Heavy quarkonium dynamics at NLO in the binding energy over temperature

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Speaker and Conference Info

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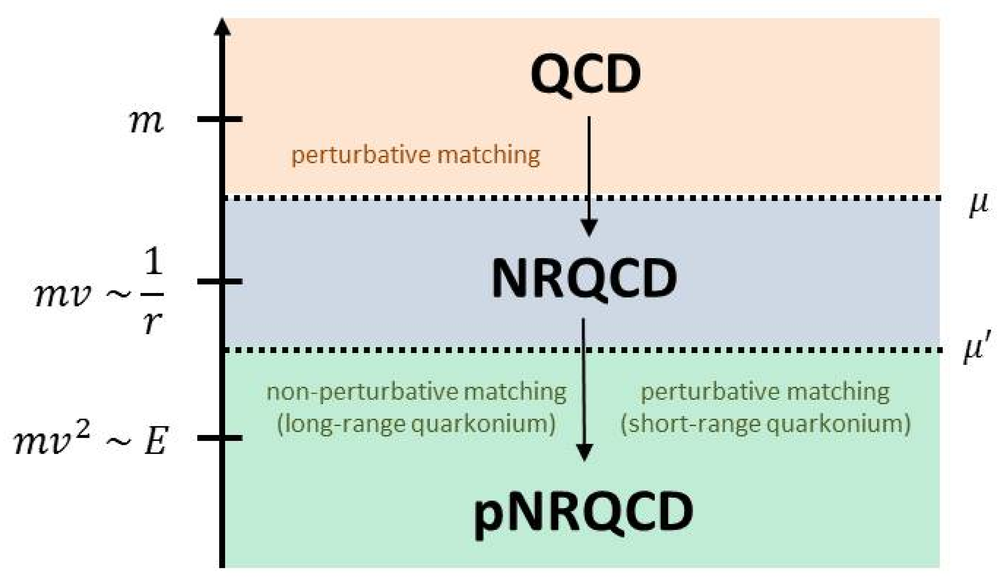
Abstract

In Ref.[1], using the potential non-relativistic quantum chromodynamics (pNRQCD) effective field theory, we derive a Lindblad equation for the evolution of the heavy-quarkonium reduced density matrix that is accurate to next-to-leading order (NLO) in the ratio of the binding energy of the state to the temperature of the medium. Finally, we make comparisons with our prior leading-order pNRQCD results and experimental data available from the ATLAS, ALICE, and CMS collaborations.

1. Heavy quarkonium as an Open Quantum System

In Open Quantum System (OQS) approach, we consider heavy quarkonium as an open quantum system/probe and light quarks and gluons as the medium.



Here, we consider the following relevant scales: temperature T, bound state mass $m \gg T$, bound state size $r \sim 1/mv \sim a_0$ (Bohr radius), Debye mass m_D, binding energy $E \sim mv^2$, medium relaxation time scale $\langle \hat{O}_M(t)\hat{O}_M(0)\rangle \sim e^{-t/t_M}$, intrinsic probe time scale $t_P \sim \frac{1}{\omega_i-\omega_j}$, probe relaxation time scale $\langle p(t)\rangle \sim e^{-t/t_{\rm rel}}$.

OQS + pNRQCD treatment gives us the following Lindblad equation

$$\xrightarrow[1/r\gg T\sim m_D\gg E]{t_{\rm rel},\, t_P\gg t_M} \frac{d\rho_{\rm probe}}{dt} = -i\left[H_{\rm probe},\rho_{\rm probe}\right] + \sum_n \left(C_n\rho_{\rm probe}C_n^\dagger - \frac{1}{2}\left\{C_n^\dagger C_n, \rho_{\rm probe}\right\}\right) \tag{1}$$

where, $H_{\rm probe}$ is Hermitian and C_n are the collapse/jump operators.
If we define a non-Hermitian effective Hamiltonian $H_{\rm eff} = H_{\rm probe} - \frac{i}{2}\Gamma$ with $\Gamma_n = C^\dagger C_n$ and $\Gamma = \sum_n \Gamma_n$, then Eq. (1) becomes

$$\frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^\dagger + \sum_n C_n\rho_{\rm probe}C_n^\dagger \tag{2}$$

