f}_f(x,p)]$,
  with $\delta f_f = f_f - f_f^0$, $\delta \bar{f}_f = \bar{f}_f - \bar{f}_f^0$.
- For a general configuration of electric and magnetic fields: $J^i = \sigma^{ij} E_j = \sigma_0 \delta^{ij} E_j + \sigma_1 \epsilon^{ijk} b_k E_j + \sigma_2 b^i b^j E_j$, with $\mathbf{b} = \frac{\mathbf{B}}{B}$.
- If electric and magnetic fields are perpendicular to each other: $J^i = \sigma^{ij} E_j = \left(\sigma_{\rm el}\delta^{ij} + \sigma_{\rm H}\epsilon^{ij}\right) E_j$.

- Relativistic Boltzmann transport (RBT) equation in the relaxation time approximation (RTA):

$$p^{\mu}\frac{\partial f_f(x,p)}{\partial x^{\mu}}+F^{\mu}\frac{\partial f_f(x,p)}{\partial p^{\mu}}=-\frac{p_{\nu}u^{\nu}}{\tau_f}\delta f_f(x,p)\,,\quad \tau_f=1/\left[5.1T\alpha_s^2\log(1/\alpha_s)(1+0.12(2N_f+1))\right],$$

$F^{\mu}=qF^{\mu\nu}p_{\nu}=(p^0\mathbf{v}\cdot\mathbf{F},p^0\mathbf{F})$, $F^{\mu\nu}$: electromagnetic field strength tensor, $\mathbf{F}=q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$.

- For an electric field along x-direction ($\mathbf{E}=E\hat{x}$) and a magnetic field along z-direction ($\mathbf{B}=B\hat{z}$), RBT equation can be expressed as

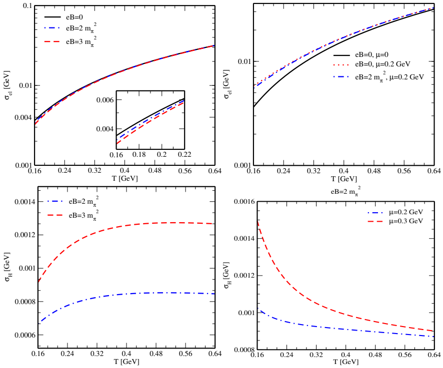
$$\tau_f qEv_x\frac{\partial f_f}{\partial p_0}+\tau_f qBv_y\frac{\partial f_f}{\partial p_x}-\tau_f qBv_x\frac{\partial f_f}{\partial p_y}=f_f^0-f_f-\tau_f qE\frac{\partial f_f^0}{\partial p_x}\ .$$

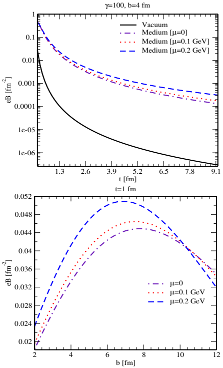
- Ansatz: $f_f=f_f^0-\tau_f q\mathbf{E}\cdot\frac{\partial f_f^0}{\partial\mathbf{p}}-\Gamma\cdot\frac{\partial f_f^0}{\partial\mathbf{p}}$.
- Quantities Γ_x and Γ_y: $\Gamma_x=\frac{qE\tau_f\left(1-\omega_c^2\tau_f^2\right)}{1+\omega_c^2\tau_f^2}$, $\Gamma_y=-\frac{2qE\omega_c\tau_f^2}{1+\omega_c^2\tau_f^2}$, where cyclotron frequency, $\omega_c=\frac{qB}{\omega_f}$.
- For quarks: $\delta f_f=2qEv_x\beta\left(\frac{\tau_f}{1+\omega_c^2\tau_f^2}\right)f_f^0\left(1-f_f^0\right)-2qEv_y\beta\left(\frac{\omega_c\tau_f^2}{1+\omega_c^2\tau_f^2}\right)f_f^0\left(1-f_f^0\right)$.
- For antiquarks: $\delta\bar{f}_f=2\bar{q}Ev_x\beta\left(\frac{\tau_{\bar{f}}}{1+\omega_c^2\tau_{\bar{f}}^2}\right)\bar{f}_f^0\left(1-\bar{f}_f^0\right)-2\bar{q}Ev_y\beta\left(\frac{\omega_c\tau_{\bar{f}}^2}{1+\omega_c^2\tau_{\bar{f}}^2}\right)\bar{f}_f^0\left(1-\bar{f}_f^0\right)$.
- Using the values of δf_f and $\delta\bar{f}_f$, and J^i expressions, the electrical conductivity and the Hall conductivity for a dense QCD medium in a weak magnetic field are obtained as