2. Lindblad equation to NLO in E/T

$$\frac{d\rho(t)}{dt} = -i\left[H,\rho(t)\right] + \sum_{n=0}^{1}\left(C_i^n\rho(t)C_i^{n\dagger} - \frac{1}{2}\left\{C_i^{n\dagger}C_i^n, \rho(t)\right\}\right), \quad H = \begin{pmatrix} h_s + {\rm Im}(\Sigma_s) & 0 \\ 0 & h_o + {\rm Im}(\Sigma_o)\end{pmatrix} \tag{3}$$

where,

$${\rm Im}\left(\Sigma_s\right) = \frac{r^2}{2}\gamma + \frac{\kappa}{4MT}\{r_i,p_i\}, \quad {\rm Im}\left(\Sigma_o\right) = \frac{N_c^2-2}{2(N_c^2-1)}\left(\frac{r^2}{2}\gamma + \frac{\kappa}{4MT}\{r_i,p_i\}\right) \tag{4}$$

$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}}\begin{pmatrix}0&1\\0&0\end{pmatrix}\left(r_i + \frac{ip_i}{2MT} + \frac{\Delta V_{os}}{4T}r_i\right) + \sqrt{\kappa}\begin{pmatrix}0&0\\1&0\end{pmatrix}\left(r_i + \frac{ip_i}{2MT} + \frac{\Delta V_{so}}{4T}r_i\right) \tag{5}$$

$$C_i^1 = \sqrt{\frac{\kappa(N_c^2-4)}{2(N_c^2-1)}}\begin{pmatrix}0&0\\0&1\end{pmatrix}\left(r_i + \frac{ip_i}{2MT}\right), \quad \rho = \begin{pmatrix}\rho_s & 0\\ 0 & \rho_0\end{pmatrix} \tag{6}$$

$$\kappa = \frac{g^2}{6N_c}\int_0^\infty {\rm d}s\,\left\langle\left\{\tilde{E}_i^a(s,\vec{0}),\tilde{E}_i^a(0,\vec{0})\right\}\right\rangle, \quad \gamma = -i\frac{g^2}{6N_c}\int_0^\infty {\rm d}s\,\left\langle\left[\tilde{E}_i^a(s,\vec{0}),\tilde{E}_i^a(0,\vec{0})\right]\right\rangle \tag{7}$$

Here, six collapse operators cover: singlet $\rightleftharpoons$ octet, octet $\rightleftharpoons$ singlet, octet $\rightleftharpoons$ octet.

The effective Hamiltonian for singlet and octet evolution is defined by $H_{s,o}^{\rm eff} = h_{s,o} + {\rm Im}(\Sigma_{s,o}) - i\Gamma_{s,o}/2$ with $\Gamma_s = \sum_{i\in\{\uparrow,\downarrow\}}\Gamma^i_{s\to o}$ and $\Gamma_o = \sum_{i\in\{\uparrow,\downarrow\}}(\Gamma^i_{o\to s} + \Gamma^i_{o\to o})$.

Acknowledgements and references

[1] N. Brambilla, M. A. Escobedo, A. Islam, M. Strickland, A. Tiwari, A. Vairo, and P. Vander Griend, *"Heavy quarkonium dynamics at next-to-leading order in the binding energy over temperature"*, (2022), arXiv:2205.10289 [hep-ph].

[2] N. Brambilla, M. Escobedo, M. Strickland, A. Vairo, and P. Vander Griend, JHEP 05(2021)136 [2012.01240].

[3] N. Brambilla, M. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J.H. Weber, PRD 104(2021) 094049 [2107.06222].

3. Numerical Solution

We solve the form of the Lindblad equation given in Eq. (2) using Quantum Trajectories Method. For details see Ref.[2,3]

4. NLO Results

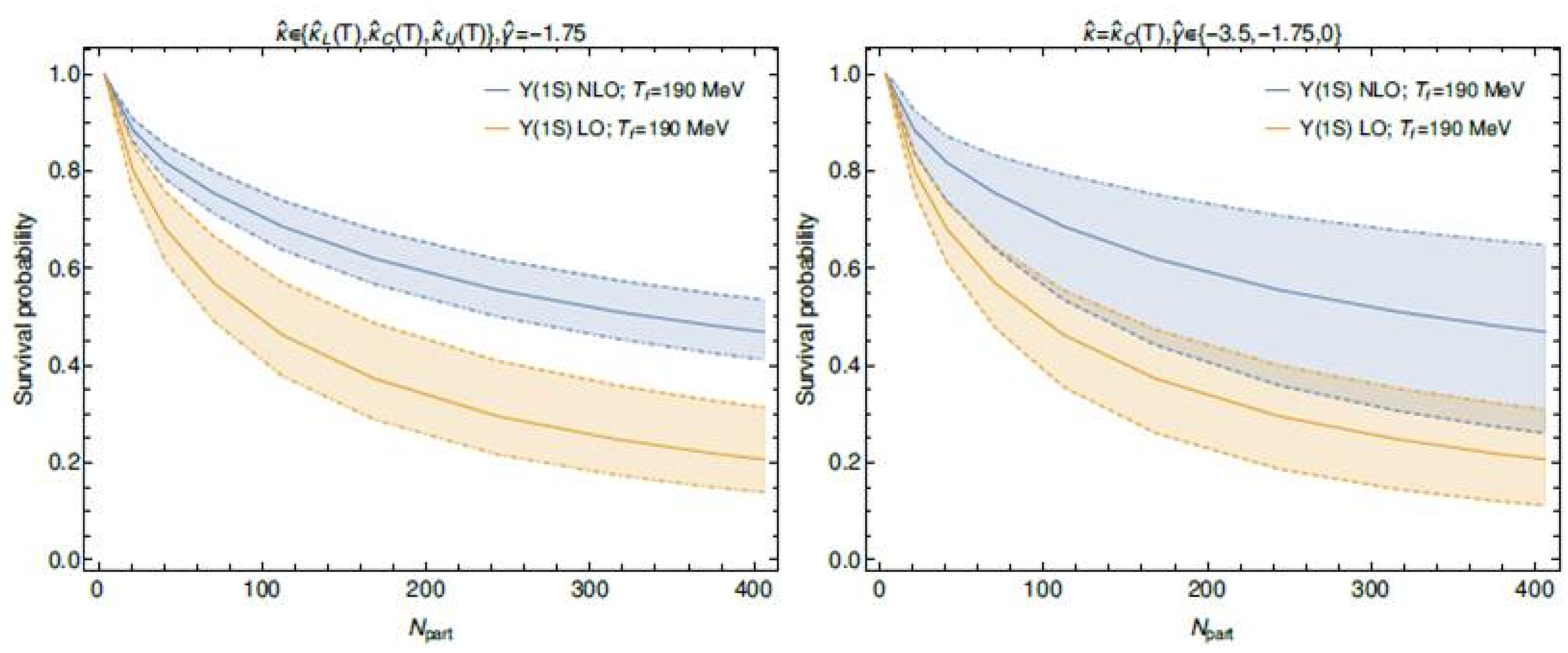


Fig. 1: The survival probability as a function of $N_{\rm part}$ of the $1S$ state calculated using $H_{\rm eff}$ evolution without jumps.

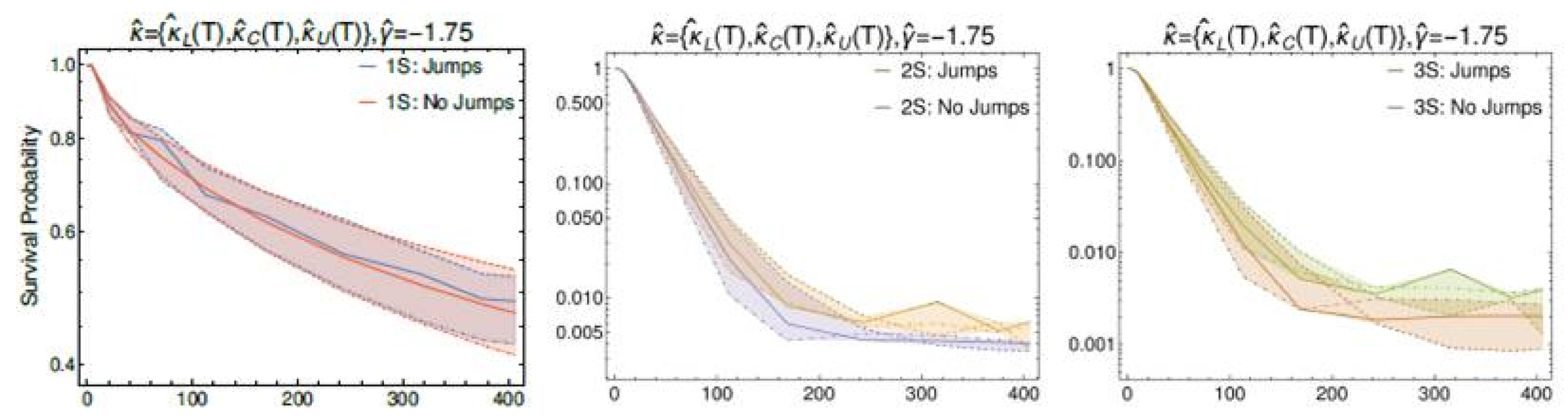


Fig. 2: The effects of quantum jumps and no jumps on NLO results.