$$\sigma_{\rm el}=\frac{\beta}{3\pi^2}\sum_f g_f q_f^2\int d{\rm p}\,\frac{{\rm p}^4}{\omega_f^2}\left[\frac{\tau_f}{1+\omega_c^2\tau_f^2}f_f^0\left(1-f_f^0\right)+\frac{\tau_{\bar{f}}}{1+\omega_c^2\tau_{\bar{f}}^2}\bar{f}_f^0\left(1-\bar{f}_f^0\right)\right],$$

$$\sigma_{\rm H}=\frac{\beta}{3\pi^2}\sum_f g_f q_f^2\int d{\rm p}\,\frac{{\rm p}^4}{\omega_f^2}\left[\frac{\omega_c\tau_f^2}{1+\omega_c^2\tau_f^2}f_f^0\left(1-f_f^0\right)+\frac{\omega_c\tau_{\bar{f}}^2}{1+\omega_c^2\tau_{\bar{f}}^2}\bar{f}_f^0\left(1-\bar{f}_f^0\right)\right].$$

Lifetime of magnetic field





- Strong magnetic fields generated in heavy ion collisions decay instantly. However in the presence of electrical conductivity, their lifetimes may be elongated.

$$e\mathbf{B}_{\rm medium} = \frac{e^2 b \sigma_{\rm el}}{8\pi(t-x)^2} e^{-\frac{b^2\sigma_{\rm el}}{4(t-x)}} \hat{\mathbf{z}},$$

$$e\mathbf{B}_{\rm vacuum} = \frac{e^2 b\gamma}{4\pi\left\{b^2+\gamma^2(t-x)^2\right\}^{3/2}} \hat{\mathbf{z}}.$$

- As compared to the strong magnetic field, the weak magnetic field can stay longer.
- The presence of chemical potential in the medium helps in elongating the lifetime of magnetic field.
- For a larger chemical potential, the magnetic field attains its highest strength at a smaller impact parameter.

- **Heat transport properties:**
- The spatial component of heat flow due to the action of external disturbance:
  $Q^i = \sum_f g_f \int \frac{d^3\mathrm{p}}{(2\pi)^3} \frac{p^i}{\omega_f} \left[(\omega_f - h_f)\delta f_f(x,p) + (\omega_f - \bar{h}_f)\delta \bar{f}_f(x,p)\right].$
- The flow of heat in a medium: $Q^i = -\left(\kappa_0 \delta^{ij} + \kappa_1 \epsilon^{ijk} b_k + \kappa_2 b^i b^j\right)\left[\partial_j T - \frac{T}{\varepsilon + P}\partial_j P\right].$
- If gradients of temperature and pressure are perpendicular to magnetic field:
  $Q^i = -\left(\kappa_0 \delta^{ij} + \kappa_1 \epsilon^{ij}\right)\left[\partial_j T - \frac{T}{\varepsilon + P}\partial_j P\right].$
- Using ansatz, the RBT equation can be rewritten as
  $\frac{\tau_f}{p_0} p^\mu \frac{\partial f_f^0}{\partial x^\mu} - \beta f_f^0 \tau_f qEv_x + \beta f_f^0 \left(\Gamma_x v_x + \Gamma_y v_y\right) - \frac{qB\tau_f \beta f_f^0}{\omega_f}\left(v_x \Gamma_y - v_y \Gamma_x\right) + \frac{\tau_f^2 qBqEv_y \beta f_f^0}{\omega_f} = 0.$
- Quantities Γ_x and Γ_y:
  $\Gamma_x = \frac{qE\tau_f\left(1-\omega_c^2\tau_f^2\right)}{1+\omega_c^2\tau_f^2} - \frac{\tau_f(\omega_f - h_f)}{T\left(1+\omega_c^2\tau_f^2\right)}\left(\partial^x T - \frac{T}{nh_f}\partial^x P\right) - \frac{\omega_c\tau_f^2(\omega_f - h_f)}{T\left(1+\omega_c^2\tau_f^2\right)}\left(\partial^y T - \frac{T}{nh_f}\partial^y P\right),$
  $\Gamma_y = -\frac{2\omega_c\tau_f^2 qE}{1+\omega_c^2\tau_f^2} - \frac{\tau_f(\omega_f - h_f)}{T\left(1+\omega_c^2\tau_f^2\right)}\left(\partial^y T - \frac{T}{nh_f}\partial^y P\right) + \frac{\omega_c\tau_f^2(\omega_f - h_f)}{T\left(1+\omega_c^2\tau_f^2\right)}\left(\partial^x T - \frac{T}{nh_f}\partial^x P\right).$
- For quarks:

$$\begin{aligned}
\delta f_f \quad = \quad & \frac{2qE\tau_f v_x \beta f_f^0\left(1 - f_f^0\right)}{1 + \omega_c^2\tau_f^2} - \frac{2qE\omega_c\tau_f^2 v_y \beta f_f^0\left(1 - f_f^0\right)}{1 + \omega_c^2\tau_f^2} - \beta^2 f_f^0\left(1 - f_f^0\right)\frac{\tau_f(\omega_f - h_f)}{\left(1 + \omega_c^2\tau_f^2\right)} \\
& \times \left[v_x\left(\partial^x T - \frac{T}{nh_f}\partial^x P\right) + v_y\left(\partial^y T - \frac{T}{nh_f}\partial^y P\right)\right] - \beta^2 f_f^0\left(1 - f_f^0\right) \\
& \times \frac{\omega_c\tau_f^2(\omega_f - h_f)}{\left(1 + \omega_c^2\tau_f^2\right)}\left[v_x\left(\partial^y T - \frac{T}{nh_f}\partial^y P\right) - v_y\left(\partial^x T - \frac{T}{nh_f}\partial^x P\right)\right].
\end{aligned}$$

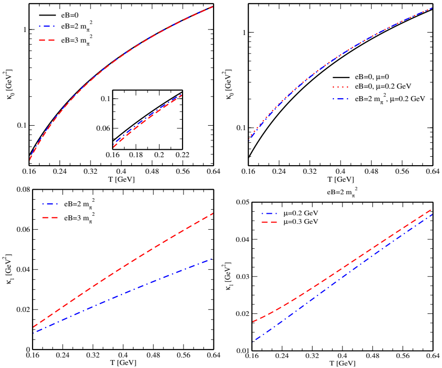
- For antiquarks:

$$\delta\bar{f}_f = \frac{2\bar{q}E\tau_{\bar{f}}v_x\beta\bar{f}_f^0\left(1-\bar{f}_f^0\right)}{1+\omega_c^2\tau_{\bar{f}}^2} - \frac{2\bar{q}E\omega_c\tau_{\bar{f}}^2v_y\beta\bar{f}_f^0\left(1-\bar{f}_f^0\right)}{1+\omega_c^2\tau_{\bar{f}}^2} - \beta^2\bar{f}_f^0\left(1-\bar{f}_f^0\right)\frac{\tau_{\bar{f}}(\omega_f-\bar{h}_f)}{\left(1+\omega_c^2\tau_{\bar{f}}^2\right)}$$
$$\times\left[v_x\left(\partial^x T-\frac{T}{n\bar{h}_f}\partial^x P\right)+v_y\left(\partial^y T-\frac{T}{n\bar{h}_f}\partial^y P\right)\right]-\beta^2\bar{f}_f^0\left(1-\bar{f}_f^0\right)$$
$$\times\frac{\omega_c\tau_{\bar{f}}^2(\omega_f-\bar{h}_f)}{\left(1+\omega_c^2\tau_{\bar{f}}^2\right)}\left[v_x\left(\partial^y T-\frac{T}{n\bar{h}_f}\partial^y P\right)-v_y\left(\partial^x T-\frac{T}{n\bar{h}_f}\partial^x P\right)\right].$$