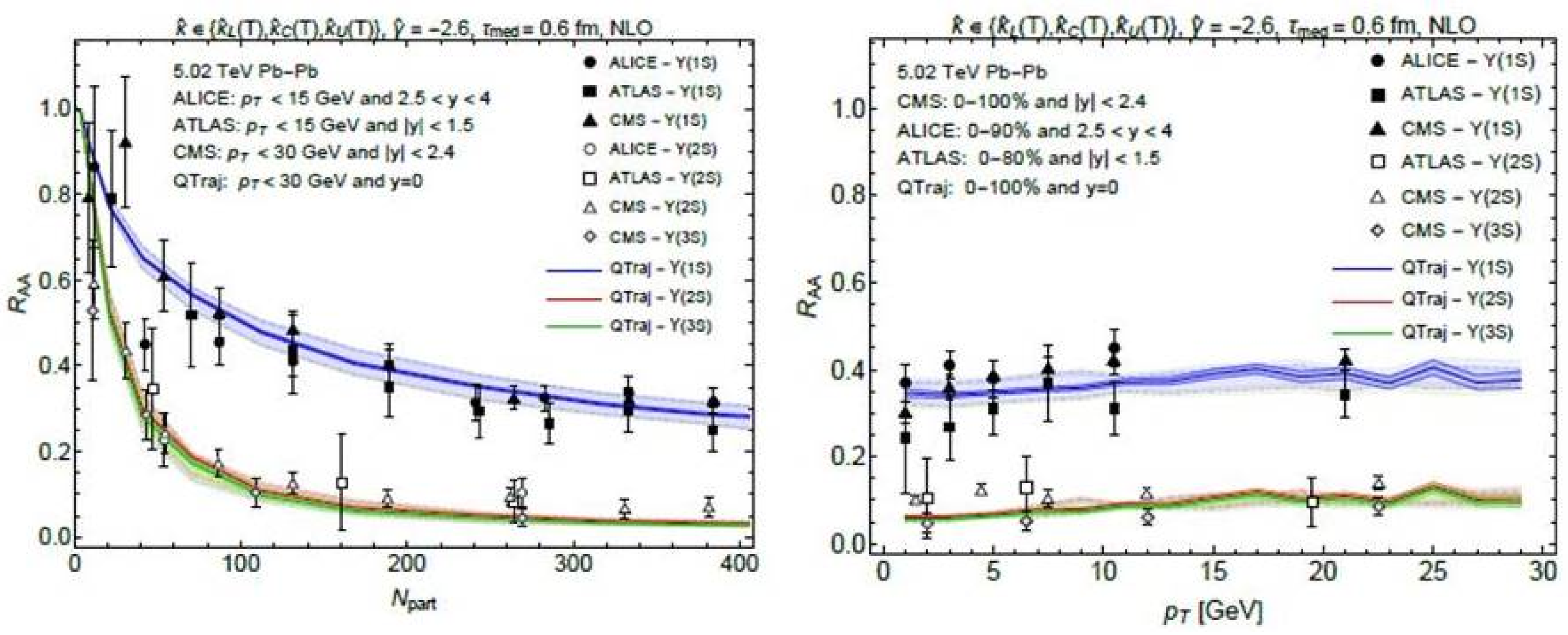


Fig. 3: R_{AA} for the $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ as a function of $N_{\rm part}$ and p_T.

5. Conclusions

- We have gone beyond prior studies [2,3] by deriving and numerically solving a Lindblad-type evolution equation that is accurate to NLO in E/T.
- We used quantum trajectories method successfully to solve the NLO Lindblad equation.
- We have shown that when going from LO to NLO there is a sizable correction to the singlet decay width for $\Upsilon(1S)$ and smaller corrections for the $\Upsilon(2S)$ and $\Upsilon(3S)$ states.
- The inclusion of NLO corrections has allowed us to extend the pNRQCD+OQS treatment down to temperatures in the vicinity of the QCD phase transition; in practice, we have lowered the decoupling temperature from $T_f = 250$ MeV at LO to $T_f = 190$ MeV at NLO.

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