- Using the values of δf_f and $\delta\bar{f}_f$, and Q^i expressions, the thermal conductivity and the Hall-type thermal conductivity for a dense QCD medium in a weak magnetic field are obtained as

$$\kappa_0 = \frac{\beta^2}{6\pi^2}\sum_f g_f\int d\mathrm{p}\,\frac{\mathrm{p}^4}{\omega_f^2}\left[\frac{\tau_f}{1+\omega_c^2\tau_f^2}(\omega_f-h_f)^2\right.$$
$$\times f_f^0\left(1-f_f^0\right)+\frac{\tau_{\bar{f}}}{1+\omega_c^2\tau_{\bar{f}}^2}(\omega_f-\bar{h}_f)^2$$
$$\left.\times\bar{f}_f^0\left(1-\bar{f}_f^0\right)\right],$$

$$\kappa_1 = \frac{\beta^2}{6\pi^2}\sum_f g_f\int d\mathrm{p}\,\frac{\mathrm{p}^4}{\omega_f^2}\left[\frac{\omega_c\tau_f^2}{1+\omega_c^2\tau_f^2}(\omega_f-h_f)^2\right.$$
$$\times f_f^0\left(1-f_f^0\right)+\frac{\omega_c\tau_{\bar{f}}^2}{1+\omega_c^2\tau_{\bar{f}}^2}(\omega_f-\bar{h}_f)^2$$
$$\left.\times\bar{f}_f^0\left(1-\bar{f}_f^0\right)\right].$$

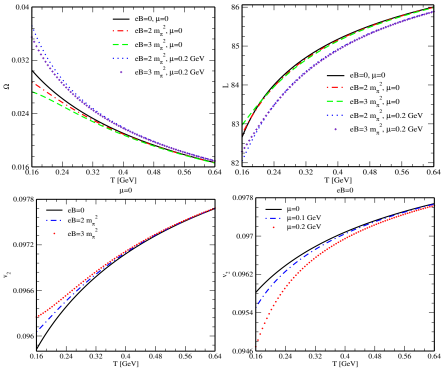


Applications:

Knudsen number, $\Omega = \frac{\lambda}{l} = \frac{3\kappa_0}{lvC_V}$,

elliptic flow, $v_2 = \frac{v_2^h}{1+\frac{\Omega}{\Omega_h}}$,

Wiedemann-Franz law, $\frac{\kappa_0}{\sigma_{\rm el}} = LT$.



$l = 4$ fm, $v = 1$, $v_2^h \approx 0.1$, $\Omega_h \approx 0.7$.

Conclusions

- Electrical conductivity and thermal conductivity decrease and Hall conductivity and Hall-type thermal conductivity increase with the magnetic field in the weak magnetic field regime.
- The emergence of finite chemical potential tends to increase the magnitudes of these charge and heat transport coefficients.
- Charge and heat transport coefficients are further used to study the Knudsen number, the elliptic flow and the Wiedemann-Franz law.
- The Knudsen number in the weakly magnetized hot and dense QCD matter retains its value much below unity. Thus, there is sufficient separation between the mean free path and the characteristic length scale for the medium to remain in the local equilibrium state.
- The elliptic flow gets increased in the presence of weak magnetic field, whereas the presence of finite chemical potential decreases it as compared to that in the absence of both magnetic field and chemical potential.
- The Lorenz number in the Wiedemann-Franz law is found to be strongly affected by the chemical potential than by the weak magnetic field. Further, with the increase of temperature, the Lorenz number is observed to increase, confirming the violation of the Wiedemann-Franz law for hot and dense QCD matter in the presence of a weak magnetic field.

Reference

- S. Rath and S. Dash, arXiv:2112.11802 [hep-ph].